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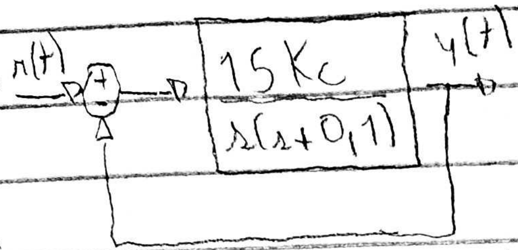
Prova 1 - Controle Digital

1) a)

Tempo contínuo ⇒



Malha fechada

$G(s) = \frac{15K_c}{s^2+0{,}1s+15K_c}$, como $t_p = 30s$ e $G(s) = \frac{w_n^2}{s^2+2\zeta w_n s+w_n^2}$

$t_p = \frac{\pi}{w_d} \Rightarrow w_d = \frac{\pi}{30} = w_n\sqrt{1-\zeta^2}$ e $w_n^2 = 15K_c$ e $2\zeta w_n = 0{,}1$

$\Downarrow$

$w_n = \frac{0{,}05}{\zeta}$

Substituindo em $w_d \Rightarrow \frac{\pi}{30} = w_n \cdot \sqrt{1 - \frac{0{,}05^2}{w_n^2}}$

quadrado nos dois lados: $\frac{\pi^2}{30^2} = w_n^2 \cdot \left(1 - \frac{0{,}05^2}{w_n^2}\right) \Rightarrow \frac{\pi^2}{30^2} + 0{,}05^2 = w_n^2$

$w_n^2 = 0{,}0135 \Rightarrow w_n = 0{,}1160$

$15K_c = 0{,}0135 \Rightarrow K_c = 0{,}00089775 \quad \zeta = 0{,}4309$

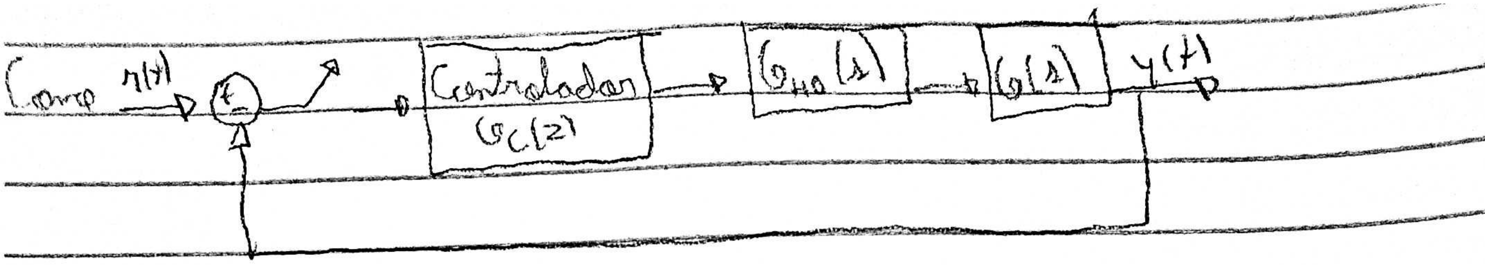
Sobressinal: $M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0{,}2231$ ou $22{,}31\%$

b) $G_c(s) = \frac{K_c}{s} = \frac{0{,}00089775}{s}$

- Regra retangular para frente: $s = \frac{z-1}{T} = \frac{z-1}{1{,}5}$ pois $T = 1{,}5s$

$G_c(z) = \frac{0{,}0013}{z-1}$

- Regra retangular para trás: $s = \frac{z-1}{Tz} \Rightarrow G_c(z) = \frac{0{,}0013z}{z-1}$

