Foundations of Graph Watermarking

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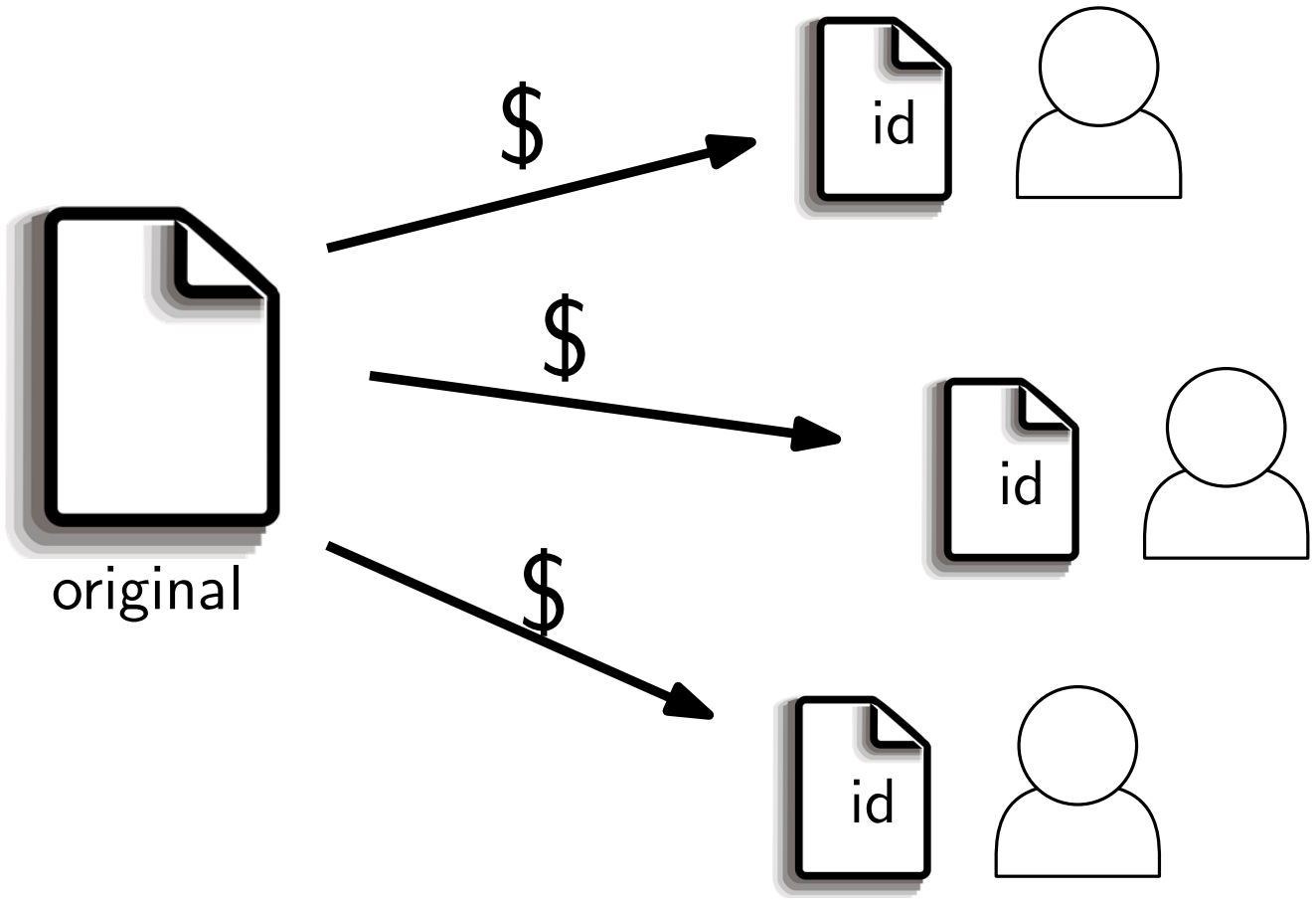
Jenny Lam, Michael Mitzenmacher

In submission at Financial Crypto 2016

problem

The digital watermarking problem



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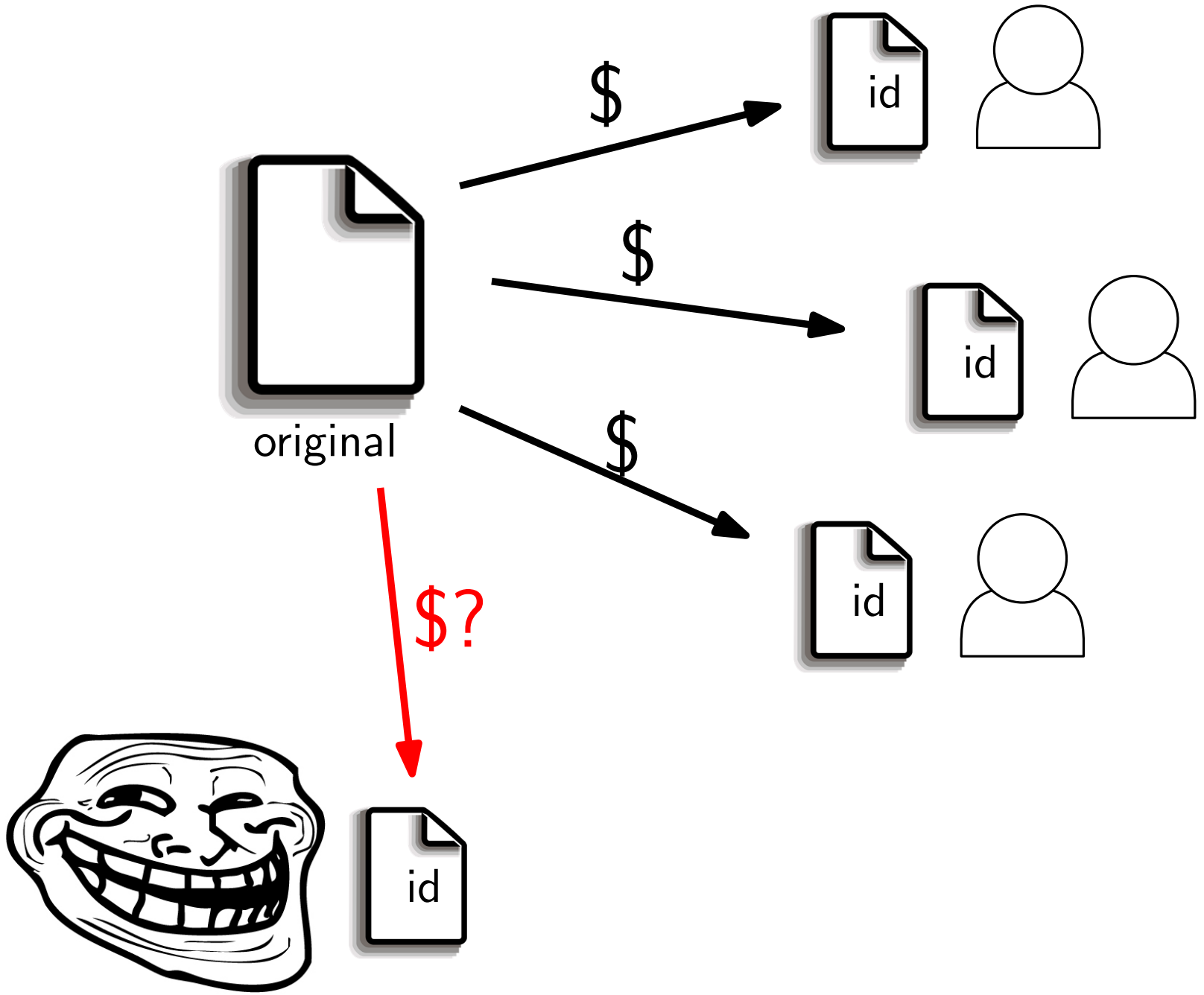

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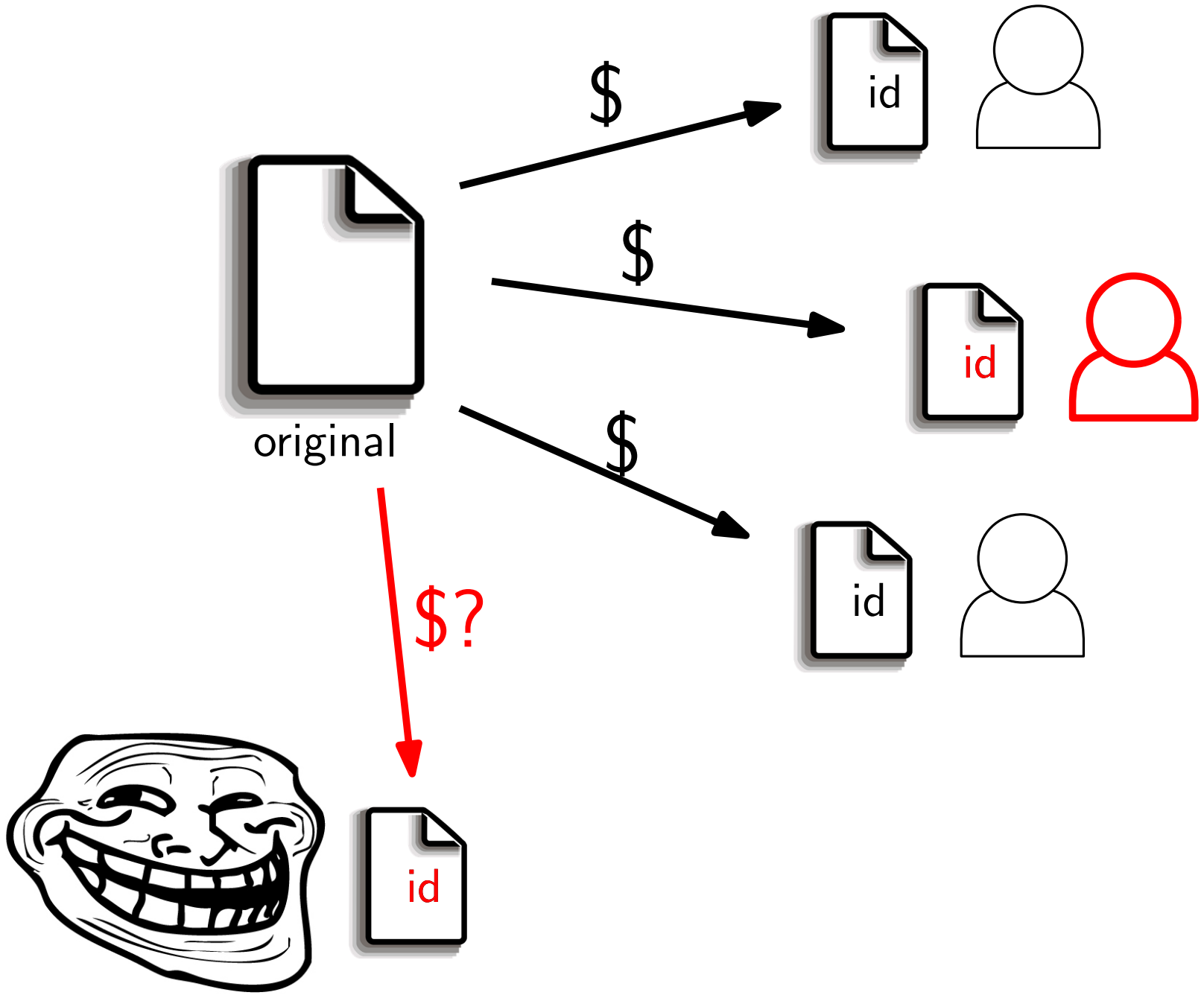


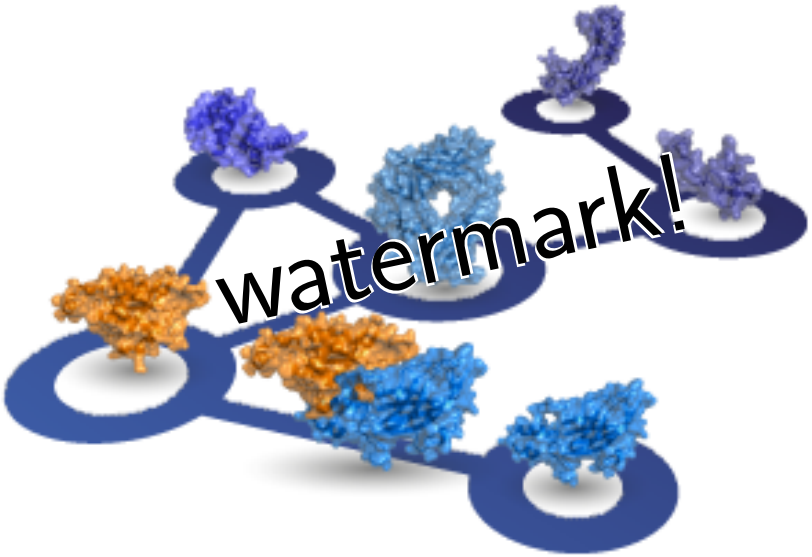

watermark!


watermark!


watermark!


watermark!


