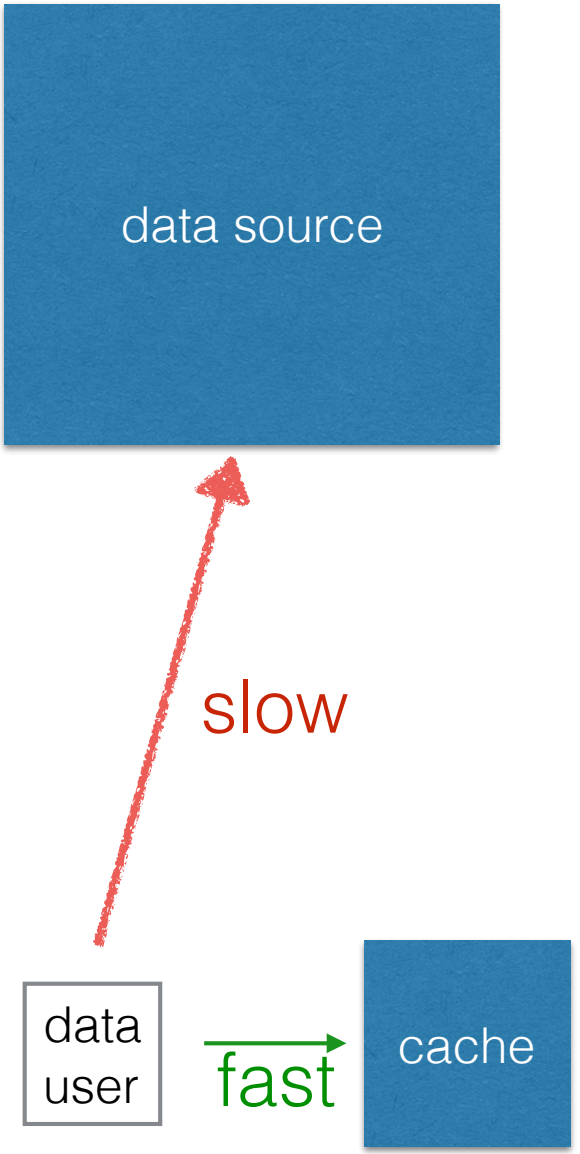
Operating Systems

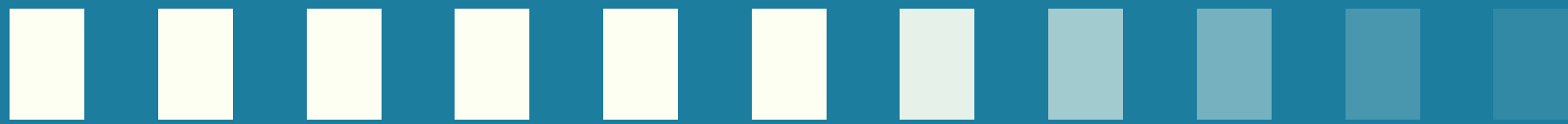
Memory: replacement policies

Last time

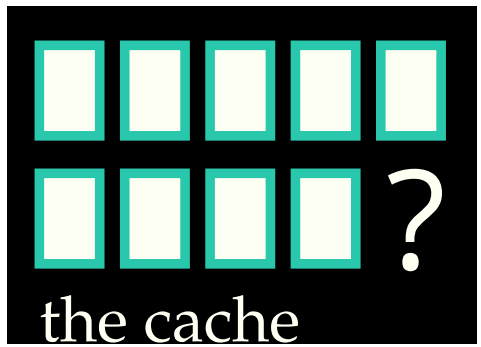
- **caching speeds up data lookups if the data is likely to be re-requested again**
- **data structures for O(1)-lookup**
  - set-associative (hardware)
  - hash tables (software)
- **cache coherence**
  - with 1 cache: buffer writes back to memory
  - with multiple caches: things get complicated
- today: **replacement policy**



THE PROBLEM



input: a sequence of requests for objects



GOAL

minimize number of cache misses

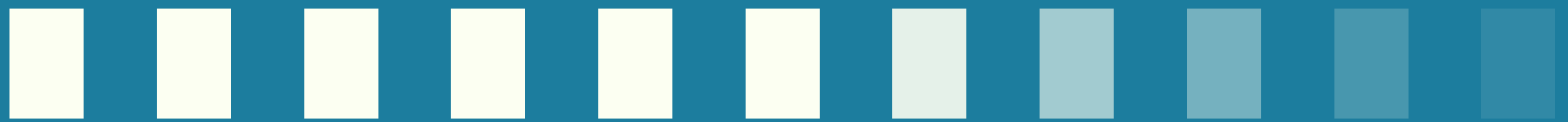
clairvoyant

LRU

FIFO

LFU

BASIC ALGORITHMS



clairvoyant

LRU

BASIC ALGORITHMS

FIFO

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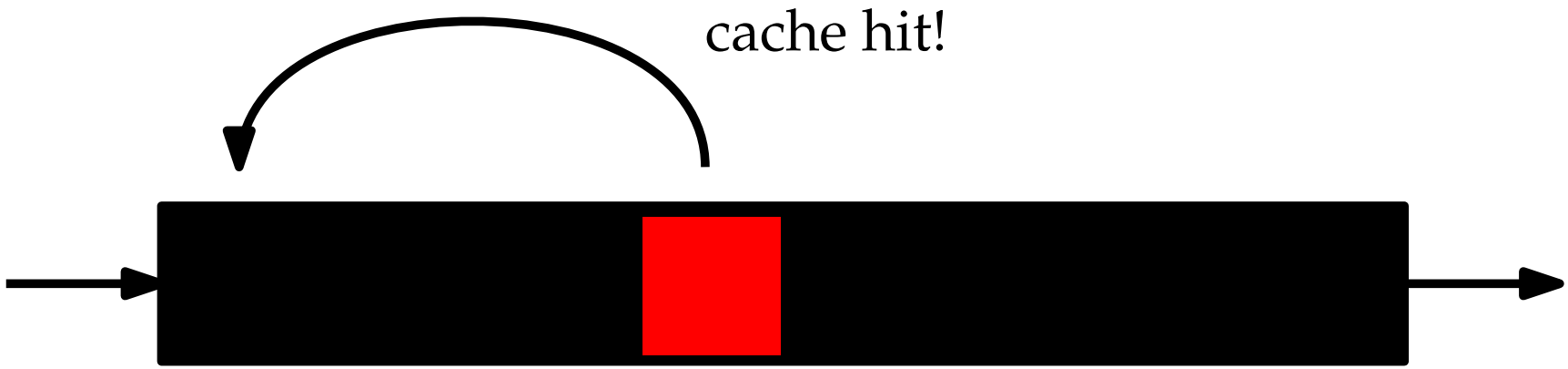

BASIC ALGORITHMS

clairvoyant

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LFU

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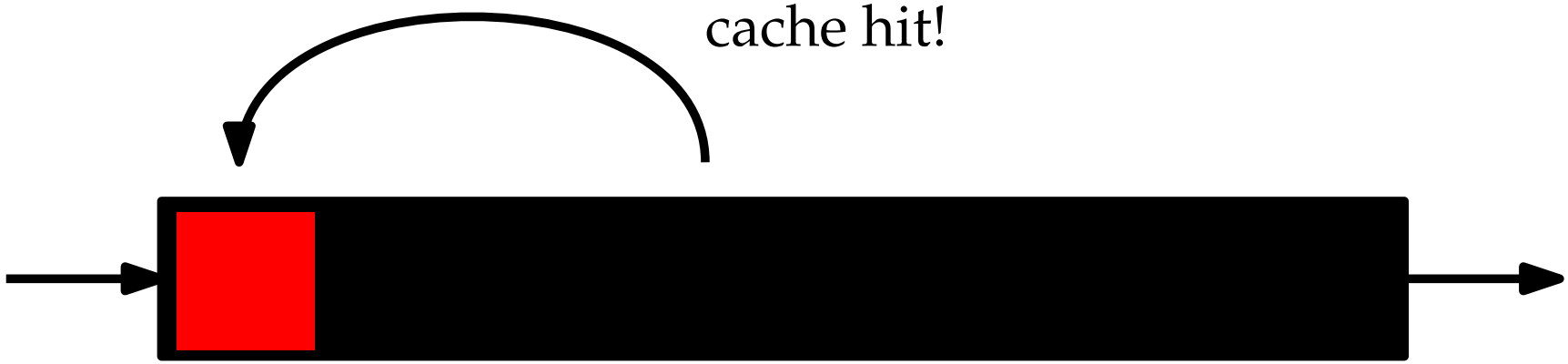

BASIC ALGORITHMS

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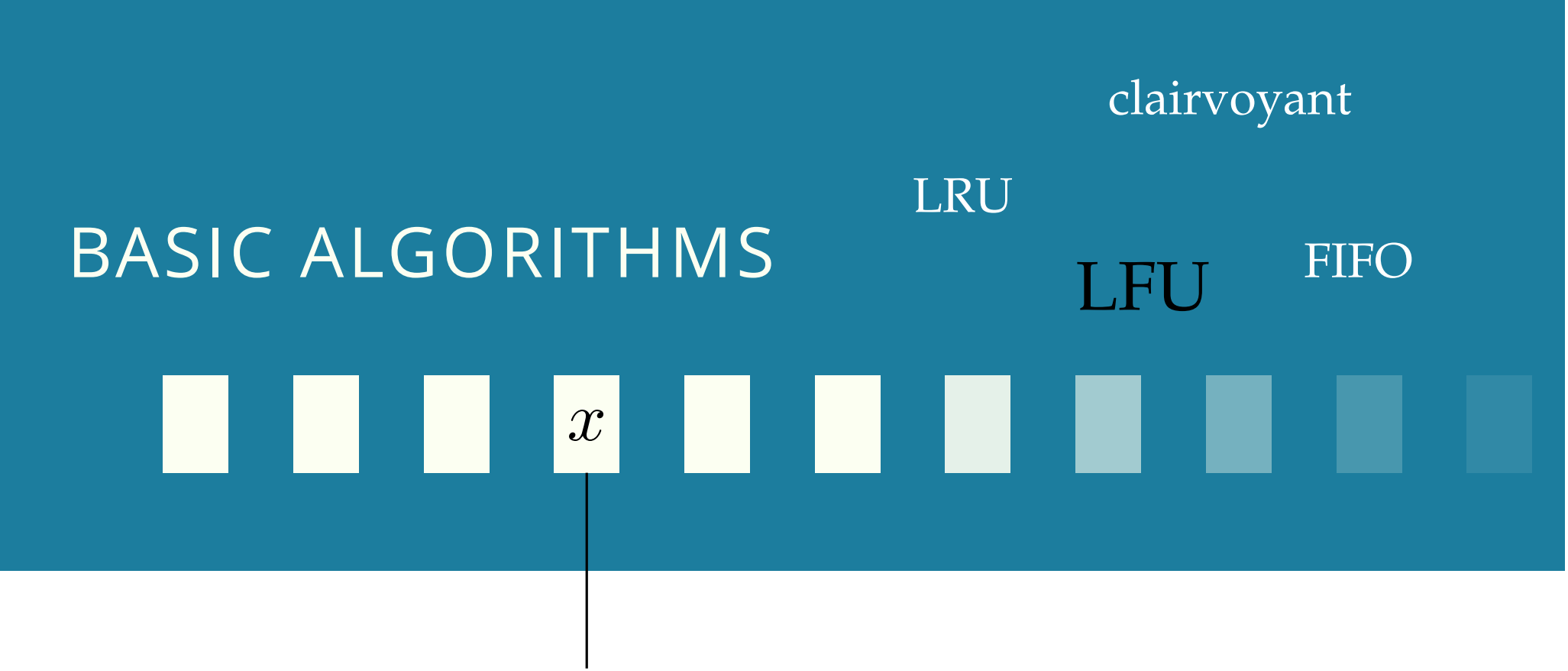
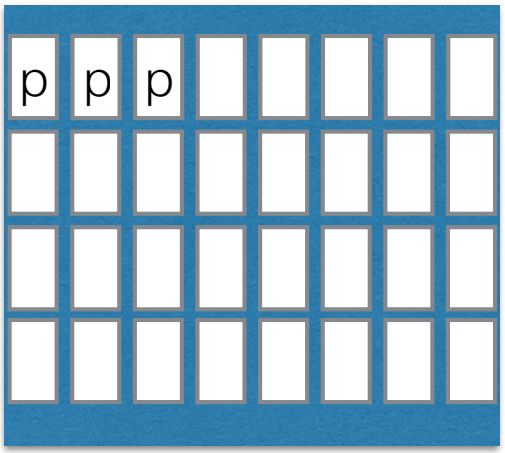


frequency(x) = number of requests

evict object with smallest frequency

Model of virtual memory

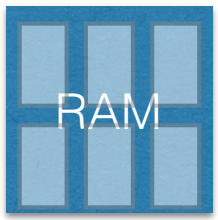


disk: slow
memory

slow

input:
request sequence
p0, p1, p2, p3, …., pn

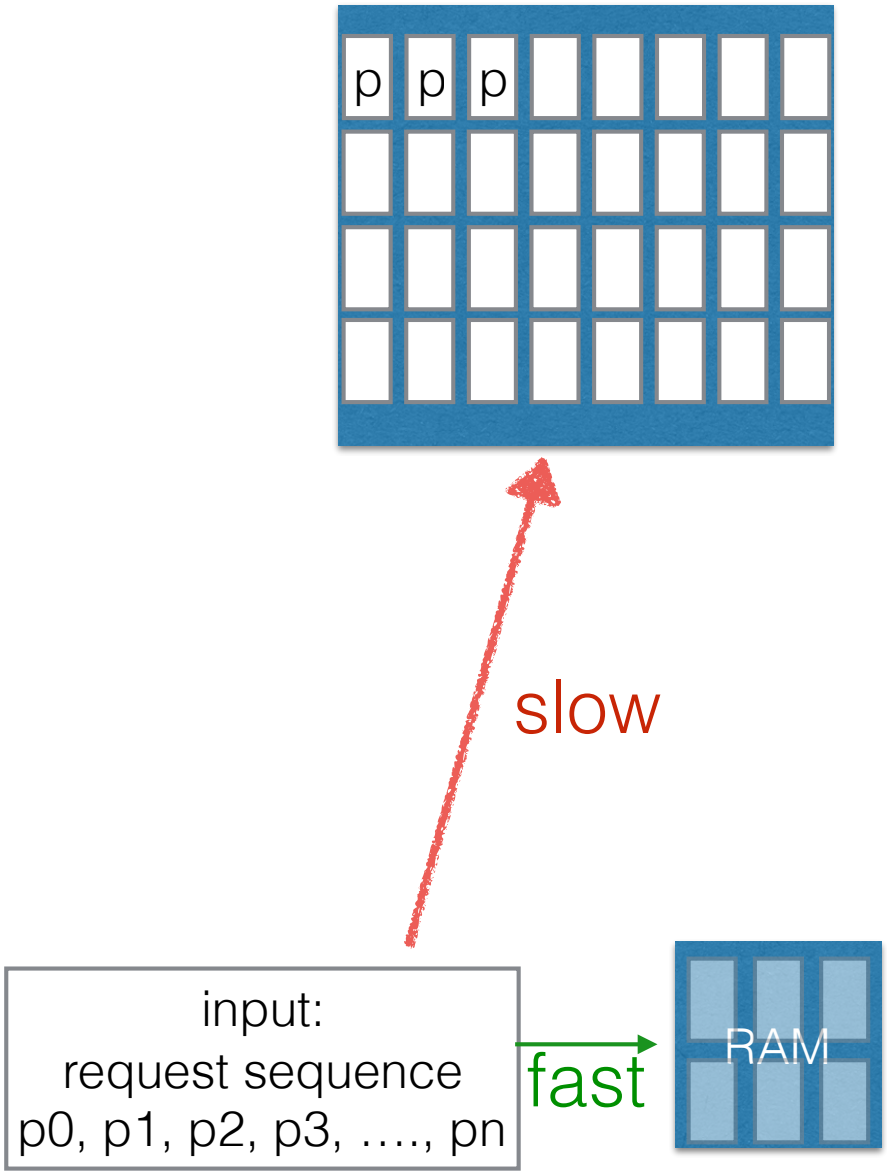
fast



cache
of size **k**

k much smaller than
# of pages in disk

Model of virtual memory

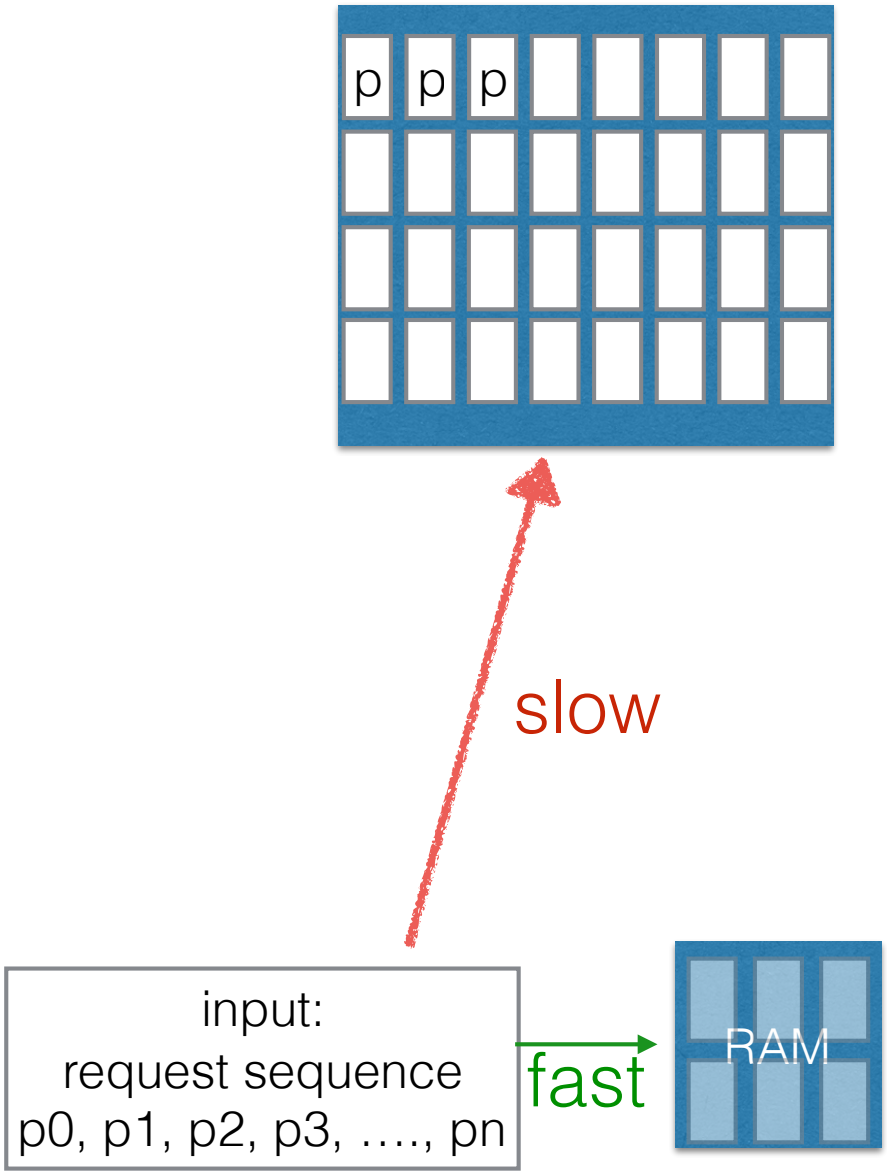


goal

minimize total cost of

bringing pages from disk

Paging problem



input

s = p0, p1, p2, p3, … pn

output

for each pi, which p to evict from cache

goal

minimize # misses

LFD is optimal

- proposed by Belady
- LFD: longest forward distance
- synonyms: OPT, MIN, clairvoyant, Belady's optimal alg
- proof given in class

LFD proof of optimality

- let ALG be any paging algorithm, s input seq. Will construct a seq of algorithms: ALG = ALG0, ALG1, ALG2, … ALGn = LFD st
- cost(ALG0, s) >= cost(ALG1, s) >= cost(ALG2, s) >= … cost(ALGn) = LFD, n = len(s)
- how? by induction: suppose ALGi is determined
- construct ALG(i+1) as follows: for the first i-1 requests, do the same eviction decisions as ALGi. On request i, ALGi evicts the page currently in its cache that will be request furthest into the future, call this page v. ALG(i) may or may not evict this same page. If it does, let ALG(i+1) = ALG(i) for the future too, so then done.
- Otherwise ALG(i) evicts page u != v. So after processing i, both caches are the same, but ALG(i) has v and ALG(i+1) has u, where u is requested before v.
- On future requests, ALG(i+1) mirrors the same decisions as ALG(i) except if ALG(i) evicts v, then ALG(i+1) evicts u. Since u will be requested first, ALG(i) will incur a miss when u is requested, at which point both may or may not be identical (off by 1). If v is requested in the future, then the costs will be identical.

