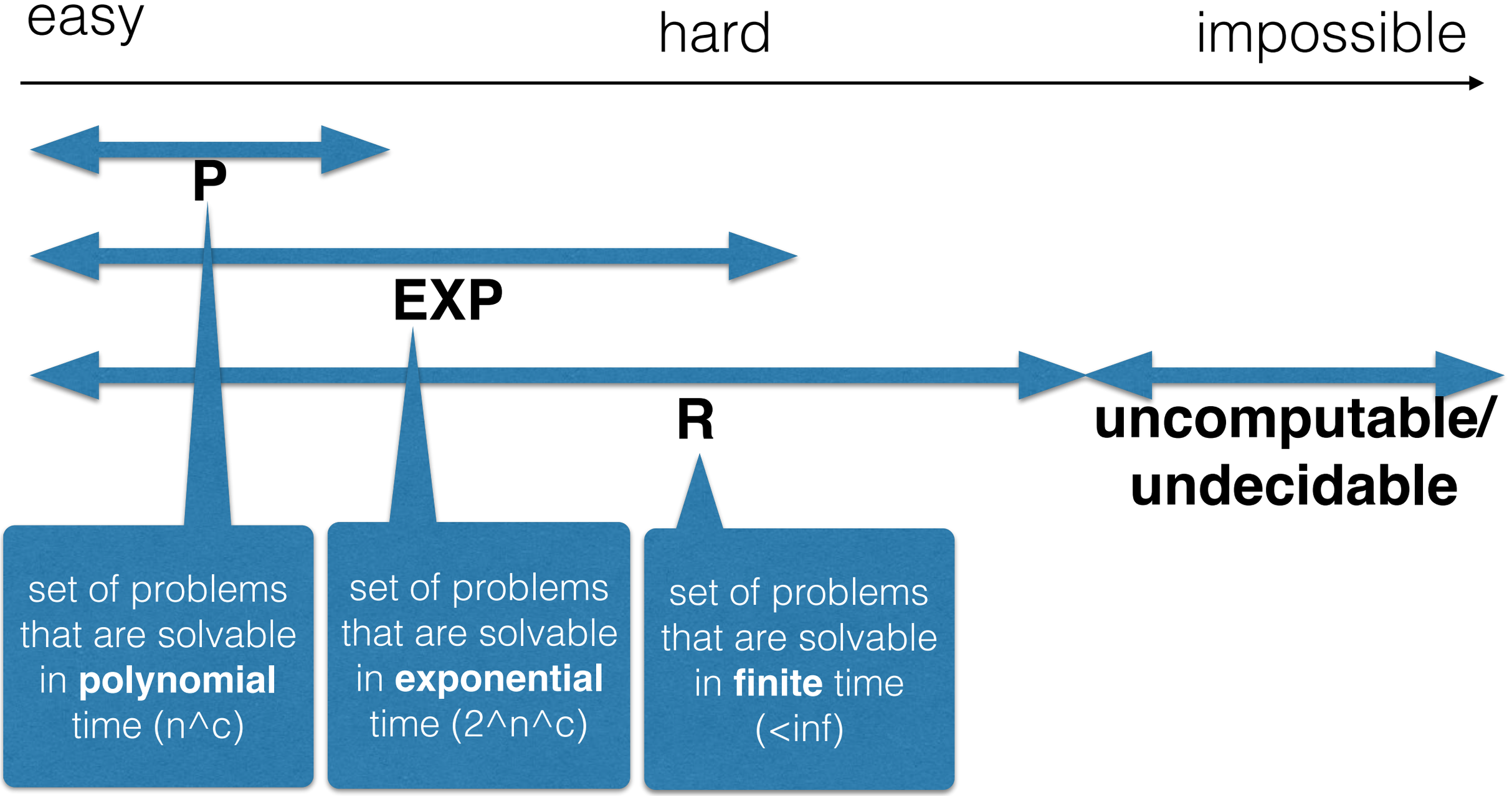
Computational complexity

CS 146 - Spring 2017

Which is harder, Tetris or Chess?

Computational difficulty



negative cycle detection: does a given graph contain a neg cycle?

easy

hard

impossible

P

EXP

R

?

uncomputable/ undecidable

n x n **chess**: who wins from a given board config?

Tetris: can player survive given board config?

