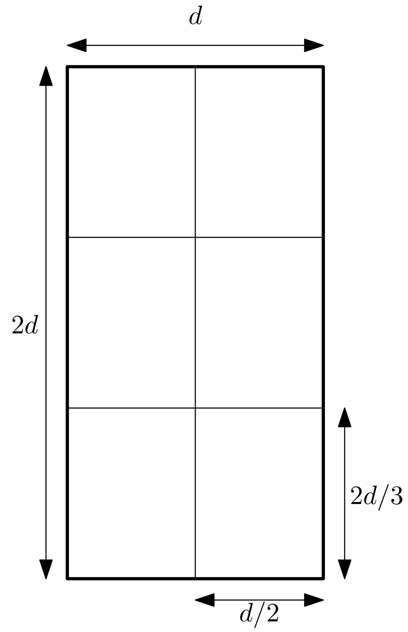
R-22.6 Prove that a rectangle of width d and height $2d$ can contain at most 6 points such that any two points are at distance at least d.

Solution. If we divide the rectangle evenly into six smaller rectangles as follows:



then each of the small rectangles has diagonals of length

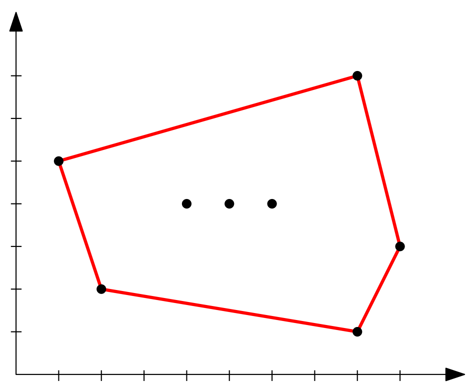
$$\sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{2d}{3}\right)^2} = \frac{5d}{6} < d.$$

Therefore there cannot be more than one point in a small rectangle, because two points in the same small rectangle would be within distance $5d/6$ of each other. So there are at most 6 points in the original rectangle. $\bullet$

R-22.8 Draw the convex hull of the following set of points:

$$\{(2,2),(4,4),(6,4),(8,1),(8,7),(9,3),(1,5),(5,4)\}.$$

Solution.



$\bullet$

C-22.1 Using the orientation test, give a pseudocode description of a method, inTriangle(p, q, r, s), which tests whether a point p is inside the interior of a triangle (q, r, s) assuming q, r and s are listed in counterclockwise order.

Solution. Check that the following three conditions hold:

(a) (p, q, r) and (s, q, r) have the same orientation,
(b) (p, r, s) and (q, r, s) have the same orientation,
(c) (p, q, s) and (r, q, s) have the same orientation.

If all three conditions hold, then return true. Otherwise, return false.

Note: this method does not require the assumption that q, r and s are listed in counterclockwise order. •

C-22.5 Describe an $O(n)$-time algorithm that tests whether a given n-vertex polygon is convex. You should not assume that P is simple.

Solution. First, check that, when traversing the polygon, either every turn is a left turn, or that every turn is a right turn.

Second, check that the polygon is not self-intersecting. There are two possible approaches:

- (a) locate the point in the polygon that has the smallest y-coordinate.
  (b) starting at this point and going all the way around the polygon, check that the sequence of y-coordinates of the points form a single monotone increasing sequence followed by a single monotone decreasing sequence.
- OR compute the sum of the internal angles of the polygon by going around and keeping a running sum of all internal angles. The polygon is non-self intersecting if and only if the sum is less than $180°/(n-2)$ (assuming the first check is passed).

If the polygon passes both checks, return "convex". Otherwise return "not convex".

Each of the steps involves running through the entire list of vertices which make up the polygon. So this algorithm has $O(n)$ running time. •