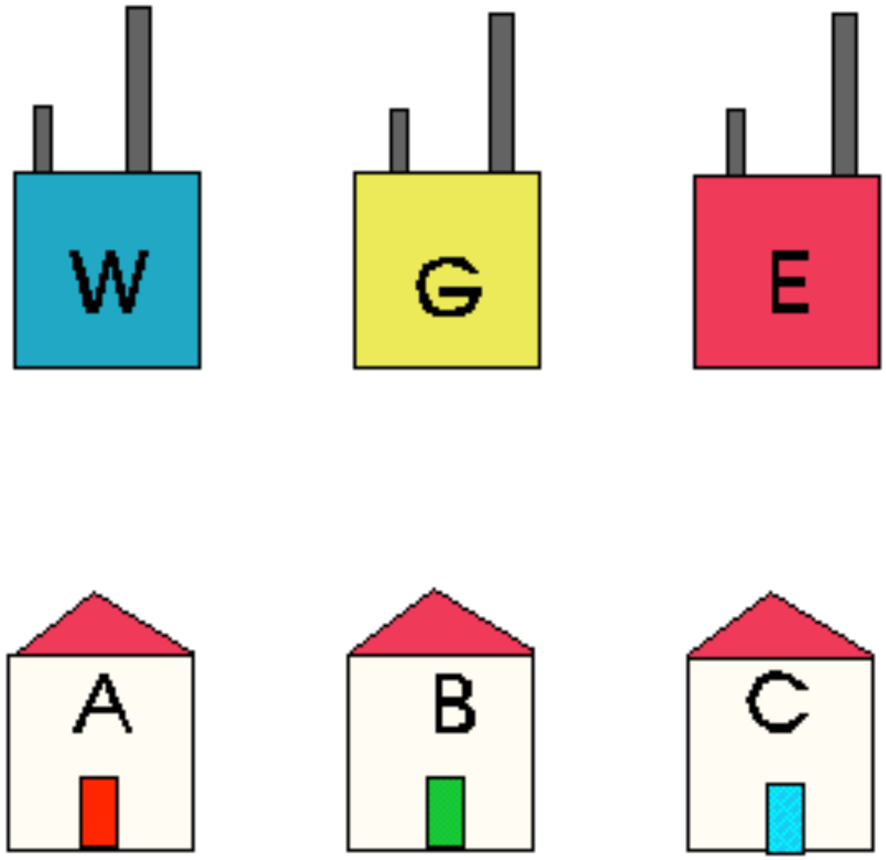


The utility problem

connect each house to all three utilities without crossing any lines

Planarity

CS 55 - Spring 2016 - Pomona College

Jenny Lam

Planarity

A graph is said to be **planar** if it can be drawn in the plane without any edge crossings. Notice that we do not require the edges to be straight lines!

Euler's Formula

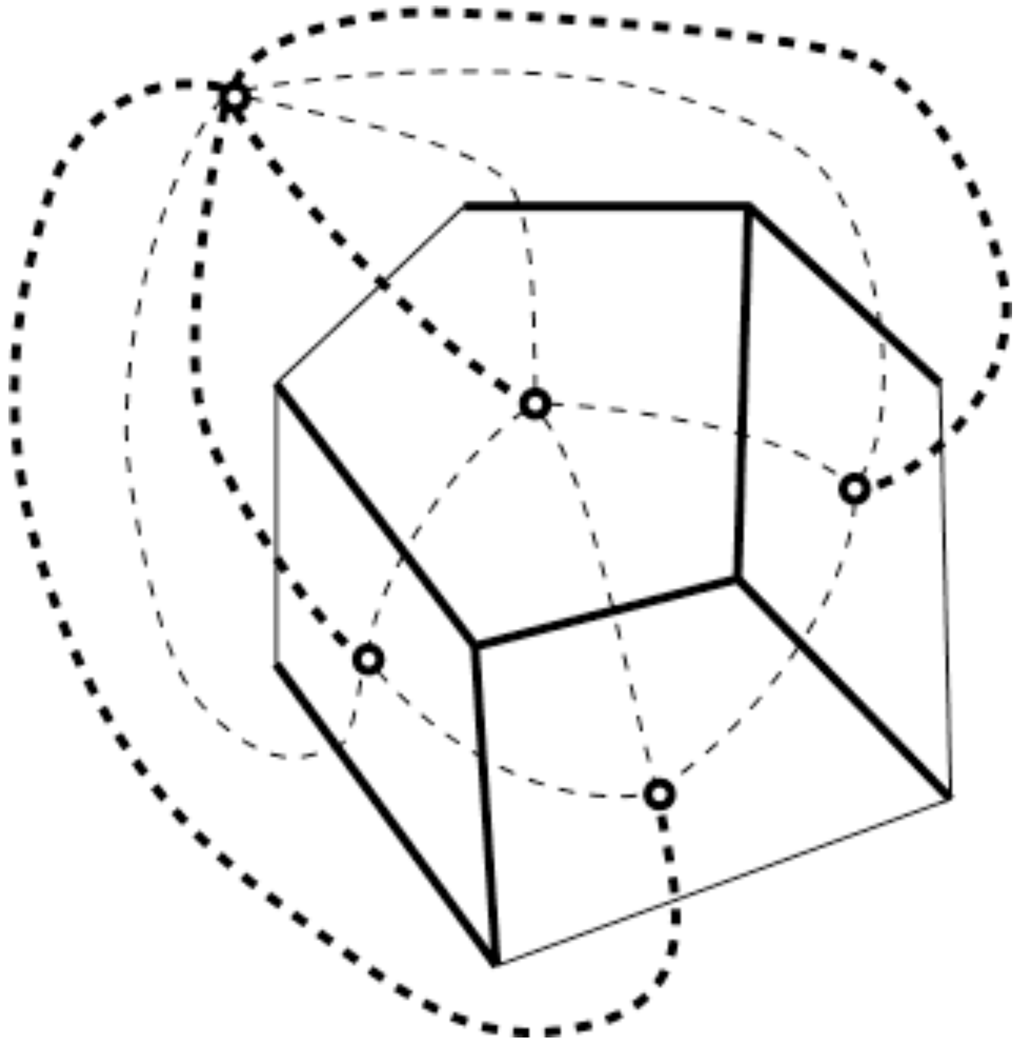
Let G be a connected planar graph with n vertices and m edges. Let k be the number of faces (regions) of the planar drawing. Then we have $k = m - n + 2$.

