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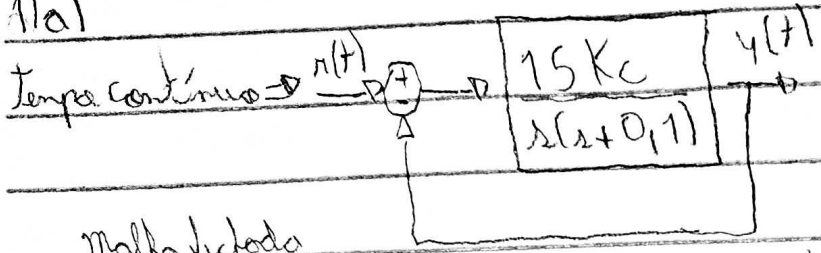
Prova 1 - Controle Digital

1)a)



Malha fechada

$$G(s) = \frac{15K_c}{s^2+0,1s+15K_c}$$, com $t_p = 30s$ e $G(s) = \frac{w_n^2}{s^2+2\zeta w_n s + w_n^2}$

$$t_p = \frac{\pi}{w_d} \Rightarrow w_d = \frac{\pi}{30} = w_n\sqrt{1-\zeta^2} \quad e \quad w_n^2 = 15K_c \quad e \quad 2\zeta w_n = 0,1$$

$$\Downarrow$$

$$w_n = 0,05\zeta$$

Substituindo em $w_d \Rightarrow \frac{\pi}{30} = w_n \cdot \sqrt{1-\frac{0,05^2}{w_n^2}}$

quadrado nos dois lados: $\frac{\pi^2}{30^2} = w_n^2 \cdot \left(1-\frac{0,05^2}{w_n^2}\right) \Rightarrow \frac{\pi^2}{30^2} + 0,05^2 = w_n^2$

$w_n^2 = 0,0135 \Rightarrow w_n = 0,1160$

$15K_c = 0,0135 \Rightarrow K_c = 0,00089775 \quad \zeta = 0,4309$

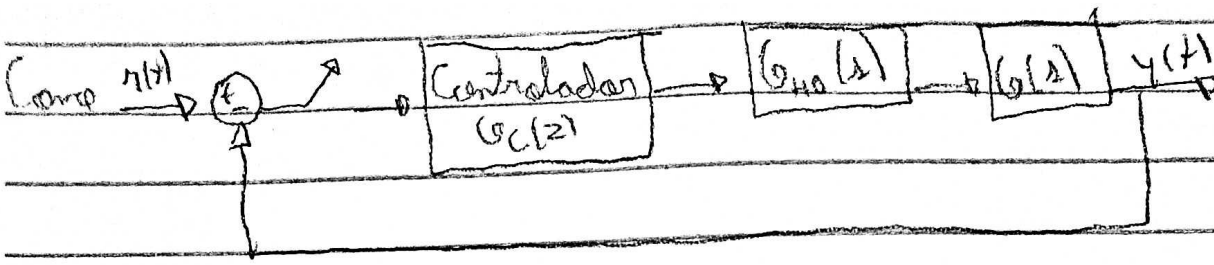
Sobressinal: $M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0,2231$ ou $22,31\%$

b) $G_c(s) = \frac{K_c}{s} = \frac{0,00089775}{s}$

- Regra retangular para frente: $s = \frac{z-1}{T} = \frac{z-1}{1,5}$ para $T = 1,5s$

$$G_c(z) = \frac{0,0013}{z-1}$$

- Regra retangular para trás: $s = \frac{z-1}{Tz} \Rightarrow G_c(z) = \frac{0,0013z}{z-1}$

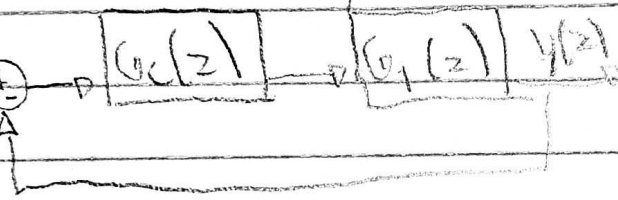


$$\frac{G(z)}{1} = Z\left\{G_{H0}(s).G(s)\right\} = (1-z^{-1})\,Z\left\{\frac{G(s)}{s}\right\} = (1-z^{-1}).Z\left\{\frac{15}{s(s+0,1)}\right\}$$ → Expansão Parcial