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Koster, Liben-Nowell 2004]

easy

hard

impossible

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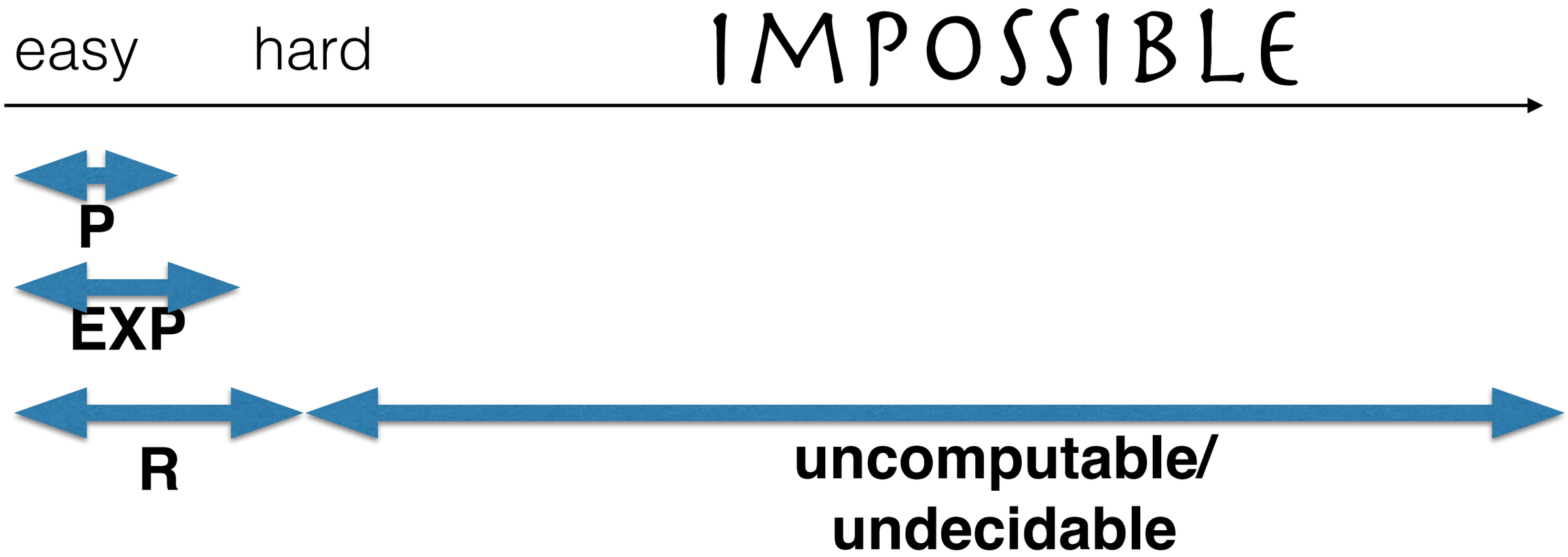
uncomputable/ undecidable

Halting problem:
will a given computer program ever stop (or will it infinite loop?)

Most decision problems are uncomputable



Proof sketch: …

Decision problem: YES/NO question	VS	**Optimization problem:** find the min/max question
negative cycle detection		maximum subarray sum
is the minimum change < x coins?		find the minimum change
is the max knapsack value > $x?		knapsack

single-source shortest paths

finding the MST

knapsack (hw 12)

