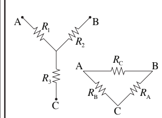
| Voltage (V) | Current (A) | Power (W) | Energy (J) | Series | Parallel | |
|---|---|---|---|---|---|---|
| $v_R = i_R R$ | $i_R = \frac{v_R}{R}$ | $P_R = \frac{v_R^2}{R} = i_R^2 R$ | $E_R = \int_{t_i}^{t} P_R \, d\tau$ | $R_{eq} = R_1 + R_2 + \cdots + R_n$ | $R_{eq}^{-1} = R_1^{-1} + R_2^{-1} + \cdots + R_n^{-1}$ | $R \ (\Omega)$ |
| $v_L = L\frac{di_L}{dt}$ | $i_L = \frac{1}{L}\int_{t_i}^{t} v_L \, d\tau + i_L(t_i)$ | $P_L = Li_L \frac{di_L}{dt}$ | $E_L = \frac{1}{2}Li^2$ | $L_{eq} = L_1 + L_2 + \cdots + L_n$ | $L_{eq}^{-1} = L_1^{-1} + L_2^{-1} + \cdots + L_n^{-1}$ | $L \ (\mathrm{H})$ |
| $v_C = \frac{1}{C}\int_{t_i}^{t} i_C \, d\tau + v_C(t_i)$ | $i_C = C\frac{dv_C}{dt}$ | $P_C = Cv_C \frac{dv_C}{dt}$ | $E_C = \frac{1}{2}Cv^2$ | $C_{eq}^{-1} = C_1^{-1} + C_2^{-1} + \cdots + C_n^{-1}$ | $C_{eq} = C_1 + C_2 + \cdots + C_n$ | $C \ (\mathrm{F})$ |

## Conservation Laws for Voltage, Current, and Power

| | |
|---|---|
| $\Sigma V_{rise} + \Sigma V_{drop} = 0$ | rise: - to +<br>drop: + to - |
| $\Sigma I_{in} + \Sigma I_{out} = 0$ | in: $\rightarrow$ node<br>out: $\leftarrow$ node |
| $\Sigma P_{del.} + \Sigma P_{con.} = 0$ | delivered: - to +<br>consumed: + to - |

## Y to Δ Transformation



$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \qquad R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \qquad R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \qquad R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$V_S = 0$ Short Circuit

$I_S = 0$ Open Circuit

## Voltage Division

$$v_j = \frac{R_j}{R_{eq}} V_S$$

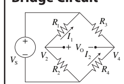
**Note:** $R_{eq}$ is the series resistance equivalent of the circuit, including $R_j$.

## Current Division

$$i_j = \frac{R_{eq}}{R_j + R_{eq}} I_S$$

**Note:** $R_{eq}$ is the parallel resistance equivalent of the circuit, not including $R_j$.

## Bridge Circuit

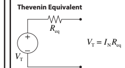


$V_O = 0$ when $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$V_O = V_2 - V_4$

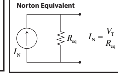
## Thevenin & Norton Equivalents

**Thevenin Equivalent**



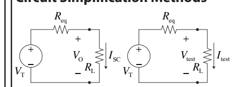
$V_T = I_N R_{eq}$

**Norton Equivalent**



$I_N = \frac{V_T}{R_{eq}}$

## Circuit Simplification Methods



$R_{eq} = \frac{V_O}{I_{SC}}$ $\quad R_{eq} = \frac{V_{test}}{I_{test}}$ $\quad P_{max}$ when $R_L = R_{eq}$; $\quad P_{R_L max}$ when $R_{eq} = 0$

$P_{R_L} = \frac{V_T^2}{4R_L}$

**Standard Method for All Sources**

1. Find $V_O$ for open circuit; $V_T = V_O$.
2. Find $I_{SC}$ for short circuit.
3. Find $R_{eq}$.

**Superposition Method for Independent Sources**

1. Suppress all independent sources except one and find $V_O$.
2. Repeat step 1 for each independent source that gives you $V_O$ for that particular source.
3. Sum all the $V_O$ values to calculate your final $V_O$.
4. Find $R_{eq}$ by removing all sources and using series and parallel simplifications.

**Test Source Method for Dependent Sources**

1. Find $V_O$ for open circuit.
2. Insert $V_{test}$ source equal to 1 in open circuit.
3. Find $I_{test}$.
4. Find $R_{eq}$.

