Sets

CS 55 - Spring 2016 - Pomona College
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Introduction to Sets

Section 1.4

Review question 1

Translate into a logical expression:

Every non-negative number x is the square of some other number.

Is this proposition true or false?

Review question 2

The set with no elements is called the **empty set** denoted by $\varnothing$.

The empty set is unique! Why?

Introduction to Sets

- empty set
- set equality
- cardinality
- subset
- power set
- tuple
- cartesian product
- set builder notation

Naive Set Theory

A **set** is thought of as collection collection of objects.

The objects in a set are also called **elements**, or **members**, of the set. A set is said to **contain** its elements.

We use the notation:

$$e \in S$$

Set Equality

Two sets are said to be **equal** if and only if they have the same elements.

$$\forall x (x \in A \leftrightarrow x \in B)$$

Cardinality

Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that

S is a **finite set**, and

n is the **cardinality** of S.

The cardinality of S is denoted by $|S|$

Puzzle

Using only the symbols { } , $\varnothing$
find a set…

- with cardinality 0
- with cardinality 1
- with cardinality 2
- with cardinality 3

Subset

A is a **subset** of B if and only if every element of A is an element of B.

$$A \subseteq B$$

The empty set is a subset of every set. Why?

Power set

Given a set S, the **power set** of S is the set of all subsets of the set S. The power set of S is denoted by

P(S)

Ordered n-tuple

An **ordered n-tuple**

$$(a_1, a_2, \ldots, a_n)$$

is the ordered collection that has

a_1 as its first element,
a_2 as its second element,
…., and
a_n as its nth element.

Set builder notation

Set of all even numbers between 0 and 10

Set of all even numbers between 0 and 100

Set of all even numbers

Set of all x such that P(x)

Set Operations

Section 1.5

Common Operations

- Set **union**: $x \in (A \cup B) \Leftrightarrow x \in A \text{ or } x \in B$
- Set **intersection**: $x \in (A \cap B) \Leftrightarrow x \in A \text{ and } x \in B$
- Set **difference**: $x \in (A \setminus B) \Leftrightarrow x \in A \text{ and } x \notin B$
- Set **complement:** $\bar{A} = U \setminus A$

We assume all sets are subsets of the universe U.

Prove the Following

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $\overline{A \cup B} = \bar{A} \cap \bar{B}$
3. $A \cap \bar{A} = \emptyset$
4. $A \cap B \subseteq A$
5. If $A \cup B = A$, then $B \subseteq A$.

   $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Next time… quiz

- Logic
- Sets