$$\frac{15}{s(s+0,1)} = \frac{A}{s} + \frac{B}{s+0,1} \Rightarrow A=150 \text{ e } B=-150 \Rightarrow \frac{150}{s} - \frac{150}{s+0,1}$$

$$Z\left\{\frac{150}{s} - \frac{150}{s+0,1}\right\} = \frac{150z}{z-1} - \frac{150z}{z-e^{-0,15}} \Rightarrow (1-z^{-1}).\left(\frac{150z}{z-1} - \frac{150z}{z-e^{-0,15}}\right)$$

$$\Rightarrow \frac{(z-1)}{z}.\left(\frac{150z}{z-1} - \frac{150z}{z-e^{-0,15}}\right) \Rightarrow \frac{150 - 150(z-1)}{z-e^{-0,15}} = \frac{20,8938}{z-0,8607} = G_1(z)$$



G(z) da Malha fechada:

- Regra retangular para frente

$$G(z) = \frac{Y(z)}{R(z)} = \frac{G_c(z).G_1(z)}{1+G_c(z).G_1(z)} = \frac{\left(\frac{0,013}{z-1}\right).\left(\frac{20,8938}{z-0,8607}\right)}{1+\frac{0,013}{z-1}.\frac{20,8938}{z-0,8607}}$$

$$G(z) = \frac{0,02814}{(z-1)(z-0,8607)+0,02814} = \frac{0,02814}{z^2-1,861z+0,8888}$$

- Regra retangular para trás

$$G(z) = \frac{Y(z)}{R(z)} = \frac{0,02814z}{z^2-1,8326z+0,8607}$$

Usando roots([1 -1,861 0,888]) e roots([1 -1,8326 0,8607]) no Matlab

Usando as raízes do polinômio característico retro para frente $\Rightarrow z = 0,9305 \pm 0,1916i$

regra para trás $\Rightarrow z = 0,9165 \pm 0,1440$

Como $z = e^{sT} = e^{-\zeta w_n T} \cdot e^{j w_d T}$ com $w_d = w_n \sqrt{1-\zeta^2}$

Logo $|z| = e^{-\zeta w_n T}$

$\angle z = w_d T$

Logo retangular para frente: $z = 0{,}9305 \pm 0{,}1516i$

$|z| = 0{,}9428$

$\angle z = 0{,}1615 \text{ rad} = 9{,}2535°$

Assim: $e^{-\zeta w_n T} = 0{,}9428 \Rightarrow \ln$ nos dois lados

$\Downarrow$

$\zeta w_n T = 0{,}0589$

$\zeta w_n = 0{,}0393 \Rightarrow \zeta = \frac{0{,}0393}{w_n}$

e $w_d T = 0{,}1615$

$w_d = 0{,}1077$ $\quad w_d = w_n \sqrt{1-\zeta^2} = 0{,}1077$

$0{,}1077^2 = w_n^2 \cdot \left(1 - \frac{0{,}0393}{w_n^2}\right)$

$\Downarrow$

$w_n = 0{,}1146 \quad \zeta = 0{,}3429$

$t_p = \frac{\pi}{w_d} = 29{,}1789\,s$ e $M_p = 0{,}3177 = 31{,}77\%$

retangular para trás: $z = 0{,}9165 \pm 0{,}144i$

$|z| = 0{,}9277$

$\angle z = 0{,}1558$

Assim $\zeta w_n = 0{,}05 \Rightarrow \zeta = \frac{0{,}05}{w_n}$ e $w_d = 0{,}1048$

$w_n^2 = 0{,}05^2 + 0{,}1048^2 \Rightarrow w_n = 0{,}1153 \Rightarrow \zeta = 0{,}4337$

