

1. *Solution 1.* Notice that for any value of j and k , i iterates $k - j + 1$ times. So given a specific value of j , since k goes from j to n , the number of times i iterates is

$$1 + 2 + \cdots + (n - j + 1) = \frac{(n - j + 1)(n - j + 2)}{2} \geq \frac{(n - j + 1)(n - j)}{2}$$

So

$$\begin{aligned} \text{total number of iterations} &\geq \sum_{j=1}^n \frac{(n - j + 1)(n - j + 2)}{2} \\ &= \frac{1}{2} \sum_{j=1}^n (n^2 + n) - (2n + 1)j + j^2 \\ &= \frac{1}{2} \left(n(n^2 + n) - (2n + 1) \frac{n(n + 1)}{2} + \frac{n(n + 1)(2n + 1)}{6} \right) \\ &= \frac{n(n^2 - 1)}{6} \end{aligned}$$

Now set

$$\frac{n(n^2 - 1)}{6} \geq cn^3$$

and solve this inequality for n . We get

$$n \geq \frac{1}{\sqrt{1 - 6c}}.$$

Therefore, if we let $c = 1/12$ and $n_0 = 2$, we have that

$$\text{total number of iterations} \geq cn^3 \quad \text{whenever } n \geq n_0.$$

So the runtime is $\Omega(n^3)$.

Solution 2. We have

$$\begin{aligned} \text{total number of iterations} &= \sum_{j=1}^n \sum_{k=j}^n \sum_{i=j}^k 1 \\ &\geq \sum_{j=1}^{n/3} \sum_{k=2n/3}^n \sum_{i=n/3}^{2n/3} 1 \\ &\geq \frac{n^3}{27}. \end{aligned}$$

Therefore, if we let $c = 1/27$ and $n_0 = 1$, we have that

$$\text{total number of iterations} \geq cn^3 \quad \text{whenever } n \geq n_0.$$

So the runtime is $\Omega(n^3)$.

Solution 3. Notice that the pseudocode iterates through all possible triplets (j, i, k) such that $j \leq i \leq k$ and such i, j, k each has a value between 1 and n . If we have ignore the duplicate values such as $(2, 2, 3)$, then the total number of such triplets is

$$\binom{n}{3} = \frac{n(n-1)(n-2)}{6}.$$

Set

$$\frac{n(n-1)(n-2)}{6} \geq cn^3.$$

Let $c = 1/12$. This simplifies to

$$n^2 - 6n + 4 \geq 0$$

Now it's easy to see that this inequality holds for all $n \geq 6$. Therefore, if we let $c = 1/12$ and $n_0 = 6$, we have that

$$\text{total number of iterations} \geq cn^3 \quad \text{whenever } n \geq n_0.$$

So the runtime is $\Omega(n^3)$.

2. If $n_0 = 59$, then $10n \log n < n^2$ whenever $n \geq n_0$.

3. All answers below are approximate.

	1 Second	1 Hour	1 Month	1 Century
$\log n$	10^{300000}	10^{10^9}	$10^{9 \times 10^{11}}$	$10^{9 \times 10^{14}}$
\sqrt{n}	10^{12}	2×10^{19}	9×10^{24}	9×10^{30}
n	10^6	4×10^9	3×10^{12}	3×10^{15}
$n \log n$	6×10^4	10^8	8×10^{10}	7×10^{13}
n^2	10^3	6×10^4	2×10^6	5×10^7
n^3	10^2	2×10^3	2×10^4	2×10^5
2^n	20	32	41	51
$n!$	9	12	15	17

4. `reverse(A) :`

```

s = stack()
for i from 1 to len(A):
    s.push(A[i])
for i from 1 to len(A):
    A[i] = s.pop()

```

The running time is $2n$ or $O(n)$.