## Natural and Step Responses of RLC Circuits

**Overdamped** $\quad \alpha^2 > \omega_0^2$

- V or I approaches its final value without oscillation.
- No Overshoot.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$x(0) = A_1 + A_2$$

$$\frac{dx(0)}{dt} = \frac{i_C(0)}{C} = A_1 s_1 + A_2 s_2$$

**Underdamped** $\quad \alpha^2 < \omega_0^2$

- V or I oscillates about its final value.
- Overshoot.

$$s_1 = -\alpha + j\omega_d \qquad s_2 = -\alpha - j\omega_d$$

$$x(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t)e^{-\alpha t}$$

$$x(0) = B_1$$

$$\frac{dx(0)}{dt} = \frac{i_C(0)}{C} = -\alpha B_1 + \omega_d B_2$$

**Critically Damped** $\quad \alpha^2 = \omega_0^2$

- V or I on the verge of oscillating about its final value.
- Almost Overshoots. $\quad s_1 = s_2 = -\alpha = -\frac{1}{2RC}$

$$x(t) = (D_1 t + D_2)e^{-\alpha t}$$

$$x(0) = D_2$$

$$\frac{dx(0)}{dt} = \frac{i_C(0)}{C} = D_1 - \alpha D_2$$

$$x(t) = \underbrace{X_f}_{\text{Step Response}} + \underbrace{(\mathrm{OD,UD,CD})}_{\text{Natural Response}}$$

$$x(0) = X_f + (\mathrm{OD,UD,CD})$$

**Characteristic Equation**

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

**Characteristic Roots**

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

| | |
|---|---|
| $\alpha_p = \frac{1}{2RC}$ | Neper $f$ for RLC Parallel |
| $\alpha_s = \frac{R}{2L}$ | Neper $f$ for RLC Series |
| $\omega_0 = \frac{1}{\sqrt{LC}}$ | Resonant Radian $f$ |
| $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ | Damped Radian $f$ |

## V-I Relationships

| | | |
|---|---|---|
| $\mathbf{V} = R\mathbf{I} = RI_m \angle \theta_i$ | Resistor | (V phase I) |
| $\mathbf{V} = j\omega L\mathbf{I} = j\omega L I_m \angle(\theta_i + 90°)$ | Inductor | (V leads I) |
| $\mathbf{V} = \frac{1}{j\omega C}\mathbf{I} = \frac{I_m}{\omega C}\angle(\theta_i - 90°)$ | Capacitor | (V trails I) |

## Sinusoidal Steady State

$$x = \underbrace{X_m \cos(\omega t + \phi)}_{\text{Sinusoidal Form}} = \underbrace{X_m \angle \phi}_{\text{Polar Form}} = \underbrace{X_m e^{j\phi}}_{\text{Phaser Form}}$$

$$\sin(\theta) = \cos(\theta - 90°) \qquad f = \frac{1}{T} \qquad \phi(t = 0) \qquad \phi_- \rightarrow \times\ \phi / \omega$$

$$\omega = 2\pi f \qquad \phi = \Delta t \times \frac{360°}{T} \qquad \phi_+ \leftarrow \times\ \phi / \omega$$

NOTE: $\Delta t$ is the distance between $t_i = 0$ and the $t_f$ where the closest positive peak amplitude is relative to the vertical axis.

| Impedance R(*Z*) | Reactance *j*(*X*) | Admittance R(*Y*) | Susceptance *j*(*W*) |
|---|---|---|---|
| $Z_R = R$ | $X_R = -$ | $Y_R = G$ | $W_R = -$ |
| $Z_L = j\omega L$ | $X_L = \omega L$ | $Y_L = j(-1/\omega L)$ | $W_L = -1/\omega L$ |
| $Z_C = j(-1/\omega C)$ | $X_C = -1/\omega C$ | $Y_C = j\omega C$ | $W_C = \omega C$ |

- Impedance is the value of resistive measure of that component.
- Admittance is the value of conductive measure of that component.

$Z = R + jX$ (ohms)

$Y = G + jW$ (siemens)

## Node Voltage Analysis

1. Identify the nodes.
2. Assign ground and circuit variables.
3. Derive equations for independent voltage sources.
4. Derive equations using current law at nodes.
5. Derive equations for dependent sources.

