LIBOR's Poker *

Jiakai Chen†

September 20, 2018

Abstract

Survey-based benchmarking is a principal-agent problem in which the principal is unaware of strategic agents' private signal distribution. Although the distribution affects agents' equilibrium reports, it does not affect the expected benchmark bias if the principal sets a harsh enough quadratic penalty, which also minimizes the reporting errors. Under such equilibrium, increasing the panel size and trimming more quotes improves the benchmark accuracy. The expected bias is not distribution-free under lenient penalty or collusion.

Keywords: Principal-agent problem; Benchmarking; Mechanism design; LIBOR

JEL classification: D43, D44, D47, G10, G21

*Many thanks for helpful discussions and suggestions to Brian Coutler, Lin William Cong, William Fuchs, Robert Helsley, Andrea Heuson, Dwight Jaffee, John Kuong, Yiqing Lu, Gustavo Manso, Dmitry Orlov, Demian Pouzo, Uday Rajan, Joel Shapiro, David Skeie, Alexei Tchistyi, James Vickery, Nancy Wallace, and Chu Zhang, as well as the participants in Oxford Financial Intermediation Theory Conference, the Berkeley-Stanford Finance Seminar, Australasian Finance and Banking Conference, Trans-Atlantic Doctoral Conference, Econometric Society European Meeting, European Finance Association Meeting, and Asian Meeting of Econometric Society. I especially wish to thank Christine Parlour for her invaluable advice. An earlier version of this paper is presented under the name "LIBOR's Poker: Discover Interbank Borrowing Costs Amid Tacit Collusion". I thank the Institute for Business and Economic Research at UC Berkeley for their financial support.

†Shidler College of Business, University of Hawaii. Email: jiakai@hawaii.edu.

1 Introduction

Surveys are widely used to make inferences about specific private information of a population. A survey administrator sends out a list of questions aimed at eliciting the private information of the population, then makes inferences using the survey responses. Nevertheless, members of the sample group may choose to submit fraudulent information to the administrator if doing so serves their benefit, even though they are supposed to respond truthfully. In this case, the survey becomes a principal-agent problem between the administrator and the sample group. A widely-adopted assumption in the principal-agent problem literature is that the statistical distribution of agents' private information is known to the principal, who can then design a direct mechanism following the revelation principle.[1] Nevertheless, if the administrator knew the distribution of the private information among the population or the sample group, she could calculate the statistical inference of the population. Hence, assuming the principal knows about the agents' private information distribution is inconsistent with the context of surveys. This paper studies surveys as a principal-agent problem without such an assumption.

Many critical economic and financial benchmarks are survey-based: notable examples include the National Compensation Survey and the University of Michigan Consumer Sentiment Index. Some of them are plagued by manipulation, the most prominent example being the London Inter-bank Offered Rate (LIBOR) and ISDAfix. LIBOR measures the interest rate at which a group of major banks can borrow unsecured funds on the interbank market. With at most a few interbank transactions daily, the benchmark administrator has to rely on an honor system in which a panel of largest banks each *honestly* reports its estimated borrowing cost should a transaction occur. The panel banks also know that a thorough verification of their responses is cost prohibitive.[2] Recent scandals showed that banks fiddled LIBOR so many times that it proved the fiddling was at least widely tolerated, if not unanimously practiced.[3] Nor was the ISDAfix index, another survey-based

[1]See classical works such as Gibbard (1973), Green and Laffont (1977), d'Aspremont and Gérard-Varet (1979), Dasgupta, Hammond, and Maskin (1979), Myerson (1979), Myerson (1981), and Maskin (1999). For correlated private signals and the correlation is a common knowledge, see in Cremer and McLean (1988), McAfee, McMillan, and Reny (1989), d'Aspremont, Cremer, and Gérard-Varet (1990), and McAfee and Reny (1992).

[2]The LIBOR benchmark administrator, which used to be the British Bankers' Association and is now Intercontinental Exchange, asks panel banks the following question: *"At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?"* The administrator then sets the LIBOR as the arithmetic inter-quartile average—a robust central moment—of banks' answers.

[3]See "The LIBOR scandal: The rotten heart of finance", The Economist, July 7, 2012, Accessed July 9, 2012. `http://www.economist.com/node/21558281`.

benchmark for cash settlement of interest rate swaptions, rigging-free.[4] Both LIBOR and ISDAfix are critical financial benchmarks that affect the value of more than 300 trillion US dollars of interest rate related derivatives and securities. Years after the LIBOR scandal unfolded, LIBOR is still a survey-based benchmark: changing a benchmark would mean amending all contracts and legal documents that refer to it. Therefore, it is important to study whether the principal can design a mechanism which can reduce the impact of the strategic reporting without significantly changing the current benchmark fixing procedure.[5]

I model a survey as a game of incomplete information between a principal and a panel of agents with private signals. The principal surveys the panel to elicit the robust mean—the benchmark—of the signal distribution. The agents have the incentive to rig the benchmark, hence report strategically. The principal cannot facilitate a direct mechanism following the revelation principle due to the lack of information on the signal distribution. To minimize the misreport from all agents, the principal chooses to randomly audit the agent's report and impose a penalty in case misreporting is discovered, whereas the amount of *ex post* penalty is subject to legal constraints.[6]

I first show the existence of a weakly increasing pure strategy Bayesian Nash equilibrium report among agents. The equilibrium report is not truth revealing, and the reporting error depends on the cross-sectional distribution of private signals.

The expected benchmark bias is distribution-free if the agents face an expected penalty as a function of the reporting error. The penalty is also proportional to the agent's sensitivity to the index upon the audit, conditional on the *ex post* penalty is severe enough to induce strictly increasing equilibrium reporting from agents. The principal can obtain an unbiased index by an

[4] ISDAfix is a global index for cash settlement of interest rate swaptions administered by ICAP plc. ICAP surveys a panel of 16 banks for the rate quotes at which they would buy and sell a standard swap each day in major currencies, then releases the ISDAfix as the inter-quartile arithmetic average of the quotes. From the press release of CFTC, `http://www.cftc.gov/PressRoom/PressReleases/pr7371-16`, CFTC Orders Citibank to Pay $250 Million for Attempted Manipulation and False Reporting of U.S. Dollar ISDAfix Benchmark Swap Rates. After the scandal had unfolded, ISDAfix has renamed the lesser known "ICE Swap Rate" in April 2015. In this paper, I use the name "ISDAfix" due to its connection with the scandals.

[5] The International Organization of Securities Commissions (IOSCO), the Market Participants Group, and Duffie and Stein (2015) recommended that key benchmarks fixing—the process of determining the index—should be "anchored" in actual market transactions or executable quotations. Nevertheless, despite some changes having been made according to IOSCO (2014), the reform of the benchmark has been slow, as in the follow-up report by IOSCO (2016): thousands of LIBOR-linked loans and ISDAfix-quoted derivative contracts would have to be redrafted and renegotiated.

[6] Notice that the principal can adopt a wider family of complex implementation strategy, if she does not concern about the potential disruption on business contracts that refer to the benchmark by significantly changing in the benchmark formation process. In fact, Coulter and Shapiro (2014) proposes a creative mechanism that leads to truthful reporting with a whistleblower bank who may contest another bank's report.

adjustment that depends only on the parameters of the benchmark calculation and the penalty function: both are attainable to the principal.

In order to minimize the agents' reporting error, the principal sets the optimal penalty function to be quadratic to the reporting error and harsh enough to ensure the agent's reporting strategy to be strictly increasing. Under the principal's equilibrium penalty function, the agents follow a symmetric Bayesian Nash equilibrium reporting strategy, which suggests that increasing the size of the panel and trimming more survey responses reduces the expected benchmark bias, whereas averaging randomly selected submissions outside the trimmed range does not.

If the penalty function is too lenient, the agent's equilibrium report will demonstrate a "bunching" region, in which agents with different private signal submit the same report. When agents manipulate the benchmark through collusion, they not only manipulate the benchmark more aggressively and effectively. Under both scenarios, the equilibrium benchmark bias becomes distribution-dependent.

This paper extends the literature of principal-agent problem by looking at the survey game with strategic agents, in which the common assumption that the principal knows the private signal distribution is inconsistent with the nature of the game.

This paper also speaks to the growing empirical literature studying benchmark manipulation. Specifically, Hou and Skeie (2014) provide a summary of the LIBOR scandal. Hartheiser and Spieser (2010) look at panel banks' submissions and estimate the clustering of LIBOR submissions. Snider and Youle (2010), and Snider and Youle (2012) study the relationship between LIBOR submissions and banks' credit default swap (CDS) spreads, in various currencies. Abrantes-Metz, Kraten, Metz, and Seow (2011) compare individual bank quotes to CDS spreads and market capitalization data from early 2007 to mid-2008. They find the data are inconsistent with a material manipulation of the US dollar 1-month LIBOR rate, although anomalies in individual quotes do exist. Kuo, Skeie, and Vickery (2012) show that during the financial crisis, banks were willing to borrow from the Fed at a higher interest rate on collateral than the interbank lending rate that the banks reported in the LIBOR fixing process. King and Lewis (2015) study the link between bank CDS spreads to the LIBOR submissions. Gandhi, Golez, Jackwerth, and Plazzi (2016) show that manipulation was initially stronger for banks incorporated outside the U.S., where enforcement is historically weaker, and that it disappeared in the aftermath of LIBOR investigations. Youle (2014) implicitly estimates the positions from the submissions: the result suggests LIBOR distortion that is consistent with Kuo et al. (2012) and Kuo, Skeie, Vickery, and Youle (2014). Eisl, Jankowitsch, and Subrahmanyam (2014) choose not to model the incentive directly but focus on the quantification of the potential effects of manipulated individual contribution to the final rate. The distribution-free expected benchmark bias may serve as a basis for the study of actual benchmark bias. Also, this study is the first to prove that increasing the panel size while retaining the share of trimmed quotes still reduces the LIBOR bias.

Finally, this paper complements the literature about transforming a survey-based benchmark into a transaction-based one, first depicted by Duffie and Stein (2015). Duffie and Dworczak (2014) study the theoretical foundation of robust and optimal benchmarking under sparse transactions. Duffie, Dworczak, and Zhu (2017) study the price-transparency role of benchmarks in a search market. Coulter, Shapiro, and Zimmerman (2017) extend the transaction-based approach by proposing a mechanism to correct the effect of manipulation-driven transactions.[7] This paper offers insights about how to reduce the impact of benchmark manipulation under the current survey-based procedure before the market completes the transition to a transaction-based benchmark.

2 The Model

2.1 Structure and Notation

I consider a game of incomplete information called *survey*, between a principal and a survey panel that contains N risk neutral asymmetric agents, $i = 1, \dots, N$.[8] Each agent first observes his i.i.d. private signal $s_i \in [\underline{s}, \overline{s}] \subset \mathbb{R}$ and then reports $b_i \in [\underline{s}, \overline{s}]$ to the principal. The private signals $\boldsymbol{s} = (s_1, \dots, s_N) \in S$ can be viewed as the panel banks' actual borrowing costs in the LIBOR fixing mechanism, or their true swap quotes in ISDAfix, while $\boldsymbol{b} = (b_1, \dots, b_N) \in S$ can be viewed as banks' reports. The density function of s_i belongs to a family of bounded and atomless distribution

[7]The International Organization of Securities Commissions (IOSCO), the Market Participants Group, and Duffie and Stein (2015) recommended that key benchmarks fixing—the process of determining the index—should be "anchored" in actual market transactions or executable quotations. Nevertheless, despite some changes having been made according to IOSCO (2014), the reform of the benchmark has been slow, as in the follow-up report by IOSCO (2016): thousands of LIBOR-linked loans and ISDAfix-quoted derivative contracts would have to be redrafted and renegotiated.

[8]Throughout the paper, I will use he/his to refer to an agent or a survey participant and she/her to the principal or the survey administrator. No association of the roles to particular genders is intended. Also, I will use "benchmark" and "index" interchangeably.

$f(s_i;\theta)$, where the parameter θ identifies the distribution function, as well as its moment conditions, within the family.[9] The cumulative distribution function is $F(s_i;\theta)$. For notational ease, I also refer to the density and cumulative density of private signals $\boldsymbol{s}$ as $\boldsymbol{f}(\boldsymbol{s};\theta)$ and $\boldsymbol{F}(\boldsymbol{s};\theta)$ respectively.

The parameter θ is common knowledge only among agents. Without knowing the distribution parameter θ, the principal tries to obtain a *robust* central moment of the distribution $f(x;\theta)$ as the benchmark through the game of incomplete information. If the principal could have access to the full information, i.e., observe the agents' private signals $\boldsymbol{s} = (s_1, \ldots, s_N)$, she would choose a trimmed average estimator (Huber 2011)

$$L_0 \equiv \sum_{j=1}^{N} w_j s_j^{(N)}, \tag{1}$$

where $s_j^{(N)}$ is the jth order statistics of reports $\boldsymbol{s}$, for example $s_1^{(N)} = \min(\boldsymbol{s}), \ldots, s_N^{(N)} = \max(\boldsymbol{s})$, and the weight $w_j = \frac{1}{N-2n}$ for $n < j \leq N-n$, $w_j = 0$ for all other j, and $0 < n < N$.[10] Nevertheless, the principal cannot observe the realization of $\boldsymbol{s}$ unless through a disruptive investigation. As a result, the principal chooses to survey agents and calculate the index L as

$$L \equiv \sum_{j=1}^{N} w_j b_j^{(N)}, \tag{2}$$

under a similar notation for the jth order statistics of $\boldsymbol{b}$ as $b_j^{(N)}$. I define the expected difference between L and L_0 to be

$$\mathbb{E}[L - L_0] \equiv \int_S \left(\sum_{i=1}^{N} w_i b_i^{(N)} - \sum_{j=1}^{N} w_j s_j^{(N)} \right) \boldsymbol{f}(\boldsymbol{s};\theta)\, d\boldsymbol{s}, \tag{3}$$

as the *expected benchmark bias*.[11] Further, I define the agent i's *reporting error* as $b_i - s_i$.

[9] One of the example is the family of generalized uniform distribution for $x \in [0,1]$, which has the distribution function $f(x;\theta) = \theta x^{\theta-1}$ where $\theta > 1$.

[10] The choice of trimmed average instead of simple average is motivated by the fixing mechanism of LIBOR and ISDAfix. A trimmed average is less efficient to a simple average in terms of estimating the central moment. Nevertheless, the trimmed average is less sensitive to fat tails and outliers in the sample.

[11] Since my focus is the effect of strategic interaction between the princial and agent, I concentrate my analysis on the difference between L and L_0, rather than on the difference between L and the central moment $\int_{[\underline{s},\overline{s}]} x f(x;\theta) dx$. The difference between the robust average of private signal L_0 and the central moment is outside the scope of this study. More details can be found in the robust statistics literature such as Huber (2011).

2.2 Agent's problem

I focus on the case in which all agents prefer to move the index towards the same direction. Without loss of generality, agent i prefers a lower index with a marginal payoff $\delta_i \in [0, \bar{\delta}] \subset \mathbb{R}$. In LIBOR benchmarking, the empirical findings in Snider and Youle (2010), Kuo et al. (2012), and Youle (2014) suggest that banks are more likely to manipulate LIBOR downwards. First, both Grove (1974) and Samuelson (1945) suggest that a lower short-term rate benchmarked by LIBOR increases the net worth of financial institutions due to their maturity transformation function.[12] The second factor is the systemic risk: one bank's failure might undermine confidence in the financial system, and cause depositors to withdraw funds from otherwise unconnected banks (Morrison and White 2013).[13] Hence, although it is ideal to model arbitrary preference of agents, and banks may also manipulate LIBOR upwards if doing so benefits their derivative positions, assuming agents prefer a lower benchmark is an essential first step.

