# M4

October 26, 2020

## 0.1 MScFE 660 Case Studies in Risk Management (C19-S3)

### 0.1.1 October 25, 2020

### 0.1.2 Collaborative Review Task M4

**Questions:** 1.Visually analyze the covariance between various factors and identify the variance explained in principle components of these factors. Next, consider the ACF and PACF of the process and its square.

2.Using PCA provide a 2-dimensional representation of the weight-space of a set of linear models representing the covariance between our factors and the different benchmark portfolios. Comment on the distribution of the benchmark portfolios across the weight-space.

3.Using linear regression test for the significance of these factors, as per the original work of Fama and French, under the equation:

`ExpectedReturns = rf + 1(rm-rf) + 2SMB + 3HML + 4RMW + 5CMA`

The five-factor asset pricing model is an extension of the Fama and French three-factor asset pricing model that include profitability and investment factors:

`ExpectedReturns = rf + 1(rm-rf) + 2SMB + 3HML + 4RMW + 5CMA`

Where rf = riskfree return

`rm = return on value-weight market portfolio`

`SMB = return on portfolio of small stocks minus portfolio of big stocks`

`HML = return on portfolio of high minus low B/M stocks`

`RMW = return on portfolio of robust minus weak profitability stocks`

`CMA = return on portfolio of low minus high investment firms`

[280]:
```python
# import modules
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

```python
from sklearn.decomposition import PCA
from sklearn.preprocessing import RobustScaler, StandardScaler
from sklearn.pipeline import Pipeline
from sklearn import linear_model

from printdescribe import print2, describe2


from functools import reduce
from operator import mul

from statsmodels.graphics.tsaplots import plot_pacf, plot_acf
from statsmodels.regression.linear_model import OLS
from statsmodels.stats.stattools import durbin_watson
from statsmodels.stats.diagnostic import acorr_ljungbox


import holoviews as hv
import hvplot
import hvplot.pandas


import warnings
warnings.filterwarnings('ignore')
```

[2]:
```python
pd.core.common.is_list_like = pd.api.types.is_list_like
import pandas_datareader.data as web
```

[281]:
```python
np.random.seed(42)
hv.extension('bokeh')
```

[283]:
```python
%opts Curve[width=900 height=400] NdOverlay [legend_position='right']
```

[18]:
```python
# Download datasets
portfolios100 = web.DataReader('100_Portfolios_10x10_Daily', 'famafrench')
factors5 = web.DataReader('F-F_Research_Data_5_Factors_2x3_Daily', 'famafrench')
```

[19]:
```python
portfolios100['DESCR']
```

[19]:
```
'100 Portfolios 10x10 Daily\n--------------------------\n\nThis file was created
by CMPT_ME_BEME_RETS_DAILY using the 202008 CRSP database. It contains value-
weighted returns for the intersections of 10 ME portfolios and 10 BE/ME
portfolios. The portfolios are constructed at the end of June. ME is market cap
at the end of June. BE/ME is book equity at the last fiscal year end of the
prior calendar year divided by ME at the end of December of the prior year.
Missing data are indicated by -99.99 or -999. The break points use Compustat
```

firms plus the firms hand-collected from the Moodys Industrial, Transportation, Utilities, and Financials Manuals. The portfolios use Compustat firms plus the firms hand-collected from the Moodys Industrial, Transportation, Utilities, and Financials Manuals. Copyright 2020 Kenneth R. French\n\n  0 : Average Value Weighted Returns -- Daily (1220 rows x 100 cols)\n  1 : Average Equal Weighted Returns -- Daily (1220 rows x 100 cols)\n  2 : Number of Firms in Portfolios (1220 rows x 100 cols)\n  3 : Average Firm Size (1220 rows x 100 cols)'

[20]:
```
factors5['DESCR']
```

[20]:
```
'F-F Research Data 5 Factors 2x3
Daily\n-------------------------------------\n\nThis file was created by
CMPT_ME_BEME_OP_INV_RETS_DAILY using the 202008 CRSP database. The 1-month TBill
return is from Ibbotson and Associates, Inc.\n\n  0 : (1220 rows x 6 cols)'
```

[22]:
```
# select the Average Value Weighted Returns -- Daily (1220 rows x 100 cols)
portfolios100 = portfolios100[0]
factors5 = factors5[0]
```

[36]:
```
# Checking for missing values porfolios dataset
print(portfolios100[portfolios100.iloc[:,0] >98.0].sum().sum(),
portfolios100.isnull().sum().sum())
```

```
0.0 0
```

[37]:
```
# Checking for missing values in factors dataset
print(factors5[portfolios100.iloc[:,0] >98.0].sum().sum(),
factors5.isnull().sum().sum())
```

```
0.0 0
```

[30]:
```
portfolios100.iloc[:, 90:100].head()
```

[30]:
```
            BIG LoBM  ME10 BM2  ME10 BM3  ME10 BM4  ME10 BM5  ME10 BM6  \
Date
2015-10-27      0.06     -0.27     -0.06      0.04     -0.03     -0.49
2015-10-28      0.54      1.67      0.21      0.99      1.23      0.74
2015-10-29     -0.01      0.43     -0.22      0.09     -0.53      0.16
2015-10-30     -0.13     -0.87     -0.59     -0.80     -1.07     -0.29
2015-11-02      0.93      0.78      0.85      1.12      1.50      1.54

            ME10 BM7  ME10 BM8  ME10 BM9  BIG HiBM
Date
2015-10-27     -0.62     -1.03     -0.39     -0.88
2015-10-28      2.79      1.37      3.05      4.68
2015-10-29     -0.28     -0.24     -0.34     -1.21
2015-10-30     -0.73     -0.56     -1.30     -1.24
```