easy

hard

impossible

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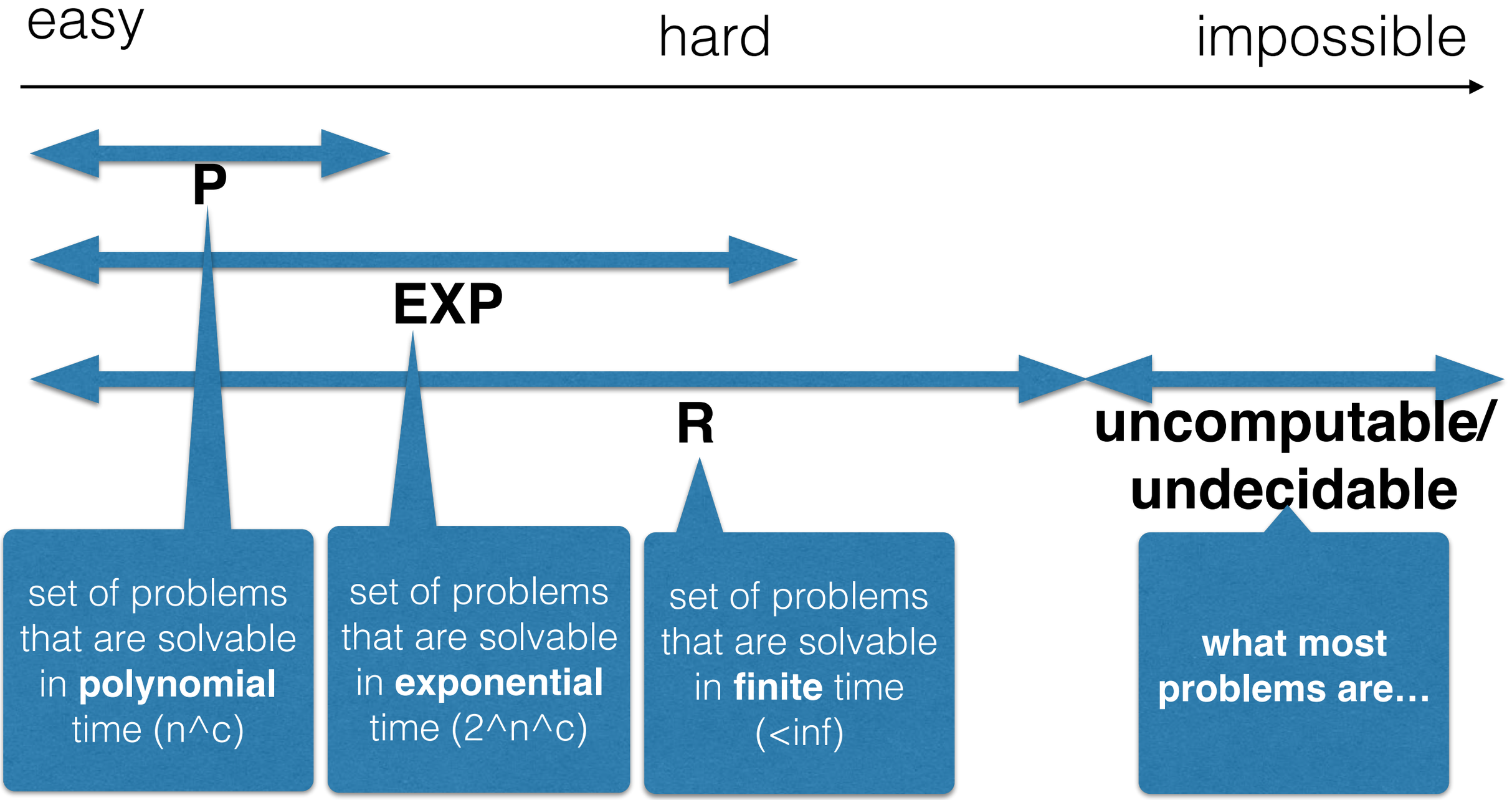
uncomputable/ undecidable