Dual of a Planar Graph

We define the **dual** of a planar graph G via a construction of one of its drawings. First, place a vertex in the middle of each face of G. Then connected vertices in adjacent faces by drawing an edge crossing their boundary.

Notices that, different drawings of G may produce different dual graphs.

Proof of Euler's Formula



http://www.ics.uci.edu/~eppstein/junkyard/euler/interdig.html

Planar Graphs are Sparse

If *G* is a connected planar graph with $n \geq 3$ vertices and *m* edge, then m $\leq$ 3n-6.

Planar Graphs are Sparse

If G is a connected planar graph with $n \geq 3$ vertices and m edges and no circuits of length 3,

then $m \leq 2n-4$.

Two Important Examples

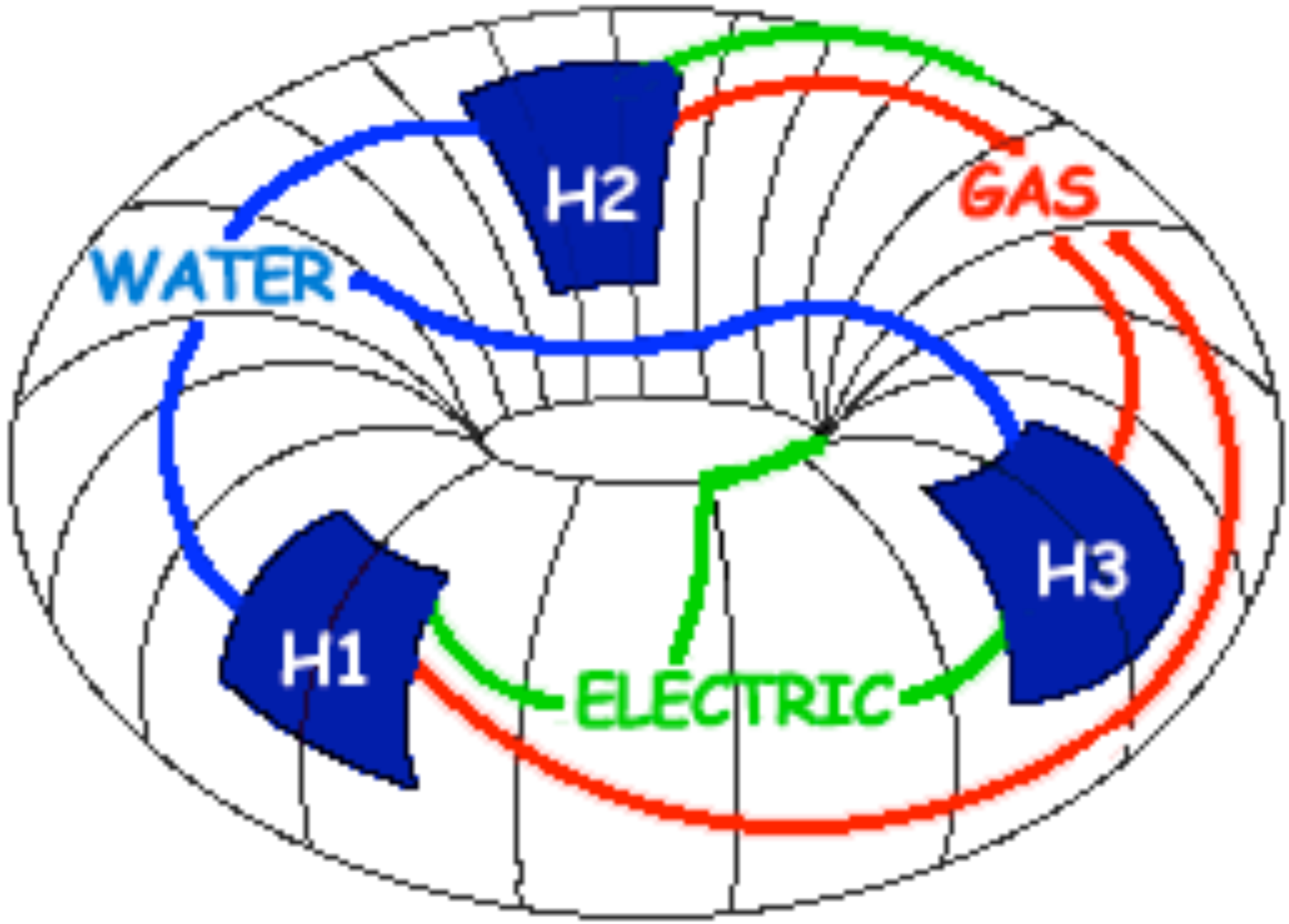
The graphs $K_{3,3}$ and K_5 are not planar!

