Tour de TNB:
Formal Proposal of Stage Safety Requirements

John Dunn and Molly Graber

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0 The Authors

Figure 1: **Racing Analyst John Dunn**

Graduating in first place from University of Zoomsabarba in California, John (Mad Max) Dunn is world renowned for his theory of S.P.E.E.D, a revolutionary paper on how the universe looks "in the fast lane." Ironically, Dr. Dunn, along with his pet sloth, enjoy long strolls in the park and napping.

John was chosen as an analyst for the Tour de TNB because of his seemly supernatural ability to see speed.

Figure 2: **Safety Analyst Molly Graber**

Graduating top of her class from the University of Wearahelmet in Norway, Miss Graber is world renowned for her work in the crashpad tech industry. Ironically, Graber enjoys to free solo climb and has scaled Halfdome in Yellowstone National Park numerous times.

Molly was chosen as an analyst for the Tour de TNB because of her acute skills with Mathematica and her guacamole making.

1 Introduction

The Tour de TNB is, mathematically, the best cycling event in the world. Its' combination of physically taxing stages and amazing NASCAR-like wrecks attracts viewers from all over. However, in recent years, health insurance premiums for riders have risen due to the speed at which the athletes travel into corners. In an effort to combat the hike in premiums, race coordinators took it upon themselves to improve the tour safety. By hiring us, they took the first step in the right direction.

Our job would be to make the adrenaline pumping routes as safe as possible for riders and spectators alike. To do this, we would place a set of hay bales on the outside of the most dangerous corner, and a feed zone six minutes long on the straightest section of road.

Though the spectators are allowed to be anywhere on the course, they typically stay within 1 km of each of the villages. Within that 1 km region, fans gravitate towards the corners, where the skill of the athletes shines through. As a result, the hay bales could not be placed within 1 km of either Chateau de Chauchy or Chateau de Laplace.

(a) Hay bales used quite appropriately as a safety device.

(b) Hay bales NOT used appropriately as a safety device.

2 The Stage Route and Spectator Viewing areas

When we were first given the parameters of stage route Figure 8, we were skeptical. We were making the assumption that all riders would be going the same speed, which is unrealistic given different technologies, race strategies, etc. Additionally, we wondered how on earth could there be a completely flat bike course on the Tour de TNB. Ignoring this, we looked at the route more closely, and other than the high curvature at (Lat: 3.5, Long: 6), the route seemed a reasonable enough curve for a race. After plotting the basic curve, we added the locations of Chateau de Chauchy, Pont de Pascal, and Chateau de Laplace. We set the start a Chateau de Chauchy to t = 0 and the finish at Chateau de Laplace to t = 1 hour. This would make it easy to see exactly how far we were along the course. Using this scale, we determined that at a time **t = 0.476 hrs**, the riders should be crossing the Pont de Pascal.

