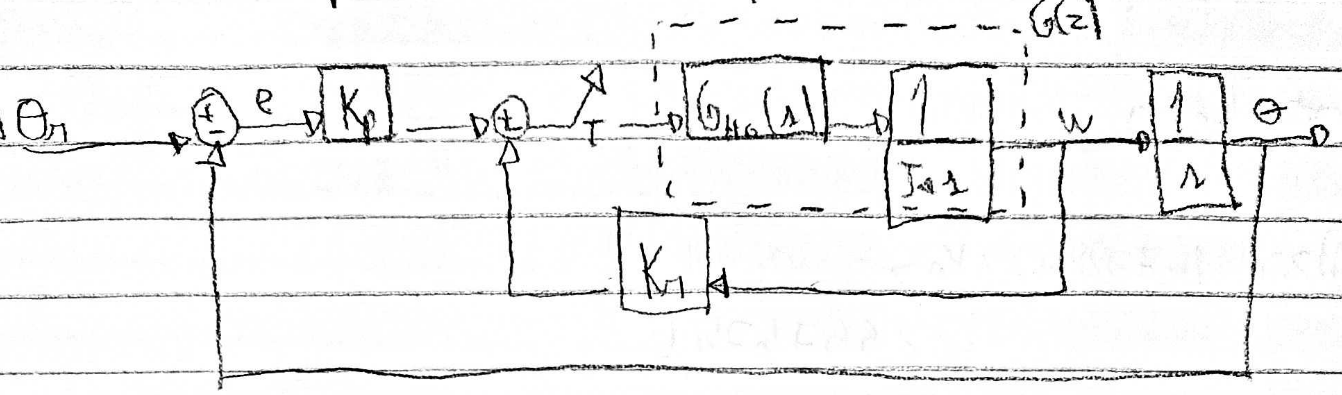
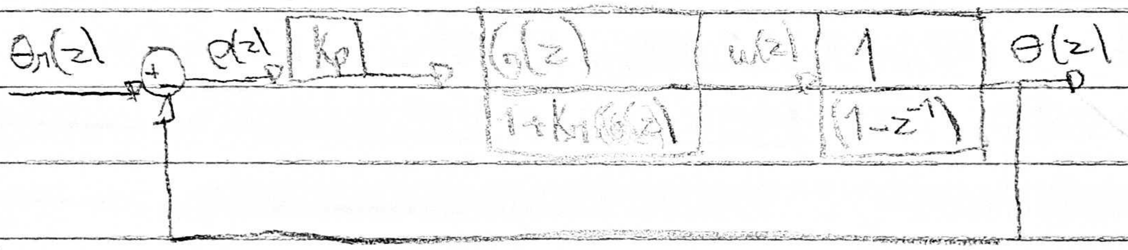
João Vítor de Carvalho Mota 160127823

3º questionário de acompanhamento

1) a)



$$G(z) = (1 - z^{-1})\, Z\left\{\frac{1}{J_w s^2}\right\} \Rightarrow \frac{w(z)}{K_p\, e(z)} = \frac{G(z)}{1 + K_r\, G(z)}, \quad G(z) = \frac{(1-z^{-1})\, T z^{-1}}{J_w (1-z^{-1})^2}$$



$$e(z) = \theta_r - \frac{K_p\, G(z)\, e(z)}{(1 + K_r\, G(z))(1 - z^{-1})}$$

$$e(z) = \frac{\theta_r}{\left(1 + \frac{K_p\, G(z)}{(1 + K_r\, G(z))(1 - z^{-1})}\right)} \Rightarrow K_v = \frac{1}{T} \lim_{z \to 1} (1 - z^{-1}) \cdot \frac{K_p\, G(z)}{(1 + K_r\, G(z))(1 - z^{-1})}$$

$$K_v = \frac{K_p}{T} \lim_{z \to 1} \frac{G(z)}{1 + K_r\, G(z)} = \frac{K_p}{T} \lim_{z \to 1} \frac{T z^{-1}}{J_w (1 - z^{-1}) + K_r\, T\, z^{-1}} = \frac{K_p}{K_r\, T} = K_v$$

Como $K_v = 10 \Rightarrow K_p = 10 K_r\, T$

b) $T = 0{,}1\,s \Rightarrow K_p = K_r$, $G_1(z) = \frac{K_p\, G(z)}{(1 - z^{-1})(1 + K_r\, G(z))}$, $G_2(z) = \frac{G_1(z)}{1 + G_1(z)}$

$$G_2(z) = \frac{K_p\, T\, z}{J_w z^2 + (K_p T + K_r T - 2 J_w) z + J_w - K_r T}, \quad K_p = K_r \text{ e } J_w = 41822$$

