② $f(x,y) = \begin{cases} e^{-(2x+y/2)}, & x \geq 0,\ y \geq 0 \\ 0 & c.c \end{cases}$

②

a)

$$\int_0^\infty \int_0^\infty e^{-(2x+y/2)}\,dx\,dy = \int_0^\infty e^{-2x}\,dx \int_0^\infty e^{-y/2}\,dy$$

$$= \frac{1}{2}\int_0^\infty 2e^{-2x}\,dx \cdot 2\int_0^\infty \frac{1}{2}e^{-y/2}\,dy = 1$$, sim se opm

$e^{(-2x+y/2)} \geq 0 \ \forall (x,y) \in \mathbb{R}^2$ e $\int_0^\infty \int_0^\infty e^{-(2x+y/2)}\,dx\,dy = 1$

c) $f_x(x) = \int_0^\infty e^{-(+2x+y/2)}\,dy = e^{-2x} \cdot \int_0^\infty e^{-y/2}\,dy = 2e^{-2x}$

$f_y(y) = \int_0^\infty e^{-(2x+y/2)}\,dx = \frac{1}{2}e^{-y/2}$

$X \sim exp(2)$, $Y \sim exp(1/2)$

b) Sim, visto que $f_x(x) \cdot f_y(y) = f_{xy}(x,y)$

d) $P(X \leq 1, Y \leq 2) = \int_0^1 \int_0^2 e^{-(2x+y/2)}\,dy\,dx = \int_0^1 -2e^{-(2x+y/2)}\Big|_0^2\,dx$

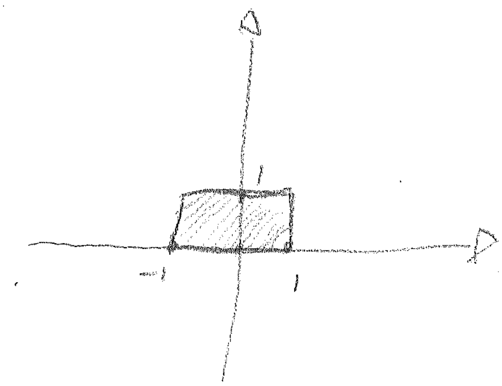
$$= \int_0^1 -2e^{-(2x+1)} + 2e^{-2x}\,dx = e^{-(2x+1)}\Big|_0^1 - e^{-(2x)}\Big|_0^1$$

$$= e^{-3} - e^{-1} - e^{-2} + 1$$ //

(4) $f(x,y) = 1/2 \quad , -1 \leq x \leq 1 \ , 0 \leq y \leq 1$

(3)

a) Obter as densidades marginais



$f_x(x) = \int_0^1 \frac{1}{2} \, dy = \frac{1}{2} \ , \ -1 \leq x \leq 1$

$f_y(y) = \int_{-1}^1 \frac{1}{2} \, dy = 1 \ , \ 0 \leq y \leq 1$

b) Sim, visto que $f_x(x) \cdot f_y(y) = f_{xy}(x,y)$

c) $F_{XY}(x,y) =: \int_{-\infty}^{x} \int_{-\infty}^{y} f_{xy}(t_1, t_2) \, dt_2 \, dt_1 = \int_0^y \int_{-1}^x \frac{1}{2} \, dt_1 \, dt_2$

$= \int_0^y \frac{1}{2} t_1 \Big|_{-1}^{x} dt_2 = \int_0^y \frac{x}{2} + \frac{1}{2} \, dt_2 = \left(\frac{x}{2} + \frac{1}{2}\right) y = \frac{xy}{2} + \frac{y}{2}$

$$F_{XY}(x,y) = \begin{cases} \frac{xy}{2} + \frac{y}{2} & \text{se } -1 \leq x \leq 1, \ 0 \leq y \leq 1 \\ 1 & \text{se } x > 1, \ y > 1 \\ 0 & \text{c.c} \end{cases}$$

