1 The Space and Time of Relativity

- $\vec{u} = \vec{u}' + \vec{v}$
- Newton's Laws are valid in one reference frame, they are also valid in any frame that moves with constant velocity relative to the first.
- Inertial Frame: A reference frame that is not accelerating.
- Postulates of relativity:

  (1) If S is an inertial frame and if a second frame S' moves with constant velocity relative to S, then S' is also and inertial frame.

  (2) In all inertial frames, light travels through the vacuum with the same speed, c = 299,792,458 m/s in any direction.

- Time dilation: $t = \gamma t_0 \geq t_0$, $\gamma = \frac{1}{\sqrt{1-\beta^2}} \geq 1$, $\beta = \frac{v}{c}$, "A moving clock is observed to run slow"
- Proper Time (t_0): Time interval measured between two events occurring in the same location.
- Length Contraction: $l = \frac{l_0}{\gamma} \leq l_0$, $\gamma = \frac{1}{\sqrt{1-\beta^2}} \geq 1$, $\beta = \frac{v}{c}$, "A moving objected is observed to be contracted"
- Proper Length(l_0): Length of an object in it's inertial rest frame.
- Transformations:

Galilean	Galilean^{-1}	Lorentz	Lorentz^{-1}
$x' = x - vt$	$x = x' + vt$	$x' = \gamma(x - vt)$	$x = \gamma(x' + vt)$
$y' = y$	$y' = y$	$y' = y$	$y' = y$
$z' = z$	$z' = z$	$z' = z$	$z' = z$
$t' = t$	$t = t'$	$t' = \gamma\Big(t - \dfrac{vx}{c^2}\Big)$	$t = \gamma\Big(t' + \dfrac{vx}{c^2}\Big)$

- The Velocity Addition Formula:

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \qquad u'_y = \frac{u_y}{\gamma(1 - \frac{u_x v}{c^2})} \qquad u'_z = \frac{u_z}{\gamma(1 - \frac{u_x v}{c^2})}$$

- Spacetime Interval: $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$
- The Doppler Effect: $f_{obs} = \sqrt{\frac{1+\beta}{1-\beta}} f_{sce}$ (approaching), $f_{obs} = \sqrt{\frac{1-\beta}{1+\beta}} f_{sce}$ (receding). $\beta = \frac{v}{c} = \frac{f_{obs}^2 - f_{sce}^2}{f_{obs}^2 + f_{sce}^2}$

2 Relativistic Mechanics

- Proper Mass: Mass of an object in its inertial rest frame.
- Relativistic Momentum: $\vec{p} = \frac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma m\vec{u}$
- Moving Total Energy: $E = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma mc^2$
- Rest Total Energy: $E = mc^2$

- Moving Energy Breakdown: $E = mc^2 + K$, $K = E - mc^2 = (\gamma - 1)mc^2$
- Pythagorean Relation: $E^2 = (pc)^2 + (mc^2)^2$
- Conversion of Mass to Energy:

  (1) Fly apart: $\Delta M = M - (m_1 + m_2)$, $\Delta Mc^2 = K_1 + K_2$

  (2) Bind together: $\Delta M = (m_1 + m_2) - M$, $\Delta Mc^2 = E_{Binding}$
- Lorentz Force: $\vec{F} = q(\mathcal{E} + \vec{u} \times \vec{B})$, $Radius = \frac{p}{qB}$
- Massless Particles ($m = 0$): $E = pc$, $\beta = 1 or u = c$

3 Atoms

- Atomic Number Z: the number of electrons in a neutral atom = the number of protons in nucleus.
- Mass Number A: (number of protons) + (number of neutrons)
- A.M.U u: $\frac{1}{12}$(mass of one neutral ^{12}C atom)
- Mass of atom with mass number A $\approxeq$ Au
- Avagadro's Number N_A: (number of atoms in 12 grams of ^{12}C) $= 6.022 \times 10^{23}$
- Ideal gas law: $pV = nRT = Nk_BT$, $N =$ number of molecules, $k_B = \frac{R}{N_A} = 1.38 \times 10^-23[J/K]$

4 Quantization of Light

- Planck: Energy of light varies with frequency and a constant: $E = hf$
- Photoelectric Effect:

  (1) Stopping Potential: $V_se = K_{max}$

  (2) If the intensity of the incident light on a metal plate is increased, the number of ejected electrons increases, but their kinetic energy does not change at all.