$P(z) = 41822z^2 + (0{,}2K_p - 83644)z + 41822 - 0{,}1K_p$

Critério de Jury

1) $|41822 - 0{,}1K_p| < 41822 \Rightarrow 0 < K_p < 836440$

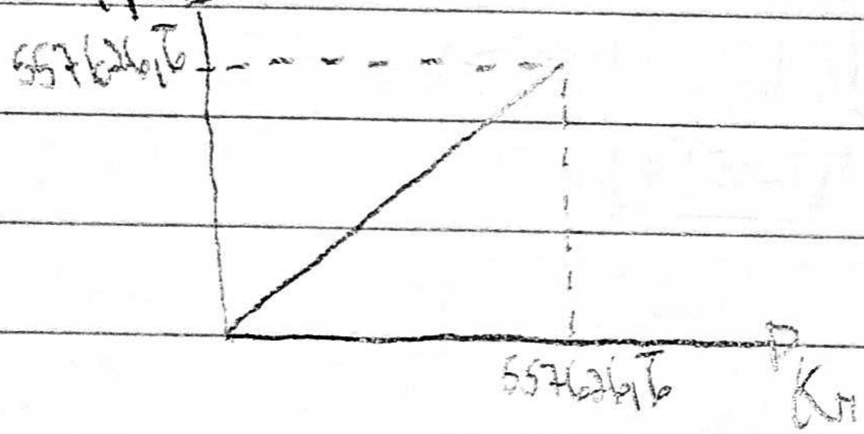
2) $P(1) > 0$, $K_p > 0$

3) $P(-1) > 0$, $167288 - 0{,}3K_p > 0$

$K_p < 557626{,}\bar{6}$

4) $n = 2$ logo não preciso da tabela.

Logo $0 < K_p = K_{rl} < 557626{,}\bar{6}$



2) $G(s) = \frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$, $\omega_n > 0$ e $0 < \xi < 1$

a) $G_c(z)\Big|_{s=\frac{z-1}{T}}$ Regra retangular p/ frente

$$G_c(z) = \frac{\omega_n^2}{\frac{z^2}{T^2} + \left(\frac{2\xi\omega_n}{T} - \frac{2}{T^2}\right)z + \omega_n^2 + \frac{1}{T^2} - \frac{2\xi\omega_n}{T}} = \frac{Y(z)}{U(z)}$$

multiplicando por $\frac{z^{-2}}{z^{-2}}$ $\Rightarrow$ $\frac{\omega_n^2 z^{-2}}{\left(\omega_n^2 + \frac{1}{T^2} - \frac{2\xi\omega_n}{T}\right)z^{-2} + \left(\frac{2\xi\omega_n}{T} - \frac{2}{T}\right)z^{-1} + \frac{1}{T^2}} = \frac{Y(z)}{U(z)}$

$$\frac{y[K]}{T^2} = y[K-1]\cdot\left(\frac{2}{T} - \frac{2\xi\omega_n}{T}\right) + y[K-2]\left(\frac{2\xi\omega_n}{T} - \frac{1}{T^2} - \omega_n^2\right) + U[K-2]\cdot\omega_n^2$$

Critério de Jury

1) $\left|\frac{1}{T^2}\right| < \omega_n^2 + \frac{1}{T^2} - \frac{2\xi\omega_n}{T} \Rightarrow \frac{1}{T^2} < \omega_n^2 + \frac{1}{T^2} - \frac{2\xi\omega_n}{T} = 1 < \omega_n^2 T^2 + 1 - 2\xi\omega_n T$

$0 < \omega_n^2 T^2 - 2\xi\omega_n T$

$2\xi\cancel{\omega_n T} < \omega_n^2 T^2 \Rightarrow 2\xi < \omega_n T \Rightarrow \frac{2\xi}{\omega_n} < T$

2) $\frac{1}{T^2} + \frac{2\xi\omega_n}{T} - \frac{2}{T} + \omega_n^2 + \frac{1}{T^2} - \frac{2\xi\omega_n}{T} > 0 \Rightarrow 1 + 2\xi\omega_n T - 2T + \omega_n^2 T^2 + 1 - 2\xi\omega_n T > 0$

$\omega_n^2 T^2 - 2T + 2 > 0$

3) $\frac{1}{T^2} - \frac{2\xi\omega_n}{T} + \frac{2}{T} + \omega_n^2 + \frac{1}{T^2} - \frac{2\xi\omega_n}{T} > 0 \Rightarrow 1 - 2\xi\omega_n \cdot T + 2\cdot T + \omega_n^2 T^2 - 2\xi\omega_n T + 1 > 0$

$2 - (4\xi\omega_n - 2)T + \omega_n^2 T^2 > 0$

b) $G_c(s)\Big|_{s=\frac{z-1}{Tz}}$ Regra retangular p/ trás

$$G_c(z) = \frac{\omega_n^2}{\frac{z^{-2}}{T^2} + \left(\frac{-2}{T^2} - \frac{2\zeta\omega_n}{T}\right)z^{-1} + \frac{1}{T^2} + \frac{2\zeta\omega_n}{T} + \omega_n^2} = \frac{Y(z)}{U(z)}$$

$$Y[K]\cdot\left(\frac{1}{T^2} + \frac{2\zeta\omega_n}{T} + \omega_n^2\right) - Y[K-1]\cdot\left(\frac{2}{T^2} + \frac{2\zeta\omega_n}{T}\right) + Y[K-2]\cdot\frac{1}{T^2} = U[K]\,\omega_n^2$$

multiplicando $\frac{z^2}{z^2} \Rightarrow$
$$\frac{\omega_n^2 z^2}{\left(\frac{1}{T^2} + \frac{2\zeta\omega_n}{T} + \omega_n^2\right)z^2 + \left(\frac{-2}{T^2} - \frac{2\zeta\omega_n}{T}\right)z + \frac{1}{T^2}}$$

Critério de Jury

1) $\left|\left(\frac{1}{T^2} + \frac{2\zeta\omega_n}{T} + \omega_n^2\right)\right| < \frac{1}{T^2} \Rightarrow 1 + 2\zeta\omega_n T + \omega_n^2 T^2 < 1$

$$\omega_n^2 T^2 < -2\zeta\omega_n T$$

$$T < \frac{-2\zeta}{\omega_n}, \text{ não é possível.}$$

logo não é estável