Agents face compliance risk when they manipulate the benchmark. The Commodity Futures Trading Commission (CFTC) decides that manipulation of LIBOR breach Sections 6(c), 6(d), and 9(a)(2) of the Commodity Exchange Act. The Financial Services Authority (FSA), the financial regulatory authority in the U.K., also requires financial institutions to ensure accurate reporting.[14] Regulators randomly audit the financial institutions, since it is prohibitively costly to investigate every single agent's report in each period.

Hence, I let each agent i might randomly be selected by the principal for an audit, upon which the agent's s_i, b_i, and δ_i are revealed. Hence, the agent faces an *expected* cost of strategic reporting $\gamma(\delta_i, s_i, b_i) \geq 0$ where the equality holds if and only if $b_i = s_i$.[15]

[12] Akella and Greenbaum (1992) and Lynge and Zumwalt (1980) show that a one basis point increase in the short-term interest rate leads to about 7.7 basis point drop in the bank stock return.

[13] The financial system faces a difficult choice between financial stability and market transparency in times of systemic banking crisis. Amid the 2008 financial crisis, managers at Barclays believed they were operating under instruction from the Bank of England to lower their LIBOR reports. Whether the Bank of England instructed Barclays—who consistently reported higher cost than others—to lower its submissions or not might never be known for sure. Nevertheless, the major banks had sufficient motive for setting a lower LIBOR, since a higher LIBOR rate would have made the Bank of England's task of saving the troubled banks, which include not only Barclays but also HBOS and RBS, more difficult. See Robert Peston, "What Did Bank of England Say to Barclays about Libor?" BBC News. 2012. Accessed December 12, 2013. `http://www.bbc.co.uk/news/business-18665080`.

[14] These requirements are derived from the FSA Handbook, which forms part of the FSA rules under the Financial Services and Markets Act 2000, in particular, principles 2, 3, and 5 of the Principles for Businesses: "A firm must conduct its business with due skill, care and diligence[it] must take reasonable care to organise and control its affairs responsibly and effectively, with adequate risk management systems,and [it] must observe proper standards of market conduct."

[15] The audit based penalty is not the only available mechanism. For example, Coulter and Shapiro (2014) proposes

The income and cost of benchmark manipulation result in the payoff function of the agent, $u_i(s_i, \boldsymbol{b}) : [\underline{s}, \overline{s}] \times S \to \mathbb{R}$ as

$$u_i(s_i, \boldsymbol{b}) \equiv -\gamma(\delta_i, s_i, b_i) - \delta_i \sum_{j=1}^{N} w_j b_j^{(N)}. \tag{4}$$

The payoff function can also be written as $u_i(s_i, b_i, b_{-i})$.

I define the objective function of agent i, $U_i : [\underline{s}, \overline{s}] \times [\underline{s}, \overline{s}] \to \mathbb{R}$, noting agent $j \neq i$'s reporting strategy as $\beta_j : [\underline{s}, \overline{s}] \to [\underline{s}, \overline{s}]$ with his private signal s_j, as

$$U_i(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) \equiv \int_{S_{-i}} u_i(s_i, b_i, \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) \boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}; \theta) \, d\boldsymbol{s}_{-i},$$

in which $\boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i}) \equiv (\ldots, \beta_{i-1}(s_{i-1}), \beta_{i+1}(s_{i+1}), \ldots)$, $\boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}; \theta)$ is the joint density function of $\boldsymbol{s}_{-i} \equiv (\ldots, s_{i-1}, s_{i+1}, \ldots)$, and S_{-i} is the support of $\boldsymbol{s}_{-i}$.

I now define the agents' pure strategy Bayesian Nash equilibrium (BNE) response.

Definition. *A pure strategy profile $\boldsymbol{\beta}(\boldsymbol{s}) = (\beta_1(s_1), \ldots, \beta_N(s_N))$ is a BNE if, for each agent i, $\beta_i(s_i)$ is the agent's best response such that $U_i(s_i, \beta_i(s_i); \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) \geq U_i(s_i, b_i'; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i}))$ for any other response b_i'.*

2.3 Principal's problem

The principal's primary objective is to induce truthful reporting from all agents, regardless their reports are used in the benchmark calculation or not. An agent who misreports a lower-than-actual quote that falls into the lowest n out of N quotes still rigs the benchmark downwards, because he pushes the nth lowest quote among the rest $N-1$ quotes, which otherwise should have been trimmed, into the benchmark calculation. In the CFTC and FSA codes, as well as in the LIBOR and ISDAfix investigation, a bank would not be acquitted merely due to his misreported quote fell into the top or bottom quartile. Hence, I let the principal find the optimal γ to minimize the squared reporting error $\int_{\underline{s}}^{\overline{s}} (\beta_i(s_i) - s_i)^2 f(s_i; \theta) \, ds_i$ for all i.

The principal also faces constraints on the γ. First, she can only audit the agents infrequently,

a sequential mechanism which leads to truthful reporting. However, the focus of this paper is to improve the current survey-based mechanism without substantially change the business process. Hence, I limit the principal's action on how to formulate a more effective penalty function.

since auditing disrupts normal business operations. Second, condition on a detection of a misreporting agent in an audit, the principal faces legal constraints on the amount of punishment she can impose. For instance, she can only punish the revealed misreporting incident, and the penalty may not exceed a finite multiple of the damage caused by the incident. Could the principal impose an unlimited penalty $\gamma(s_i, b_i) = +\infty$ for all $b_i \neq s_i$, the agent's equilibrium report $\beta_i(s_i) = s_i$, which leads to the equilibrium expected penalty for all agents $\int_{\underline{s}}^{\overline{s}} \gamma\left(\delta_i, s_i, \beta_i\left(s_i\right)\right) f\left(s_i; \theta\right) ds_i = 0$. Therefore, the practical limits on γ lead to a strictly positive *lower* bound on the equilibrium expected penalty for all agents.[16]

Nor can the principal set the equilibrium expected penalty for all agents $\int_{\underline{s}}^{\overline{s}} \gamma\left(\delta_i, s_i, \beta_i\left(s_i\right)\right) f\left(s_i; \theta\right) ds_i$ too high, or the participating agents may choose not to join the survey at all. In the specific case of LIBOR and ISDAfix, the panel banks enjoy the prestige since only the most credible and influential financial institutions are invited to the panel. However, if the compliance risk is too high, the panel may not be able to attract enough participating banks. Together with the existence of the lower bound, the principal faces an equality constraint on the equilibrium expected penalty, i.e. $\int_{\underline{s}}^{\overline{s}} \gamma\left(\delta_i, s_i, \beta_i\left(s_i\right)\right) f\left(s_i; \theta\right) ds_i = \delta_i K$.

I also assume $\gamma(s_i, b_i)$ to be a bounded, continuous, and concave C^2 function such that $\partial^2\gamma(\delta_i, s_i, b_i)/\partial b_i^2 > 0$. In addition, the penalty increases with the report when the agent sends in a number higher than the truth, that is, $\partial\gamma(\delta_i, s_i, b_i)/\partial b_i \geq 0$ if and only if $b_i > s_i$, and decreases with the report otherwise.

Finally, since the principal does not know the distribution parameter θ, the optimal penalty γ cannot be θ dependent. It is also desirable to have the expected benchmark error $\mathbb{E}\left[L - L_0\right] = \int_S \left(\sum_{i=1}^N w_i \beta_i(s_i)^{(N)} - \sum_{j=1}^N w_j s_j^{(N)}\right) \boldsymbol{f}\left(\boldsymbol{s}; \theta\right) d\boldsymbol{s}$, to be θ-independent.

I summarize the principal's problem as to find the equilibrium penalty function $\gamma\left(\delta_i, s_i, \beta_i\left(s_i\right)\right)$:

Definition. *The principal's equilibrium penalty function $\gamma\left(\delta_i, s_i, \beta_i\left(s_i\right)\right)$ leads to an θ-independent expected benchmark bias $\mathbb{E}\left[L - L_0\right]$ and minimizes the squared reporting error*

$$\inf_{\gamma} \int_{\underline{s}}^{\overline{s}} \left(\beta_i\left(s_i\right) - s_i\right)^2 f\left(s_i; \theta\right) ds_i \ \textit{subject to} \ \int_{\underline{s}}^{\overline{s}} \gamma\left(\delta_i, s_i, \beta_i\left(s_i\right)\right) f\left(s_i; \theta\right) ds_i = \delta_i K, \tag{5}$$

for all i.

[16] Since it is the expected penalty that drives the equilibrium result, I choose not to model the audit probability and the magnitude of *ex post* penalty separately.

3 Equilibrium analysis

3.1 Existence and monotonicity of equilibrium reporting strategy under γ

I first show the existence and monotonicity of the agents' reporting strategy.

Proposition 1. *A non-decreasing BNE $\boldsymbol{\beta}(\boldsymbol{s})$ exists.*

Proof. See Appendix. $\square$

Proposition 1 shows that the agents have a weakly increasing equilibrium reporting strategy, which makes the subsequent equilibrium analysis easier. The non-decreasing equilibrium strategy also suggests that the order among signals is the same as the order among equilibrium bids. Therefore, the expected benchmark bias becomes

$$\mathbb{E}\left[L - L_0\right] = \int_S \sum_{j=1}^{N} w_j \left(\beta_j \left(s_j^{(N)}\right) - s_j^{(N)}\right) \boldsymbol{f}\left(\boldsymbol{s}\right) d\boldsymbol{s}. \tag{6}$$

I present a few sample equilibrium best responses below to illustrate the economic intuition before I proceed to further analysis.

Example. *Consider three surveys, all of which have agents $i = 1, 2, 3$ with independent and identically distributed (i.i.d.) private signals $s_i \in [0, 1]$. Under $\theta = \{\theta_1, \theta_2\}$, the private signal distribution is $f\left(s_i; \theta\right) = 1 + \theta_1(s_i - \frac{1}{2}) + \theta_2 \frac{1}{2}\pi \cos\left(\frac{\pi s_i}{2} - 1\right)$. Let $\theta_1 = \theta_2 = 0$ in the first survey and $\theta_1 = 2, \theta_2 = 0$ in the second: the private signal follows uniform distribution $f_i(s_i) = 1$ and triangle distribution $f_i(s_i) = 2s_i$, respectively. In the third survey, $\theta_1 = 0, \theta_2 = 1$ so s_i has a density function $f_i(s_i) = \frac{1}{2}\pi \cos\left(\frac{\pi s_i}{2}\right)$. The index is calculated as the median of the reports $b = (b_1, b_2, b_3)$, so $L = b_2^{(3)}$. The objective function for agent i is $U_i = \int_{S_{-i}} (-(s_i - b_i)^2 - L)\boldsymbol{f}(\boldsymbol{s}_{-i})d\boldsymbol{s}_{-i} = -(s_i - b_i)^2 - \int_{S_{-i}} b_2^{(3)} \boldsymbol{f}(\boldsymbol{s}_{-i})d\boldsymbol{s}_{-i}$.*

From Proposition 1, agent i has an increasing pure strategy equilibrium: this means agent i's report b_i becomes the median (and therefore, $b_i = L$) if and only if agent i's signal $\min(\boldsymbol{s}) \leq s_i \leq \max(\boldsymbol{s})$*, which happens with probability $1 - s_i^2 - (1 - s_i)^2 = 2s_i(1 - s_i)$ in the first survey. Hence the first order condition suggests $0 = \frac{\partial}{\partial b_i} U_i = \frac{\partial}{\partial b_i}(-(s_i - b_i)^2 - 2s_i(1 - s_i)b_i)$, and gives the strategy for agents in the first survey as $\beta(s_i) = s_i^2$. Similarly, the equilibrium strategy for the second survey is $\beta(s_i) = s_i\left(s_i^3 - s_i + 1\right)$, and for the third survey $\beta(s_i) = s_i - \left(1 - \sin\left(\frac{\pi s_i}{2}\right)\right) \sin\left(\frac{\pi s_i}{2}\right)$.*

It is intuitive to observe why $\beta_i(s_i) \leq s_i$ under not only all three distributions above but also more general cases. Since the marginal cost of lying is zero at the true borrowing cost value, truth-telling cannot be an equilibrium. Indeed, consider a small downward deviation: the marginal benefit from reporting a lower number is strictly positive at least for some realization of the signal. Therefore, equilibrium strategies involve some misreporting, and given that reporting a cost strictly above the true value is a strictly dominated strategy, any distortion has to be towards lower values.

The numerical example suggests that an agent's reporting error $\beta_i(s_i) - s_i$ depends on a few factors: the parameters of the panel N and n, the penalty function γ, and $\boldsymbol{f}(\boldsymbol{s};\theta)$–the distribution function of the signals. Among these factors, the parameter θ of the density function of $\boldsymbol{f}(\boldsymbol{s};\theta)$ is unknown to the principal. Hence, the principal cannot implement a truth-telling mechanism using the revelation principle under the survey mechanism.[17]

3.2 θ-independent benchmark bias

As the numerical example shows, the unknown θ prevents the principal from implementing a direct mechanism. Hence, from the principal's viewpoint, it is important to design a survey in which the expected benchmark bias $\mathbb{E}[L - L_0]$ does not depend on the signal distribution, therefore much easier to correct.

Proposition 2. *If the expected penalty function $\gamma(\delta_i, s_i, b_i)$ takes the form $\delta_i \hat{\gamma}(b_i - s_i)$, and the second order derivative of $\hat{\gamma}$ satisfies*

$$\hat{\gamma}'' > \frac{1}{N-2n} \frac{(N-1)!}{(n-1)!\,(N-n-1)!} \frac{\left[(1-F(s_i;\theta))^{N-2n} - F(s_i;\theta)^{N-2n}\right]}{\left[F(s_i;\theta)(1-F(s_i;\theta))\right]^{1-n}} f(s_i;\theta) \tag{7}$$

for all $s_i \in [\underline{s}, \overline{s}]$, then the equilibrium report β is symmetric, strictly increasing, and the expected benchmark bias depends only on the panel size N, the trimming window n, and the penalty function γ, but not on the parameter θ of the private signal distribution.

Proof. See Appendix. □

[17] The revelation principle suggests that, with a transfer between the agents and the regulators, we can eliminate the bias induced by the equilibrium reporting strategy. Unfortunately, this usually requires knowledge about the distribution of private signals. Also, although the regulator can and does impose penalties on the misreporting banks, a daily transfer between the regulator and agents is difficult to implement in reality.

Proposition 2 shows that if the penalty, scaled down by the agent's sensitivity δ_i, is a function of the reporting error $b_i - s_i$ but not s_i and b_i, then the penalty leads to a θ-independent expected benchmark bias $\mathbb{E}\left[L - L_0\right]$. The expected benchmark bias only depends only on the panel size N, the trimming window n, and the penalty function γ, all fixed to the principal.

Since adding a fixed number to the benchmark in the payoff function does not change an agent's optimal strategy β, the principal can induce an unbiased expected benchmark more easily. Notice in the previous example, the objective function for agent i is $U_i = \int_{S_{-i}} (-(s_i - b_i)^2 - L)\boldsymbol{f}(\boldsymbol{s}_{-i})d\boldsymbol{s}_{-i} = -(s_i - b_i)^2 - \int_{S_{-i}} b_2^{(3)}\boldsymbol{f}(\boldsymbol{s}_{-i})d\boldsymbol{s}_{-i}$, which implies the penalty function is $(s_i - b_i)^2$. I then verify such penalty function indeed leads to θ-independent benchmark bias.