watermark!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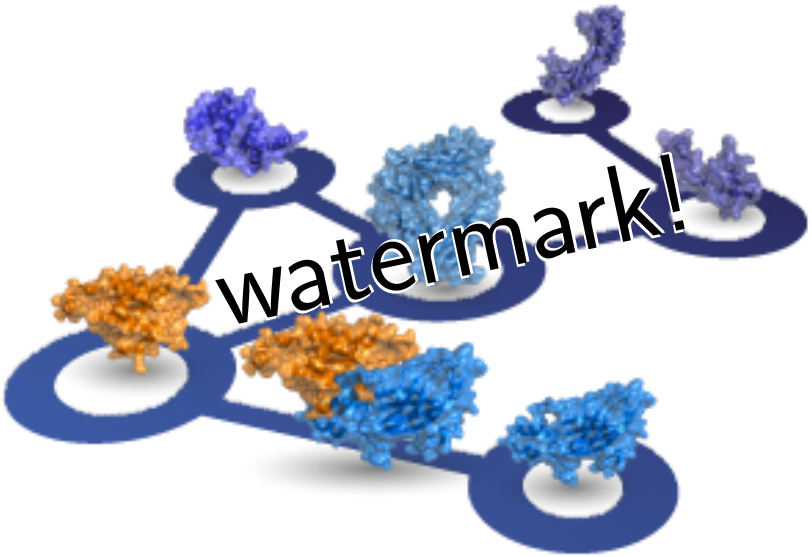
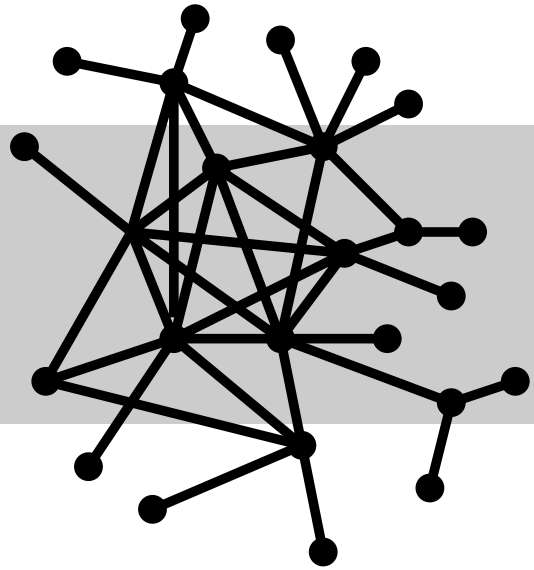
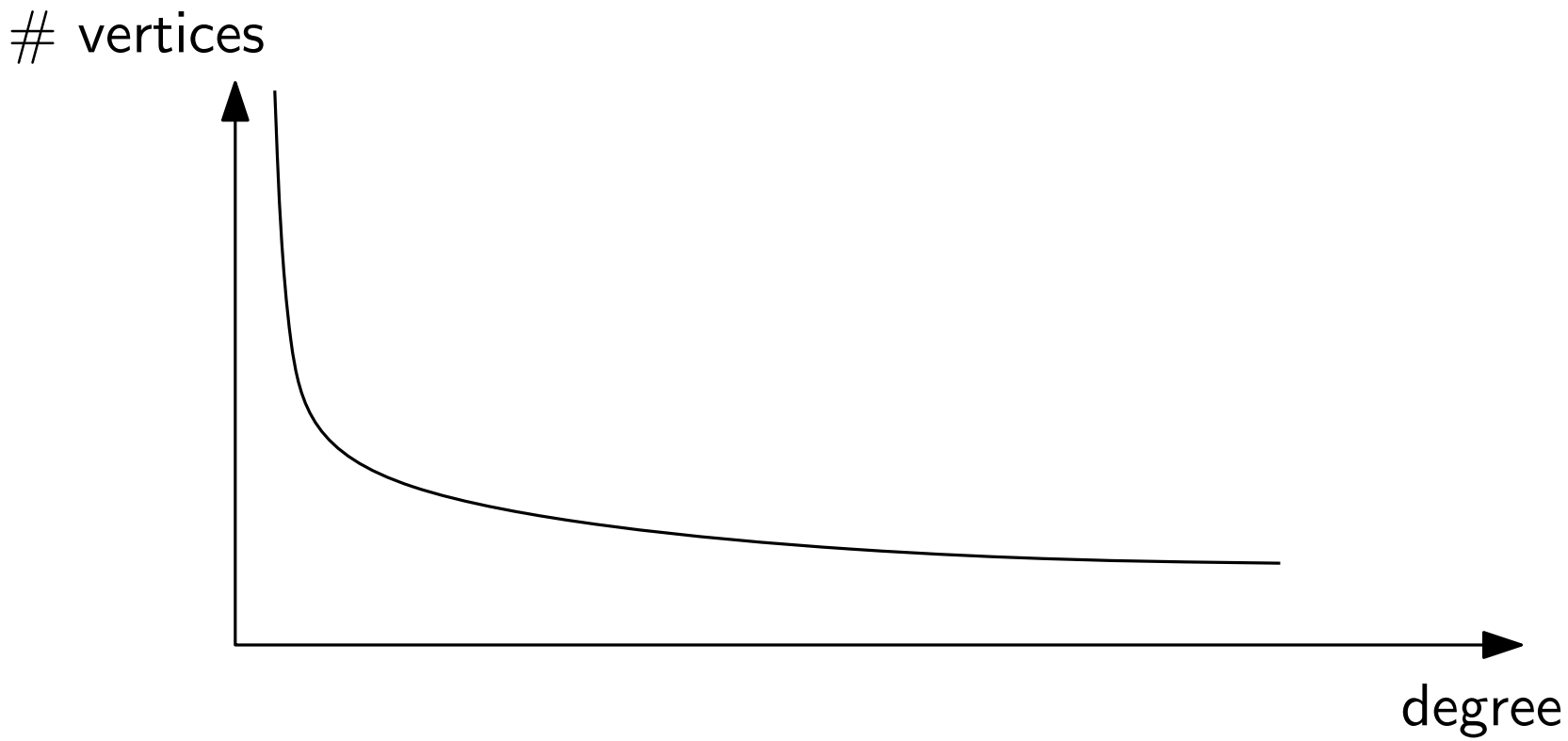

watermark!


watermark!

Scale-free networks

fraction of vertices with degree k $\propto$ $\frac{1}{k^\gamma}$ $2 < \gamma < 3$





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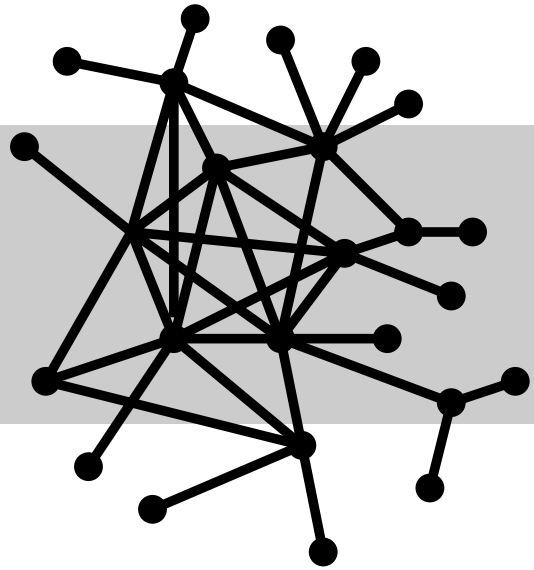

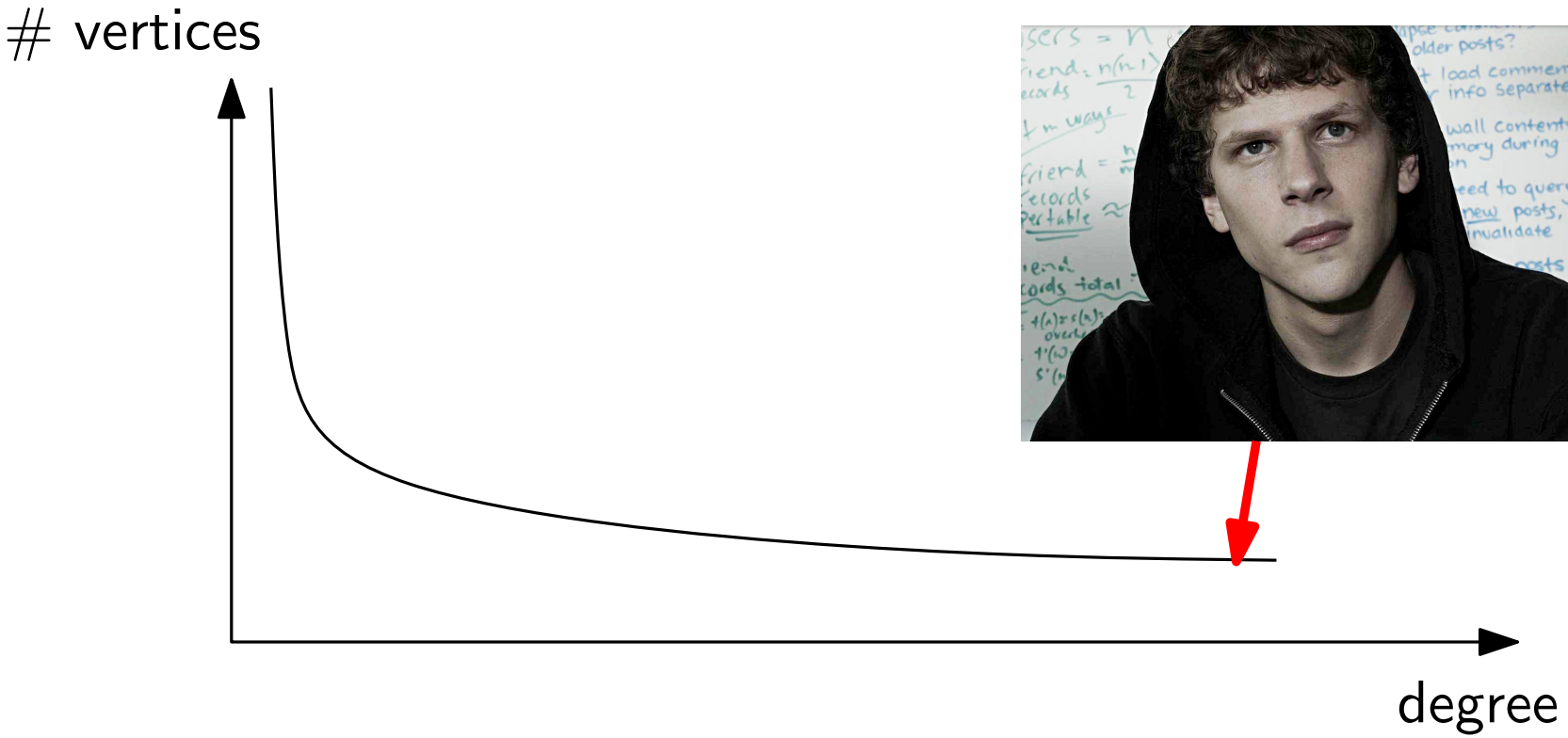

Scale-free networks

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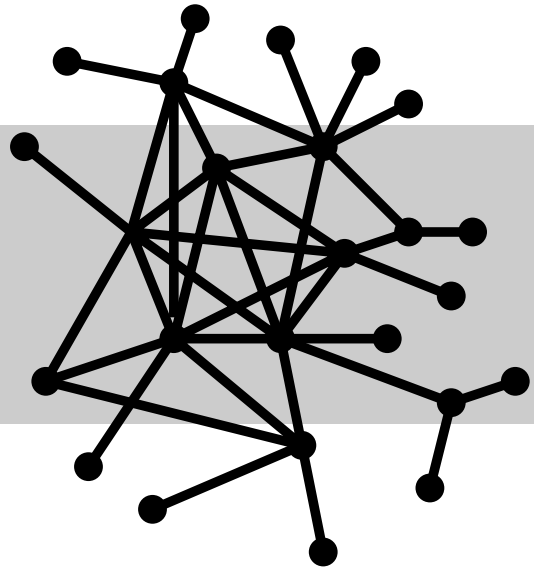
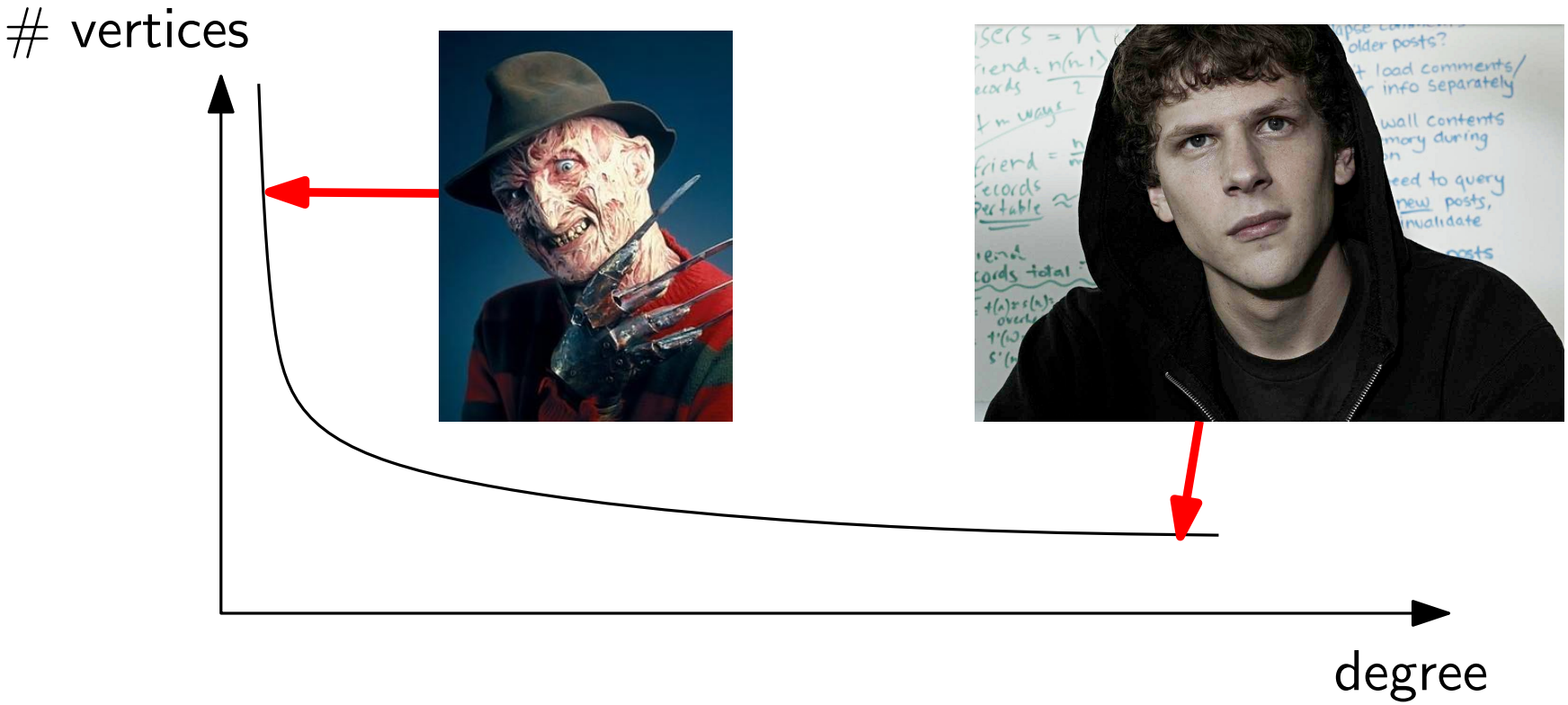
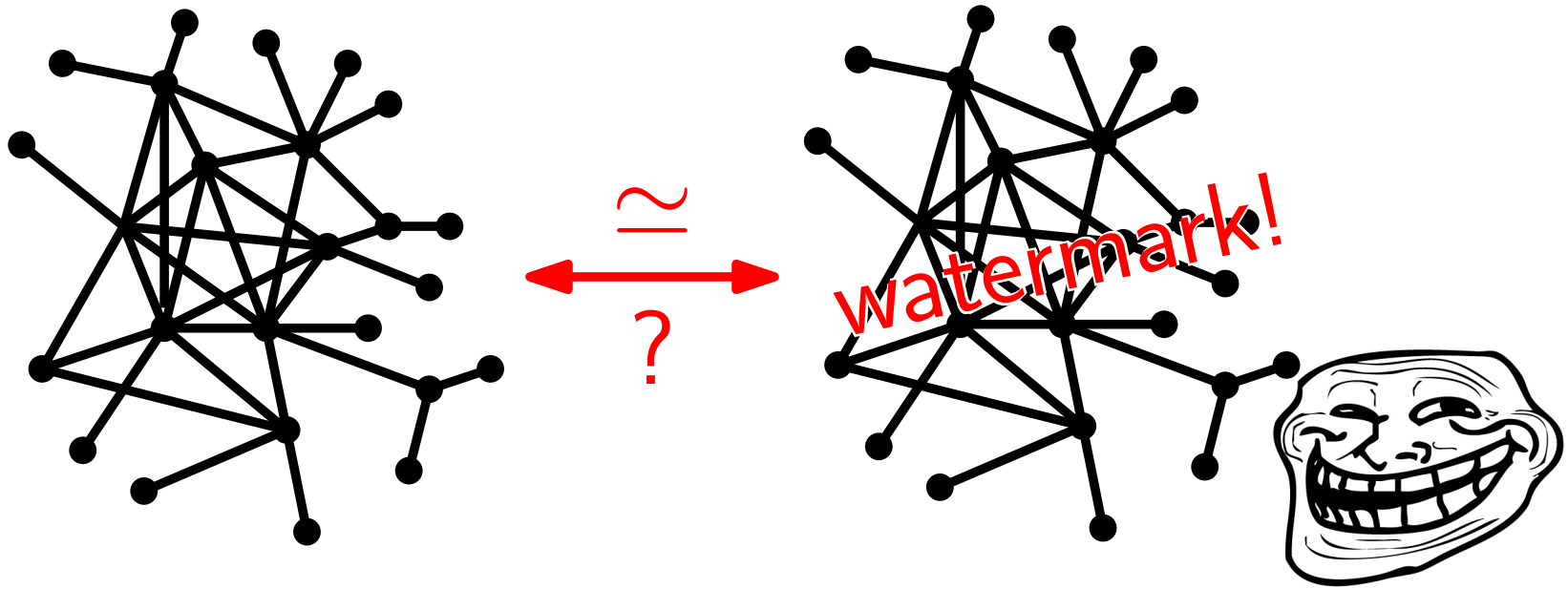





