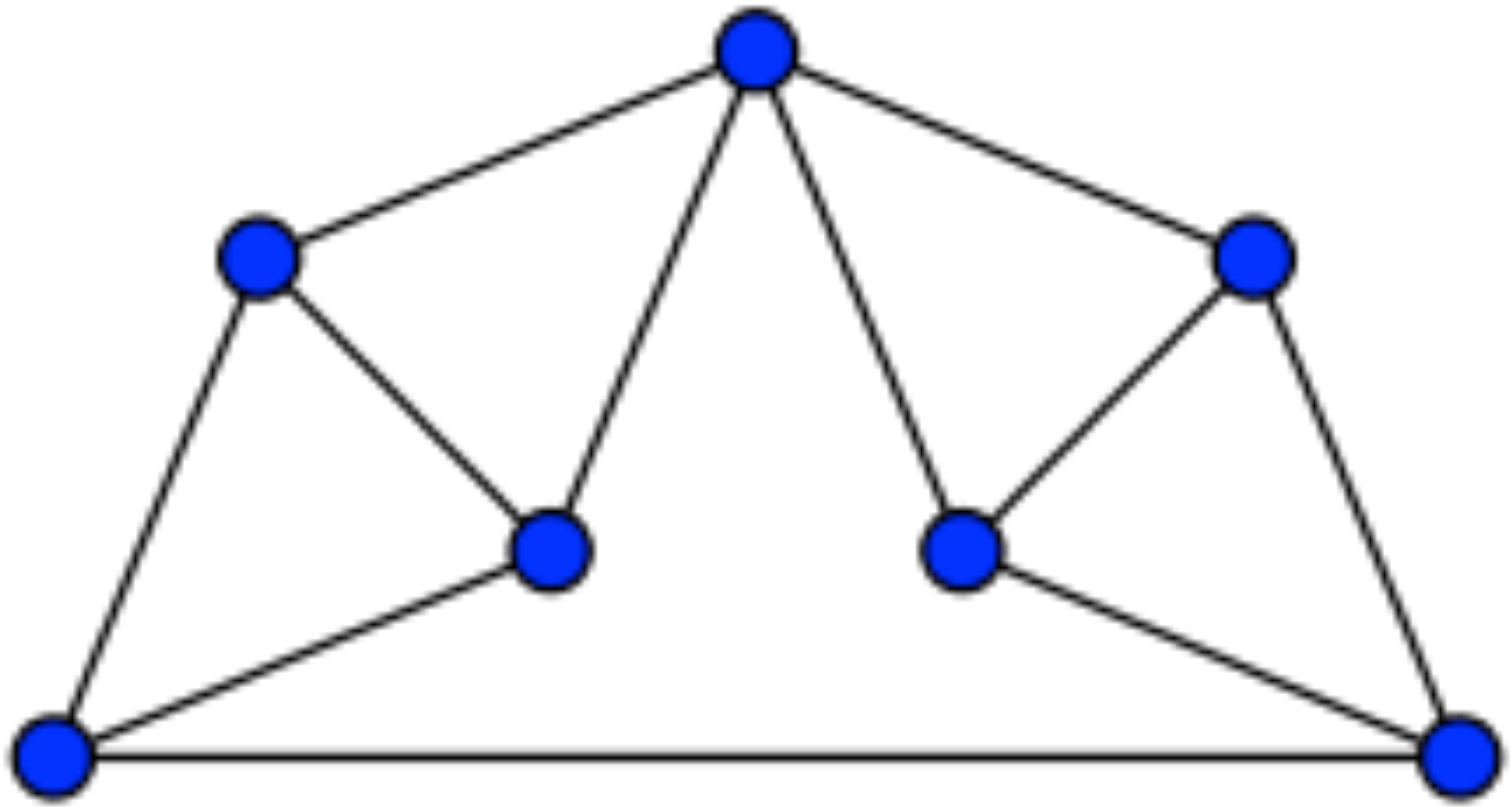


How many colors are needed to color every vertex in such a way that neighbors have different colors?