Example. *In this example, I verify Proposition 2, which suggests that all three equilibria with different signal distributions in the example in Section 2 should lead to the same expected benchmark bias. In the first survey with uniform distribution, the density of the median is* $f_2^{(3)}(s_i) = \frac{1}{6}s_i(1 - s_i)$ *and the expected benchmark bias* $\mathbb{E}\left[b_2^{(3)} - s_2^{(3)}\right]$ *equals to* $\int_0^1 (\beta(s_i) - s_i) f_2^{(3)}(s_i) = \int_0^1 \left(s_i^2 - s_i\right)\frac{1}{6}s_i(1 - s_i)\,ds_i = -\frac{1}{5}$. *Under the second survey with triangle distribution, in which* $f_2^{(3)}(s_i) = \frac{1}{3}s_i^2\left(1 - s_i^2\right)s_i$, *and the equilibrium report as* $s_i\left(1 - s_i + s_i^3\right)$, *we can verify that the expected benchmark bias is also* $-\frac{1}{5}$. *Finally, when* $s_i \in [0,1]$ *has a density function* $f_i(s_i) = \frac{1}{2}\pi\cos\left(\frac{\pi s_i}{2}\right)$, $f_2^{(3)}(s_i) = 3\pi\cos\left(\frac{\pi s_i}{2}\right)\left(1 - \sin\left(\frac{\pi s_i}{2}\right)\right)\sin\left(\frac{\pi s_i}{2}\right)$. *With the equilibrium report as* $s_i - \left(1 - \sin\left(\frac{\pi s_i}{2}\right)\right)\sin\left(\frac{\pi s_i}{2}\right)$, *the expected benchmark bias is again* $-\frac{1}{5}$. *Therefore, the principal can restore the proper level of index in expectations by adding* $\frac{1}{5}$ *to the index without worrying about whether the distribution is uniform, triangular, or any other absolutely continuous distribution.*

Finally, Proposition 2 suggests that under $\gamma(\delta_i, s_i, b_i) = \delta_i\hat{\gamma}(b_i - s_i)$, the agents follow a symmetric equilibrium strategy. Hence, I drop the i subscript and use β to note the equilibrium report of agents in the following analysis.

3.3 Equilibrium penalty $\gamma(\delta_i, s_i, \beta(s_i))$

Based on the functional form suggested in Proposition 2, I present the equilibrium penalty function that minimizes the agents' reporting error in Proposition 3.

Proposition 3. *To minimize the agents' reporting error, the principal follows the equlibrium ex-*

pected penalty function $\gamma(\delta_i, s_i, \beta(s_i)) = \delta_i \hat{\gamma}(\beta(s_i) - s_i) = \delta_i \zeta(\beta(s_i) - s_i)^2$, *where*

$$\zeta = \frac{1}{2K} \int_0^{\frac{1}{2}} \left[\frac{1}{N-2n} \frac{(N-1)!}{(n-1)!\,(N-n-1)!} \int_0^x \frac{\left[(1-y)^{N-2n} - y^{N-2n}\right]}{\left[y\,(1-y)\right]^{1-n}} dy \right]^2 dx, \tag{8}$$

assuming the ζ *satisfies the strictly increasing equilibrium condition in Equation (7).*

Proof. See Appendix. □

Proposition 3 suggests that, under a constraint on how harsh the penalty can be, a quadratic expected penalty function minimizes the reporting error. The equilibrium quadratic penalty function also leads to a closed-form equilibrium report β among agents, as Proposition 4 shows.

Proposition 4. *Under the equilibrium penalty function* $\gamma(\delta_i, s_i, \beta(s_i)) = \delta_i \hat{\gamma}(\beta(s_i) - s_i) = \delta_i \zeta(\beta(s_i) - s_i)^2$, *and if* ζ *satisfies the condition*

$$\zeta > \zeta^*(s_i) = \frac{1}{2(N-2n)} \frac{(N-1)!}{(n-1)!\,(N-n-1)!} \frac{\left[(1-F(s_i;\theta))^{N-2n} - F(s_i;\theta)^{N-2n}\right]}{\left[F(s_i;\theta)\,(1-F(s_i;\theta))\right]^{1-n}} f(s_i;\theta) \tag{9}$$

for all $s_i \in (\underline{s}, \overline{s})$, *then agent* i *has a strictly increasing BNE report* $\beta : [\underline{s}, \overline{s}] \to [\underline{s}, \overline{s}]$ *such that*

$$\beta(s_i) = s_i - \frac{1}{2\zeta(N-2n)} \Delta(s_i;\theta)$$

where $\Delta(s_i;\theta) = \frac{(N-1)!}{(n-1)!(N-n-1)!} \int_0^{F(s_i;\theta)} \frac{\left[(1-y)^{N-2n} - y^{N-2n}\right]}{\left[y(1-y)\right]^{1-n}} dy$.

Intuitively, $\Delta(s_i)$ is the probability that agent i's equilibrium submission is used in the index calculation, where $s_i - \frac{1}{2\zeta(N-2n)}$ characterizes the "first-best" report of the agent if he knows his submission will be used. Second, $F(s_i;\theta) \in [0,1]$ suggests that the integral $\int_0^{F(s_i;\theta)} \frac{(1-y)^{N-2n} - y^{N-2n}}{\left[y(1-y)\right]^{1-n}} dy$ is non-negative; hence, in equilibrium, an agent always reports $\beta(s_i) \leq s_i$. Finally, when an agent has the lowest possible borrowing cost, he reports $\beta(\underline{s}) = \underline{s}$. Figure 1 plots the strictly increasing equilibrium.

Next, I evaluate how regulation shapes an agent's submission strategy.

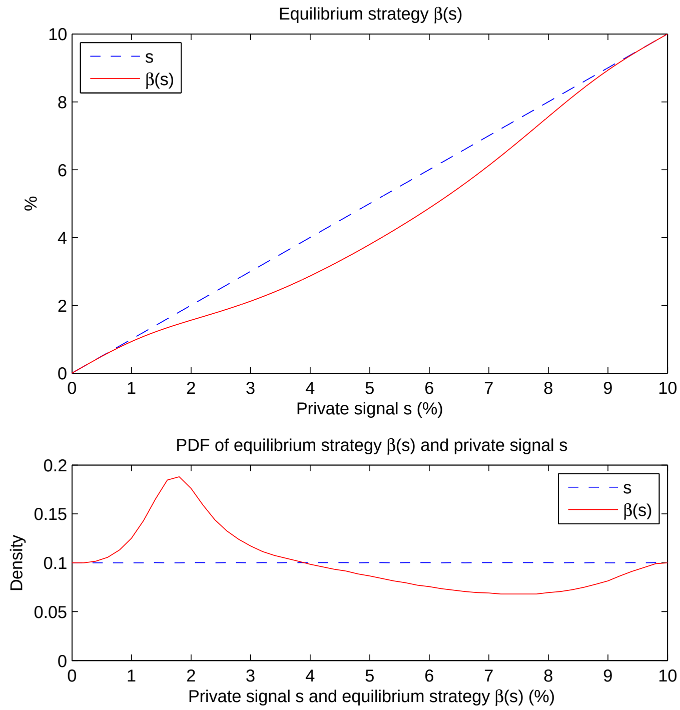

Figure 1: **Equilibrium strategy with non-binding monotonicity constraint.** This figure shows the agent's equilibrium submission strategy β when the panel contains $N = 16$ agents with i.i.d. types uniformly distributed on $[0\%, 10\%]$. The lowest and highest $n = 4$ bids are trimmed. The penalty is $\gamma(\delta_i, s_i, b_i) = \delta_i \zeta (s_i - b_i)^2$ where $\zeta = 0.05$ and the interest rate sensitivity δ_i is normalized to $\delta_i = 1$ for all i. The upper subplot shows the equilibrium strategy β, and the lower subplot shows the distribution of signals and of equilibrium submissions. The optimal strategy causes the submissions of agents to cluster around the lower half of the support.

3.4 Comparative Statics

Since the LIBOR manipulation scandal unfolded, there has been much discussion about possible options for strengthening the benchmark fixing mechanism. For instance, Wheatley (2012) considers three options: increase the number of participating agents, increase the number of trimmed reports n, or calculate the benchmark by taking the average of a few randomly selected submissions from the central quartiles. In this subsection, I first discuss the comparative statistics of the equilibrium result in Proposition 4, which comment on the options proposed in Wheatley (2012).

Figure 2 addresses the first and the second options under a survey in which the support of private signal is $[0\%, 10\%]$, the penalty function as $\gamma(\delta_i, s_i, b_i) = \delta_i \zeta (s_i - b_i)^2$ in which δ_i normalized to 1 and $\zeta = 0.05$. The figure shows the expected benchmark bias under different panel sizes N

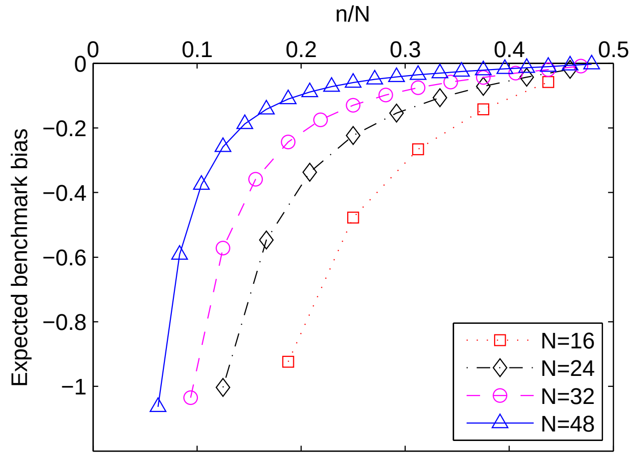

Figure 2: **Expected LIBOR bias.** This figure shows the expected LIBOR bias with N agents, each with $s_i \in [0\%, 10\%]$. The lowest and highest n bids are trimmed. The penalty is $\gamma(\delta_i, s_i, b_i) = \delta_i \zeta (s_i - b_i)^2$ where $\zeta = 0.05$ and the interest rate sensitivity δ_i is normalized to $\delta_i = 1$ for all i. The Y-axis is the expected LIBOR bias in percentages. The X-axis is the proportion of trimmed bids. For example, the point at $n/N = 0.25$ on the $N = 16$ curve shows the expected LIBOR bias when $n = 4$.

from 16 to 48 and different fractions of trimmed bids n/N. When the fraction of bids trimmed remains unchanged, a larger panel (higher N) leads to a lower expected bias. Is this property true under all possible combination of parameters? Proposition 5 confirms this conjecture.

Proposition 5. *Increasing both the size of the panel N and the number of trimmed reports, and keeping the proportion n/N the same, reduces the reporting error. In other words, note the equilibrium strategy under panel size N and trimmed quotes n as $\beta_{N,n}(s_i)$, for positive rational number m such that mn and mN are integers, $\beta_{N,n}(s_i) \leq \beta_{mN,mn}(s_i)$.*

Proposition 6 shows that given the size of the panel N, the magnitude of expected benchmark bias decreases when n increases.

Proposition 6. *Increasing the size of panel N and the number of trimmed reports, and keeping the included reports to be the same, reduces the reporting error, so that $\beta_{N,n}(s_i) \leq \beta_{N+2m,n+m}(s_i)$ for positive rational number m.*

The last option, taking the average of randomly selected submissions from the central quartiles, might not work at all. Suppose that each agent in the central quartile has a probability q of being selected. This option reduces each agent's probability of changing the index by a factor of q. On the other hand, if agent i is selected in the averaging, his submission now has $1/q$ times more weight. Therefore, the agent's objective function $U_i\left(s_i, \boldsymbol{b}\right)$ is unchanged, as is the equilibrium.

4 Lenient penalty

The θ-independence property described in Proposition 2 assumes each agent faces a high enough compliance risk such that his equilibrium report $\beta\left(s_i\right)$ is strictly increasing in s_i. Nevertheless, there might be practical limitation such that the principal cannot impose such a harsh penalty, either due to the limit on the frequency of audits or on the maximum fine she can impose. What happens when the compliance risk is not high enough? Further, although the principal can obtain a θ-independent benchmark and minimize reporting errors condition on a strictly increasing equilibrium, the condition *per se* is still θ-dependent. Hence, it is essential to study the equilibrium in which the $\zeta \leq \zeta^*\left(s_i\right)$ for some intervals of s_i—ζ is too low to induce a strictly increasing β for all s_i—and whether the equilibrium can remind the principal that her penalty is lenient.

Proposition 7. *If $\zeta \leq \zeta^*\left(s_i\right)$ for some intervals of s_i with positive measure, agent i BNE strategy β is characterized by the region $[\underline{\sigma}, \bar{\sigma}]$ in which the strategy is "bunching" and the region $[\underline{s}_i, \bar{s}_i] \setminus [\underline{\sigma}, \bar{\sigma}]$ in which the strategy is strictly increasing. The BNE strategy of agent i with $s_i \notin [\underline{\sigma}, \bar{\sigma}]$ is the same as Proposition 4, i.e., $\beta\left(s_i\right) = s_i - \frac{1}{2\zeta(N-2n)}\Delta\left(s_i;\theta\right)$ with $\Delta\left(s_i;\theta\right) = \frac{(N-1)!}{(n-1)!(N-n-1)!}\int_0^{F(s_i;\theta)} \frac{\left[(1-y)^{N-2n}-y^{N-2n}\right]}{[y(1-y)]^{1-n}}dy$. The BNE strategy for agents with $s_i \in [\underline{\sigma}, \bar{\sigma}]$ is*

$$\bar{b} = \frac{\int_{\underline{\sigma}}^{\bar{\sigma}} s_i f(s_i;\theta) ds_i}{\int_{\underline{\sigma}}^{\bar{\sigma}} f(s_i;\theta) ds_i} - \frac{\bar{\Delta}}{2\zeta\left(N-2n\right)}, \tag{10}$$

with $\bar{\Delta} = \frac{(N-1)!}{(n-1)!(N-n-1)!}\left[\int_0^{F(\underline{\sigma};\theta)} \frac{(1-y)^{N-n-1}}{y^{1-n}}dy - \int_0^{F(\bar{\sigma};\theta)} \frac{y^{N-n-1}}{(1-y)^{1-n}}dy\right]$. Finally, $\underline{\sigma} = \min\beta^{-1}\left(\bar{b}\right)$, and $\bar{\sigma} = \max\beta^{-1}\left(\bar{b}\right)$.

Proposition 7 shows when the penalty is not harsh enough, the agent's response becomes "cheap talk" such that β does not vary with the private signal for some s_i. Such equilibrium creates an alert to the principal that if she notices the agents' equilibrium reports cluster together, her penalty is not imposing enough compliance risks to strategic agents. The expected benchmark bias is no longer distribution free when the equilibrium message is "bunching," as the following example shows.