watermark!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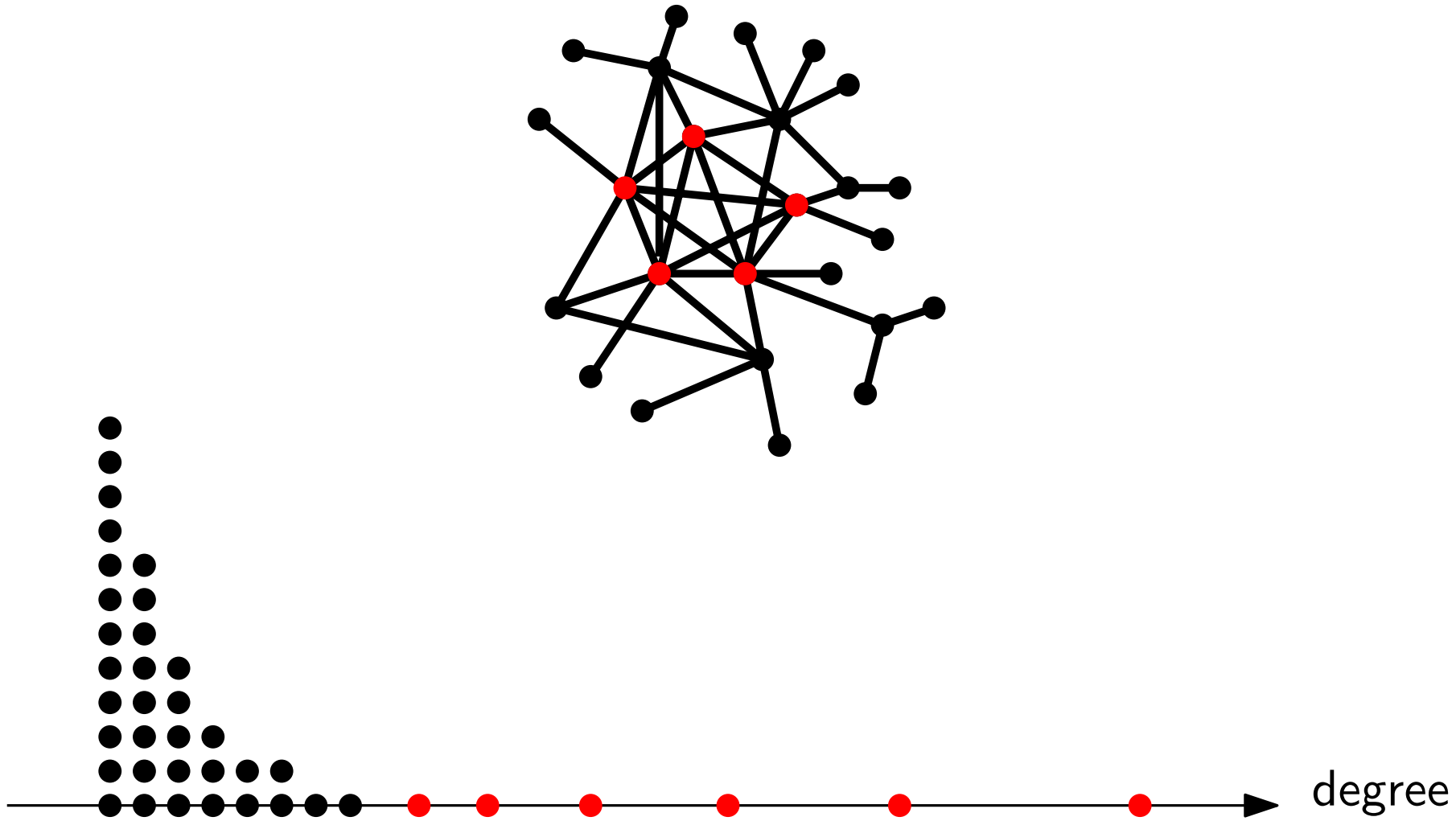
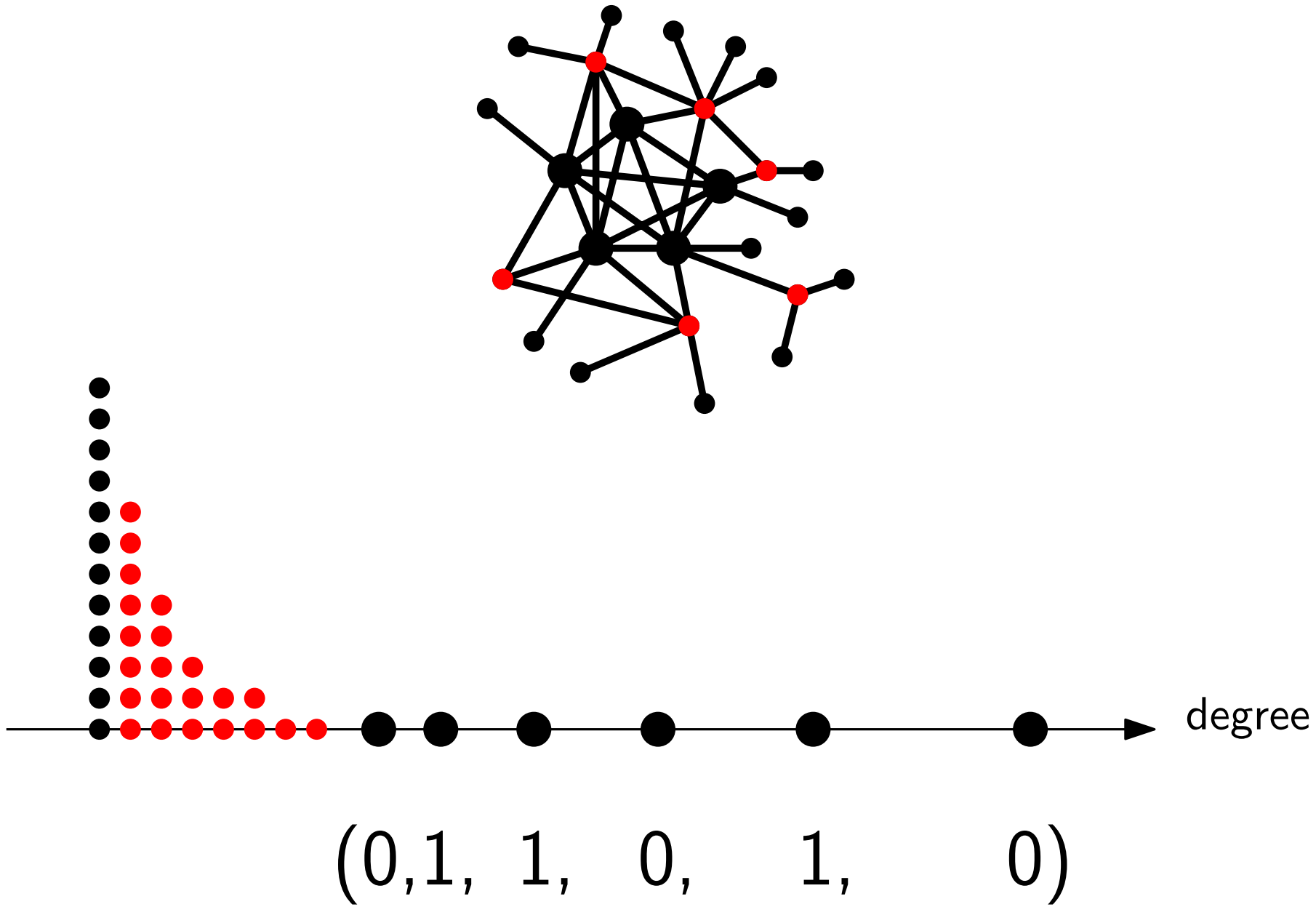
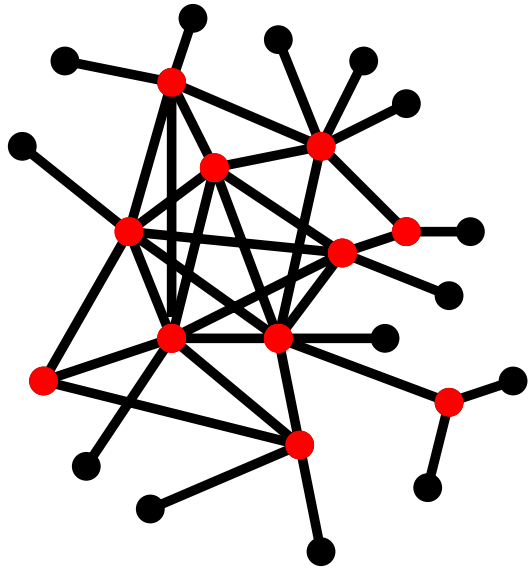
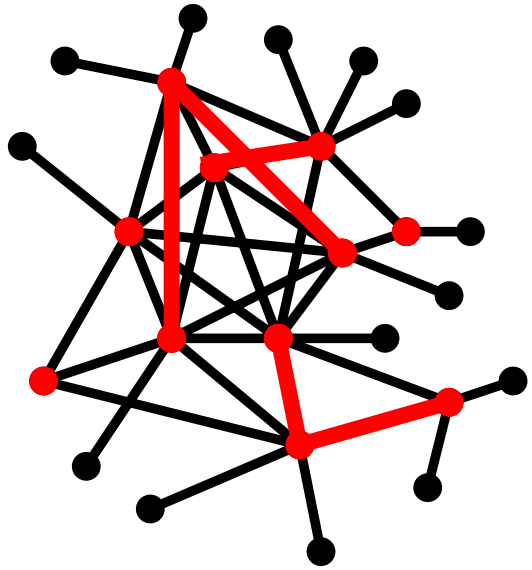
watermark!

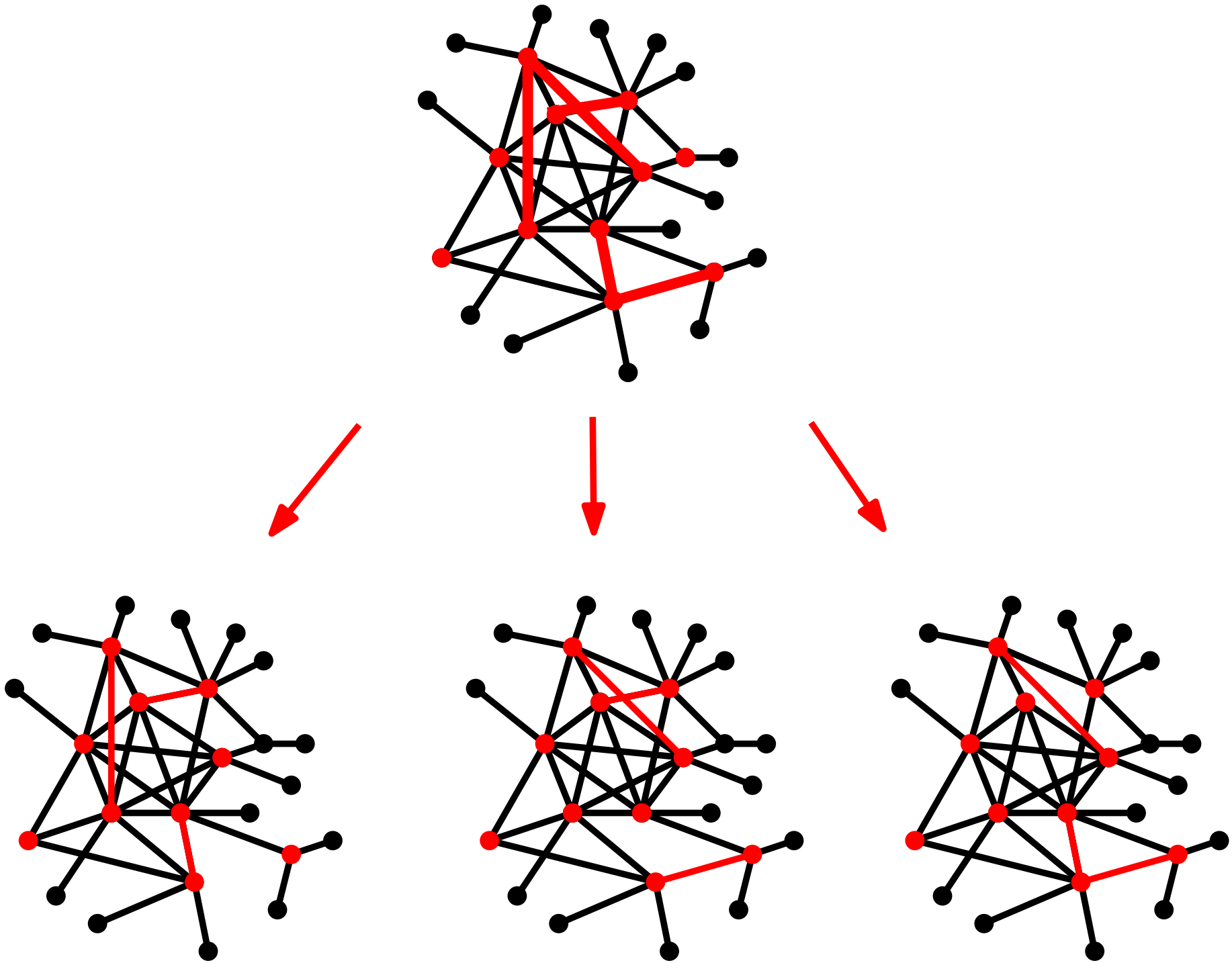
solution


degree


degree
(0,1, 1, 0, 1, 0)

