Combined Complementary Filter For Inertial Navigation System

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***Abstract*—This paper presents an analysis of complementary algorithm for inertial navigation systems. Proposed suboptimal algorithm for sensors data fusion of navigation system, this method employs combined complementary filter approach and attitude error equation of inertial navigation system. Simulation results are presented to demonstrate the performance of the proposed approach.**

***Index Terms*—complementary filtering, inertial navigation system, data fusion**

I. Introduction

The traditional approach to navigation systems employ Inertial Navigation System (INS) and Global Navigation Satellite System(GNSS), different data fusion algorithms are used.

Extended Kalman filter. The EKF linearizes both the process and the observation functions errors of the EKF [5], since the approximated none-linear problem is actually solved optimally by the corresponding linear Kalman filter. Besides it is difficult to know exact values for covariance matrix of process noise. Together, these factors contribute to filter divergence.

Leading researchers solve this problem in most cases using own unique approaches. In particular in order to overcome divergence large amount Kalman filter modification was developed, for instance: Yasvinsky algorithms, different robust [3] and adaptive extensions [4].

At the present time besides optimal state vector estimation (Kalmna filter), there are other methods of data fusion, which is well proven in practice, in particular complementary filters. The feasibility of this method is due to the fact that measurement of navigation data based on different physical principles, and measurement errors remain in different frequency ranges.

Complementary filter approach allows estimating only measured components of navigation system, in case of Kalman filter there is capability to estimate all components of state vector, in particular the angular orientation and sensor's instrumental errors (gyro and accelerometer biases).

II. Complementary filter

This paper proposes the combined complementary filter: measured components of state vector estimated with complementary filter approach and indirectly measured attitude extrapolate by means of INS error equation.

As data fusion of redundant navigation information, complementary filter is proposed, which is well known from

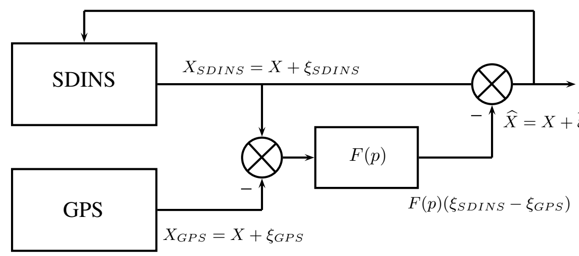

Figure 1. Complementary filter

Doppler inertial navigation systems (Fig. 1), rather than reduced Kalman filter.

Instead of the classical aperiodic filter $F(p)$ in compensation schemes, authors propose to use a third order filter with variable structure.

$$F(p) = \begin{cases} \dfrac{1}{T_1 p + 1} x & \text{if } t_{up} \leq T_1; \\ \dfrac{T_2 p + 1}{(T_2 p + 1)(T_2 p + 1)(T_2 p + 1)} & \text{if } T_1 < t_{up} \leq T_2; \\ \dfrac{T_3 p + 1}{(T_3 p + 1)(T_3 p + 1)(T_3 p + 1)} & \text{if } T_2 < t_{up}. \end{cases} \quad (1)$$

here t_{up} - uptime of compensation complementary filter.

Studies have shown [2], such data fusion algorithm gives results not worse than the Kalman filtering, without affecting the stability of estimation algorithms.

However, this algorithm does not estimate observable state vector components, in particular angular orientation.

State vector of complex inertial satellite navigation system $\mathbf{x_{ISNS,k}}$ based on error equation of inertial and stattelite systems $\mathbf{x}_{ISNS;k} = \begin{vmatrix} \mathbf{x}_{INS,k} \\ \mathbf{x}_{GNSS,k} \end{vmatrix}$, by using the optimal Kalman filter, the generalized state space equation of a complex system errors can be written as:

$$\mathbf{x}_{ISNS;k} = \Phi_{ISNS,k} \mathbf{x}_{ISNS;k-1} + \xi_{ISNS} \quad (2)$$

where $\Phi_{ISNS;k} = \begin{vmatrix} \Phi_{INS,k} & 0 \\ 0 & \Phi_{GNSS,k} \end{vmatrix}$ – known state propagation matrix, which is formed on the basis of theΦ_{ISNS}, Φ_{GNSS} matrices, models of correlated components of GNSS

