Lecture 3: Integer representation

September 6, 2019

601.229 Computer System Fundamentals

Representing integers

- We've seen how to represent unsigned (nonnegative) integers
  - Bit string intrepreted as a binary (base 2) number
- How to represent signed integers?
  - Sign magnitude
  - Ones' complement
  - Two's complement
- In examples that follow, we'll use 4-bit words
  - Ideas will generalize to larger word sizes

Desired features for signed representation

What we want in a representation for signed integers:

- About half of encoding space used for negative values
- Each represented integer has a unique encoding as bit string
- Straightforward way to do arithmetic

Sign magnitude representation

Let most significant bit be a sign bit: **0**→positive, **1**→negative

Bit string	value	Bit string	value
0000	0	**1**000	-0
0001	1	**1**001	-1
0010	2	**1**010	-2
0011	3	**1**011	-3
0100	4	**1**100	-4
0101	5	**1**101	-5
0110	6	**1**110	-6
0111	7	**1**111	-7

Downsides: two representations of 0, arithmetic complicated by sign bit

Ones' complement

Ones' complement: to represent `-x`, invert all of the bits of `x`

Bit string	value	Bit string	value
0000	0	1000	-7
0001	1	1001	-6
0010	2	1010	-5
0011	3	1011	-4
0100	4	1100	-3
0101	5	1101	-2
0110	6	1110	-1
0111	7	1111	-0

Downside: two representations of 0, slightly complicated arithmetic

Sign magnitude and ones' complement are obsolete

- Sign magnitude and ones' complement representations are not used for integer representation by modern computers
  - But, sign magnitude is used in floating point representation
- The rest of this lecture will discuss *two's complement*

Two's complement

Two's complement: in w-bit word, the most significant bit represents -2^{w-1}

E.g., when $w = 4$,

Representation	Bit 3	Bit 2	Bit 1	Bit 0
Unsigned	8	4	2	1
Two's complement	-8	4	2	1

Given bit string 1011,

- Unsigned, 1011 is $8 + 2 + 1 = 11$
- Two's complement, 1011 is $-8 + 2 + 1 = -5$

Two's complement

Two's complement: in w-bit word, the most significant bit represents -2^{w-1}

Bit string	value	Bit string	value
0000	0	1000	-8
0001	1	1001	-7
0010	2	1010	-6
0011	3	1011	-5
0100	4	1100	-4
0101	5	1101	-3
0110	6	1110	-2
0111	7	1111	-1

Note asymmetry of negative and positive ranges: -8 is represented, 8 isn't

Thinking about two's complement

Useful way to think about a w-bit two's complement representation:

- Bit $w - 1$ is the sign bit, 0→positive, 1→negative
- If sign bit is 0, usual unsigned interpretation
- If sign bit is 1, bits $w - 2 \mathinner{\ldotp\ldotp} 0$ indicate the "offset" from -2^{w-1}

Two's complement example

Given $w = 4$, example bit string is $\boxed{1\underline{011}}$

- Sign bit is 1
- Offset from -2^3 is 011, which is 3 (2+1)
- -8 + 3 = -5

So, 1011 represents -5

Clicker quiz

Clicker quiz omitted from public slides