(5) $f(x,y) = K \quad , A = \{(x,y) \in \mathbb{R}^2 / x^2 + y^2 \leq 1$

a) Visto que $\iint_A dR = \pi$, então $\boxed{K = \frac{1}{\pi}}$

b) Distribuição marginal

$f_x(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi} \, dy = \frac{1}{\pi} \cdot 2\sqrt{1-x^2} = \begin{cases} \frac{2}{\pi}\sqrt{1-x^2} & \text{se } -1 \leq x \leq 1 \\ 0 & \text{c.c} \end{cases}$

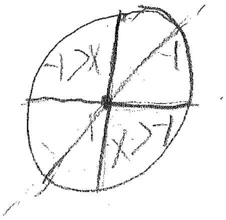
$$f_y(y) = \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{1}{\pi}\, dx = \begin{cases} \frac{2}{\pi}\sqrt{1-y^2} & \text{se } -1<y<1 \\ 0 & \text{c.c} \end{cases}$$ (4)

c) não, visto que $f_y(y)\, f_x(x) \neq f_{xy}(x,y)$

(6) ¯_(ツ)_/¯

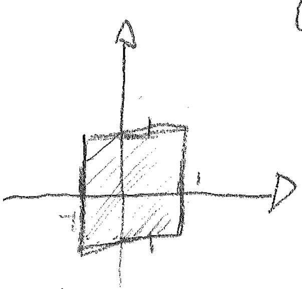
(7) A mesma que o item (5). $f_{xy}(x,y) = \begin{cases} \frac{1}{\pi} & \text{se } (x,y) \in A \\ 0 & \text{c.c} \end{cases}$

b) $P(X<Y)$, $P(X>Y)$ e $P(X=Y)$



$P(X<Y) = P(X>Y) = \frac{1}{2}$. $P(\underbrace{X=Y}_{\text{origem}}) = 0$

(8)



a) $f_{xy}(x,y) = \frac{1}{\text{Área}} = \frac{1}{4}$

b) $f_x(x) = \int_{-1}^{1} \frac{1}{4}\, dy = \frac{1}{4} y \Big|_{-1}^{1} = \begin{cases} \frac{1}{2} & \text{se } -1<x<1 \\ 0 & \text{c.c} \end{cases}$

$f_y(y) = \int_{-1}^{1} \frac{1}{4}\, dx = \begin{cases} \frac{1}{2} & \text{se } -1<y<1 \\ 0 & \text{c.c} \end{cases}$

c) Sim, visto que $f_x(x) \cdot f_y(y) = f_{xy}(x,y)$

5

(9) Seja $R: \{(x,y,z) \in \mathbb{R}^3 / x^2+y^2+z^2 \leq 1\}$

a) Vejamos $\iiint_R = \frac{4}{3}\pi$. Então, $f_{xyz}(x,y,z) = \frac{3}{4\pi}$

b) $f_{xy}(x,y) = \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} \frac{3}{4\pi} dz = \frac{3}{4\pi} \cdot 2\sqrt{1-x^2-y^2} = \begin{cases} \frac{3}{2\pi}\sqrt{1-x^2-y^2} \\ 0 \quad c.c \end{cases}$

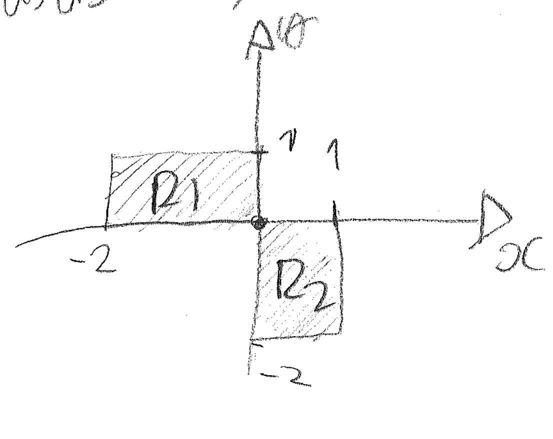
de fato: $\int_{-1}^{1}\int_{-(1-x^2)^{1/2}}^{(1-x^2)^{1/2}} \frac{3}{2\pi}\sqrt{1-\underbrace{x^2-y^2}_{r^2}} = \int_0^{2\pi}\int_0^1 \frac{3}{2\pi}(1-r^2)^{1/2} r \, dr \, d\theta$

