

1. Here is a solution to [https://github.com/jnylam/SJSU-cs146-s17/blob/master/02\\_Recursion2/src/cc/jennyham/cs146/TowersOfHanoi.java](https://github.com/jnylam/SJSU-cs146-s17/blob/master/02_Recursion2/src/cc/jennyham/cs146/TowersOfHanoi.java):

```

public void solve() {
    moveStack(n, 0, 2);
}

public void moveStack(int n, int fromPeg, int toPeg) {
    if (n == 0) return;
    int otherPeg = 3 - fromPeg - toPeg; // sum of the peg indices is 0
        + 1 + 2 = 3
    moveStack(n-1, fromPeg, otherPeg);
    moveTopDisk(fromPeg, toPeg);
    moveStack(n-1, otherPeg, toPeg);
}

// (This part was given.)
public void moveTopDisk(int fromPeg, int toPeg) {
    int disk = pegs.get(fromPeg).pop();
    Stack<Integer> peg = pegs.get(toPeg);
    assert (peg.isEmpty() || peg.peek() > disk); // valid move
    peg.push(disk);
}

```

which produces the following output when  $n = 4$ :

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2. Here are the completed implementations of methods in [https://github.com/jnylam/SJSU-cs146-s17/blob/master/02\\_Recursion2/src/cc/jennylam/cs146/TreeNode.java](https://github.com/jnylam/SJSU-cs146-s17/blob/master/02_Recursion2/src/cc/jennylam/cs146/TreeNode.java)

```
/*
 * Return the max value among all nodes in the tree rooted at the
 * current node.
 */
public E max() {
    E max = value;
    if (left != null) {
        E maxOfLeftSubtree = left.max();
        if (max.compareTo(maxOfLeftSubtree) < 0)
            max = maxOfLeftSubtree;
    }
    if (right != null) {
        E maxOfRightSubtree = right.max();
        if (max.compareTo(maxOfRightSubtree) < 0)
            max = maxOfRightSubtree;
    }
    return max;
}

/*
 * Return the number of nodes in the tree rooted at the current node.
 * The current node is counted.
 */
public int size() {
    int size = 1;
    if (left != null)
        size += left.size();
    if (right != null)
        size += right.size();
    return size;
}

/*
 * Return a list of the nodes in the tree rooted at the current node,
 * ordered by post-order traversal.
 */
public List<E> postorder() {
    List<E> list = new ArrayList<>();
    // left subtree
    if (left != null) {
        List<E> leftList = left.inorder();
        list.addAll(leftList);
    }
    // right subtree
    if (right != null) {
        List<E> rightList = right.inorder();
        list.addAll(rightList);
    }
    // do a little work
    list.add(value);
    return list;
}
```

3. a) The outer loop is called 567 times. The innermost loop is called  $j$  times, with  $j$  ranging between 0 and  $n - 1$  (inclusive). The number of calls is  $567(n - 1)n/2$  times, which is  $O(n^2)$ . The amount of space used is constant.
- b) Let  $n$  be the length of the input array. `doSomething()` is called  $n/2$  times in the for-loop and  $\log_2 n$  times in the while-loop for a total of  $n/2 + \log_2 n$  or  $O(n/2)$  calls. The space complexity is  $O(1)$ .
- c) There is one call ( $O(1)$ ) to `doSomething()`, and  $O(\log n)$  amount of space used.
- d) There are  $\log_{10} n$  or  $O(\log n)$  to `doSomething()`, and  $O(\log n)$  amount of space used.
- e) The answer is the same as in (d).
- f) There are  $n/3$  or  $O(n)$  calls to `doSomething()` and  $O(n)$  amount of space used.
- g) If  $n < m$ , there are  $m - n$  calls to `doSomething()` and  $O(m - n)$  amount of space used. Otherwise there is a single call and a constant amount of space used.

4. a) There are  $\log_{13} n$  calls to `meow()`.

```
b) void makeSomeNoise(int n) {
    int m = n;
    for (int i = 0; i < m; i++) {
        while (n > 1) {
            meow();
            n /= 13;
        }
    }
}
```

```
c) void makeSomeNoise(int n) {
    if (n <= 1)
        return;
    meow();
    makeSomeNoise(n/13);
}
```

5. a) 

```
int powerOf2(int n) {
    int p = 1;
    for (int i = 0; i < n; i++)
        p *= 2;
    return p;
}
```

The time complexity of this function is  $O(n)$ .

- b) They compute the same value since, on input  $n$ , both return the square of `foo(n/2)`.
- c) Every call to `foo()` makes at most one recursive call, whereas `bar()` makes two. Specifically, `foo` makes  $\log_2(n + 1)$  calls, so runs in  $O(\log n)$ . On the other hand, `bar()`, on input  $n = 2^k$ , makes

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1$$

or  $O(n)$  calls, each of which does a constant amount of work, for a total of  $O(n)$  time.

```
d) int powerOf2(int n) {  
    if (n == 0)  
        return 1;  
    int p = powerOf2(n/2);  
    return (n%2 == 0) ? p*p : 2*p*p;  
}
```