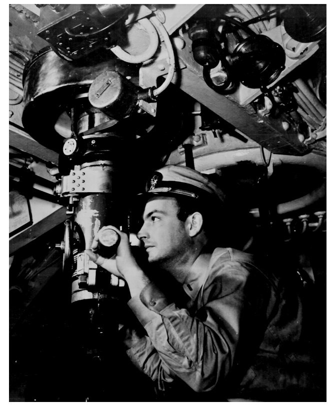
1 Introduction

A periscope is an optical system which is typically used to view objects from a taller point of view. For example, if a soldier was entrenched, but wished to view the battlefield above his/her head, a periscope would be the perfect tool. Similarly, naval submarines utilize sophisticated periscope systems to see above the water.

(a) Soldier using a periscope to see above a trench. [1]



(b) Naval officer using a periscope to see above the water.[2]

Figure 1

For trench periscopes, shown in Fig 1b, the optical system is typically very simple; No more than two mirrors at 45° on the ends of a tube. A similar system is shown in Fig 2a. On the other hand, submarine periscopes can be much more complicated. Partially, this is due to the restrictions of the optical system. In a trench, the tube length of the periscope is typically less than 1 meter. In a submarine, the tube length can be considerably larger, ranging from 3 to 5 meters long. This restriction, along with many others, complicates the optical system for submarine periscopes.

In this project, I will be attempting to design a medium complexity periscope. Here are some basic assumptions I will make of the system:

- The system will only have to work in the day time. (i.e brightness will not be an issue)
- The height of the system should be around 1.5 meters. When held, the user should be able to see over a 3 meter high wall.
- The system must have a magnification level similar to a $40 pair of binoculars (e.g $M_{ang} = 20$).

2 Design

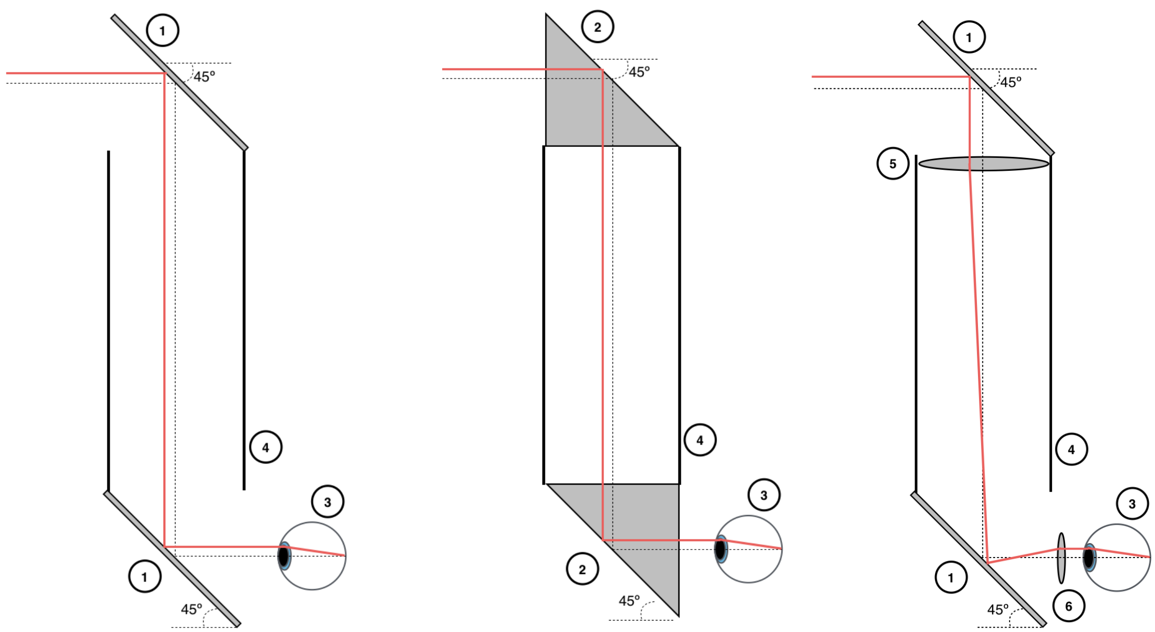
2.1 Basic Designs

In Fig. 2, there are a few of the simplest persicope designs. Fig. 2a utilizes two mirrors to fold (reflect) the incoming rays to the eye of the user. The right angle prism periscope shown in Fig. 2b produces the same effect, but through refraction. One might choose the prism based design if the system requires accurate optomechanical placement, or, if scratching of optical surfaces might be an issue. Prisms are easier to properly align with fittings, and are less susceptible to image deformation from minor scratches. However, prisms are much worse when it's comes to chromatic abberations. In fact, mirrors produce no aberrations at all. The advantages of systems like these two in Fig. 2 are: cost, weight, and simplicity. However, if angular magnification of objects is a requirement, then these systems would not suffice.