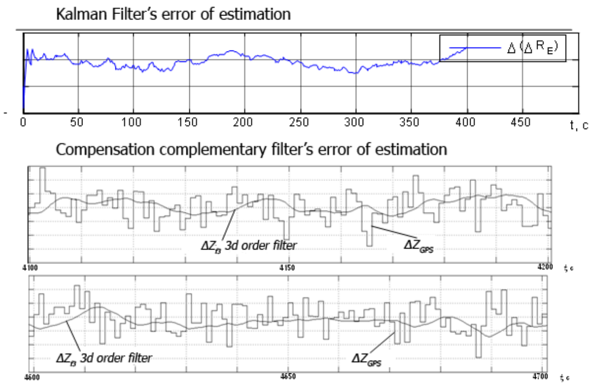

Figure 2. Kalman and complementary filter performance

and INS; $\xi_{ISNS;k} = \begin{vmatrix} \xi_{INS,k} \\ \xi_{SNS,k} \end{vmatrix}$ – vector of zero mean white Gaussian noise with covariance matrix $Q_{ISNS;k}$ (process noise matrix), of two corresponding navigation systems.

Equation for estimation of $\hat{\mathbf{x}}_{ISNS;k}$, with certain assumptions, derived from the general equations of optimal filtering and have the form:

$$\begin{aligned} \hat{\mathbf{x}}_{ISNS;k} &= \tilde{\mathbf{x}}_{ISNS;k|k-1} + K_k(z_{GNSS,k} - \widehat{z}_{ISNS;k}); \\ \widehat{z}_{ISNS,k} &= G(z_{INS,k} - M_{INS,k}\tilde{\mathbf{x}}_{INS,k|k-1}) + \\ &\quad + M_{SNS,k}\tilde{\mathbf{x}}_{SNS,k|k-1} \end{aligned} \tag{3}$$

$$\tilde{\mathbf{x}}_{ISNS;k|k-1} = \Phi_{ISNS;k}\hat{\mathbf{x}}_{ISNS;k-1}; \tag{4}$$

$$K_k = P_{k|k-1}H_k^{\mathrm{T}}(H_k P_{k|k-1} H_k^{\mathrm{T}} + N_k)^{-1} \tag{5}$$

$$P_{k|k-1} = \Phi_{ISNS;k}P_{k-1}\Phi^{\mathrm{B}}_{ISNS;k} + Q_{ISNS;k} \tag{6}$$

$$P_k = P_{k|k-1} - K_k H_k P_{k|k-1} \tag{7}$$

$$\begin{aligned} H_k = &\frac{\partial}{\partial V_{ISNS;k}}[G(Z_{INS,k} - M_{INS,k}\mathbf{x}_{INS,k}) + \\ &+ M_{SNS,k}\mathbf{x}_{SNS,k}]_{\mathbf{x}_{ISNS,k} = \tilde{\mathbf{x}}_{ISNS,k|k-1}} \end{aligned} \tag{8}$$

where z_{GNSS}, z_{INS} – observation vectors of GNSS and INS; $\mathbf{G}$ – known matrix of vector function $\mathbf{G}(\hat{\mathbf{x}}_{ISNS;k})$, that connects the radio navigation signal parameters with the estimated state vector$\hat{\mathbf{x}}_{ISNS;k}$; M_{GNSS}, M_{INS} – known matrices error of observation process from GNSS and INS; $\widehat{z}_{ISNS}$ – estimation of observation vector; $\tilde{\mathbf{x}}_{ISNS}$, $\tilde{\mathbf{x}}_{\mathrm{INS}}$, $\tilde{\mathbf{x}}_{\mathrm{SNS}}$ – errors of estimation of complex system, INS and GNSS errors; $\tilde{\mathbf{x}}_{ISNS;k|k-1}$ and $P_{k|k-1}$ – corresponding errors of INS, GNSS and covariance matrix for moment P_k, calculated based on measurements in previous time stamps $k-1$, $k-2$; H – measurement matrix; N – measurement noise covariance matrix.