## Mesh Current Analysis

1. Identify the meshes.
2. Assign mesh current variables.
3. Derive equations for independent current sources.
4. Derive equations using voltage law for each mesh.
5. Derive equations for dependent sources.

### Dot Assignment Method for Mutual Inductance

1. Select one terminal of either coil and mark with a dot.
2. Assign current $I_1$ into that dot.
3. Use Right-Hand Rule to determine direction of magnetic field $ø_1$.
4. Select one terminal of the other coil.
5. Assign current $I_2$ into that terminal.
6. Use Right-Hand Rule to determine direction of magnetic field $ø_2$.
7. Compare the direction of $ø_1$ and $ø_2$ as follows:

- If $ø_1$ and $ø_2$ are in the same direction, place the second dot where $I_2$ enters.
- If $ø_1$ and $ø_2$ are not in the same direction, place the second dot where $I_2$ leaves.

### RL and RC Circuit Method of Analysis

| | $t=\infty$ | $t \geq 0$ |
|---|---|---|
| $L$ | S.C. | $I_S$ |
| $C$ | O.C. | $V_S$ |

1. Identify the variable of interest.
- For $RC$, choose $V_C$.
- For $RL$, choose $I_L$.
2. Deterine the initial value(s) by drawing circuit at $t=0$ and solve for $R$, $L$, or $C$ equivalent.
- $t_i$ : $x(0^-) = x(0) = x(0^+)$
3. Determine the final value(s) by drawing circuit at $t = 0^+$ and $t = \infty$.
- $t_f$ : $x(\infty)$
4. Calculate the time constant. $X(t) = X(\infty) + \left[X(0) - X(\infty)\right]e^{\frac{-(t-t_i)}{\tau}}$

NOTE: $R$ values could be negative if dependent sources are involved.

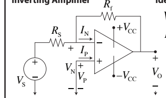
**Time Constants**

$$\tau_L = \frac{L}{R}$$

$$\tau_C = RC$$

## Operational Amplifers

### Inverting Amplifier



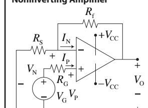
$$V_O = \frac{-R_f}{R_S}V_S$$

### Ideal Op-Amp Laws

$$V_P = V_N$$

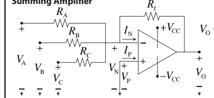
$$I_P = I_N = 0$$

### Noninverting Amplifier



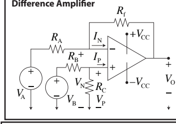
$$V_O = \frac{R_S + R_f}{R_S}V_G$$

### Summing Amplifier



$$V_O = -\left(\frac{R_f}{R_A}V_A + \frac{R_f}{R_B}V_B + \frac{R_f}{R_C}V_C\right)$$

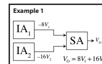
### Difference Amplifier



$$V_O = \frac{R_f}{R_A}(V_B - V_A)$$

$$\frac{R_A}{R_f} = \frac{R_B}{R_C}$$

**Example 1**



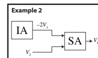
$V_O = 8V_1 + 16V_2$

$R_{f1} = 80\ \Omega$ $R_{S1} = 10\ \Omega$

$R_{f2} = 160\ \Omega$ $R_{S2} = 10\ \Omega$

$R_A = R_B = R_f = 100\ \Omega$

**Example 2**



$V_O = 2V_1 - \frac{1}{2}V_2$

$R_{f1} = 200\ \Omega$ $R_{S1} = 100\ \Omega$

$R_f = R_A = 10\ \Omega$ $R_B = 20\ \Omega$

### Straight Line Equation

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

| Prefix | Symbol | Power |
|---|---|---|
| atto | a | $10^{-18}$ |
| femto | f | $10^{-15}$ |
| pico | p | $10^{-12}$ |
| nano | n | $10^{-9}$ |
| micro | $\mu$ | $10^{-6}$ |
| milli | m | $10^{-3}$ |
| centi | c | $10^{-2}$ |
| deci | d | $10^{-1}$ |
| deka | da | 10 |
| hecto | h | $10^{2}$ |
| kilo | k | $10^{3}$ |
| mega | M | $10^{6}$ |
| giga | G | $10^{9}$ |
| tera | T | $10^{12}$ |

### Rectangular | Polar | Exponential Conversions

$$a + jb = \sqrt{a^2 + b^2}\,e^{j\theta}$$

$$ce^{j\theta} = c\angle\theta° \qquad \tan^{-1}\left(\frac{b}{a}\right) = \theta°$$

$$ce^{j\theta} = c(\cos\theta + j\sin\theta)$$

$$c\angle\theta° = c(\cos\theta + j\sin\theta)$$

**Euler's Identity**

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

### Standard Circuit Simplification for One Source

1a. If $V_S$ is known, use mesh current analysis to find source current.
1b. If $I_S$ is known, use node voltage analysis to find source voltage.
2. Calculate $R_{eq}$.