  (3) If the frequency f of the incident light is reduced below a certain critical frequency f_0, no electrons are ejected, no matter how intense the light may be.

  (4) Work Function: $K_{max} = hf - \phi$
- Compton Effect: $\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}(1 - cos(\theta))$
- Particle Wave Duality: $E = hf$ and $p = \frac{h}{\lambda}$

5 Quantization of Atomic Energy Levels

- Absorption spectrum: graph that shows which wavelengths are absorbed by a material
- Emission spectrum: graph that shows which wavelengths are emitted from a material
- The Balmer-Rydberg Formula:

  (1) $\frac{1}{\lambda} = R(\frac{1}{n'^2} - \frac{1}{n^2}))$, $(n > n'$, both integers)

  (2) $E_\lambda = hcR(\frac{1}{n'^2} - \frac{1}{n^2})$, $(n > n'$, both integers)

- The Bohr Model:

  (1) $E = K + U = \frac{1}{2}U = -\frac{1}{2}\frac{ke^2}{r}$, r can have range of $0 < r < \infty$, E can have range of $-\infty < E < 0$

  (2) Angular Momentum is quantized: $L = n\hbar$, $(n = 1, 2, 3, ...)$

  (3) Allowed Radii: $r = n^2 a_B$, $(n = 1, 2, 3, ...)$

  (4) Bohr Radius: $a_B = \frac{\hbar^2}{ke^2 m}$

  (5) Rydberg Energy: $E_R = hcR = \frac{ke^2}{2a_B} = \frac{m(ke^2)^2}{2\hbar^2} = 13.6eV$

  (6) Allowed Energies: $E_n = -\frac{E_R}{n^2}$

  (7) Lyman Series: $n' = 1$, Balmer Series: $n' = 2$, Paschen Series: $n' = 3$

- Hydrogen-Like Ions:

  (1) Allowed Radii: $r = n^2 \frac{\hbar^2}{Zke^2 m} = n^2 \frac{a_B}{Z}$

  (2) Reduced Mass: $\mu = \frac{m_e}{1 + \frac{m_e}{m_{nuc}}} \rightarrow E_R = \frac{\mu(ke^2)^@}{2\hbar^2}$

  (3) Allowed Energies: $E_n = -Z^2 \frac{E_R}{n^2}$, may or may not need reduced mass E_R

6 Matter Waves

- DeBroglie's Hypothesis: $E = hf = \hbar\omega$, $p = \frac{p}{\lambda} = \hbar k$
- (Probability number of electrons in dV at $\vec{r}$) $\propto [\Psi(\vec{r}, t)]^2 dV$
- $\Psi = \Psi_{real} + \Psi_{imag}$, $|\Psi| = \sqrt{\Psi_{real}^2 + \Psi_{imag}^2}$
- $|\Psi(\vec{r}, t)|^2 dV = P($finding particle in dV at $\vec{r})$
- $|\Psi(\vec{r}, t)|^2 = P($finding particle at $\vec{r})$
- Sinusoidal Waves:

  (1) $y(x, t) = Asin(2\pi(\frac{x}{\lambda} + \frac{t}{T})) = Asin(kx - \omega t)$

  (2) Wavelength: λ, Wave Number: $k = \frac{2\pi}{\lambda}$, Period: T, Frequency: $f = \frac{1}{T}$, Angular Frequency: $\omega = 2\pi f = \frac{2\pi}{T}$

  (3) $v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$

- Wave Packets and Fourier Analysis:

(1) Fourier Series: $f(x) = \sum_{n=0}^{\infty} A_n cos\left(\frac{2\pi n}{\lambda}x\right)$, $A_n = \frac{2}{\pi n} sin\left(\frac{\pi a n}{\lambda}\right)$

(2) Fourier Integral: $f(x) = \int A(k) cos(kx) dk$

(3) Uncertainty Principles: $\Delta x \Delta k \geq \frac{1}{2}$, $\Delta t \Delta \omega \geq \frac{1}{2}$

- Heisenburg Uncertainty Principles: $\Delta x \Delta p \geq \frac{\hbar}{2}$, $\Delta t \Delta E \geq \frac{\hbar}{2}$, $\Delta p = \Delta v m$
- Wave Velocity: $v_{pack} = \frac{d\omega}{dk}$

7 The Schrodinger Equation in One Dimension

- Standing waves: if a is the length of the wire, then $\lambda = \frac{2a}{n}$ where $n = 1, 2, 3....$ The ends of this string are fixed.
- Standing waves in quantum mechanics (Stationary States):

(1) Classical Standing wave: $\Psi(x,t) = Asin(kx)cos(\omega t) = \psi(x)cos(\omega t) = \psi(x)e^{-i\omega t}$

(2) $|\Psi(x,t)|^2 = |\psi(x)|^2$

- Particle in a rigid box (No Schrodinger):

(1) Allowed Energies: $E_n = n^2 \frac{\pi^2 \hbar^2}{2ma^2} = n^2 E_1$, $n = 1, 2, 3, ...$

(2) Ground State Energy: $E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$

- Time Independant Schrodinger Equation: $\frac{d^2\phi}{dx^2} = \frac{2m}{\hbar^2}[U(x) - E]\psi$
- Particle in a rigid box (With Schrodinger):

(1) $\psi(x) = \sqrt{\frac{2}{a}} sin\left(\frac{n\pi x}{a}\right)$

(2) Normalization Constraint: $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

(3) Expectation Value: $<x> = \frac{2}{a} \int_0^a x sin^2\left(\frac{\pi x}{a}\right) dx$

- The Free Particle:

(1) $\Psi(x,t) = \psi(x)e^{-i\omega t} = Ce^{i(kx-\omega t)} + De^{-i(kx+\omega t))}$

(2) This is a superposition of two travelling waves, one moving to the right (with coefficient C) and the other moving to the left (with Coefficient D). If we choose the coefficient $D = 0$, then (1) represents a particle with definite momentum $\hbar k$ to the right; if we choose the coefficient $C = 0$, then (1) represents a particle with definite momentum $\hbar k$ to the left.

- The Nonrigid Box:

(1)

$$U(x) = \begin{cases} 0, & 0 \leq x \leq a \\ U_0, & x < 0 \text{ and } x > a \end{cases}$$

(2)

$$\psi(x) = \begin{cases} Ce^{\alpha x}, & x < 0 \\ Fsin(kx) + Gcos(kx) \\ Be^{-\alpha x}, & x > a \end{cases}$$

(3) The parts of the piecewise function must be continuous and differentiable.

- Simple Harmonic Oscillator: $E_n = (n + \frac{1}{2})\hbar\omega_c$
- Tunneling Penetration: $\alpha = \sqrt{\frac{2m(U_0-E)}{\hbar^2}}$, (Probability of Tunneling) $= e^{-2\alpha L}$
- Time Dependant Schrodinger Equation: $i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\big)\Psi(x,t)$

8 The Three Dimensional Schrodinger Equation

- 3DSE: $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{2M}{\hbar^2}[U-E]\psi$
- 2D square box:

(1) 2DSE: $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{2M}{\hbar^2}[U-E]\psi$

(2) Setup:

$$U(x) = \begin{cases} 0, & 0 \le x \le a \text{ and } 0 \le y \le a \\ \infty, & \text{Otherwise} \end{cases}$$