$$r(t) = 6 * cos(2.98\pi t)\mathbf{i} + (cos(3.5\pi t) + 5t)\mathbf{j} \tag{1}$$

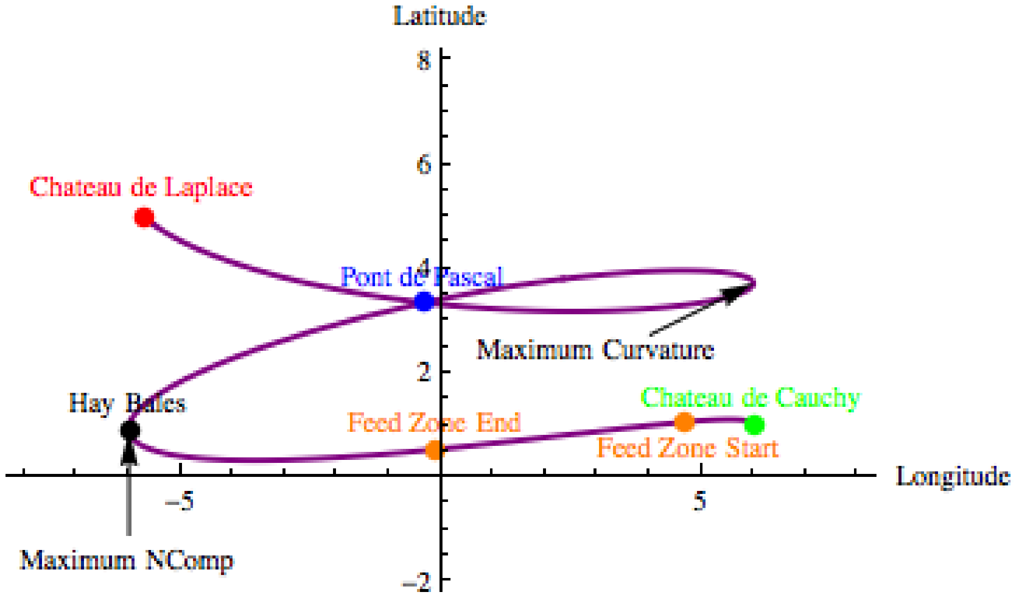

Figure 4: Overview of the route including the start point at Chateau de Chauncy, the Pont de Pascal, the finish at Chateau de Laplace, and Safety Requirements.

3 Analysis of Riders on the Stage

It was important to realize the speed at which riders were traveling down the course. With the information shown in Figure 5, we could see exactly where the riders were being more cautious. It might be inferred that the time of lowest speed would correspond with the sharpest turn.

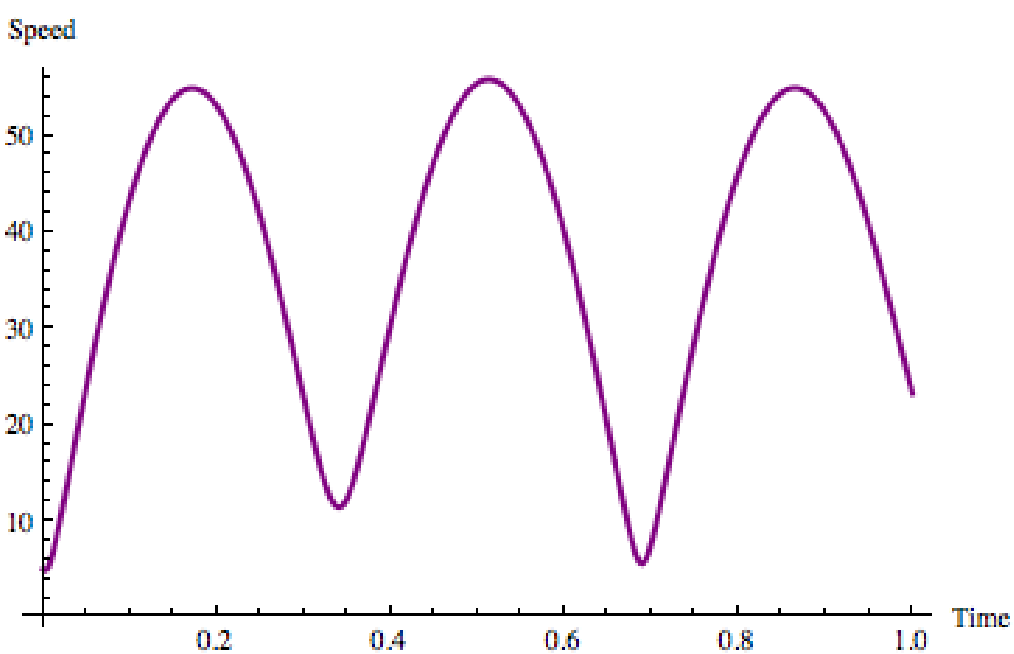

Figure 5: The (projected) speed of the rider along the course as a function of time.