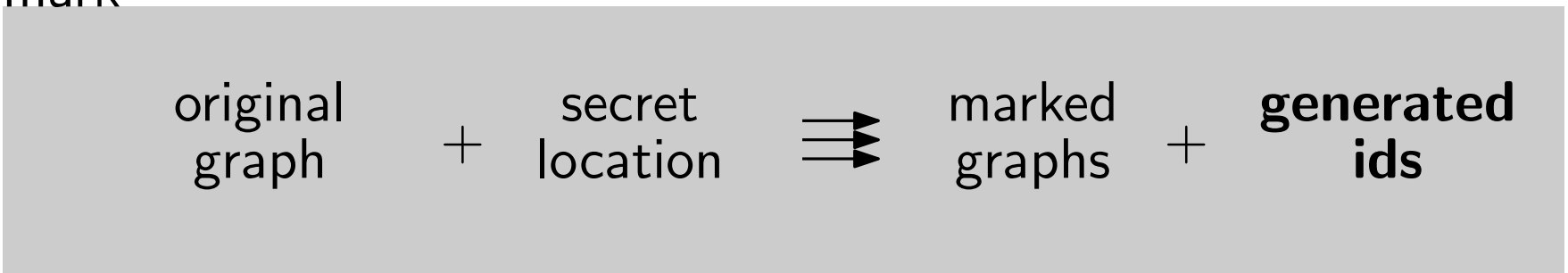




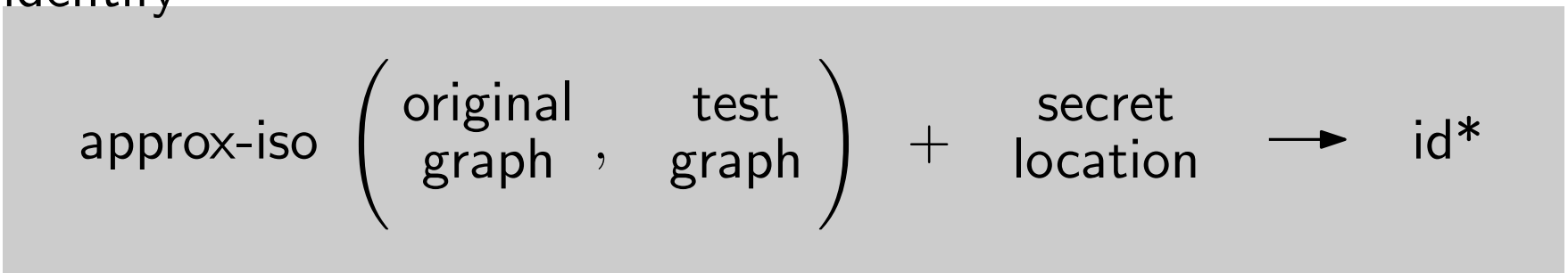


Watermarking scheme

mark



identify



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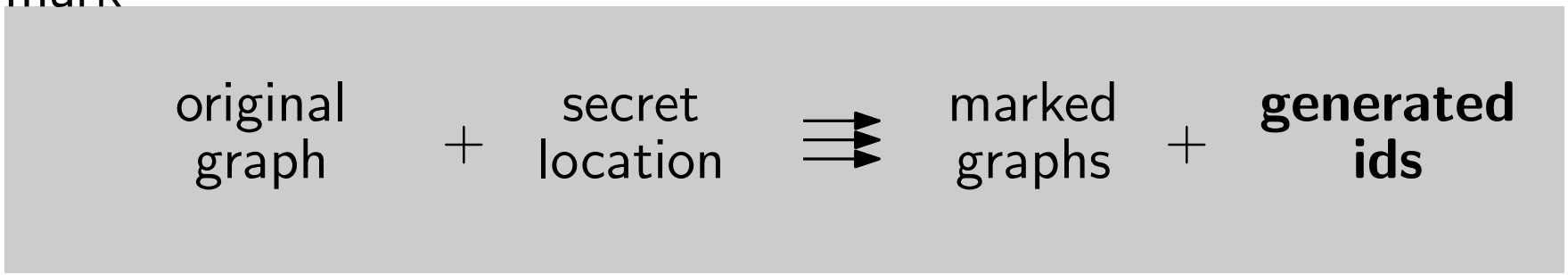

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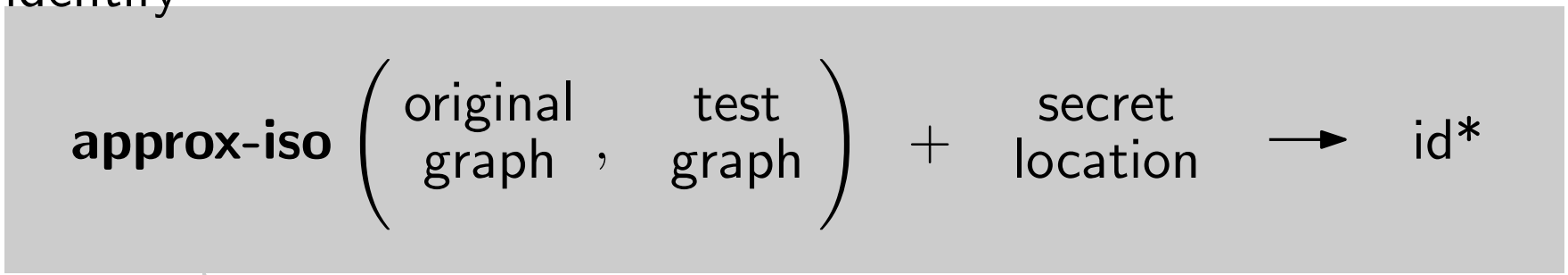


high-degree

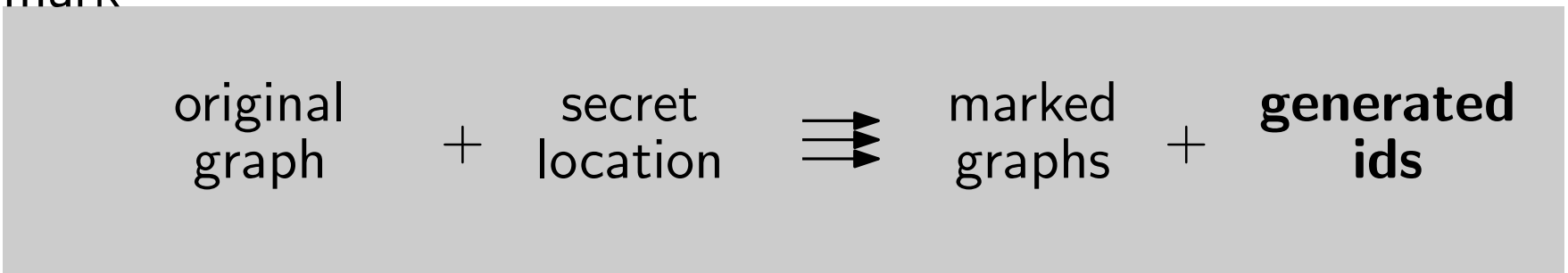
match by degree rank

medium-degree

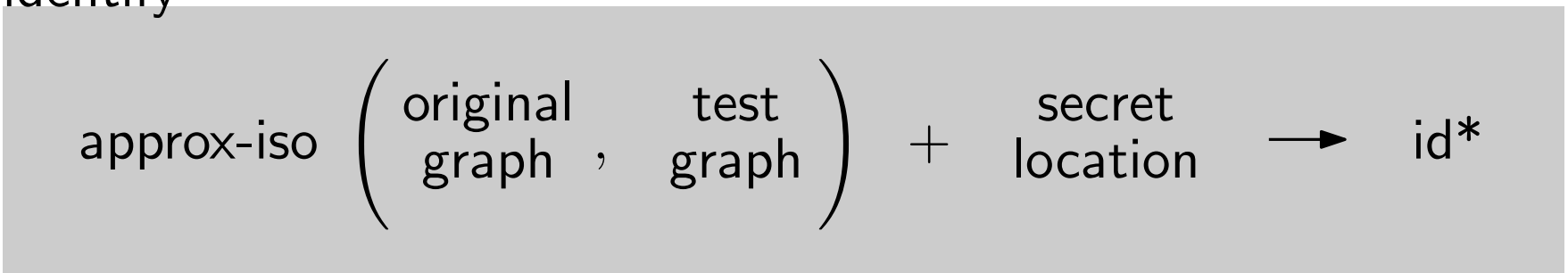
match closest pairs

Watermarking scheme

mark



identify



Watermarking scheme

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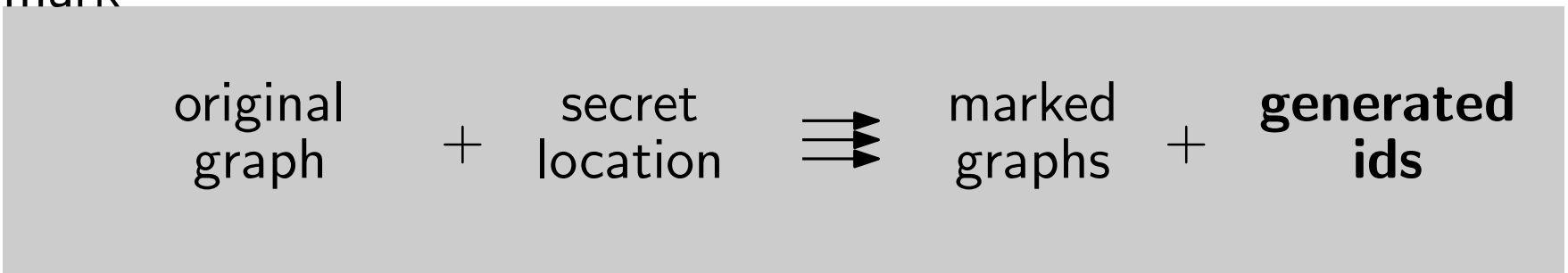

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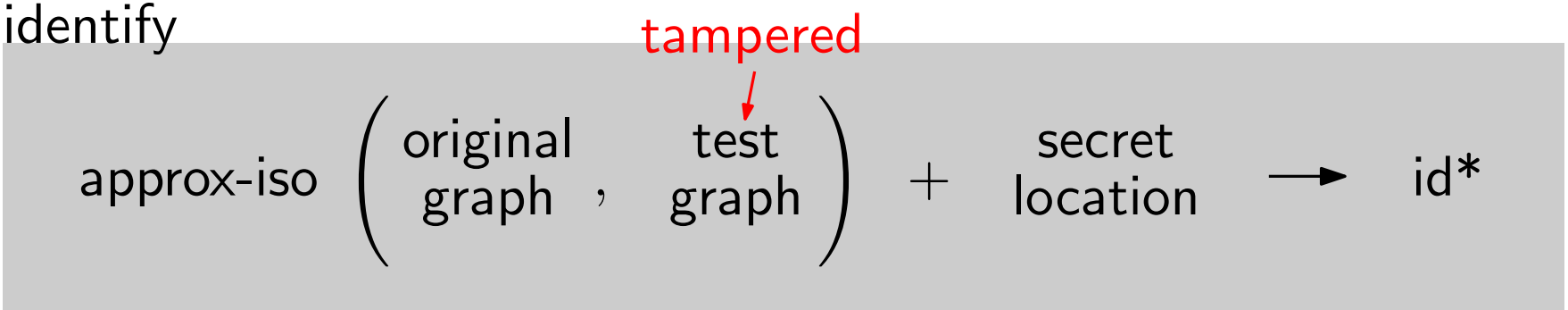
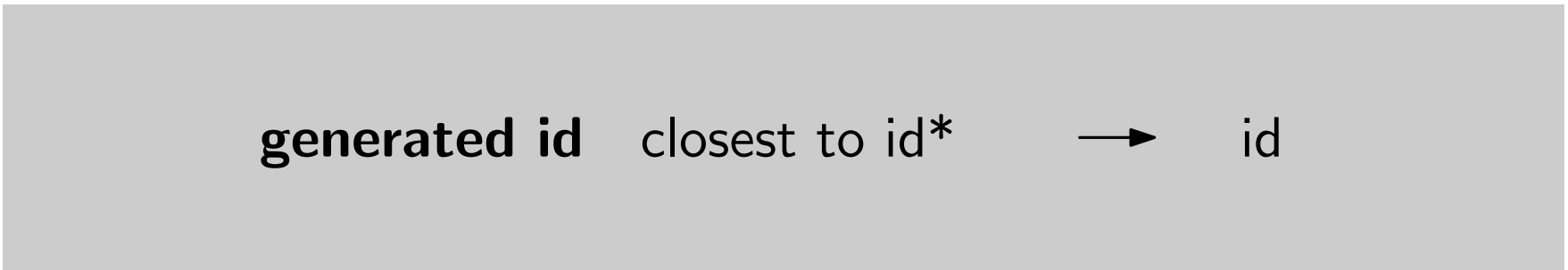




evaluation

Security definition

experiment(adversary: $G \mapsto G'$)

random graph distribution $\longrightarrow G$

generate secret location

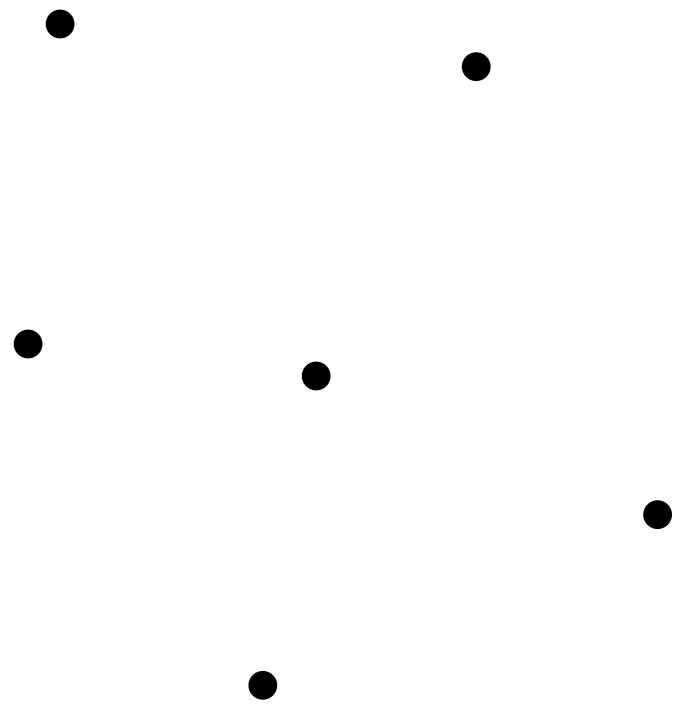
mark (G, secret location) $\longrightarrow (G_1, \mathrm{id}_1), \ldots (G_k, \mathrm{id}_k)$