Example 8. *Now consider two surveys, both of which have agents $i = 1, 2, 3$ with independent and identically distributed (i.i.d.) private signals $s_i \in [0, 1]$. The index is calculated as the median of the reports $b = (b_1, b_2, b_3)$, so $L = b_2^{(3)}$. The private signal follows uniform distribution $f_i(s_i) = 1$ in the first survey, and trignomial distribution $f_i(s_i) = 2 - 2s_i$ in the second. The objective function is $U_i = \int_{[\underline{s},\bar{s}]_{-i}} (-(s_i - b_i)^2/2 - L)\boldsymbol{f}(\boldsymbol{s}_{-i})d\boldsymbol{s}_{-i} = -(s_i - b_i)^2/2 - \int_{[\underline{s},\bar{s}]_{-i}} b_2^{(3)}\boldsymbol{f}(\boldsymbol{s}_{-i})d\boldsymbol{s}_{-i}$: so the expected penalty is more lenient. It is easy to calculate that $\zeta = \frac{1}{2} \leq \zeta^*(s_i) = 1 - 2s_i$ for all $s_i \in [0, \frac{1}{4}]$ in the first survey, and $\zeta = \frac{1}{2} \leq \zeta^*(s_i) = 2 - 10s_i + 12s_i^2 - 4s_i^3$ for all $s_i \in \left[0, 1 - \frac{\sqrt[3]{9-\sqrt{57}}}{2 \cdot 3^{2/3}} - \frac{1}{\sqrt[3]{3(9-\sqrt{57})}}\right]$ in the second.*

It is easy to see the equilibrium for the first survey is $\beta(s_i) = \bar{b} = 0$ for $s_i \in [0, \frac{1}{2}]$, and $\beta(s_i) = 2s_i^2 - s_i$ for $s_i \in (\frac{1}{2}, 1]$. As a result, the expected benchmark bias is $\frac{57}{160}$. For the second survey, $\beta(s_i) = \bar{b} = 0$ for $s_i \in \left[0, \frac{1}{3}\left(4 - \frac{\sqrt[3]{23-3\sqrt{57}}}{2^{2/3}} - \frac{2^{2/3}}{\sqrt[3]{23-3\sqrt{57}}}\right)\right]$, and $\beta(s_i) = 2s_i^4 - 8s_i^3 + 10s_i^2 - 3s_i$ for $s_i \in \left(\frac{1}{3}\left(4 - \frac{\sqrt[3]{23-3\sqrt{57}}}{2^{2/3}} - \frac{2^{2/3}}{\sqrt[3]{23-3\sqrt{57}}}\right), 1\right]$. Hence the expected benchmark bias is around 0.256078, which is different from that of the first survey as well as the $\frac{1}{5}$ as in the previous examples. Therefore, the expected benchmark bias is distribution-dependent under lenient penalty with $\zeta < \zeta^*(s_i)$ for some (s_i).

5 Collusion

Does the distribution-free property still hold when agents collude? I focus on the scenario that only a subset of agents forms a collusion ring since the trimming mechanism suggests that it is not necessary to have a ring with more than $N - n$ agents. The agent with the weakly highest sensitivity to the benchmark acts as the leader and calls upon other $m-1$ agents to collude. Figure 3 illustrates the survey with colluding agents. Without loss of generality, I allow the agent 1 to be the leader. The rest of the colluding agents participate in the collusion ring in the hope that they will be able to become the leader later when their sensitivity to the benchmark becomes higher.[18]

[18] As shown in the article "What the UBS Libor emails said", BBC, December 19, 2012. Accessed January 20, 2013. `http://www.bbc.com/news/business-20781763`. The brokerage arm of a panel bank, driven by some major trading clients, persuades the LIBOR submission personnel in other banks to submit fabricated numbers jointly. Favors are returned over time among the brokers.

> Trader:Alright, well make sure he [the UBS trader] knows
> Broker: Yeah, he will know mate. Definitely, definitely, definitely

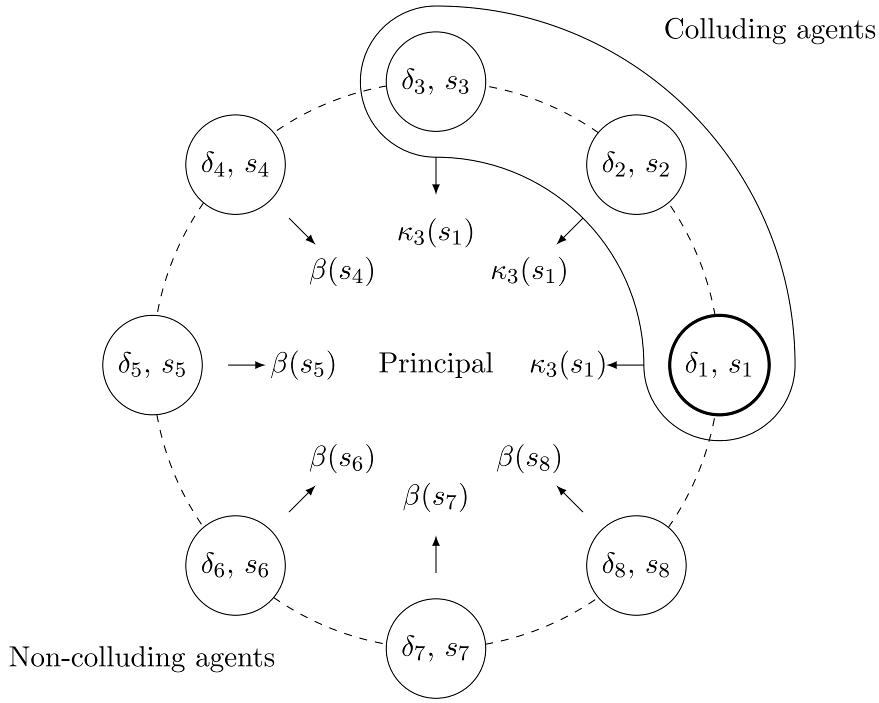

Figure 3: **A sample survey panel with colluding agents.** This figure shows a sample benchmark survey panel with $N = 8$ agents each with private information s_i. Among them, agent 1, 2, and 3 are colluding, which means that $m = 3$. The agent 1, with a higher incentive $\delta_1 > \max(\delta_2, \delta_3)$, acts as the leader among the colluding agents and decides the submission $\kappa_3(s_1)$ for all three colluding agents. The rest of the panel, unaware of the collusion, submit their individual strategic bids $\beta(s_i)$.

Therefore, I limit my study to cases without side payments among colluding banks.

Without side payments among the colluding agents, the effective collusion strategy is to have all participants submit the same bids, as in McAfee and McMillan (1992). There is no credible way for a colluding agent to signal his private type: each agent has the incentive to claim that he suffers severely in order to join the collusion, hence demanding a large side payment. I allow the leader to choose a uniform submission κ for all m colluding agents within the ring, while κ maximizes the leader's utility given his type s_i. Thus, the agents' reports are $\boldsymbol{b} = \{\kappa_m(s_1), \ldots, \kappa_m(s_1), b_{m+1}, \ldots, b_N\}$. The payoff of the leading agent 1 and colluding banks $i \in$

> Submitter: You know, scratch my back yeah an all
> Broker: Yeah oh definitely, yeah, play the rules.

One trader told other parties:

> "Anytime i can return the favour let me know ...".

Although there is evidence that the external trader paid back his broker, there was little evidence showing that the trader has paid back all the brokers that participated in the collusion.

$\{2, \ldots, m\}$ are

$$\begin{aligned} u_1(s_1, \boldsymbol{b}) &\equiv -\zeta\delta_1 (s_1 - \kappa_m(s_1))^2 - \delta_1 \sum_{j=1}^{N} w_j b_j^{(N)} \\ u_i(s_i, \boldsymbol{b}) &\equiv -\zeta\delta_i (s_i - \kappa_m(s_1))^2 - \delta_i \sum_{j=1}^{N} w_j b_j^{(N)}, \end{aligned}$$

respectively. The rest of the agents $j \in \{m+1, \ldots, N\}$ are not aware of the collusion.

Proposition 9. *If the equilibrium strategy $\beta(\cdot)$ for each non-colluding agent is strictly increasing, the leading agent decides the equilibrium colluding submission $\kappa_m(\cdot)$ for all the m colluding agents, which satisfies*

$$\kappa_m(s_i) = s_i - \frac{\Delta(\kappa_m(s_i); \theta)}{2\zeta (N - 2n)}$$

where

$$\Delta(\kappa_m(s_i); \theta) = \sum_{j=1}^{m} \sum_{r=n-m+j}^{(N-m)-(n-m)-j} \binom{N-m}{r} F^r \left(\beta^{-1}(\kappa_m(s_i)); \theta\right) \left[1 - F\left(\beta^{-1}(\kappa_m(s_i)); \theta\right)\right]^{N-m-r}.$$

In the case when $m = 1$, the equilibrium degenerates into the case with no collusion, so $\kappa_1(s_i) = \beta(s_i)$.

Proof. See Appendix. □

Figure 4 shows the collusive equilibrium strategy κ_m. It shows that the leading agent manipulates the benchmark more aggressively when more agents join the collusion. This is because the leading agent can manipulate the index more effectively since his efforts are helped by other colluding banks.

Collusion makes correcting the benchmark more difficult, since the magnitude of expected benchmark bias is no longer distribution-free, as the following proposition shows.

Proposition 10. *The equilibrium strategy $\kappa_m(\cdot)$ characterized in Proposition 9 suggests that the excess benchmark bias introduced by collusion does not satisfy the distribution-free property in Proposition 2.*

Proof. See Appendix. □

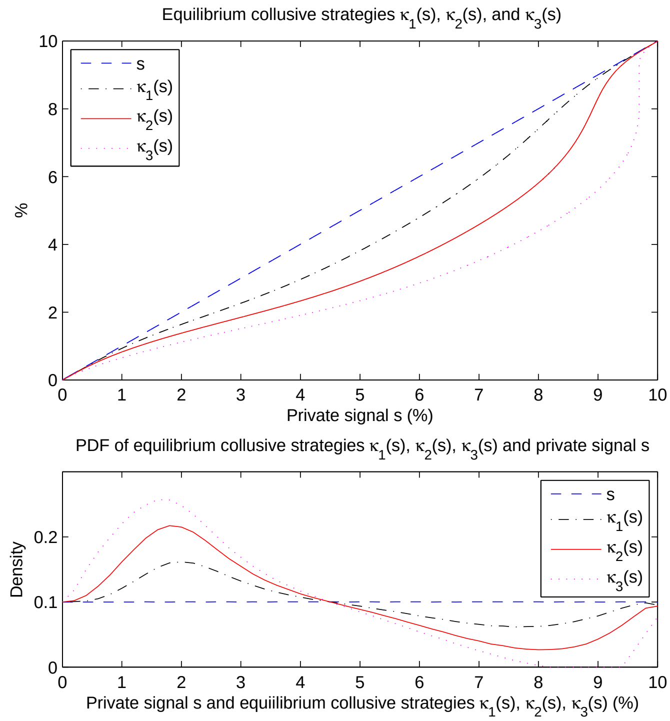

Figure 4: **Equilibrium collusive strategy under various m.** This figure shows the equilibrium strategy of the leading agent when $m = 1$, 2, or 3 agents colluding, which are $\kappa_1(s)$, $\kappa_2(s)$, and $\kappa_3(s)$. The survey contains $N = 16$ agents with i.i.d. private types uniformly distributed on $[0\%, 10\%]$, and the lowest and highest $n = 4$ bids are trimmed. The penalty is $\gamma(\delta_i, s_i, b_i) = \delta_i \zeta (s_i - b_i)^2$ where $\zeta = 0.1$ and the interest rate sensitivity δ_i is normalized to $\delta_i = 1$ for all i. The upper subplot shows the equilibrium strategy β, and the lower subplot shows the distribution of signals and of equilibrium submissions. The optimal strategy causes the submissions of agents to cluster around the lower half of the support. The figure shows that when more agents are involved in the collusion, the leading agent submits a more aggressive strategic bid.

The collusion can be blocked by not releasing the individual agent's report: when the leader cannot verify each agent's submission, it is not individually rational for the non-leading agents in the ring to follow the leader and submit $\kappa \neq \beta(s_i)$. Nevertheless, the principal may want to release the individual agents' submissions, especially when agents are major players in the market. As a compromise, the principal may choose to release the agents' submissions with a delay. For instance, in July 2013, following the recommendations set out in Wheatley (2012), the British Bankers' Association decided that the publication of individual banks' LIBOR submissions would be embargoed for 3 months. Although this will make collusion more difficult, the leader will still be able to verify 3 months later whether other colluding agents complied.

Instead of discussing collusive bidding in a repeated game setting, I focus on a static setting assuming the non-leading agents in the bidding ring comply with the lead bank's request. The intuition remains the same under a repeated game setting: when agents are colluding, the leader manipulates LIBOR more aggressively, causing a more substantial benchmark bias and a distribution dependent expected benchmark bias.

6 Conclusion

I study survey as a principal-agent problem when agents respond strategically, and their private signal distribution is common knowledge only among agents but not to the principal. The principal, whose objective is to minimize agents' reporting error, randomly audit the agents and impose penalty once she discovers misreporting. Such a game depicts a widely adopted method of benchmarking a group of agents' private information of interest. In particular, the benchmark is the trimmed average of solicited reports, as in many survey-based benchmarks including LIBOR and ISDAfix.

Agents' equilibrium reporting strategy exists and depends on the signal distribution. Further, the principal minimizes the agents' reporting error when the penalty function is proportional to the agent's sensitivity to benchmark and quadratic to the agent's reporting error. Such a penalty function also leads to a distribution-free expected index bias. Hence, the change in the unadjusted index is an unbiased estimator of the change in the average private signals. Finally, the principal can restore the impartiality in the benchmark without the knowledge of private signal distribution.

Comparative statics suggest that expanding the size of the panel as well as the number of the trimming quotes would reduce the reporting error. Using a smaller range for benchmark calculation also helps. Collusion increases the bias and eliminates the distribution-free property of expected index bias, though a recent LIBOR mechanism reform has largely fixed this problem by embargoing individual agents' reports.

Future research should focus on more general cases such as the agents have the incentive to rig benchmark towards different directions.

A Appendix

A.1 Proofs for the Propositions and Lemmas

This section contains all the proofs of Propositions. For notational ease, I drop the θ in the distribution function $f(\cdot;\theta)$ and $F(\cdot;\theta)$ when doing so does not cause ambiguity.

Proof of the existence of equilibrium Before proving the Propositions, I first define the single crossing property of incremental returns of an objective function and the single crossing condition for games of incomplete information, as in Athey (2001) and Milgrom and Shannon (1994).

Definition. *(Athey (2001); Milgrom and Shannon (1994)) The objective function $U_i(s_i, b_i)$ satisfies the single crossing property of incremental returns in (s_i, b_i) if, for all $s_i < s_i'$ and $b_i < b_i'$, $U_i(s_i, b_i') - U_i(s_i, b_i) \geq (>) 0$ implies $U_i(s_i', b_i') - U_i(s_i', b_i) \geq (>) 0$.*

Definition. *(Athey (2001)) The single crossing condition for games of incomplete information is satisfied if for each $i = 1, \ldots, N$, whenever every opponent $j \neq i$ uses a nondecreasing strategy $\beta_j(s_j)$, agent i's objective function $U_i(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i}))$ satisfies single crossing properties of incremental returns in (s_i, b_i).*

Before I prove the existence of equilibrium, I first verify that the objective function of a bank does exist and has bounded value in Lemma A.1.

