

# First-Order Logic and Sets

CS 55 - Spring 2016 - Pomona College  
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## Review questions

What is a shorter expression that is logically equivalent to:

$$\neg P \vee Q$$

Find another logically equivalent expression by using

1. the double negation identity, and

2. De Morgan's law

## Review question

Is this a proposition?

$$2x + 1 = 4$$

## First-Order Logic

Section 1.3

## Propositional Functions

If we allow variables in a proposition, we get what is called a **propositional functions**. *The truth value of a propositional function is determined when all variables have been assigned a value.*

Ex:  $P(x): x > 3$

$P(5)$ : True    and     $P(2)$ : False

## Universe of Discourse

The domain of a propositional function is called the **universe of discourse** and consists of all values the variables may be assigned.

## Universal Quantification

The **universal quantification** of  $P(x)$  is the proposition:

“ $P(x)$  is true for all values of  $x$  in the universe of discourse”

We use the notation:

$$\forall x P(x)$$

## Existential Quantification

The **existential quantification** of  $P(x)$  is the proposition:

“There exists an element  $x$  in the universe of discourse such that  $P(x)$  is true.”

We use the notation:

$$\exists x P(x)$$

## Examples on Board

## Translating to English

$C(x)$ : "x is a computer science major"

$F(x,y)$ : "x and y are friends"

$$\forall x(C(x) \vee \exists y(C(y) \wedge F(x,y)))$$

## Practice: Translating from English

1. Every computer science student needs a course in discrete mathematics.
2. All students in the class has received an email or text message from another student in the class
3. There is a computer science student who needs a course in curling.
4. There exists a unique real number that is neither positive nor negative.

## Expressing uniqueness

"There exists a unique x such that  $P(x)$ "

is equivalent to the two combined statements:

there exists x such that  $P(x)$  and  
for all y such that  $P(y)$ ,  $x = y$ .

$$\exists x(P(x) \wedge \forall y(P(y) \rightarrow (x = y)))$$

## Quantification Order

$$\forall x \exists y (\dots) \neq \exists y \forall x (\dots)$$

## De Morgan's Law

$$\neg \forall x P(x) \Leftrightarrow \exists x \neg P(x)$$

$$\neg \exists x Q(x) \Leftrightarrow \forall x \neg Q(x)$$

## Introduction to Sets

Section 1.4

## Naive Set Theory

A **set** is thought of as collection collection of objects.

The objects in a set are also called **elements**, or **members**, of the set. A set is said to **contain** its elements.

We use the notation:

$$e \in S$$

## Set Equality

Two sets are said to be **equal** if and only if they have the same elements.

$$\forall x(x \in A \leftrightarrow x \in B)$$