Graph Minors

A graph H is called a **minor** of the graph G if H can be formed from G by deleting edges, deleting vertices and by contracting edges.

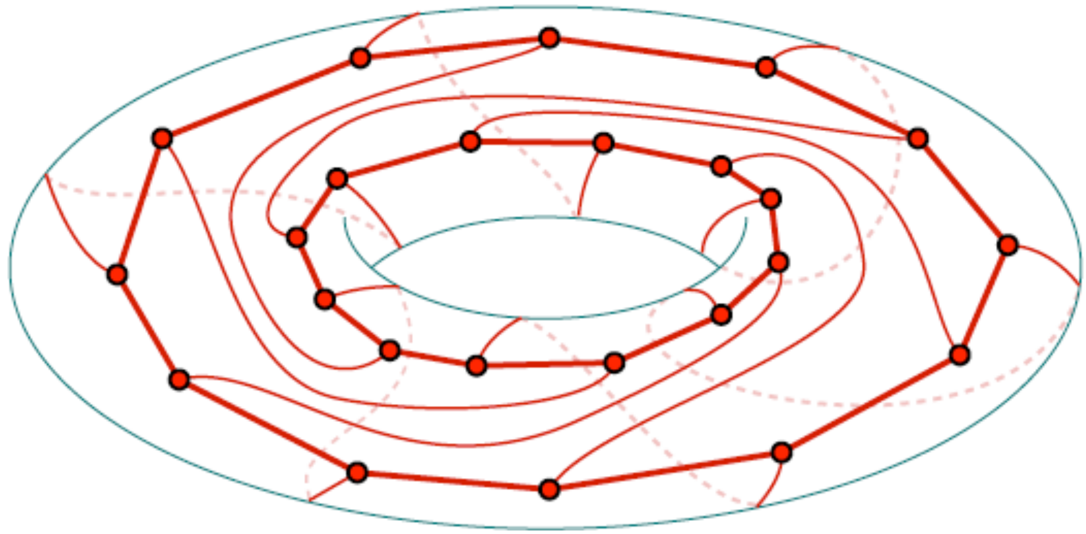
Wagner's Theorem

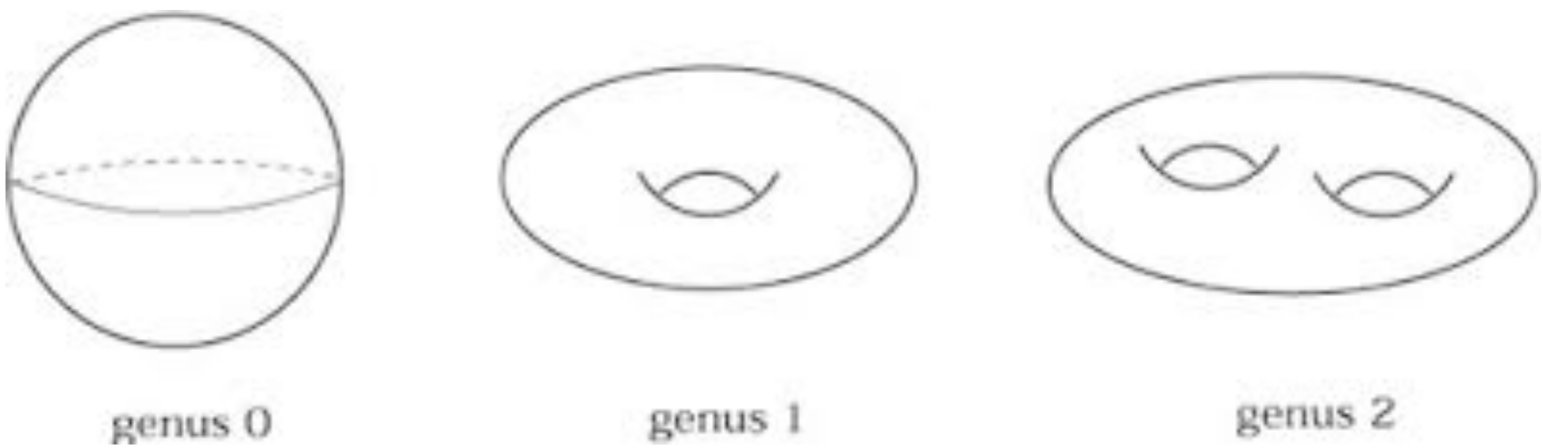
A graph is planar if and only if it does not contain either $K_{3,3}$ or K_5 as a minor.

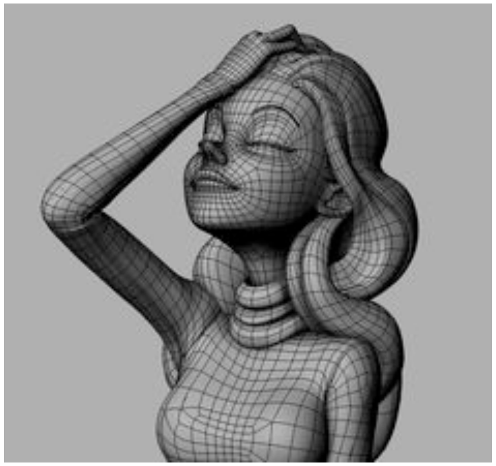