## Root Mean Square and Power

Converts AC to DC equivalent. $\text{rms} = \text{eff}$ (effective value)

$$X_{\text{rms}} = \sqrt{\frac{1}{T}\int_{t_i}^{T} x^2\, dt}$$

**Apparent Power (VA)**

$$|S| = \sqrt{P^2 + Q^2}$$

**Power Source for RMS**

$$S_S = -V_{\text{eff}}I_{\text{eff}}^{*} = -V_{\text{eff}}I_{\text{eff}}\angle(\theta_v - \theta_i)$$

NOTE: Results in a $P$ and $Q$ value.
* is the conjugate form of that complex expression; i.e. change the sign of the complex value.

**Average and Reactive Power for RMS**

$$P = V_{\text{eff}}\, I_{\text{eff}}\, \cos(\theta_v - \theta_i)$$

$$Q = V_{\text{eff}}\, I_{\text{eff}}\, \sin(\theta_v - \theta_i)$$

**Power of Impedance Component for RMS**

$$S_Z = |I_{\text{eff}}|^2 Z = \frac{|V_{\text{eff}}|^2}{Z^*}$$

$$P_Z = |I_{\text{eff}}|^2 R = \frac{I_m^2 R}{2} \qquad Q_Z = |I_{\text{eff}}|^2 X = \frac{I_m^2 X}{2}$$

$$P_Z = \frac{|V_{\text{eff}}|^2}{R} = \frac{V_m^2}{2R} \qquad Q_Z = \frac{|V_{\text{eff}}|^2}{X} = \frac{V_m^2}{2X}$$

$$X_{\text{eff}} = \frac{X_m}{\sqrt{2}} \quad \text{Sine}$$

$$X_{\text{eff}} = X_m \quad \text{Square}$$

$$X_{\text{eff}} = \frac{X_m}{\sqrt{3}} \quad \text{Triangle}$$

$$X_{\text{eff}} = \frac{X_m}{\sqrt{3}} \quad \text{Sawtooth}$$

## Power for Sinusoidal Steady-State

**Average (Real) Power (W), Energy Released**

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

**Reactive (Complex) Power (VARS), Energy Stored**

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

**Complex Power (VA)**

$$S = P + jQ = \frac{V_m I_m}{2}\angle(\theta_v - \theta_i)$$

**Power for Purely Resistive Circuits**

$$P_R = \frac{V_m I_m}{2} \quad Q_R = - \quad (\text{V phase I})$$

**Instantaneous Power**

$$p = \pm vi$$

**Power for Purely Inductive Circuits**

$$P_L = -Q_L \sin 2\omega t \quad Q_L = \frac{V_m I_m}{2} \quad (\text{V leads I by } 90°)$$

**Power for Purely Capacitive Circuits**

$$P_C = -Q_C \sin 2\omega t \quad Q_L = -\frac{V_m I_m}{2} \quad (\text{V trails I by } 90°)$$

**Power Factor**

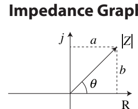
$$\text{pf} = \cos(\theta_v - \theta_i)$$

**Reactive Factor**

$$\text{rf} = \sin(\theta_v - \theta_i)$$

NOTE: If $\theta v$ is greater than $\theta i$, then pf is **lagging**. If $\theta v$ is less than $\theta i$, then pf is **leading**.

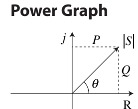
## Impedance Graph




$$Z = a + jb$$

$$|Z| = \sqrt{a^2 + b^2}$$

## Power Graph




$$S = P + jQ$$

$$|S| = \sqrt{P^2 + Q^2}$$

## Maximum Power Transfer

The maximum transfer of power between the source components and the load components is achieved when...

$$Z_L = Z_{Th}^{*} \qquad R_L = R_{Th}$$

If the magnitude of ZL can be varied but its phase angle cannot, maximum transfer of power to the load is...