Lemma A.1. *$\int_\Omega u_i(s_i, b_i, \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) \boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}|s_i)\, d\boldsymbol{s}_{-i}$ exists and is bounded for all convex $\Omega \subseteq S_{-i}$.*

Proof. With $[\underline{s}, \overline{s}] \subset \mathbb{R}$ as the type space of agent i, then the type space for all agents except i, S_{-i}, is convex and bounded on $\mathbb{R}^{N-1}$; thus, all convex set $\Omega \subseteq S_{-i}$ is bounded. Next, since both $\gamma_i(s_i, b_i)$ and $b_i \in [\underline{s}, \overline{s}]$ are bounded for all i, u_i is finite as well. Together with the fact that $\boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}|s_i)$ is a bounded and atomless probability density function, we know that $\int_\Omega u_i(s_i, b_i, \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) \boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}|s_i)\, d\boldsymbol{s}_{-i}$ exists and is finite for all convex $\Omega \subseteq S_{-i}$. $\square$

Lemma A.2 shows that the object function satisfies single crossing properties of incremental returns in (s_i, b_i), which is a critical step that I use later in the proof of equilibrium existence.

Lemma A.2. *For each $i = 1, \ldots, N$, whenever every opponent $j \neq i$ uses a nondecreasing strategy $\beta_j(s_j)$, agent i's objective function $U_i(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i}))$ satisfies single crossing properties of incremental returns in (s_i, b_i).*

Proof. First, $u_i(s_i, \boldsymbol{b})$ is supermodular in (s_i, b_i), since with $s_i' > s_i$ and $b_i' > b_i$,

$$\begin{aligned} & u_i(s_i', (b_i', \boldsymbol{b}_{-i})) - u_i(s_i', (b_i, \boldsymbol{b}_{-i})) - (u_i(s_i, (b_i', \boldsymbol{b}_{-i})) - u_i(s_i, (b_i, \boldsymbol{b}_{-i}))) \\ = & -\int_{s_i}^{s_i'} \int_{b_i}^{b_i'} \frac{\partial^2}{\partial x \partial y} \gamma_i(x, y) dx dy > 0. \end{aligned}$$

Second, $u_i(s_i, \boldsymbol{b})$ is supermodular in (s_i, b_k) for all $k \neq i$, because with $s_i' > s_i$ and $b_k' > b_k$,

$$\begin{aligned} & u_i\left(s_i', (b_k', \boldsymbol{b}_{-k})\right) - u_i\left(s_i', (b_k, \boldsymbol{b}_{-k})\right) - \left(u_i\left(s_i, (b_k', \boldsymbol{b}_{-k})\right) - u_i\left(s_i, (b_k, \boldsymbol{b}_{-k})\right)\right) \\ = \quad & -\gamma_i\left(s_i', b_i\right) + \gamma_i\left(s_i, b_i\right) - \left(-\gamma_i\left(s_i', b_i\right) + \gamma_i\left(s_i, b_i\right)\right) = 0. \end{aligned}$$

In summary, $u_i(s_i, \boldsymbol{b})$ is supermodular in (s_i, b_k) for all k. Based on the fact (v) on the page 872 in Athey (2001), and similar results in Athey (1998) and Athey (2002), if $u_i(s_i, \boldsymbol{b})$ is supermodular in (s_i, b_k) for all k, and $\boldsymbol{s}$ is affiliated, then $U_i\left(s_i, b_i; \boldsymbol{\beta}_{-i}\left(\boldsymbol{s}_{-i}\right)\right) = \int_{[\underline{s}, \overline{s}]^{N-1}} u_i\left(s_i, b_i, \boldsymbol{\beta}_{-i}\left(\boldsymbol{s}_{-i}\right)\right) \boldsymbol{f}_{-i}\left(\boldsymbol{s}_{-i}\right) d\boldsymbol{s}_{-i}$ is supermodular in (s_i, b_i). Then based on fact (i) on page 871 of Athey (2001), since $U_i\left(s_i, b_i; \boldsymbol{\beta}_{-i}\left(\boldsymbol{s}_{-i}\right)\right)$, which is a function of (s_i, b_i), is supermodular in (s_i, b_i), then it satisfies SCP-IR in $(b_i; s_i)$. Since this is true for all $i = 1, \ldots, N$, the single crossing condition is satisfied. □

Now I am ready to prove the Proposition 1.

Proposition 1. *A non-decreasing BNE $\boldsymbol{\beta}(\boldsymbol{s})$ exists.*

Proof. Lemma A.1 shows that the assumption A1 in Athey (2001) is satisfied. Lemma A.2 implies that for each $i = 1, \ldots, N$, whenever every opponent $j \neq i$ uses a nondecreasing strategy $\beta_j(s_j)$, agent i's objective function $U_i\left(s_i, b_i; \boldsymbol{\beta}_{-i}\left(\boldsymbol{s}_{-i}\right)\right)$ satisfies single crossing of incremental returns in (s_i, b_i). This is exactly the single crossing condition for games of incomplete information. Together with the fact that for all i, $u_i\left(s_i, b_i; \boldsymbol{b}_{-i}\right)$ is continuous in both b_i and $\boldsymbol{b}_{-i}$, the Corollary 2.1 in Athey (2001) suggests that a non-decreasing PSBNE exists. □

Proof of Proposition 2 I first prove Lemma A.3, which will be used in the proof of Proposition 2 as well as others.

Lemma A.3. *Under the setup in Proposition 2, if agent i reports b_i, and all other agents submit their equilibrium reports, so $b = (\ldots, \beta(s_{i-1}), b_i, \beta(s_{i+1}), \ldots)$, then under any strictly increasing $\beta(\cdot)$,*

$$\frac{\partial}{\partial b_i} \int_{S_{-i}} b_j^{(N)} \boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}) d\boldsymbol{s}_{-i} = F_{j-1}^{(N-1)}\left(s_i\right) - F_j^{(N-1)}\left(s_i\right),$$

where $b_j^{(N)}$ is the jth-order statistics of all reports $\boldsymbol{b}$, and $F_j^{(N-1)}(\cdot)$ is the cumulative density function of jth order statistics of $\boldsymbol{s}_{-i}$.

Proof. The agent i's report b_i can either happen to be the jth smallest among $\boldsymbol{b}$, or lower or higher than

$b_j^{(N)}$, and nothing else. So I can split the set S_{-i} into three partitions such that

$$\begin{aligned}
\Omega_1 &= \{\boldsymbol{s}_{-i} : b_i < \beta_{j-1}^{(N-1)}(\boldsymbol{s}_{-i})\} \\
\Omega_2 &= \{\boldsymbol{s}_{-i} : \beta_j^{(N-1)}(\boldsymbol{s}_{-i}) < b_i\} \\
\Omega_3 &= \{\boldsymbol{s}_{-i} : \beta_{j-1}^{(N-1)}(\boldsymbol{s}_{-i}) < b_i < \beta_j^{(N-1)}(\boldsymbol{s}_{-i})\},
\end{aligned}$$

where $\beta_j^{(N-1)}(\boldsymbol{s}_{-i})$ is the jth smallest equilibrium report among $N-1$ reports from agents other than i. From the strictly increasing equilibrium strategy, we know that

$$\begin{aligned}
\Omega_1 &= \{\boldsymbol{s}_{-i} : s_i < s_{j-1}^{(N-1)}\} \\
\Omega_2 &= \{\boldsymbol{s}_{-i} : s_j^{(N-1)} < s_i\} \\
\Omega_3 &= \{\boldsymbol{s}_{-i} : s_{j-1}^{(N-1)} < s_i < s_j^{(N-1)}\},
\end{aligned}$$

where $s_j^{(N-1)}$ is the jth smallest signal among $N-1$ signals of agents other than i. Hence,

$$\begin{aligned}
\int_{S_{-i}} b_j^{(N)} \boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}) d\boldsymbol{s}_{-i} &= \sum_{p=1,2,3} \int_{\Omega_p} b_j^{(N)} \boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}) d\boldsymbol{s}_{-i} \\
&= \int_{b_i}^{\overline{s}} x f_{j-1}^{(N-1)}(x)\, dx + \int_{\underline{s}}^{b_i} x f_j^{(N-1)}(x)\, dx + b_i \left[F_{j-1}^{(N-1)}(b_i) - F_j^{(N-1)}(b_i)\right].
\end{aligned}$$

The regularity of distribution function ensures $\int_{S_{-i}} b_j^{(N)} \boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}) d\boldsymbol{s}_{-i}$ to be differentiable with respect to b_i, and the derivative is just $F_{j-1}^{(N-1)}(s_i) - F_j^{(N-1)}(s_i)$.

□

Proposition 2. *If the expected penalty function $\gamma(\delta_i, s_i, b_i)$ takes the form $\delta_i \hat{\gamma}(b_i - s_i)$, and the second order derivative of $\hat{\gamma}$ satisfies*

$$\hat{\gamma}'' > \frac{1}{N-2n} \frac{(N-1)!}{(n-1)!\,(N-n-1)!} \frac{\left[(1-F(s_i;\theta))^{N-2n} - F(s_i;\theta)^{N-2n}\right]}{\left[F(s_i;\theta)(1-F(s_i;\theta))\right]^{1-n}} f(s_i;\theta) \tag{7}$$

for all $s_i \in [\underline{s}, \overline{s}]$, then the equilibrium report β is symmetric, strictly increasing, and the expected benchmark bias depends only on the panel size N, the trimming window n, and the penalty function γ, but not on the parameter θ of the private signal distribution.

Proof. Under i.i.d. distribution, using the notation $b_j^{(N)}$ as defined in Lemma A.3, the agent's objective function is

$$U_i(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) = \int_{S_{-i}} -\zeta \delta_i \hat{\gamma}(b_i - s_i) - \delta_i \sum_{j=1}^{N} w_j b_j^{(N)} \boldsymbol{f}_{-i}(\boldsymbol{s}_{-i}) d\boldsymbol{s}_{-i}.$$

Using Lemma A.3, the first order condition of agent i's objective function $U_i\left(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})\right)$ w.r.t. b_i is

$$0 = -\delta_i \zeta \hat{\gamma}'(b_i - s_i) - \delta_i \sum_{j=1}^{N} w_j \left[F_{j-1}^{(N-1)}(s_i) - F_j^{(N-1)}(s_i)\right], \tag{11}$$

and the second order condition confirms the maximization of objective function. It is clear that the equilibrium b_i is symmetric since δ_i does not affect the first order condition.

Together with the property of order statistics, this suggests that the equilibrium strategy $\beta(\cdot)$ is

$$\begin{aligned}
\beta(s_i) &= s_i - \hat{\gamma}'^{-1}\left(\sum_{j=1}^{N} \frac{w_j}{\zeta}\left[F_{j-1}^{(N-1)}(s_i) - F_j^{(N-1)}(s_i)\right]\right) \\
&= s_i - \hat{\gamma}'^{-1}\left(\sum_{j=1}^{N} \frac{w_j}{\zeta}\left[\frac{\int_0^{F(s_i;\theta)} t^{n-2}(1-t)^{N-n+1}\,dt}{B(n-1, N-n+2)} - \frac{\int_0^{F(s_i;\theta)} t^{n-1}(1-t)^{N-n}\,dt}{B(n, N-n+1)}\right]\right).
\end{aligned}$$

I define the error term as $\rho(s_i) = \beta(s_i) - s_i$: so $\rho(s_i)$ is a function of $F(s_i;\theta)$. Under strictly increasing The expected index error $\mathbb{E}[L - L_0]$ becomes

$$\int_{\underline{s}}^{\overline{s}} \left[... \left[\int_{\underline{s}}^{x_2} w_1 \rho(x_1) f_1^{(N)}\left(x_1 | s_2^{(N)} < x_2, ..., s_N^{(N)} < x_N\right) dx_1\right] ... + w_N \rho(x_N)\right] f_N^{(N)}(x_N)\, dx_N.$$

With the Markov property of order statistics,

$$f_1^{(N)}\left(x_1 | s_2^{(N)} < x_2, ..., s_N^{(N)} < x_N\right) = f_1^{(N)}\left(x_1 | s_2^{(N)} < x_2\right) = \frac{f(x_1;\theta)}{1 - F(x_2;\theta)},$$

plugging in the formula of $\rho(x)$ it is easy to see we only need to check

$$\begin{aligned}
&\int_{\underline{s}}^{x_2} \left[\int_0^{F(x_1)} t^{n-2}(1-t)^{N-n+1}\,dt\right] f_1^{(N)}\left(x_1 | s_2^{(N)} < x_2\right) dx_1 \\
=& \int_{\underline{s}}^{x_2} \left[\int_0^{F(x_1)} t^{n-2}(1-t)^{N-n+1}\,dt\right] \frac{1}{1 - F(x_2;\theta)} dF(x_1;\theta),
\end{aligned}$$

which is indeed a function of $F(x_2)$. I can write

$$\left[\int_{\underline{s}}^{x_2} w_1 \rho(x_1) f_1^{(N)}\left(x_1 | s_2^{(N)} < x_2\right) dx_1\right] = G\left(F(x_2;\theta)\right),$$

then plug it back to $\mathbb{E}[L - L_0]$ and get

$$\begin{aligned}
&\mathbb{E}[L - L_0] \\
=& \int_{\underline{s}}^{\overline{s}} \left[\int_{\underline{s}}^{x_j} ... \int_{\underline{s}}^{x_3} \left[G\left(F(x_2;\theta)\right) + w_2 \rho(x_2)\right] f_2^{(N)}\left(x_2 | s^{(2)} < x_3\right) dx_2 ... + w_N \rho(x_N)\right] f_N^{(N)}(x_N)\, dx_N.
\end{aligned}$$

Similarly, $\int_0^{x_3} \left[G\left(F\left(x_2;\theta\right)\right)+w_2\rho\left(x_2\right)\right] f^{(2)}\left(x_2|s^{(2)}<x_3\right) dx_2$ is a function of $F\left(x_3;\theta\right)$. By repeatedly using the Markov property of the order statistics, I get

$$\mathbb{E}\left[L-L_0\right] = \int_{\underline{s}}^{\overline{s}} \left[H\left(F\left(x_N;\theta\right)\right)+w_N\rho\left(x_N\right)\right] \frac{F^{N-1}\left(x_N\right)}{B\left(N,1\right)} dF\left(x_N;\theta\right).$$

Consider that $\rho\left(x_N\right)$ is an integral with upper bound as $F\left(x_N;\theta\right)$. I can treat $F\left(x_N;\theta\right)$ as an integration variable, and the result is a function of $F\left(\overline{s};\theta\right)$ and $F\left(\underline{s};\theta\right)$. Since any distribution CDF function $F(\cdot;\theta)$ has $F\left(\overline{s};\theta\right)=1$ and $F\left(\underline{s};\theta\right)=0$, by construction, the functional form of F does not change $\mathbb{E}\left[L-L_0\right]$. Hence, the expected LIBOR bias $\mathbb{E}\left[L-L_0\right]$ only depends on ζ, N, and $\{w_i\}$.