Simulations of proposed filtering approach were done with simplified variant of inertial navigation system with following equation of motion:

$$\begin{aligned} &\dot{\varphi} = \frac{V_N}{R_E + h}; \dot{h} = V_U; \dot{\vartheta} = \omega - \frac{V_N}{R_E + h}; \\ &\dot{V}_N = a_N - \frac{V_N}{R_E + h}V; \dot{V}_U = a_U + \frac{V_N}{R_E + h}V_N - g; \\ &a_N = a_y \cos\vartheta - a_z \sin\vartheta; a_U = a_y \sin\vartheta + a_z \cos\vartheta; \end{aligned} \tag{9}$$

φ,h- latitude and height; V_N, V_U- North and Up velocity; a_N, a_U - North and Up acceleration; a_y, a_z - acceleration in body frame (accelerometers output); ϑ - pitch angle;ω- angular velocity in body frame (gyro output); R_E- Earth radius;

For simulation purposes next parameters of inertial sensors was used:

Table I
SIMULATION PARAMETERS

Parameter	Value
Gyro bias	$100°/hr$
Angular random walk	$1.2°/\sqrt{hr}$
Accelerometer bias	$10^{-2}g$
Velocity random walk	$0.18m/s/\sqrt{hr}$
GNSS position precision	$7m(1\sigma)$
GNSS velocity precision	$0.05m/s(1\sigma)$

Performance of complementary filter presented on fig.3, 4, 5, and 6. In particular fig. 6 shows the extrapolation of the INS angular orientation errors.

III. ATTITUDE ERROR EXTRAPOLATOR

In studies of the combined complementary filter was made the following assumption: since from the output of complementary filter we can observe the errors evolution of inertial navigation system, it becomes possible to use this information to construct the attitude error extrapolator. Attitude errors can be described with following equation:

$$\delta\dot{\vartheta} = -\frac{1}{R_E + h}\delta V_N + \frac{V_N}{(R_E + h)^2}\delta h + \varepsilon \tag{10}$$

δh- INS height error; δV_N- INS North velocity error; $\delta\vartheta$- attitude error; ε- gyro bias.

This approach makes possible to predict the INS attitude error, like Kalman filter does on predict stage. In case of precise initial conditions, it is possible to get quite accurate predictions for the pitch error on a long time period, even for very noisy sensors. In our case, the angular random walk was chosen$1.2°/\sqrt{hr}$, for example inertial measurement unit Navchip ($ 1,000) from InterSense includes micromechanical gyroscopes with an angular random walk$0.18°/\sqrt{hr}$, and the device has a one order of magnitude lower the noise density.

This method has several drawbacks: large sensitivity to initial conditions of attitude INS errors. If gyro bias set with an accuracy of 10%, the results of the extrapolation of attitude error significantly deteriorate (fig. 5). Since no stage correction, extrapolated attitude INS error diverges with the time. Approach can be used quite successful in case if UAVs operation time or attitude correction time (for instance steady flight without acceleration) less then diverge time of filter.