$= \int_0^{2\pi} \frac{3}{2\pi}\int_0^1 r(1-r^2)^{1/2} dr\,d\theta = \int_0^{2\pi} \frac{1}{2\pi} d\theta = \frac{1}{2\pi}\Big|_0^{2\pi} = 1$

c) $f_x(x) = \int_{-(1-x^2)^{1/2}}^{(1-x^2)^{1/2}} \frac{3}{2\pi}\sqrt{1-x^2-y^2}\, dy$

(10) $f(x,y) = \begin{cases} 1/4, & -2 \leq x \leq 0,\ 0 \leq y \leq 1 \\ 1/4, & 0 \leq x \leq 1,\ -2 \leq 0 \\ 0 & c.c \end{cases}$

a) distribuições marginais



se R1:
$f_x(x) = \int_0^1 \frac{1}{4} dy = \frac{1}{4}$

se R2:
$f_x(x) = \int_{-2}^{0} \frac{1}{4} dy = \frac{1}{2}$

0 c.c

SE R1 SE R2

$f_y(y) = \int_{-2}^{0} \frac{1}{4} dx = \frac{1}{2}$ $f_y(y) = \int_{0}^{1} \frac{1}{4} dx = \frac{1}{4}$ ⑥

então

$$f_x(x) = \begin{cases} \frac{1}{4} & \text{se } -2 \leq x \leq 0 \\ \frac{1}{2} & \text{se } 0 \leq x \leq 1 \\ 0 & \text{c.c} \end{cases}$$

$$f_y(y) = \begin{cases} \frac{1}{2} & \text{se } 0 \leq y \leq 1 \\ \frac{1}{4} & \text{se } -2 \leq y \leq 0 \\ 0 & \text{c.c} \end{cases}$$

b) não, visto que $f_x(x) \cdot f_y(y) \neq f_{xy}(x,y)$

c) sim

d) $F_{xy}(x,y) = \int\int_R f_{xy}(x,y)\, dA$

Se R_1:

$$\int_{-2}^{x}\int_{0}^{y} \frac{1}{4}\, dt_1 dt_2 = \int_{-2}^{x} \frac{y}{4} dt_2 = \frac{y}{4}(x+2) = \frac{yx}{4} + \frac{y}{2}$$

Se R_2:

$$\int_{0}^{x}\int_{-2}^{y} \frac{1}{4}\, dt_2 dt_1 = \int_{0}^{x} \frac{1}{4}(y+2)\, dt_2 = \frac{x}{4}(y+2) = \frac{xy}{4} + \frac{x}{2}$$

$$F_x(x,y) = \begin{cases} \frac{yx}{4} + \frac{y}{2} & \text{se } -2 \leq x \leq 0 \text{ e } 0 \leq y \leq 1 \\ \frac{xy}{4} + \frac{x}{2} & \text{se } 0 \leq x \leq 1 \text{ e } -2 \leq y \leq 0 \\ 1 & \text{se } x > 1 \text{ e } y > 1,\ 0 \text{ c.c} \end{cases}$$

(11) $f_{XY}(x,y) = \begin{cases} \lambda^2 e^{-\lambda y}, & 0 < x < y \\ 0 & c.c \end{cases}$, calcular as densidades marginais

(7)

$$f_X(x) = \int_x^{\infty} \lambda^2 e^{-\lambda y}\, dy = \lambda^2 \int_x^{\infty} e^{-\lambda y}\, dy = \frac{\lambda^2}{\lambda}(-e^{-\lambda y})\Big|_x^{\infty}$$

$$= \lambda e^{-\lambda x} \text{ se } x \geq 0$$

$$f_Y(y) = \int_0^{y} \lambda^2 e^{-\lambda y}\, dx = y\lambda^2 e^{-\lambda y} \quad y \geq 0$$