Belady's anomaly

- increasing the cache size may result in more misses
- hw: FIFO suffers from this phenomenon
- hw: LRU doesn't

Competitive analysis

Which replacement policy is better?

$$\textbf{competitive ratio of ALG} = \max_{\text{sequences } s} \frac{\text{misses(ALG, s)}}{\text{misses(OPT, s)}}$$

ALG is said to be **k-competitive** if
its competitive ratio is $\leq k$

LRU is k-competitive

for any input sequence s:

misses(LRU, s) ≤ k misses(OPT, s)

Proof sketch: divide s into k-phases, where each phase consists of references to exactly k distinct pages
then misses(LRU, s) ≤ k
and misses(OPT, s) ≥ 1

FIFO is k-competitive

for any input sequence s:

misses(FIFO, s) ≤ k misses(OPT, s)

Proof: homework

LFU is not competitive

there exists a sequence of input sequences s1, s2, s3, …

$$\lim_{i \to \infty} \frac{\text{misses(LFU, si)}}{\text{misses(OPT, si)}} = \infty$$

Proof: let si = p0 (x i+1), p1 (x i+1), … p(k-1) (x i+1),
[pk, p(k+1)] (x i)
then misses(LFU, si) = k-1 + 2i
and misses(OPT, si) = k

LIFO is not competitive

there exists a sequence of input sequences s1, s2, s3, …

$$\lim_{i \to \infty} \frac{\text{misses(LIFO, si)}}{\text{misses(OPT, si)}} = \infty$$

Proof: homework

A timeline of paging analysis

- Belady's paper shows that **MIN is optimal**
- **competitive analysis,** Sleator and Tarjan
- 1995: 1st analysis of paging with **locality of reference**, Borodin et al. using access graphs
- 1996: strongly competitiveness, Irani et al.
- 1999: LRU is better than FIFO
- 2000: markov paging, Karlin et al.
- 2007: relative worst order, Boyar et al
- 2009: relative dominance, Dorrigiv et al
- 2010: parameterized analysis, Dorrigiv et al
- 2012: access graph model under worst order analysis, Boyar et al
- 2013: access graph model under relative interval analysis, Boyar et al.
- 2015: competitiveness in terms of locality parameters, Albers et al.

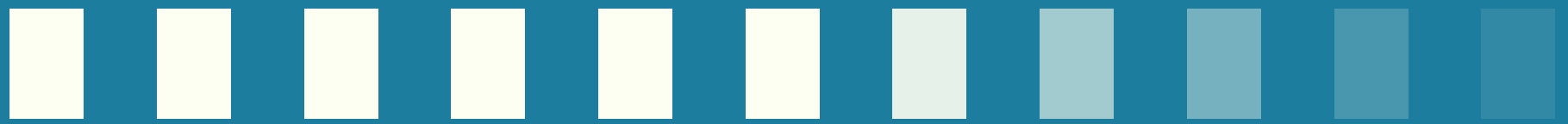
over the years:
- alternative performance measures proposed
- various frameworks for locality of reference proposed

LRU

FIFO

LFU

STRENGTHS AND WEAKNESSES



STRENGTHS AND WEAKNESSES

LRU

good if popularity changes quickly

unpopular objects stay too long

LFU

FIFO

STRENGTHS AND WEAKNESSES

LRU

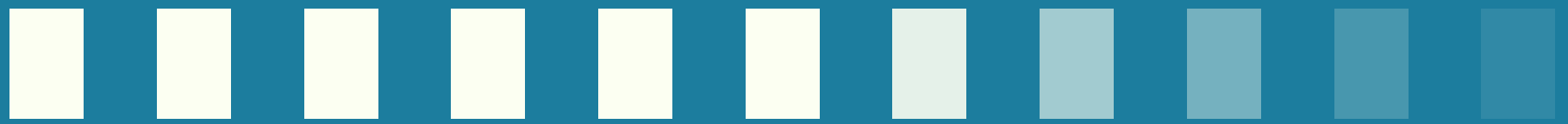
good if popularity changes quickly
unpopular objects stay too long

LFU

good if popularity is static
suffers from cache pollution over time

FIFO

STRENGTHS AND WEAKNESSES



good if popularity changes quickly
unpopular objects stay too long

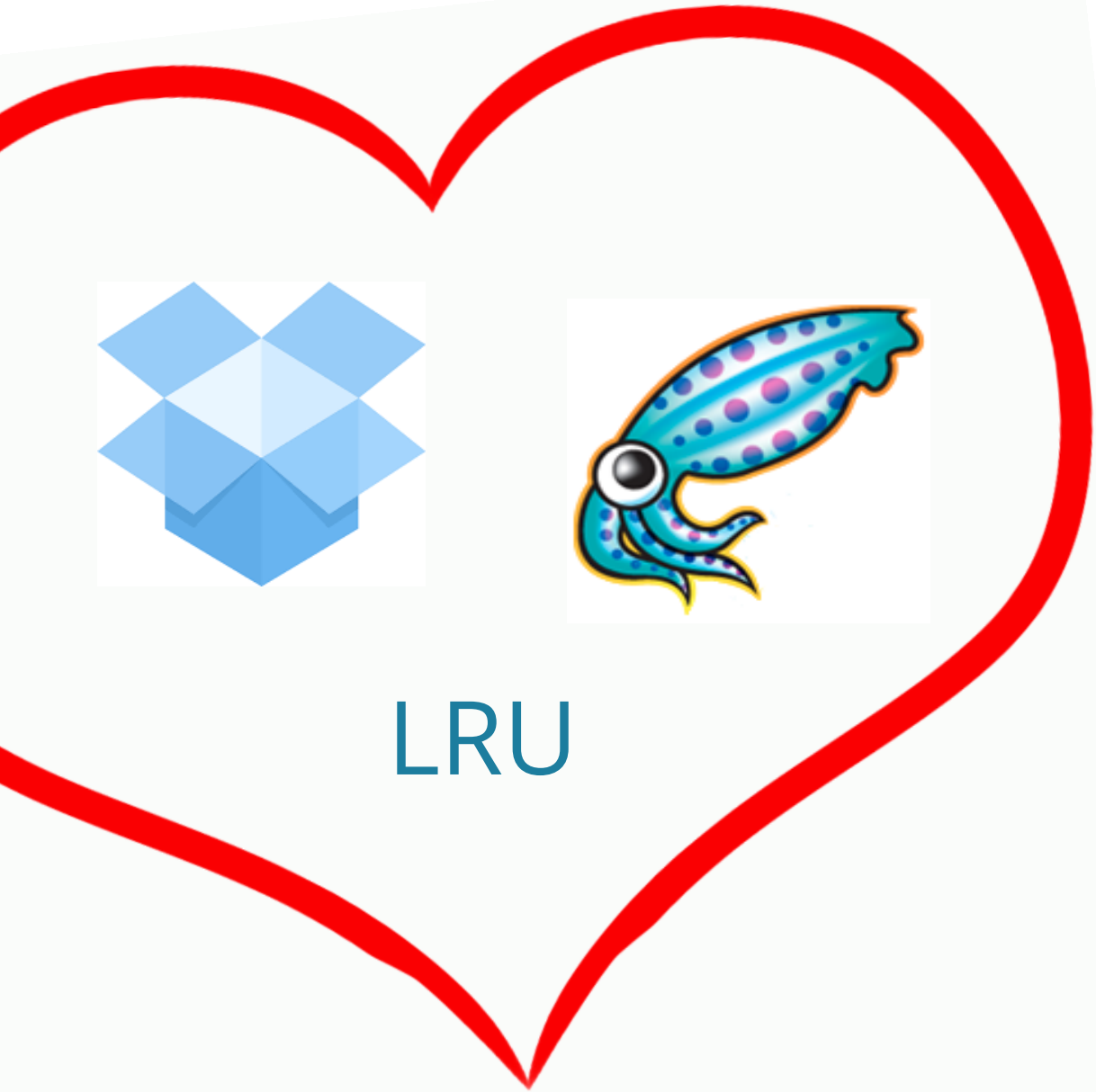
LRU

good when
fragmentation
is an issue

FIFO

LFU

good if popularity is static
suffers from cache pollution over time



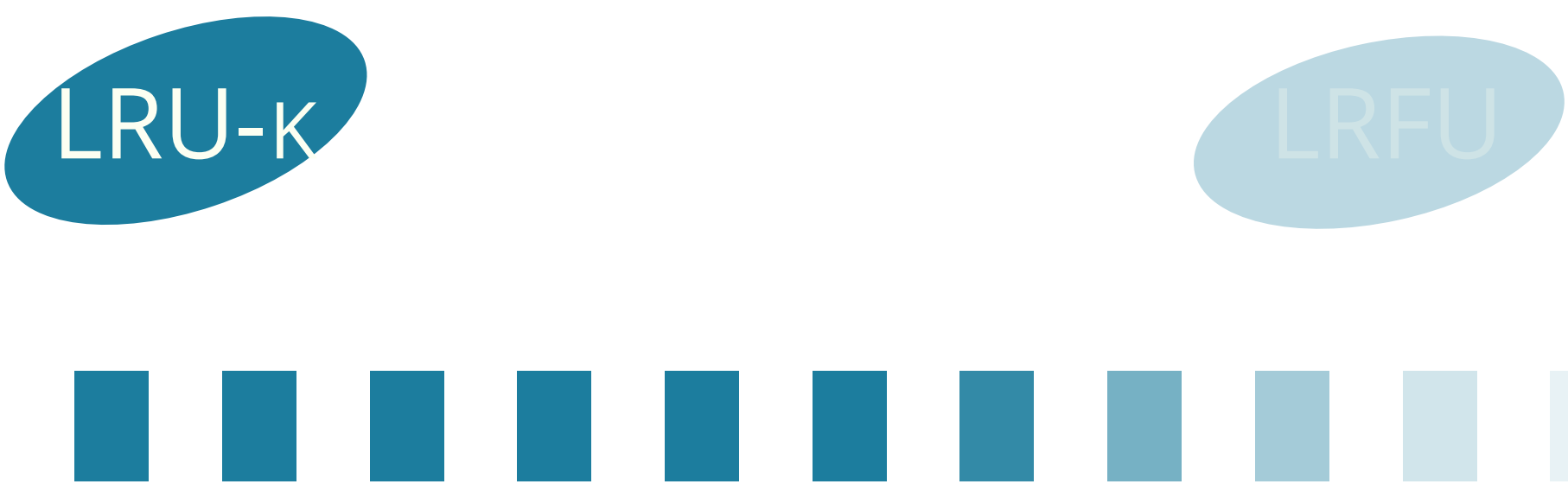
LRU

FIXING IT

admission control

find a middle ground between LRU and LFU

LRU-LFU HYBRID EVICTION ALGORITHMS



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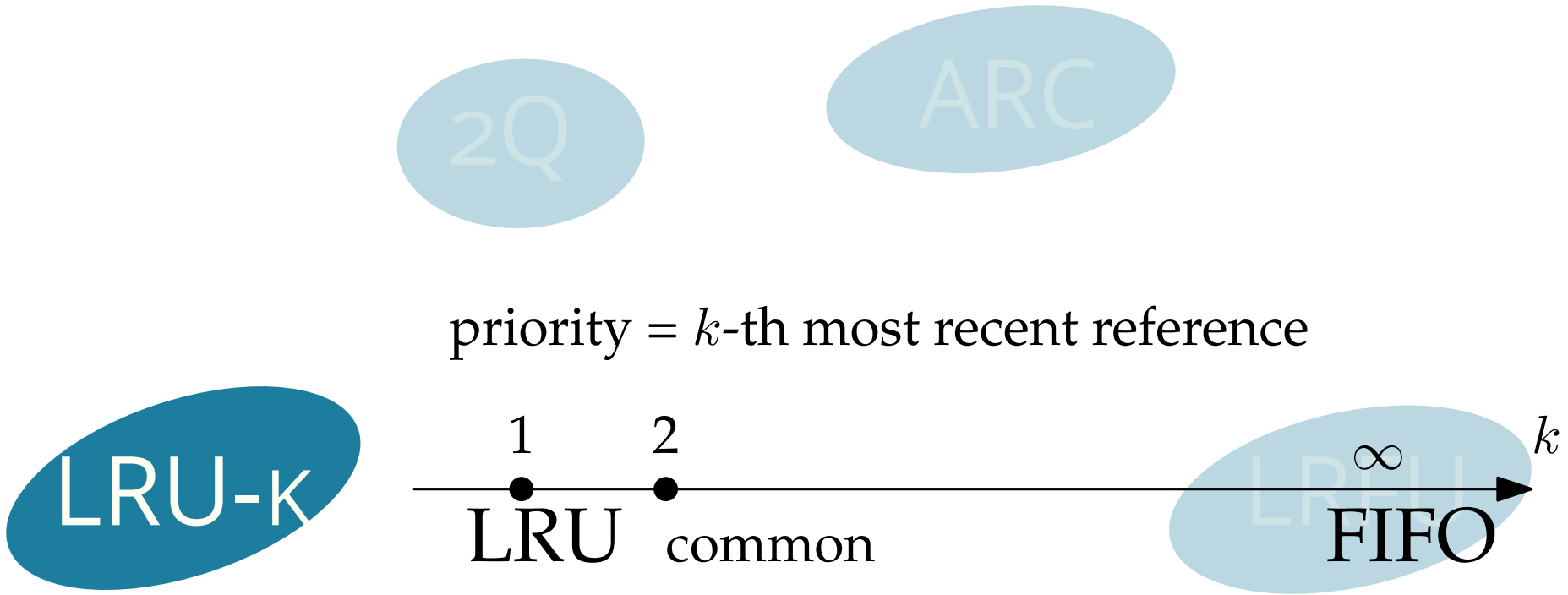

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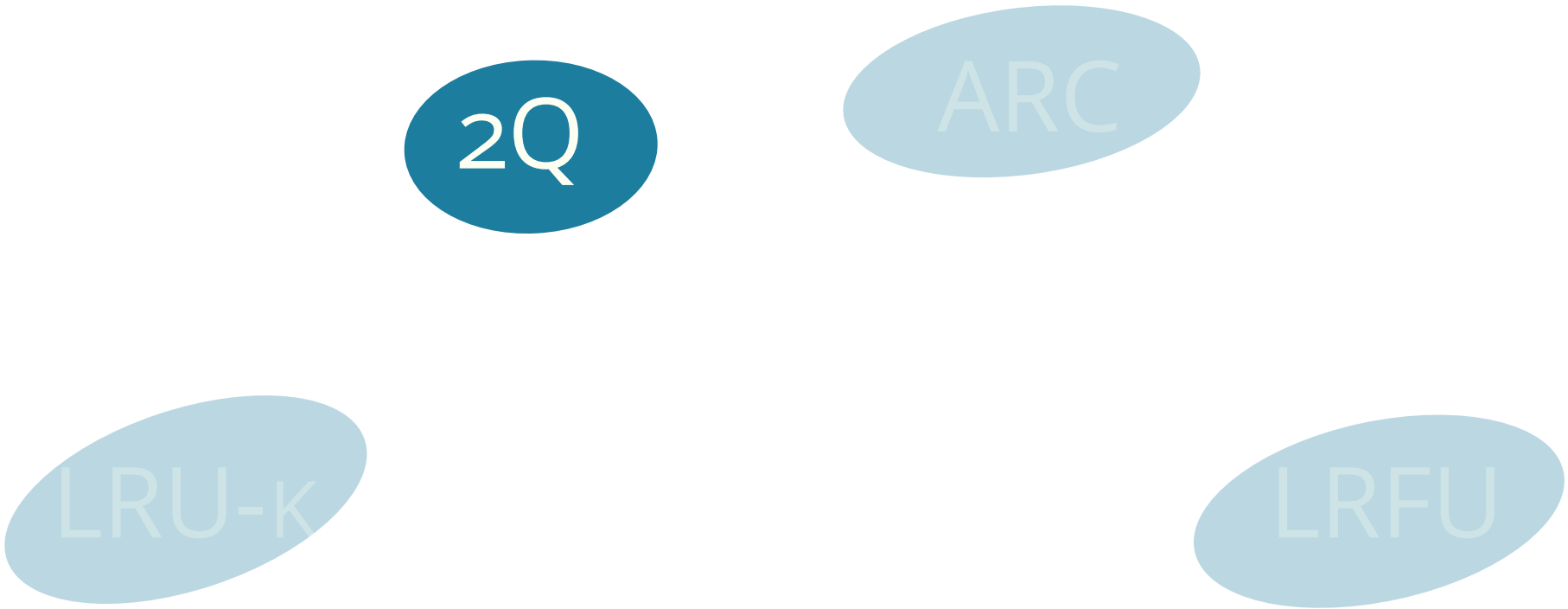