Another interesting piece of data was the average speed of the riders. To determine the average speed, we integrated the speed and divided by the change in time, shown in Equation 2.

$$\overline{Speed} = \int_0^1 |\underline{v}|\mathrm{d}t = \mathbf{37.1789}\ [\frac{\mathbf{km}}{\mathbf{hr}}] \tag{2}$$

Since the time to complete the stage is 1 hour, and we know that the riders average 37.1789 $[\frac{km}{hr}]$, we can determine that the length of the race is 37.1789 km.

$$Stage\ length = 37.1789\ [\frac{km}{hr}] * 1\ [\frac{hr}{1}] = \mathbf{37.1789}\ [\mathbf{km}]$$

Using the arc length formula, we concluded the same stage length.

$$\int_0^1 |\underline{v}|\mathrm{d}t = \mathbf{37.1789}\ [\mathbf{km}]$$

The while the length of the stage might be 37.1789 km, the distance as the crow flies is very different. To calculate the point to point distance, we simply used the distance formula, given in Equation **??**. In our case, (x_0, y_0) and (x, y) are the Longitude and Latitude of Chateau de Chauchy and Chateau de Laplace respectively.

$$Distance = \sqrt{(x - x_0)^2 + (y - y_0)^2} = \mathbf{12.3709}\ [\mathbf{km}]$$

4 Analysis Forces Experienced on the Stage

Speed is a key component of any rider's race, but other aspects of like curvature of the course and the normal component of acceleration contribute a great deal to the rider's experience. The magnitude of curvature corresponds to the tightness of corners and the magnitude of the normal component of acceleration corresponds to the centripetal force felt by riders going around an arc. From our Calc 3 class, we knew that the curvature of a path is given by Equation 5.

$$\hat{T} = \frac{\underline{v(t)}}{|\underline{v(t)}|} \tag{3}$$

$$\hat{N} = \frac{\hat{T}'(t)}{|\hat{T}'(t)|} \tag{4}$$

$$Curvature = k = \frac{|\hat{T}'(t)|}{|\underline{v(t)}|} \tag{5}$$

Plotting the curvature of the stage, we noticed that peaks in curvature coincided with sharp turns and low speeds. This was to be expected, as riders most likely would slow going into sharp corners. Overall, the plot of curvature told us that the course was very straight except for a few sharp corners at $t \approx 0.35\ hrs$ and $t \approx 0.7\ hrs$.

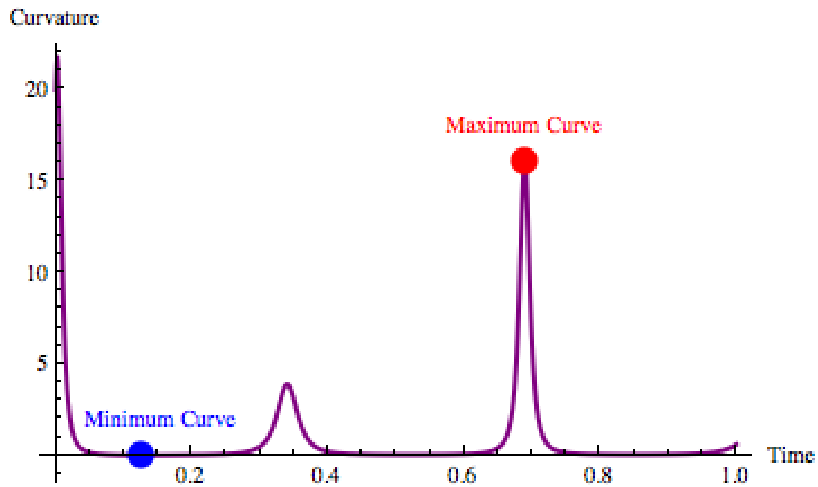

Figure 6: The (projected) curvature of the course as a function of time.

The time of maximum curvature was found using the Mathematica function FindMaximum[ ] and came out to be:

$$t_{max\ k} = \mathbf{0.6889hrs}$$

$$\textit{Point of maximum curvature on course}: \ (\mathbf{Long}:\mathbf{5.9998},\ \mathbf{Lat}:\mathbf{3.7200})$$

Another mathematical component that indicates force is the normal component of acceleration. The normal component of acceleration indicates how hard the riders are turning into a turn. As we know acceleration can be given by both tangental and normal vector components. The tangential component is the analogous to being pushed into the back of you seat when you press hard on the pedals. The normal component is analogous to the centripetal force of taking a bike around a corner. For our purposes, we focused on the normal component of acceleration, given by Equation 7.

