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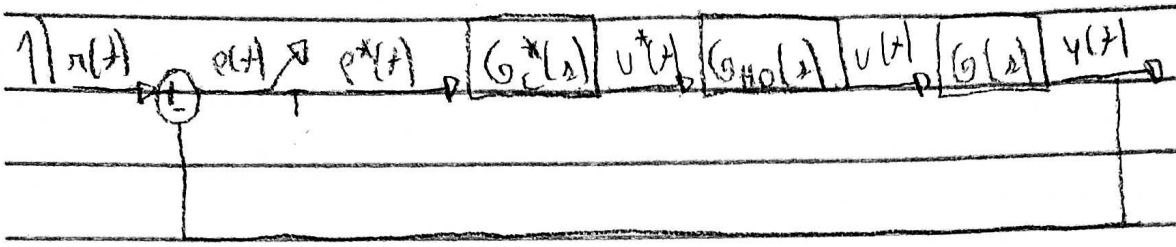
6º questionário de acompanhamento

1) 

$$G(s) = \frac{10}{s(s+2)} \qquad T = 0,1\,s$$

a) $Z\{G_{ZOH} \cdot G(s)\} = (1-z^{-1})\,Z\left\{\frac{10}{s^2(s+2)}\right\} \Rightarrow \frac{10}{s^2(s+2)} = \frac{A(s+2) + Bs(s+2) + Cs^2}{s^2(s+2)}$

$A = 5,\ C = 2,5,\ B = -2,5 \Rightarrow \frac{5}{s^2} - \frac{2,5}{s} + \frac{2,5}{(s+2)} \Rightarrow \frac{0,5z}{(z-1)^2} - \frac{2,5z}{z-1} + \frac{2,5z}{z-e^{-0,2}}$

$$\Rightarrow \frac{z-1}{z}\left(\frac{0,5z}{(z-1)^2} - \frac{2,5z}{z-1} + \frac{2,5z}{z-e^{-0,2}}\right) = \frac{0,5}{z-1} - 2,5 + \frac{2,5(z-1)}{z-e^{-0,2}}$$

$$\frac{0,5(z-e^{-0,2}) - 2,5(z-1)(z-e^{-0,2}) + 2,5(z-1)^2}{(z-1)(z-e^{-0,2})} = \frac{0,04683z + 0,04381}{z^2 - 1,819z + 0,8187} = G(z)$$

$n = 2-1 = 1 \qquad K = m = 1 \qquad p = \max\left(m_p[G(z)]_{em\ z=1}, m_p(G_{(z)em\ z=1}\right) = \max(1,1) = 1$

$M(z) = (1+0,9355z^{-1})(m_1 z^{-1}) \Rightarrow 1 - (1+0,9355z^{-1})(m_1 z^{-1}) = (1-z^{-1})(1+a_1 z^{-1})$

$1 - M(z) = (1-z^{-1})\cdot(1+a_1 z^{-1})$

não haverá oscilação na saída

$$1 - m_1 z^{-1} - 0,9355 m_1 z^{-2} = 1 + (a_1 - 1)z^{-1} - a_1 z^{-2}$$

$$\begin{cases} (a_1 - 1) = -m_1 \\ -0,9355 m_1 = -a_1 \end{cases} \Rightarrow \begin{cases} a_1 + m_1 = 1 \\ a_1 = 0,9355 m_1 \end{cases} \Rightarrow m_1 = 0,5167 \quad a_1 = 0,4833$$

Logo $M(z) = 0,5167z^{-1} + 0,4833z^{-2} = \frac{0,5167z + 0,4833}{z^2}$

$1 - M(z) = (1 - z^{-1})(1 + 0{,}4833 z^{-1}) = 1 - 0{,}5167 z^{-1} - 0{,}4833 z^{-2} = \frac{z^2 - 0{,}5167 z - 0{,}4833}{z^2}$

$G_c(z) = \frac{1}{G(z)} \cdot \frac{M(z)}{1 - M(z)} = \frac{(z-1)(z-0{,}8187)}{0{,}04683(z+0{,}9355)} \cdot \frac{0{,}5167(z+0{,}9355)}{(z-1)(z+0{,}4833)}$

$G_c(z) = \frac{11{,}0335(z - 0{,}8187)}{(z + 0{,}4833)}$

b) Resposta ao degrau:

$Y(z) = M(z) R(z) = \frac{0{,}5167(z+0{,}9355)}{z^2} \cdot \frac{z}{z-1} = \frac{0{,}5167(z+0{,}9355)}{z(z-1)}$

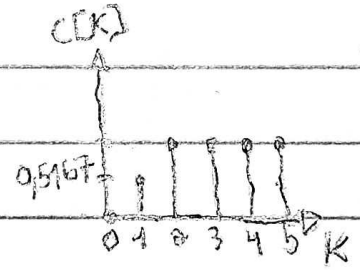
$0{,}5162z + 0{,}4833 \;|\; z^2 - z$

$0{,}5167z - 0{,}5167 \quad 0{,}5167z^{-1} + z^{-2} + z^{-3} \ldots \quad Y(z) = 0{,}5167 z^{-1} + z^{-2} + z^{-3} + \ldots$