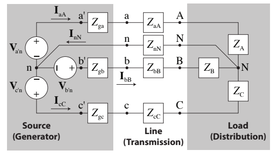
$$|Z_L| = |Z_{Th}|$$

NOTE: If one component is set as a **variable component**, the other component will achieve maximum power when the variable component is set to zero.

$$p_{\max} = \frac{|V_{\text{eff}}|^2}{4R_L} = \frac{V_m^2}{8R_L}$$

## Three-Phase Circuits (Balanced)

Three-Phase Circuit




Balanced Circuit

$$\mathbf{V}_{a'n} = \mathbf{V}_{b'n} = \mathbf{V}_{c'n}$$
$$Z_{ga} = Z_{gb} = Z_{gc}$$
$$Z_{aA} = Z_{bB} = Z_{cC}$$
$$Z_A = Z_B = Z_C$$
$$\mathbf{V}_n = \mathbf{V}_N = \mathbf{I}_{nN} = 0$$

Line Currents for Wye-Load

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n} - \mathbf{V}_N}{Z_A + Z_{aA} + Z_{ga}} = \frac{\mathbf{V}_{a'n}}{Z_\phi}$$
$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{b'n} - \mathbf{V}_N}{Z_B + Z_{bB} + Z_{gb}} = \frac{\mathbf{V}_{b'n}}{Z_\phi}$$
$$\mathbf{I}_{cC} = \frac{\mathbf{V}_{c'n} - \mathbf{V}_N}{Z_C + Z_{cC} + Z_{gc}} = \frac{\mathbf{V}_{c'n}}{Z_\phi}$$

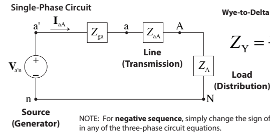
Line Voltages, Positive Sequence

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = V_\phi\angle 0° - V_\phi\angle -120° = \sqrt{3}V_\phi\angle 30°$$
$$\mathbf{V}_{BC} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = V_\phi\angle -120° - V_\phi\angle +120° = \sqrt{3}V_\phi\angle -90°$$
$$\mathbf{V}_{CA} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = V_\phi\angle +120° - V_\phi\angle 0° = \sqrt{3}V_\phi\angle 150°$$

Line Currents for Delta-Load, Positive Sequence

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = I_\phi\angle 0° - I_\phi\angle 120° = \sqrt{3}I_\phi\angle -30°$$
$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB} = I_\phi\angle -120° - I_\phi\angle 0° = \sqrt{3}I_\phi\angle -150°$$
$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = I_\phi\angle 120° - I_\phi\angle -120° = \sqrt{3}I_\phi\angle 90°$$

Single-Phase Circuit




Wye-to-Delta Load

$$Z_Y = \frac{Z_\Delta}{3}$$

Load Currents for Delta-Load

$$\mathbf{I}_{AB} = \left(\frac{1}{\sqrt{3}}\angle 30°\right)\mathbf{I}_{aA}$$
$$\mathbf{I}_{BC} = \left(\frac{1}{\sqrt{3}}\angle 30°\right)\mathbf{I}_{bB}$$
$$\mathbf{I}_{CA} = \left(\frac{1}{\sqrt{3}}\angle 30°\right)\mathbf{I}_{cC}$$

Power for Wye-Load

$$V_\phi = |\mathbf{V}_{AN}| = |\mathbf{V}_{BN}| = |\mathbf{V}_{CN}| \qquad I_\phi = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|$$
$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}$$

Power for Delta-Load

$$V_\phi = |\mathbf{V}_{AB}| = |\mathbf{V}_{BC}| = |\mathbf{V}_{CA}| \qquad I_\phi = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$
$$\theta_\phi = \theta_{vAB} - \theta_{iAB} = \theta_{vBC} - \theta_{iBC} = \theta_{vCA} - \theta_{iCA}$$

NOTE: For **negative sequence**, simply change the sign of the angles in any of the three-phase circuit equations.