```
2015-11-02      1.61      2.09      1.68      1.39
```

[33]:
```
portfolios100.shape
```

[33]:
```
(1220, 100)
```

[34]:
```
factors5.head()
```

[34]:
```
            Mkt-RF   SMB   HML   RMW   CMA   RF
Date
2015-10-27   -0.42 -1.06 -0.93 -0.45 -0.51  0.0
2015-10-28    1.43  1.49  0.68 -0.71 -0.25  0.0
2015-10-29   -0.20 -0.89  0.08  0.43 -0.20  0.0
2015-10-30   -0.42  0.21 -0.56  0.36  0.08  0.0
2015-11-02    1.25  0.90  0.00 -0.51 -0.05  0.0
```

[35]:
```
factors5.shape
```

[35]:
```
(1220, 6)
```

# 1 1a. Visually analyze the covariance between various factors and identify the variance explained in principle components of these factors.

## 1.1 1b. Next, consider the ACF and PACF of the process and its square.

[284]:
```
pd.melt(factors5.add(1).cumprod().reset_index(), id_vars=["Date"]).hvplot.
↪line(x='Date', y='value', by='variable')
```

[284]:
```
:NdOverlay   [variable]
   :Curve   [Date]   (value)
```

[ ]:
```

```

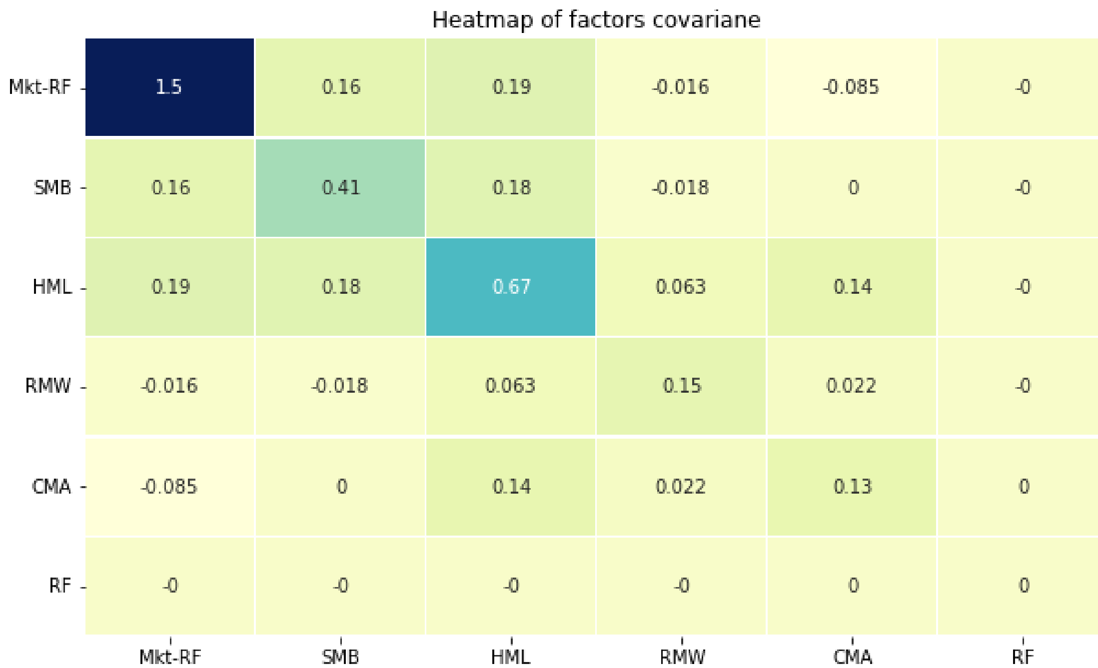
[38]:
```
factors_cov = factors5.cov()
factors_cov
```

[38]:

| | Mkt-RF | SMB | HML | RMW | CMA | RF |
|---|---|---|---|---|---|---|
| Mkt-RF | 1.491448 | 0.161363 | 0.192485 | -0.016157 | -0.085303 | -0.000145 |
| SMB | 0.161363 | 0.405510 | 0.185286 | -0.017541 | 0.000443 | -0.000089 |
| HML | 0.192485 | 0.185286 | 0.668600 | 0.063133 | 0.139938 | -0.000059 |
| RMW | -0.016157 | -0.017541 | 0.063133 | 0.151251 | 0.022233 | -0.000026 |
| CMA | -0.085303 | 0.000443 | 0.139938 | 0.022233 | 0.132428 | 0.000002 |
| RF | -0.000145 | -0.000089 | -0.000059 | -0.000026 | 0.000002 | 0.000011 |

[ ]:
```

```

[46]:
```
plt.figure(figsize = [10, 6])
# Visualize the covariance matrix using a heatmap
sns.heatmap(round(factors_cov,3),annot=True, linewidths=.5, cmap="YlGnBu",␣
↪cbar=False)
plt.yticks(rotation=0, fontsize="10", va="center")
plt.title('Heatmap of factors covariane');
```