Coloring

CS 55 - Spring 2016 - Pomona College

Jenny Lam

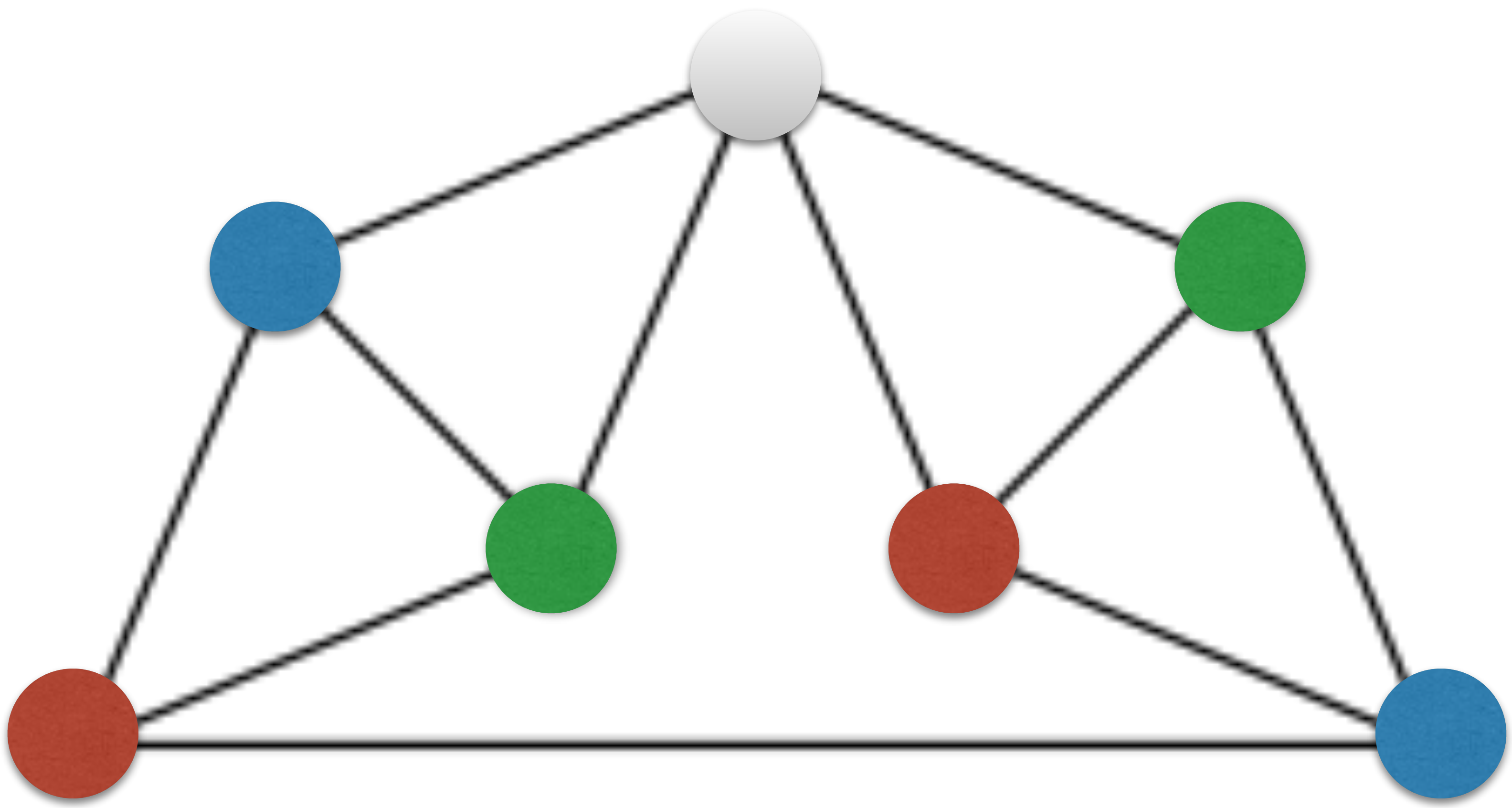
The Coloring Problem

A **coloring** of a graph is an assignment of colors (often represented using natural numbers) to the vertices of a graph such that no two adjacent vertices have the same color.

