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Questionário 2 - Controle Digital

1) $y[k] = 0{,}5 - 0{,}5.(0{,}6)^k,\ k \geq 0 \quad T = 1s \quad u(t) \Rightarrow U(z) = \dfrac{z}{z-1}$

a) Usando $u(kT) \Leftrightarrow \dfrac{z}{z-1}$ e $a^k u[k] \Longleftrightarrow \dfrac{z}{z-a}$

$$Y(z) = Z\{y[k]\} = \frac{0{,}5.z}{z-1} - \frac{0{,}5.z}{z-0{,}6} = \frac{0{,}5.z.(z-0{,}6) - 0{,}5z.(z-1)}{(z-1)(z-0{,}6)}$$

$$= \frac{0{,}5z((z-0{,}6)-(z-1))}{(z-1)(z-0{,}6)} = \frac{z.0{,}4.0{,}5}{(z-1)(z-0{,}6)} = \frac{0{,}2z}{(z-1)(z-0{,}6)}$$

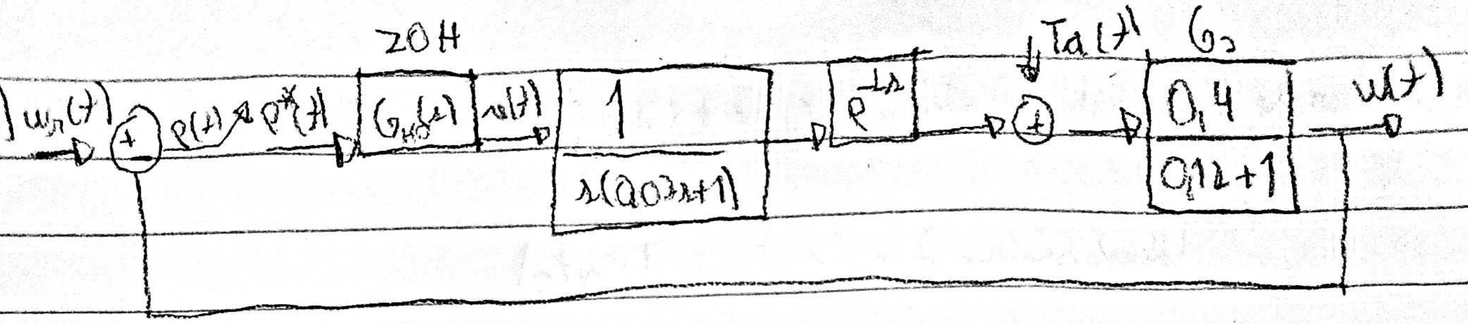
b) $U(z) = \dfrac{z}{z-1} \Rightarrow \dfrac{Y(z)}{U(z)} = \dfrac{0{,}2\cancel{z}}{\cancel{(z-1)}(z-0{,}6)} \cdot \dfrac{\cancel{(z-1)}}{\cancel{z}} = \dfrac{0{,}2}{z-0{,}6} = G(z)$

Frações parciais

c) $\dfrac{Y(z)}{U(z)} = \dfrac{0{,}2}{z-0{,}6} \Rightarrow \dfrac{0{,}2}{z-0{,}6} = \dfrac{Az}{z-0{,}6} + B \Rightarrow A = \dfrac{1}{3}, B = -\dfrac{1}{3} \Rightarrow \dfrac{1}{3}\dfrac{z}{(z-0{,}6)} - \dfrac{1}{3}$

$$Z^{-1}\left\{\frac{Y(z)}{U(z)}\right\} = \frac{0{,}6^k}{3} - \frac{\delta[k]}{3} \Rightarrow \frac{Y[k]}{U[k]} = \frac{0{,}6^k}{3} - \frac{\delta[k]}{3}$$

2)



$L = 0{,}1s$, $T = 0{,}05s$ $\quad G_{HO} = \frac{1-e^{-sT}}{s} \quad G_2 = \frac{0{,}4}{0{,}1s+1}$

a) Considerando $T_d(t)=0$, $G(s) = \frac{1}{s(0{,}02s+1)} \cdot e^{-0{,}1s} \cdot \frac{0{,}4}{0{,}1s+1}$

Malha Aberta: $E^*(s) \cdot G_{HO}(s) \cdot G(s) = \Omega_1(s) \Rightarrow E^*(s) \cdot (G_{HO}G(s))^* = \Omega_1^*(s)$

$$\Rightarrow \frac{\Omega_1^*(s)}{E^*(s)} = G_{HO}G(s)^* \Rightarrow \frac{\Omega_1(z)}{E(z)} = (1-z^{-1}) \cdot Z\left\{\left[\frac{G^*(s)}{s}\right]\right\}$$

$$G^*(s) = \frac{e^{-0{,}1s} \cdot 0{,}4}{s(0{,}1s+1)(0{,}02s+1)} \Rightarrow G^*(s) = \frac{e^{-0{,}1s} \cdot 0{,}4}{0{,}002s^3+0{,}12s^2+s}$$