$1, 2, \ldots k \longrightarrow j$

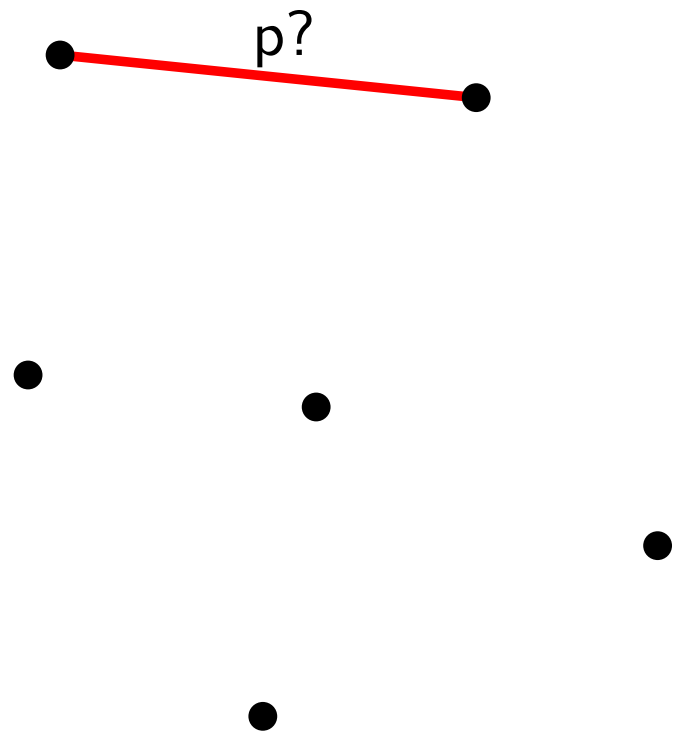
 (G_j) $\longrightarrow G'$

advantage of 

$$\Pr\left[\, \mathrm{dist}(G, G') \leq d, \quad \mathrm{identify}\left(G, G', \begin{matrix}\text{secret}\\ \text{location}'\end{matrix} \begin{matrix}\text{generated}\\ \text{ids}\end{matrix}\right)\right]$$



Erdös-Rényi
G(n, p)



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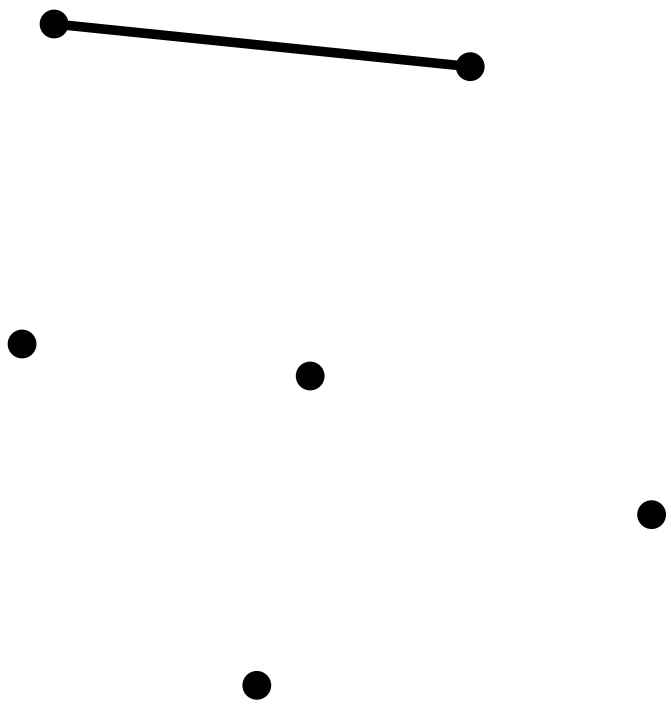

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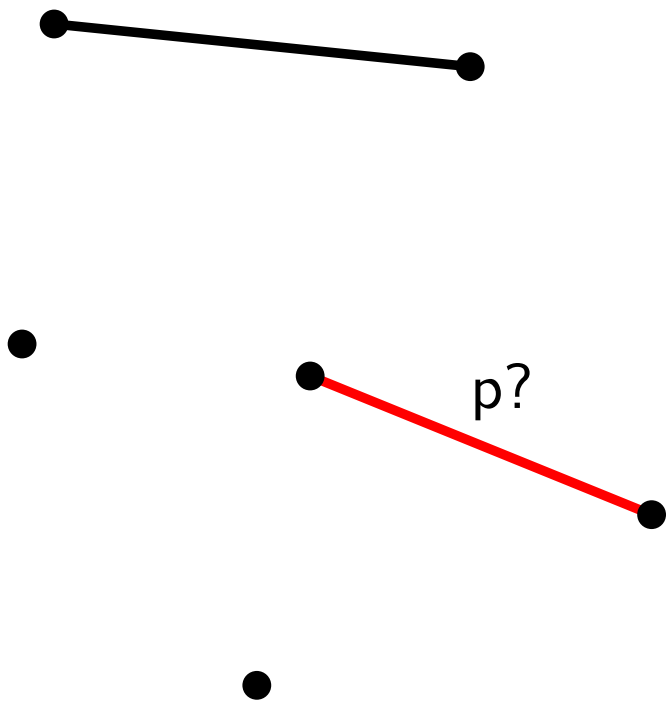
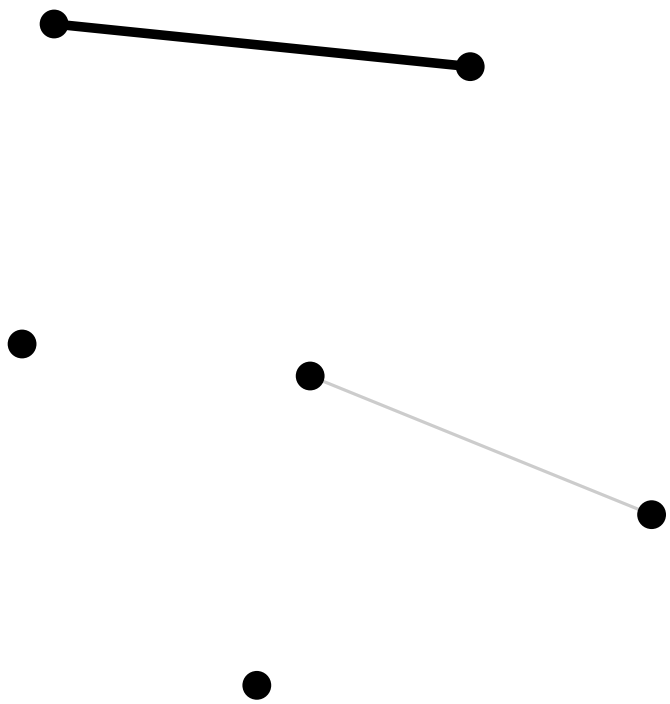
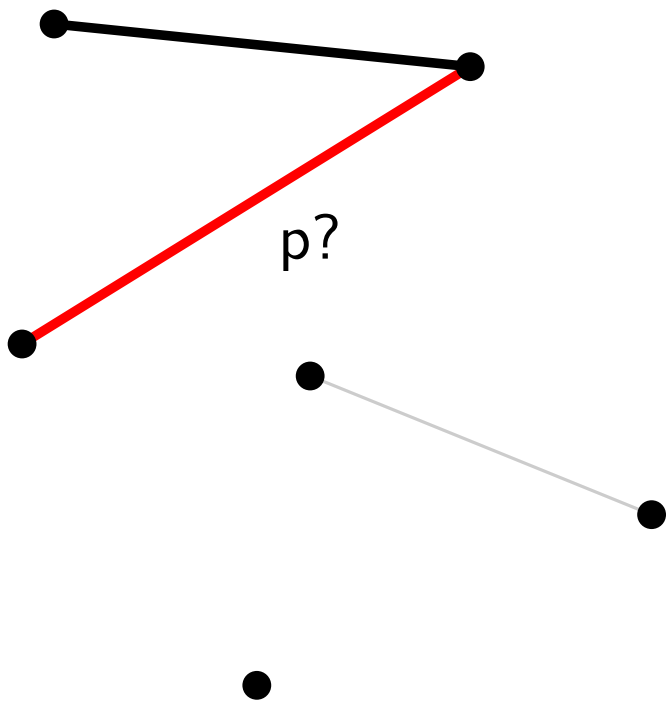


Erdös-Rényi

G(n, p)



Erdös-Rényi
G(n, p)



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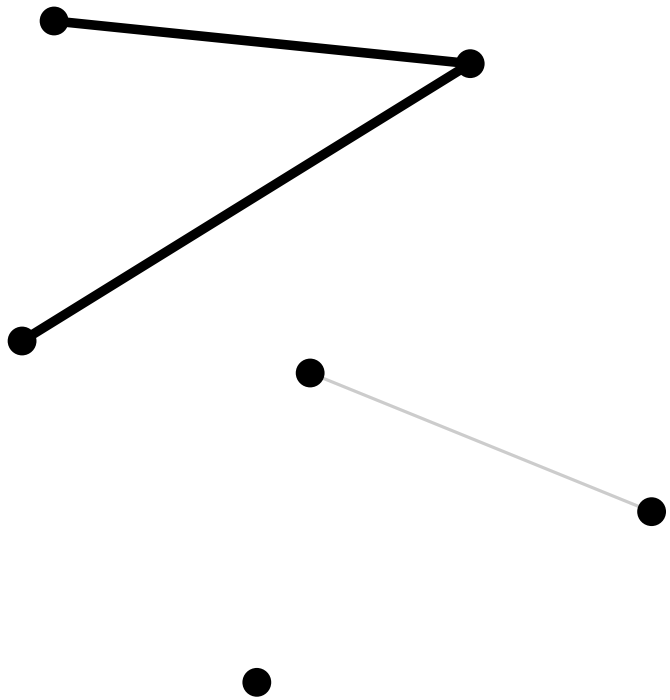

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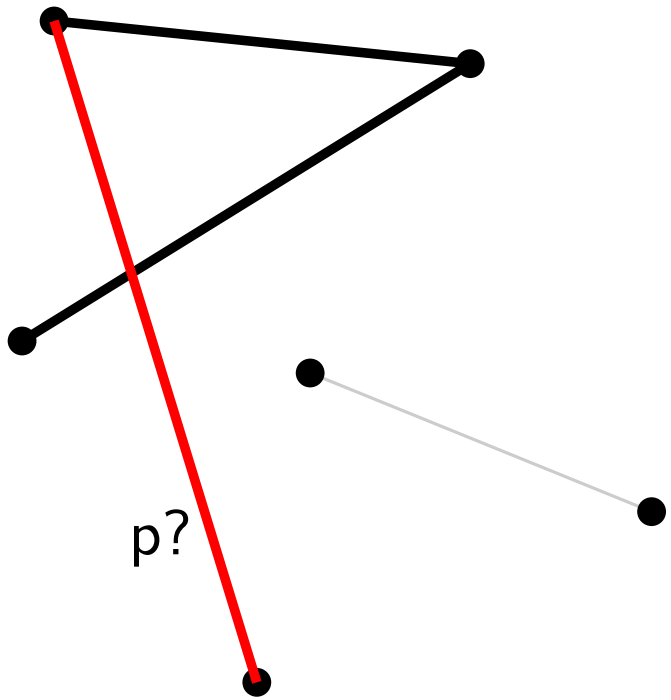

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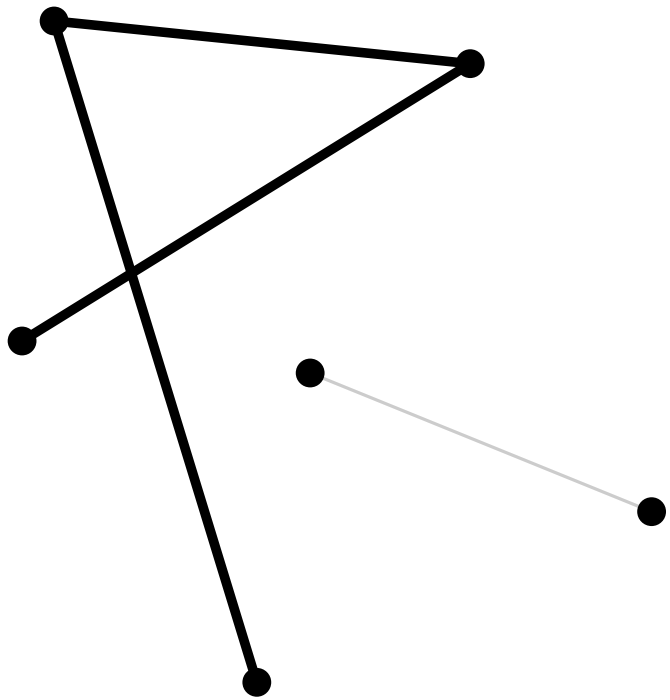