From the heatmap, there mostly positive low covariances. We notice zero or negative zero covariance of riskfree return with other factors.

[52]:
```
# calculate the variance
pd.DataFrame(factors5.var().reset_index().values,columns=["Factors",␣
↪"Variance"])
```

[52]:

| | Factors | Variance |
|---|---|---|
| 0 | Mkt-RF | 1.49145 |
| 1 | SMB | 0.40551 |
| 2 | HML | 0.6686 |
| 3 | RMW | 0.151251 |
| 4 | CMA | 0.132428 |
| 5 | RF | 1.13173e-05 |

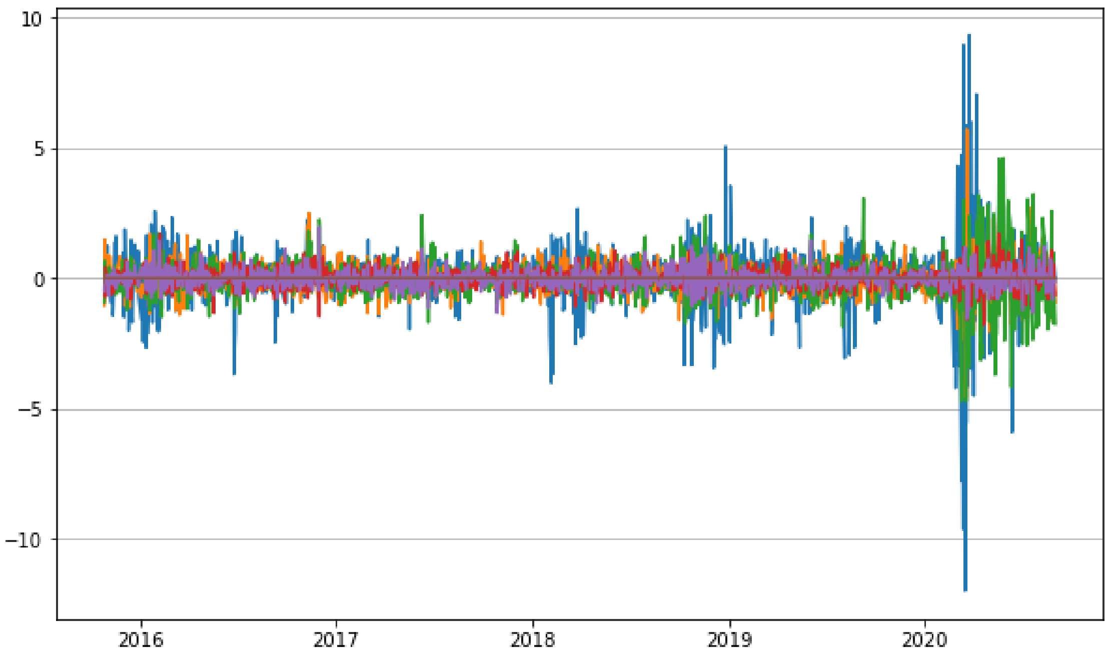
[53]:
```
# compute the correlation matrix
factors5.corr()
```

[53]:

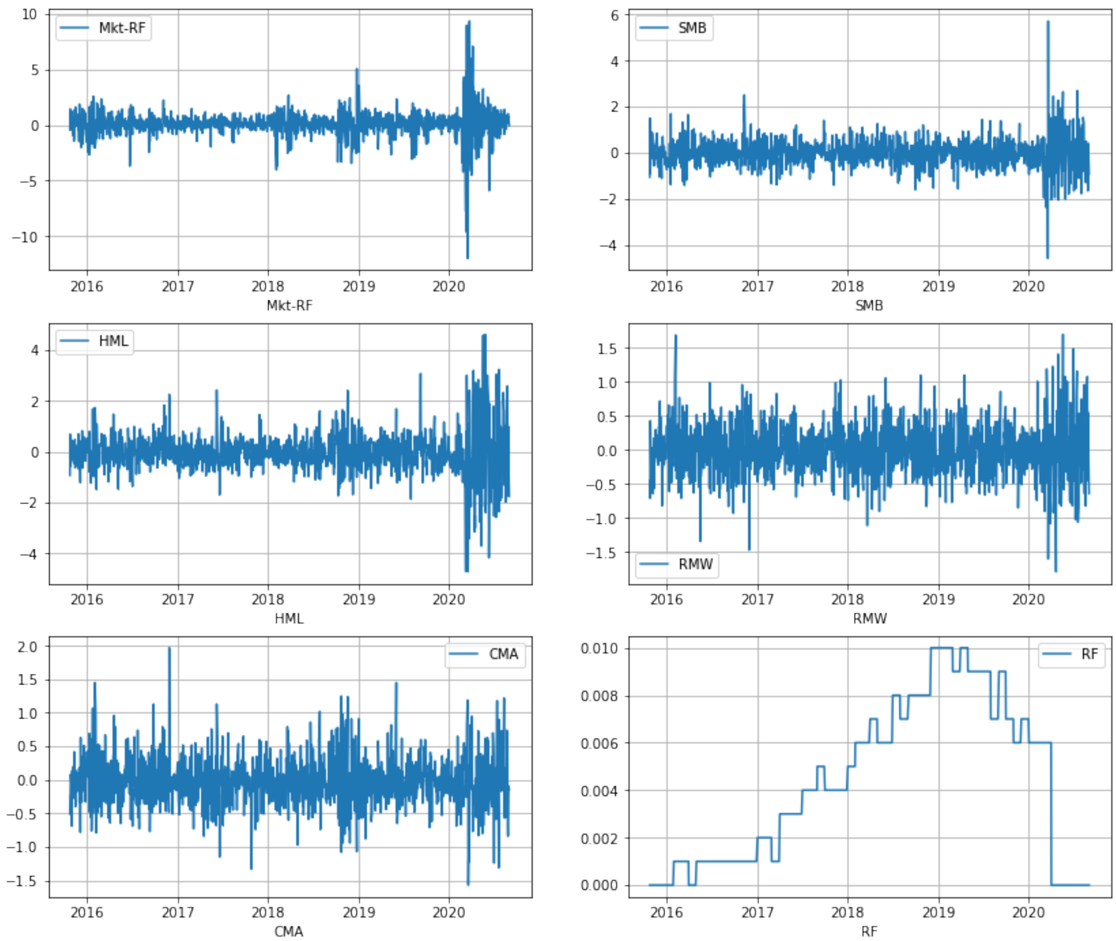
| | Mkt-RF | SMB | HML | RMW | CMA | RF |
|---|---|---|---|---|---|---|
| Mkt-RF | 1.000000 | 0.207491 | 0.192757 | -0.034018 | -0.191941 | -0.035188 |
| SMB | 0.207491 | 1.000000 | 0.355844 | -0.070827 | 0.001912 | -0.041452 |
| HML | 0.192757 | 0.355844 | 1.000000 | 0.198528 | 0.470286 | -0.021627 |
| RMW | -0.034018 | -0.070827 | 0.198528 | 1.000000 | 0.157096 | -0.019564 |
| CMA | -0.191941 | 0.001912 | 0.470286 | 0.157096 | 1.000000 | 0.001654 |
| RF | -0.035188 | -0.041452 | -0.021627 | -0.019564 | 0.001654 | 1.000000 |

### 1.2 Next, consider the ACF and PACF of the process.

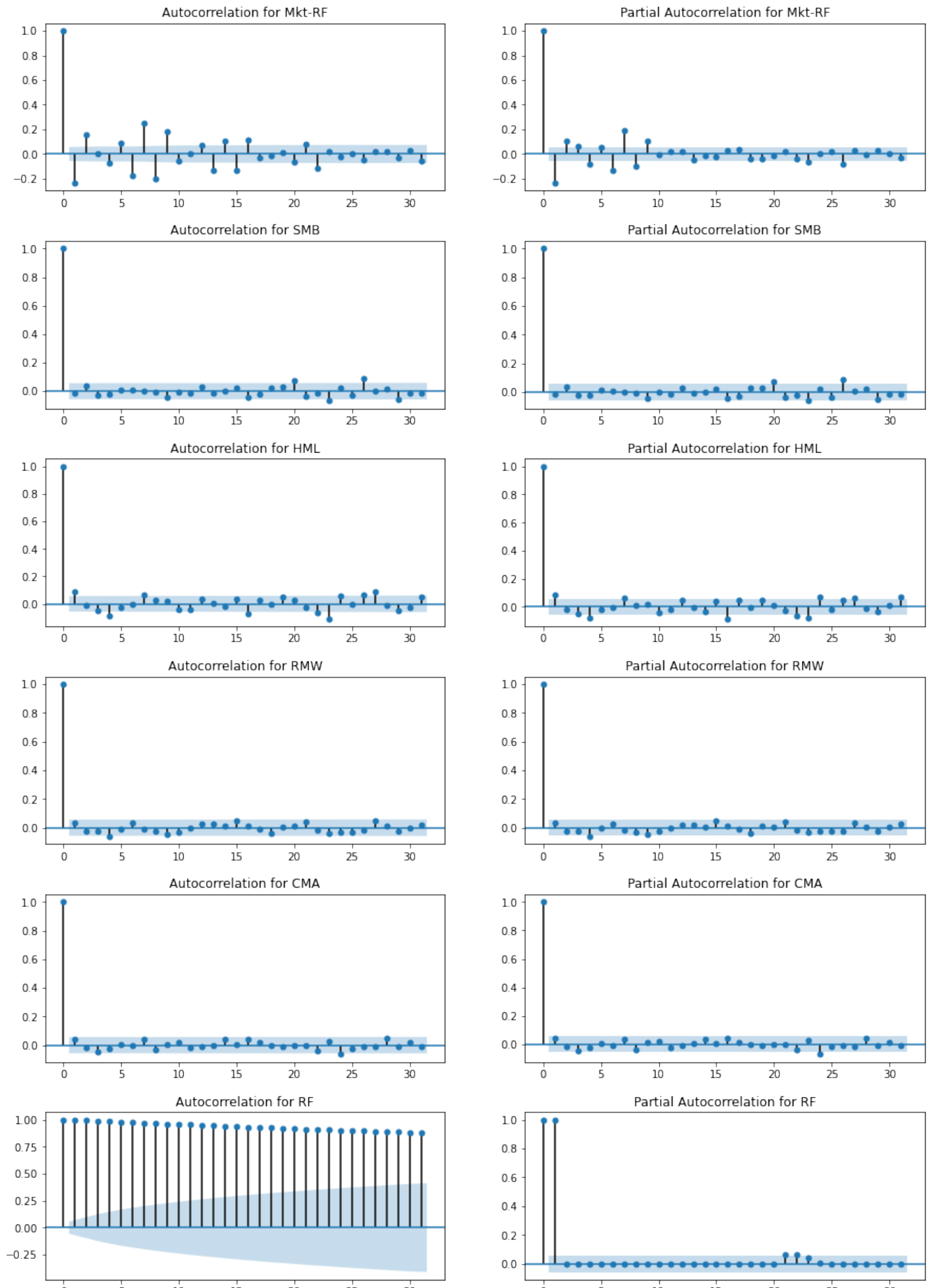
[88]:
```
# plot the factors together
plt.figure(figsize = [10, 6])
plt.plot(factors5)
plt.grid(axis='y');
```




[113]:
```
# plot the factors
factor_labels = factors5.columns.tolist()
fig, ax = plt.subplots(nrows=3, ncols=2, figsize = (14,12))
for idx, ax in enumerate(ax.flatten(),start=0):
    ax.plot(factors5.iloc[:,idx], label=factor_labels[idx])
    ax.set_xlabel(factor_labels[idx])
    ax.grid()
    ax.legend()
```

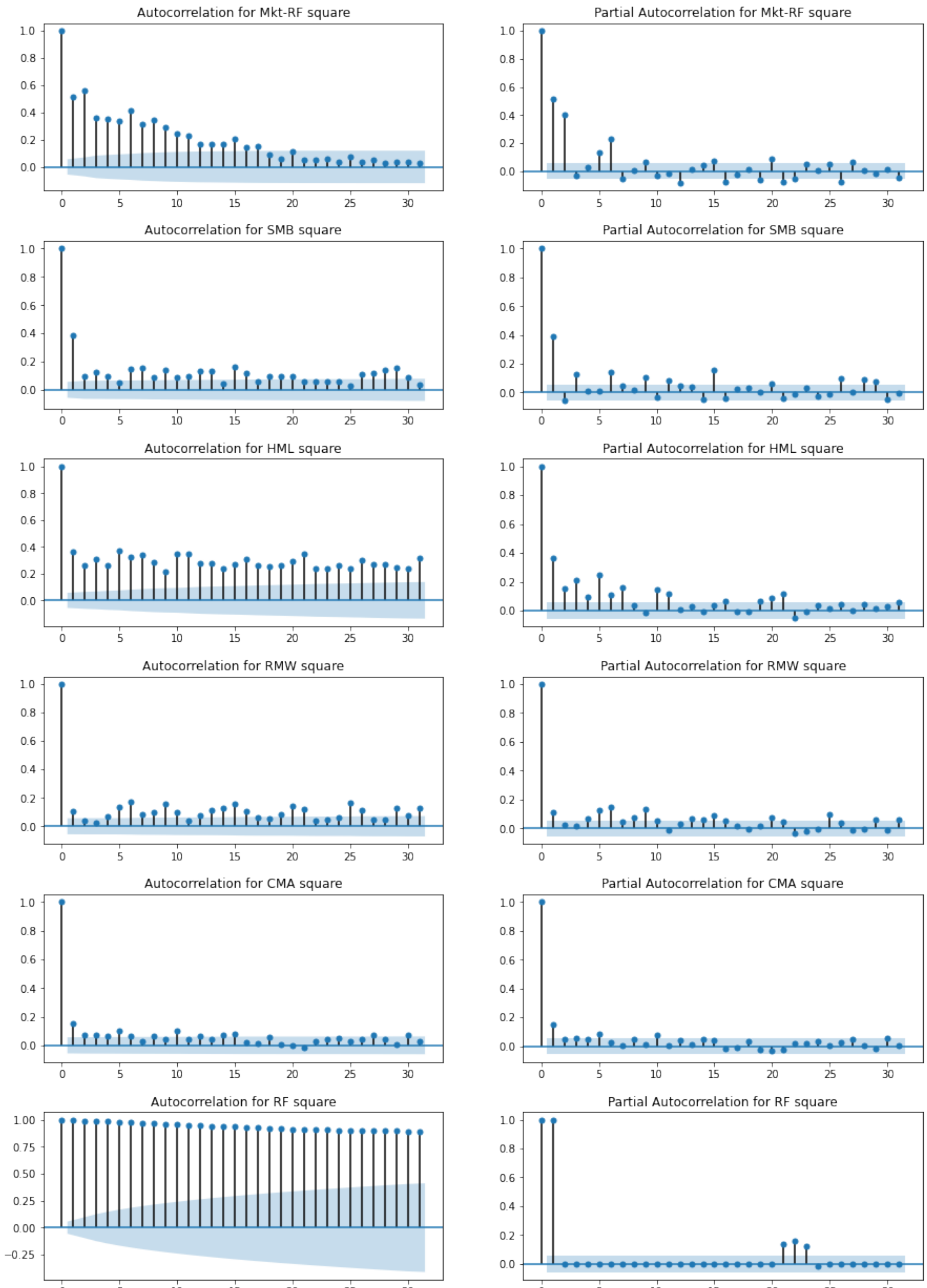



[167]:
```python
# plot the acf and pacf for the process
fig, ax = plt.subplots(nrows=6, ncols=2, figsize = (15,22))
p = ax.flatten().tolist()
for indx, colname in enumerate(factors5.columns):
    ba =  indx*2
