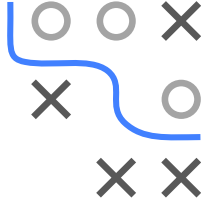
Introduction to Machine Learning

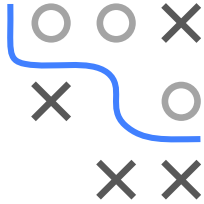


CART
Growing a Tree

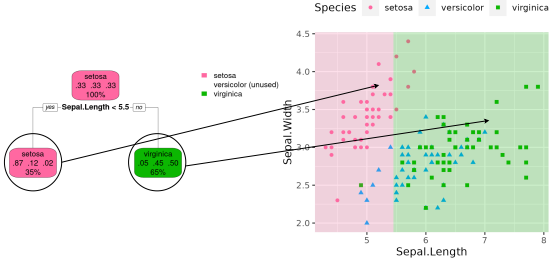
Learning goals

- Understand how a tree is grown by an exhaustive search
- Know where and how the split point is set

TREE GROWING

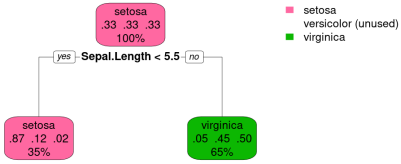


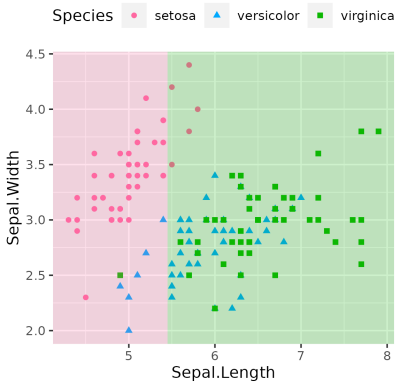
- We start with an empty tree, a root node that contains all the data. Trees are then grown by recursively applying **greedy** optimization to each node $\mathcal{N}$.
- Greedy means we do an **exhaustive search**: Ideally, all possible splits of $\mathcal{N}$ on all possible points t for all features x_j are compared in terms of their empirical risk $\mathcal{R}(\mathcal{N}, j, t)$.
- The training data is then distributed to child nodes according to the optimal split and the procedure is repeated in the child nodes.



TREE GROWING

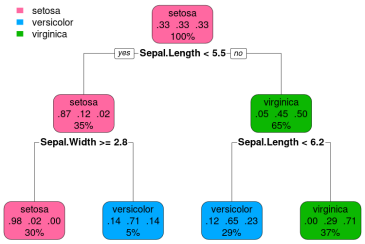
1. Start with a root node of all data.
2. Search for feature and split point that minimizes the empirical risk in child nodes – makes label distribution more homogenous.

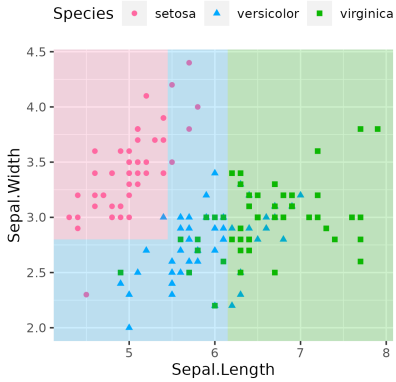




TREE GROWING

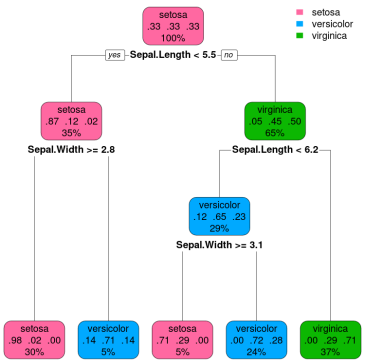
❸ Proceed recursively for each child node: Select best split and divide data from parent node into left and right child nodes.

