First-Order Logic and Sets

CS 55 - Spring 2016 - Pomona College
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Review questions

What is a shorter expression that is logically equivalent to:

$$\neg P \wedge Q$$

Find another logically equivalent expression by using
1. the double negation identity, and

2. De Morgan's law

Review question

Is this a proposition?

$2x + 1 = 4$

First-Order Logic

Section 1.3

Propositional Functions

If we allow variables in a proposition, we get what is called a **propositional functions**. *The truth value of a propositional function is determined when all variables have been assigned a value.*

Ex: $P(x)$: $x > 3$

$P(5)$: True and $P(2)$: False

Universe of Discourse

The domain of a propositional function is called the **universe of discourse** and consists of all values the variables may be assigned.

Universal Quantification

The **universal quantification** of P(x) is the proposition:

“P(x) is true for all values of x in the universe of discourse”

We use the notation:

$$\forall x \; P(x)$$

Existential Quantification

The **existential quantification** of P(x) is the proposition:

“There exists an element x in the universe of discourse such that P(x) is true.”

We use the notation:

$$\exists x \; P(x)$$

Examples on Board

Translating to English

C(x): “x is a computer science major”
F(x,y): “x and y are friends”

$$\forall x(C(x) \lor \exists y(C(y) \land F(x,y)))$$

Practice: Translating from English

1. Every computer science student needs a course in discrete mathematics.

2. All students in the class has received an email or text message from another student in the class

3. There is a computer science student who needs a course in curling.

4. There exists a unique real number that is neither positive nor negative.

Expressing uniqueness

"There exists a unique x such that P(x)"

is equivalent to the two combined statements:

there exists x such that P(x) and
for all y such that P(y), x = y.

$$\exists x(P(x) \land \forall y(P(y) \rightarrow (x = y))$$

Quantification Order

$$\forall x \exists y (\cdots) \neq \exists y \forall x (\cdots)$$

De Morgan's Law

$$\neg \forall x P(x) \Leftrightarrow \exists x \neg P(x)$$

$$\neg \exists x Q(x) \Leftrightarrow \forall x \neg Q(x)$$

Introduction to Sets

Section 1.4

Naive Set Theory

A **set** is thought of as collection collection of objects.

The objects in a set are also called **elements**, or **members**, of the set. A set is said to **contain** its elements.

We use the notation:

$$e \in S$$

Set Equality

Two sets are said to be **equal** if and only if they have the same elements.

$$\forall x (x \in A \leftrightarrow x \in B)$$