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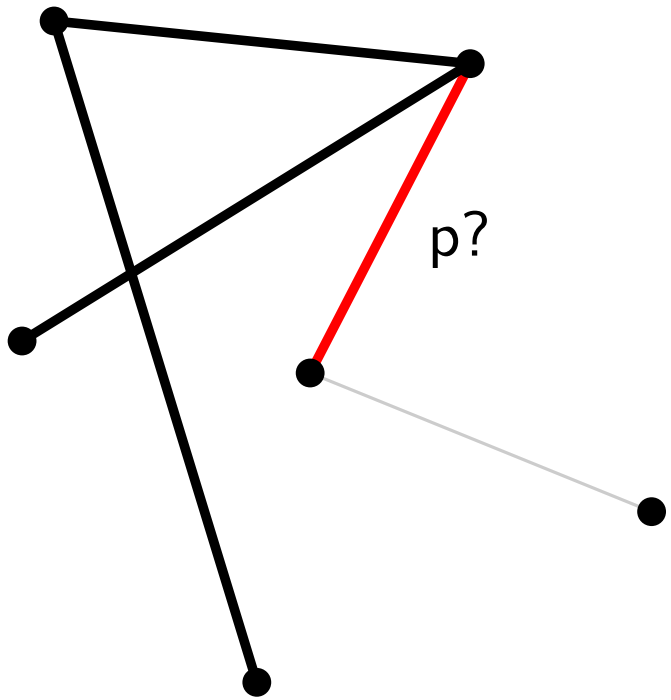

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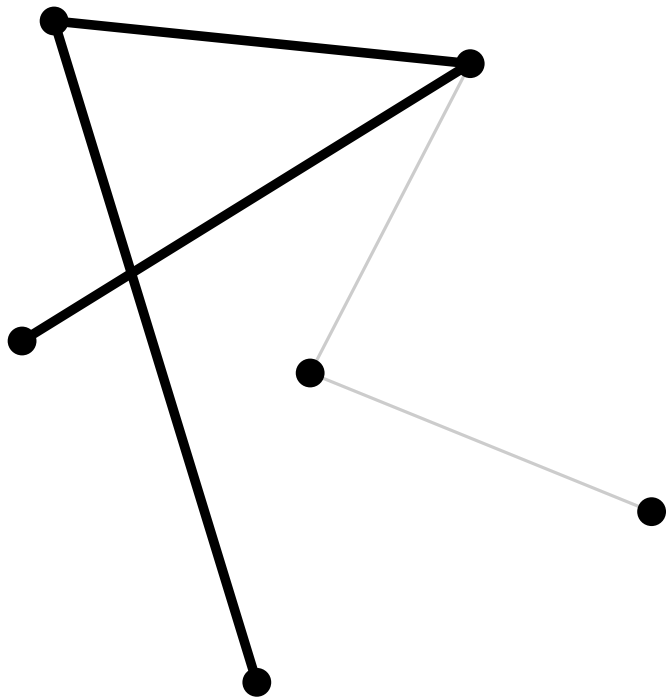

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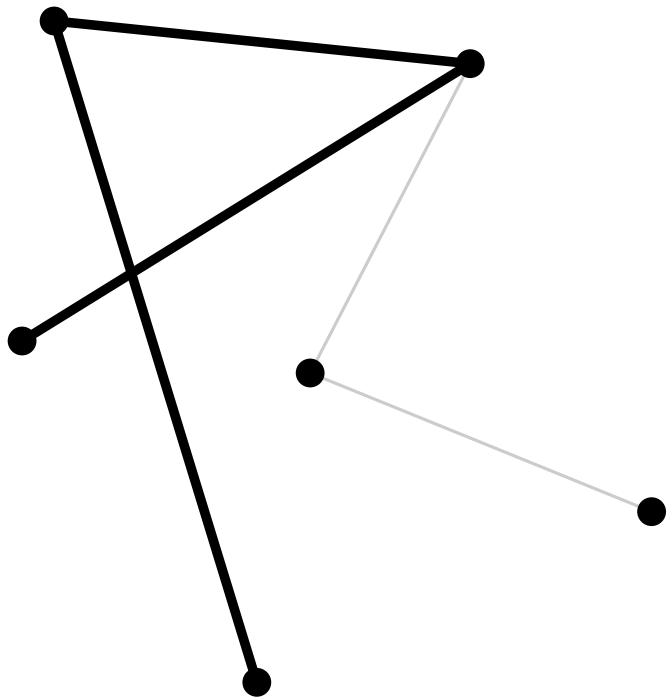

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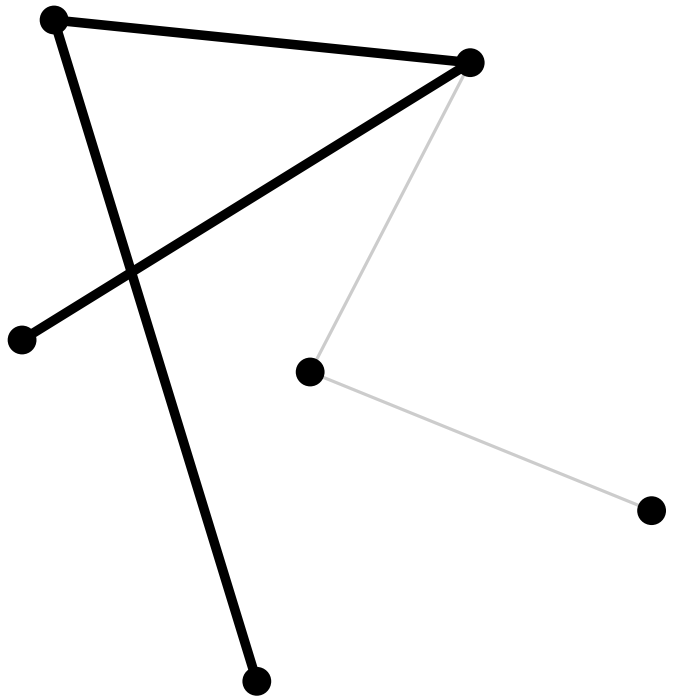

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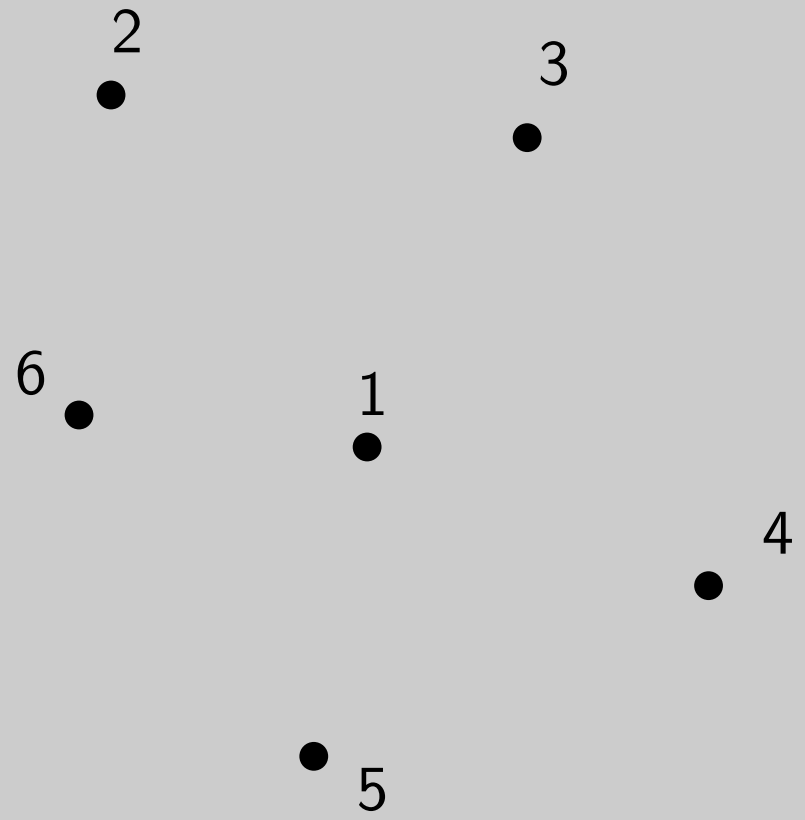
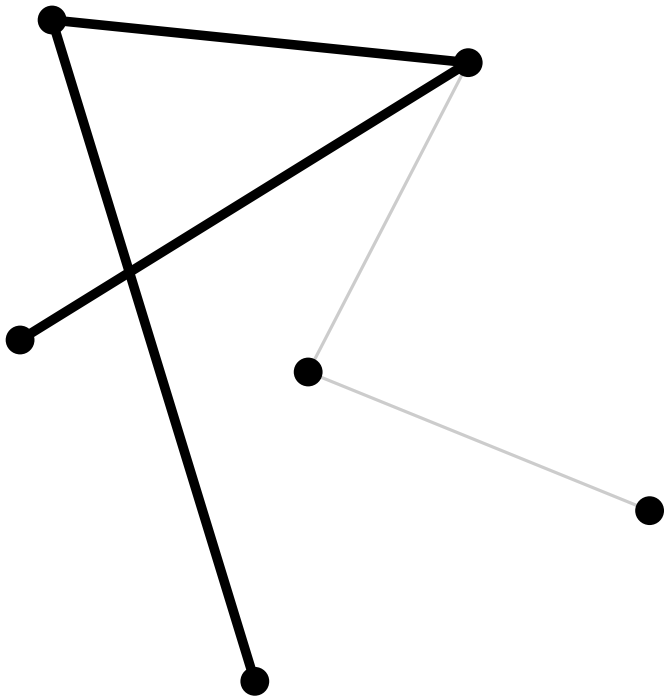
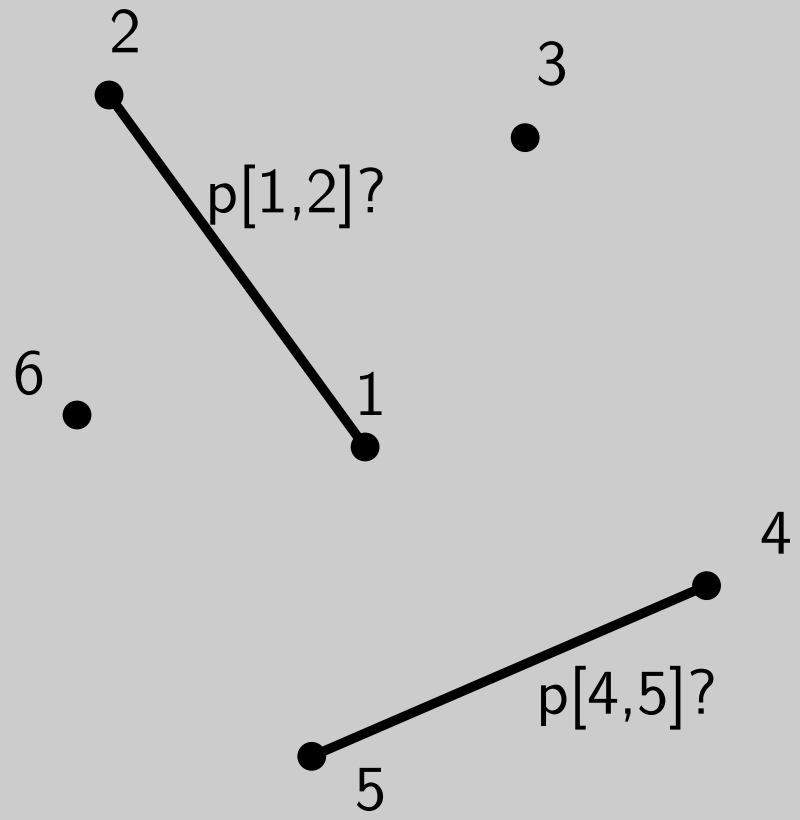
G(n, p)



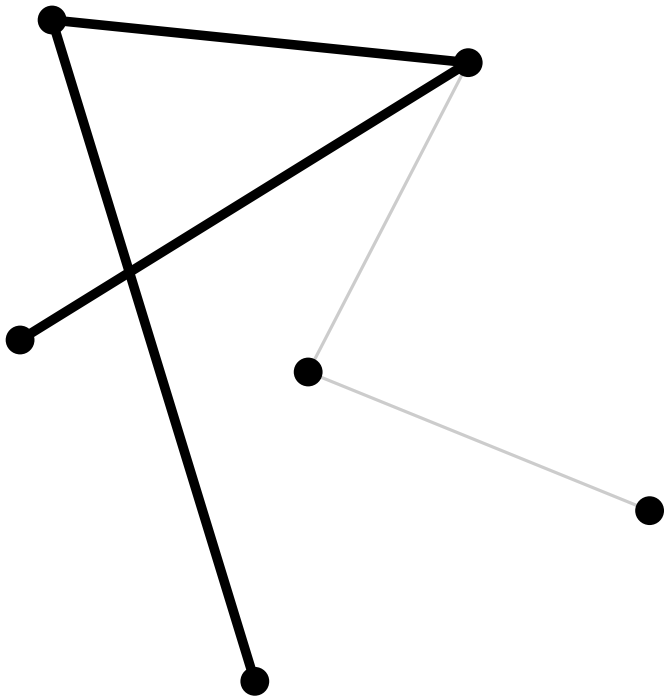
Rand power-law graph
G(w^{γ})



Erdös-Rényi
G(n, p)

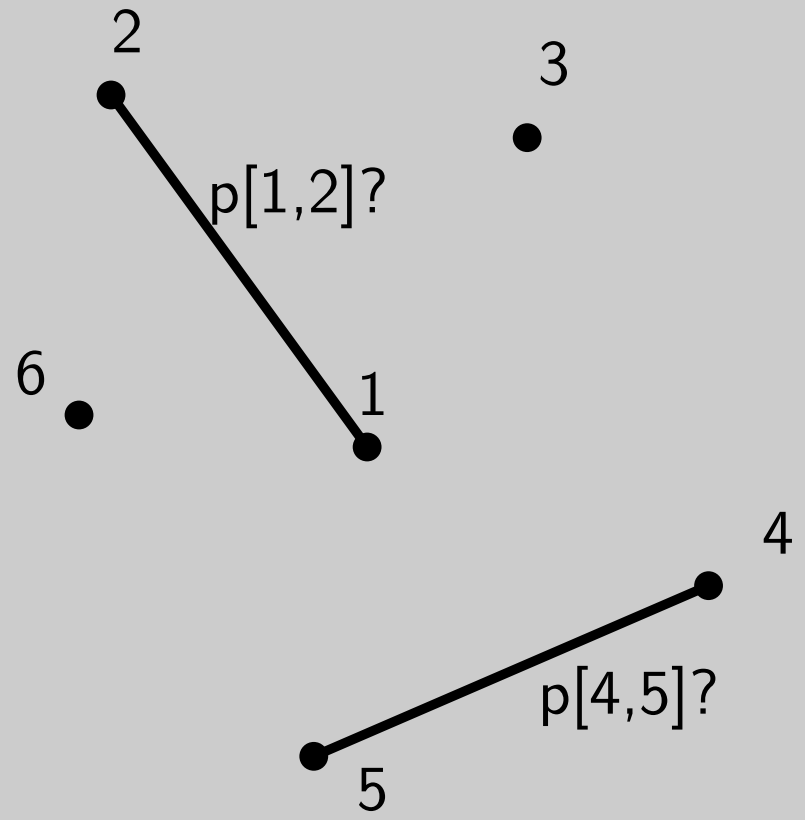


Rand power-law graph
G(w^{γ})