$$\underline{a} = \frac{d|\underline{v}|}{dt}\hat{T} + \kappa|\underline{v}|^2\hat{N} \tag{6}$$

$$\underline{a}_N = \kappa|\underline{v}|^2 \tag{7}$$

Looking at the graph of the normal component of acceleration, we noticed similarities between it and the graph of curvature. **The time of the maximum values were not the same**. This is because the speed of the riders around the first corner is larger than than their speeds around the second corner. In relation to the speed graph, the normal component graph seems to increase when the speed of the riders decrease. Again, this makes sense because as riders enter sharper turns, they slow down, but still feel a normal component force. One thing worth noting is that the scale of the normal component is much larger than the scale of curvature.

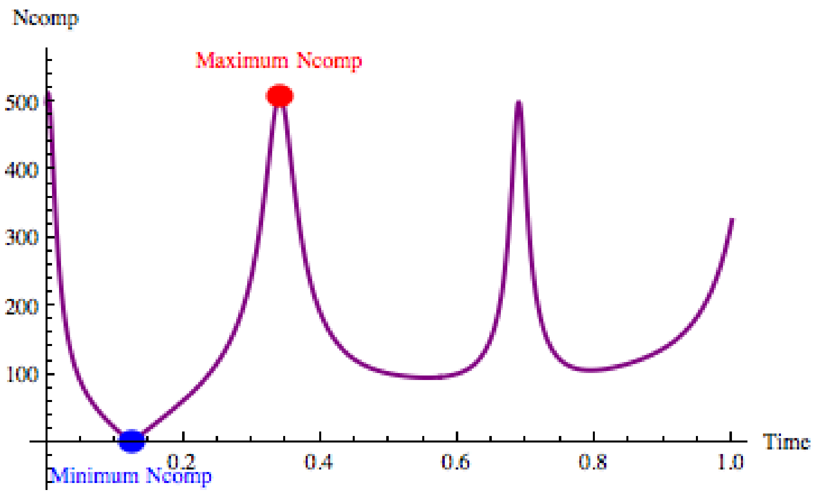

Figure 7: The (projected) normal component of acceleration of the course as a function of time.

There is no way to change the curvature of the route, as it is a property of the arc length rate of change of direction, and does not depend on the speed of the rider.

One way that the peak of the normal component of acceleration would not coincide with a sharp turn on the course is if the racers went much faster on the slight curves and much slower on the tight curves of the course.

The time of maximum $\hat{N}$ component of acceleration was found using the Mathematica function FindMaximum[ ] and came out to be:

$$t_{max\ Ncomp} = \mathbf{0.3403hrs}$$

$$Point\ of\ maximum\ N_{comp}\ on\ course: \ (\mathbf{Long}: \mathbf{-5.9950},\ \mathbf{Lat}: \mathbf{0.8768})$$

5 Safety Requirements for the Stage

The first requirement for the safety devices, including the feed zone and hay bale wall, was that they could not be within the most popular spectating areas. Here we defined the most popular spectating areas as areas within 1 km of either village. If the length of the stage was 37.1789, we wanted to know at what times the riders would reach 1 km (1 km outside of Chateau de Chauchy), and 36.1789 km (1 km before Chateau de Laplace). Setting the arclength equation to 1 km and 36.1789 km, we were able to solve for the respective ride times.

$$1\ [km] = \int_0^t |\underline{v}| \mathrm{d}t, \quad \mathbf{t = 0.0630\ [hr]} \tag{8}$$

$$36.1789\ [km] = \int_0^t |\underline{v}| \mathrm{d}t, \quad \mathbf{t = 0.9659\ [hr]} \tag{9}$$

In finding these times, we had a window in which we could place safety devices. **The hay bales should be placed at the maximum $\hat{N}$ component of acceleration**. We chose to use the maximum $\hat{N}$ component as a point instead of the maximum curvature because the $\hat{N}$ component is indicative of the forces the riders are under; Curvature is a property of the route and can result in varying forces depending on the speed of a rider.

$$t_{Hay\ Bales} = \mathbf{0.3403hrs}$$

$$Hay\ Bales:\ (\textbf{Long}: \mathbf{-5.9950},\ \textbf{Lat}: \mathbf{0.8768})$$