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{AN}}{Z_A} \quad \mathbf{V}_{an} = Z_{aA}\mathbf{I}_{aA} + \mathbf{V}_{AN} \quad \mathbf{V}_{a'n} = Z_{ga}\mathbf{I}_{aA} + \mathbf{V}_{an}$$
$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{BN}}{Z_B} \quad \mathbf{V}_{bn} = Z_{bB}\mathbf{I}_{bB} + \mathbf{V}_{BN} \quad \mathbf{V}_{b'n} = Z_{gb}\mathbf{I}_{bB} + \mathbf{V}_{bn}$$
$$\mathbf{I}_{cC} = \frac{\mathbf{V}_{CN}}{Z_C} \quad \mathbf{V}_{cn} = Z_{cC}\mathbf{I}_{cC} + \mathbf{V}_{CN} \quad \mathbf{V}_{c'n} = Z_{gc}\mathbf{I}_{cC} + \mathbf{V}_{cn}$$

ABC (Positive) Phase Sequence

$$\mathbf{V}_a = V_m\angle 0°$$
$$\mathbf{V}_b = V_m\angle -120°$$
$$\mathbf{V}_c = V_m\angle +120°$$

ACB (Negative) Phase Sequence

$$\mathbf{V}_a = V_m\angle 0°$$
$$\mathbf{V}_b = V_m\angle +120°$$
$$\mathbf{V}_c = V_m\angle -120°$$

Power for Three-Phase Circuits

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi\mathbf{I}_\phi^* \quad S_\phi = \mathbf{V}_{AN}\mathbf{I}_{aA}^* = \mathbf{V}_{BN}\mathbf{I}_{bB}^* = \mathbf{V}_{CN}\mathbf{I}_{cC}^* \quad S_T = 3S_\phi = 3V_\phi I_\phi\angle\theta_\phi^\circ = \sqrt{3}V_L I_L\angle\theta_\phi^\circ$$
$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi\cos\theta_\phi \quad P_T = 3P_\phi = 3V_\phi I_\phi\cos\theta_\phi = \sqrt{3}V_L I_L\cos\theta_\phi$$
$$Q_A = Q_B = Q_C = Q_\phi = V_\phi I_\phi\sin\theta_\phi \quad Q_T = 3Q_\phi = 3V_\phi I_\phi\sin\theta_\phi = \sqrt{3}V_L I_L\sin\theta_\phi$$

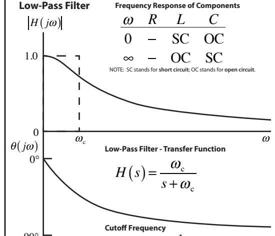
NOTE: The variables $V_L$ and $I_L$ represent the line voltage and line current in rms respectively.

Instantaneous Power for Three-Phase Circuits

$$p_A = v_{AN}i_{aA} = V_m I_m\cos\omega t\cos(\omega t - \theta_\phi) \qquad p_T = p_A + p_B + p_C = 1.5V_m I_m\cos\theta_\phi \quad V_m = \sqrt{2}V_\phi \quad I_m = \sqrt{2}I_\phi$$
$$p_B = v_{BN}i_{bB} = V_m I_m\cos(\omega t - 120°)\cos(\omega t - \theta_\phi - 120°)$$
$$p_C = v_{CN}i_{cC} = V_m I_m\cos(\omega t + 120°)\cos(\omega t - \theta_\phi + 120°)$$

## Frequency Selective Circuits (Passive)

### Low-Pass Filter




Frequency Response of Components

| $\omega$ | $R$ | $L$ | $C$ |
|---|---|---|---|
| 0 | – | SC | OC |
| $\infty$ | – | OC | SC |

NOTE: SC stands for **short circuit**; OC stands for **open circuit**.

Low-Pass Filter - Transfer Function

$$H(s) = \frac{\omega_c}{s + \omega_c}$$

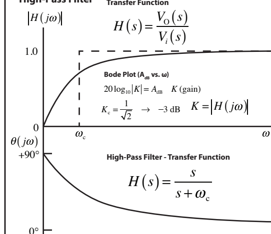
Cutoff Frequency

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}}H_{max}$$

### High-Pass Filter

Transfer Function

$$H(s) = \frac{V_O(s)}{V_i(s)}$$




Bode Plot ($A_{dB}$ vs. ω)

$$20\log_{10}|K| = A_{dB} \quad K \text{ (gain)}$$
$$K_c = \frac{1}{\sqrt{2}} \rightarrow -3\text{ dB} \quad K = |H(j\omega)|$$