Erdös-Rényi
G(n, p)

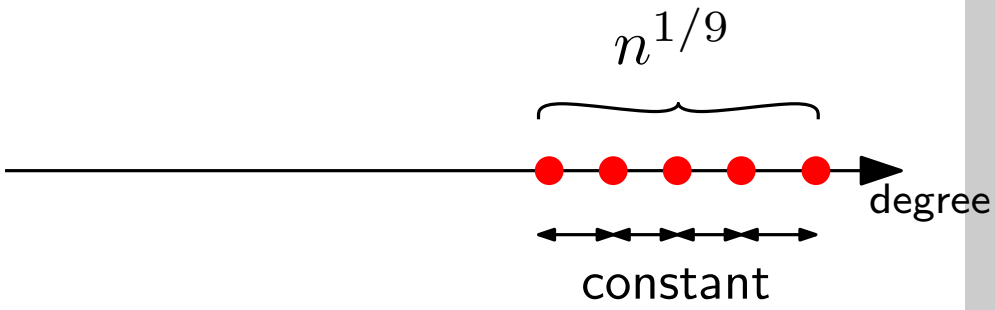
$$p[i,j] \sim \left(\frac{1}{n^{\gamma-3}ij}\right)^{\frac{1}{\gamma-1}}$$

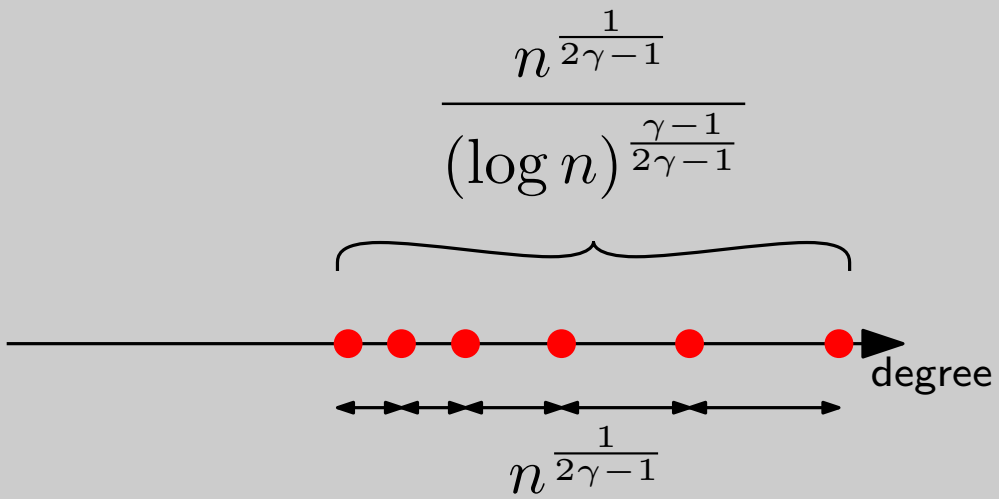


Rand power-law graph
G(w^{γ})

Separation of vertex labels

Separation of vertex labels

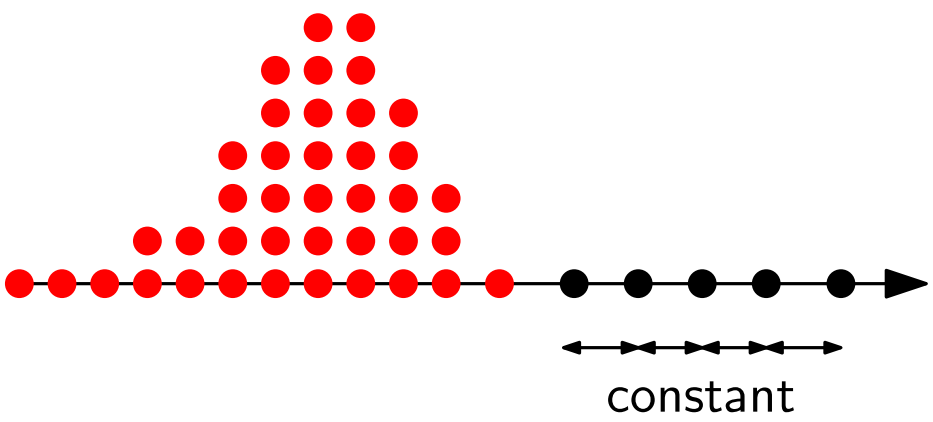


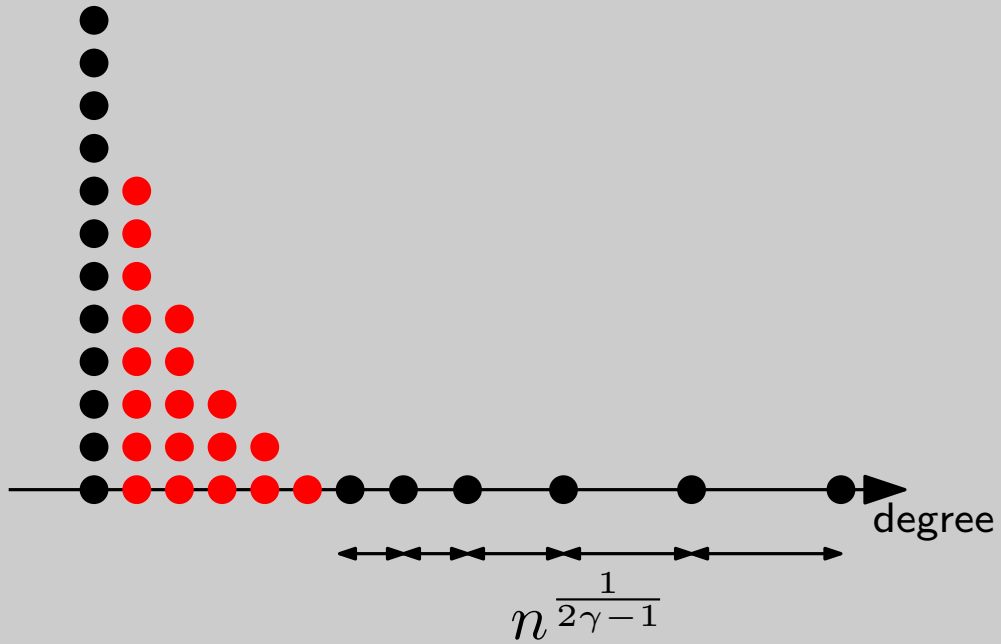


Erdos-Renyi
G(n, p)

Rand power-law graph
G(w$^{\gamma}$)

Separation of vertex labels

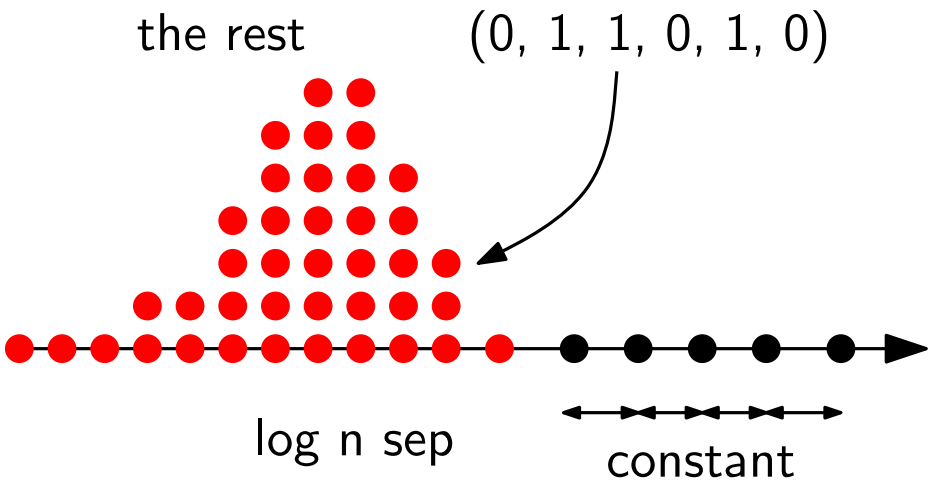




Erdos-Renyi G(n, p)

Rand power-law graph $G(w^{\gamma})$

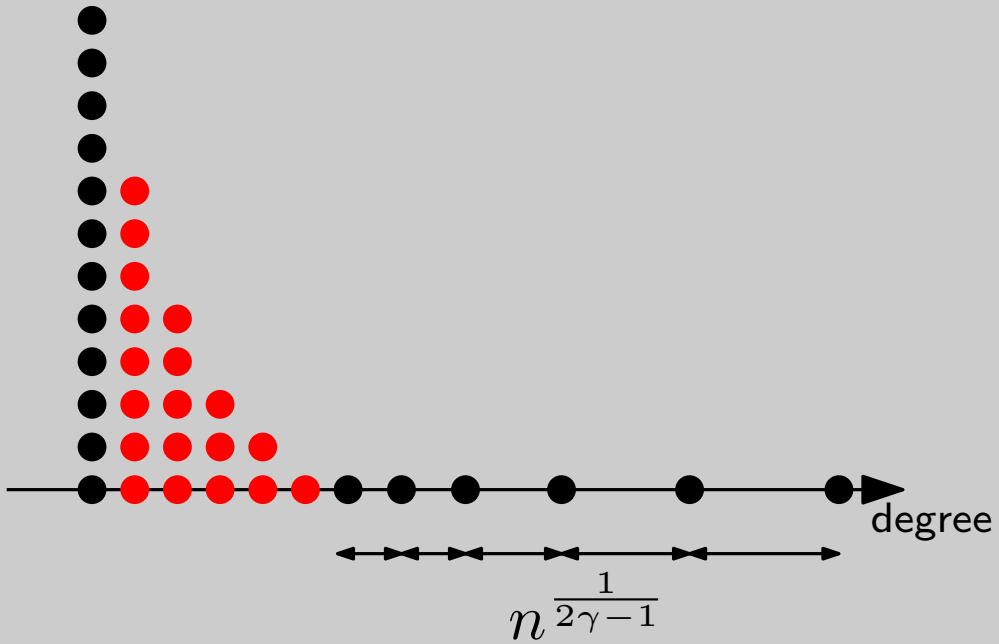
Separation



$$p = \omega(n^{-\epsilon} \log n)$$

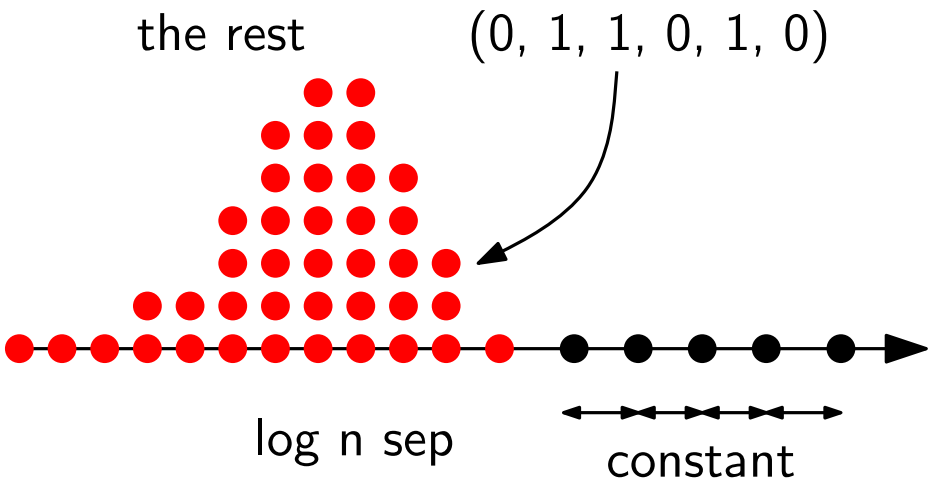
Erdos-Renyi
G(n, p)

of vertex labels



Rand power-law graph
G(w^{γ})