Notice that to show the θ-independency I used Lemma A.3, which depends on the assumption that β is strictly increasing. I now derive the necessary and sufficient condition for strictly increasing β when $\gamma\left(\delta_i,s_i,b_i\right)$ takes the form $\delta_i\hat{\gamma}\left(b_i-s_i\right)$. Plug in the weight w_i to the first order condition in Equation (11) suggests that the equilibrium $\beta\left(s_i\right)$

$$0=-\zeta\hat{\gamma}'(\beta\left(s_i\right)-s_i)-\frac{1}{N-2n}\left[F_n^{(N-1)}\left(s_i\right)-F_{N-n}^{(N-1)}\left(s_i\right)\right].$$

Writing $\Delta\left(s_i\right)=F_n^{(N-1)}\left(s_i\right)-F_{N-n}^{(N-1)}\left(s_i\right)$, and with the results in order statistics, noting the incomplete Beta function as $I_x\left(a,b\right)=\frac{B(x,a,b)}{B(a,b)}$, I have $0=-\zeta\hat{\gamma}'(\beta\left(s_i\right)-s_i)-\frac{\Delta(s_i)}{N-2n}$, where

$$\begin{aligned}\Delta\left(s_i\right) &= I_{F\left(s_i;\theta\right)}\left(n,N-n\right)-I_{F\left(s_i;\theta\right)}\left(N-n,n\right)\\ &= \frac{(N-1)!}{(n-1)!\left(N-n-1\right)!}\int_0^{F\left(s_i;\theta\right)}\frac{\left(1-y\right)^{N-2n}-y^{N-2n}}{\left[y\left(1-y\right)\right]^{1-n}}dy.\end{aligned}$$

With

$$\frac{\partial\beta(s_i)}{\partial s_i}=-\frac{\zeta\hat{\gamma}''-\frac{\Delta'(s_i)}{N-2n}}{-\zeta\hat{\gamma}''},$$

the equilibrium β is strictly increasing if and only if

$$\hat{\gamma}''>\frac{1}{N-2n}\frac{(N-1)!}{(n-1)!\left(N-n-1\right)!}\frac{\left[\left(1-F\left(s_i;\theta\right)\right)^{N-2n}-F\left(s_i;\theta\right)^{N-2n}\right]}{\left[F\left(s_i;\theta\right)\left(1-F\left(s_i;\theta\right)\right)\right]^{1-n}}f\left(s_i;\theta\right)$$

for all s_i.

□

Proof of Proposition 3

Proposition 3. *To minimize the agents' reporting error, the principal follows the equlibrium expected penalty* *function* $\gamma(\delta_i, s_i, \beta(s_i)) = \delta_i \hat{\gamma}(\beta(s_i) - s_i) = \delta_i \zeta(\beta(s_i) - s_i)^2$*, where*

$$\zeta = \frac{1}{2K} \int_0^{\frac{1}{2}} \left[\frac{1}{N-2n} \frac{(N-1)!}{(n-1)!\,(N-n-1)!} \int_0^x \frac{\left[(1-y)^{N-2n} - y^{N-2n}\right]}{\left[y\,(1-y)\right]^{1-n}} dy \right]^2 dx, \tag{8}$$

assuming the ζ *satisfies the strictly increasing equilibrium condition in Equation (7).*

Proof. With the equilibrium choice of β we have

$$\hat{\gamma}'\left(s_i - \beta\left(s_i\right)\right) = \frac{1}{N-2n} \Delta\left(s_i\right).$$

Let $\frac{1}{N-2n}\Delta\left(s_i\right) = \frac{1}{N-2n}\frac{(N-1)!}{(n-1)!(N-n-1)!}\int_0^{F(s_i;\theta)} \frac{\left[(1-y)^{N-2n} - y^{N-2n}\right]}{\left[y(1-y)\right]^{1-n}} dy = g\left(F\left(s_i;\theta\right)\right)$, it is easy to see $g\left(F\left(s_i;\theta\right)\right) = g\left(1 - F\left(s_i;\theta\right)\right)$, so $g\left(\cdot\right)$ is symmetric along $\frac{1}{2}$, $g\left(0\right) = g\left(1\right) = 0$, and g obtains a maximum at $F\left(s_i;\theta\right) = \frac{1}{2}$ for all θ. It is easy to see that we can replace $F\left(s_i;\theta\right)$ with x and have

$$\int_{\underline{s}}^{\bar{s}} \left[\hat{\gamma}'^{-1}\left(\frac{1}{N-2n}\Delta\left(s_i\right)\right)\right]^2 f\left(s_i;\theta\right) ds_i = 2\int_0^{\frac{1}{2}} \left[\hat{\gamma}'^{-1}\left(g\left(x\right)\right)\right]^2 dx$$

and the I.R. constraint of banks become

$$\int_{\underline{s}}^{\bar{s}} \hat{\gamma}\left(\hat{\gamma}'^{-1}\left(\frac{1}{N-2n}\Delta\left(s_i\right)\right)\right) f\left(s_i;\theta\right) ds_i = 2\int_0^{\frac{1}{2}} \hat{\gamma}\left(\hat{\gamma}'^{-1}\left(g\left(x\right)\right)\right) dx$$

So I can rewrite the principal's program is to find the optimal $\hat{\gamma}$

$$\hat{\gamma} = \arg\min_{\hat{\gamma}} \int_0^{\frac{1}{2}} \left[\hat{\gamma}'^{-1}\left(g\left(x\right)\right)\right]^2 dx$$

subject to

$$\int_0^{\frac{1}{2}} \hat{\gamma}\left(\hat{\gamma}'^{-1}\left(g\left(x\right)\right)\right) dx \le \frac{K}{2}$$

as well as the feasibility constraints of $\hat{\gamma}$.

The Lagrangian is

$$\mathcal{L} = \int_0^{\frac{1}{2}} \left\{\left[\hat{\gamma}'^{-1}\left(g\left(x\right)\right)\right]^2 - \lambda\hat{\gamma}\left(\hat{\gamma}'^{-1}\left(g\left(x\right)\right)\right)\right\} dx$$

which is strictly increasing in $x \in \left(0, \frac{1}{2}\right)$. The principal's program is a calculus of variations problem to find the $\hat{\gamma}$ that minimizes the $\mathcal{L}$.

The strict monotonicity of $g\left(x\right)$ when $x \in \left(0, \frac{1}{2}\right)$ allows me to carry out a change of variable $u =$

$\hat{\gamma}'^{-1}(g(x))$ so $x = g^{-1}(\hat{\gamma}'(u))$ and

$$\begin{aligned} dx &= \frac{dg^{-1}(\hat{\gamma}'(u))}{d\hat{\gamma}'(u)} \frac{d\hat{\gamma}'(u)}{du} du \\ &= \frac{\hat{\gamma}''(u)}{g'(g^{-1}(\hat{\gamma}'(u)))} du \end{aligned}$$

so the principal's program can be converted to a classical calculus of variations problem with an integral constraint, i.e., an isoperimetric problem, as

$$\min \int_{\hat{\gamma}'^{-1}(g(0))}^{\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)} \frac{\hat{\gamma}''(u)\, u^2}{g'(g^{-1}(\hat{\gamma}'(u)))} du$$

subject to

$$\int_{\hat{\gamma}'^{-1}(g(0))}^{\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)} \frac{\hat{\gamma}''(u)\, \hat{\gamma}(u)}{g'(g^{-1}(\hat{\gamma}'(u)))} du \leq \frac{K}{2},$$

and $\hat{\gamma}$ has a fixed ending point at $\hat{\gamma}(0) = 0$ and a variable ending point at $\hat{\gamma}(u_{max})$ where $u_{max} = \hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)$, which is dependent on the choice of $\hat{\gamma}$. The variable ending point satisfies the condition $\hat{\gamma}'(u_{max}) = \hat{\gamma}'\left(\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)\right) = g\left(\frac{1}{2}\right)$.

Following the standard solution, I write down the Lagrangian as

$$\mathcal{L} = \int_{\hat{\gamma}'^{-1}(g(0))}^{\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)} Z(u, \hat{\gamma}(u), \hat{\gamma}'(u), \hat{\gamma}''(u))\, du$$

where

$$Z(u, \hat{\gamma}(u), \hat{\gamma}'(u), \hat{\gamma}''(u)) = \frac{\hat{\gamma}''(u)\left[u^2 - \lambda\hat{\gamma}(u)\right]}{g'(g^{-1}(\hat{\gamma}'(u)))},$$

which gives the Euler-Lagrangian Equation

$$\frac{d^2}{du^2}\left(\frac{\partial Z}{\partial \hat{\gamma}''}\right) - \frac{d}{du}\left(\frac{\partial Z}{\partial \hat{\gamma}'}\right) + \frac{\partial Z}{\partial \hat{\gamma}} = 0.$$

There is some tedious algebraic operations, and eventually I have

$$\begin{aligned} 0 =& \frac{d^2}{du^2}\left(\frac{\partial Z}{\partial \hat{\gamma}''}\right) - \frac{d}{du}\left(\frac{\partial Z}{\partial \hat{\gamma}'}\right) + \frac{\partial Z}{\partial \hat{\gamma}} \\ =& \frac{g'\left(g^{-1}(\hat{\gamma}'(u))\right)^2 [2 - \lambda\hat{\gamma}''(u)] - \hat{\gamma}''(u) g''\left(g^{-1}(\hat{\gamma}'(u))\right)[-\lambda(\hat{\gamma}'(u) + \hat{\gamma}(u)) + u(u+2)]}{g'(g^{-1}(\hat{\gamma}'(u)))^3}. \end{aligned}$$

Notice at the variational ending point $u = \hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)$ I have $\hat{\gamma}'(u) = g\left(\frac{1}{2}\right)$, or $\delta\hat{\gamma} = g\left(\frac{1}{2}\right)\delta u$. Hence the fundamental necessary condition for an extremum takes the form

$$\lim_{u\uparrow\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)}\left(Z-\hat{\gamma}'\frac{\partial Z}{\partial\hat{\gamma}'}\right)\delta u+\frac{\partial Z}{\partial\hat{\gamma}'}\delta\hat{\gamma}=0$$

which gives us

$$\begin{aligned}\lim_{u\uparrow\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)} Z+\left[g\left(\frac{1}{2}\right)-\hat{\gamma}'\right]\frac{\partial Z}{\partial\hat{\gamma}'} &= \lim_{u\uparrow\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)} Z\\ &= \lim_{u\uparrow\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)} \frac{\hat{\gamma}''(u)\left[u^2-\lambda\hat{\gamma}(u)\right]}{g'\left(g^{-1}\left(\hat{\gamma}'(u)\right)\right)}\\ &= 0.\end{aligned}$$

Notice that $\hat{\gamma}''(u)=0$ cannot be a solution since the constraint $\int_{\hat{\gamma}'^{-1}(g(0))}^{\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)}\frac{\hat{\gamma}''(u)\hat{\gamma}(u)}{g'(g^{-1}(\hat{\gamma}'(u)))}du=0$. To summarize, the optimal $\hat{\gamma}(u)$ together with λ satisfies the non-linear ODE sytem:

$$\begin{aligned}g'\left(g^{-1}\left(\hat{\gamma}'\right)\right)^2\left[2-\lambda\hat{\gamma}''\right]-\left[-\lambda\left(\hat{\gamma}'+\hat{\gamma}\right)+u(u+2)\right]\hat{\gamma}''g''\left(g^{-1}\left(\hat{\gamma}'\right)\right)&=0\\ \int_{\hat{\gamma}'^{-1}(g(0))}^{\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)}\frac{\hat{\gamma}''(u)\,\hat{\gamma}(u)}{g'\left(g^{-1}\left(\hat{\gamma}'(u)\right)\right)}du&=\frac{K}{2}\end{aligned}$$

with boundary condition

$$\begin{aligned}\hat{\gamma}(0)&=0,\\ \lim_{u\uparrow\hat{\gamma}'^{-1}\left(g\left(\frac{1}{2}\right)\right)} u^2-\lambda\hat{\gamma}(u)&=0.\end{aligned}$$

It is easy to see there is a special solution to the non-linear ODE system satisfies both $2-\lambda\hat{\gamma}''=0$ and $-\lambda\left(\hat{\gamma}'+\hat{\gamma}\right)+u(u+2)=0$, which suggests the special solution is $\hat{\gamma}(u)=\zeta u^2$ where $\zeta=\frac{1}{\lambda}$. It is easy to verify that both boundary conditions are satisfied.

To solve for λ, I plug it back to the constraint, and with $\hat{\gamma}'^{-1}(x)=\frac{x}{2a}=\frac{\lambda}{2}x$, the I.R. constraint becomes

$$\begin{aligned}\frac{K}{2}&=\int_0^{\frac{1}{2}}\hat{\gamma}\left(\hat{\gamma}'^{-1}\left(g\left(x\right)\right)\right)dx\\ &=\int_0^{\frac{1}{2}}\hat{\gamma}\left(\frac{\lambda g(x)}{2}\right)dx\\ &=\int_0^{\frac{1}{2}}\frac{\lambda g^2(x)}{4}dx,\end{aligned}$$

which gives us $\hat{\gamma}(u) = \zeta u^2$ where

$$\zeta = \frac{1}{2K}\int_0^{\frac{1}{2}}\left[\frac{1}{N-2n}\frac{(N-1)!}{(n-1)!\,(N-n-1)!}\int_0^x \frac{\left[(1-y)^{N-2n} - y^{N-2n}\right]}{\left[y\,(1-y)\right]^{1-n}}dy\right]^2 dx.$$

□

Proof of Proposition 4

Proposition 4. *Under the equilibrium penalty function $\gamma(\delta_i, s_i, \beta(s_i)) = \delta_i\hat{\gamma}(\beta(s_i) - s_i) = \delta_i\zeta(\beta(s_i) - s_i)^2$, and if ζ satisfies the condition*

$$\zeta > \zeta^*(s_i) = \frac{1}{2(N-2n)}\frac{(N-1)!}{(n-1)!\,(N-n-1)!}\frac{\left[(1-F(s_i;\theta))^{N-2n} - F(s_i;\theta)^{N-2n}\right]}{\left[F(s_i;\theta)\,(1-F(s_i;\theta))\right]^{1-n}}f(s_i;\theta) \tag{9}$$

for all $s_i \in (\underline{s}, \overline{s})$, then agent i has a strictly increasing BNE report $\beta : [\underline{s}, \overline{s}] \to [\underline{s}, \overline{s}]$ such that

$$\beta(s_i) = s_i - \frac{1}{2\zeta(N-2n)}\Delta(s_i;\theta)$$

where $\Delta(s_i;\theta) = \frac{(N-1)!}{(n-1)!(N-n-1)!}\int_0^{F(s_i;\theta)} \frac{\left[(1-y)^{N-2n} - y^{N-2n}\right]}{[y(1-y)]^{1-n}}dy$.

Proof. Use Lemma A.3, plug in the weight w_i and $\gamma(\delta_i, s_i, b_i) = \delta_i\zeta(b_i - s_i)^2$ into the first order condition in Equation (11), I get the equilibrium $\beta(s_i)$

$$0 = -2\zeta(\beta(s_i) - s_i) - \frac{1}{N-2n}\left[F_n^{(N-1)}(s_i) - F_{N-n}^{(N-1)}(s_i)\right].$$

Writing $\Delta(s_i) = F_n^{(N-1)}(s_i) - F_{N-n}^{(N-1)}(s_i)$, and with the results in order statistics, I have $\beta(s_i) = s_i - \frac{1}{2\zeta(N-2n)}\Delta(s_i;\theta)$ and

$$\begin{aligned}\Delta(s_i) &= I_{F(s_i;\theta)}(n, N-n) - I_{F(s_i;\theta)}(N-n, n)\\ &= \frac{(N-1)!}{(n-1)!\,(N-n-1)!}\int_0^{F(s_i;\theta)} \frac{(1-y)^{N-2n} - y^{N-2n}}{\left[y\,(1-y)\right]^{1-n}}dy,\end{aligned}$$

□

Proof of Proposition 5

Proposition 5. *Increasing both the size of the panel N and the number of trimmed reports, and keeping the proportion n/N the same, reduces the reporting error. In other words, note the equilibrium strategy under* *panel size N and trimmed quotes n as $\beta_{N,n}(s_i)$, for positive rational number m such that mn and mN are integers, $\beta_{N,n}(s_i) \leq \beta_{mN,mn}(s_i)$.*

Proof. Since under the given setup the equilibrium report $\beta(s_i) < s_i$, the proposition is equivalent to the claim that the ratio

$$
\begin{aligned}
& \frac{s_i - \beta_{mN,mn}(s_i)}{s_i - \beta_{N,n}(s_i)} \\
= \quad & m\frac{B(mn, mN-mn)}{B(n, N-n)} \frac{\int_0^{F(s_i;\theta)} \left[t^{mn-1}(1-t)^{mN-mn-1} - t^{mN-mn-1}(1-t)^{mn-1}\right] dt}{\int_0^{F(s_i;\theta)} \left[t^{n-1}(1-t)^{N-n-1} - t^{N-n-1}(1-t)^{n-1}\right] dt} < 1.
\end{aligned}
$$

The inequality of Beta function suggests that $\frac{mB(mn,mN-mn)}{B(n,N-n)} \leq m\left[\left(\frac{n}{N}\right)^n \left(\frac{N-n}{N}\right)^{N-n}\right]^{(m-1)}$. Checking the logarithm of the ratio, and using Jensen's inequality for the logarithm function, it is easy to see that $\log\left\{m\left[\left(\frac{n}{N}\right)^n \left(\frac{N-n}{N}\right)^{N-n}\right]^{(m-1)}\right\} < 0$ for all $m \geq 1$. Therefore, $\frac{mB(mn,mN-mn)}{B(n,N-n)} < 1$.