High-Pass Filter - Transfer Function

$$H(s) = \frac{s}{s + \omega_c}$$

Low-Pass Filter - Type A

$$V_i \cdots sL \cdots R$$
$$V_O = V_R$$
$$\omega_c = R/L$$

Low-Pass Filter - Type B

$$V_i \cdots R \cdots \frac{1}{sC}$$
$$V_O = V_{1/sC}$$
$$\omega_c = 1/RC$$

High-Pass Filter - Type A

$$V_i \cdots \frac{1}{sC} \cdots R$$
$$V_O = V_R$$
$$\omega_c = 1/RC$$

High-Pass Filter - Type B

$$V_i \cdots R \cdots sL$$
$$V_O = V_{sL}$$
$$\omega_c = R/L$$

Low-Pass Filter - Transfer Function (Magnitude & Angle)

$$|H(j\omega)| = \frac{\omega_c}{\sqrt{\omega^2 + \omega_c^2}}$$
$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

High-Pass Filter - Transfer Function (Magnitude & Angle)

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \omega_c^2}}$$
$$\theta(j\omega) = 90° - \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

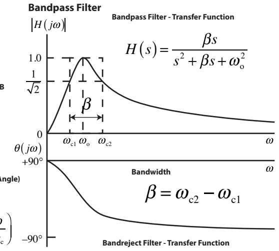
Bandpass Filter - Transfer Function (Magnitude & Angle)

$$|H(j\omega)| = \frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega\beta^2}}$$
$$\theta(j\omega) = 90° - \tan^{-1}\left(\frac{\omega\beta}{\omega_o^2 - \omega^2}\right)$$

Bandreject Filter - Transfer Function (Magnitude & Angle)

$$|H(j\omega)| = \frac{|\omega_o^2 - \omega^2|}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega\beta^2}}$$
$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega\beta}{\omega_o^2 - \omega^2}\right)$$

### Bandpass Filter




Bandpass Filter - Transfer Function

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_o^2}$$

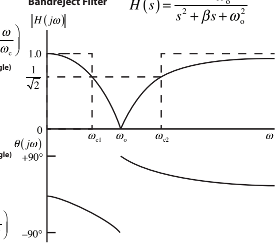
Bandwidth

$$\beta = \omega_{c2} - \omega_{c1}$$

### Bandreject Filter

Bandreject Filter - Transfer Function

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$




Bandpass Filter - Type A

$$V_i \cdots sL \cdots \frac{1}{sC} \cdots R \qquad V_O = V_R$$
$$\omega_o = \sqrt{1/LC} \quad \beta = R/L$$

Bandpass Filter - Type B

$$V_i \cdots R \cdots \left(\frac{1}{sC} \parallel sL\right) \qquad V_O = V_{sL}$$
$$\omega_o = \sqrt{1/LC} \quad \beta = 1/RC$$

Bandreject Filter - Type A

$$V_i \cdots R \cdots sL \cdots \frac{1}{sC} \qquad V_O = V_{sL} + V_{1/sC}$$
$$\omega_o = \sqrt{1/LC} \quad \beta = R/L$$

Bandreject Filter - Type B

$$V_i \cdots \left(sL \parallel \frac{1}{sC}\right) \cdots R \qquad V_O = V_R$$
$$\omega_o = \sqrt{1/LC} \quad \beta = 1/RC$$

Cutoff Frequency, Lower

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Cutoff Frequency, Upper

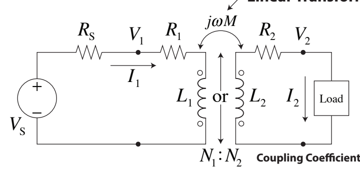
$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Quality Factor

$$Q = \frac{\omega_o}{\beta} = \sqrt{\frac{L}{R^2 C}}$$

# Transformers

Linear Transformer




Ideal Transformer

NOTE: $L_{SC}$ and $L_{OC}$ is the inductor in the source loop when the other inductor is in the load loop.

