Exercise 1: Overfitting & underfitting

Assume a polynomial regression model with a continuous target variable y and a continuous, p-dimensional feature vector \mathbf{x} and polynomials of degree d, i.e.,

$$f\left(\mathbf{x}^{(i)}\right) = \sum_{j=1}^{p} \sum_{k=0}^{d} \theta_{j,k}(\mathbf{x}_{j}^{(i)})^{k},$$

and $y^{(i)} = f(\mathbf{x}^{(i)}) + \epsilon^{(i)}$ where the $\epsilon^{(i)}$ are iid with $\mathsf{Var}(\epsilon^{(i)}) = \sigma^2 \ \forall i \in \{1, \dots, n\}$.

- a) For each of the following situations, indicate whether we would generally expect the performance of a flexible polynomial learner (high d) to be better or worse than an inflexible one (low d). Justify your answer.
 - (i) The sample size n is extremely large, and the number of features p is small.
 - (ii) The number of features p is extremely large, and the number of observations n is small.
 - (iii) The true relationship between the features and the response is highly non-linear.
 - (iv) The variance of the error terms, σ^2 , is extremely high.
- b) Are overfitting and underfitting properties of a learner or of a fixed model? Explain your answer.
- c) Should we aim to completely avoid both overfitting and underfitting?

Exercise 2: Resampling strategies

- a) Why would we apply resampling rather than a single holdout split?
- b) Classify the german_credit data into solvent and insolvent debtors using logistic regression. Compute the training error w.r.t. MCE.

(Python Hint: Read the already preprocessed file german_credit_for_py.csv.)

- c) In order to evaluate your learner, compare test MCE using
 - i) three times ten-fold cross validation (3x10-CV)
 - ii) 10x3-CV
 - iii) 3x10-CV with stratification for the feature foreign_worker to ensure equal representation in all folds
 - iv) a single holdout split with 90% training data

(R Hint: you will need rsmp, resample and aggregate.)

(Python Hint: you will need RepeatedKFold, RepeatedStratifiedKFold and train_test_split.)

- d) Discuss and compare your findings from c) and compare them to the training error from b).
- e) Would you consider LOO-CV to be a good alternative?