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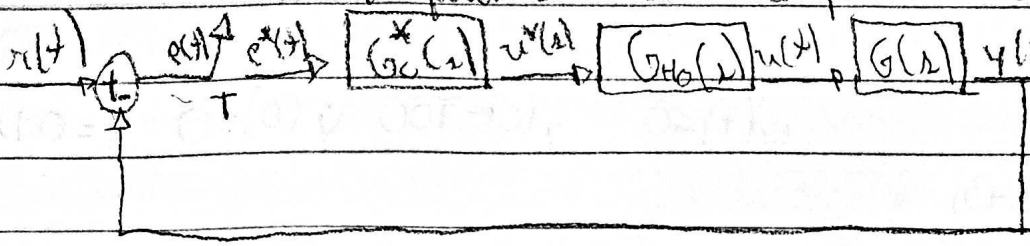
4º questionário de acompanhamento



$G_c(s) = K_c > 0$

$T = 0,1\ s$

1) $Z\{G_{H0}(s)G(s)\} = (1-z^{-1})\,Z\left\{\frac{G(s)}{s}\right\}$ $\qquad G(s) = \frac{1}{s(s+10)}$

$= (1-z^{-1})\,Z\left\{\frac{1}{s^2(s+10)}\right\}$ = Fração parcial:

$\frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+10} = \frac{1}{s^2(s+10)} \Rightarrow A = \frac{1}{10},\ B = -\frac{1}{100},\ C = \frac{1}{100} \Rightarrow$

$= Z\left\{\frac{1}{10s^2} - \frac{1}{100s} + \frac{1}{100(s+10)}\right\} \quad T = 0,1\ s$

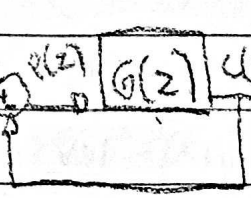
$= \frac{Tz}{10(z-1)^2} - \frac{z}{100(z-1)} + \frac{z}{100(z-e^{-10T})}$

$Z\{G_{H0}(s)G(s)\} = \frac{(z-1)}{z}\left(\frac{z}{100(z-1)^2} - \frac{z}{100(z-1)} + \frac{z}{100(z-e^{-1})}\right)$

$= \frac{1}{100(z-1)} - \frac{1}{100} + \frac{(z-1)}{100(z-e^{-1})} = \frac{0,3679z + 0,2642}{10^2(z-1)(z-e^{-1})}$

Plano direto:

$G(z) = \frac{K_c(0,003679z + 0,002642)}{z^2 - 1,3679z + 0,3679}$



Polinômio característico:

$1 + G(z) = 0 \Rightarrow K_c = \frac{-z^2 + 1,3679z - 0,3679}{0,003679z + 0,002642} \Rightarrow \frac{dK_c}{dz} = 0 \Rightarrow$

$-z^2 \cdot 0,003679 - 0,005284z + 0,004967 = 0$ $\qquad$ Matlab: roots([0,003679...])

$z = -2,0841$ e $z = 0,6478$

$\Rightarrow K_c\big|_{z=-2,0841} = 1504,8$ e $K_c\big|_{z=0,6478} = 19,6171$ e $K_c\big|_{z=-1} = 2638,2$

Cruzamento com eixo imaginário: $z = jw$

$$1 + G(z) = 0 = z^2 + (K_c \cdot 0{,}003679 - 1{,}3679)z + (0{,}3679 + 0{,}002642 K_c)$$

$$\begin{cases} -w^2 + 0{,}3679 + 0{,}002642 K_c = 0 \\ 0{,}003679 K_c - 1{,}3679 = 0 \end{cases} \Rightarrow K_c = 371{,}813 \quad w = \pm 1{,}162$$

$$G(z) = \frac{K_c(0{,}003679 z + 0{,}002642)}{(z-1)(z-e^{-1})}$$

Condição do ângulo

$$\angle G(z) = \angle 0{,}003679 z + 0{,}002642 - \angle z - 1 - \angle z - e^{-1} = 180$$

$z = \sigma + jw$

$$= \angle((0{,}003679\sigma + 0{,}002642) + 0{,}003679 w j - \angle \sigma + jw - 1 - \angle \sigma + jw - e^{-1} = 180$$

$$= \arctan\left(\frac{0{,}003679 w}{0{,}003679\sigma + 0{,}002642}\right) - \arctan\left(\frac{w}{\sigma - 1}\right) - \arctan\left(\frac{w}{\sigma - e^{-1}}\right) = 180^\circ$$