**Linear Transformer Laws**

$$Z_{11} = R_S + R_1 + j\omega L_1$$

$$Z_{22} = R_2 + j\omega L_2 + R_L$$

$$I_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2} V_S$$

$$I_2 = \frac{j\omega M}{Z_{11}Z_{22} + \omega^2 M^2} V_S$$

**Coupling Coefficient**

$$k = \sqrt{1 - \frac{L_{SC}}{L_{OC}}}$$

$$M = k\sqrt{L_1 L_2}$$

**Mutual Inductance**

**Reflected Impedance**

$$Z_L = R_L + jX_L$$

$$Z_r = \frac{\omega^2 M^2}{R_2 + R_L + j(\omega L_2 + X_L)}$$

$$Z_r = \frac{\omega^2 M^2}{\|Z_{22}\|^2}\left[(R_2 + R_L) - j(\omega L_2 + X_L)\right]$$

$$\|Z_{22}\|^2 = (R_2 + R_L)^2 - (\omega L_2 + X_L)^2$$

**Ideal Transformer Laws** $k = 1$

$$(9.76) \quad \frac{V_L}{V_S} = \pm\frac{v_L}{v_S} \qquad \frac{V_1}{N_1} = \pm\frac{V_2}{N_2}$$

$$(9.77) \quad I_S i_S = \pm I_L i_L \qquad I_1 N_1 = \pm I_2 N_2$$

**Impedance Matching**

$$Z_{11} = \frac{V_1}{I_1} = \frac{1}{a^2}\frac{V_2}{I_2} \qquad a = \frac{N_2}{N_1}$$

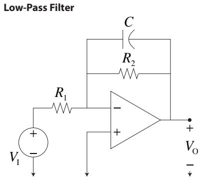
## Dot Assignment Rules

Assuming all currents are referenced as clockwise:

- If the dots are diagonal from each other, then *M* is positive.
- If the dots are across from each other, then *M* is negative.
- If the coil voltages V1 and V2 are both positive or negative at the dot-marked terminal, then Eq. 9.76 is positive; otherwise negative.
- If the coil curretns I1 and I2 are both directed into or out of the dot-makred terminal, then Eq. 9.77 is negative; otherwise positive.

# Frequency Selective Circuits (Active)

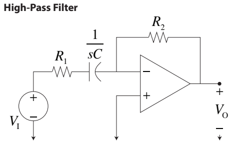
### Low-Pass Filter




$$H(s) = -K\frac{\omega_c}{s + \omega_c} \qquad K = \frac{R_2}{R_1}$$

$$\omega_c = \frac{1}{R_2 C} \qquad \omega_c = 2\pi f_c$$

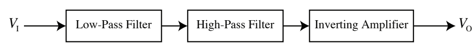
### High-Pass Filter




$$H(s) = -K\frac{s}{s + \omega_c} \qquad K = \frac{R_2}{R_1}$$

$$\omega_c = \frac{1}{R_1 C} \qquad \omega_c = 2\pi f_c$$

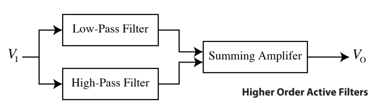
### Bandpass Filter




$$H(s) = -K\frac{\omega_{c2}s}{s^2 + \omega_{c2}s + \omega_{c1}\omega_{c2}} \qquad K = \frac{R_f}{R_i}$$

$$\omega_{c1} = \frac{1}{R_H C_H} \qquad \omega_{c2} = \frac{1}{R_L C_L}$$

### Bandreject Filter




$$H(s) = K\frac{s^2 + 2\omega_{c1}s + \omega_{c1}\omega_{c2}}{(s + \omega_{c1})(s + \omega_{c2})} \qquad K = \frac{R_f}{R_i}$$

$$\omega_{c1} = \frac{1}{R_L C_L} \qquad \omega_{c2} = \frac{1}{R_H C_H}$$

### Broadband Filter

$$\frac{\omega_{c2}}{\omega_{c1}} \geq 2$$

### Higher Order Active Filters

| Order | Slope |
|---|---|
| 1 | ±20 dB/dec |
| 2 | ±40 dB/dec |
| 3 | ±60 dB/dec |
| 4 | ±80 dB/dec |

NOTE: The order of the filter is determined by the number of LFP and HPF used which in turn are connected in series to increase the slope of the cutoff frequency.

Example:
A Bandpass Filter with two LFP and two HPF is therefore a second-order filter.

### Scaling

$$R' = k_m R \qquad L' = k_m L \qquad C' = C / k_m$$

$$R' = R \qquad L' = L / k_f \qquad C' = C / k_f \qquad k_f = \frac{\omega'_c}{\omega_c}$$

$$R' = k_m R \qquad L' = \frac{k_m}{k_f} L \qquad C' = \frac{1}{k_m k_f} C$$

### Bode Plot (Low-Pass Filter)

Gain (dB) = $20\log_{10}\frac{V_O}{V_I}$