To see whether $\frac{\int_0^{F(s_i;\theta)}\left[t^{mn-1}(1-t)^{mN-mn-1} - t^{mN-mn-1}(1-t)^{mn-1}\right]dt}{\int_0^{F(s_i;\theta)}\left[t^{n-1}(1-t)^{N-n-1} - t^{N-n-1}(1-t)^{n-1}\right]dt} \leq 1$, first notice both the numerator and denominator equal to 0 when $F(s_i;\theta) = 0$ or $F(s_i;\theta) = 1$. Also,

$$
\begin{aligned}
& \frac{\int_1^{1-F(s_i;\theta)} \left[t^{mn-1}(1-t)^{mN-mn-1} - t^{mN-mn-1}(1-t)^{mn-1}\right] dt}{\int_1^{1-F(s_i;\theta)} \left[t^{n-1}(1-t)^{N-n-1} - t^{N-n-1}(1-t)^{n-1}\right] dt} \\
= \quad & \frac{\int_0^{F(s_i;\theta)} \left[t^{mn-1}(1-t)^{mN-mn-1} - t^{mN-mn-1}(1-t)^{mn-1}\right] dt}{\int_0^{F(s_i;\theta)} \left[t^{n-1}(1-t)^{N-n-1} - t^{N-n-1}(1-t)^{n-1}\right] dt},
\end{aligned}
$$

therefore, I only need to check whether the inequality holds for $F(s_i;\theta) \leq \frac{1}{2}$, where $t^{n-1}(1-t)^{N-n-1} - t^{N-n-1}(1-t)^{n-1} > 0$.

I can check the ratio of the derivative of numerator and denominator with respect to $F(s_i;\theta)$ instead. Since the case for $m = 1$ is trivially true, I focus on the case in which $m \geq 2$, and write

$$
\frac{t^{mn-1}(1-t)^{mN-mn-1} - t^{mN-mn-1}(1-t)^{mn-1}}{t^{n-1}(1-t)^{N-n-1} - t^{N-n-1}(1-t)^{n-1}} = \frac{x_1^m - x_2^m}{x_1 - x_2},
$$

where $x_1 = t^n(1-t)^{N-n}$, and $x_2 = t^{N-n}(1-t)^n$. Notice that x_1 attains its maximum when $t = n/N$, so $\log x_1 = \log\left[t^n(1-t)^{N-n}\right] \leq \log\left[\left(\frac{n}{N}\right)^n\left(\frac{N-n}{N}\right)^{N-n}\right]$. Use the similar sequence of inequalities as above, I have $x_1 < \frac{1}{e}$. Similarly, $x_2 < \frac{1}{e}$ as well. Since $dx^m/dx = mx^{m-1} < me^{1-m} < 1$ for all $0 < x < 1/e$, Hence we have $x_1^m - x_2^m = \int_{x_1}^{x_x}(x^m)' dx < \int_{x_1}^{x_x} dx = x_1 - x_2$. Therefore

$$
t^{mn-1}(1-t)^{mN-mn-1} - t^{mN-mn-1}(1-t)^{mn-1} < t^{n-1}(1-t)^{N-n-1} - t^{N-n-1}(1-t)^{n-1},
$$

and the inequality holds. Together with the inequality $\frac{mB(mn,mN-mn)}{B(n,N-n)} < 1$, it is easy to see the proposition holds. $\square$

Proof of Proposition 6

Proposition 6. *Increasing the size of panel N and the number of trimmed reports, and keeping the included reports to be the same, reduces the reporting error, so that $\beta_{N,n}(s_i) \leq \beta_{N+2m,n+m}(s_i)$ for positive rational number m.*

Proof. Notice that from the properties of incomplete Beta function,

$$
\begin{aligned}
& s_i - \beta_{N+2,n+1}(s_i) - [s_i - \beta_{N,n}(s_i)] \\
= \ & \frac{1}{2\zeta(N-2n)} \left\{ I_{F(s_i;\theta)}(n+1, N-n+1) - I_{F(s_i;\theta)}(N-n+1, n+1) \right\} \\
& - \frac{1}{2\zeta(N-2n)} \left\{ I_{F(s_i;\theta)}(n, N-n) - I_{F(s_i;\theta)}(N-n, n) \right\} \\
= \ & \frac{1}{2\zeta(N-2n)} \left[\frac{x^n(1-x)^{N-n}}{(N-n)B(n,N-n)} \left(1 - \frac{N}{n}(1-x)\right) + \frac{x^{N-n}(1-x)^n}{(N-n)B(N-n,n)} \left(1 - \frac{N}{n}x\right) \right] < 0.
\end{aligned}
$$

In other words, increasing the size of the panel bank and the trimmed reports by one, while calculating the index using the same number of reports, strictly lowers reporting bias. $\square$

Proof of Proposition 7

Proposition 7. *If $\zeta \leq \zeta^*(s_i)$ for some intervals of s_i with positive measure, agent i BNE strategy β is characterized by the region $[\underline{\sigma}, \bar{\sigma}]$ in which the strategy is "bunching" and the region $[\underline{s}_i, \bar{s}_i] \setminus [\underline{\sigma}, \bar{\sigma}]$ in which the strategy is strictly increasing. The BNE strategy of agent i with $s_i \notin [\underline{\sigma}, \bar{\sigma}]$ is the same as Proposition 4, i.e., $\beta(s_i) = s_i - \frac{1}{2\zeta(N-2n)}\Delta(s_i;\theta)$ with $\Delta(s_i;\theta) = \frac{(N-1)!}{(n-1)!(N-n-1)!}\int_0^{F(s_i;\theta)} \frac{\left[(1-y)^{N-2n} - y^{N-2n}\right]}{[y(1-y)]^{1-n}} dy$. The BNE strategy for agents with $s_i \in [\underline{\sigma}, \bar{\sigma}]$ is*

$$
\bar{b} = \frac{\int_{\underline{\sigma}}^{\bar{\sigma}} s_i f(s_i;\theta) ds_i}{\int_{\underline{\sigma}}^{\bar{\sigma}} f(s_i;\theta) ds_i} - \frac{\bar{\Delta}}{2\zeta(N-2n)}, \tag{10}
$$

with $\bar{\Delta} = \frac{(N-1)!}{(n-1)!(N-n-1)!}\left[\int_0^{F(\underline{\sigma};\theta)} \frac{(1-y)^{N-n-1}}{y^{1-n}} dy - \int_0^{F(\bar{\sigma};\theta)} \frac{y^{N-n-1}}{(1-y)^{1-n}} dy\right]$. Finally, $\underline{\sigma} = \min \beta^{-1}(\bar{b})$, and $\bar{\sigma} = \max \beta^{-1}(\bar{b})$.

Proof. The constrained problem is to find $\beta(\cdot)$, using its derivative $\mu(\cdot)$ as a constrained control, to maximize $U_i(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i}))$. In other words,

$$
\beta(s_i) = \arg\max_{\beta} \int_{[\underline{s},\bar{s}]} U_i(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) f(s_i) ds_i,
$$

s.t. $\partial\beta(s_i)/\partial s_i = \mu(s_i) \geq 0$.

According to Gelfand and Fomin (1963), the Hamiltonian is

$$\mathcal{H}(s_i, \beta(s_i), \lambda(s_i), \mu(s_i)) = U_i(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) f(s_i) + \lambda(s_i)\mu(s_i)$$

and Pontryagin's principle suggests an optimum $\beta(s_i)$ and $\mu^*(s_i)$ are given by

1. $\mathcal{H}(s_i, \beta(s_i), \lambda(s_i), \mu^*(s_i)) \geq \mathcal{H}(s_i, \beta(s_i), \lambda(s_i), \mu(s_i))$
2. In the region that derivative exists, $\frac{\partial\lambda(s_i)}{\partial s_i} = -\frac{\partial U_i(s_i, b_i; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i}))}{\partial\beta(s_i)}$.
3. The transversality condition $\lambda(\underline{s}) = \lambda(\bar{s}) = 0$ are satisfied.

From complementary slackness, when constraint is not binding, I have $\lambda(s_i) = 0$, so $\partial\lambda(s_i)/\partial s_i = 0$. Therefore, for $s \notin [\underline{\sigma}, \bar{\sigma}]$, the FOC is the same as before:

$$2\zeta(s_i - \beta(s_i)) = -\frac{1}{N-2n}\left[G_n^{(N-1)}(\beta(s_i)) - G_{N-n}^{(N-1)}(\beta(s_i))\right].$$

Given the strictly increasing equilibrium strategy for $s_i \in [\underline{s}, \bar{s}] \setminus [\underline{\sigma}, \bar{\sigma}]$, there is a one-to-one mapping from $F(s_i)$ to $G(\beta(s_i))$, therefore,

$$2\zeta(s_i - \beta(s_i)) = -\frac{1}{N-2n}\left[F_n^{(N-1)}(s_i) - F_{N-n}^{(N-1)}(s_i)\right] \quad \forall s_i \in [\underline{s}, \bar{s}] \setminus [\underline{\sigma}, \bar{\sigma}]$$

which is the same as the unconstrained optimization problem.

When the condition for strictly increasing equilibrium does not hold, the monotonicity of the best response function suggests that there exists $s_i^1 \neq s_i^2$ such that $\beta(s_i^1) = \beta(s_i^2)$. Around the area where the condition does not hold, I can write a convex set $[\underline{\sigma}, \bar{\sigma}] \subseteq [\underline{s}, \bar{s}]$ in which the $\beta(s_i) = \bar{b}$ for all $s_i \in [\underline{\sigma}, \bar{\sigma}]$.

I now prove that $\beta(\cdot)$ is continuous. It is straightforward to see $\beta(s_i)$ is continuous for $s_i \in (\underline{\sigma}, \bar{\sigma})$, as well as $s_i \in [0, \underline{\sigma}) \cup (\bar{\sigma}, \bar{s}]$. The continuity of Hamiltonian $\mathcal{H}(s_i, \beta(s_i), \lambda(s_i), \mu^*(s_i))$ as well as the costate variable $\lambda(s_i)$ when $s_i \in \{\underline{\sigma}, \bar{\sigma}\}$ suggests $\beta(s_i)$ is also continuous at point $\underline{\sigma}$ and $\bar{\sigma}$.

I now focus on how to determine $\bar{b}$. The continuity of $\beta(\cdot)$ together with the constraint suggests that $\beta(\underline{\sigma}) = \beta(\bar{\sigma}) = \bar{b}$, the bunching level. In order to determine the bunching level $\bar{b}$, I write

$$\begin{aligned} J(\bar{b}) &= \int_{\underline{s}}^{\underline{\sigma}} U_i(s_i, \beta(s_i); \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) f(s_i)\, ds_i \\ &\quad + \int_{\underline{\sigma}}^{\bar{\sigma}} U_i\left(s_i, \bar{b}; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})\right) f(s_i)\, ds_i \\ &\quad + \int_{\bar{\sigma}}^{\bar{s}} U_i(s_i, \beta(s_i); \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})) f(s_i)\, ds_i \end{aligned}$$

in which $\{\underline{\sigma}, \bar{\sigma}\}$ are the minimum and maximum values of $\beta^{*-1}\left(\bar{b}\right)$. If $\bar{b}$ is indeed the optimal bunching level, variation of the level should not change $J\left(\bar{b}\right)$ in the first order. This leads to

$$\frac{\partial}{\partial \bar{b}} J\left(\bar{b}\right) = \int_{\underline{\sigma}}^{\bar{\sigma}} \frac{\partial}{\partial \bar{b}} U_i\left(s_i, \bar{b}; \boldsymbol{\beta}_{-i}(\boldsymbol{s}_{-i})\right) f\left(s_i\right) ds_i = 0,$$

and after simplification, I have

$$\bar{b} = \frac{\int_{\underline{\sigma}}^{\bar{\sigma}} s_i f(s_i) ds_i}{\int_{\underline{\sigma}}^{\bar{\sigma}} f(s_i) ds_i} - \frac{\bar{\Delta}}{2\zeta\left(N-2n\right)},$$

with $\bar{\Delta} = \frac{(N-1)!}{(n-1)!(N-n-1)!}\left[\int_0^{F(\underline{\sigma})} \frac{(1-y)^{N-n-1}}{y^{1-n}} dy - \int_0^{F(\bar{\sigma})} \frac{y^{N-n-1}}{(1-y)^{1-n}} dy\right]$. Together with $\beta\left(\underline{\sigma}\right) = \beta\left(\bar{\sigma}\right) = \bar{b}$, with three equations and three unknowns, $\bar{\sigma}$, $\underline{\sigma}$, and $\bar{\sigma}$ are characterized. □

Proof of Proposition 9

Proposition 9. *If the equilibrium strategy $\beta(\cdot)$ for each non-colluding agent is strictly increasing, the leading agent decides the equilibrium colluding submission $\kappa_m(\cdot)$ for all the m colluding agents, which satisfies*

$$\kappa_m(s_i) = s_i - \frac{\Delta(\kappa_m(s_i);\theta)}{2\zeta\left(N-2n\right)}$$

where

$$\Delta(\kappa_m(s_i);\theta) = \sum_{j=1}^{m} \sum_{r=n-m+j}^{(N-m)-(n-m)-j} \begin{pmatrix} N-m \\ r \end{pmatrix} F^r\left(\beta^{-1}\left(\kappa_m(s_i)\right);\theta\right)\left[1-F\left(\beta^{-1}\left(\kappa_m(s_i)\right);\theta\right)\right]^{N-m-r}.$$

In the case when $m=1$, the equilibrium degenerates into the case with no collusion, so $\kappa_1(s_i) = \beta(s_i)$.