(12) $f_{XY}(x,y) = \begin{cases} 4xy, & 0 < x < 1,\ 0 < y < 1 \\ 0 & c.c \end{cases}$

a) $\int_0^1 \int_0^1 4xy\, dx\, dy = \int_0^1 2y\, dy = 2\frac{y^2}{2}\Big|_0^1 = 1$

b) $f_X(x) = \int_0^1 4xy\, dy = 4x\frac{y^2}{2}\Big|_0^1 = 2x$ se $0 < x < 1$

$f_Y(y) = \int_0^1 4xy\, dx = 4y\frac{x^2}{2}\Big|_0^1 = 2y$ se $0 < y < 1$

c) Sim, visto que $f_X(x) \cdot f_Y(y) = f_{XY}(x,y)$

d) $P(X < \frac{1}{2}) = \int_0^{1/2} f_X(x)\, dx = \int_0^{1/2} 2x\, dx = 2\frac{x^2}{2}\Big|_0^{1/2} = 1/4$

e) $P(X < 1/2, Y < \frac{1}{2}) = \int_0^{1/2}\int_0^{1/2} 4xy\, dx\, dy = \int_0^{1/2} \cancel{4}^{2}\, y \frac{x^2}{\cancel{2}}\Big|_0^{1/2} dy$

$= \int_0^{1/2} \frac{y}{2}\, dy = \frac{1}{2}\frac{y^2}{2}\Big|_0^{1/2} = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$

(13) $f(x_1, x_2) = \frac{1}{3}(3 - x_1 - x_2)$, $0 \le x_1 \le 1$, $0 \le x_2 \le 2$ (8)

$f_{x_1}(x_1) = \int_0^2 \frac{1}{3}(3 - x_1 - x_2)\,dx_2 = \frac{1}{3}\int_0^2 3 - x_1 - x_2\,dx_2$

$= \frac{1}{3}\left(3x_2\Big|_0^2 - x_1 x_2\Big|_0^2 - \frac{x_2^2}{2}\Big|_0^2 \right) = \frac{1}{3}(6 - 2x_1 - 2)$

$\boxed{\frac{4 - 2x_1}{3} \text{ se } 0 < x_1 < 1}$

$\frac{1}{3}\int_0^1 4 - 2x_1\,dx = \frac{1}{3}(4x_1 - x_1^2)\Big|_0^1 = \frac{1}{3}(4 - 1) = 1$

$f_{x_2}(x) = \frac{1}{3}\int_0^1 3 - x_1 - x_2\,dx_1 = \frac{1}{3}\left(3x_1\Big|_0^1 - \frac{x_1^2}{2}\Big|_0^1 - x_2 x_1\Big|_0^1 \right)$

$\frac{1}{3}\left(3 - \frac{1}{2} - x_2\right) = \boxed{1 - \frac{1}{6} - \frac{x_2}{3} \text{ c.c.}}$ $\int_0^2 1 - \frac{1}{6} - \frac{x_2}{3}\,dx_2$

$x_2\Big|_0^2 - \frac{x_2}{6}\Big|_0^2 - \frac{x_2^2}{6}\Big|_0^2$

$\left(\frac{4}{3} - \frac{2x_1}{3}\right)\left(1 - \frac{1}{6} - \frac{x_2}{3}\right)$

$2 - \frac{2}{6} - \frac{4}{6} = 1$

$\frac{4}{3} - \frac{4}{18} - \frac{4x_2}{9} - \frac{2x_1}{3} + \frac{2x_1}{18} + \frac{2x_1 x_2}{9}$ mas

Vemos são dependentes visto que $f_{x_1}(x_1) f_{x_2}(x_2) \neq f_{x_1, x_2}(x_1, x_2)$