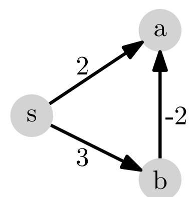
C-14.2 Give an example of an weighted directed graph, G, with negative-weight edges but no negative-weight cycle, such that Dijkstra's algorithm incorrectly computes the shortest-path distances from some start vertex v.

Solution. In this example, $s \to a$ and $s \to b$ will be relaxed first (these two can be relaxed in any order). Then a, having the lower distance estimate to s, will be pulled off the priority queue, with no outgoing edges to relax. Finally, b will be pulled off, again with no edges to relax since its only outgoing edge is to a vertex that has already been examined. This implies that Dijkstra's algorithm will determine a's shortest path to be $s \to a$ rather than $s \to b \to a$.



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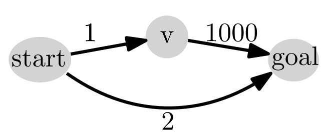
C-14.4 Consider the following greedy strategy for finding a shortest path from vertex *start* to vertex *goal* in a given connected graph.

(a) Initialize *path* to *start*.
(b) Initialize *VisitedVertices* to $\{start\}$.
(c) If $start = goal$, return *path* and exit. Otherwise, continue.
(d) Find the edge $(start, v)$ of minimum weight such that v is adjacent to *start* and v is not in *VisitedVertices*.
(e) Add v to *path*.
(f) Add v to *VisitedVertices*.
(g) Set *start* equal to v and go to Step 3.

Does this greedy strategy always find a shortest path from *start* to *goal*? Either explain intuitively why it works, or give a counter example.

Solution. In the following example, the greedy algorithm will return path *start*→v→*goal*, which has length 1001 and is not the shortest path, which is actually the direct path *start*→*goal* with length 2.



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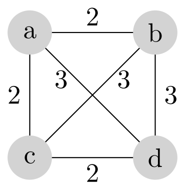
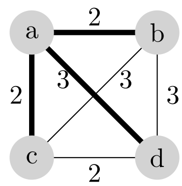
R-15.9 Give an example of weighted, connected, undirected graph, G, such that the minimum spanning tree for G is different from every shortest-path tree rooted at at a vertex of G.

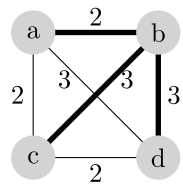
Solution. Here is an example.



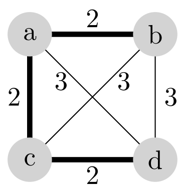
original graph



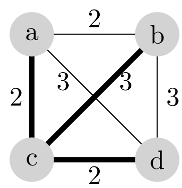
shortest path tree rooted at a



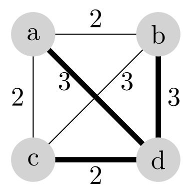
shortest path tree rooted at b



minimum spanning tree (the only one in this graph)



shortest path tree rooted at c



shortest path tree rooted at d

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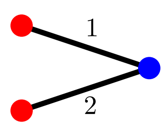
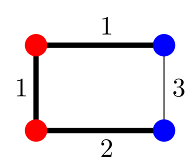
R-15.10 Let G be a weighted, connected, undirected graph, and let V_1 and V_2 be a partition of the vertices of G into two disjoint nonempty sets. Furthermore, let e be an edge in the minimum spanning tree for G such that e has one endpoint in V_1 and the other in V_2. Give an example that shows that e is not necessarily the smallest-weight edge that has one endpoint in V_1 and the other in V_2.

Solution. Here are two valid examples in which the edge of weight 2 is the edge e required by the problem. The edges of the minimum spanning tree are bold and the vertex partitioning is represented by the vertex colors. (Notice that the minimum spanning tree of a tree is the tree itself.)





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