Notes on Analysis

Jonathan E. Fuzaro Alencar

Chapter 1

Preliminaries

Greek Alphabet

alpha	α	A
beta	β	B
gamma	γ	Γ
delta	δ	Δ
epsilon	ϵ	E
zeta	ζ	Z
eta	η	H
iota	ι	I
kappa	κ	K
lambda	λ	Λ
mu	μ	M
nu	ν	N
xi	ξ	Ξ
omicron	o	O
pi	π	Π
rho	ρ	P
sigma	σ	Σ
tau	τ	T
upsilon	υ	Υ
phi	ϕ, φ	Φ
chi	χ	X
psi	ψ	Ψ
omega	ω	Ω

1 Sums & Products

Sum Notation: $\displaystyle\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$;

E.g.: $\sum_{i=1}^{3}\sum_{j=2}^{4}(i+j) = (\sum_{j=2}^{4} 1+j) + (\sum_{j=2}^{4} 2+j) + (\sum_{j=2}^{4} 3+j)$

Product Notation: $\displaystyle\prod_{i=1}^{n} a_i = a_1 \cdot a_2 \cdots a_n$;

E.g.: $\prod_{i=1}^{n} i = n!$

2 Logic

Universal Quantifier:	$\forall$	"for all"
Existential Quantifier:	$\exists$	"there exists"
Uniqueness Quantifier:	$\exists!$	"there exists one and only one"
Material Implication:	$\Longrightarrow$	"implies"
Material Equivalence:	$\Longleftrightarrow$	"if and only if (iff)"
Logical Complement:	$\neg$	"not"
Logical Conjuction:	$\wedge$	"and"
Logical Disjunction:	$\vee$	"or"
Exclusive Disjucntion:	$\oplus$	"exclusive or (xor)"

3 Set Theory

3.1 Notation

Set Relations:	$\{a, b\}$	"the set containing the elements a and b"
	$\{a \mid p(a)\}$	"the set of a such that $p(a)$ is true"
	$\in$	"is an element of"; e.g. $3 \in \{1, 2, 3\}$
	$\notin$	"is not an element of"; e.g. $4 \notin \{1, 2, 3\}$
	$=$	"equals"; $(A = B) \iff \forall x \mid (x \in A \iff x \in B)$
	$\neq$	"does not equal"; $A \neq B \iff \neg(A = B)$
	$\subseteq$	"is a subset of"; $(A \subseteq B) \iff \forall x \mid (x \in A \implies x \in B)$
	$\subset, \subsetneq$	"is a proper subset of"; $(A \subset B) \iff (A \subseteq B \wedge A \neq B)$
	$\supseteq$	"is a superset of"; $A \supseteq B \iff B \subseteq A$
	$\supset, \supsetneq$	"is a proper superset of"; $A \supset B \iff (B \supseteq A \wedge B \neq A)$
Set Operations:	$\overline{A}$	"the complement of A"; $\overline{A} := \{x \mid x \notin A\}$
	$\cap$	"intersect"; $A \cap B := \{x \mid x \in A \wedge x \in B\}$
	$\cup$	"union"; $A \cup B := \{x \mid x \in A \vee x \in B\}$
	$\times$	"cross"; $A \times B := \{(x, y) \mid x \in A \wedge y \in B\}$
	$-, \setminus$	"complement, slash"; $A - B := \{x \mid x \in A \wedge x \notin B\}$

3.2 Indexed Operations

If S_i is a collection of sets indexed by $i \in I$ then,

Indexed Intersetion: $\bigcap_{i \in I} S_i := S_{I_1} \cap S_{I_2} \cap ... = \{x \mid \forall i \in I; x \in S_i\}$

Indexed Union: $\bigcup_{i \in I} S_i := S_{I_1} \cup S_{I_2} \cup ... := \{x \mid \exists i \in I; x \in S_i\}$

3.3 Common Sets

$\varnothing$	the empty set	$\{\}$
$\mathbb{N}$	the *natural numbers*	$\{1, 2, 3, ...\}$
$\mathbb{Z}$	the *integers*	$\{..., -1, 0, 1, ...\}$
$\mathbb{Q}$	the *rational numbers*	$\{p/q \mid p, q \in \mathbb{Z} \wedge q \neq 0\}$
$\mathbb{R}$	the *real numbers*	infinite decimals
$\mathbb{C}$	the *complex numbers*	$\{i = \sqrt{-1}, a + ib \mid a, b \in \mathbb{R}\}$

4 Functions

Definition 1 (**Function as a rule**). A *function* consists of two sets, called the *domain* and the *codomain*, and a rule that associates to any element in the domain exactly one element in the codomain.

Definition 2 (**Set theoretic function**). A *function* $f : X \to Y$ is a subset $\Gamma_f \subseteq X \times Y$ having the property that for every $x \in X$, there exists a unique $y \in Y$ such that $(x, y) \in \Gamma_f$. That is, $\Gamma_f = \{(x, y) \mid (\forall x \in X)(\exists! y \in Y)\}$.

Definition 3 (**Image**). The set of all values of f is called its *image*: y is an element of the image of a function $f : X \to Y$ if there exists an $x \in X$ such that $f(x) = y$. The image of $f : X \to Y$ is written $f(X)$; it is a subset of Y. That is, the image of f is $\{f(x) \mid x \in X\} = f(X)$.