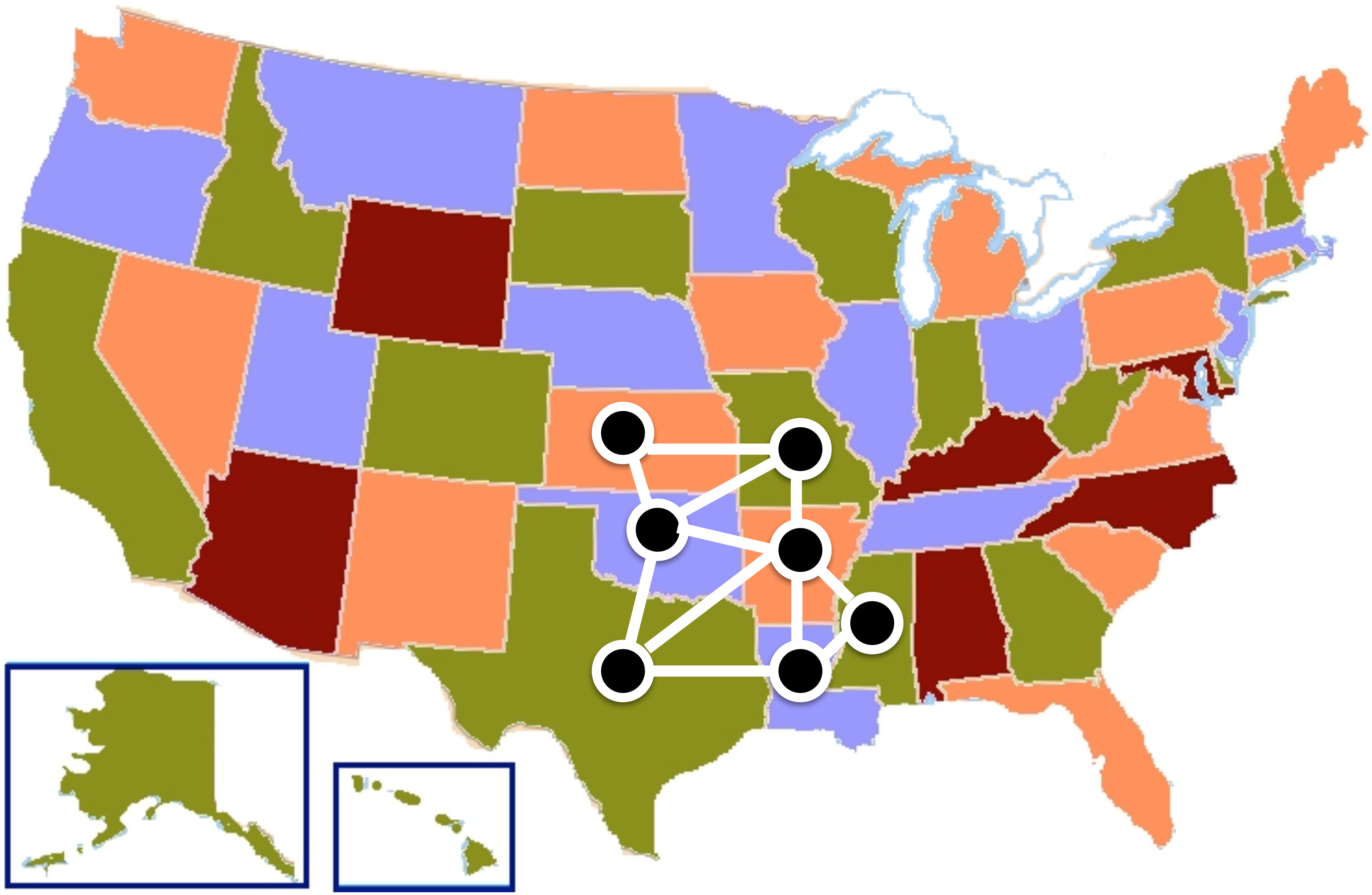
Chromatic Number

The **chromatic number** of a graph is the minimal number of colors needed to produce a valid coloring.