$\frac{1}{1 - z^{-1}}$

$\frac{}{z^{-1}}$

$Y[K] = 0{,}5167\,\delta[K-1] + \delta[K-2] + \delta[K-3] + \ldots$



$E(z) = R(z) - Y(z) \Rightarrow G_c(z) Y(z) + G(z) R(z) = U(z)$

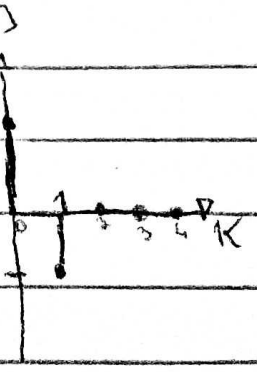
$U(z) = \frac{0{,}5167(z+0{,}9355)}{z(z-1)} \cdot \frac{11{,}0335(z-0{,}8187)}{(z+0{,}4833} + \frac{11{,}0335(z-0{,}8187)}{(z+0{,}4833)} \cdot \frac{z}{z-1}$

$U(z) = \frac{5{,}701(z+0{,}9355)(z-0{,}8187)}{z(z-1)(z+0{,}4833)} + \frac{11{,}0335 z (z-0{,}8187)}{(z+0{,}4833)(z-1)}$

Resolvendo no Wolfram alpha calculando $U(z)$ nb:

$U(z) = 11{,}0335 - 9{,}033 z^{-1}$

$U(K) = 11{,}0335\,\delta[K] - 9{,}0338\,[K-1]$



a) $G(s) = \frac{e^{-1}}{s+0,5}$

$Z\{G_{HO}(s) \cdot G(s)\} = \frac{(z-1)}{z} z^{-1} \cdot Z\left\{\frac{1}{s(s+0,5)}\right\} \Rightarrow A(s+0,5) + Bs = 1$

$A = 2 \quad B = -2$

$\Rightarrow \frac{(z-1)}{z^2} Z\left\{\frac{2}{s} - \frac{2}{s+0,5}\right\} = \frac{(z-1)}{z^2}\left(\frac{2z}{(z-1)} - \frac{2z}{(z-e^{-0,5})}\right) = \frac{2}{z} - \frac{2(z-1)}{z(z-e^{-0,5})}$

$\Rightarrow \frac{2(z-e^{-0,5}) - 2(z-1)}{z(z-e^{-0,5})} = \frac{2z - 1,2131 - 2z + 2}{z(z-0,6065)} = \frac{0,7869}{z(z-0,6065)}$

a) $y(t) = 0$ para $t \leq 0 \quad y(1) = 0,7 \quad y(t) = 1$ para $t \geq 3$

$y(k) = 0,7\delta(k-1) + 1\delta[k-3] + \delta[k-4] + \ldots$

$y(z) = 0,7z^{-1} + z^{-3} + z^{-4} + \ldots$

$y(z) = 0,7z^{-1} + pq \Rightarrow 0,7z^{-1} + lm = 0,7z^{-1} + \frac{z^{-3}}{1-z^{-1}}$

$y(z) = \frac{0,7z^{-2} + z^{-3}}{1-z^{-1}} = \frac{0,7z + 1}{z^3 - z^2} = \frac{0,7(z-1) + 1}{z^2(z-1)} = \frac{0,7z - 0,7 + 1}{z^2(z-1)}$

$y(z) = \frac{0,7z + 0,3}{z^2(z-1)} = m(z) \cdot R(z) \Rightarrow R(z) = \frac{z}{z-1}$ Luego $m(z) = \frac{0,7z + 0,3}{z^3}$

$m(z) = 0,7z^{-2} + 0,3z^{-3}$

$1 - m(z) = 1 - 0,7z^{-2} - 0,3z^{-3}$

$1 - m(z) = 1 - \frac{0,7z + 0,3}{z^3}$

$1 - m(z) = \frac{z^3 - 0,7z - 0,3}{z^3}$

$G_c(z) = \frac{z(z-0,6065)}{0,7869} \cdot \frac{0,7z + 0,3}{z^3 - 0,7z - 0,3} = \frac{0,7(z^2 - 0,6065z)(z + 0,4286)}{0,7869(z^3 - 0,7z - 0,3)}$

$G_c(z) = \frac{0,8896z(z-0,6065)(z+0,4286)}{(z-1)(z^2+z+0,3)}$

Usando Matlab: d2c(G,'zoh')

$G_c(s) = \frac{0,8896s^3 + 1,148s^2 + 5,221s + 1,689}{s^3 + 1,204s^2 + 7,767s}$

b) $U(z) = G_c(z)R(z) - G_c(z)Y(z)$

$$U(z) = \frac{0,8896z(z-0,6065)(z+0,4286)}{(z-1)(z^2+z+0,3)} \cdot \frac{z}{z-1} \quad \frac{0,8896z(z-0,6065)(z+0,4286)(0,7z+0,3)}{z^2(z-1)}$$

Utilizando Wolfram Alpha!

$$U(z) = 0,8896 + 0,7313z^{-1} + 0,5z^{-2} + 0,5z^{-3}$$

Logo $U[k] = 0,8896\delta[k] + 0,7313\delta[k-1] + 0,5\delta[k-2] + 0,5\delta[k-3]$