(3) Complete wave function:

$$\psi(x,y) = X(x)Y(y) = Bsin(\frac{n_x\pi}{a}x)Csin(\frac{n_y\pi}{a}y) = Asin(k_x x)sin(k_y y) = (\frac{2}{a})sin(k_x x)sin(k_y y)$$

(4) Allowed Energies: $E = E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2Ma^2}(n_x^2 + n_y^2) = E_0(n_x^2 + n_y^2)$, $E_0 = \frac{\hbar^2\pi^2}{2Ma^2}$

(5) 2D Probability: $|\psi(x,y)|^2 = A^2 sin^2\big(\frac{\pi x}{a}\big)sin^2\big(\frac{\pi y}{a}\big)$

- Two Dimensional Central Force problem:

(1) Solution is of the form: $\psi(r,\phi) = R(r)\Phi(\phi) = R(r)e^{im\phi}$

(2) 2DSE in polar form: $\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = \frac{2M}{\hbar^2}[U-E]\psi$

(3) Differential Equations from 2DSE in polar form:

$$\Phi''(\phi) = -m^2\Phi(\phi)$$

$$R'' + \frac{R'}{r} - \Big[\frac{m^2}{r^2} + \frac{2M}{\hbar^2}(U-E)\Big]R = 0$$

(4) $\Phi(\phi) = e^{im\phi}$, $m = 0, \pm1, \pm2, ...$

(5) $L = m\hbar$

- Three Dimensional Central Force Problem

(1) Solution is of the form: $\psi(r,\phi) = R(r)\Phi(\phi)\Theta(\theta) = R(r)\Theta_{l,m}e^{im\phi}$

(2) 3DSE in polar form: $\frac{1}{r}\frac{\partial^2 r\psi}{\partial r^2} + \frac{1}{r^2 sin(\theta)}\frac{\partial}{\partial\theta}\big(sin(\theta)\frac{\partial\psi}{\partial\theta}\big) + \frac{1}{r^2 sin^2(\theta)}\frac{\partial^2\psi}{\partial\theta^2} = \frac{2M}{\hbar^2}[U-E]\psi$

(3) Differential Equations from 3DSE in polar form:

$$\Phi''(\phi) = -m^2\Phi(\phi)$$

$$\frac{1}{sin(\theta)}\frac{\partial}{\partial theta}\big(sin(\theta)\frac{\partial\Theta}{\partial\theta}\big) + \big(k - \frac{m^2}{sin^2(\theta)}\big)\Theta = 0$$

$$\frac{\partial^2}{\partial r^2}(rR) = \frac{2M}{\hbar^2}\big[U(r) + \frac{k\hbar^2}{2Mr^2} - E\big](rR)$$

(4) $\Phi(\phi) = e^{im\phi}$, $m = 0, \pm1, \pm2, ...$

(5) $L_Z = m\hbar$, $m = l, l-1, ...l$

(6) m gives the angular momentum around the z-axis

(7) l given the magnitude of the angular momentum $|L| = \sqrt{l(l+1)}\hbar$

- The Energy Levels of the Hydrogen Atom:

(1) Allowable Energies: $E = -\frac{E_R}{n^2}$

(2) Choose n. n creates allowable values for l, $l < n$, $l \in \mathbb{Z}$. l creates allowable values for m, $|m| \leq l$, $m, l \in \mathbb{Z}$.

(3) Code Letters for l: s - 0, p - 1, d - 2, f - 3, g - 4, ...

(4) Degeneracy without spin m_s is n^2

- Spin:

(1) Spin quantum number: $s = \mathbb{N} * \frac{1}{2}$

(2) s determines the magnitude of the spin, $S = \frac{h}{2\pi}\sqrt{s(s+1)}$

(3) m_s determines the spin around the z-axis. $S_z = m_s\hbar$

(4) Degeneracy with spin $m_s = 2n^2$

(3) Total Angular Momentum: $J = L + S$