$$\chi(G) = \text{“The chromatic number of } G\text{”}$$



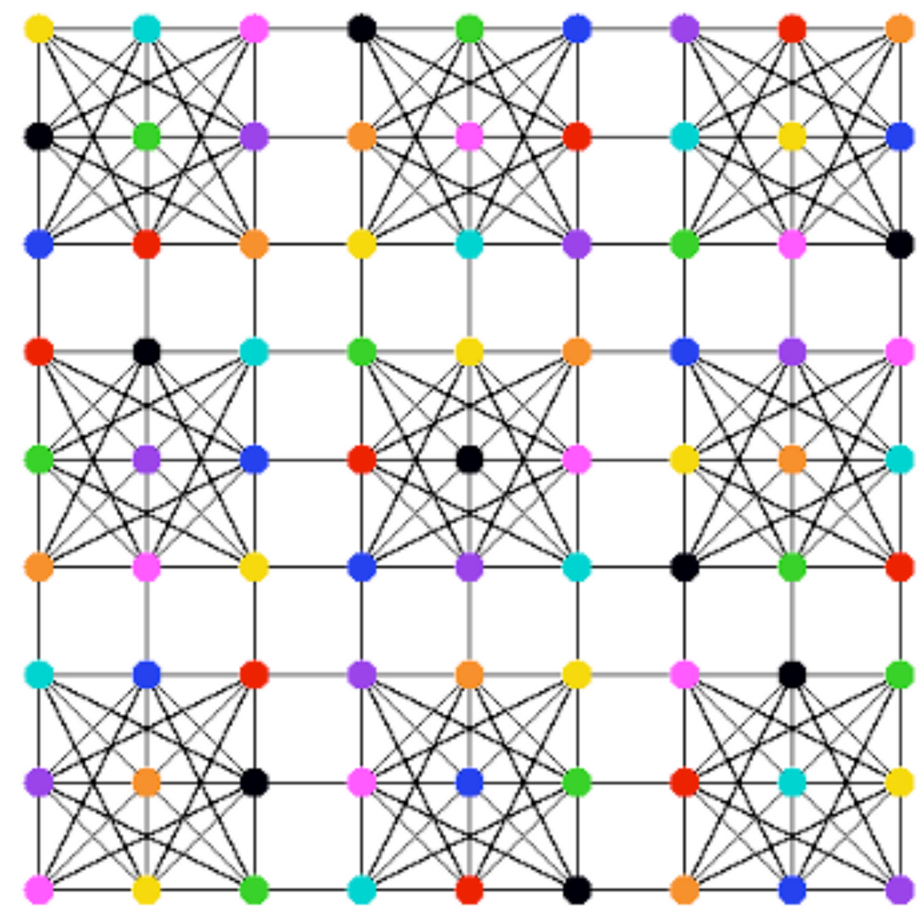
χ (Mosers' spindle) = 4



4-color theorem:
the chromatic number of a planar graph is at most 4

Sudoku is a graph coloring problem

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9



Note: this graph is not complete. Why not?

Register allocation

a = c + d
e = a + b
f = e - d

r1 = r2 + r3
r1 = r1 + r4
r1 = r1 - r3

register
interference
graph

Graph Coloring

The following are believed to require exponential time to solve algorithmically:

- determine chromatic number
- determine if chromatic number is 3
- approximating the chromatic number