Tangente dos lados

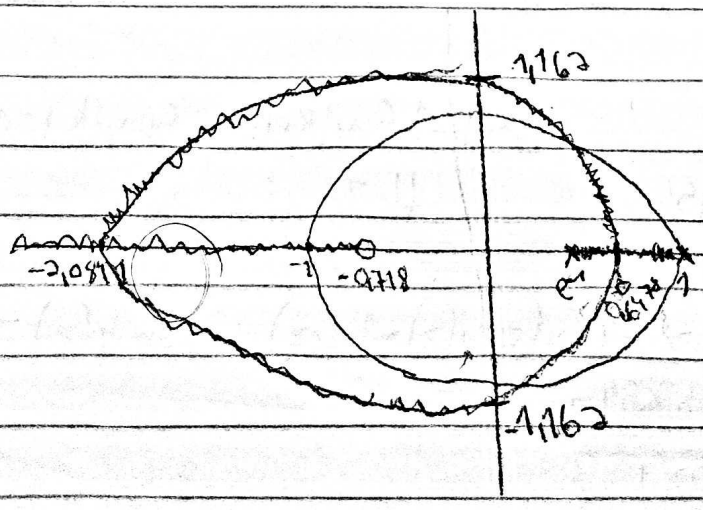
$$\frac{\frac{0{,}003679 w}{0{,}003679\sigma + 0{,}002642} - \frac{w}{\sigma - 1}}{1 + \frac{0{,}003679 w}{0{,}003679\sigma + 0{,}002642} \cdot \frac{w}{\sigma - 1}} = \tan 180^\circ + \frac{w}{\sigma - e^{-1}}$$

$$1 + \tan 180^\circ \cdot \frac{w}{\sigma - e^{-1}}$$

↓ → feita em rascunho e Matlab

$$w^2 + \sigma^2 + 1{,}4363\sigma = 1{,}3591$$

$$w^2 + (\sigma + 0{,}7181)^2 = 1{,}3729^2 = \text{equação da circunferência}$$



b) $w^2 + (\sigma + 0{,}7181)^2 = 1{,}8848$ $\quad w = e^{-0{,}1\zeta w_n} \cdot \operatorname{sen}(0{,}1 w_d)$ $\quad w_d = \pi,\ T_p = 1s$

$\sigma = e^{-0{,}1\zeta w_n} \cdot \cos(0{,}1 w_d)$

$$e^{-0{,}2\zeta w_n} \operatorname{sen}^2(0{,}1 w_d) + \left(e^{-0{,}2\zeta w_n} \cdot \cos^2(0{,}1 w_d) + 2 \cdot 0{,}7181 \cdot e^{-0{,}1\zeta w_n} \cdot \cos(0{,}1 w_d) + 0{,}7181^2\right) = 1{,}8848$$

$$e^{-0{,}2\zeta w_n} \left(\underbrace{\operatorname{sen}^2(0{,}1 w_d) + \cos^2(0{,}1 w_d)}_{1}\right) + 1{,}4362\, e^{-0{,}1\zeta w_n} \cdot \underbrace{\cos(0{,}1\pi)}_{0{,}9511} + 0{,}5157 = 1{,}8848$$

$$e^{-0{,}2\zeta w_n} + 1{,}4362 \cdot e^{-0{,}1\zeta w_n} \cdot 0{,}9511 = 1{,}3691$$

$$e^{-0{,}2\zeta w_n} + 1{,}3656\, e^{-0{,}1\zeta w_n} = 1{,}3691$$

$e^{-0{,}1\zeta w_n} = x$

$$x^2 + 1{,}3656x - 1{,}3691 = 0$$

$x = -2{,}0307$

$x = 0{,}6742$

$e^{-0{,}1\zeta w_n} = -2{,}0307$, logaritmo imaginário

$e^{-0{,}1\zeta w_n} = -0{,}6743$, ln não existe

$-0{,}1\zeta w_n = -0{,}3943$

$\zeta w_n = 3{,}9423$

Logo $z = e^{sT} = e^{-3{,}9423T} \cdot e^{j w_d T} = e^{-0{,}3943} \cdot e^{j 0{,}1\pi}$

$|z| = 0{,}6743 \quad \angle z = 0{,}1\pi \quad , |z| \cdot \cos(0{,}1\pi) + |z| \cdot \operatorname{sen}(0{,}1\pi) \cdot i$

$z = 0{,}6421 + 0{,}2083\, i$

$$\left|G(z)\right| = 1 \Rightarrow \left| \frac{K_c(0{,}00879 z + 0{,}00264)}{|z^2 - 1{,}3679 z + 0{,}3679)|} \right|_{z = 0{,}6421 + 0{,}2083 i} = 1$$

$$\frac{K_c \cdot 0{,}0051}{0{,}1425} = 1 \Rightarrow K_c = 28{,}1475$$

c) $t_s = 1,25\ s \Rightarrow t_s = \frac{4}{\zeta \omega_n} \Rightarrow \zeta\omega_n = 3,2$