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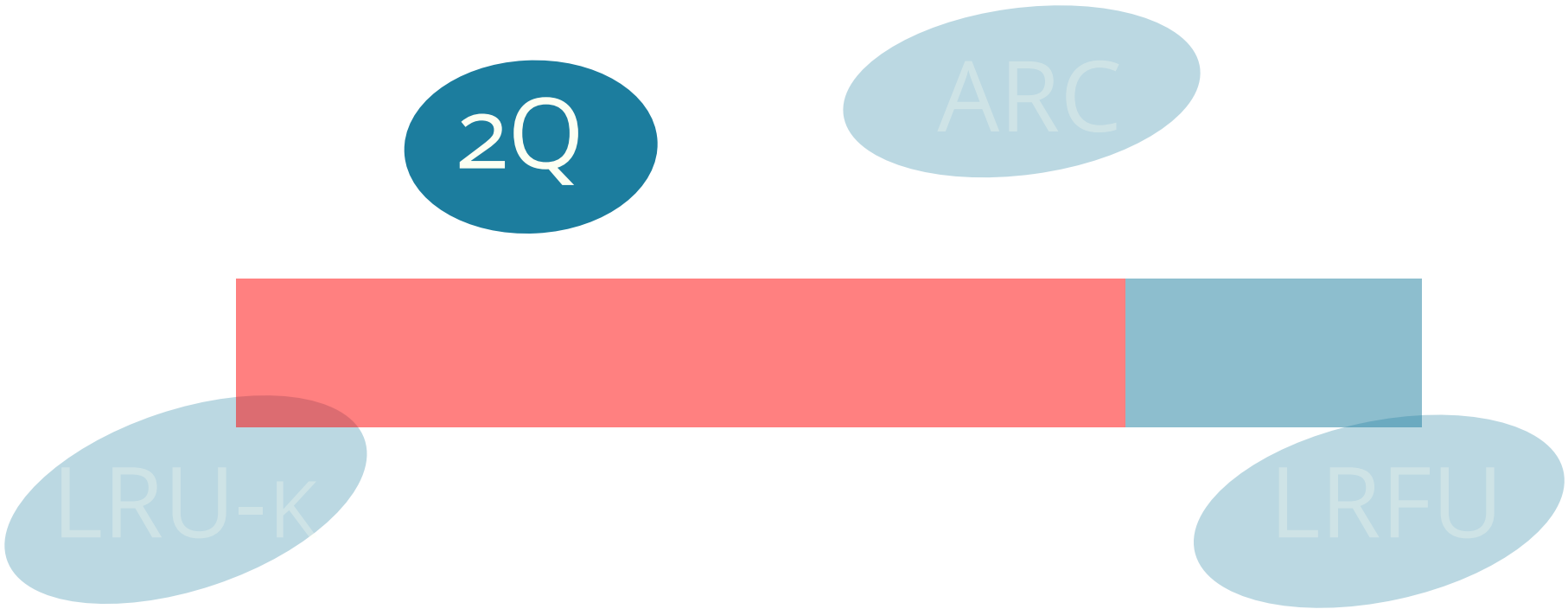
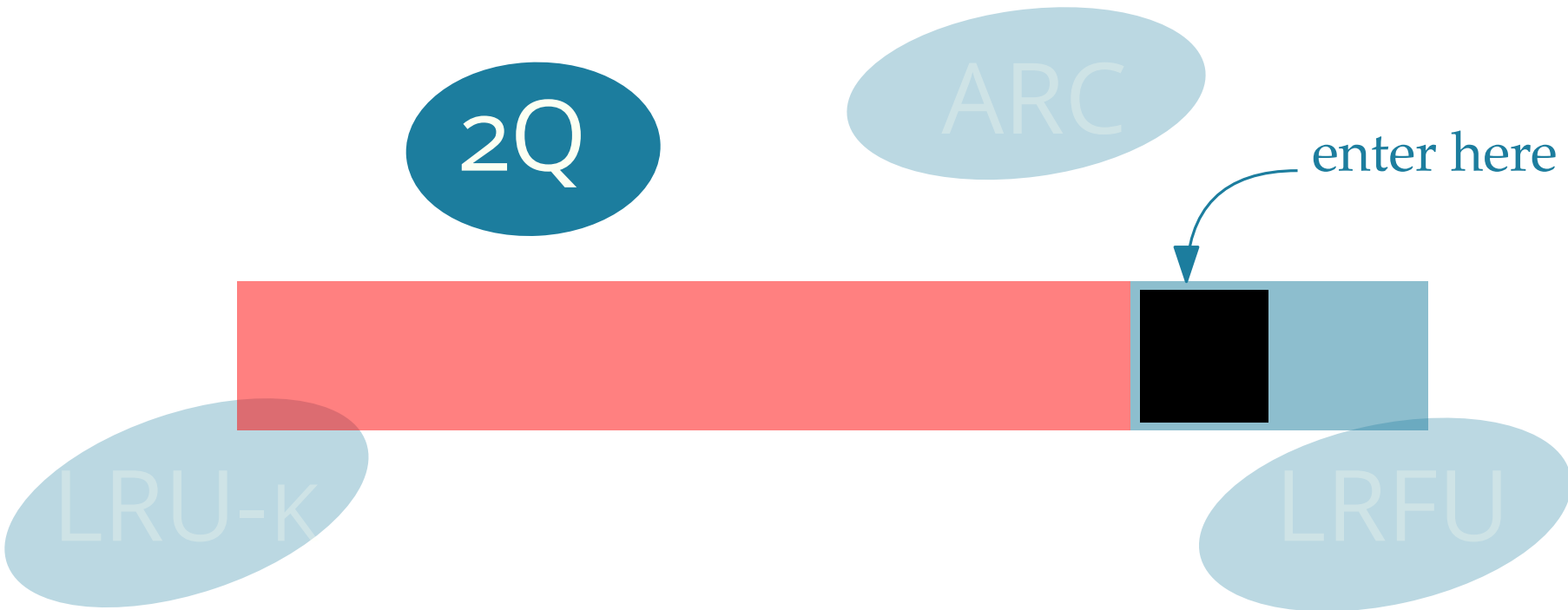


LRU-LFU HYBRID EVICTION ALGORITHMS



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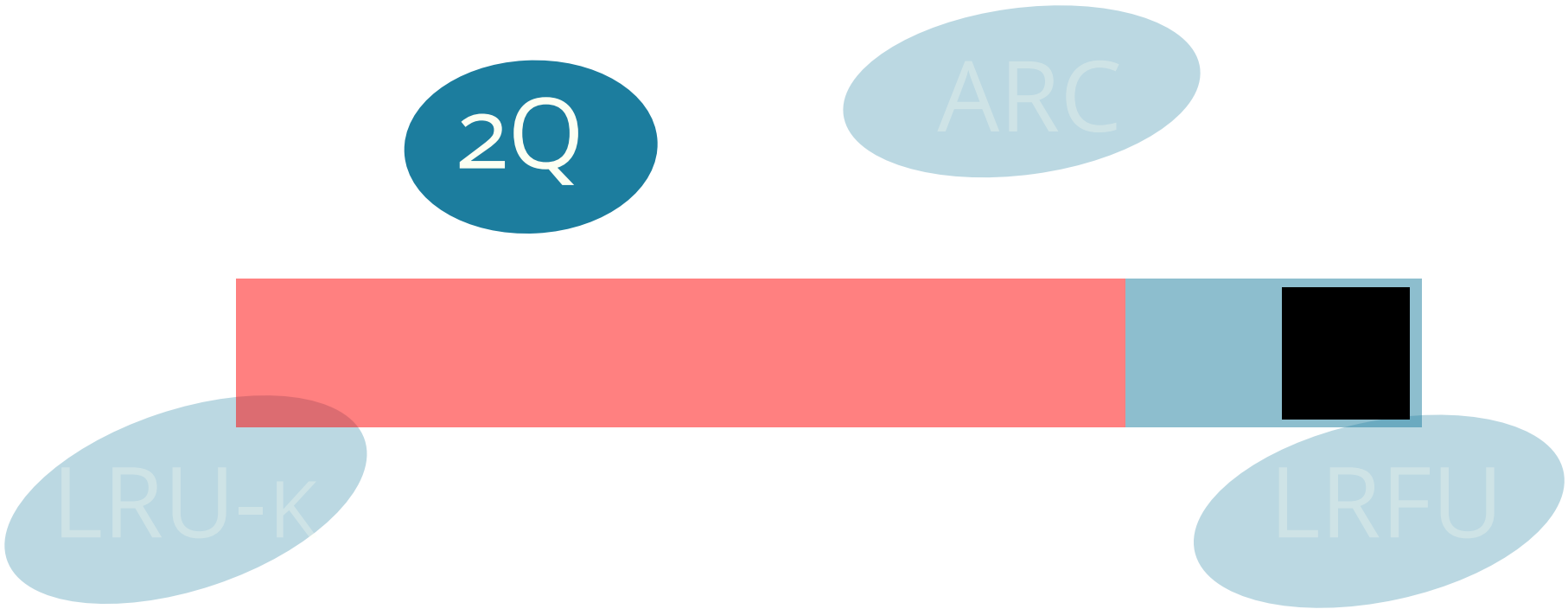

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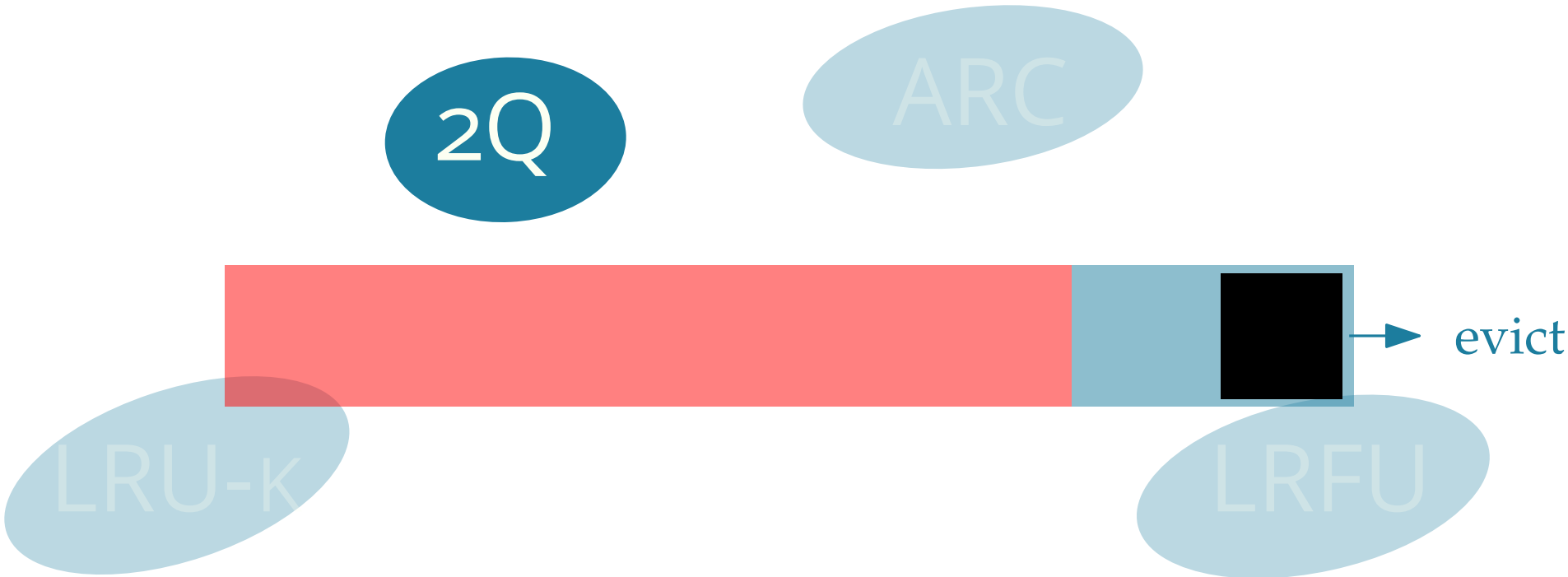

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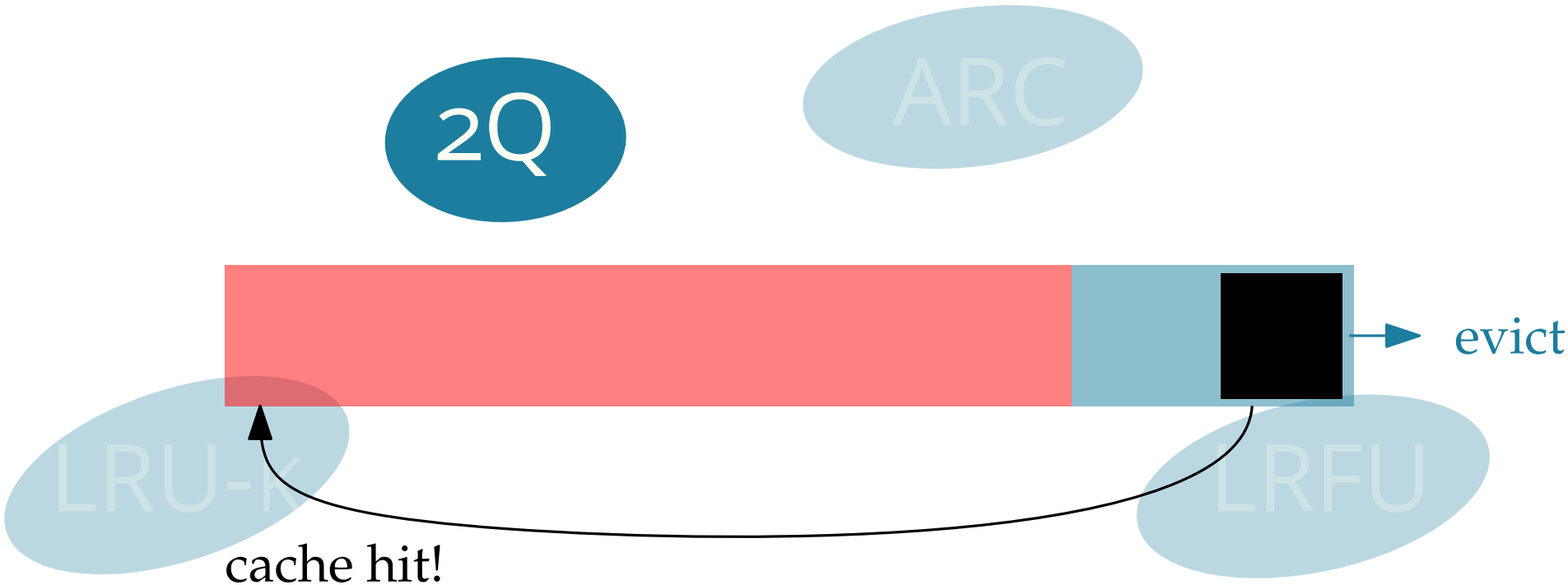

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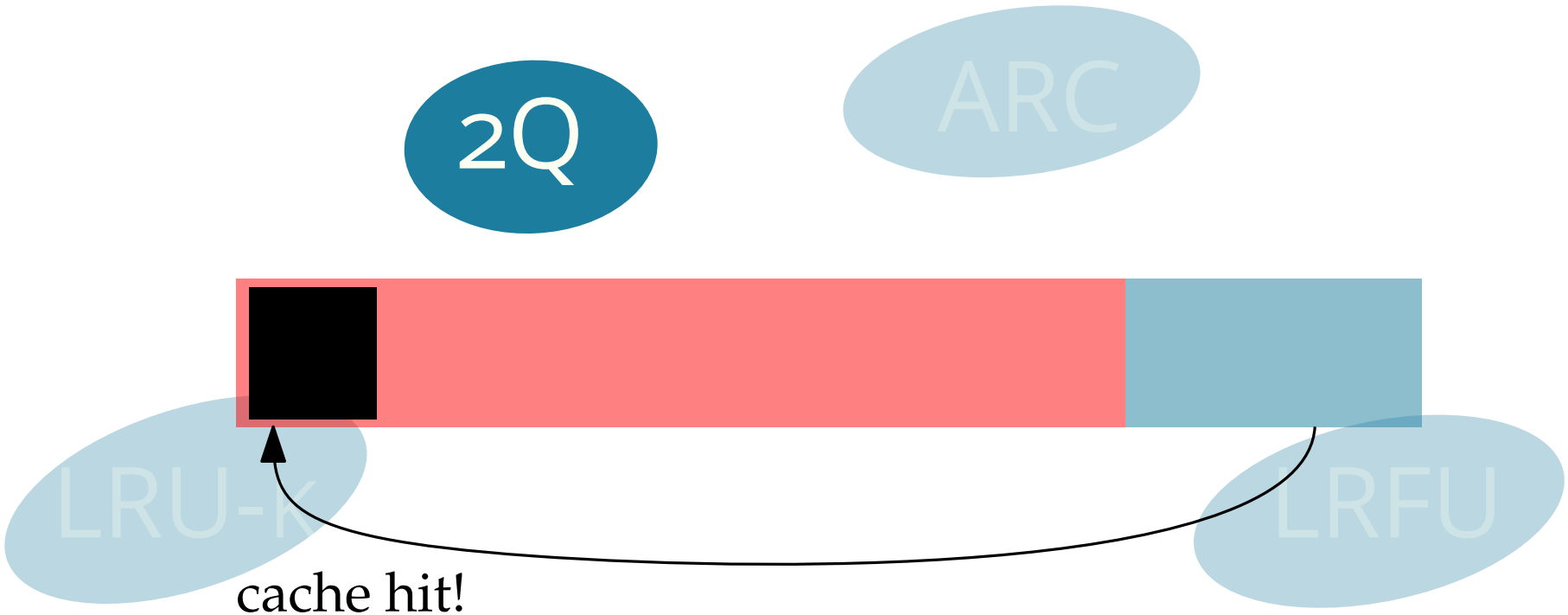

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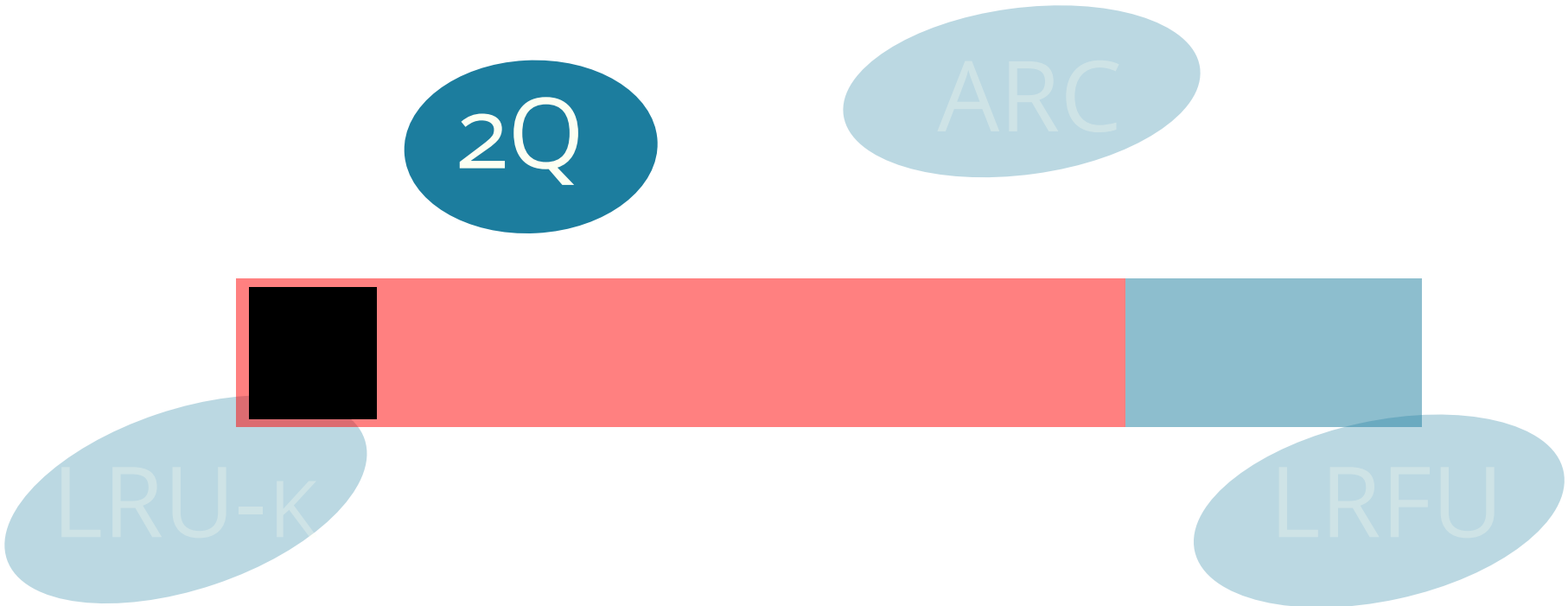

