

Optimum reject options for multiclass classification

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Bachelor Thesis

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Abstract

Contents

1	Introduction	1
2	Reject Options	3
2.1	Two Classes	3
2.1.1	Reject Strategy	3
2.1.2	Optimal Θ	3
2.2	Multi class Classification	4
2.2.1	Global Reject	4
2.2.2	Local Reject	4
2.2.3	Optimal local Reject	4
2.2.4	Computation by Dynamic Programming	4
2.2.5	Greedy Computation	4
2.2.6	Evaluation	4
3	Application for SVM	5
4	Conclusions	7

Introduction

1

Reject Options

2.1 Two Classes

To make our way towards optimal rejects for multi class classification, we start of small by looking at a general two class classifier f that divides the space via a decision boundary.

$$f : \mathbb{R}^n \rightarrow \{1, 2\}$$

Let r be a measure of confidence that a point is part of its respective class, e.g. distance to the decision boundary. If $r(\bar{x})$ is large it means that \bar{x} is likely in the class it was assigned to.

2.1.1 Reject Strategy

We look for a threshold Θ that defines for a given point \bar{x} whether it is rejected or not:

$$r(\bar{x}) < \Theta : \bar{x} \text{ rejected}$$

2.1.2 Optimal Θ

In order to find an optimal threshold we need to decide how to evaluate it. Optimally only wrongly classified data points are rejected (now referred to as true rejects). But generally there will be rejected although correctly classified points (false rejects). A given labeled (by y) data set X is divided into the two sets L and E by applying f as a classifier with:

$$L = \{\forall \bar{x} \in X \mid f(\bar{x}) = y(\bar{x})\} : \text{correctly classified points} \quad (2.1)$$

$$E = \{\forall \bar{x} \in X \mid f(\bar{x}) \neq y(\bar{x})\} : \text{incorrectly classified points} \quad (2.2)$$

And by applying our reject strategy with the threshold Θ , X is divided into A and R with:

$$A_{\Theta} = \{\forall \bar{x} \in X \mid r(\bar{x}) \geq \Theta\} : \text{accepted points} \quad (2.3)$$

$$R_{\Theta} = \{\forall \bar{x} \in X \mid r(\bar{x}) < \Theta\} : \text{rejected points} \quad (2.4)$$

The set T contains all true rejects and F all false rejects.

$$T_{\Theta} = R \cap E \quad (2.5)$$

$$F_{\Theta} = R \cap L \quad (2.6)$$

Naturally we want $|T_{\Theta}|$ to be large and $|F_{\Theta}|$ to be small. Since these two goals often contradict each other, e.g. more true rejects most times bring more false ones (TODO: graph), there is no single optimal choice of Θ in general. Still there is a set of values that we consider optimal. As shown above each Θ corresponds to a tuple $(|T_{\Theta}|, |F_{\Theta}|)$. Θ is optimal if

$$\nexists \Theta' : |F_{\Theta'}| \leq |F_{\Theta}|, |T_{\Theta'}| \geq |T_{\Theta}|$$

and at least one term is unequal (TODO: wording?).

2.2 Multi class Classification

Let us now expand the problem to a classifier for multiple classes. By using a one vs all strategy, we get i binary classifiers f_i like in chapter 2.1 and an according measure of confidence r_i . Points are now classified in the class where confidence is maximal among all classes. This gives us a multi class classifier f with

$$f : \mathbb{R}^n \rightarrow \{1, \dots, i\} \quad (2.7)$$

$$\bar{x} \mapsto i \mid \arg \max_i r_i(\bar{x}) \quad (2.8)$$

2.2.1 Global Reject

2.2.2 Local Reject

2.2.3 Optimal local Reject

2.2.4 Computation by Dynamic Programming

2.2.5 Greedy Computation

2.2.6 Evaluation

Application for SVM

3

Conclusions

4

List of Figures

List of Tables

