CMPSCI 611 - Advanced Algorithms
Homework 1

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Estimated amount of hours invested in the assignment: 30

Question 1

Question 2

Proposed Algorithm

Question 3

For (E, I) to be a matroid, it has to satisfy the exchange property and the cardinality property.

   The Exchange Property: According to the problem statement:

For $A, B \in I$

$\mid A \mid = k \Rightarrow \sum_{i=1}^{N} (\mid A \cap E_i \mid = 1) = k,$

$\mid B \mid = j < k \Rightarrow \sum_{i=1}^{N} (\mid B \cap E_i \mid = 1) < k$

Thus, $\exists e \in E_i \cdot (A \cap E_i = \{e\} \wedge B \cap E_i = \phi)$

Thus $B \cup \{e\} \in I$

Thus the Exchange Property is maintained

   The Cardinality Property: According to the problem statement:

For $A, B \in I, \mid A \mid = \mid B \mid = \mid I \mid = k$

$\sum_{i=1}^{N} (\mid A \cap E_i \mid = 1) = \sum_{i=1}^{N} (\mid B \cap E_i \mid = 1) = k$

Thus, $\neg \exists E_i \in I \cdot (A \cap E_i = \phi \vee B \cap E_i = \phi)$

Thus A and B are maximal and of equal size, and maintain the Cardinality property.

Question 4

Question 5

Question 6