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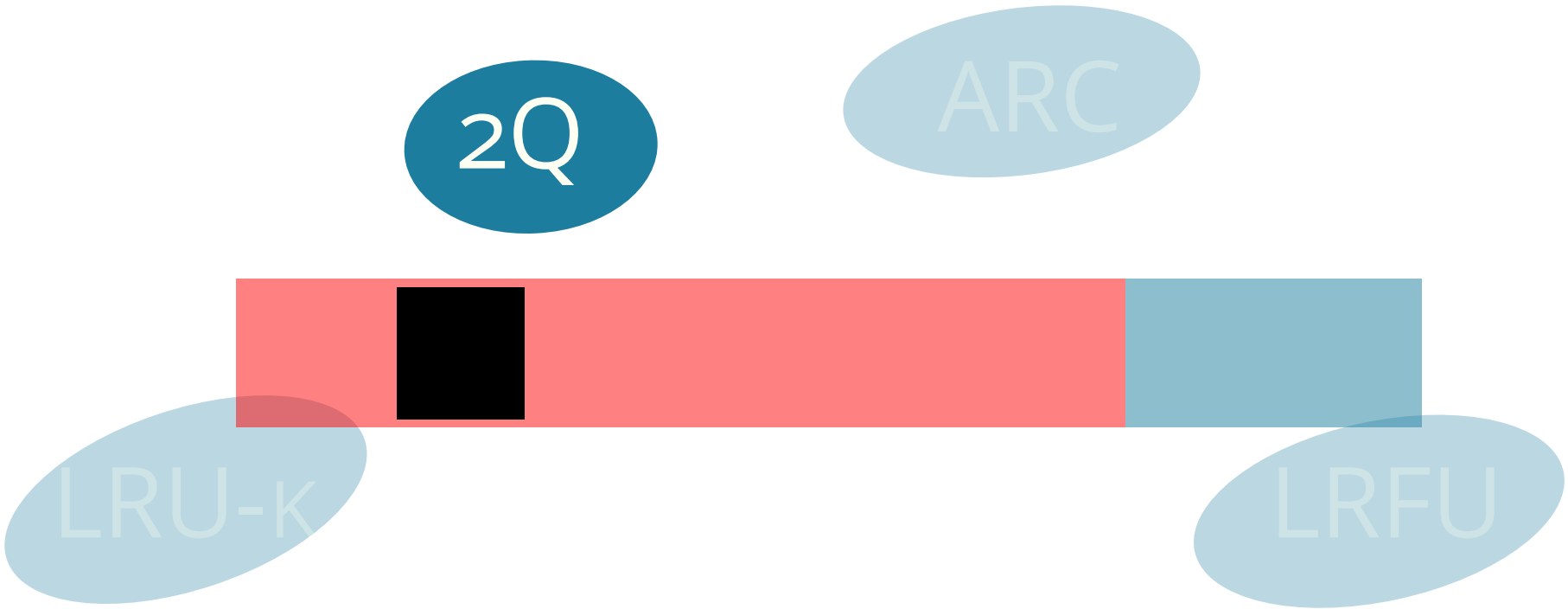

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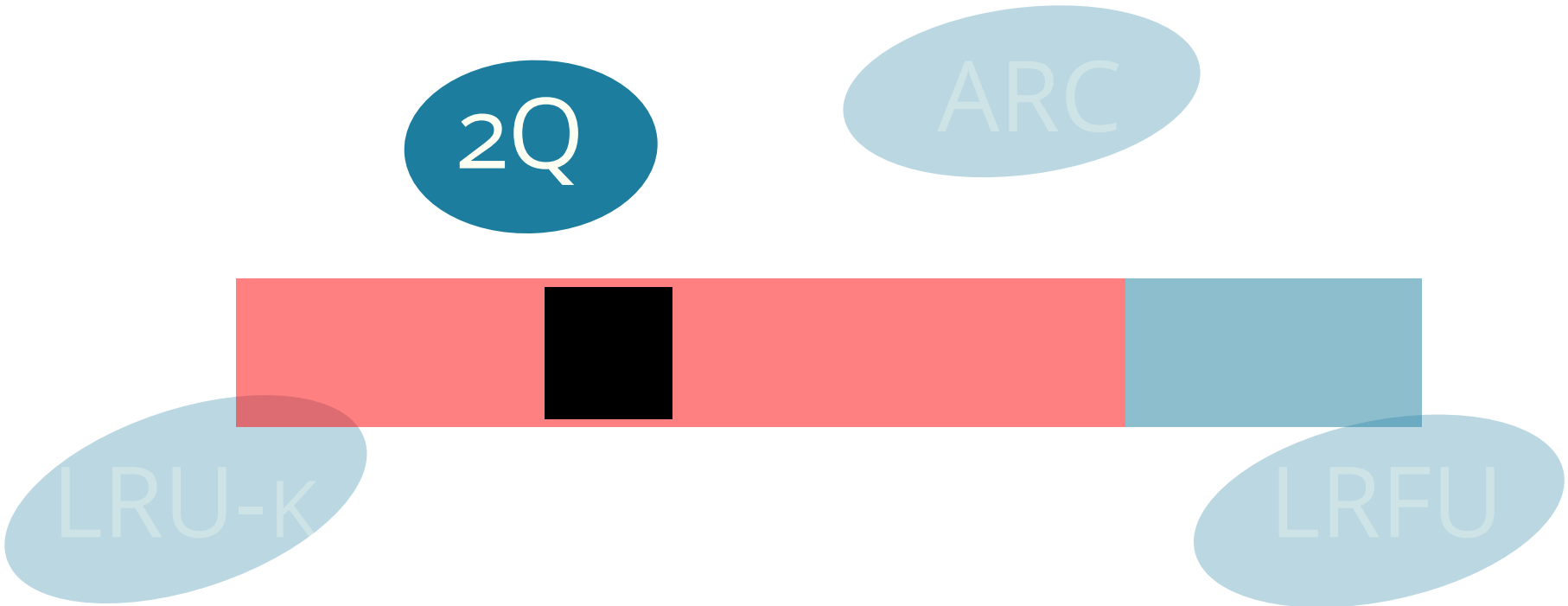

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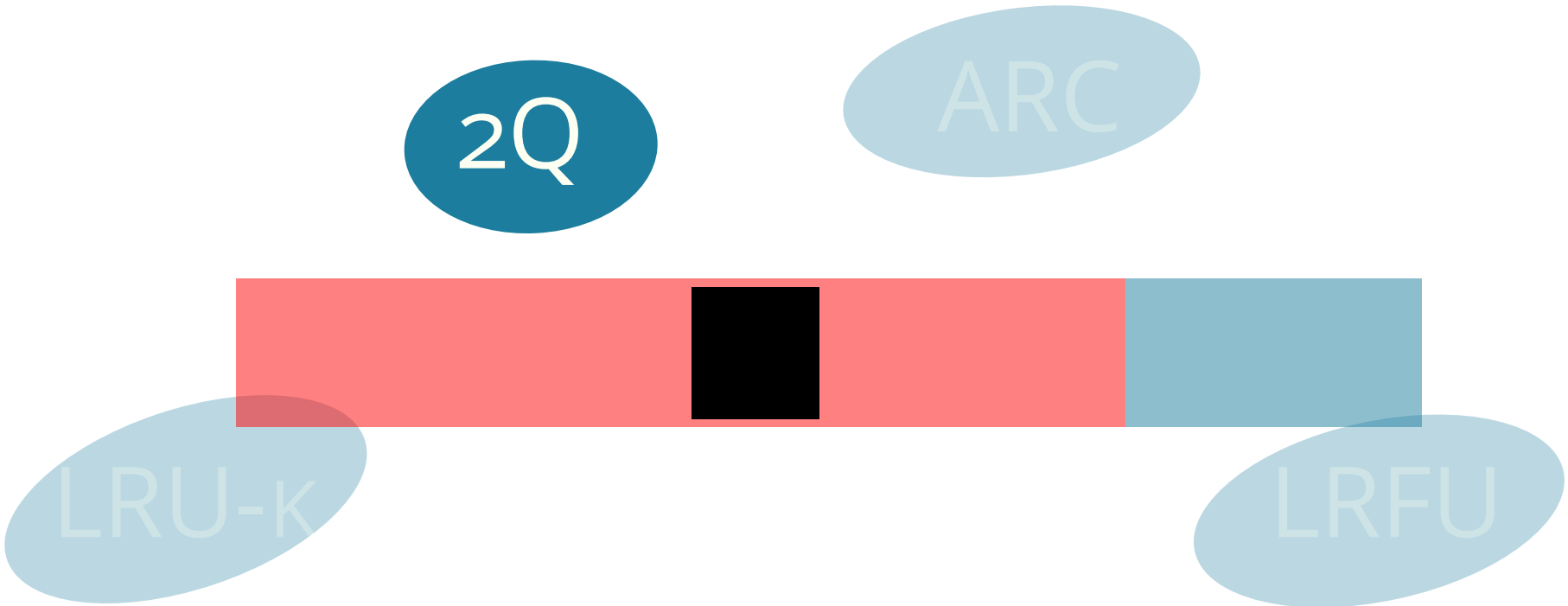

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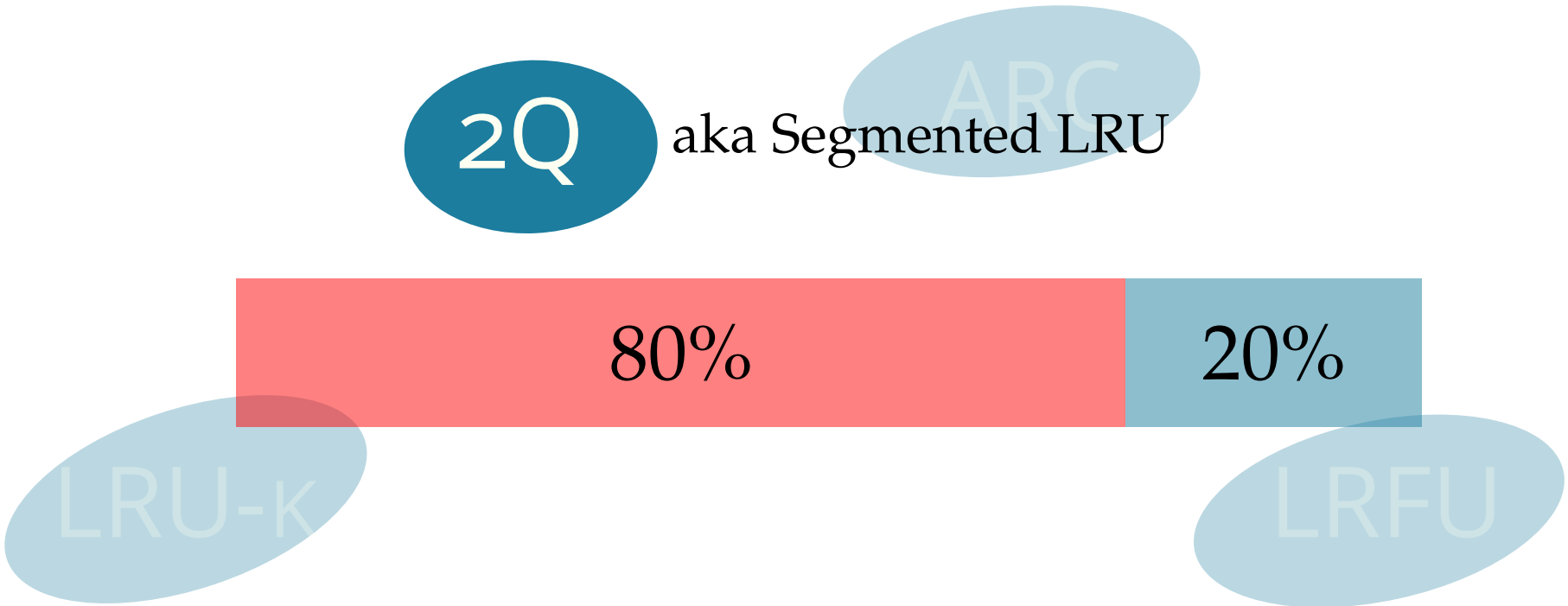

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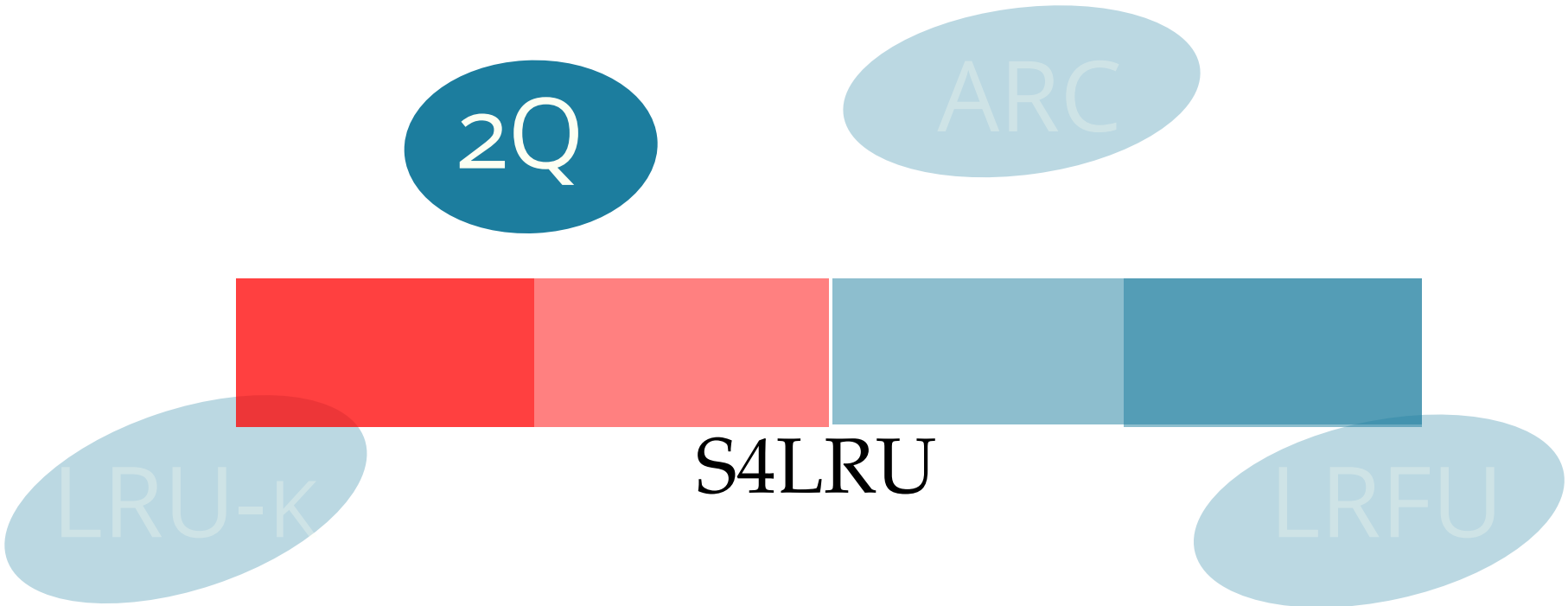

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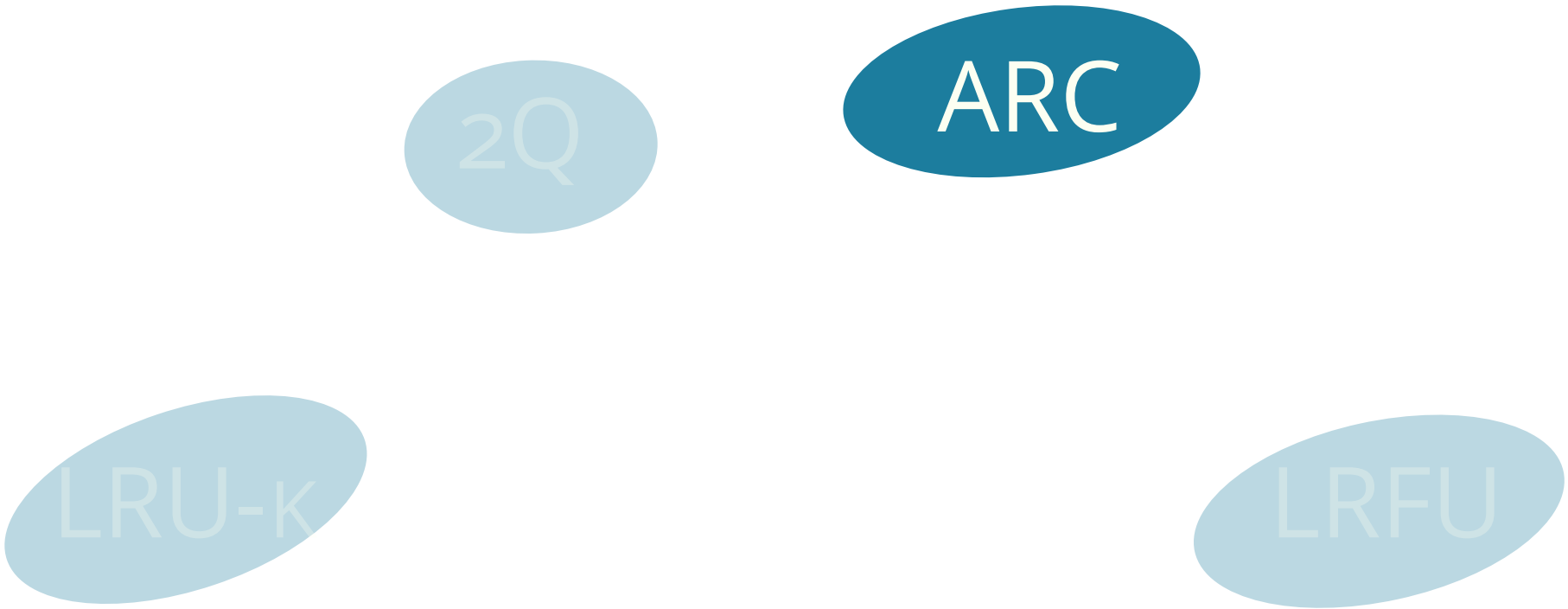

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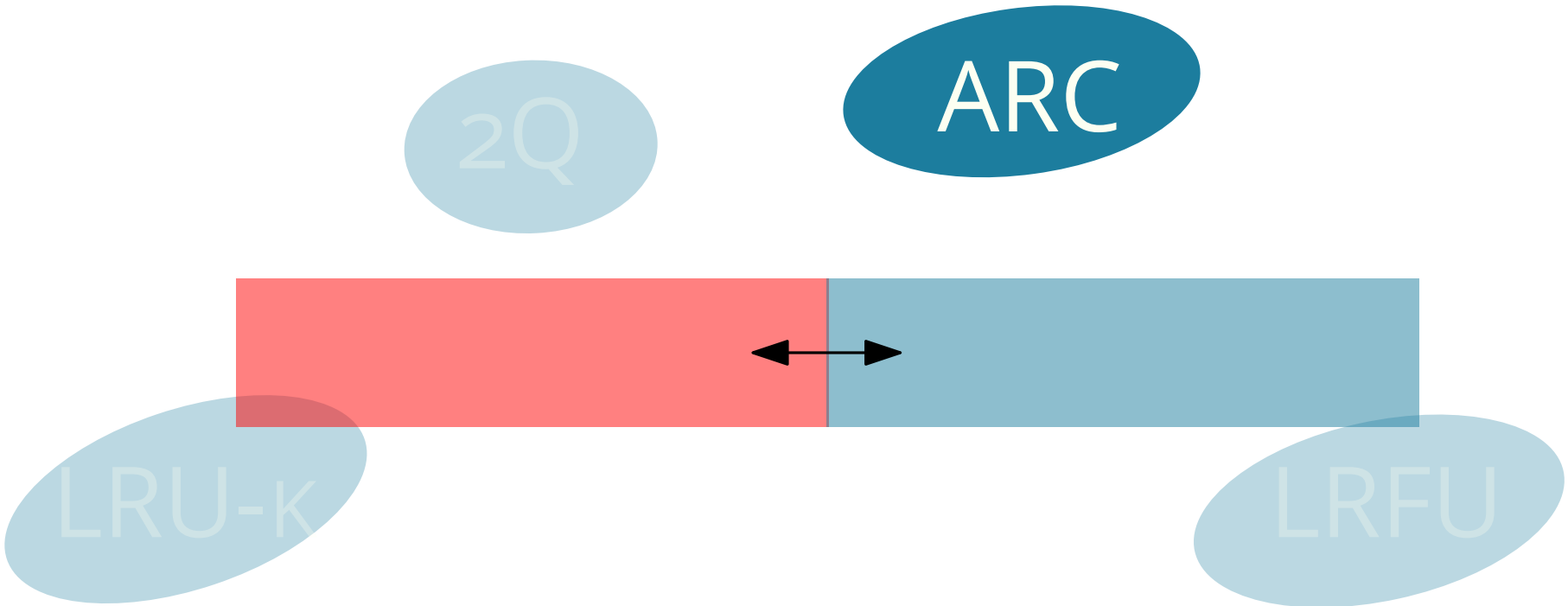