    plot_acf(factors5.iloc[:,indx], title = f'Autocorrelation for {colname}',␣
↪ax=p[ba])
    plot_pacf(factors5.iloc[:, indx], title = f'Partial Autocorrelation for␣
↪{colname}', ax=p[ba+1])
    fig.subplots_adjust(hspace=.3)
```


Autocorrelation for Mkt-RF
Partial Autocorrelation for Mkt-RF
Autocorrelation for SMB
Partial Autocorrelation for SMB
Autocorrelation for HML
Partial Autocorrelation for HML
Autocorrelation for RMW
Partial Autocorrelation for RMW
Autocorrelation for CMA
Partial Autocorrelation for CMA
Autocorrelation for RF
Partial Autocorrelation for RF

### 1.3 Next, consider the ACF and PACF of the process square.

[169]:
```
# plot the acf and pacf for the process square
factor5_sq = factors5**2
fig, ax = plt.subplots(nrows=6, ncols=2, figsize = (15,22))
p = ax.flatten().tolist()
for indx, colname in enumerate(factors5.columns):
    ba =  indx*2
    plot_acf(factor5_sq.iloc[:,indx], title = f'Autocorrelation for {colname}␣
↪square', ax=p[ba])
    plot_pacf(factor5_sq.iloc[:, indx], title = f'Partial Autocorrelation for␣
↪{colname} square', ax=p[ba+1])
    fig.subplots_adjust(hspace=.3)
```


Autocorrelation for Mkt-RF square
Partial Autocorrelation for Mkt-RF square
Autocorrelation for SMB square
Partial Autocorrelation for SMB square
Autocorrelation for HML square
Partial Autocorrelation for HML square
Autocorrelation for RMW square
Partial Autocorrelation for RMW square
Autocorrelation for CMA square
Partial Autocorrelation for CMA square
Autocorrelation for RF square
Partial Autocorrelation for RF square

### 1.4 2.Using PCA provide a 2-dimensional representation of the weight-space of a set of linear models representing the covariance between our factors and the different benchmark portfolios. Comment on the distribution of the benchmark portfolios across the weight-space.

[184]:
```
def pca_function(dataframe, transformer=StandardScaler()):

