João Viktor de Carvalho Mota 160127823

8º Questionário de Acompanhamento

1) $x[K+1] = Gx[K] + H_1 u[K] + H_2 v[K]$

$y[K] = Cx[K]$

$x[K] = \begin{bmatrix} x_1[K] \\ x_2[K] \end{bmatrix}$, $G = \begin{bmatrix} 0{,}5 & 1 \\ 0{,}5 & 0{,}7 \end{bmatrix}$, $H_1 = \begin{bmatrix} 0{,}2 \\ 0{,}1 \end{bmatrix}$, $H_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

a) $u[K] = N_1 r[K] - Fx[K]$

Como $x[K+1] = Gx[K] + H_1 u[K] + H_2 v[K]$ e $v[K] = 0$

$x[K+1] = Gx[K] + H_1 N_1 r[K] - H_1 F x[K] = (G - H_1 F)x[K] + H_1 N_1 r[K]$

sem realimentação: $\Delta(z) = \det(zI - G) = \det\left(\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0{,}5 & 1 \\ 0{,}5 & 0{,}7 \end{bmatrix} \right)$

$\det\left(\begin{bmatrix} z-0{,}5 & -1 \\ -0{,}5 & z-0{,}7 \end{bmatrix} \right) = (z-0{,}5)(z-0{,}7) - 0{,}5 = z^2 - 1{,}2z - 0{,}15 \Rightarrow z_1 = 1{,}3141$

$z_2 = -0{,}1141$

$\mathcal{L} = \begin{bmatrix} 0{,}2 & 0{,}2 \\ 0{,}1 & 0{,}17 \end{bmatrix}$ $\det \mathcal{L} = 0{,}014 \neq 0 \Rightarrow$ posto $\mathcal{L} = 2 = n \Rightarrow$ Controlável

Com realimentação: $\det(zI - G + H_1 F) = \det\left(\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0{,}5 & 1 \\ 0{,}5 & 0{,}7 \end{bmatrix} + \begin{bmatrix} 0{,}2 \\ 0{,}1 \end{bmatrix} \cdot \begin{bmatrix} f_1 & f_2 \end{bmatrix} \right)$

$\det\left(\begin{bmatrix} z-0{,}5 & -1 \\ -0{,}5 & z-0{,}7 \end{bmatrix} + \begin{bmatrix} 0{,}2f_1 & 0{,}2f_2 \\ 0{,}1f_1 & 0{,}1f_2 \end{bmatrix} \right) = \det\left(\begin{bmatrix} z-0{,}5+0{,}2f_1 & -1+0{,}2f_2 \\ -0{,}5+0{,}1f_1 & z-0{,}7+0{,}1f_2 \end{bmatrix} \right)$

$$\frac{f_2}{20}-\frac{f_1}{25}-\frac{6z}{5}+\frac{f_1 z}{5}+\frac{f_2 z}{10}+z^2-\frac{3}{20}=0$$

Desejamos polos em $z=0,5\pm 0,5j \Rightarrow \Delta_d(z)=z^2-z+0,5$, logo

$$\begin{cases} z^2-z+0,5=0 \\ z^2+(0,2f_1+0,1f_2-1,2)z+(0,05f_2-0,04f_1-0,15)=0 \end{cases}$$

$$\begin{cases} 0,2f_1+0,1f_2-1,2=-1 \\ 0,05f_2-0,04f_1-0,15=0,5 \end{cases} \Rightarrow \begin{cases} f_1+0,5f_2=1 \Rightarrow 0,5f_2=1-f_1 \\ -0,04f_1+0,05f_2=0,65 \Rightarrow -0,4f_1+0,5f_2=6,5 \end{cases}$$

$$\Downarrow$$

$$-0,4f_1+1-f_1=6,5$$

$$-1,4f_1=5,5$$

$$f_1=-3,9286$$

$$f_2=9,8571$$

Assim $F=[f_1\ f_2]=[-3,9286\ \ 9,8571]$

Usando $e[\infty]=1-NC(I-G+H_1F)^{-1}H_1$, $e[\infty]=0$, $x[0]=0$, $v[k]=0$, $u[k]$ degrau

$$NC(I-G+H_1F)^{-1}H_1=1 \Rightarrow N\cdot[1\ 0]\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}0,5&1\\0,5&0,7\end{bmatrix}+\begin{bmatrix}0,2\\0,1\end{bmatrix}\begin{bmatrix}-3,9286&9,8571\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}0,2\\0,1\end{bmatrix}$$

$$N\cdot[1\ 0]\cdot\left(\begin{bmatrix}-0,2857&0,9714\\-0,8929&1,2857\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}0,2\\0,1\end{bmatrix}=1 \Rightarrow N\cdot[0,32]=1$$

$$N=3,1249$$

Logo $u[k]=3,1249\,r[k]-[-3,9286\ \ 9,8571]\cdot x[k]$ para $v[k]=0$, $r[k]=$ degrau

erro estacionário nulo e polos de malha fechada $z=0,5\pm 0,5j$

b) Como $e[\infty] = \underset{\substack{\downarrow \\ r[\infty]}}{0} - NC(I - G + H_1 F)^{-1} H_2$ quando $v[k]$ for degrau e $r[k] = 0$ e $x[0] = 0$

$$e[\infty] = 0 - 3{,}1249 \cdot \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0{,}5 & 1 \\ 0{,}5 & 0{,}7 \end{bmatrix} + \begin{bmatrix} 0{,}2 \\ 0{,}1 \end{bmatrix} \begin{bmatrix} -3{,}92 & 9{,}86 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$e[\infty] = 0 - \begin{bmatrix} 3{,}1249 & 0 \end{bmatrix} \cdot \begin{pmatrix} -3{,}4286 & 8{,}8571 \\ -0{,}5 & 0{,}3 \end{pmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$e[\infty] = 0 - 2{,}5715$$

$$e[\infty] = -2{,}5715$$

a) $x[k+1] = Gx[k] + H_1 u[k] + H_2 v[k] \Rightarrow x[k+1] = Gx[k] + H_1(k_0 x_0[k] - Fx[k]) + H_2 v[k]$

