R13.2 Let G be a simple connected graph with n vertices and m edges. Explain why $O(\log m)$ is $O(\log n)$.

Solution. Since $m \leq n^2$, applying the logarithm to both sides,

$$\log m \leq 2 \log n.$$

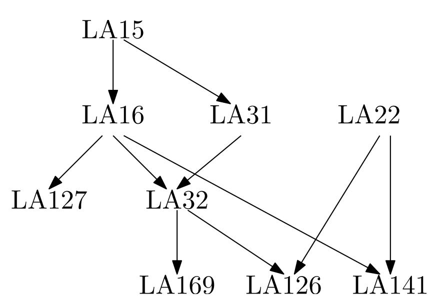
So $O(\log m)$ is $O(\log n)$. •

R13.4 Bob loves foreign languages and wants to plan his course schedule to take the following nine language courses with the following prerequisites:

- LA15: (none)
- LA16: LA15
- LA22: (none)
- LA31: LA15
- LA32: LA16, L31
- LA126: LA22, LA32
- LA127: LA16
- LA141: LA22, LA16
- LA169: LA32.

Find a sequence of courses that allows Bob to satisfy all the prerequisites.

Solution. One possible sequence is LA15, LA16, LA31, LA22, LA127, LA32, LA169, LA126, LA141.



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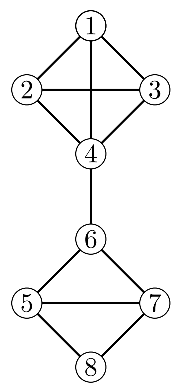
R13.6 Let G be a graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below:

vertex	adjacent vertices
1	(2, 3, 4)
2	(1, 3, 4)
3	(1, 2, 4)
4	(1, 2, 3, 6)
5	(6, 7, 8)
6	(4, 5, 7)
7	(5, 6, 8)
8	(5, 7)

Assume that, in a traversal of G, the adjacent vertices of a given vertex are returned in the same order as they are listed in the above table.

(a) Draw G.

Solution.



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(b) Order the vertices as they are visited in a DFS traversal starting at vertex 1.

Solution. There are many possible DFS traversals. One DFS traversal is 1, 2, 3, 4, 6, 5, 7, 8. •

(c) Order the vertices as they are visited in a BFS traversal starting at vertex 1.

Solution. There are many possible BFS traversals. One BFS traversal is 1, 2, 3, 4, 6, 5, 7, 8. •

R13.7 Would you use the adjacency list structure or the adjacency matrix structure in each of the following cases? Justify your choice.

(a) The graph has 10,000 vertices and 20,000 edges, and it is important to use as little space as possible.

Solution. An adjacency list will use 10,000 + 20,000 = 30,000 units of space whereas an adjacency matrix will use $(10{,}000)^2 = 100{,}000{,}000$ units of space, so I would use the adjacency list. •

(b) The graph has 10,000 vertices and 20,000,000 edges, and it is important to use as little space as possible.

Solution. An adjacency list will use $10{,}000 + 20{,}000{,}000 \simeq 20{,}000{,}000$ units of space whereas an adjacency matrix will use $(10{,}000)^2 = 100{,}000{,}000$ units of space, so I would still use the adjacency list (because I want to use as little space as possible). •

(c) You need to answer the query areAdjacent as fast as possible, no matter how much space you use.

Solution. Each list of neighbors in the adjacency list has length at most 10,000, so a query areAdjacent requires a scan through a list that is this long in the worse case. In contrast, the adjacency matrix makes constant time areAdjacent queries. So I would use the adjacency matrix. •