    portfolios_standard_ = transformer.fit_transform(dataframe)
    portfolios_standard = pd.DataFrame(portfolios_standard_, columns=dataframe.
 ↪columns, index=dataframe.index)

    n = 2

    equities_ = dataframe.columns
    n_tickers = len(equities_)

    pca = None
    cov_matrix = pd.DataFrame(data=np.ones(shape=(n_tickers, n_tickers)),␣
 ↪columns=equities_)

    cov_matrix = portfolios_standard.cov()
    pca = PCA()
    pca.fit(cov_matrix)

    return portfolios_standard, pca.explained_variance_ratio_


def explain_com(pca_explained_variance_ratio):
    # cumulative variance explained
    var_threshold_list = [0.80, 0.85, 0.90, 0.95, 0.96, 0.97, 0.99, 0.9999]

    for xx in var_threshold_list:
        var_explained = np.cumsum(pca_explained_variance_ratio)
        num_comp = np.where(np.logical_not(var_explained < xx))[0][0] + 1  # +1␣
 ↪due to zero based-arrays
        print(f'{num_comp} components explain {round(100* xx,2)}% of variance')

def pca_plot(dataframe, variance_e):

    bar_width = 0.9
    n_asset = int((1 / 10) * dataframe.shape[1])

    if n_asset > len(dataframe.columns):
        n_asset = len(dataframe.columns)

    elif n_asset <= 0:
        n_asset = len(dataframe.columns)
```

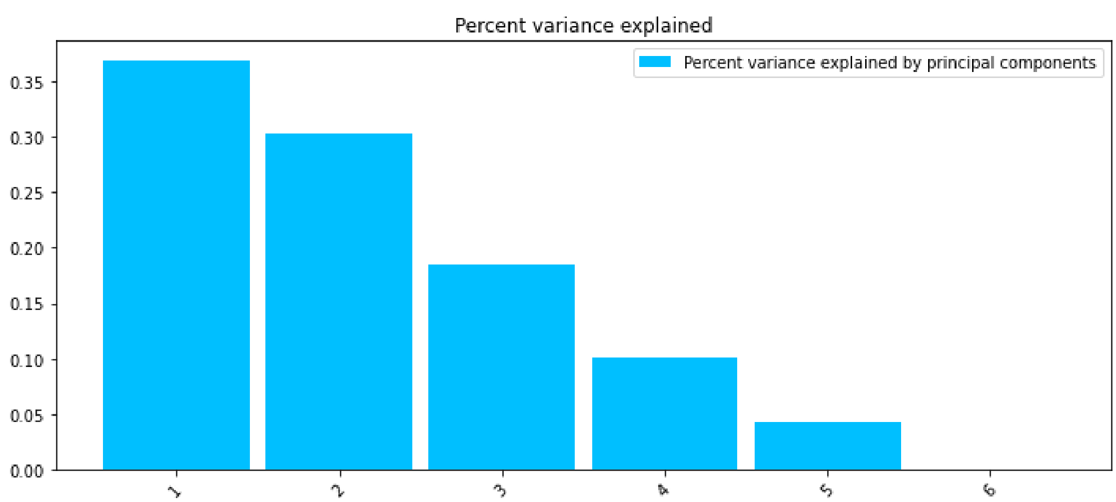
```
    x_indx = np.arange(n_asset)

    fig, ax = plt.subplots()
    fig.set_size_inches(12, 5)

    # Eigenvalues are measured as percentage of explained variance.
    rects = ax.bar(x_indx, variance_e[:n_asset], bar_width, color='deepskyblue')
    ax.set_xticks(x_indx)
    ax.set_xticklabels(list(range(1,n_asset+1)), rotation=45)
    ax.set_title('Percent variance explained')
    ax.legend((rects[0],), ('Percent variance explained by principal␣
↪components',))
```

[185]:
```
std2, pca22 = pca_function(factors5)
```

[186]:
```
explain_com(pca22)
```