The utility problem is solvable on a torus!

Higher Genus Graphs







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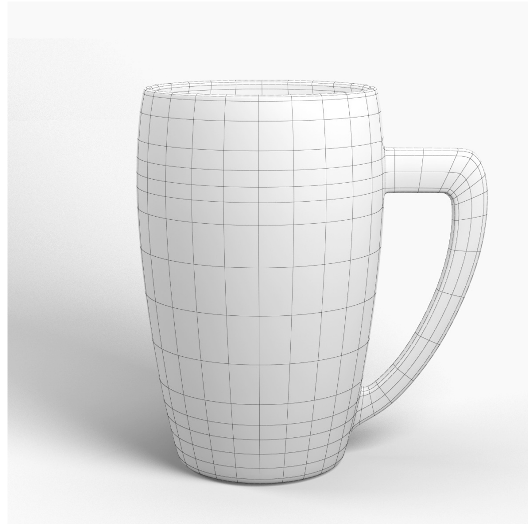

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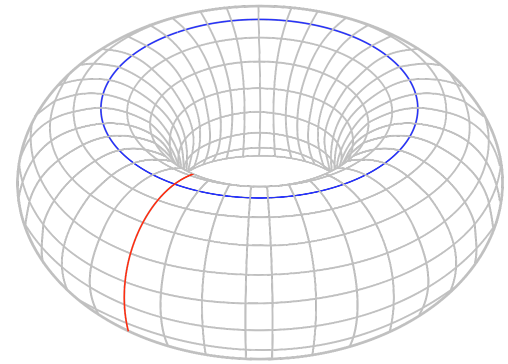
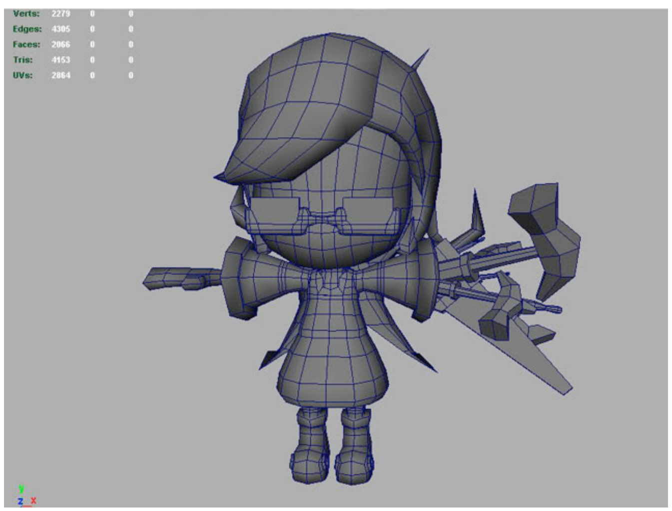


homologically equivalent surfaces



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