The simplest way of creating angular magnification is shown in Fig.2c First, a large objective lens is placed at the top of the periscope tube, Fig.2c(5) with an extremely long effective focal length. Second a smaller, matched lens, Fig.2c(6), is placed after the second mirror. By matching, separating the two lenses by the sum of their effective focal lengths, the two lenses maintain the afocal nature of the system, while providing a angular magnification, given by:

$$M_{ang} = \frac{1}{M_{lin}} = \frac{\text{Height of Exiting Ray}}{\text{Height of Incoming Ray}} = \frac{h_0}{h_k}$$

Notice that angular magnification is the inverse of linear magnification.



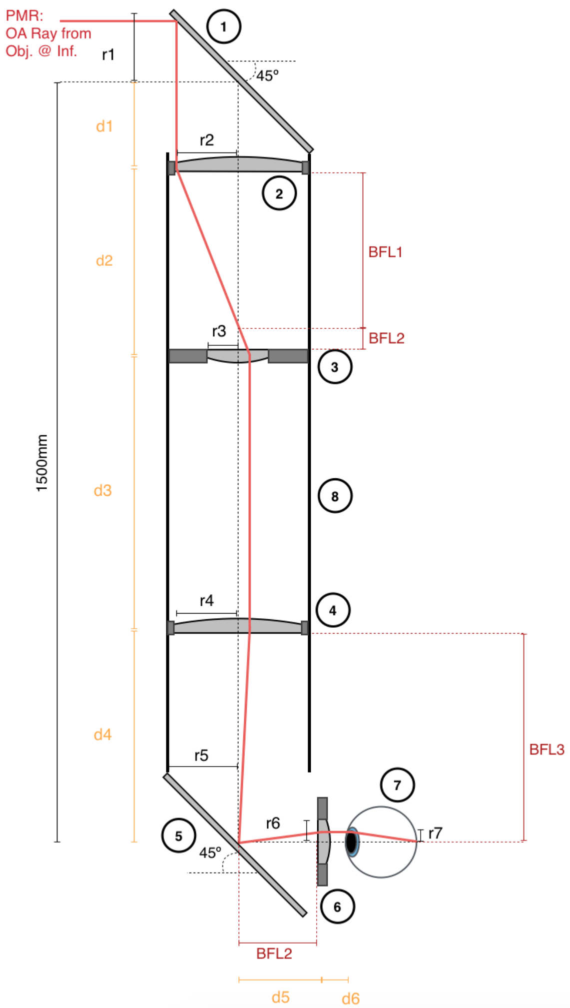
(a) Basic Mirror Periscope. (b) Basic Prism Periscope. (c) Mirror Periscope with linear magnification.

Figure 2: Some basic periscope designs: (1) Elliptical Mirrors, (2) Prisms, (3) Eye, (4) Periscope Tube, (5) Objective Lens, (6) Eye Lens.

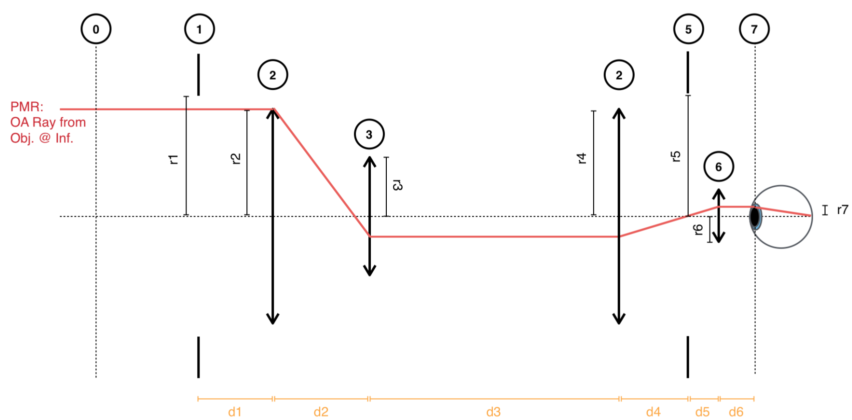
2.2 Chosen Design

The design shown in Fig.2c is close to the system requirements, but it fails to satisfy the length requirement of the periscope tube, which is approximately 1.5 meters. Unfortunately there is not a cost effective lens that: a) Has an effective focal length of over 1 meter, and, b) Has a diameter that would fit into a reasonable sized tube. The solution is to add two relay lenses to the system, shown in Fig.3.

The objective lens, Fig.3(2), is matched with Relay Lens 1, Fig.3(3), and Relay Lens 2, Fig.3(4), is matched with Eye Lens, Fig.3(6). Essentially, the whole periscope system is comprised of two afocal subsystems. The first provides a moderate amount of linear magnification over a relatively short distance, while the second provides additional linear magnification such that the outgoing rays of the system are pupil matched to a human eye.