$s = -3,2 \pm \omega_d j \Rightarrow z = e^{-0,32} \cdot e^{j\omega_d 2\pi} = u + jv \qquad |z| = e^{-0,32}$

$|z| = \sqrt{u^2+v^2} = e^{-0,32} \Rightarrow u^2+v^2 = 0,5273$

$v^2 = 0,5273 - u^2$

$z = \sigma + \omega j = u + vj \Rightarrow \sigma = u \ e \ v = \omega$

equação da circunferência:

$$v^2 + (\sigma + 0,7781)^2 = 1,8848$$

$$\Downarrow$$

$$(0,5273 - u^2) + (u + 0,7781)^2 = 1,8848$$

$$2u \cdot 0,7781 = 0,8418$$

$$u = 0,5403 \Rightarrow v = 0,4286$$

$$z = 0,5403 + 0,4286j$$

$$|G(z)| = 1 \Rightarrow K_c \left| \frac{0,00367z + 0,00262}{z^2 - 1,4439z + 0,5679} \right|_{z = 0,5403 + 0,4286j} = 1$$

$$\frac{K_c \cdot 0,0051}{0,2866} = 1 \Rightarrow K_c = 56,1872$$

2) $T=1_s \quad G(s)=\frac{e^{-s}}{s+2} \qquad \zeta=0,6$

$e_{ss}=0$ para degrau

$n=8$

$$G(z)=Z\{G_{H0}(s)G(s)\}=(1-z^{-1}).z^{-1}.Z\left\{\frac{1}{s(s+2)}\right\}$$

$$\frac{1}{s(s+2)}=\frac{A}{s}+\frac{B}{s+2} \Rightarrow A(s+2)+sB=1 \Rightarrow \frac{1}{2s}-\frac{1}{2(s+2)}$$

$$A=\frac{1}{2} \quad B=-\frac{1}{2}$$

$$G(z)=\frac{(z-1)}{z}\cdot\frac{1}{2}\cdot\left(\frac{z}{z-1}-\frac{z}{z(z-e^{-2})}\right)$$

$$G(z)=\frac{1}{2}\left[\frac{(z-1)}{z-1}-\frac{(z-e^{-2})+(z-1)}{z(z-e^{-2})}\right]=\frac{-e^{-2}+1}{2z(z-e^{-2})}=\frac{0,4323}{z(z-0,1353)}$$

$$n=\frac{2\pi}{w_dT}=\frac{2\pi}{w_d}=\frac{2\pi}{w_n\sqrt{1-\zeta^2}} \Rightarrow \frac{2\pi}{w_n\sqrt{1-0,6^2}}=8 \Rightarrow 2\pi=8w_n.0,8$$

$$w_n=0,9817$$

$$\zeta=0,6$$

$$w_d=w_n\sqrt{1-\zeta^2}=0,7854$$

$$w_n.\zeta=0,5890$$

Logo $s=-\zeta w_n \pm j\,w_d=-0,589+0,7854j$

$$z=e^{-0,589}.e^{j0,7854}=e^{-0,589}(\cos(0,7854)+i.\sin(0,7854))$$

$$=0,3924+0,3924i$$

$$\angle G(z)=-\theta_1-\theta_2 \qquad \theta_1=\text{atan}\frac{0,3924}{0,3924-0,1353}=56,7672$$

$$\theta_2=45°$$

$$\angle G(z)=-101,7672 \Rightarrow \text{Logo } \angle G_c=-78,2328$$

Considerando $G_c=\frac{K_c(z-z_c)}{z-1}$ para erro estacionário $=0$ para degrau

$$\angle G_c=\phi_c-\theta_c=\phi_c-180+\text{atan}\left(\frac{0,3924}{1-0,392}\right)=-78,2328$$

$$\phi_c=68,9720$$

$\phi_c = 68,917 \Rightarrow \tan \frac{0,3934}{0,3924 - z_c} = 68,917$ K.

$2,5932 \cdot 0,3924 - 2,5932 z_c = 0,3934$

$\therefore z_c = 0,2411$

Logo $G_c(z) = \frac{K_c (z - 0,2411)}{(z - 1)}$

$G(z) \cdot G_c(z) = \frac{K_c\, 0,4323\, (z - 0,2411)}{z(z - 0,1353)(z - 1)}$

$\left| \frac{K_c\, 0,4323 \cdot (z - 0,2411)}{z(z-1)(z-0,1353)} \right|_{z = 0,3934 + 0,3934i} = 1$

$\frac{K_c \cdot 0,4323 \cdot 0,4506}{0,5549 \cdot 0,7333 \cdot 0,4691} = 1 \Rightarrow K_c = 1,0355$

Com isso $G_c(z) = 1,0355 \cdot \frac{(z - 0,2411)}{(z - 1)}$