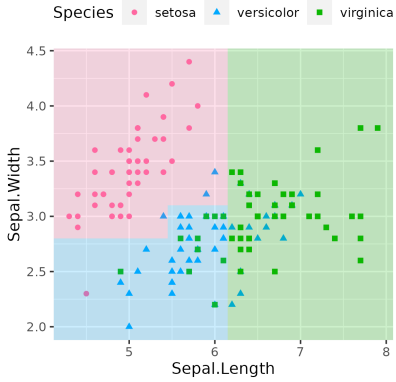


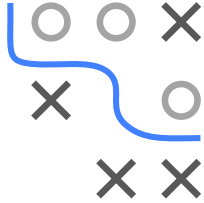


TREE GROWING

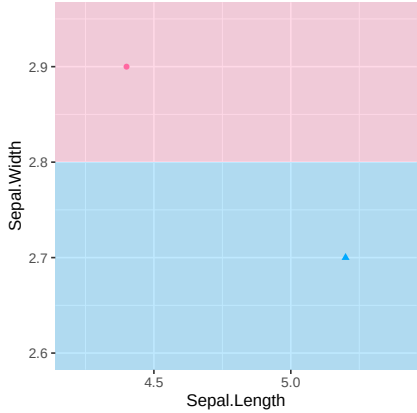
4. Repeat until we reach a stop criterion, e.g., until each leaf cannot be split further.

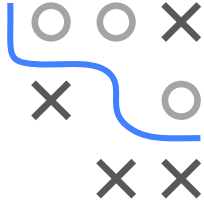






SPLIT PLACEMENT

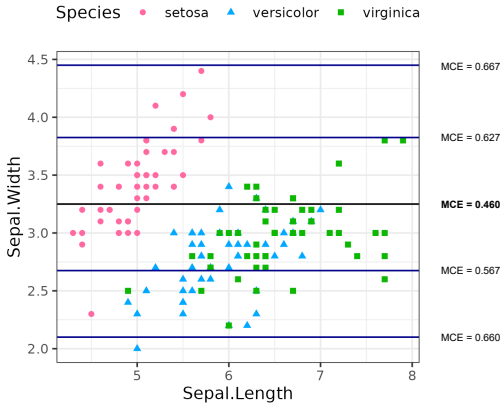


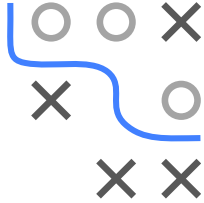


Splits are usually placed at the mid-point of the observations they split: the large margin to the next closest observations makes better generalization on new, unseen data more likely.

FINDING THE SPLIT

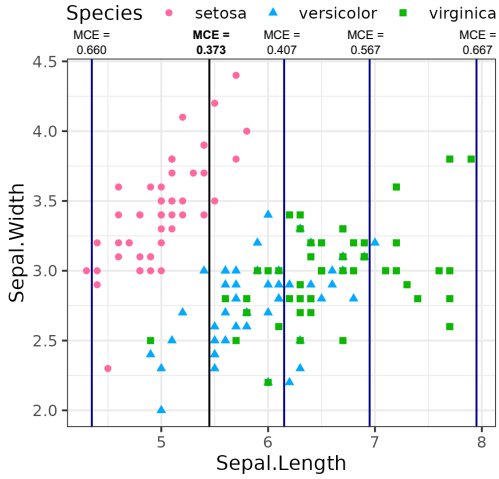
Assume we split the data so that the misclassification error (MCE) is minimal through the splitting.
First, we check a set of potential splits for `Sepal.Width`

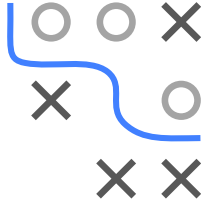




FINDING THE SPLIT

Then we check a set of potential splits for `Sepal.Length`





FINDING THE SPLIT

- We take the split with lowest MCE: `Sepal.Length` = 5.5
- In real life, we actually search over many more splitting points. Common strategies involve: a) Searching over all possible split points (exhaustive search), b) searching quantile-wise
- MCE is rarely used, we will cover split criteria in detail later.