Make change

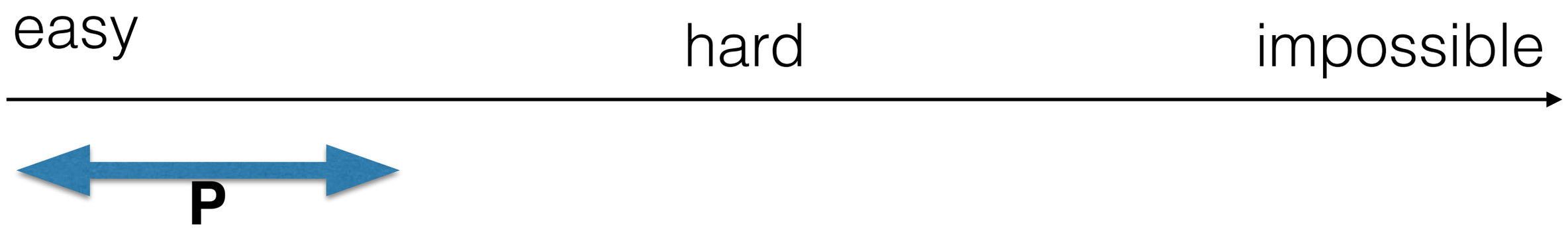
longest increasing subsequence

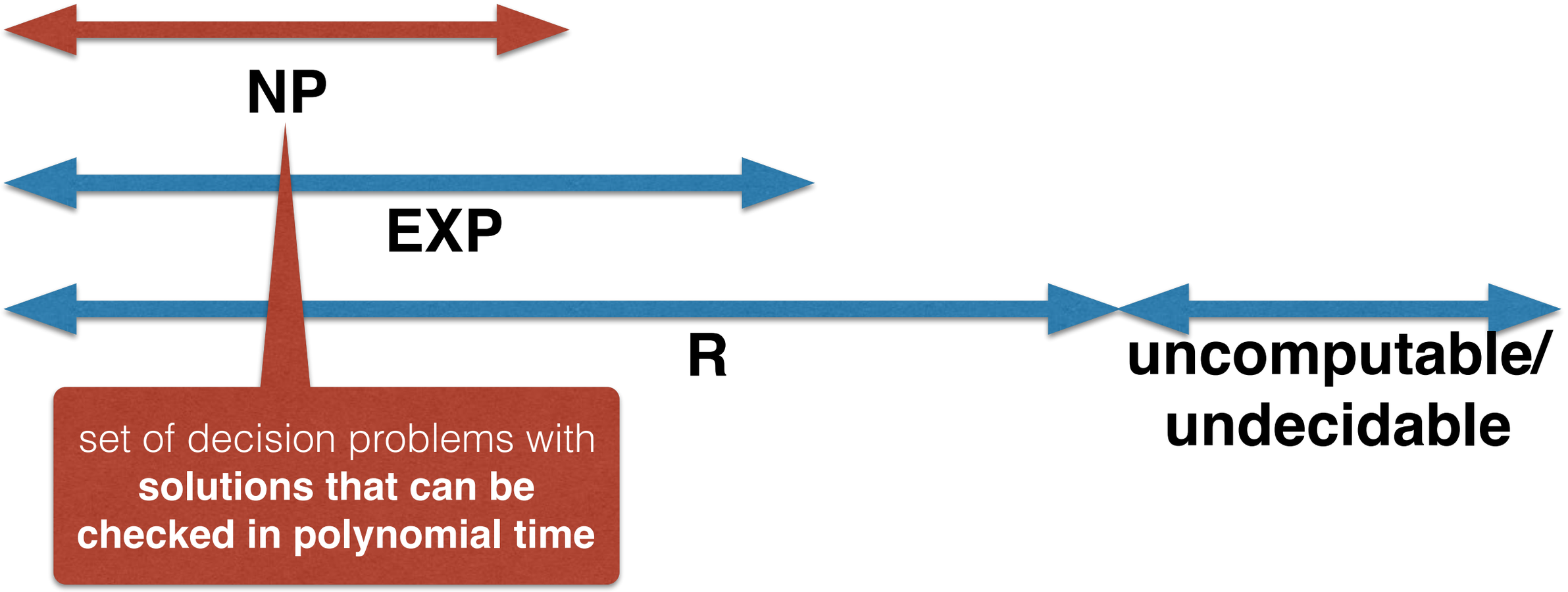