Usando c2d do Matlab com o metodo 'zoh' chegamos em:

$$\Omega_1(z) = \underbrace{z^{-2} \cdot \left(\frac{0{,}002162z^2+0{,}004567z+0{,}0004943}{z^3-1{,}689z^2+0{,}7384z-0{,}04979}\right)}_{G_1^*(z) = G_{HO} \cdot \text{Motor CC} \cdot \text{Atraso} \cdot \text{Motor do automóvel.}} \cdot E(z) \quad \text{com } T_d = 0$$

Com $\omega_r(t)=0$, $\omega(t) = -e^*(t)$

$$E^*(s) \cdot G_{HO}(s) \cdot G(s) + T_d(s) \cdot G_2 = \Omega_2(s)$$

$$\Downarrow$$

$$-\Omega_2^*(s) \cdot G_{HO} \cdot G(s)^* + T_dG_2(s)^* = \Omega_2^*(s) \Rightarrow \Omega_2(s) = \frac{T_dG_2(s)^*}{1+G_{HO}G(s)^*}$$

Sobreposição: $\Omega^*(s) = G_{MF}(s)^* \cdot \Omega_r^*(s) + \frac{T_dG_2(s)^*}{1+G_{HO}G(s)^*}$

$\Omega^*(s) = \Omega_1^*(s) + \Omega_2^*(s)$

Como $E^*(s) = \Omega_r^*(s) - \Omega_1^*(s)$ e $E^*(s) \cdot G^*(s) = \Omega_1^*(s)$

Logo $(\Omega_r^*(s) - \Omega_1^*(s)) \cdot G^*(s) = \Omega_1^*(s) \Rightarrow \Omega_r^*(s) \cdot G^*(s) = \Omega_1^*(s)(1 + G^*(s))$

Então em malha fechada com $T_d = 0$: $\dfrac{\Omega_1^*(s)}{\Omega_r^*(s)} = \dfrac{G^*(s)}{1 + G^*(s)} = G_{1MF}^*(s) \Rightarrow$

$\Rightarrow \Omega_1^*(s) = G_{1MF}^*(s) \cdot \Omega_r^*(s)$

Como teremos $\frac{\Omega(z)}{\Omega_r(z)}$ e $\frac{\Omega(z)}{E(z)}$ e $\Omega^*(s) = G_{1MF}^*(s) \cdot \Omega_r^*(s) + \frac{Td\,G_2(s)^*}{(1+G_{H0}G(s)^*)}$

Dividindo os dois lados por $\Omega_r^*(s)$:

$$\frac{\Omega^*(s)}{\Omega_r^*(s)} = G_{1MF}^*(s) + \frac{Td\,G_2(s)^*}{(1+G_{H0}G(s)^*) \cdot \Omega_r^*(s)}, \quad \frac{\Omega(z)}{\Omega_r(z)} = G_{1MF}(z) + Z\left\{\frac{Td\,G_2^*(s)}{1+G_{H0}G(s)^* \cdot \Omega_r^*(s)}\right\}$$

Dividindo os dois lados por $E^*(s)$

$$\frac{\Omega^*(s)}{E^*(s)} = G_{1MF}^*(s) \cdot \frac{\Omega_r^*(s)}{E^*(s)} + \frac{Td\,G_2^*(s)}{(1+G_{H0}G(s)^*) \cdot E^*(s)}, \quad E^*(s) = \Omega_r^*(s) - \Omega^*(s)$$

$$E^*(s) = \Omega_r^*(s) - G_{1MF}^*(s) \cdot \Omega_r^*(s) - \frac{Td\,G_2(s)^*}{(1+G_{H0}G(s)^*)}$$

$$* G_{1MF}(z) = \frac{0{,}002162z^2 + 0{,}004567z + 0{,}0004943}{z^5 - 1{,}689z^4 + 0{,}7384z^3 - 0{,}04762z^2 + 0{,}004567z + 0{,}0004943}$$

Feito no Matlab

feedback $(G_1^*(z))$

b) Com $w_r = 0$ e $Td(s) = \frac{Td}{s}$

$$G^*(s) = \frac{e^{-0{,}1s} \cdot 0{,}4}{0{,}002s^3 + 0{,}12s^2 + s}$$

$$\Omega^*(s) = \frac{\frac{Td}{s} \cdot \frac{0{,}4}{(0{,}1s+1)}}{1 + G^*(s)} \Rightarrow \Omega(z) = \frac{Td \cdot \left(\frac{0{,}004261z + 0{,}003608}{z^2 - 1{,}607z + 0{,}6065}\right)}{1 + \left(z^{-2} \cdot \left(\frac{0{,}002162z^2 + 0{,}004567z + 0{,}0004943}{z^3 - 1{,}689z^2 + 0{,}7384z - 0{,}04979}\right)\right)}$$

$$w(kT), k \to \infty \Rightarrow \lim_{z \to 1} (1 - z^{-1}) \cdot \Omega(z)$$

$$w(kT) \text{ com } k \to \infty \Rightarrow \frac{Td \cdot 0{,}0079}{0{,}4 \cdot 0{,}0072} \cong 2{,}7431\,Td$$