$y[k] = Cx[k]$

$u[k] = k_0 x_0[k] - Fx[k]$, $\quad x_0[k+1] = x_0[k] + r[k] - \underset{Cx[k]}{\underset{\shortparallel}{y[k]}}$

$$\tilde{x}[k] = \begin{bmatrix} x[k] \\ x_0[k] \end{bmatrix} \Rightarrow \tilde{x}[k+1] = \begin{bmatrix} x[k+1] \\ x_0[k+1] \end{bmatrix}$$

$$\tilde{x}[k+1] = \overset{\tilde{G}}{\begin{bmatrix} G - H_1 F & H_1 k_0 \\ -C & I \end{bmatrix}} \cdot \tilde{x}[k] + \overset{\tilde{H}_2}{\begin{bmatrix} H_2 \\ 0 \end{bmatrix}} v[k] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r[k]$$

$$y[k] = \begin{bmatrix} C & 0 \end{bmatrix} \tilde{x}[k]$$

$$\tilde{G} = \begin{bmatrix} 0{,}5 - 0{,}2k_1 & 1 - 0{,}2k_2 & 0{,}2k_0 \\ 0{,}5 - 0{,}1k_1 & 0{,}7 - 0{,}1k_2 & 0{,}1k_0 \\ -1 & 0 & 1 \end{bmatrix}$$

, Como $\tilde{x}[k+1] = \tilde{G}\tilde{x}[k] + \tilde{H}_2 v[k] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r[k]$

considerando $\tilde{x}[0] = 0$

$$x(z) = (zI - \tilde{G})^{-1}\left(\tilde{H}_2 v[z] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(z)\right)$$

Logo: $\det(zI - \tilde{G}) = \det\left(\begin{bmatrix} z - 0{,}5 + 0{,}2k_1 & -1 + 0{,}2k_2 & -0{,}2k_0 \\ -0{,}5 + 0{,}1k_1 & z - 0{,}7 + 0{,}1k_2 & -0{,}1k_0 \\ 1 & 0 & z - 1 \end{bmatrix}\right)$

chegamos em

$$z^3 + (-2{,}2 + 0{,}2k_1 + 0{,}1k_2)z^2 + (0{,}2k_0 - 0{,}05k_2 - 0{,}24k_1 + 1{,}05)z + (0{,}04k_1 - 0{,}05k_2 - 0{,}04k_0 + 0{,}15) = 0$$

Como queremos polos em $z = 0$ e $z = 0{,}5 \pm 0{,}5j \Rightarrow \Delta_d(z) = z(z - 0{,}5 - 0{,}5j)(z - 0{,}5 + 0{,}5j)$

$= z^3 - z^2 + 0{,}5z$

Igualando chegamos em:

$$\begin{cases} -2{,}2 + 0{,}2k_1 + 0{,}1k_2 = -1 \Rightarrow 2k_1 + k_2 = 12 \\ 0{,}2k_0 - 0{,}05k_2 - 0{,}24k_1 + 1{,}05 = 0{,}5 \Rightarrow 2{,}4k_1 + 0{,}5k_2 - 2k_0 = 5{,}5 \\ 0{,}04k_1 - 0{,}05k_2 - 0{,}04k_0 + 0{,}15 = 0 \Rightarrow -4k_1 + 5k_2 + 4k_0 = 15 \end{cases}$$

Resolvendo o sistema chegamos em $\Rightarrow k_0 = 3{,}125$

matlab $\Rightarrow$ rslve

$k_1 = 4{,}1071$

$k_2 = 3{,}7857$

Com isso chegamos em $v[K] = 3{,}125\,x_a[K] - [4{,}1071 \ \ 3{,}7857] \cdot x[K]$

b) Como $\tilde{x}(z) = (zI - \tilde{G})^{-1}\left(\hat{H}_2 w[z] + \begin{bmatrix}0\\0\\1\end{bmatrix} r(z)\right)$

e $y(K) = [C \ 0]\,\tilde{x}[K]$

logo $y(z) = [C \ 0] \cdot (zI - \tilde{G})^{-1}\left(\tilde{H}_2 w(z) + \begin{bmatrix}0\\0\\1\end{bmatrix} r(z)\right)$

Considerando $r(K)$ degrau e $w[K] = 0$

$$e(\infty) = 1 - [C \ 0](I - \tilde{G})^{-1} \cdot \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$e(\infty) = 1 - [1 \ 0 \ 0]\begin{pmatrix} 1-0{,}5+0{,}2l_1 & -1+0{,}2l_2 & -0{,}2l_0 \\ -0{,}5+0{,}1l_1 & 1-0{,}7+0{,}1l_2 & -0{,}1l_0 \\ 1 & 0 & 1-1 \end{pmatrix}^{-1} \cdot \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$e(\infty) = 1 - [1 \ 0 \ 0]\begin{bmatrix} 0 & 0 & 1 \\ -0{,}625 & 1{,}25 & 0{,}9375 \\ -1{,}3571 & -0{,}4857 & 1{,}75 \end{bmatrix} \cdot \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$e(\infty) = 0$

Considerando $r(K) = 0$ e $w(K) = $ degrau

$$\tilde{H}_2 = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$e(\infty) = 0 - [1 0 0](I - \tilde{G})^{-1} \cdot \tilde{H}_2$$

$e(\infty) = 0 - 0 = 0$