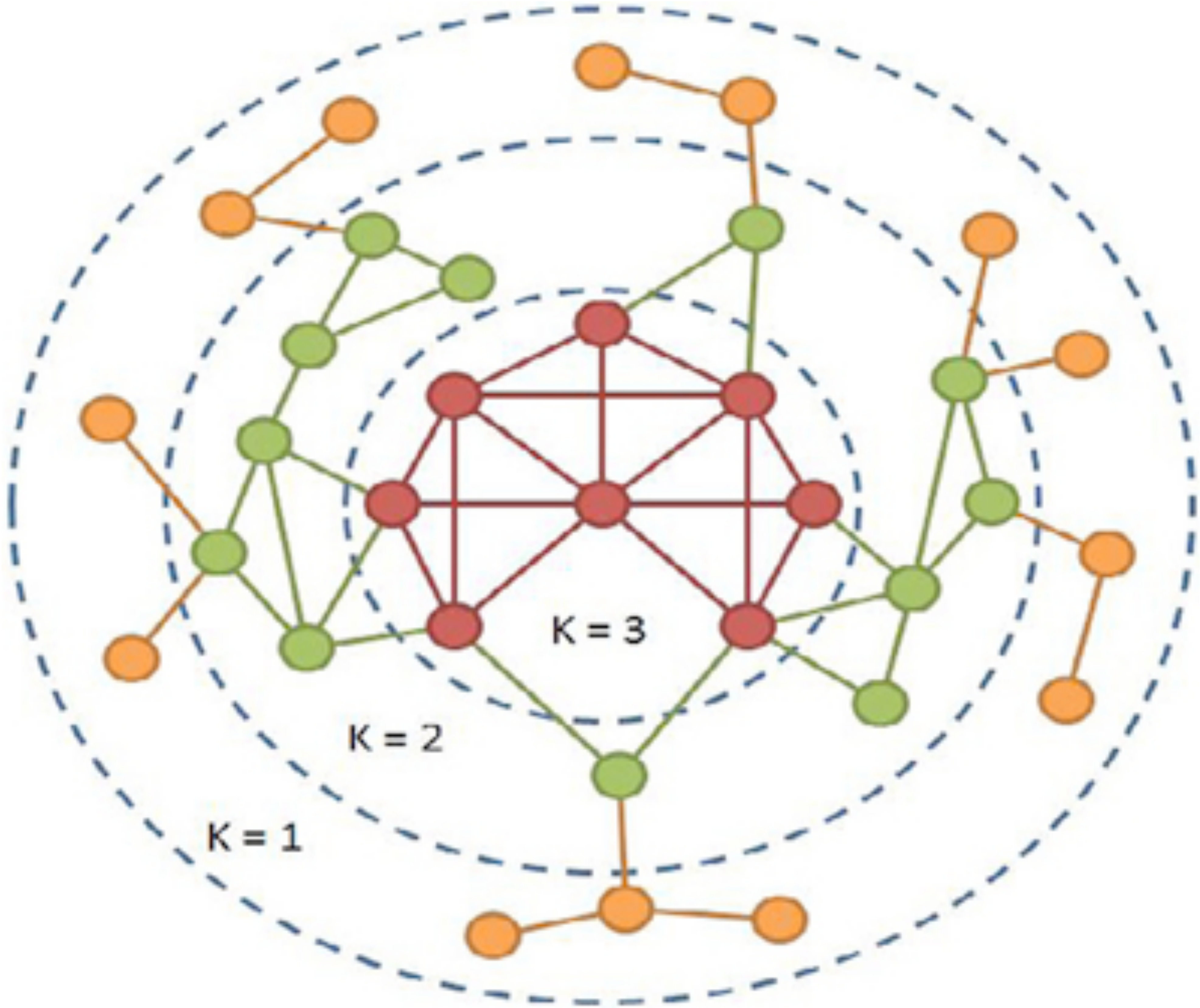
Coloring is a canonical example of a very difficult algorithmic problem.

Degeneracy

The **degeneracy** of a graph G is the value of k after running the following algorithm:

```
Let k equal 1
while G still has vertices:
    while G still has vertices of degree ≤ k:
        remove a vertex from G of degree k along with all
        of its incident edges
    increment k
```

The order is which the vertices were removed is called a **degeneracy ordering**.