longest common subsequence

Computational difficulty

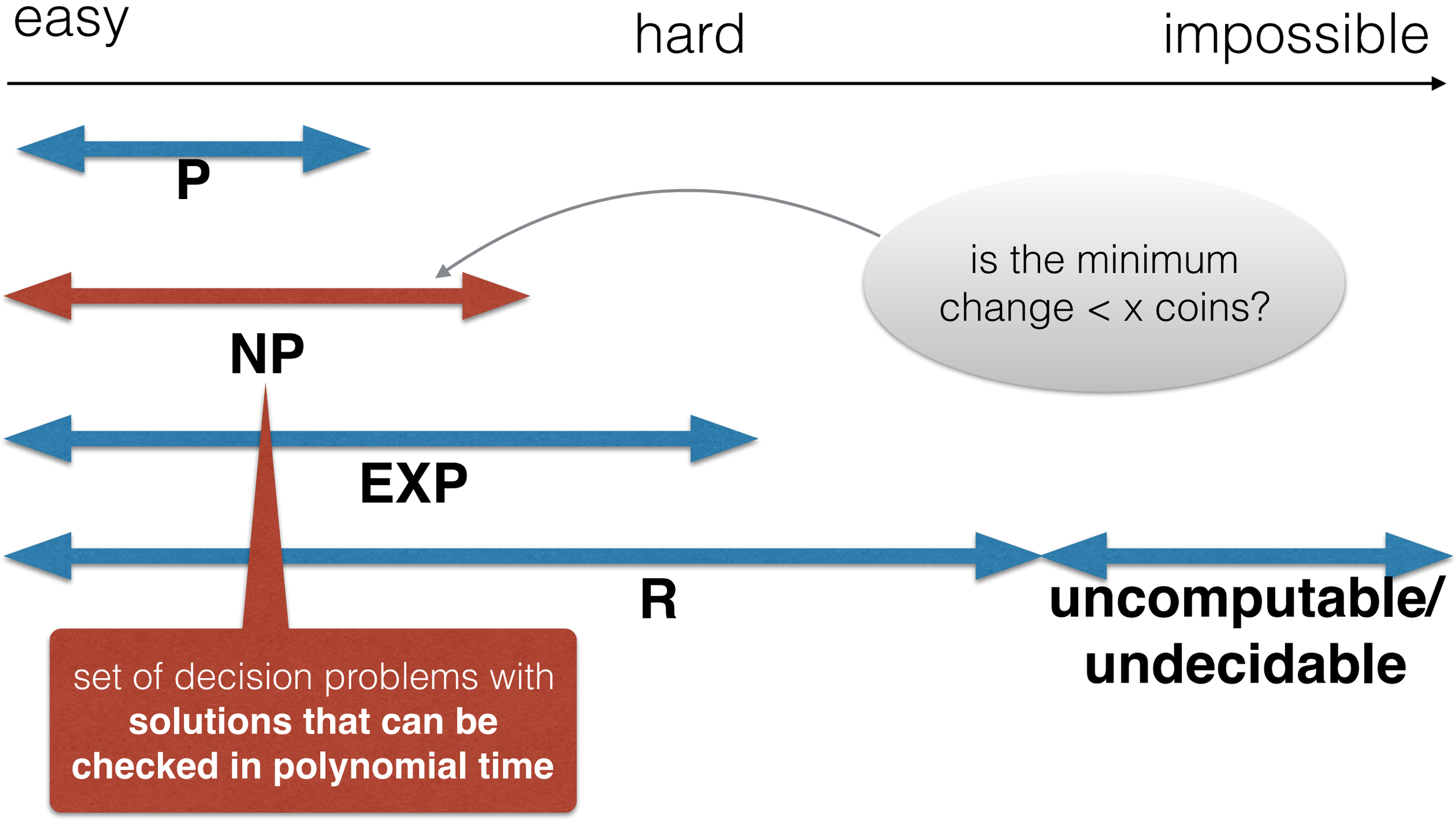


Computational difficulty