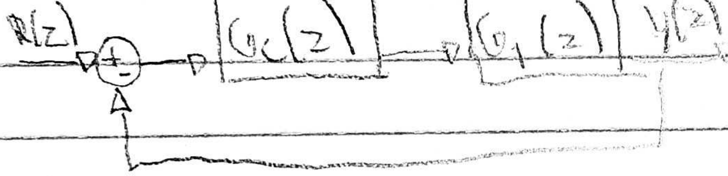


$$G_1(z) = Z\{G_{HO}(s)\cdot G(s)\} = (1-z^{-1})\, Z\left\{\frac{G(s)}{s}\right\} = (1-z^{-1})\cdot Z\left\{\frac{15}{s(s+0{,}1)}\right\}$$ ⇒ Frações Parciais

$$\frac{15}{s(s+0{,}1)} = \frac{A}{s} + \frac{B}{s+0{,}1} \Rightarrow A = 150 \text{ e } B = -150 \Rightarrow \frac{150}{s} - \frac{150}{s+0{,}1}$$

$$Z\left\{\frac{150}{s} - \frac{150}{s+0{,}1}\right\} = \frac{150z}{z-1} - \frac{150z}{z-e^{0{,}15}} \Rightarrow (1-z^{-1})\cdot\left(\frac{150z}{z-1} - \frac{150z}{z-e^{0{,}15}}\right)$$

$$\Rightarrow \frac{(z-1)}{z}\cdot\left(\frac{150z}{z-1} - \frac{150z}{z-e^{0{,}15}}\right) \Rightarrow 150 - \frac{150(z-1)}{z-e^{0{,}15}} = \frac{20{,}8938}{z-0{,}8607} = G_1(z)$$



G(z) de Malha Fechada:

• Regra retangular para frente

$$G(z) = \frac{Y(z)}{R(z)} = \frac{G_c(z)\cdot G_1(z)}{1+G_c(z)\cdot G_1(z)} = \frac{\left(\frac{0{,}0013}{z-1}\right)\cdot\left(\frac{20{,}8938}{z-0{,}8607}\right)}{1+\frac{0{,}0013}{z-1}\cdot\frac{20{,}8938}{z-0{,}8607}}$$

$$G(z) = \frac{0{,}02814}{(z-1)(z-0{,}8607)+0{,}02814} = \frac{0{,}02814}{z^2-1{,}861z+0{,}8888}$$

• Regra retangular para trás

$$G(z) = \frac{Y(z)}{R(z)} = \frac{0{,}02814z}{z^2-1{,}8326z+0{,}8607}$$

Usando roots([1 -1,861 0,888]) e roots([1 -1,8326 0,8607]) no Matlab

Chegamos nas raízes do polinômio característico regra para frente ⇒ $z = 0{,}9305 \pm 0{,}1916i$

regra para trás ⇒ $z_1 = 0{,}9165 \pm 0{,}1440$

Como $z = e^{sT} = e^{-\zeta w_n T} \cdot e^{j w_d T}$ com $w_d = w_n\sqrt{1-\zeta^2}$

Logo $|z| = e^{-\zeta w_n T}$

$\angle z = w_d T$

Logo retangular para frente: $z = 0{,}9305 \pm 0{,}1516 i$

$|z| = 0{,}9428$

$\angle z = 0{,}1615 \text{ rad} = 9{,}2535°$

Assim: $e^{-\zeta w_n T} = 0{,}9428 \Rightarrow \ln$ nos dois lados

$\Downarrow$

$\zeta w_n T = 0{,}0589$

$\zeta w_n = 0{,}0393 \Rightarrow \zeta = \frac{0{,}0393}{w_n}$

e $w_d T = 0{,}1615$

$w_d = 0{,}1077$ $\quad w_d = w_n\sqrt{1-\zeta^2} = 0{,}1077$

$0{,}1077^2 = w_n^2 \cdot \left(1 - \frac{0{,}0393}{w_n^2}\right)$

$\Downarrow$

$w_n = 0{,}1146$ $\zeta = 0{,}3429$

$t_p = \frac{\pi}{w_d} = 29{,}1789 s$ e $M_p = 0{,}3177 = 31{,}77\%$

retangular para trás: $z = 0{,}9165 \pm 0{,}144 i$

$|z| = 0{,}9277$

$\angle z = 0{,}1558$

Assim $\zeta w_n = 0{,}05 \Rightarrow \zeta = \frac{0{,}05}{w_n}$ e $w_d = 0{,}1048$

$w_n^2 = 0{,}05^2 + 0{,}1048^2 \Rightarrow w_n = 0{,}1153 \Rightarrow \zeta = 0{,}4337$

$t_p = 30{,}2464$ e $M_p = 0{,}2204 = 22{,}04\%$