```
3 components explain 80.0% of variance
3 components explain 85.0% of variance
4 components explain 90.0% of variance
4 components explain 95.0% of variance
5 components explain 96.0% of variance
5 components explain 97.0% of variance
5 components explain 99.0% of variance
5 components explain 99.99% of variance
```

[187]:
```
pca_plot(std2, pca22)
```



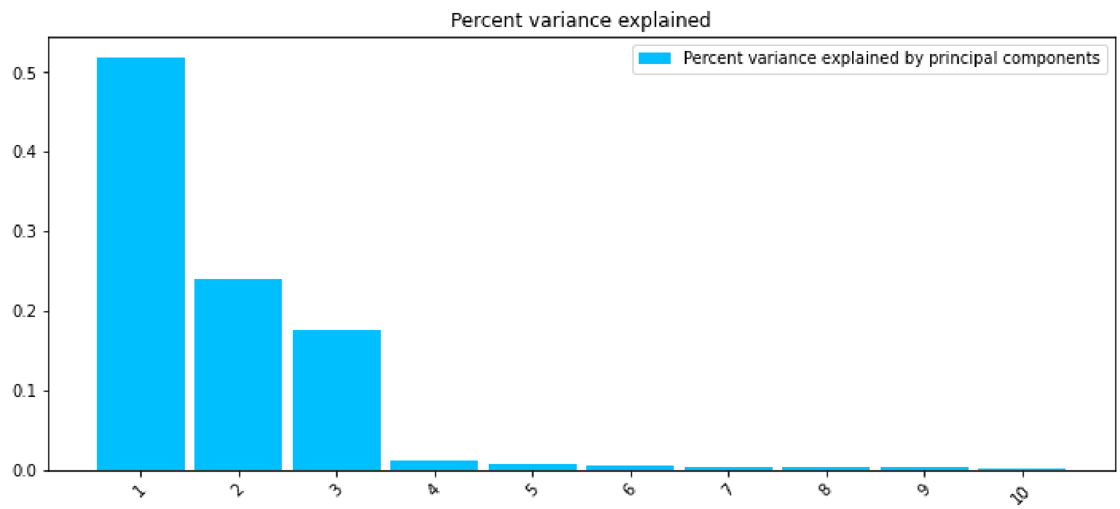

[ ]:
```

```

[196]:
```
std2, pca22 = pca_function(portfolios100)
```

[197]:
```
explain_com(pca22)
```

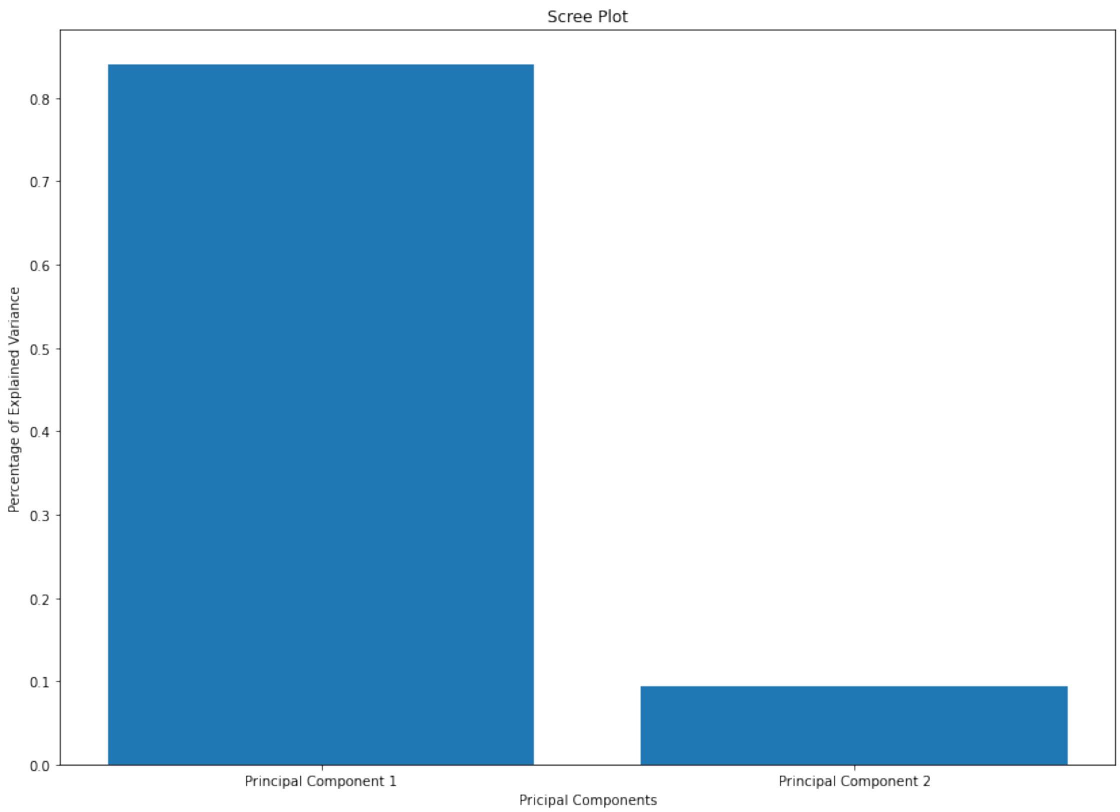
```
3 components explain 80.0% of variance
3 components explain 85.0% of variance
3 components explain 90.0% of variance
5 components explain 95.0% of variance
7 components explain 96.0% of variance
10 components explain 97.0% of variance
30 components explain 99.0% of variance
95 components explain 99.99% of variance
```

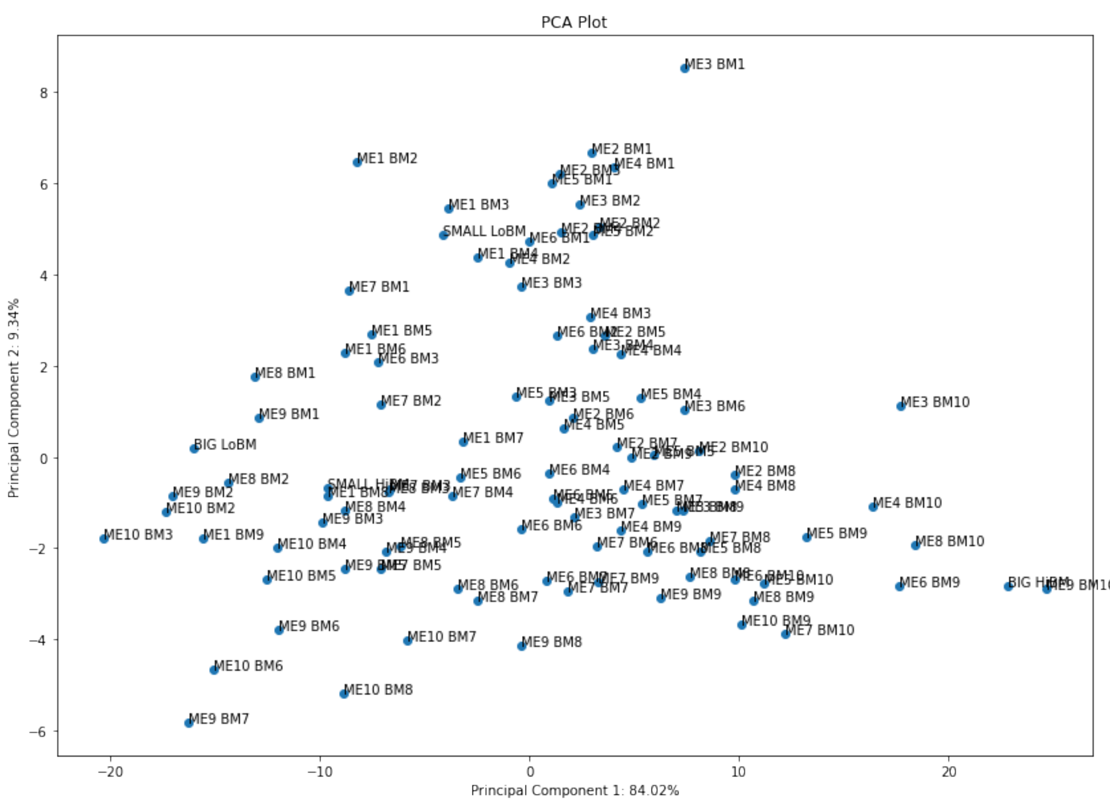
[199]:
```
pca_plot(std2, pca22)
```




[273]:
```
colname = portfolios100.columns
n = 2

# Using Pipeline
pipe3 = Pipeline([('scaler', StandardScaler()),
             ('pca', PCA(n_components=n))])

# Fit it to the dataset and extract the component vectors
pcomp_pro = pipe3.fit_transform(portfolios100.cov())
pcompfactors= pipe3.fit(portfolios100.cov())
# pcomp_pro = pipe3.fit_transform(portfolios100)
# pcompfactors= pipe3.fit(portfolios100)

plt.figure(figsize = [14, 10])
# plt.xticks(rotation=45)
```

```
per_var = np.round(pipe3.steps[1][1].explained_variance_ratio_, 4)
labels = [f"Principal Component {i}" for i in range(1,len(per_var)+1)]
plt.bar(labels,per_var)
plt.ylabel("Percentage of Explained Variance")
plt.xlabel("Pricipal Components")
plt.title("Scree Plot")
plt.show()


plt.figure(figsize = [14, 10])
pca_df = pd.DataFrame(pcomp_pro, index=colname,
                      columns=labels)
# pca_df = pd.DataFrame(pcomp_pro,
#                        columns=labels)
colname2 = pca_df.columns.tolist()
plt.scatter(pca_df[colname2[0]], pca_df[colname2[1]])

for sample in pca_df.index:
    plt.annotate(sample, (pca_df[colname2[0]].loc[sample],
                          pca_df[colname2[1]].loc[sample]))