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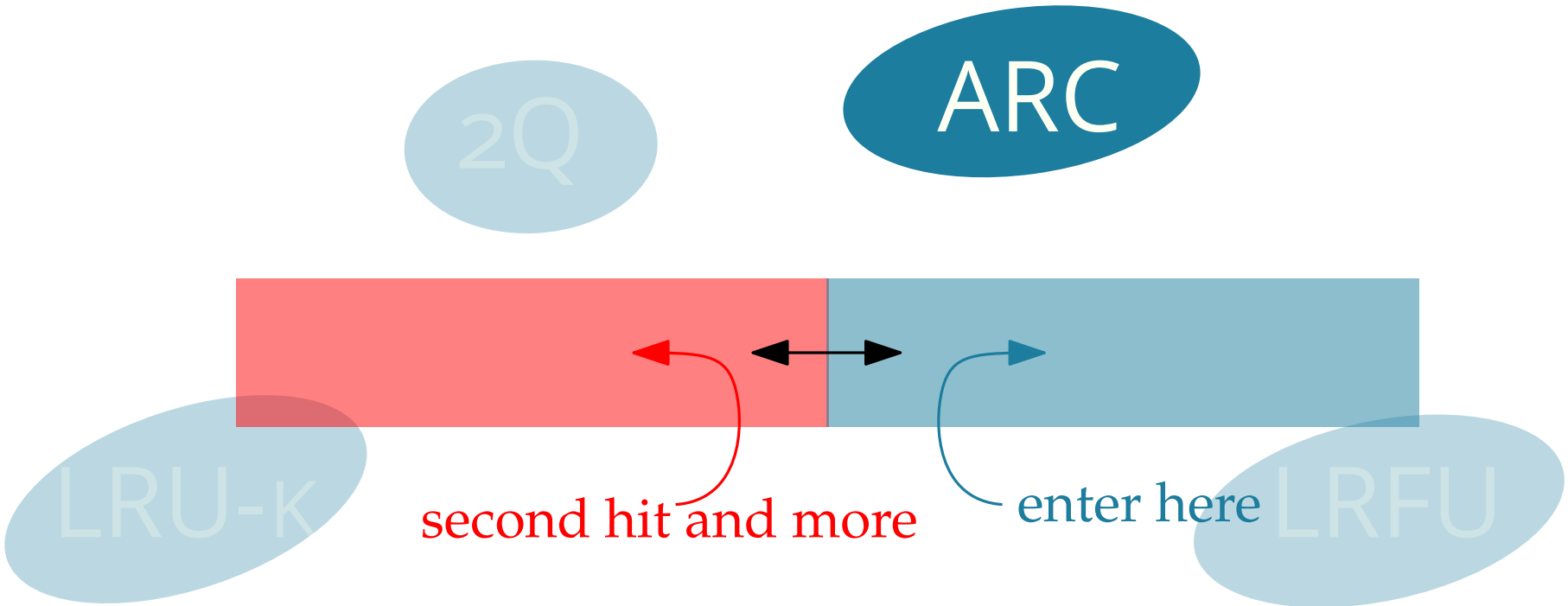

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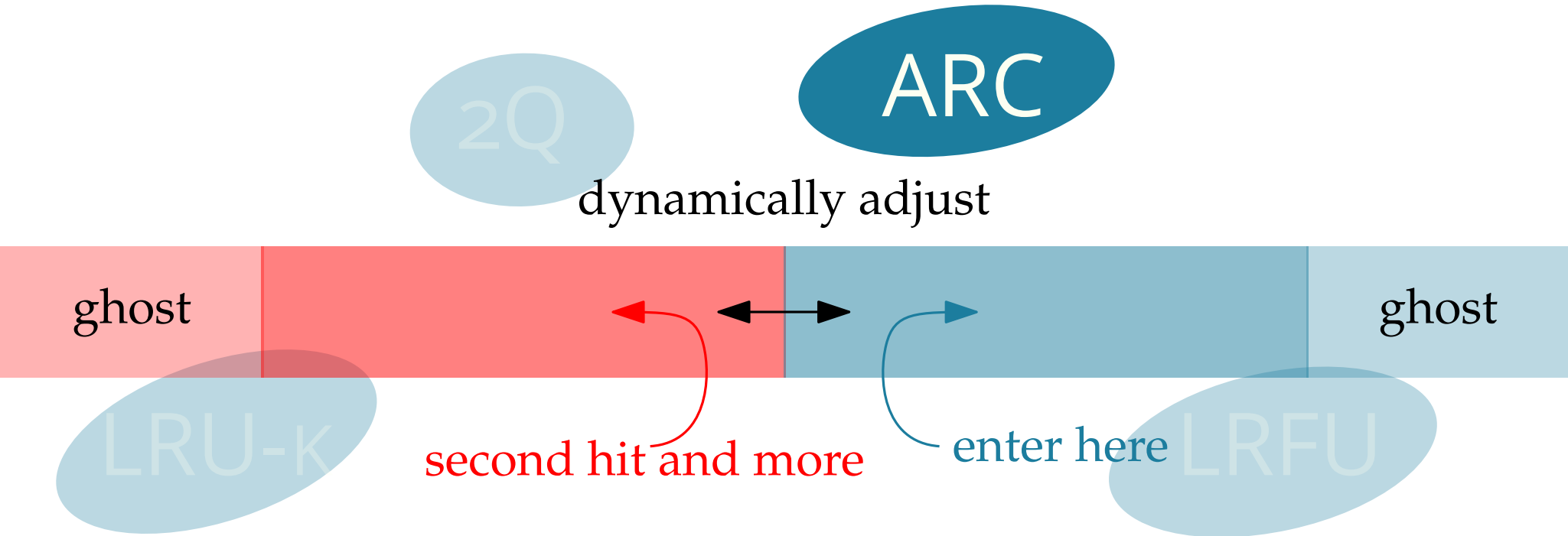

LRU-LFU HYBRID EVICTION ALGORITHMS

LRU-LFU HYBRID EVICTION ALGORITHMS

combined recency and frequency

$$\mathrm{CRF(x)} = \sum_i \alpha^{-t(x,i)}$$

time since last i-th request to x

2Q

ARC

LRU-K

LRFU

LRU-LFU HYBRID EVICTION ALGORITHMS

combined recency and frequency

$$\mathrm{CRF(x)} = \sum_i \alpha^{-t(x,i)}$$

2Q

ARC

time since last i-th request to x

LRU-LFU HYBRID EVICTION ALGORITHMS

combined recency and frequency

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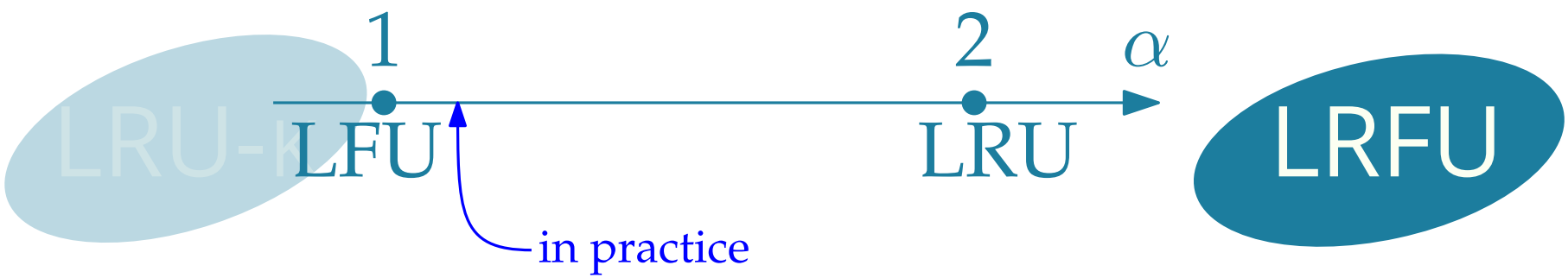

LRU-LFU HYBRID EVICTION ALGORITHMS

HYBRID SOLUTIONS TEND TO DO WELL

Evaluation of Caching Strategies Based on Access Statistics of Past Requests, Hasslinger, Ntougias, MMB & DFT 2014

UNDERSTANDING THE WORKLOAD


LRU
LFU
handles bursts in popularity well
the colder can push out the warmer
suffers from cache pollution
optimal if popularity is static

LRU ◄——————————————————————————► LFU

handles bursts in popularity well

the colder can push out the warmer

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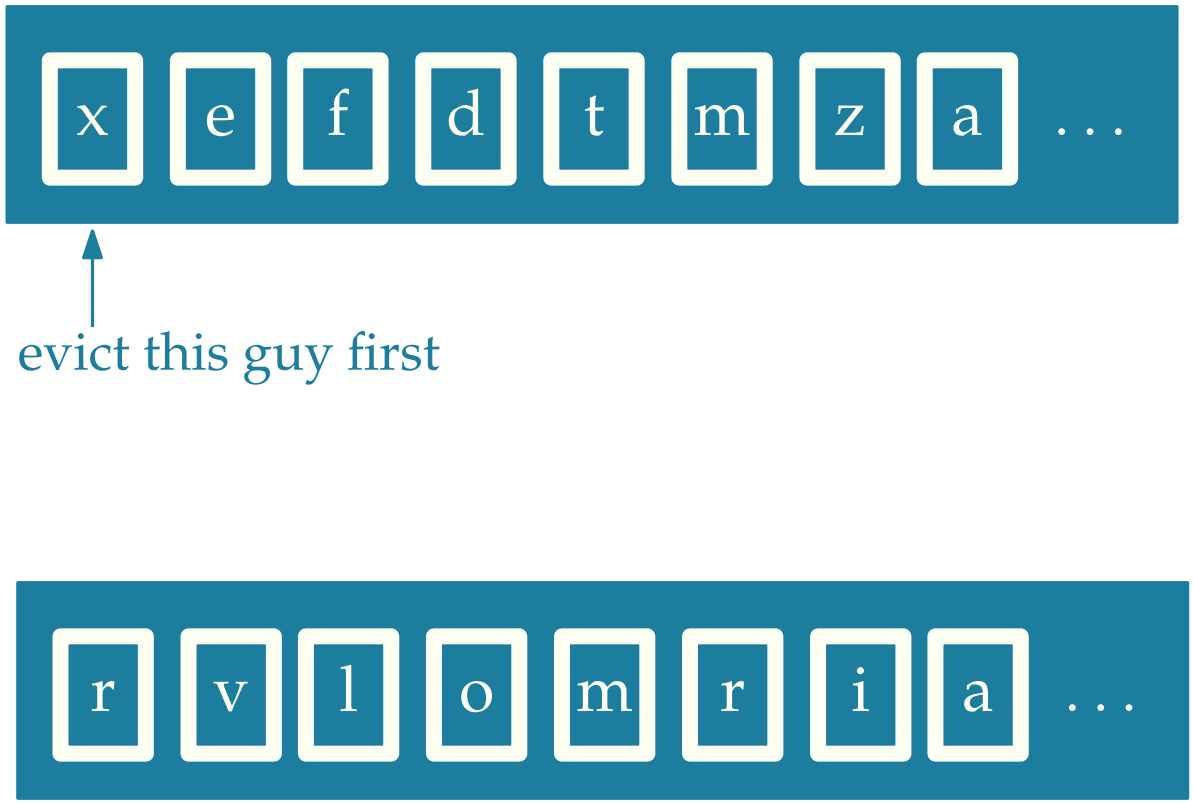
optimal if popularity is static

what a caching algorithm boils down to



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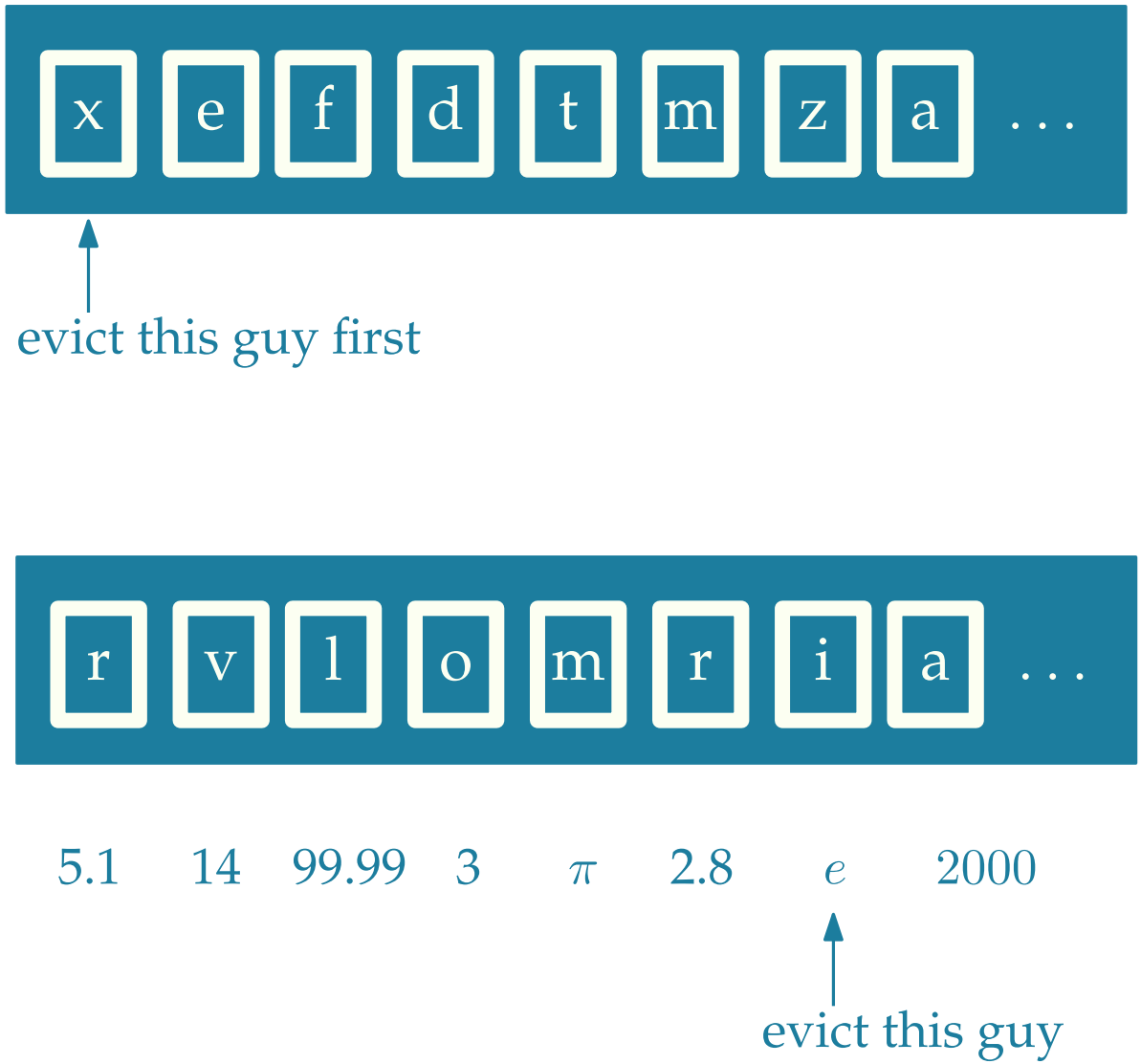
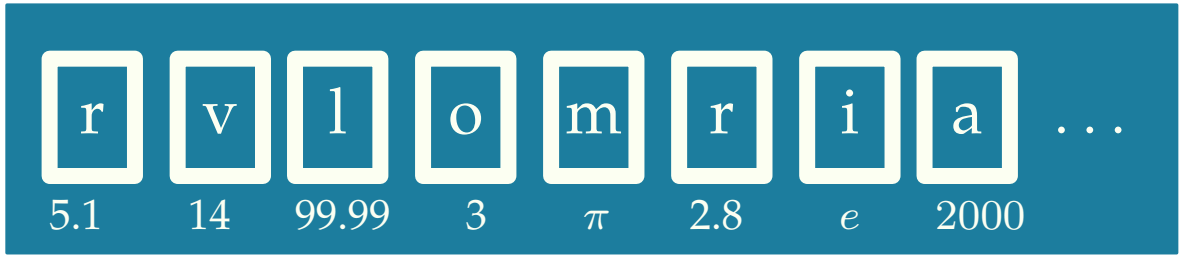


what a caching algorithm boils down to

caching algorithm template



handle-request(object o):