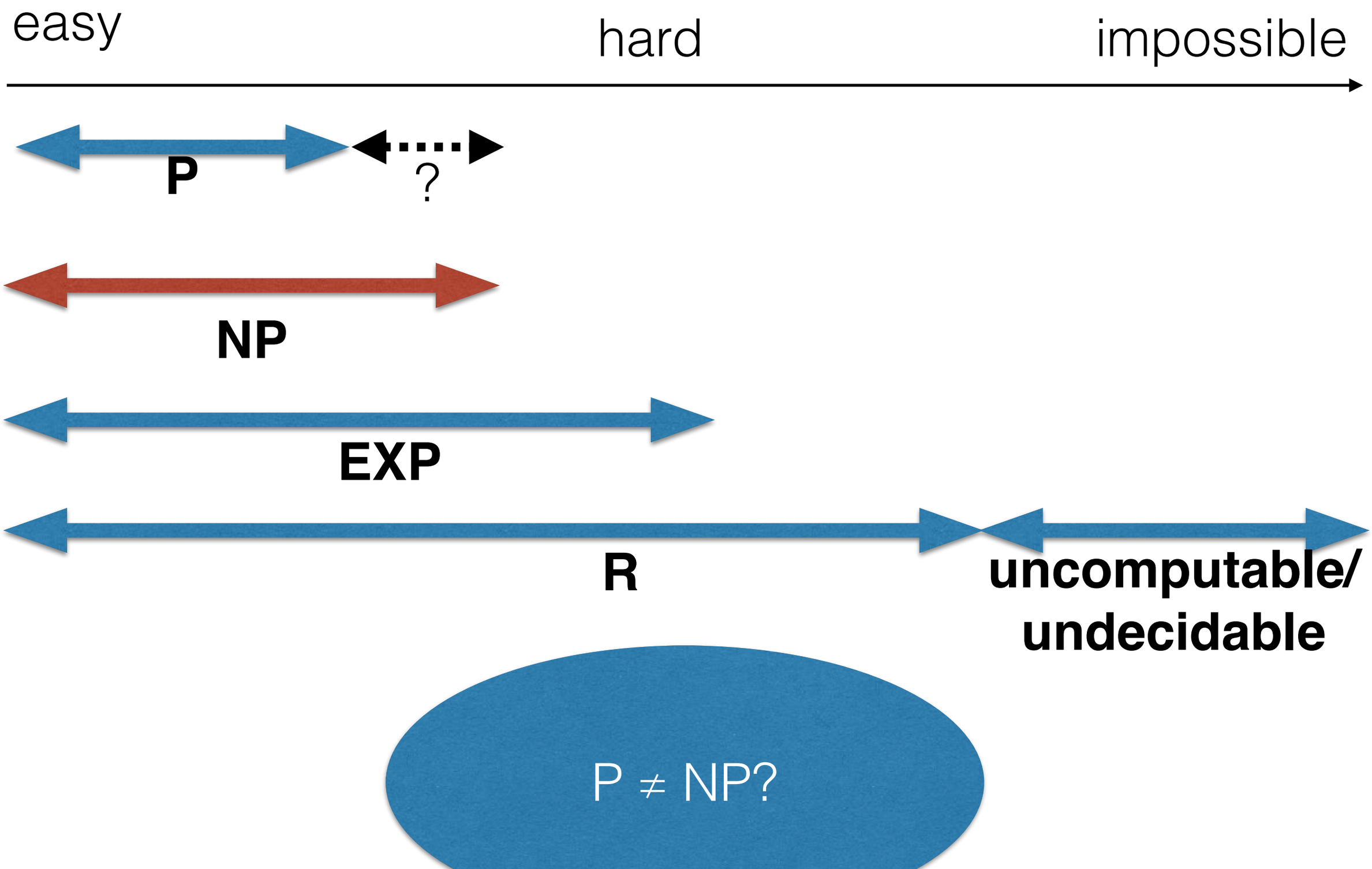




Computational difficulty



$1,000,000 conjecture

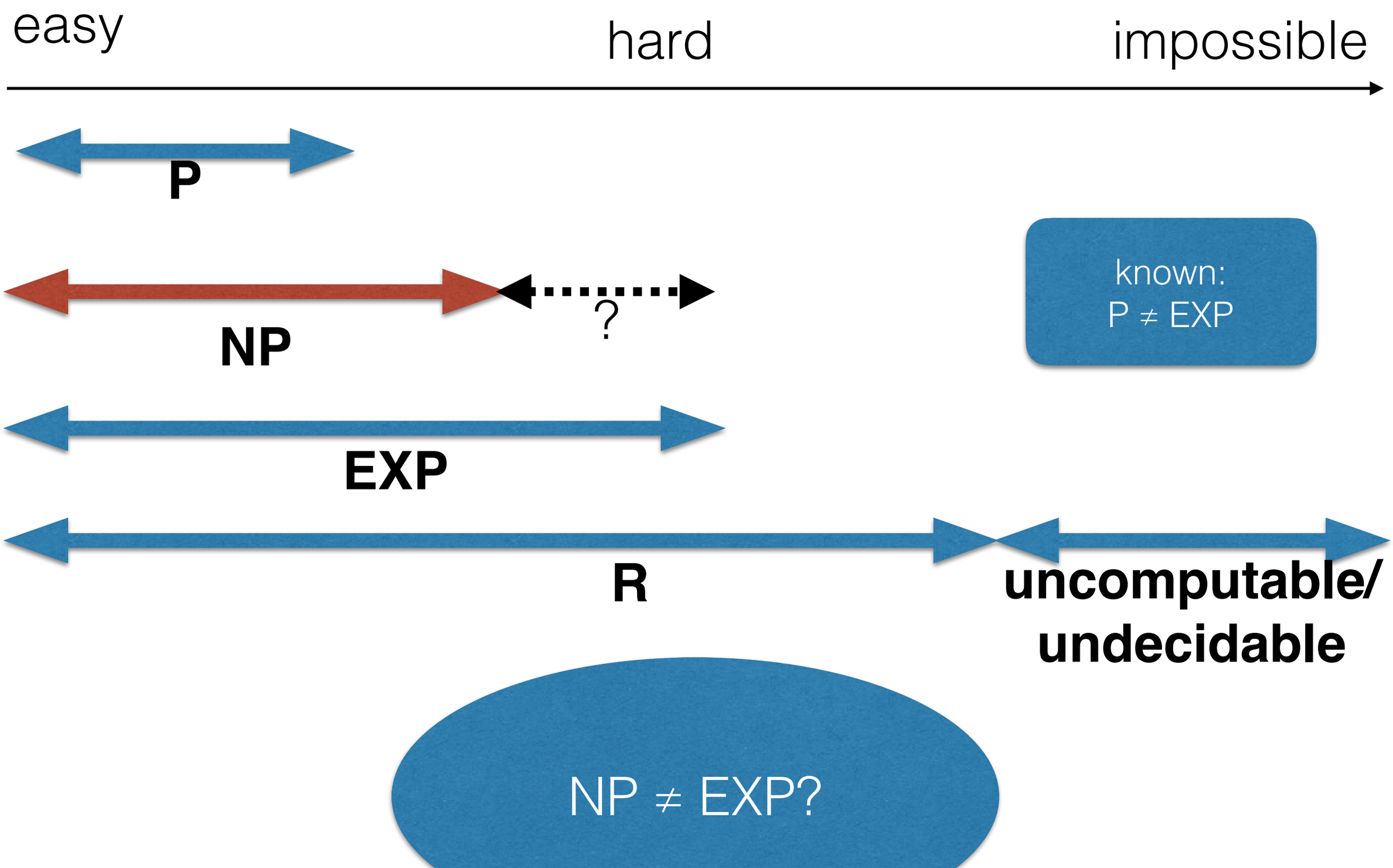


Warning!