Separation



$$p = \omega(n^{-\epsilon} \log n)$$

Erdos-Renyi
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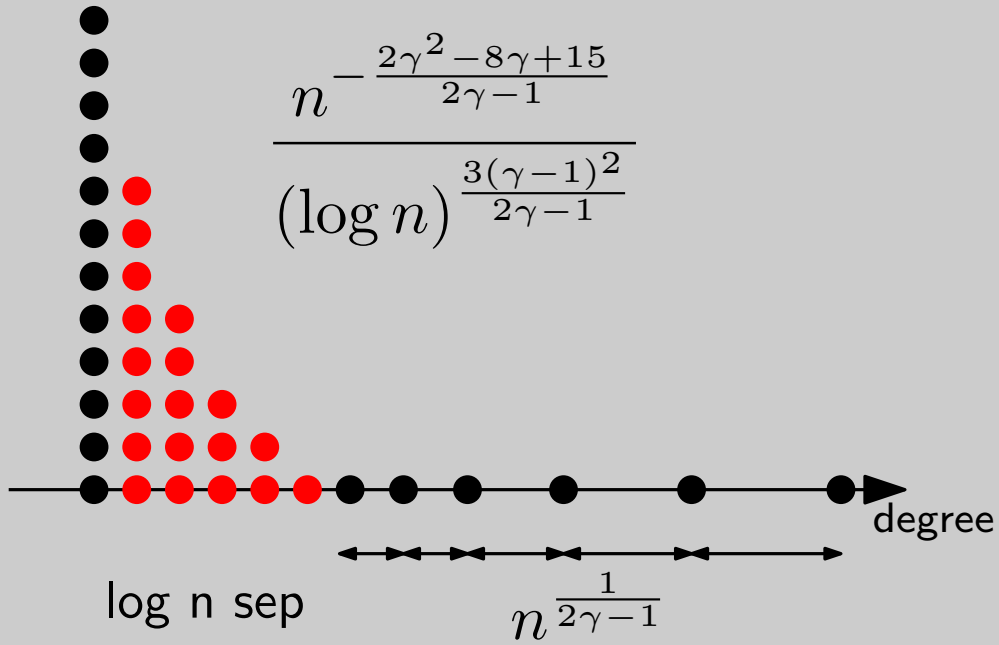
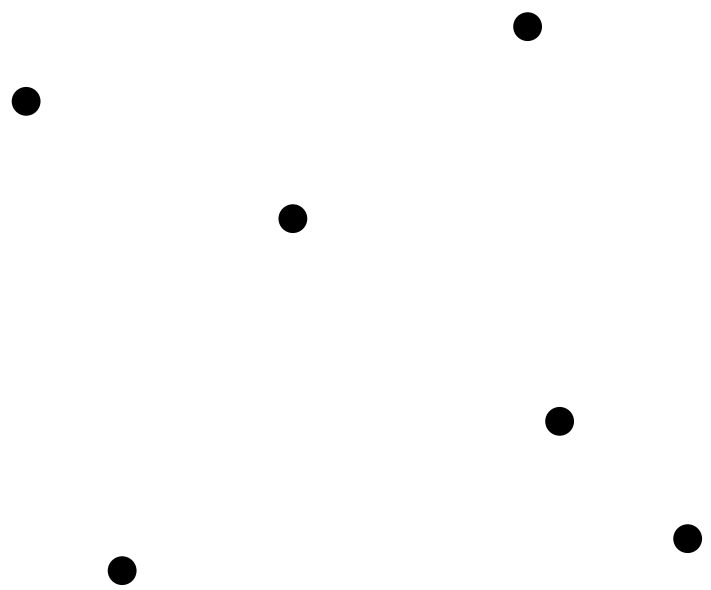
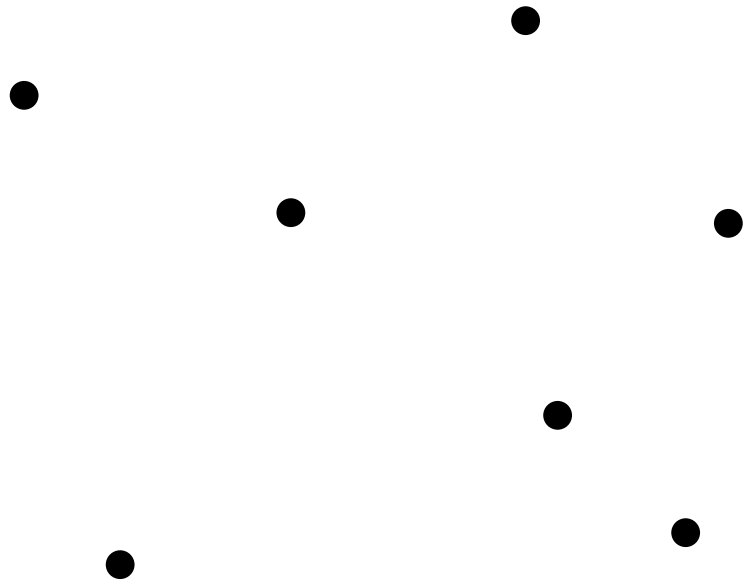


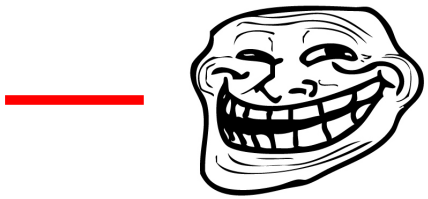
Rand power-law graph
G(w^{γ})

It's hard to guess the secret location



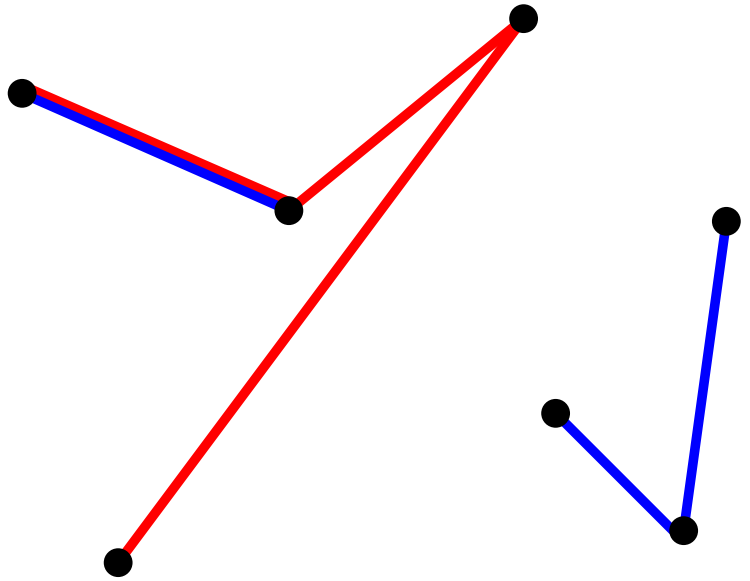
It's hard to guess the secret location

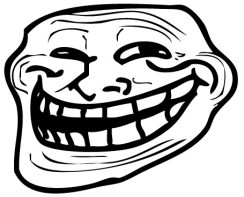




secret location

It's hard to guess the secret location



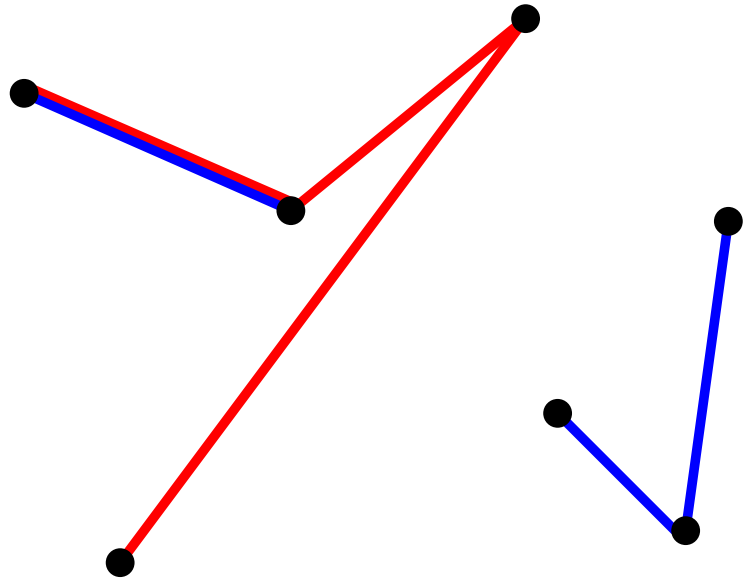


secret location

degree constraint

no more than t edges incident to any vertex

It's hard to guess the secret location



secret location

degree constraint

no more than t edges
incident to any vertex

$\ell \ll N$ Pr [secret location contains $8\ell r/N^2$ red edges] is small

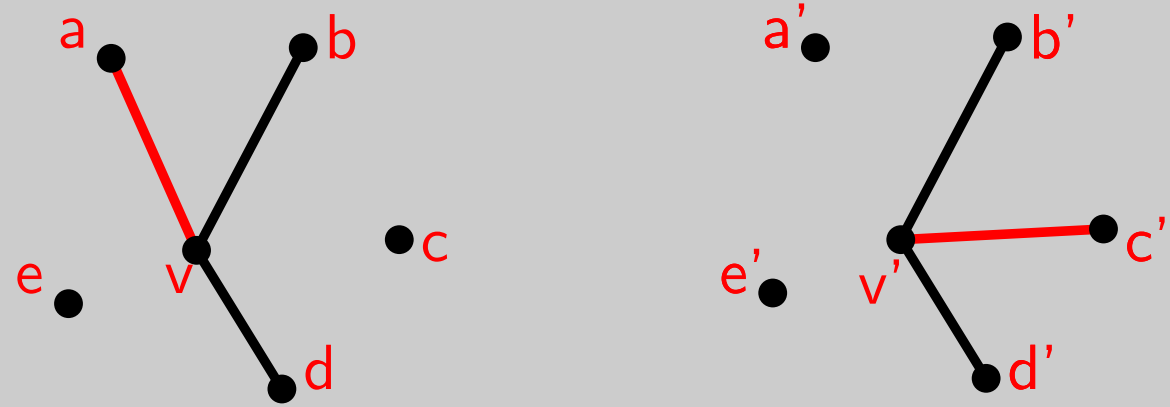
$\frac{\ell r}{N^2} \longrightarrow 0$ Pr [secret location contains 1 red edge] is small

Distance between graphs

graph edit distance

min number of edges to flip to go from G to H

vertex distance



min (over mappings) max(#edges/vertex to flip, G to H)

Security definition

experiment(adversary: $G \mapsto G'$)

random graph distribution $\longrightarrow G$

generate secret location

mark (G, secret location) $\longrightarrow (G_1, \mathrm{id}_1), \ldots (G_k, \mathrm{id}_k)$

$1, 2, \ldots k \longrightarrow j$

 (G_j) $\longrightarrow G'$

advantage of 

$$\Pr\left[\, \mathrm{dist}(G, G') \leq d, \quad \mathrm{identify}\left(G, G', \begin{matrix}\text{secret}\\ \text{location}'\end{matrix} \begin{matrix}\text{generated}\\ \text{ids}\end{matrix}\right)\right]$$

Security definition

edge flipper

experiment(adversary: $G \mapsto G'$)

random graph distribution $\longrightarrow G$

generate secret location

mark (G, secret location) $\longrightarrow (G_1, \mathrm{id}_1), \ldots (G_k, \mathrm{id}_k)$

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$$\Pr\left[\, \mathrm{dist}(G, G') \leq d, \quad \mathrm{identify}\left(G, G', \begin{matrix}\text{secret}\\ \text{location}'\end{matrix}, \begin{matrix}\text{generated}\\ \text{ids}\end{matrix}\right)\right]$$

Security definition

edge flipper

experiment(adversary: $G \mapsto G'$)

random graph distribution $\longrightarrow$ G G(n,p) or G(w^{γ})

generate secret location

mark (G, secret location) $\longrightarrow$ (G_1, id_1), . . . (G_k, id_k)

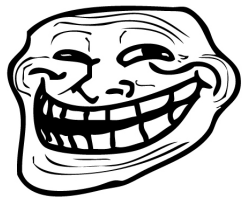
1, 2, . . . k $\longrightarrow$ j

(G_j) $\longrightarrow$ G'

advantage of 

$$\Pr\left[\text{dist}(G, G') \leq d, \quad \text{identify}\left(G, G', \begin{matrix}\text{secret} \\ \text{location'}\end{matrix} \quad \begin{matrix}\text{generated} \\ \text{ids}\end{matrix}\right)\right]$$

Security definition



edge flipper

experiment(adversary: $G \mapsto G'$)

random graph distribution $\longrightarrow$ G — G(n,p) or G(w^{γ})

generate secret location — small, degree constraint

mark (G, secret location) $\longrightarrow$ $(G_1, id_1), \ldots (G_k, id_k)$

$1, 2, \ldots k \longrightarrow j$

 (G_j) $\longrightarrow$ G'

advantage of 

$$\Pr\left[\text{dist}(G, G') \leq d, \quad \text{identify}\left(G, G', \begin{matrix}\text{secret} \\ \text{location}'\end{matrix} \quad \begin{matrix}\text{generated} \\ \text{ids}\end{matrix}\right)\right]$$

Security definition

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experiment(adversary: $G \mapsto G'$)

random graph distribution $\longrightarrow$ G — G(n,p) or G(w^{γ})

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1, 2, ... k $\longrightarrow$ j

 (G_j) $\longrightarrow$ G'

advantage of 

$$\Pr\left[\text{dist}(G, G') \leq d, \quad \text{identify}\left(G, G', \begin{matrix}\text{secret}\\ \text{location}'\end{matrix} \quad \begin{matrix}\text{generated}\\ \text{ids}\end{matrix}\right)\right]$$

graph edit distance

vertex distance

Erdos-Renyi G(n, p)

secret location	
edges	dn
edges/vertex	d
adversary	
edges	dn
edges/vertex	d
marked graphs	poly(n)

Rand power-law graph $G(w^{\gamma})$

secret location	
edges	log n / p
edges/vertex	log n
adversary	
edges	p (# identifiable vertices)^2
edges/vertex	log n
marked graphs	poly (n)

Erdos-Renyi G(n, p)

secret location

edges	dn
edges/vertex	d

adversary

edges	dn
edges/vertex	d

marked graphs poly(n)

Rand power-law graph $G(w^{\gamma})$

secret location

edges	log n / p
edges/vertex	log n

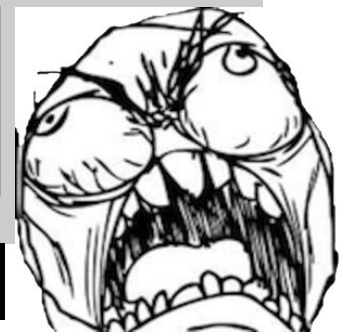
adversary

edges	p (# identifiable vertices)^2
edges/vertex	log n

marked graphs poly (n)

advantage of 

$$\Pr\left[\text{dist}(G, G') \leq d, \quad \text{identify}\left(G, G', \underset{\text{location}}{\text{secret}}, \underset{\text{ids}}{\text{generated}}\right)\right]$$



small

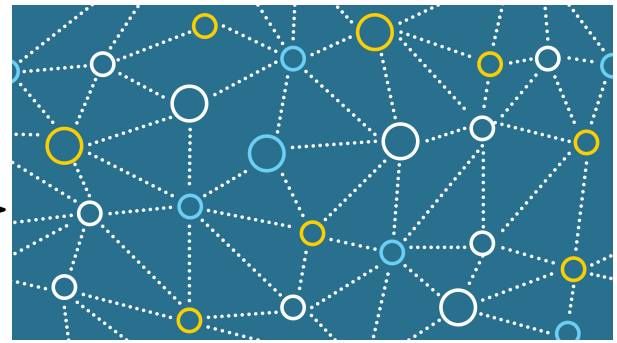
future work

Experiments

real-world network

Experiments

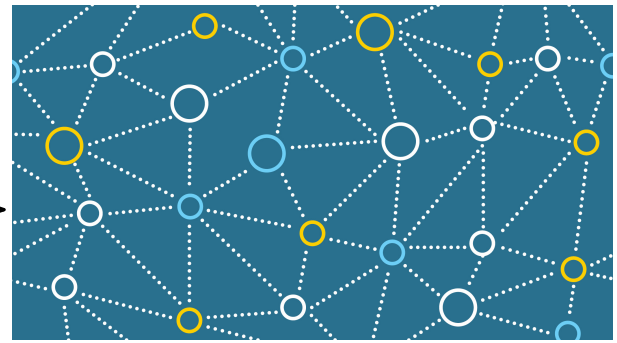
real-world network



random power-law model

Experiments

real-world network



random power-law model

experiments