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if o is in cache:
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        (for all p in cache: update priority(p))

else:
        while there is not enough room in cache for o:
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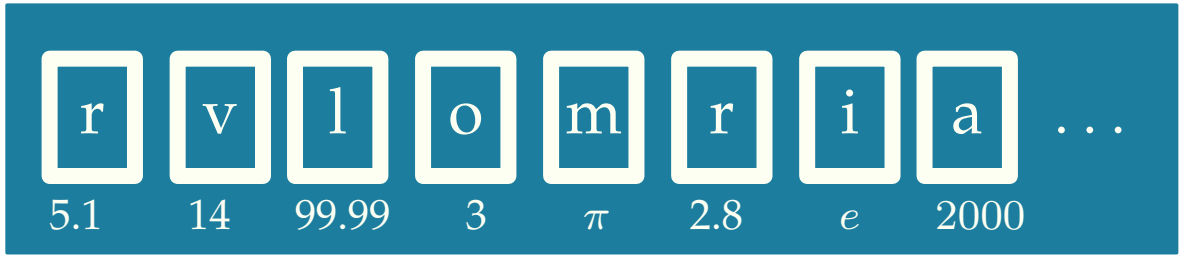
        bring o in cache

        compute priority(o)
        (for all p in cache: update priority(p))
```

caching algorithm template



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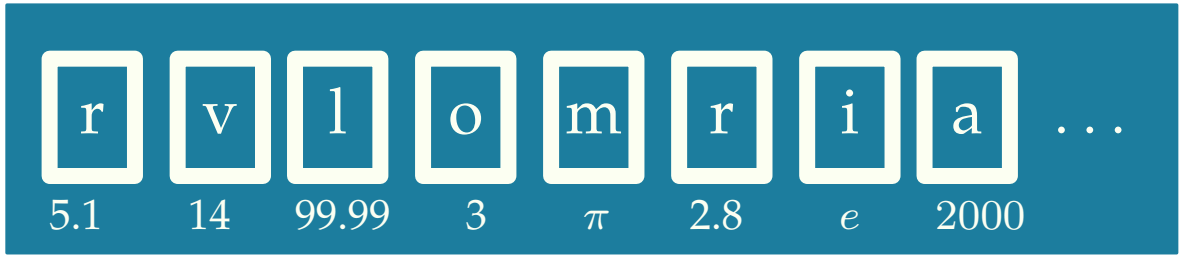

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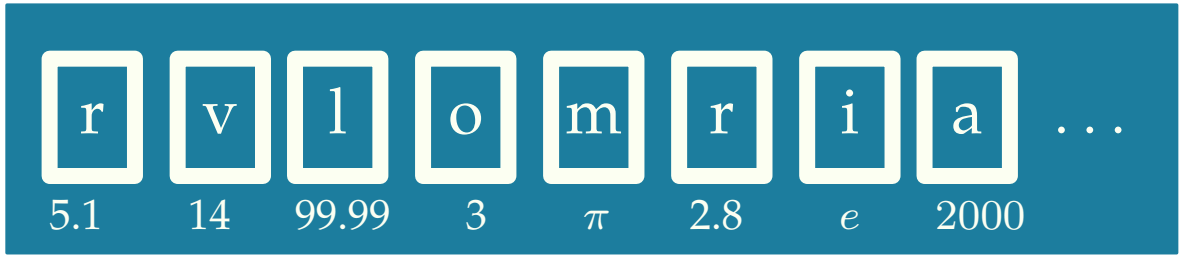
caching algorithm template



handle-request(object o):

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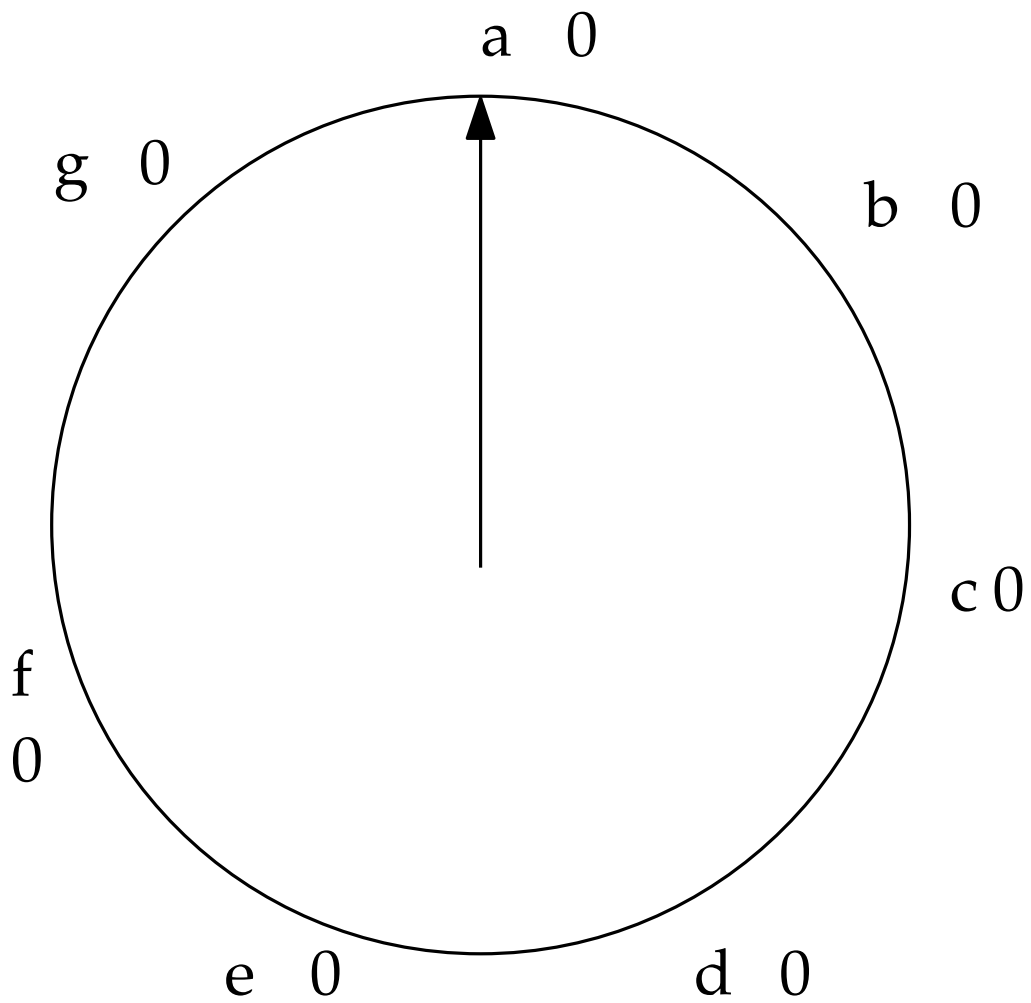
while there is not enough room in cache for o:
find and evict object in cache with smallest priority
O(1)
bring o in cache

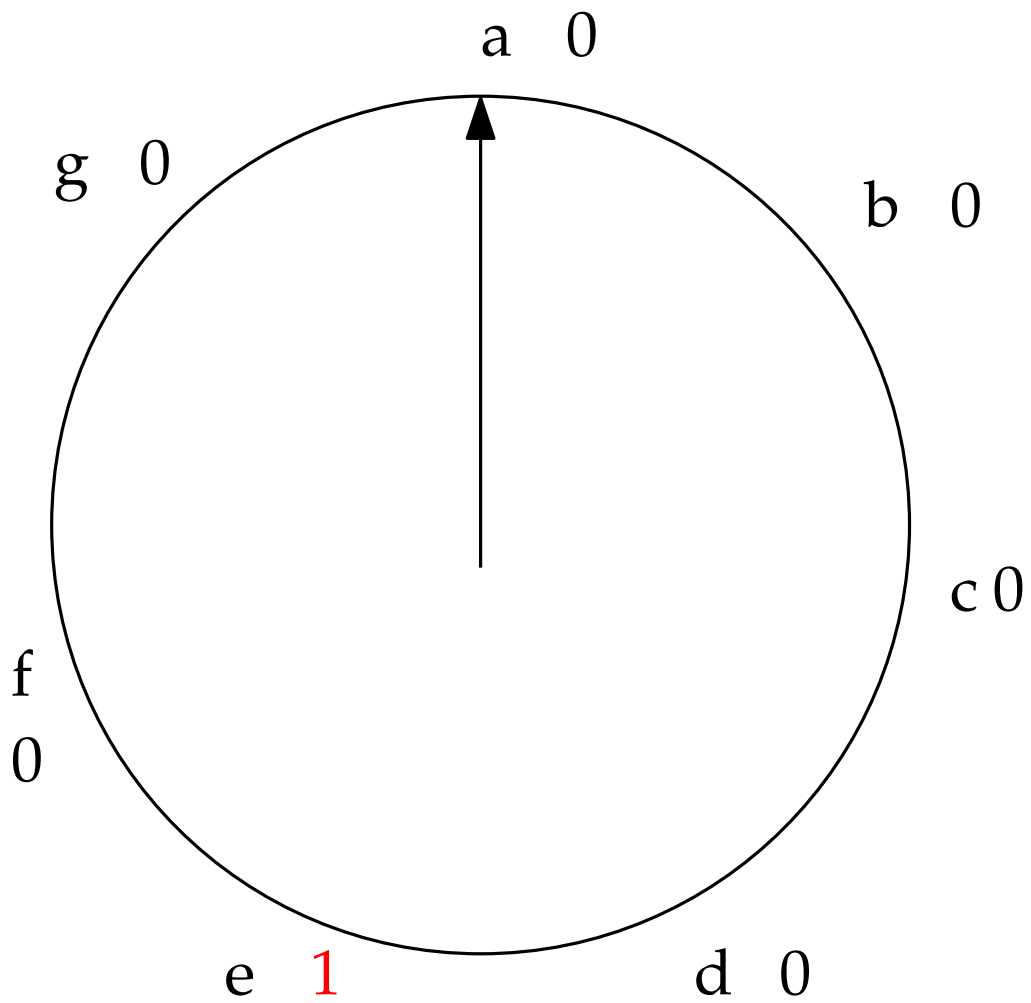
compute priority(o) O(1)
(for all p in cache: update priority(p))

clock: an approximation of LRU



e d a e b h

clock: an approximation of LRU



e d a e b h