plt.ylabel(f"Principal Component 2: {per_var[1]*100}%")
plt.xlabel(f"Principal Component 1: {per_var[0]*100}%")
plt.title("PCA Plot")
plt.show()
```


Scree Plot
Percentage of Explained Variance
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Principal Component 1
Principal Component 2
Pricipal Components

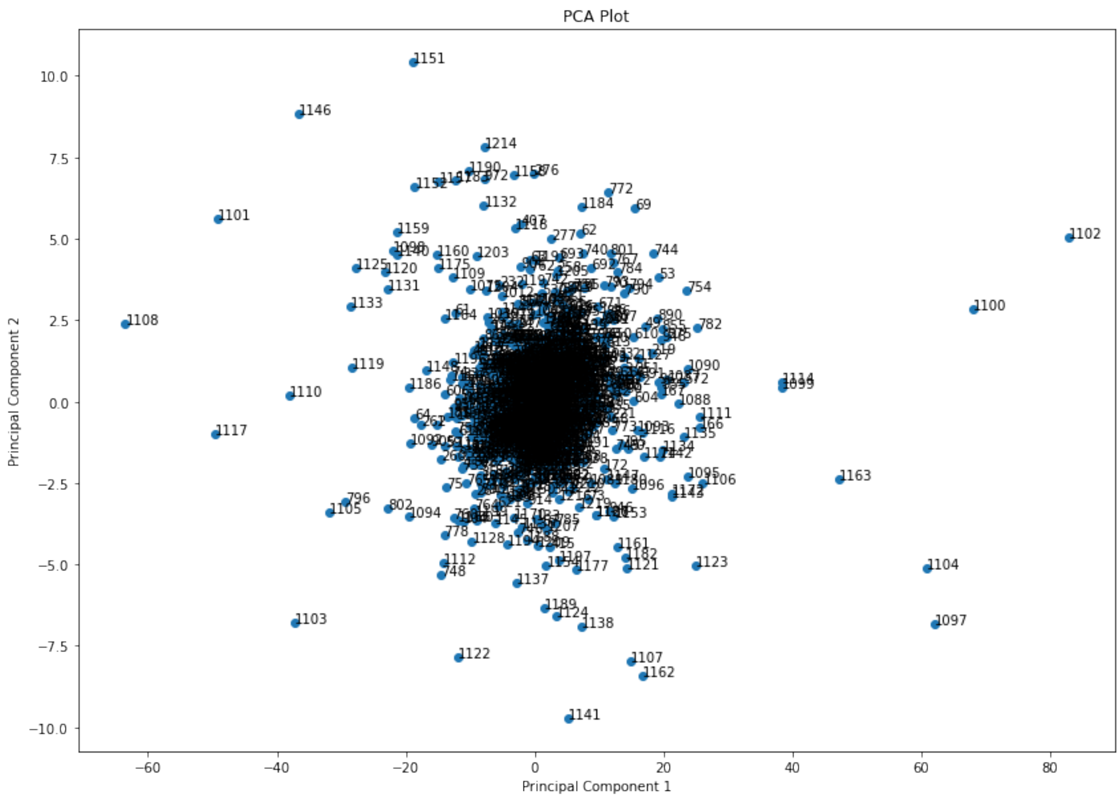

PCA Plot
Principal Component 2: 9.34%
Principal Component 1: 84.02%
−20
−10
0
10
20
−6
−4
−2
0
2
4
6
8

[ ]:

```

```

[400]:

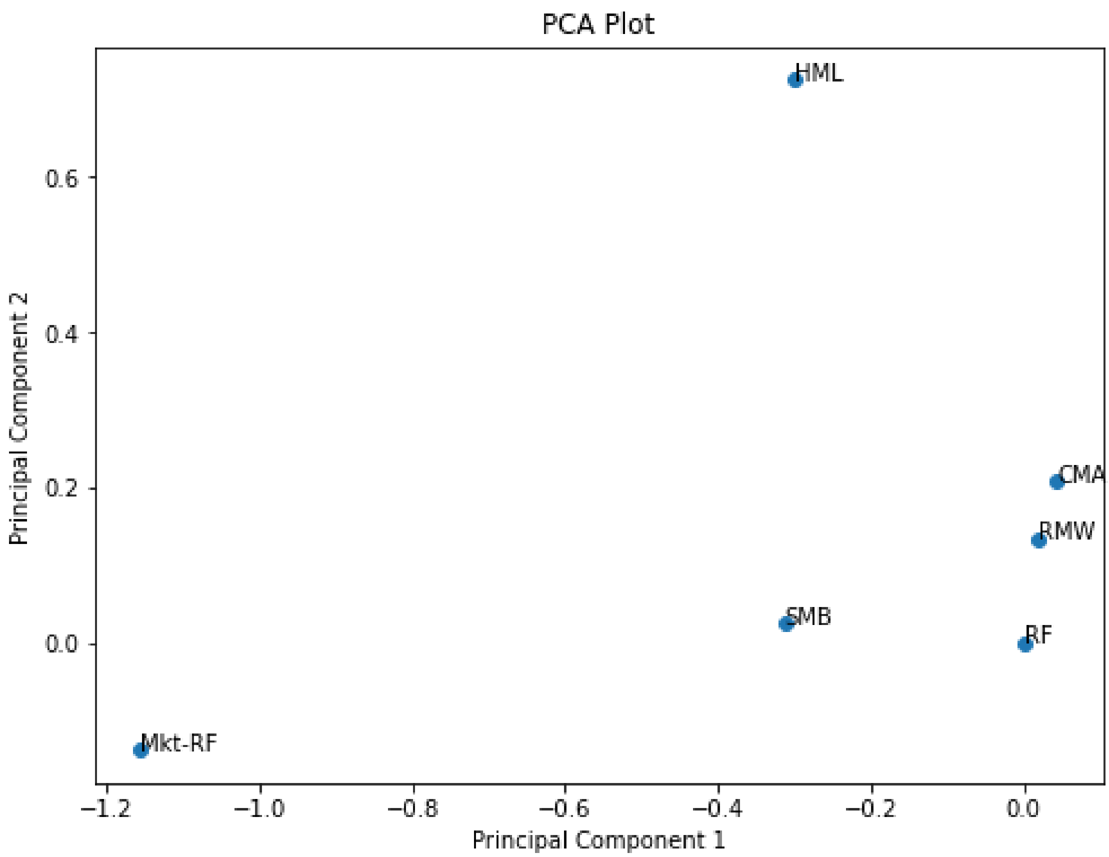
```
colname = portfolios100.columns
n = 2

# Using Pipeline
pipe3 = Pipeline([('scaler', StandardScaler()),
            ('pca', PCA(n_components=n))])

# Fit it to the dataset and extract the component vectors
# pcomp_pro = pipe3.fit_transform(portfolios100.cov())
# pcompfactors= pipe3.fit(portfolios100.cov())
pcomp_pro = pipe3.fit_transform(portfolios100)
pcompfactors= pipe3.fit(portfolios100)

# plt.figure(figsize = [14, 10])
# # plt.xticks(rotation=45)
# per_var = np.round(pipe3.steps[1][1].explained_variance_ratio_, 4)
# labels = [f"Principal Component {i}" for i in range(1,len(per_var)+1)]
# plt.bar(labels,per_var)
# plt.ylabel("Percentage of Explained Variance")
# plt.xlabel("Pricipal Components")
# plt.title("Scree Plot")
# plt.show()


plt.figure(figsize = [14, 10])
# pca_df = pd.DataFrame(pcomp_pro, index=colname,
#                       columns=labels)
pca_df = pd.DataFrame(pcomp_pro,
                      columns=labels)
colname2 = pca_df.columns.tolist()
plt.scatter(pca_df[colname2[0]], pca_df[colname2[1]])

for sample in pca_df.index:
    plt.annotate(sample, (pca_df[colname2[0]].loc[sample],
                          pca_df[colname2[1]].loc[sample]))

plt.ylabel(f"Principal Component 2")
plt.xlabel(f"Principal Component 1")
plt.title("PCA Plot")
plt.show()
```




```
[ ]:
```

```
[272]: # Using Pipeline
lreg = Pipeline([('scaler', StandardScaler()),
            ('lnReg', linear_model.LinearRegression())])