$t_p = 30{,}2464$ e $M_p = 0{,}2204 = 22{,}04\%$

Controle Digital - Questao 1 - P1

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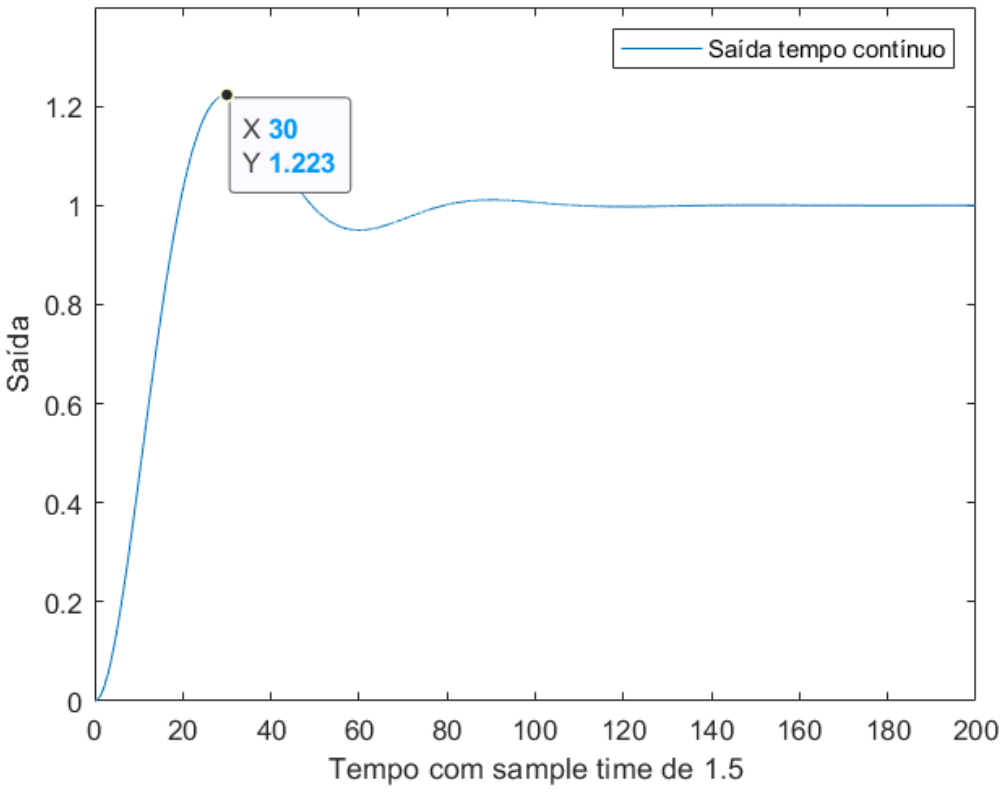
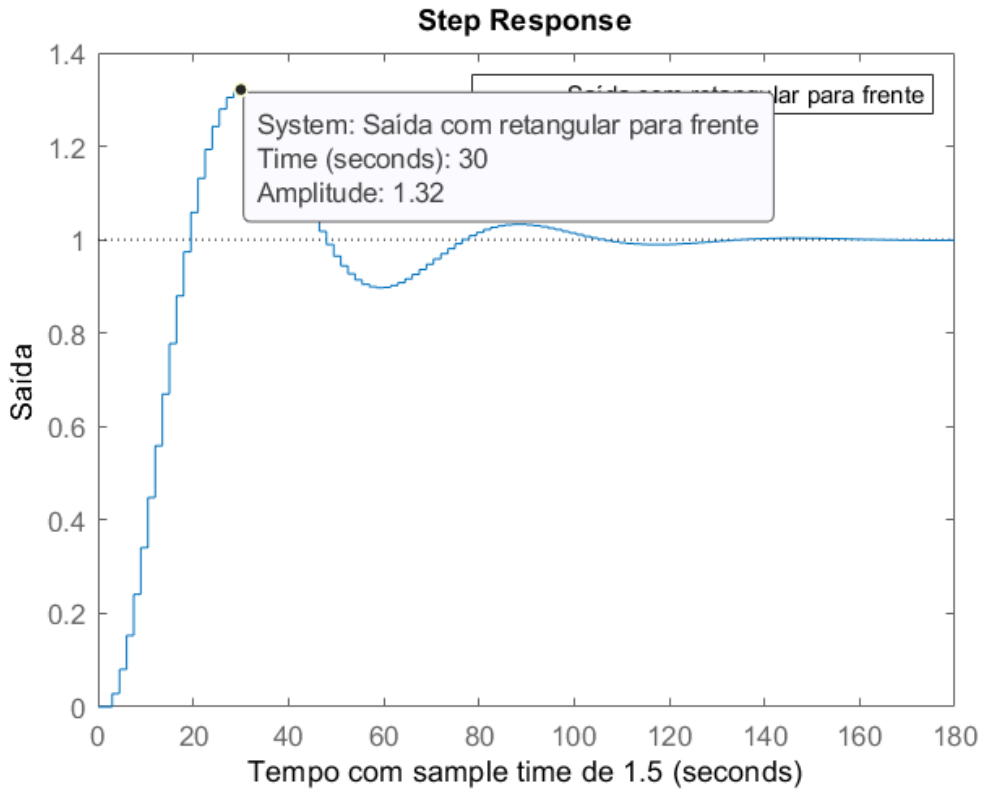
1.

```
wn=0.116;
zeta=0.4309;
np=[wn^2];
dp=[1 2*zeta*wn wn^2];
sys=tf(np,dp)
t=0:0.01:200;
y=step(sys,t);
plot(t,y)
xlabel('Tempo com sample time de 1.5')
ylabel('Saída')
legend('Saída tempo contínuo')
hold on
```



```
kc = (((pi^2)/(30^2))+0.05^2)/15
e = exp(-0.15)
G_c_frente = tf([kc*1.5],[1 -1],1.5)
G_c_tras = tf([kc*1.5 0],[1 -1],1.5)
syms s z
G_ghz= tf([15],[1 0.1]);
G_ghz_d = c2d(G_ghz,1.5)
g1 = G_ghz_d*G_c_frente
g1f = feedback(g1,1)
step(g1f)
xlabel('Tempo com sample time de 1.5')
ylabel('Saída')
legend('Saída com retangular para frente')
hold on
```




```
g2 = G_ghz_d*G_c_tras
g2f = feedback(g2,1)
step(g2f)
xlabel('Tempo com sample time de 1.5')
ylabel('Saída')
legend('Saída com retangular para tras')
hold on
```