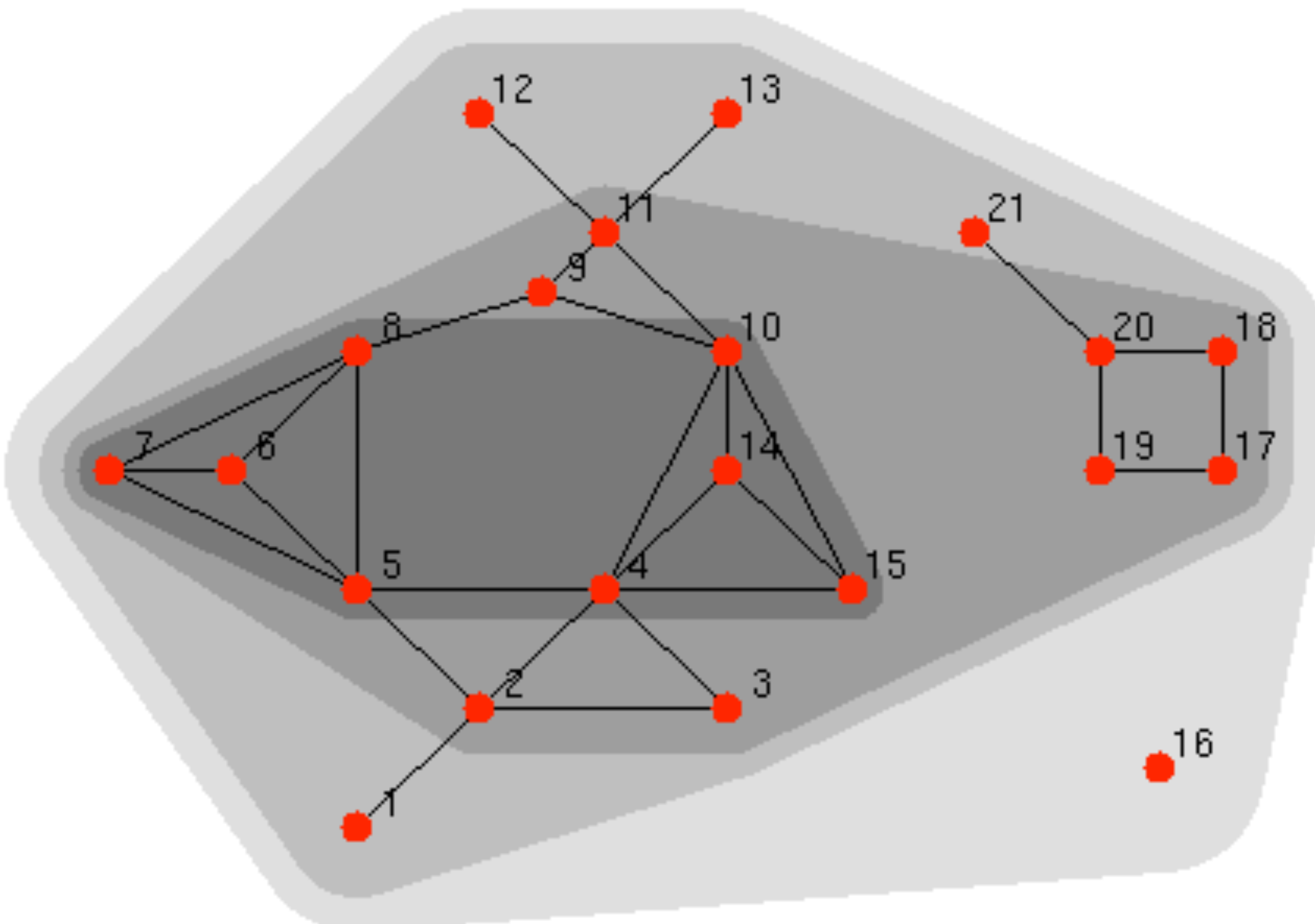
K = 3
K = 2
K = 1

k-cores

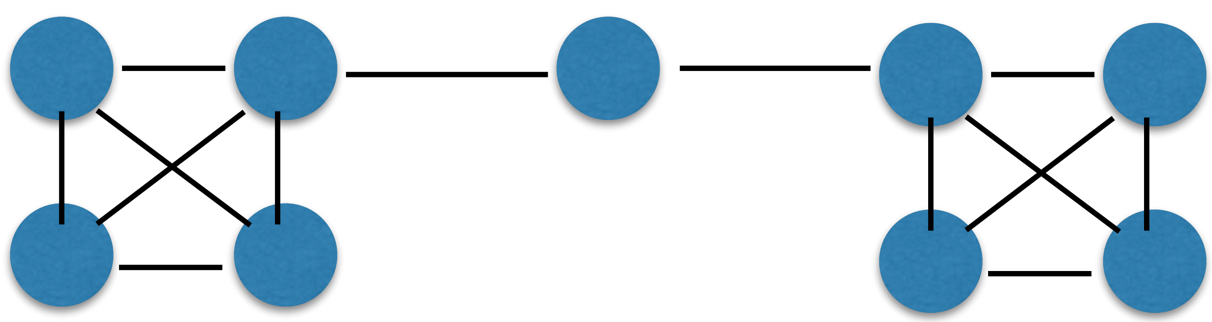
The **k-cores** of a graph are the connected components of a graph that are left after iteratively removing all vertices of degree less than k.

The k-cores are implicitly found when using the degeneracy algorithm on the previous slide.

The concept of a k-core was first introduced to study clustering of communities in social networks.


12
13
11
9
21
8
10
20
18
7
6
14
19
17
5
4
15
2
3
16
1

The k-core can be disconnected



Coloring in Degeneracy Order

Greedily coloring in degeneracy order will produce a coloring using $k + 1$, where k is the degeneracy.

Thus the chromatic number of a graph is at most one more than the degeneracy.

This coloring is rarely optimal, but is often the best we can do.