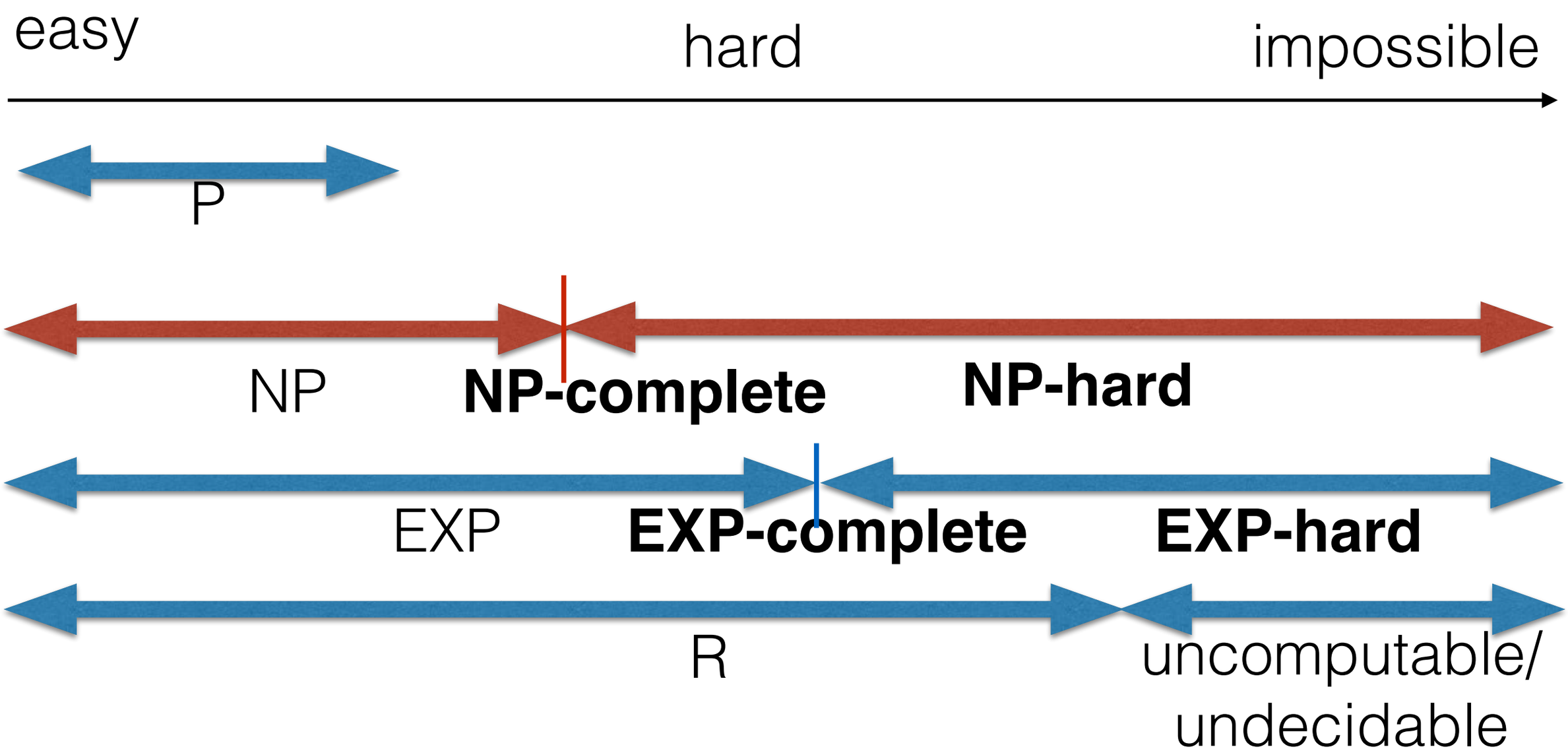
NP stands for **N**on-deterministic **P**olynomial

(not non-polynomial)

Also open, but no money



Completeness, hardness



EXP-**hard**: as hard as any problem in EXP

EXP-**complete**: in EXP and as hard as any problem in EXP

Reduction

- convert your problem into a problem you already know how to solve, instead of solving your problem from scratch

easy

hard

impossible

P

?