Step Response
Saída com retangular para tras
System: Saída com retangular para tras
Time (seconds): 29.9
Amplitude: 1.22
Saída
Tempo com sample time de 1.5 (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
20
40
60
80
100
120
140

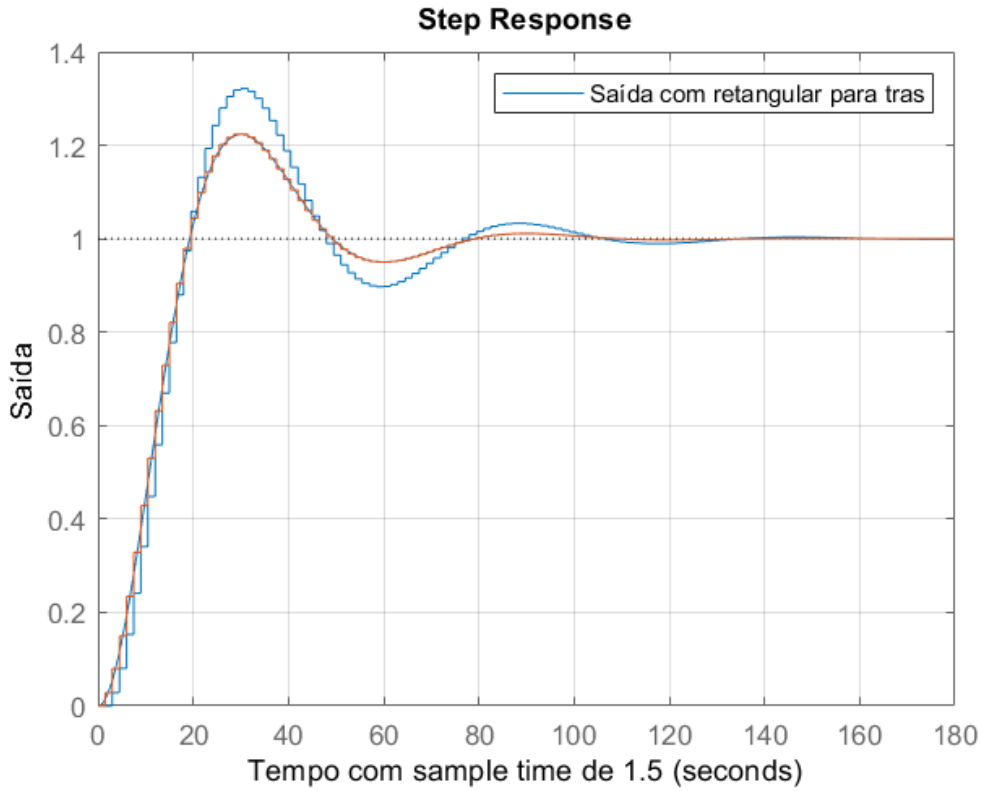


Figure 1. Todos Graficos juntos