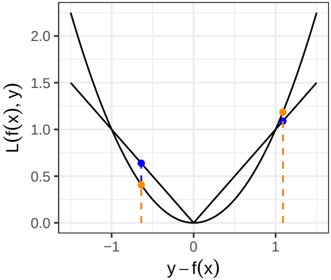
Introduction to Machine Learning

Supervised Regression: Linear Models with $L1$ Loss

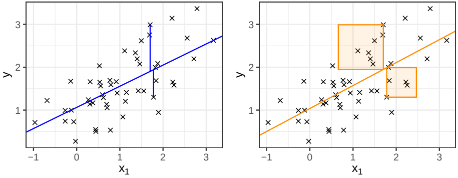


Learning goals

- Understand difference between $L1$ and $L2$ regression
- See how choice of loss affects optimization & robustness

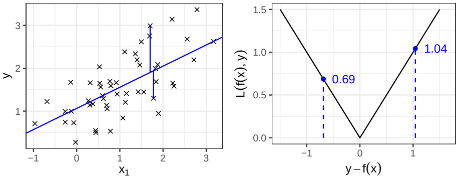
ABSOLUTE LOSS

- $L2$ regression minimizes quadratic residuals – wouldn't **absolute** residuals seem more natural?

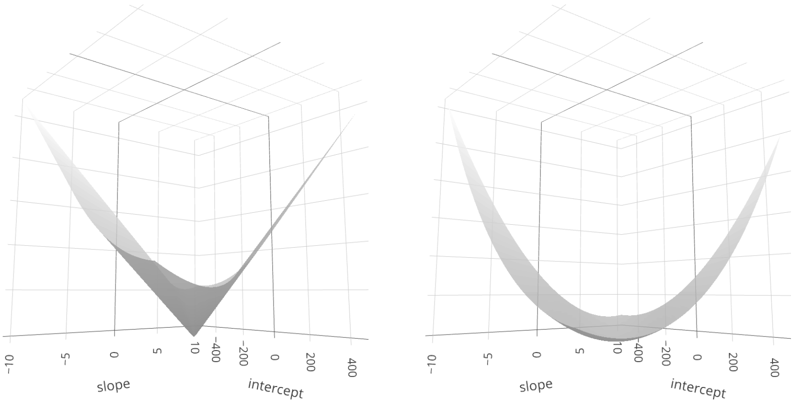


- $L1$ **loss / absolute error / least absolute deviation (LAD)**

$$L\left(y, f(\mathbf{x})\right) = |y - f(\mathbf{x})|$$



$L1$ VS $L2$ – LOSS SURFACE



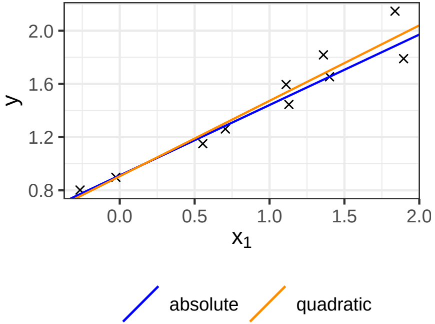
$L1$ loss (left) harder to optimize than $L2$ loss (right)

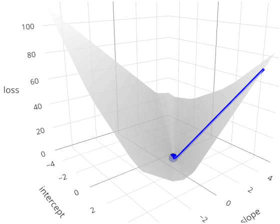
- Convex but **not differentiable** in $y - f(\mathbf{x}) = 0$
- No analytical solution

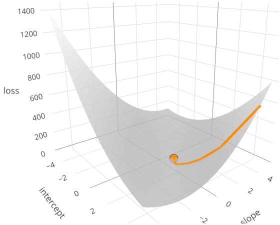
$L1$ VS $L2$ – ESTIMATED PARAMETERS

- Results of $L1$ and $L2$ regression often not that different
- Simulated data: $y^{(i)} = 1 + 0.5x_1^{(i)} + \epsilon^{(i)}, \quad \epsilon^{(i)} \overset{i.i.d}{\sim} \mathcal{N}(0, 0.01)$

	intercept	slope
$L1$	0.91	0.53
$L2$	0.91	0.57

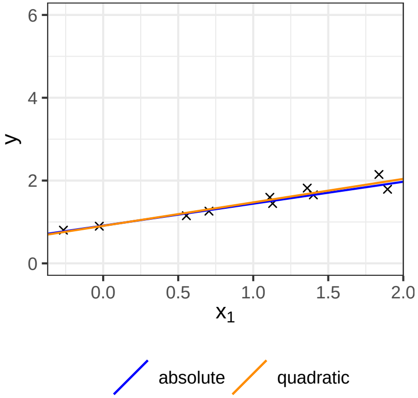


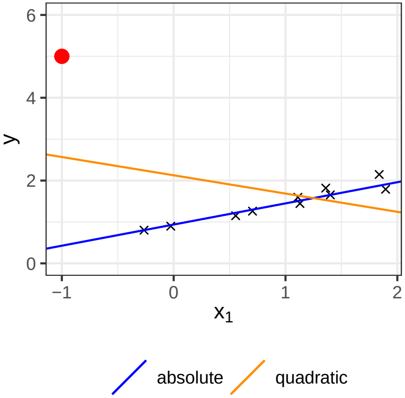




$L1$ VS $L2$ – ROBUSTNESS

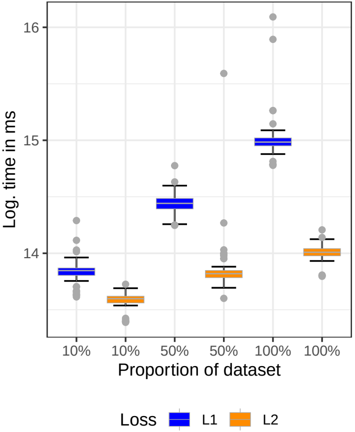
- $L2$ quadratic in residuals $\rightsquigarrow$ outlying points carry lots of weight
- E.g., $3\times$ residual $\Rightarrow$ $9\times$ loss contribution
- $L1$ more **robust** in presence of outliers (example ctd.):





$L1$ VS $L2$ – OPTIMIZATION COST

- Real-world `weather` problem $\rightsquigarrow$ predict mean temperature
- Compare **time** to fit $L1$ (`quantreg::rq()`) vs $L2$ (`lm::lm()`) for different dataset proportions (repeat $50\times$)



Loss

	Fitted: $L1$	Fitted: $L2$
Total $L1$ loss	8.98×10^4	8.99×10^4
Total $L2$ loss	5.83×10^6	5.81×10^6

Estimated coefficients

x_j	$L1$: $\hat{\theta}_j$	$L2$: $\hat{\theta}_j$
`Max_temperature`	0.553	0.563
`Min_temperature`	0.441	0.427
`Visibility`	0.026	0.041
`Wind_speed`	0.002	0.010
`Max_wind_speed`	-0.026	-0.039
`(Intercept)`	-0.380	-0.102

$L1$ **slower** to optimize!