Proof. With a little abuse of notation, I let $s^{(i)} = s^{i:N-m}$ as the ith lowest bid from the $N-m$ banks that are not in the ring, instead of the N banks as in previous sections. The banks in the ring submit the same bid $\rho = \beta\left(k\right)$, pretending they are strategically behaved (but without collusion) as type k.

$$\begin{aligned}
\left(N-2n\right) E\left[L\right] \quad &= \quad \sum_{i=n+1}^{N-m-n} \beta\left(s^{(i)}\right) + \\
&+ \quad \sum_{j=1}^{m}\left\{\int_{\underline{s}}^{k} \beta\left(x\right) f_{n-m+j}^{(N-m)}\left(x\right) dx + \int_{k}^{\overline{s}} \beta\left(x\right) f_{(N-m)-(n-m)-j+1}^{(N-m)}\left(x\right) dx\right\} \\
&+ \quad \sum_{j=1}^{m} \beta\left(k\right)\left(F_{n-m+j}^{(N-m)}\left(k\right) - F_{(N-m)-(n-m)-j+1}^{(N-m)}\left(k\right)\right),
\end{aligned}$$

so

$$\frac{\partial}{\partial k}(N-2n)\,E[L]=\sum_{j=1}^{m}\beta'(k)\left(F_{n-m+j}^{(N-m)}(k)-F_{(N-m)-(n-m)-j+1}^{(N-m)}(k)\right).$$

Hence the optimal $\kappa=\beta(k)$ is that

$$\frac{\partial}{\partial\beta(k)}\left[E[L]-E\left[\gamma(\beta(k)-s)^2+\xi(\beta(k))\right]\right]=0,$$

from which I can get the equilibrium submission $\kappa=\beta(k)$. □

Proof of Proposition 10

Proposition 10. *The equilibrium strategy $\kappa_m(\cdot)$ characterized in Proposition 9 suggests that the excess benchmark bias introduced by collusion does not satisfy the distribution-free property in Proposition 2.*

Proof. Since $\kappa_m(s_i)$ depends on $\beta^{-1}(\cdot)$, and

$$\beta(s_i)=s_i-\frac{1}{2\zeta(N-2n)}\Delta(s_i)\leq s_i,$$

where

$$\Delta(s_i)=\frac{(N-1)!}{(n-1)!\,(N-n-1)!}\int_0^{F(s_i;\theta)}\frac{(1-y)^{N-2n}-y^{N-2n}}{[y(1-y)]^{1-n}}dy,$$

$\kappa_m(s_i)-s_i$ in general cannot be written as a function of only $F(s_i)$, but also as a function of s_i. As a result, I cannot treat $F(s_i;\theta)$ as an integration variable in the the expectation $\int_{\underline{s}}^{\overline{s}}\kappa_m(s_i)\,dF(s_i;\theta)$. Therefore, the integral depends on the parameter θ: the expected excess LIBOR bias introduced by collusion is not distribution-free. □

A.2 Discussion about Signaling

During the LIBOR benchmark fixing process, a bank might reveal his credit riskiness by reporting a high interbank borrowing cost, in addition to the level of systemic risk. Such a signal might concern the bank's short-term funding providers, who may close positions abruptly, leading to a costly fire sale of long-term assets as in Shleifer and Vishny (2011). Hence, banks reported lower than actual borrowing costs when the market was concerned about their creditworthiness.

To the best of my knowledge, signaling has only happened in LIBOR fixing so far. Nevertheless, in any survey, agents are able to signal through reporting as long as the public can identify an agent's report. Hence, it is worthwhile to check whether the distribution-free property of the expected benchmark bias still holds in a survey with a quadratic penalty.

Let agent i, by submitting b_i, incurs a signaling cost $\xi(b_i)$. I assume that $\xi : [\underline{s}, \overline{s}] \to \mathbb{R}^+$ is a strictly increasing, concave, and C^2 function that satisfies $\xi'(b_i) \to 0$ as $b_i \to 0$. In addition, I normalize $\xi(0) = 0$. I use a normalized quadratic functional form $\xi(b_i) = \xi b_i^2$ in the following analysis. Hence, the agent's payoff function under signaling becomes $u_i : [\underline{s}, \overline{s}] \times S \to \mathbb{R}$ as

$$u_i(s_i, b) \equiv -\zeta\delta(s_i - b_i)^2 - \xi b_i^2 - \delta \sum_{j=1}^{N} w_j b_j^{(N)}, \tag{12}$$

where the coefficient $w_j = \frac{1}{N-2n}$ for $n < j \leq N - n$, and $w_j = 0$ for all the other j.

Proposition A.4 characterizes the equilibrium submission strategy under signaling.

Proposition A.4. *If the probability density function of the nth-order statistics $f_n^{(N-1)}(\cdot)$ and $N-n$th-order statistics $f_{N-n}^{(N-1)}(\cdot)$ satisfies the condition for strictly increasing equilibrium, then the equilibrium strategy $\beta : [\underline{s}, \overline{s}] \to [\underline{s}, \overline{s}]$ for each agent i is strictly increasing and*

$$\beta(s_i) = \frac{\zeta\delta s_i}{\zeta\delta + \xi} - \frac{\delta}{2(\zeta\delta + \xi)(N - 2n)}\Delta(s_i)$$

where $\Delta(s_i)$ is

$$\Delta(s_i) = \frac{(N-1)!}{(n-1)!(N-n-1)!}\int_0^{F(s_i)} \frac{\left[(1-y)^{N-2n} - y^{N-2n}\right]}{[y(1-y)]^{1-n}} dy.$$

Proof. The first order derivative of the payoff function suggests the equilibrium is

$$\beta(s_i) = \frac{\zeta\delta s_i}{\zeta\delta + \xi} - \frac{\delta}{2(\zeta\delta + \xi)(N - 2n)}\left[F_n^{(N-1)}(s_i) - F_{N-n}^{(N-1)}(s_i)\right],$$

where

$$\Delta(s_i) = \frac{(N-1)!}{(n-1)!(N-n-1)!}\int_0^{F(s_i)} \frac{(1-y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy.$$

□

A sample equilibrium strategy is plotted in Figure 5.

Signaling causes an agent to report a lower borrowing cost, which leads to a larger benchmark bias. Signaling has another negative consequence: in general, the expected benchmark bias loses its distribution-free property when agents signal their credit quality in the benchmark fixing process, as shown in Proposition A.5. Therefore it is harder to correct the benchmark when agents have the incentive to signal.

Proposition A.5. *The equilibrium strategy $\beta(\cdot)$ characterized in Proposition A.4 suggests that the expected excess benchmark bias under signaling does not satisfy the distribution-free property in Proposition 2.*

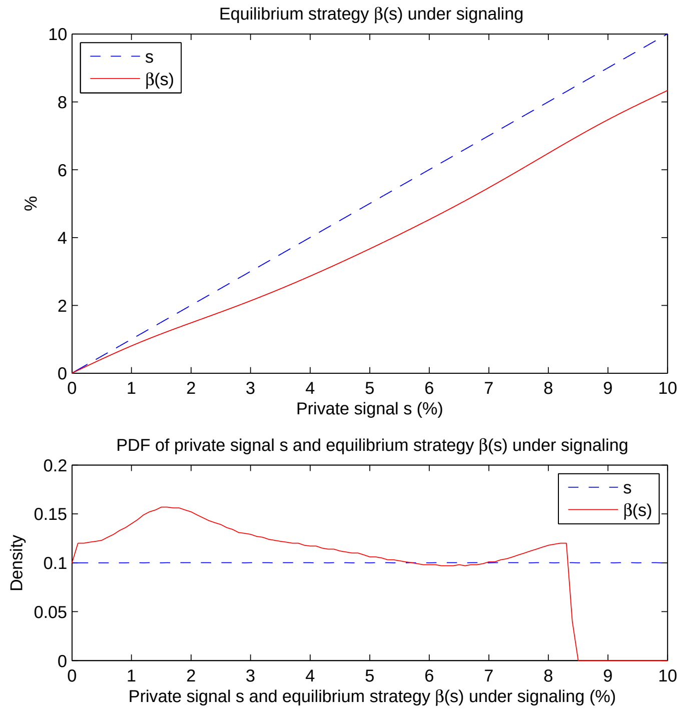

Figure 5: **Equilibrium strategy with signaling.** This figure shows the equilibrium strategy when the survey contains $N = 16$ agents with i.i.d. borrowing costs uniformly distributed on $[0\%, 10\%]$. The lowest and highest $n = 4$ bids are trimmed. The penalty is $\gamma(s_i, b_i) = 0.1 \cdot (s_i - b_i)^2$, the signaling parameter $\xi = 0.02$, and the sensitivity is normalized to $\delta = 1$. The upper subplot shows the equilibrium strategy $\beta(s)$ as a function of borrowing cost s. The lower subplot shows the distribution of borrowing costs and equilibrium submissions. The signaling causes the submissions of agents to be even lower.

Proof. As in the proof of Proposition 10, the benchmark error under signaling $E[L - L_0]$ is

$$\int_{\underline{s}}^{\overline{s}} \left[\ldots \left[\int_{\underline{s}}^{x_2} w_1 \rho(x_1) f_1^{(N)} \left(x_1 | s_2^{(N)} < x_2, \ldots, s_N^{(N)} < x_N \right) dx_1 \right] \ldots + w_N \rho(x_N) \right] f_N^{(N)}(x_N) \, dx_N,$$

where

$$\begin{aligned} \rho(x_1) &= \frac{\xi x_1}{\zeta\delta + \xi} + \frac{\delta}{2(\zeta\delta + \xi)(N - 2n)} \Delta(x_1) \\ &\ldots \\ \rho(x_N) &= \frac{\xi x_N}{\zeta\delta + \xi} + \frac{\delta}{2(\zeta\delta + \xi)(N - 2n)} \Delta(x_N). \end{aligned}$$

Although $\Delta(s_i^{(N)})$ can be represented as a function of $F(s_i)$, the integration variable x_i above suggests that the integral contains a term like $\int_0^{\overline{s}} x dF(x)$. Therefore, the expectation does depend on the distribution function $F(\cdot)$. $\square$

Similar to collusion, signaling could be fixed easily by limiting access to the agent's benchmark reports to regulators only. Without the information about agents' borrowing costs, the market participants would not be able to infer agents' types. Therefore, the principal can improve the accuracy of the benchmark by *reducing* the information available to the market.

References

Abrantes-Metz, R., M. Kraten, A. Metz, and G. Seow (2011). Libor manipulation? *Journal of Banking & Finance*.

Akella, S. R. and S. I. Greenbaum (1992). Innovations in interest rates, duration transformation, and bank stock returns. *Journal of Money, Credit and Banking 24*(1), 27–42.

Athey, S. (1998). Characterizing properties of stochastic objective functions. *MIT Department of Economics Working Paper*.

Athey, S. (2001). Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica 69*(4), 861–889.

Athey, S. (2002). Monotone comparative statics under uncertainty. *quarterly Journal of Economics*, 187–223.

Coulter, B., J. Shapiro, and P. Zimmerman (2017). A mechanism for libor. *Review of Finance*.

Coulter, B. and J. D. Shapiro (2014). A mechanism for libor. *Available at SSRN 2256952*.

Cremer, J. and R. P. McLean (1988). Full extraction of the surplus in bayesian and dominant strategy auctions. *Econometrica: Journal of the Econometric Society*, 1247–1257.

Dasgupta, P., P. Hammond, and E. Maskin (1979). The implementation of social choice rules: Some general results on incentive compatibility. *The Review of Economic Studies 46*(2), 185–216.

d'Aspremont, C., J. Cremer, and L.-A. Gérard-Varet (1990). Incentives and the existence of pareto-optimal revelation mechanisms. *Journal of Economic Theory 51*(2), 233–254.

d'Aspremont, C. and L. Gérard-Varet (1979). Incentives and incomplete information. *Journal of Public economics 11*(1), 25–45.

Duffie, D. and P. Dworczak (2014). Robust benchmark design. Technical report, National Bureau of Economic Research.

Duffie, D., P. Dworczak, and H. Zhu (2017). Benchmarks in search markets. *The Journal of Finance*.

Duffie, D. and J. C. Stein (2015). Reforming libor and other financial market benchmarks. *The Journal of Economic Perspectives 29*(2), 191–212.

Eisl, A., R. Jankowitsch, and M. G. Subrahmanyam (2014). The manipulation potential of libor and euribor. *Available at SSRN 2201013*.

Gandhi, P., B. Golez, J. C. Jackwerth, and A. Plazzi (2016). Financial market misconduct and public enforcement: The case of libor manipulation. *Available at SSRN 2342075*.

Gelfand, I. and S. Fomin (1963). Calculus of variations.

Gibbard, A. (1973). Manipulation of voting schemes: a general result. *Econometrica: journal of the Econometric Society*, 587–601.

Green, J. and J.-J. Laffont (1977). Characterization of satisfactory mechanisms for the revelation of preferences for public goods. *Econometrica: Journal of the Econometric Society*, 427–438.

Grove, M. A. (1974). On "duration" and the optimal maturity structure of the balance sheet. *The Bell Journal of Economics and Management Science*, 696–709.

Hartheiser, A. and P. Spieser (2010). Libor rate and financial crisis: has the libor rate been manipulated? *Journées de finance interécoles de commerces, Lyon: France*, 38.

Hou, D. and D. R. Skeie (2014). LIBOR: origins, economics, crisis, scandal, and reform. *FRB of New York Staff Report* (667).

Huber, P. J. (2011). *Robust statistics*. Springer Berlin Heidelberg.

IOSCO (2014, July). Review of the Implementation of IOSCO's Principles for Financial Benchmarks by Administrators of EURIBOR, LIBOR and TIBOR. Technical report.

IOSCO (2016, February). Second Review of the Implementation of IOSCO's Principles for Financial Benchmarks by Administrators of EURIBOR, LIBOR and TIBOR. Technical report.

King, T. and K. Lewis (2015). Credit risk, liquidity, and lies. *Working Paper, Federal Reserve Bank of Chicago*.

Kuo, D., D. Skeie, and J. Vickery (2012). A comparison of libor to other measures of bank borrowing costs. *Working Paper, Federal Reserve Bank of New York*.

Kuo, D., D. R. Skeie, J. I. Vickery, and T. Youle (2014). Identifying term interbank loans from fedwire payments data. *FRB of New York Staff Report* (603).

Lynge, M. J. and J. K. Zumwalt (1980). An empirical study of the interest rate sensitivity of commercial bank returns: A multi-index approach. *Journal of Financial and Quantitative analysis 15*(03), 731–742.

Maskin, E. (1999). Nash equilibrium and welfare optimality. *The Review of Economic Studies 66*(1), 23–38.

McAfee, R. P. and J. McMillan (1992). Bidding rings. *The American Economic Review*, 579–599.

McAfee, R. P., J. McMillan, and P. J. Reny (1989). Extracting the surplus in the common-value auction. *Econometrica: Journal of the Econometric Society*, 1451–1459.

McAfee, R. P. and P. J. Reny (1992). Correlated information and mecanism design. *Econometrica: Journal of the Econometric Society*, 395–421.

Milgrom, P. and C. Shannon (1994). Monotone comparative statics. *Econometrica: Journal of the Econometric Society*, 157–180.

Morrison, A. D. and L. White (2013). Reputational contagion and optimal regulatory forbearance. *Journal of Financial Economics 110*(3), 642–658.

Myerson, R. B. (1979). Incentive compatibility and the bargaining problem. *Econometrica: journal of the Econometric Society*, 61–73.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of operations research 6*(1), 58–73.

Samuelson, P. A. (1945). The effect of interest rate increases on the banking system. *The American Economic Review 35*(1), 16–27.

Shleifer, A. and R. Vishny (2011). Fire sales in finance and macroeconomics. *Journal of Economic Perspectives 25*(1), 29–48.

Snider, C. and T. Youle (2010). Does the libor reflect banks' borrowing costs? *Working Paper*.

Snider, C. A. and T. Youle (2012). The fix is in: Detecting portfolio driven manipulation of the libor.

Wheatley, M. (2012). *The Wheatley Review of LIBOR: final report.* HM Treasury.

Youle, T. (2014). How much did manipulation distort the libor? Technical report, Working Paper, University of Minnesota.