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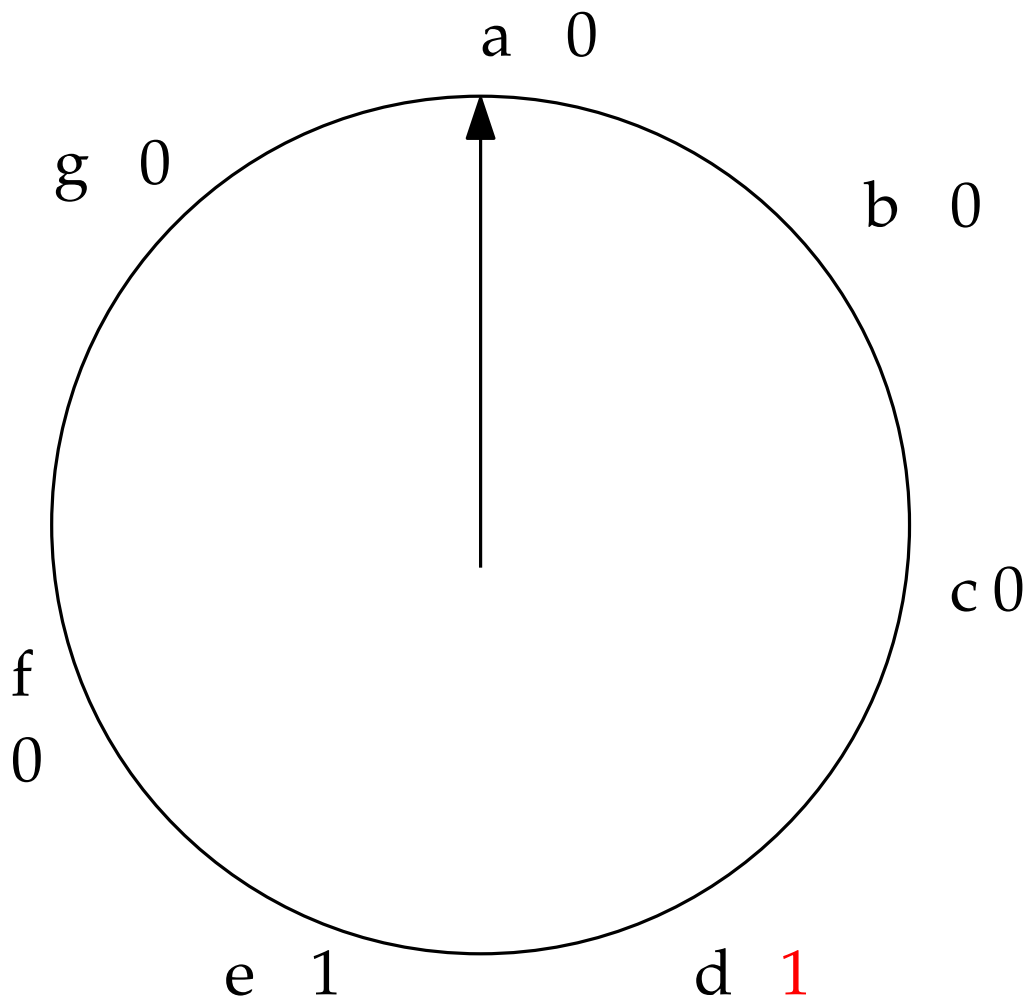

e d a e b h

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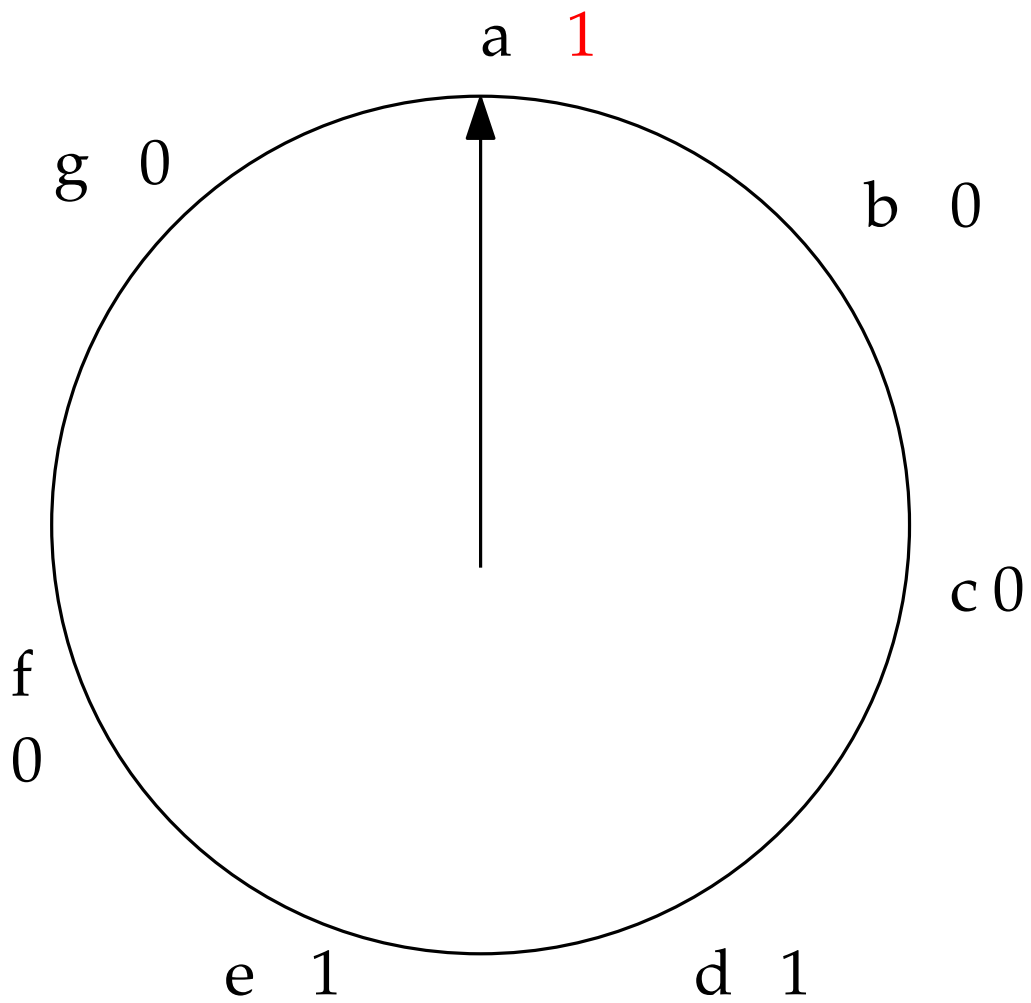

e d a e b h

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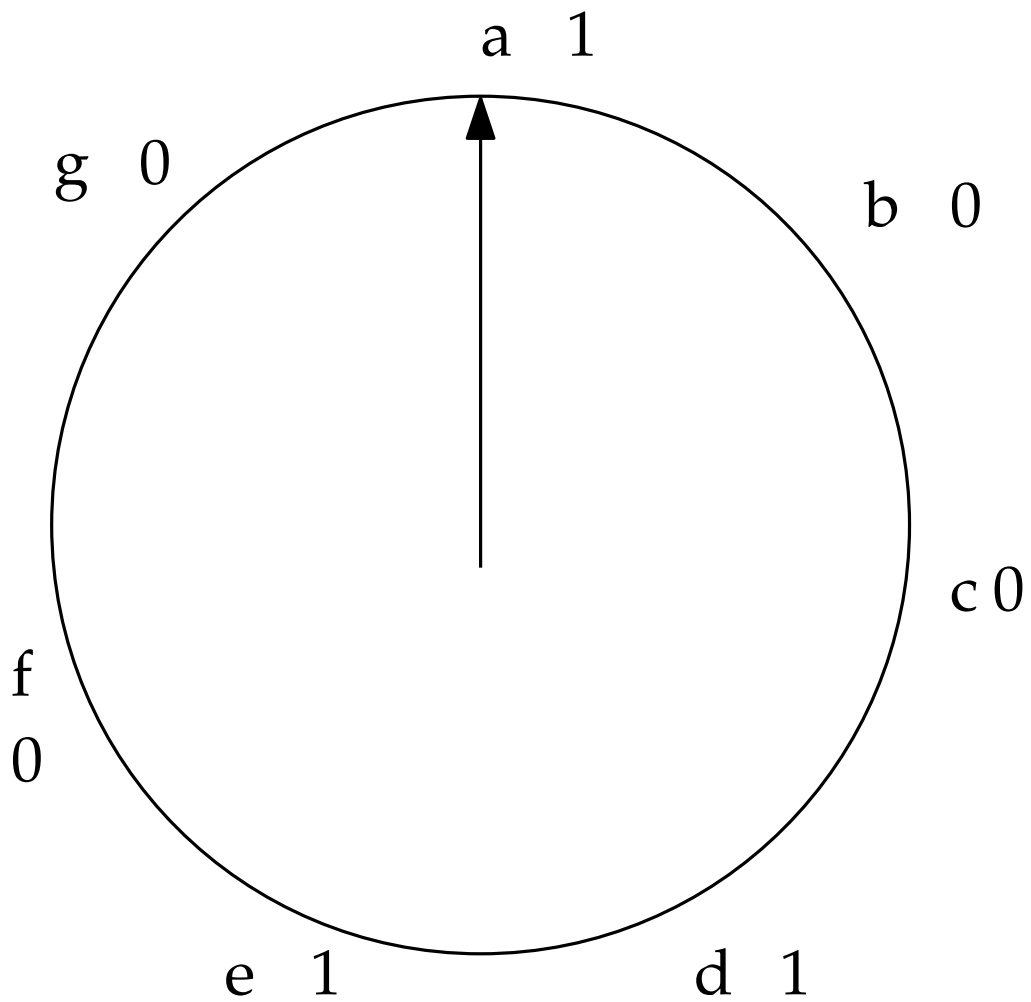

e d a e b h

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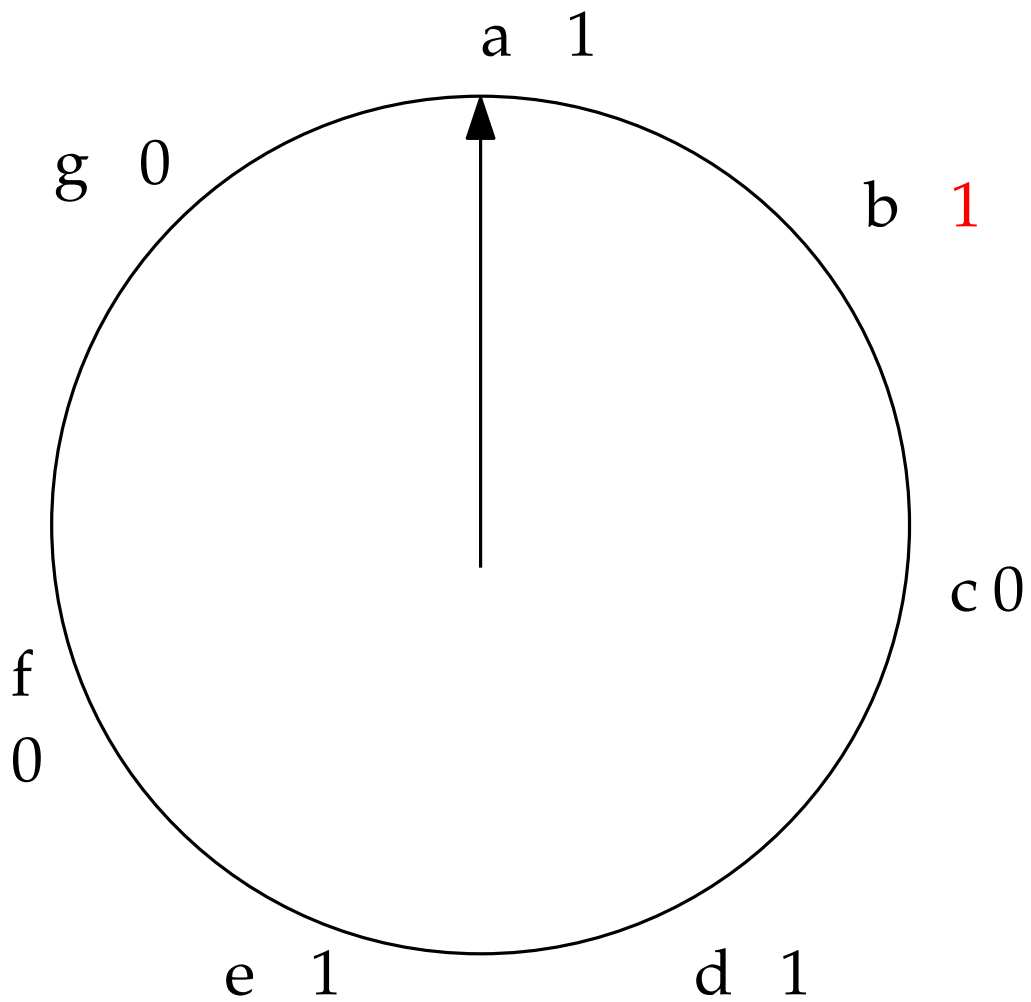
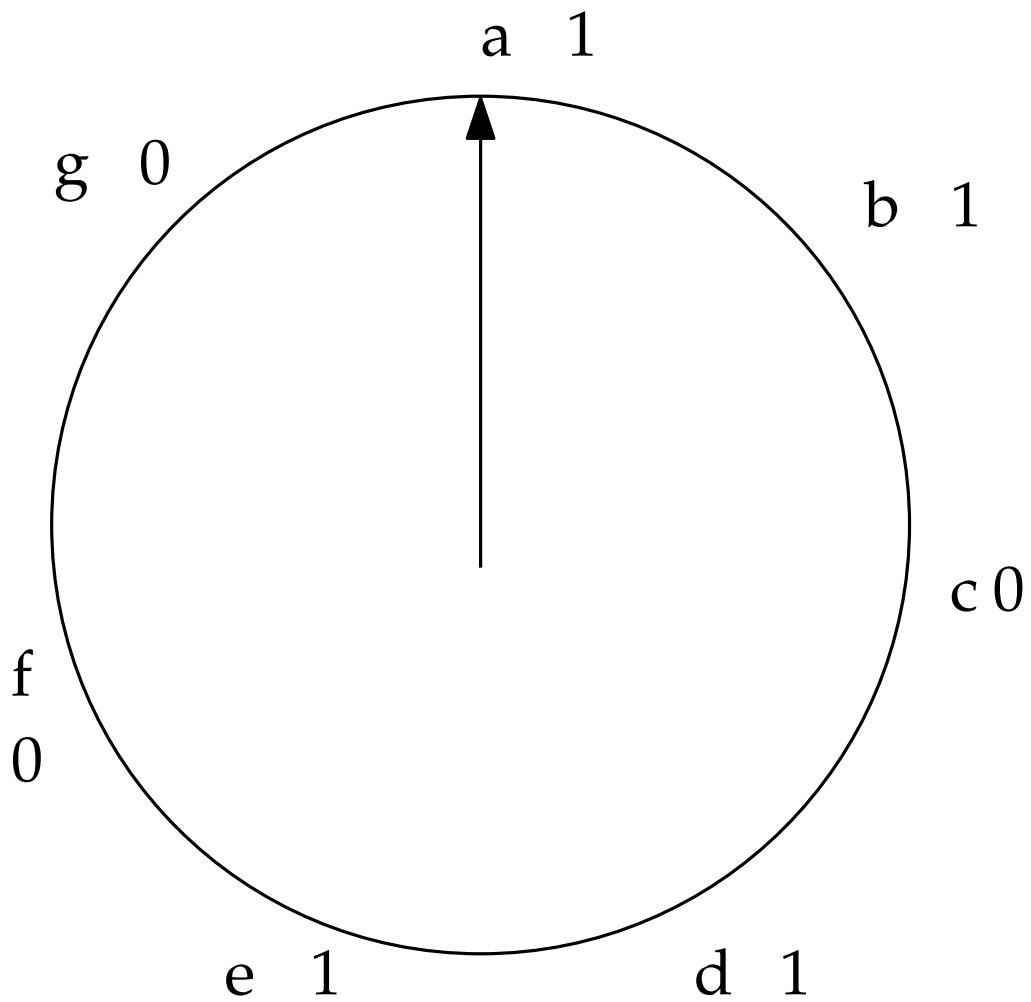


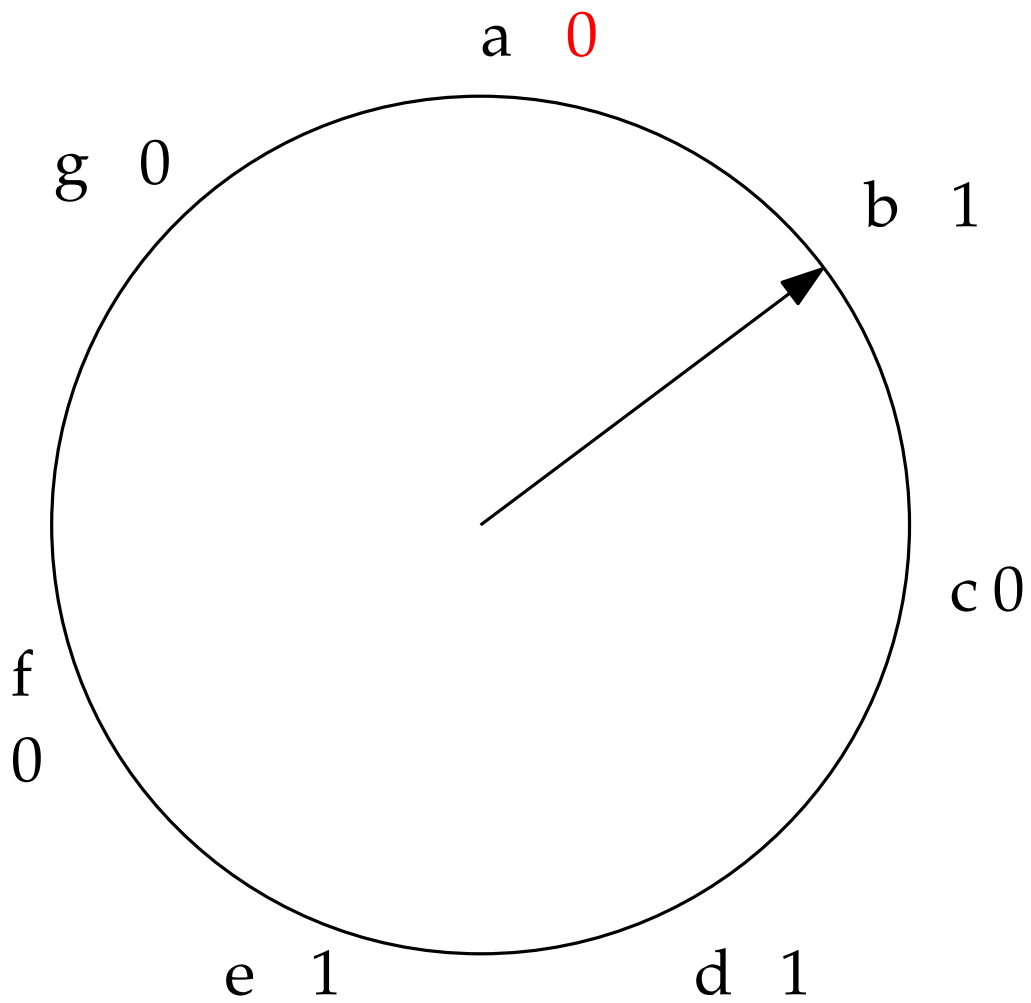
e d a e b h

clock: an approximation of LRU



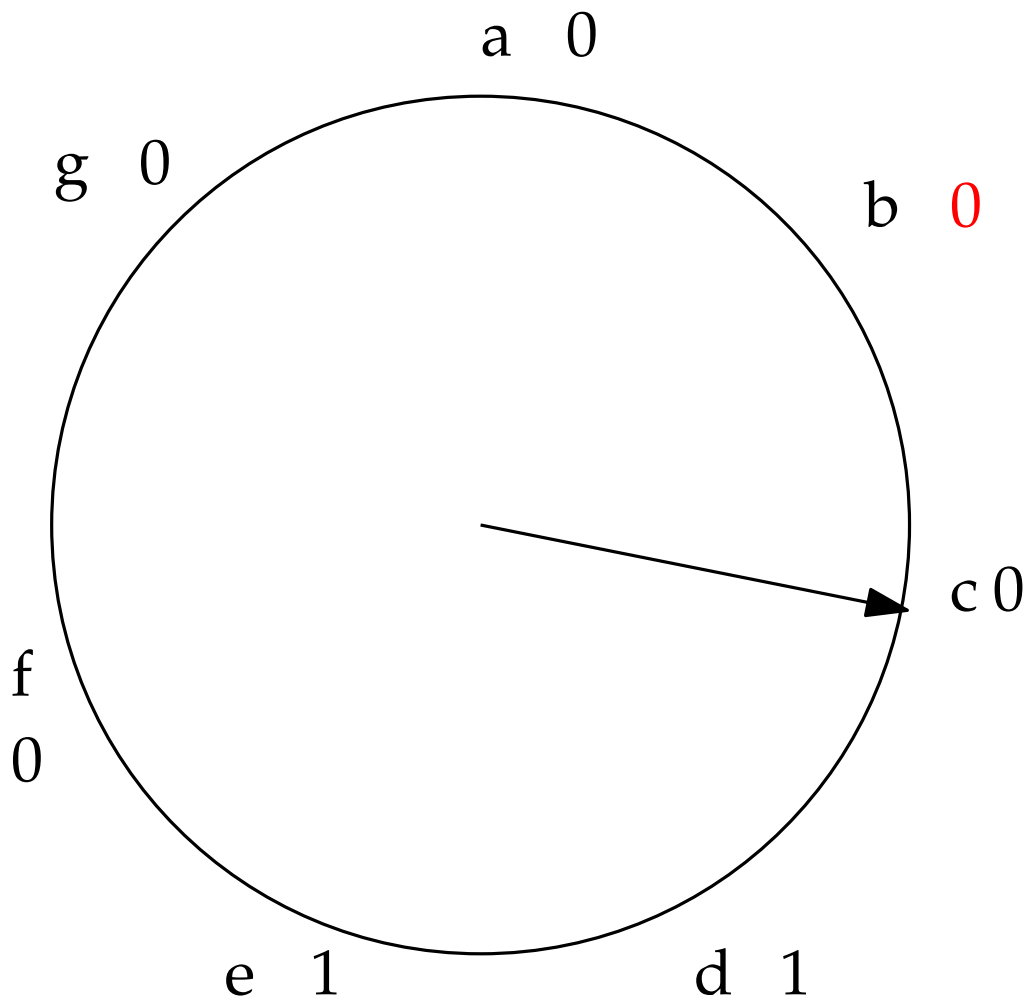
e d a e b h

clock: an approximation of LRU



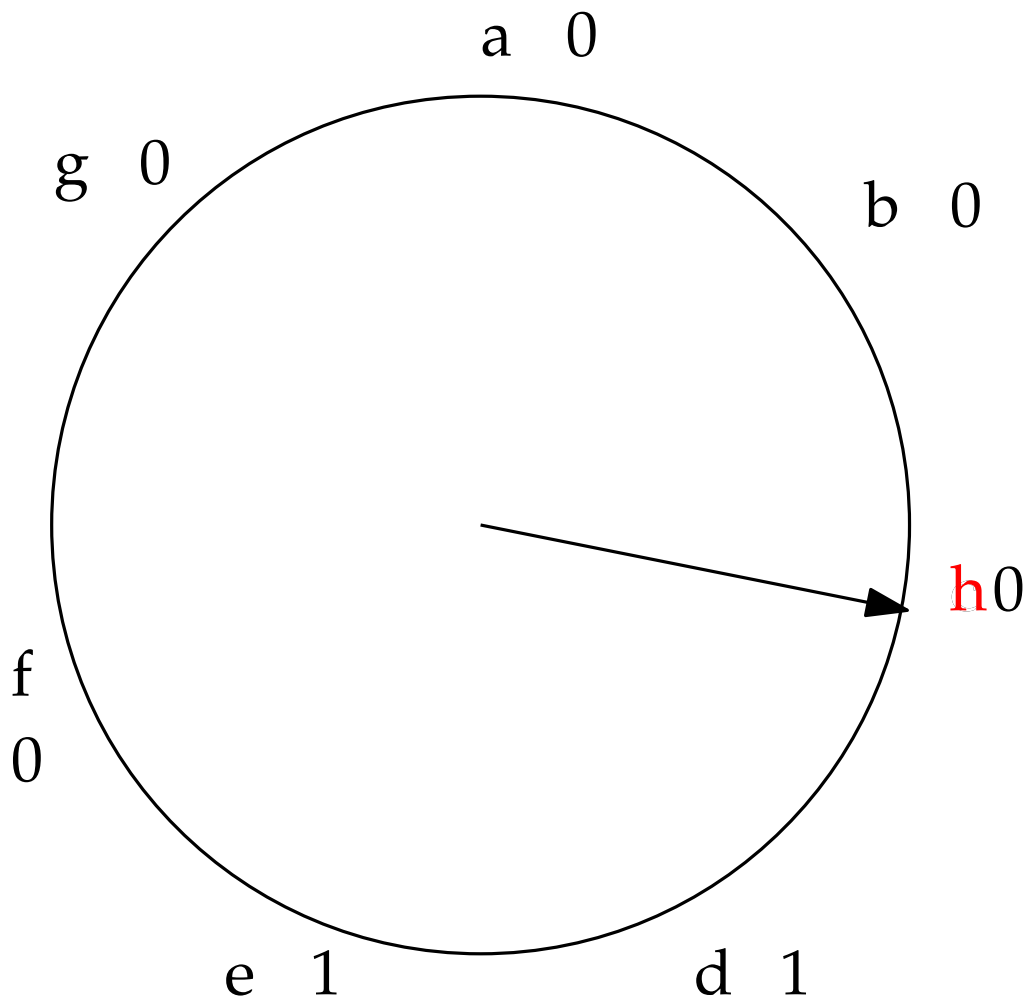
e d a e b h

clock: an approximation of LRU



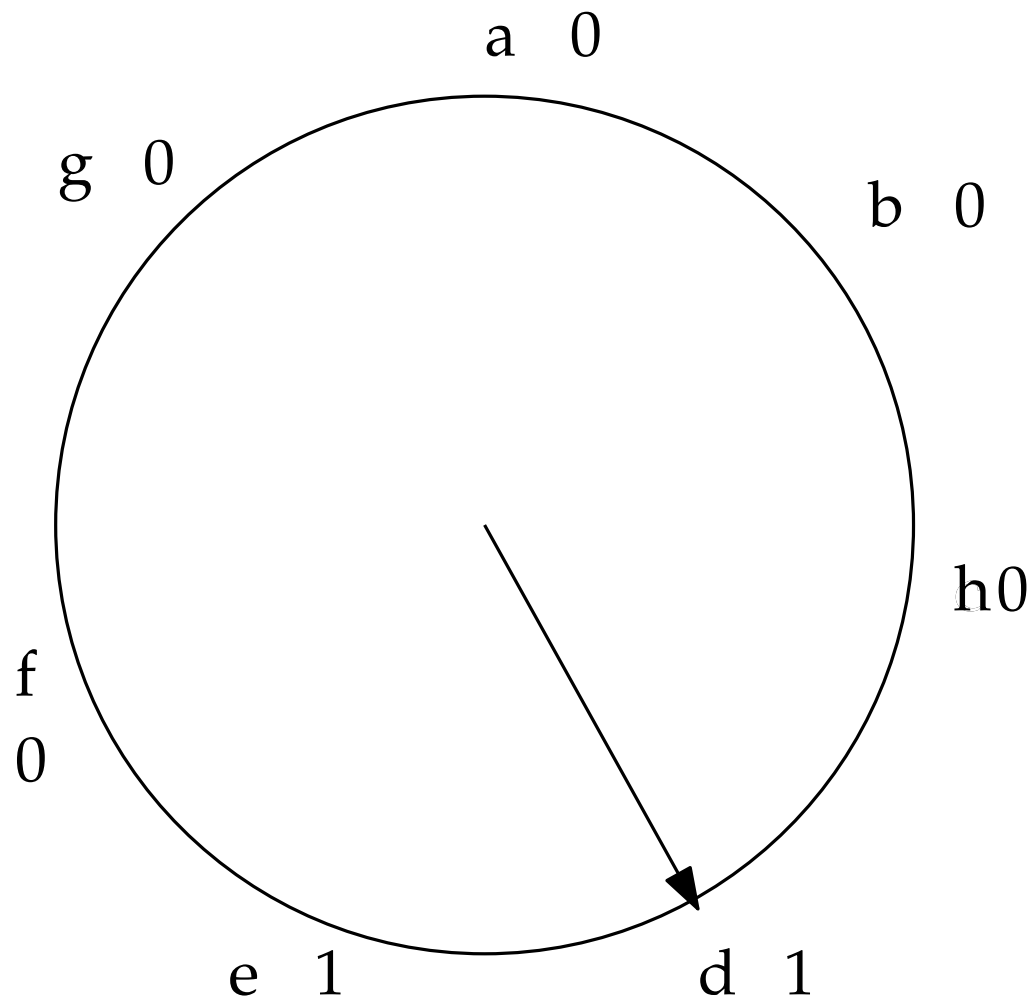
e d a e b h

clock: an approximation of LRU



e d a e b h

clock: an approximation of LRU



e d a e b h

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handle-request(object o):

    if o is in cache:
        update priority(o)


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            find and evict object in cache with smallest priority

        bring o in cache

        compute priority(o)
```