# Fit it to the dataset and extract the component vectors
pcomp_pro = lreg.fit(pca_df, factors5)
per_var = np.round(lreg.steps[1][1].coef_, 4)
labels = [f"Pc_reg {i}" for i in range(1,len(per_var)+1)]

colname = factors5.columns.tolist()
reg_df = pd.DataFrame(per_var, index=colname,
                      columns=["x", "y"])

colname2 = reg_df.columns.tolist()

plt.figure(figsize = [8, 6])
plt.scatter(reg_df[colname2[0]], reg_df[colname2[1]])

for sample in reg_df.index:
    plt.annotate(sample, (reg_df.x.loc[sample],
```

```
                                reg_df.y.loc[sample]))

plt.ylabel(f"Principal Component 2")
plt.xlabel(f"Principal Component 1")
plt.title("Distribution of factors across the weight space")
plt.show()
```




[ ]:

```

```

## 2 3.Using linear regression test for the significance of these factors, as per the original work of Fama and French.

[311]:

```
import statsmodels.api as sm

factors6, _ = pca_function(factors5)
portfolio2, _ = pca_function(portfolios100)

# factors6 = sm.add_constant(factors6, prepend=False)
```

```python
# Fit and summarize OLS model
model = sm.OLS(portfolio2.iloc[:,0], factors6)

result = model.fit()

print(result.summary())
```

```
                            OLS Regression Results
================================================================================
=======
Dep. Variable:             SMALL LoBM   R-squared (uncentered):
0.728
Model:                            OLS   Adj. R-squared (uncentered):
0.727
Method:                 Least Squares   F-statistic:
541.7
Date:                Mon, 26 Oct 2020   Prob (F-statistic):
0.00
Time:                        05:07:41   Log-Likelihood:
-936.79
No. Observations:                1220   AIC:
1886.
Df Residuals:                    1214   BIC:
1916.
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Mkt-RF         0.6790      0.016     41.844      0.000       0.647       0.711
SMB            0.4147      0.017     24.997      0.000       0.382       0.447
HML           -0.1400      0.020     -7.144      0.000      -0.179      -0.102
RMW           -0.1335      0.015     -8.621      0.000      -0.164      -0.103
CMA            0.0044      0.018      0.245      0.807      -0.031       0.040
RF             0.0119      0.015      0.791      0.429      -0.018       0.041
==============================================================================
Omnibus:                       81.561   Durbin-Watson:                   2.116
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              303.551
Skew:                           0.204   Prob(JB):                     1.22e-66
Kurtosis:                       5.409   Cond. No.                         2.18
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[347]:
```
n_portfolios = len(portfolios100.columns)
rfree = np.array([0.] * n_portfolios)
beta1 = np.array([0.] * n_portfolios)
beta2 = np.array([0.] * n_portfolios)
beta3 = np.array([0.] * n_portfolios)
beta4 = np.array([0.] * n_portfolios)
beta5 = np.array([0.] * n_portfolios)
```

[353]:
```
for i in range(portfolios100.shape[1]):
    model = sm.OLS(portfolio2.iloc[:,i], factors6)
    r_ = model.fit()
    pval = round(r_.pvalues, 3)
    beta1[i], beta2[i], beta3[i] , beta4[i], beta5[i], rfree[i] = pval.T.values
```

[ ]:

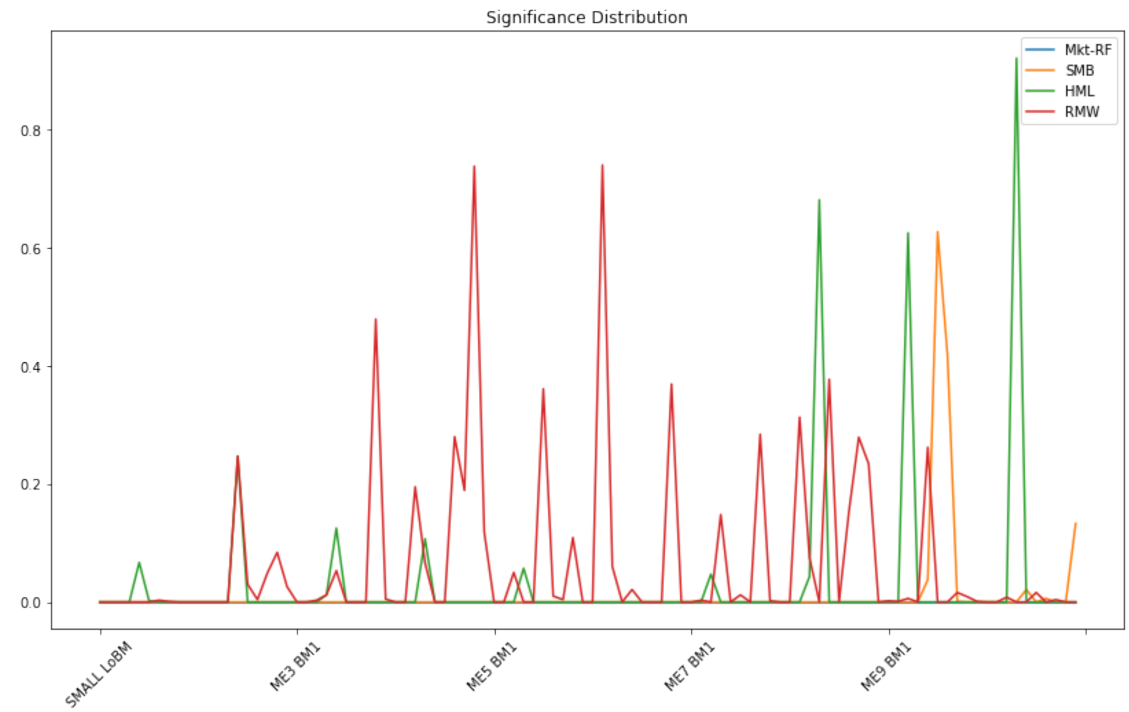
[393]:
```
data = pd.DataFrame(data={'RF':rfree, 'Mkt-RF':beta1, 'SMB':beta2, 'HML':beta3,␣
↪'RMW':beta4, 'CMA':beta5},
                    index = portfolios100.columns) #.reset_index().
↪rename(columns={"index":"Porfolios"})
```

[394]:
```
data.head()
```

[394]:

| | RF | Mkt-RF | SMB | HML | RMW | CMA |
|---|---|---|---|---|---|---|
| SMALL LoBM | 0.429 | 0.0 | 0.0 | 0.000 | 0.0 | 0.807 |
| ME1 BM2 | 0.853 | 0.0 | 0.0 | 0.000 | 0.0 | 0.367 |
| ME1 BM3 | 0.157 | 0.0 | 0.0 | 0.000 | 0.0 | 0.599 |
| ME1 BM4 | 0.588 | 0.0 | 0.0 | 0.000 | 0.0 | 0.557 |
| ME1 BM5 | 0.355 | 0.0 | 0.0 | 0.067 | 0.0 | 0.480 |

[398]:
```
data.iloc[:,1:5].plot(figsize=(14,8))
plt.xticks(rotation=45)
plt.title("Significance Distribution");
```




Clearly only Mkt-RF, SMB, HML and RWL are statistically significant.