The feed zone should be placed at the minimum curvature, or straightest part of the route, within the safety device region.

$$t_{min\ k} = 0.1240hrs$$

$$Point\ of\ minimum\ curve\ on\ course:\ (Long: 2.5546,\ Lat: 0.8251)$$

If we know that the window for safety regions is (0.0630 hrs, 0.9659 hrs), then we can test if the minima of curvature at $t_{min\ k} = 0.1240\ hrs$ can be used for a feed zone. To do this, we subtract 3 minutes = 0.05 hrs from $t_{min\ k} = 0.1240\ hrs$. This gives us the lower bound of the feed zone.

$$t_{FZlowerBound} = 0.1240 - 0.050 = 0.074$$

Since $0.074 > 0.0630$, and the upper bound is before the first curve or the route, the feed zone can be centered at the point of minimum curve.

$$t_{FZStart} = \mathbf{0.074\ hrs}$$

$$t_{FZEnd} = \mathbf{0.174\ hrs}$$

$$Feed\ Zone\ Start:\ (\textbf{Long}: \mathbf{4.6825},\ \textbf{Lat}: \mathbf{1.0564})$$

$$Feed\ Zone\ End:\ (\textbf{Long}: \mathbf{-0.0943},\ \textbf{Lat}: \mathbf{0.5335})$$

6 Conclusion

After many hours of measuring track and doing racing simulations, our team was reaching its mental limit of calculus. Just kidding, that will never happen. In the end we investigated the projected speed of riders, normal component of acceleration on riders, and curvature of the route. From this information we determined the point where hay bales should be placed and where a feed zone would be stationed.

Point	Time (Hr)	Longitude	Latitude
Hay Bales	0.3403	-5.9998	0.8768
Feed Zone Start	0.0740	4.6825	1.0564
Feed Zone End	0.1740	-0.094	0.5335

Table 1: Coordinates and times of the safety devices to be placed.

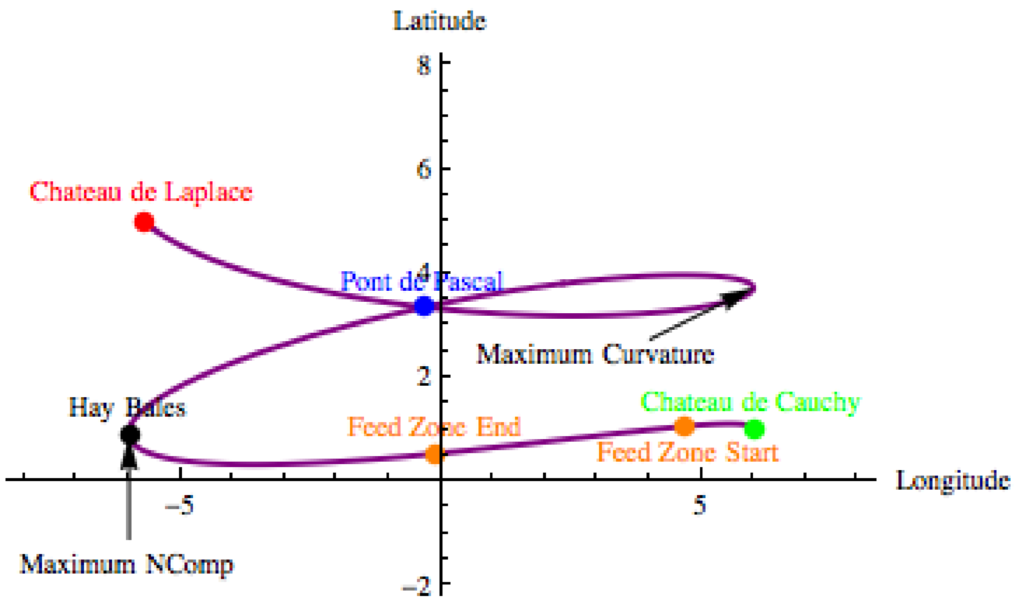

Figure 8: Overview of the route including the start point at Chateau de Chauncy, the Pont de Pascal, the finish at Chateau de Laplace, and Safety Requirements.