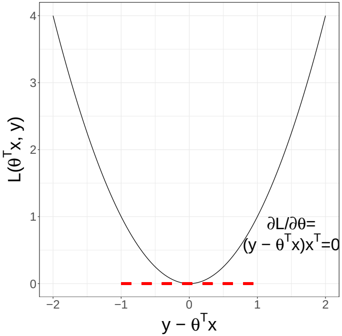
Introduction to Machine Learning

Supervised Regression
In a Nutshell



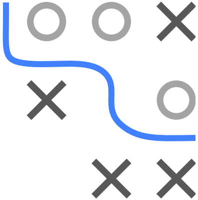
Learning goals

- Understand basic concept of regressors
- Understand difference between L1 and L2 Loss
- Know basic idea of OLS estimator

LINEAR REGRESSION TASKS

- Learn linear combination of features for predicting the target variable
- Find best parameters of the model by training w.r.t. a loss function

  $CreditBalance = \theta_0 + \theta_1 Rating + \theta_2 Income + \theta_3 CreditLimit$



Training

Input: Labeled data

Rating	Income	Credit Limit	Credit Card Balance
283	14.891	3606	333
483	106.025	6645	903
514	104.593	7075	580

Learning

Output

Regressor

Prediction

Input: Unlabeled data

Rating	Income	Credit Limit	Credit Card Balance
107	32.318	4351	?
471	88.180	5042	?
512	121.218	8101	?

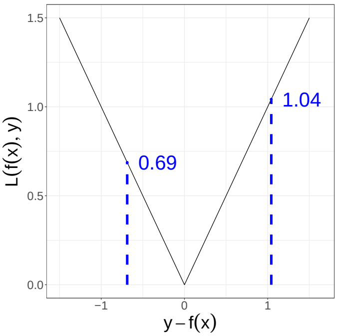
Regressor

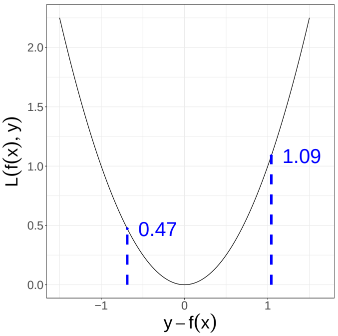
Prediction

Credit Card Balance
482
720
987

LINEAR MODELS: L1 VS L2 LOSS

Loss can be characterized as a function of residuals $r = y - f(\mathbf{x})$





- **L1** penalizes the **absolute** value of residuals
- $L(r) = |r|$
- Robust to outliers

- **L2** penalizes the **quadratic** value of residuals
- $L(r) = r^2$
- Easier to optimize

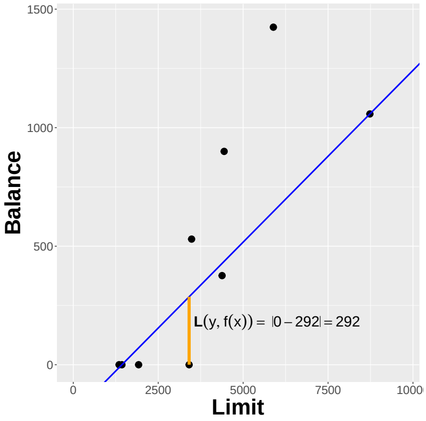
LINEAR MODELS: L1 VS L2 LOSS

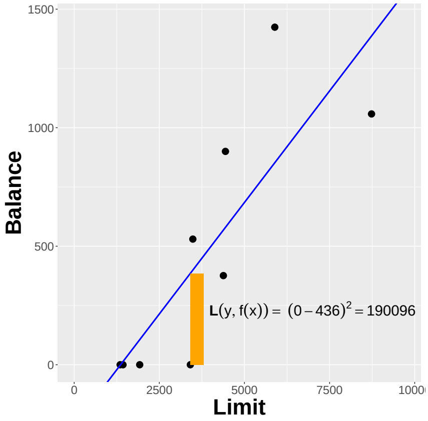
- **L1** Loss is not differentiable in $r = 0$
- Optimal parameters are computed numerically

- **L2** is a smooth function hence it is differentiable everywhere
- Optimal parameters can be computed analytically or numerically

LINEAR MODELS: L1 VS L2 LOSS

- The parameter values of the best model depend on the loss type



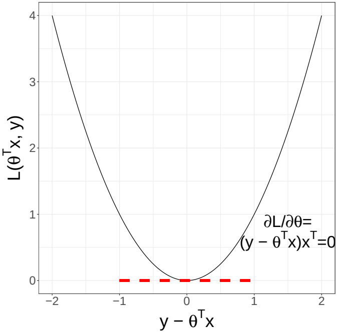


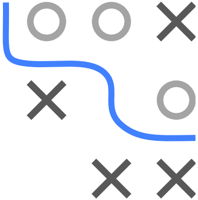
- $\hat{\theta}_{L_1} = 0.14 \rightarrow$ if the Credit Limit increases by 1\$ the Credit Balance increases by 14 Cents
- $\hat{\theta}_{L_2} = 0.19 \rightarrow$ if the Credit Limit increases by 1\$ the Credit Balance increases by 19 Cents

OLS ESTIMATOR

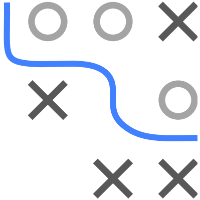
Ordinary-Least-Squares (OLS) estimator:

- Analytical solution for linear models with L2 loss
- Best parameters can be computed via derivation of the empirical risk
- Solution: $\hat{\theta} = (\mathbf{X}^\top \mathbf{X})^{-1}\mathbf{X}^\top \mathbf{y}$





OLS ESTIMATOR



Components of **OLS** estimator:

- **X**: Features + extra column for intercept
- **y**: Label vector

X

Intercept	Rating	Income	Credit Limit
1	283	14.891	3606
1	483	106.025	6645
1	514	104.593	7075

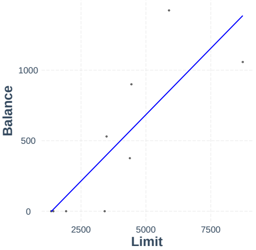
y

Credit Card Balance
333
903
580

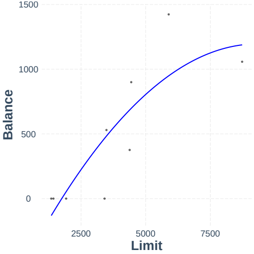
POLYNOMIAL REGRESSION

- Adding polynomial terms to the linear combination leads to more flexible regression functions
- Too high degrees can lead to overfitting

$Balance = \theta_0 + \theta_1 Limit$



$Balance = \theta_0 + \theta_1 Limit + \theta_2 Limit^2$



$Balance = \theta_0 + \theta_1 Limit + \theta_2 Limit^2 + \theta_3 Limit^3$