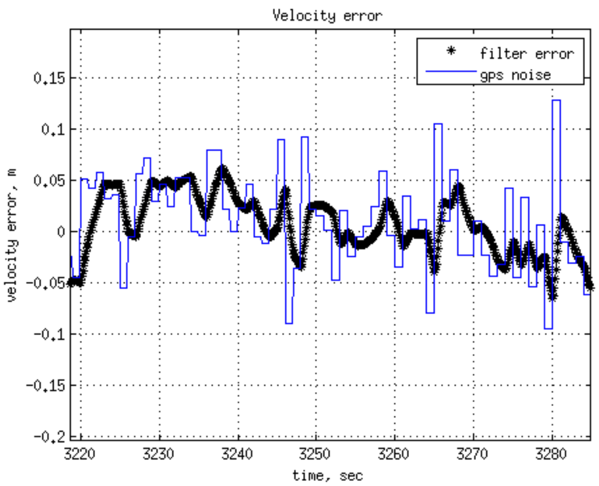

Figure 3. Estimation error of velocity

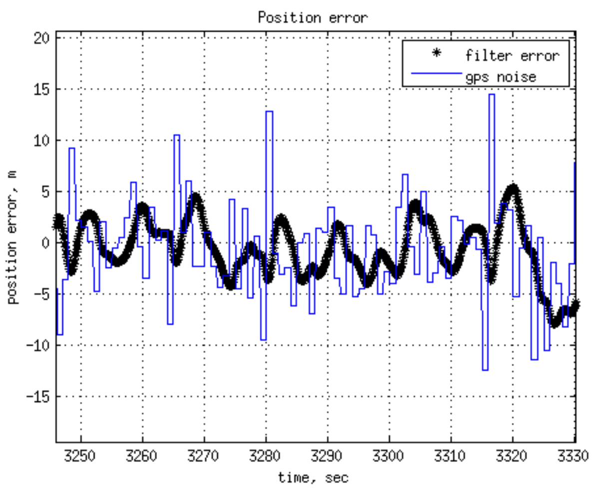

Figure 4. Estimation error of positon

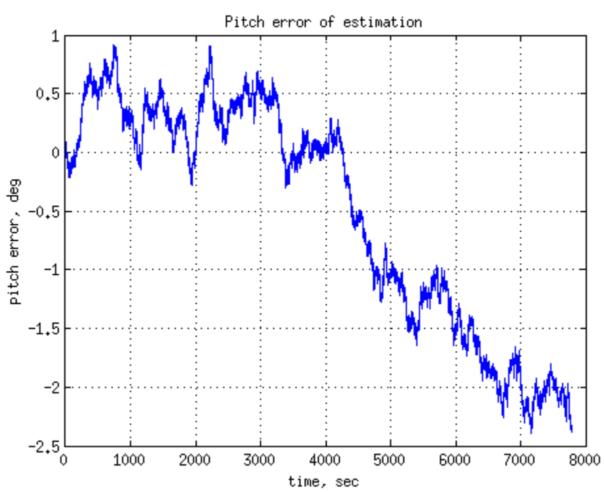

Figure 5. Attitude estimation error

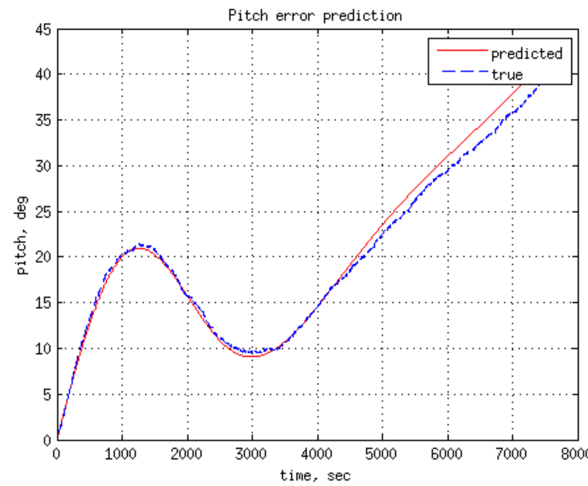

Figure 6. Prediction of INS attitude error

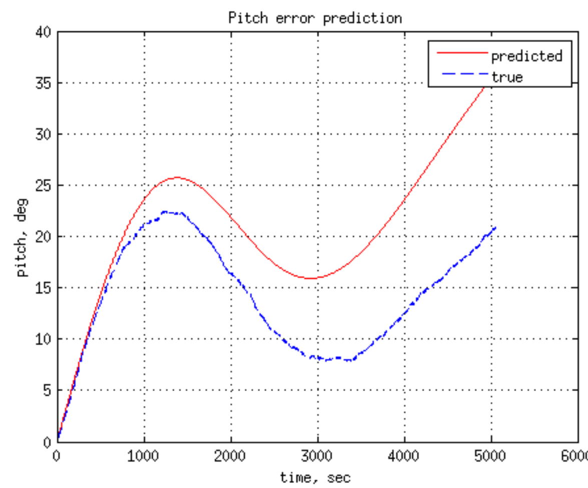

Figure 7. Prediction of INS attitude error in case of 10% error in initial condition for gyro bias

IV. Conclusions

The proposed approach to state estimation and data fusion for inertial and satellite navigation systems is faster operating and robust to non-stationary random processes, which are for instance sensor's scale factors or biases and in other hand it can be quite easy implemented in the onboard digital computer.

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