NP

NP-complete

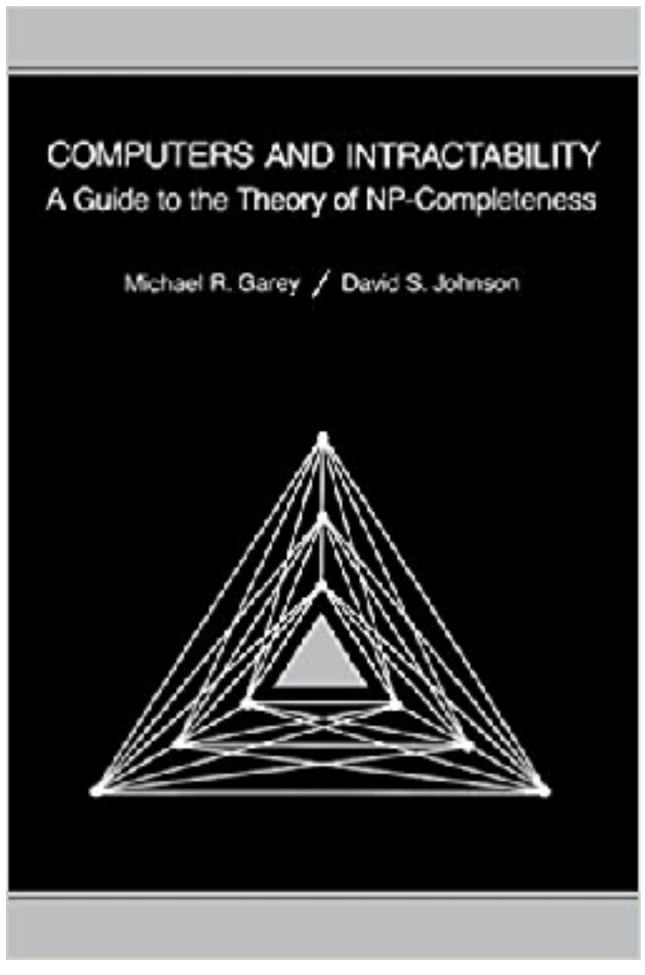
NP-hard

EXP

EXP-hard

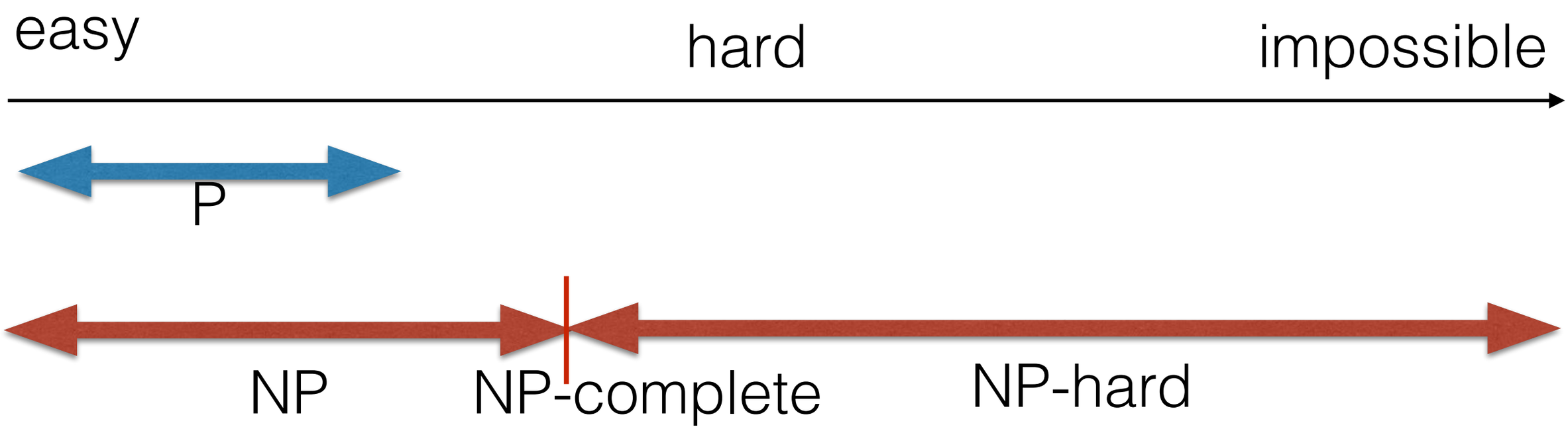
R

uncomputable/
undecidable



a lot of problems were
found to be NP-complete

More open problems



almost all interesting problems are known to be in P or are NP-complete

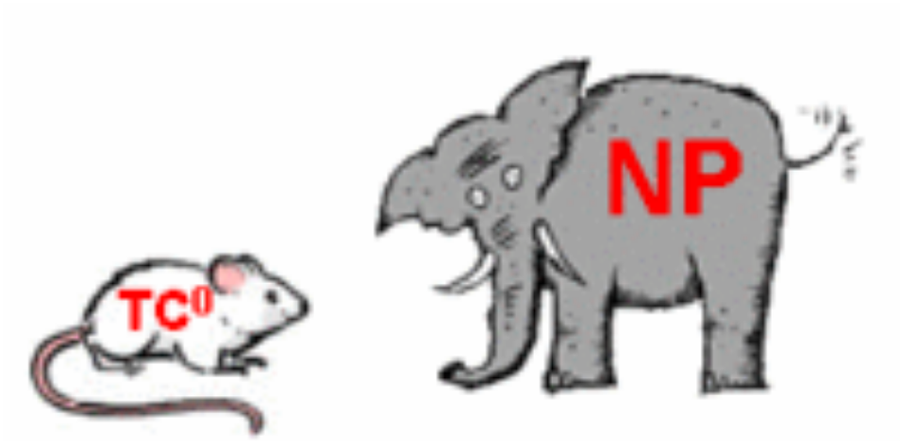
except…

is factoring in P?

is graph isomorphism in P?

But wait, there's more…

http://complexityzoo.com



→ exponential-time algorithms

what to do then? → approximation algorithms

→ fixed-parameter tractable algorithms

→ heuristic search

Acknowledgements

lecture based on Erik Demaine's

http://courses.csail.mit.edu/6.006/fall11/notes.shtml