LRU

handle-request(object o):

if o is in cache:

update priority(o) O(1)

else:

while there is not enough room in cache for o:

find and evict object in cache with smallest priority O(1)

bring o in cache

compute priority(o) O(1)

LRU clock

handle-request(object o):

if o is in cache:

update priority(o) O(1) move to front bit =1

else:

while there is not enough room in cache for o:

find and evict object in cache with smallest priority O(1) remove from tail move hand set bits to 0

bring o in cache

compute priority(o) O(1) place in front bit = 0

handle-request(object o):

if o is in cache:

update priority(o)

else:

while there is not enough room in cache for o:

find and evict object in cache with smallest priority

bring o in cache

compute priority(o)

what about
in general

using a priority queue (heap)

handle-request(object o):

if o is in cache:

update priority(o) O(log n)

else:

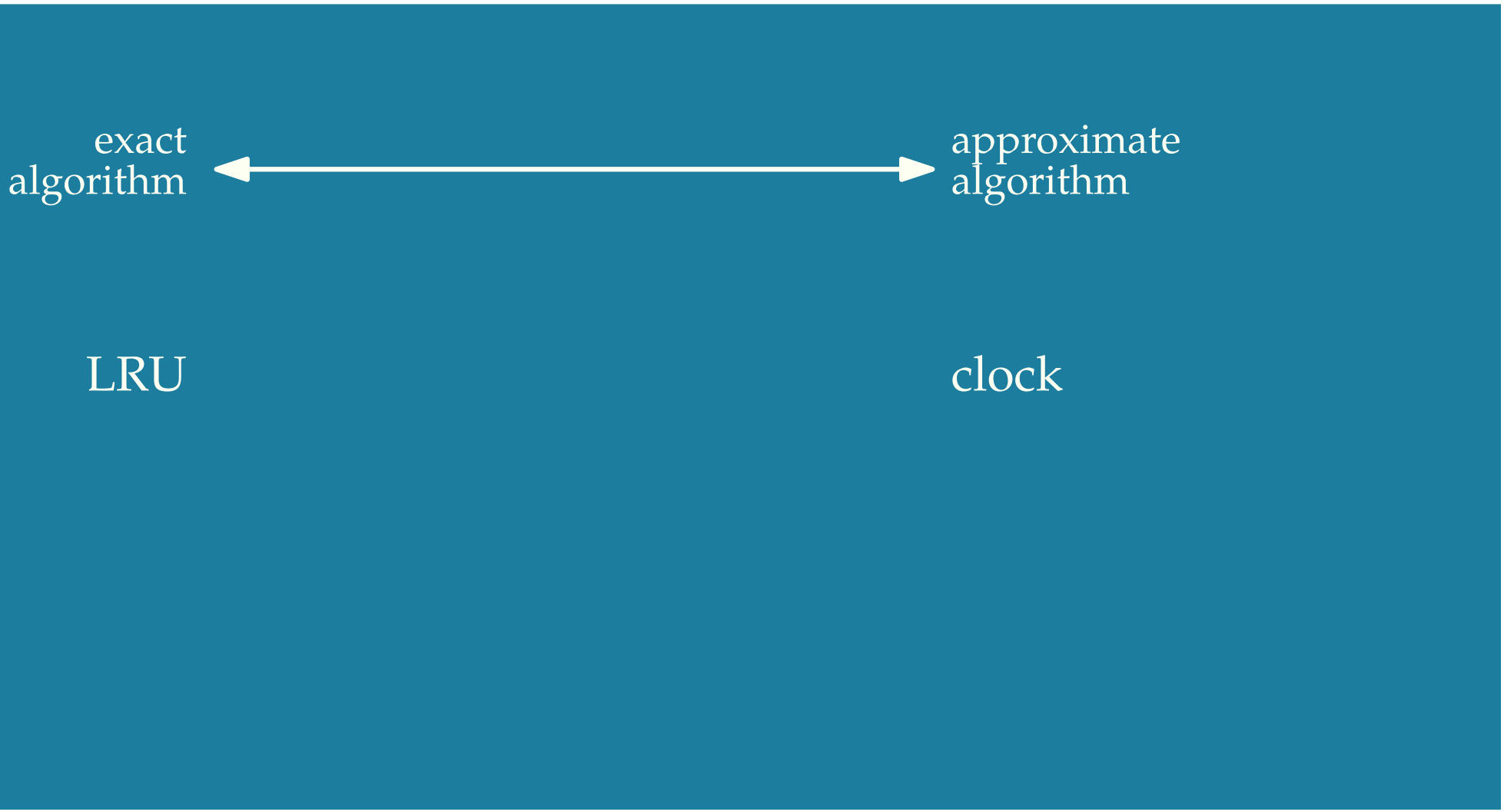
while there is not enough room in cache for o:

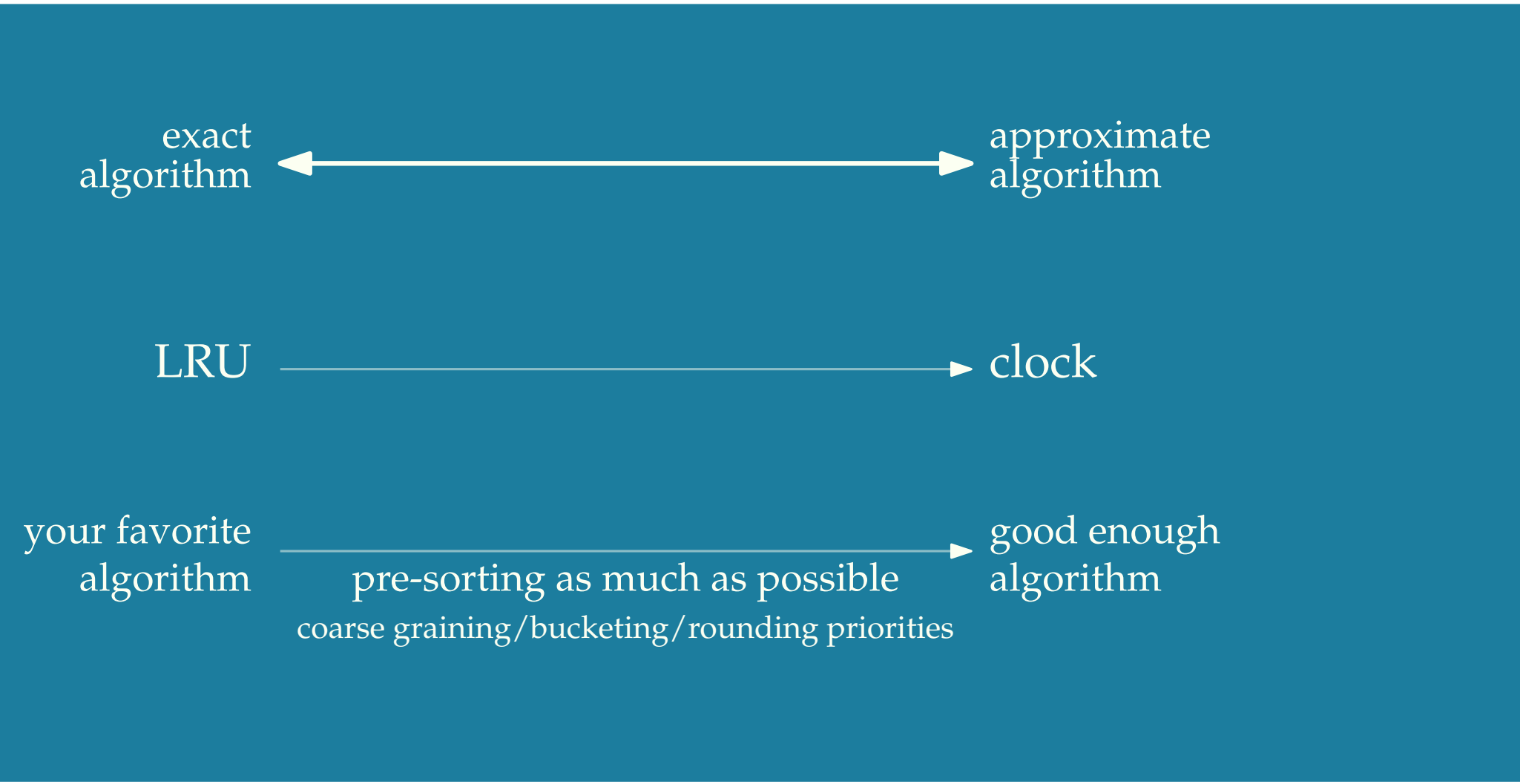
find and evict object in cache with smallest priority O(log n)

bring o in cache

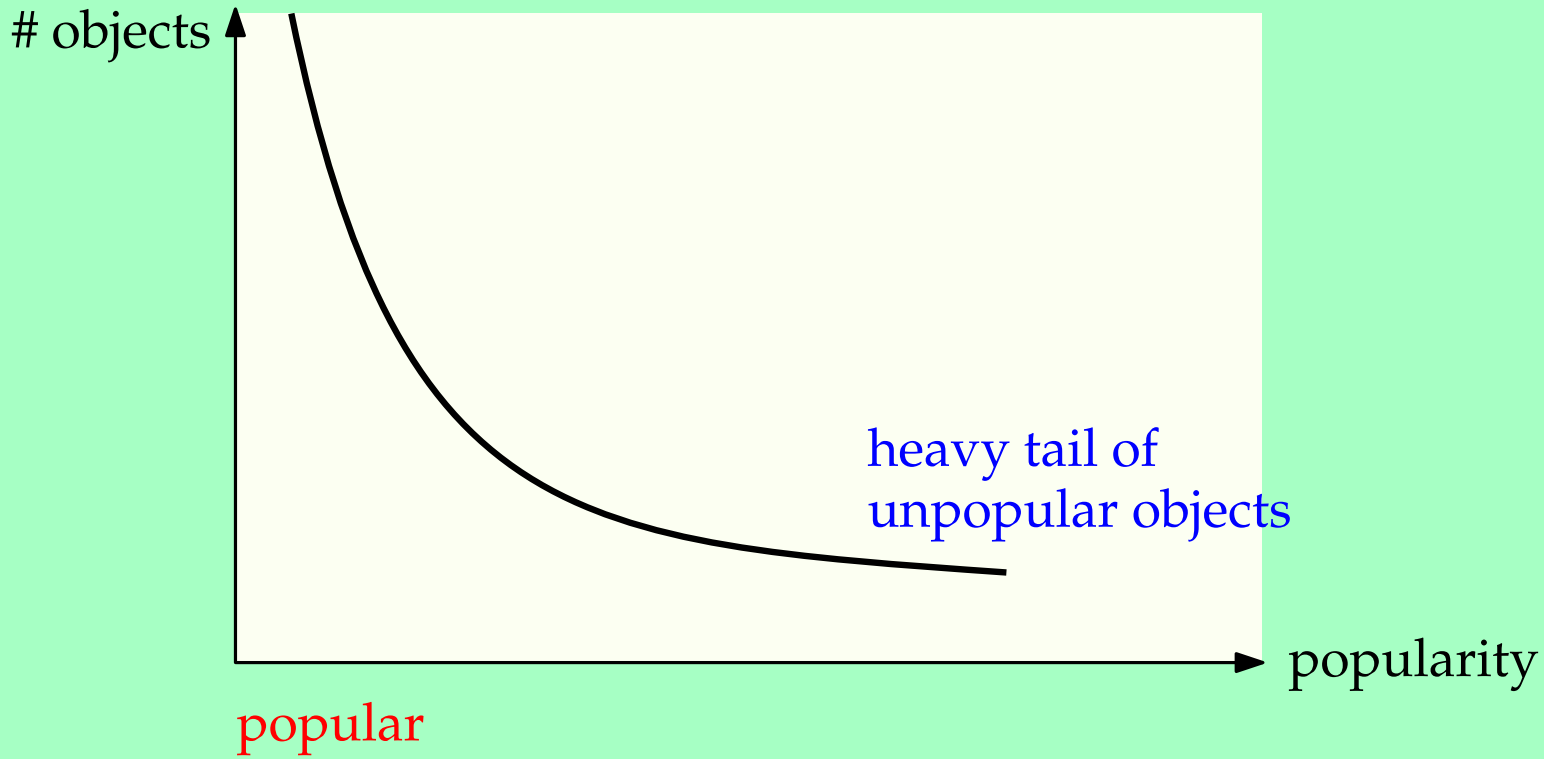
compute priority(o) O(log n)

n = nb objects in cache

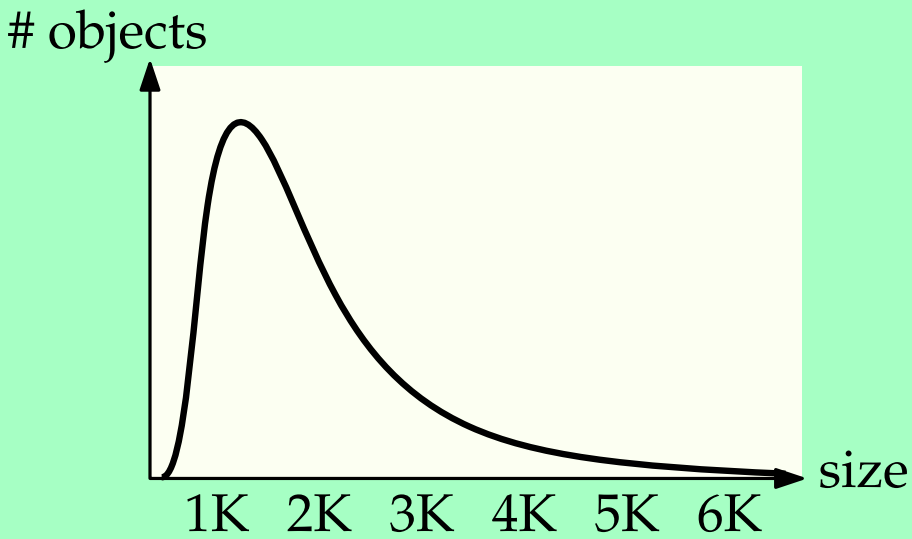

exact
algorithm
approximate
algorithm
LRU
clock

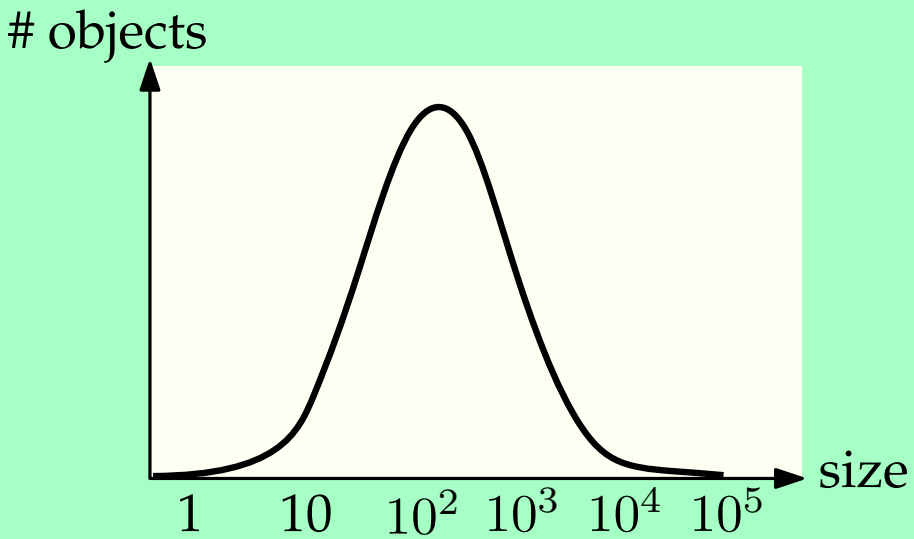

exact algorithm
approximate algorithm
LRU
clock
your favorite algorithm
pre-sorting as much as possible
coarse graining/bucketing/rounding priorities
good enough algorithm

The popularity of objects follows Zipf's law.

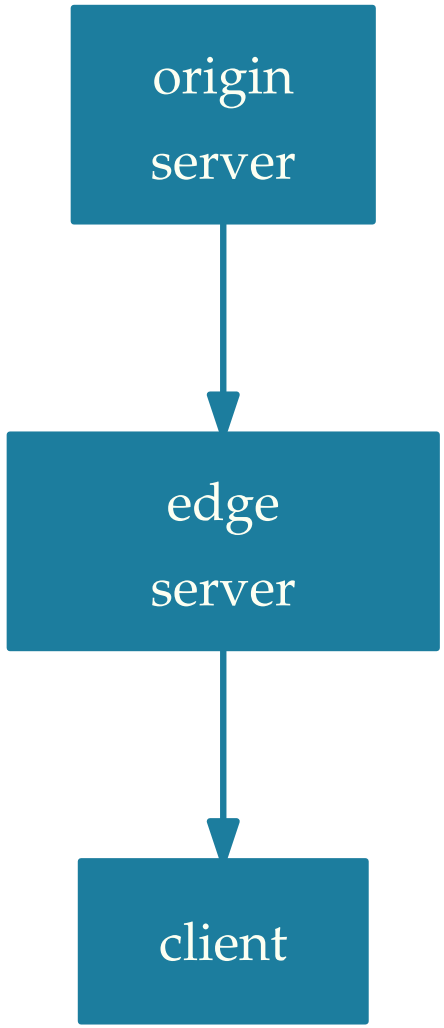


The object size follows a log-normal distribution





FIGURES OF MERIT



reduce load

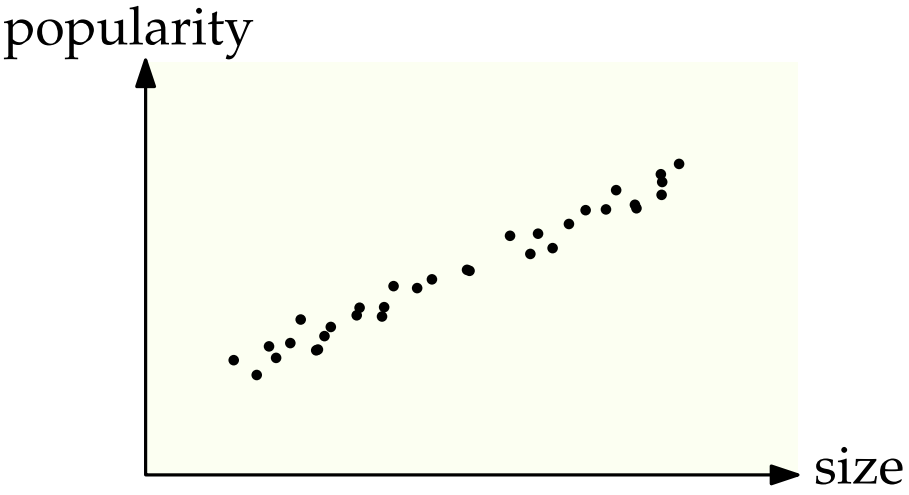
→ maximize byte hit ratio

sum of sizes of objects that were request hits

reduce perceived delay

→ maximize hit ratio

request hits



prefer to keep
small objects

large objects

better ← hit ratio → worse

same ← byte hit ratio → same

ifferent popularity-size correlations

prefer to keep
small objects

large objects

worse ← hit ratio → better

worse ← byte hit ratio → better

will yield different performances

Recap

- LFD, LRU, FIFO, LFU
- provable results:
  - LFD is optimal
  - LRU and FIFO are k-competitive, LFU is not
- practice: LRU-LFU hybrids are preferred because workloads tend to exhibit locality of reference
- practice: clock approximates LRU with low overhead
- practice: best algorithm will depend on the workload

Cache replacement considered less important

large caches: decreasing storage cost

reduction of cacheable traffic and rate of change
(changes to objects over time reduce
value of having a large cache to store them longer)

plenty of good enough algorithms

Extra

Locality of reference

An input sequence of references exhibits
locality of reference if
a page that is referenced is likely
to be referenced again in the near future.

Modeling locality of reference

Characteristic vector of an input sequence s

$$C = (c_0, \ldots, c_{p-1})$$

p: number of distinct pages referenced

c_i: number of distance-i requests

distance-i request: a request to a page such that the number of distinct pages requested since the last time this page was requested is i

Ref: Quantifying Competitiveness in Paging with Locality of Reference, Albers, Frascaria, 2015

LRU beats FIFO in the presence of locality of reference

Characteristic vector of an input sequence s

$$C = (c_0, \ldots, c_{p-1})$$

What does the characteristic vector of a sequence with high locality of reference look like?

Ref: Quantifying Competitiveness in Paging with Locality of Reference, Albers, Frascaria, 2015