(a) Basic Design: (1) Elliptical Mirror 1, (2) Objective Lens, (3) Relay Lens 1, (4) Relay Lens 2, (5) Elliptical Mirror 2, (6) Eye Lens, (7) Eye, (8) Periscope Tube.



(b) Unfolded System: (0) Object Plane (1) Elliptical Mirror 1 Aperture, (2) Objective Lens, (3) Relay Lens 1, (4) Relay Lens 2, (5) Elliptical Mirror 2 Aperture, (6) Eye Lens, (7) Image Plane

Figure 3: Red text: Back Focal Lengths of lenses, Orange text: distances between principle planes of elements, Black text: heights of elements.

3 Specification

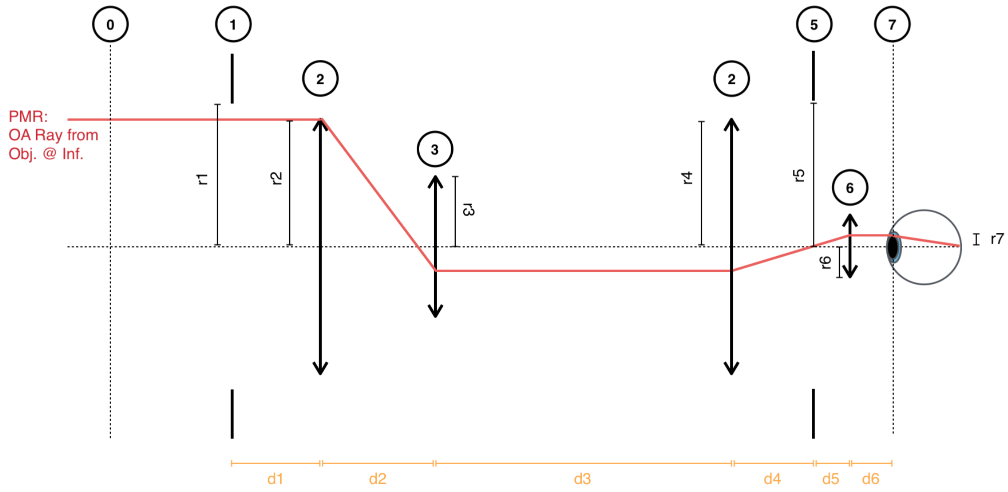


Figure 4: *Note*: r_i indicates the radius of an aperture or lens at a plane. h_i indicates the height of a ray at a plane. u_i' indicates the refracted ray angle at a plane.

Some approximations:

- **Pupil Diameter Approximation** - The average human pupil dilates from 3mm to 8mm in diameter ($r_7 = 1.5mm - 4mm$ in radius). Since this system is specified for use in the daytime, this means that $r_7 \approxeq 2mm$. To achieve pupil matching, the PMR, an optical axis ray from an object at infinity, must exit the system parallel to the optical axis with a height $h_7 = r_7 \approxeq 2mm$.

- **Observation Distance** - Obviously, the distance from the Eye Lens to the Eye, d_6, cannot be zero. Otherwise, the user would have their eye directly in contact with the eye lens. 2cm is a reasonable distance to hold one's eye from the eye lens, so $d_6 = 20mm$.

- **Refraction Index** - The system is in air (i.e. $n = 1$).

The primary specification for this system is linear magnification, given by:

$$M_{ang} = \frac{1}{M_{lin}} = \frac{h_0}{h_7}$$

Since h_7 is fixed by the Pupil Diameter Approximation at $h_7 \approxeq 1.5mm$, it is required that $h_0 \geq 30mm$. After a bit of research, I found a lens with radius, $r_2 = 50mm$, and power, $k = 5$ (See Fig. 5a). With optomechanical fittings taken into account, the lens has an effective radius of $r_2 = 49mm$. With this lens, the magnification of the system is given by:

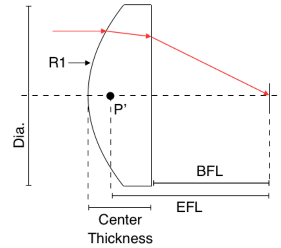
$$M_{ang} = \frac{1}{M_{lin}} = \frac{h_0}{h_7} = \frac{49mm}{1.5mm} = 32.6$$

Using the objective lens as a basis for the system, I found similar planoconvex lenses that limited vignetting, and maximized linear magnification and angular field of view (given by the $2 \times u_1'$ of the PPR). Luckily, I was able to find lenses that did not require me to create a field stop to limit vignetting. After selecting the lenses, the angular magnification of the system had changed to:

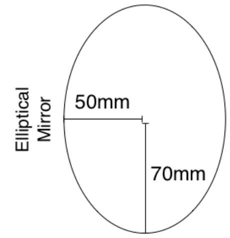
$$M_{ang} = \frac{1}{M_{lin}} = \frac{h_0}{h_7} = \frac{49mm}{2.45mm} = 20$$

	Objective Lens [4]	Relay Lens 1 [5]	Relay Lens 2 [6]	Eye Lens [7]
Type	Plano-Convex	Plano-Convex	Plano-Convex	Plano-Convex
Diameter (mm)	100.0000	50.0000	100.0000	30.0000
EFL (mm)	200.0000	50.0000	300.0000	60.0000
K (m^-1)	5.0000	20.0000	3.3333	16.6667
BFL (mm)	189.0000	43.2800	291.0000	56.0300
Center Thickness (mm)	17.0000	12.0000	12.5000	6.0000
Radius R2	103.5000	39.2400	155.0000	31.0100
f/#	2.0000	1.0000	3.0000	2.0000
NA	0.2500	0.5000	0.1700	0.2500

(a) Lenses of the system.



(b) Lens Specification Reference



(c) Elliptical Mirrors Specification

Figure 5: System Elements Specifications

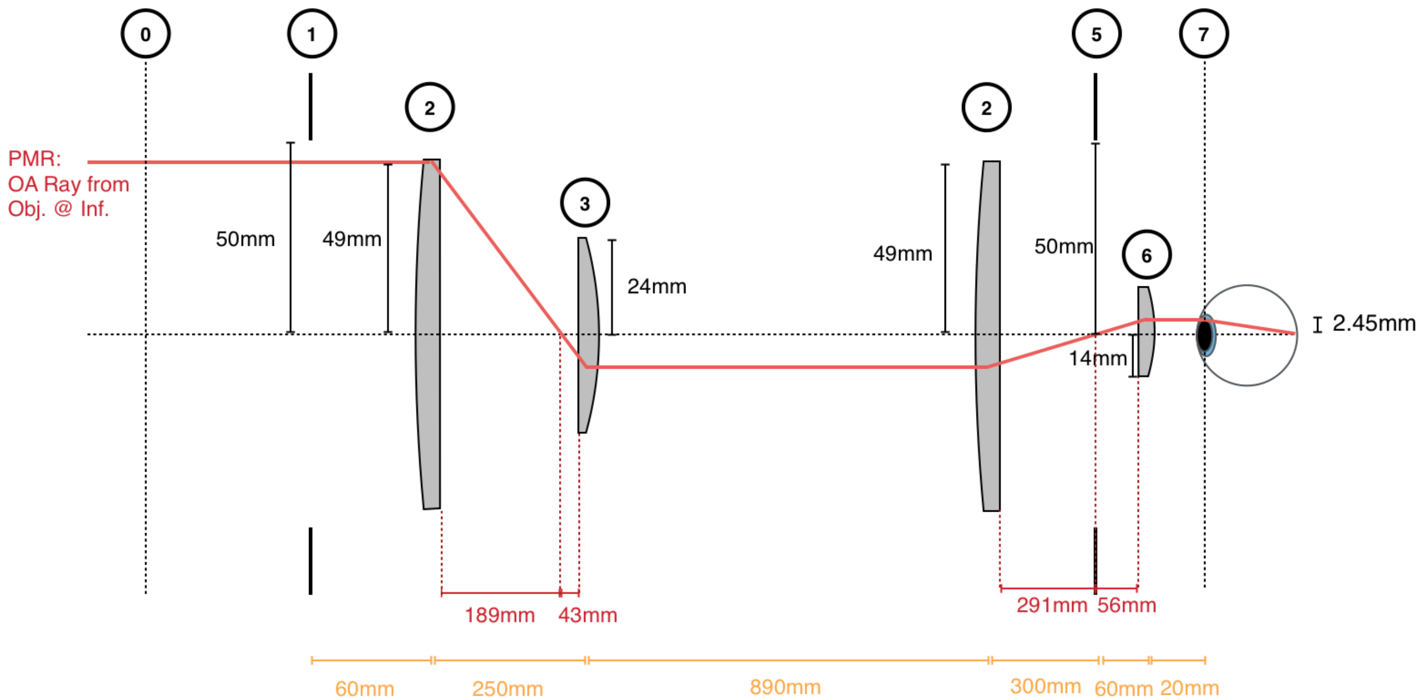


Figure 6: The Optical System: Unfolded and Dimensioned. (0) Object Plane, (1) Elliptical Mirror 1 Aperture, (2) Objective Lens, (3) Relay Lens 1, (4) Relay Lens 2, (5) Elliptical Mirror 2 Aperture, (6) Eye Lens, (7) Image Plane. Red text: Back Focal Lengths of lenses, Orange text: distances between principle planes of elements, Black text: heights of elements.

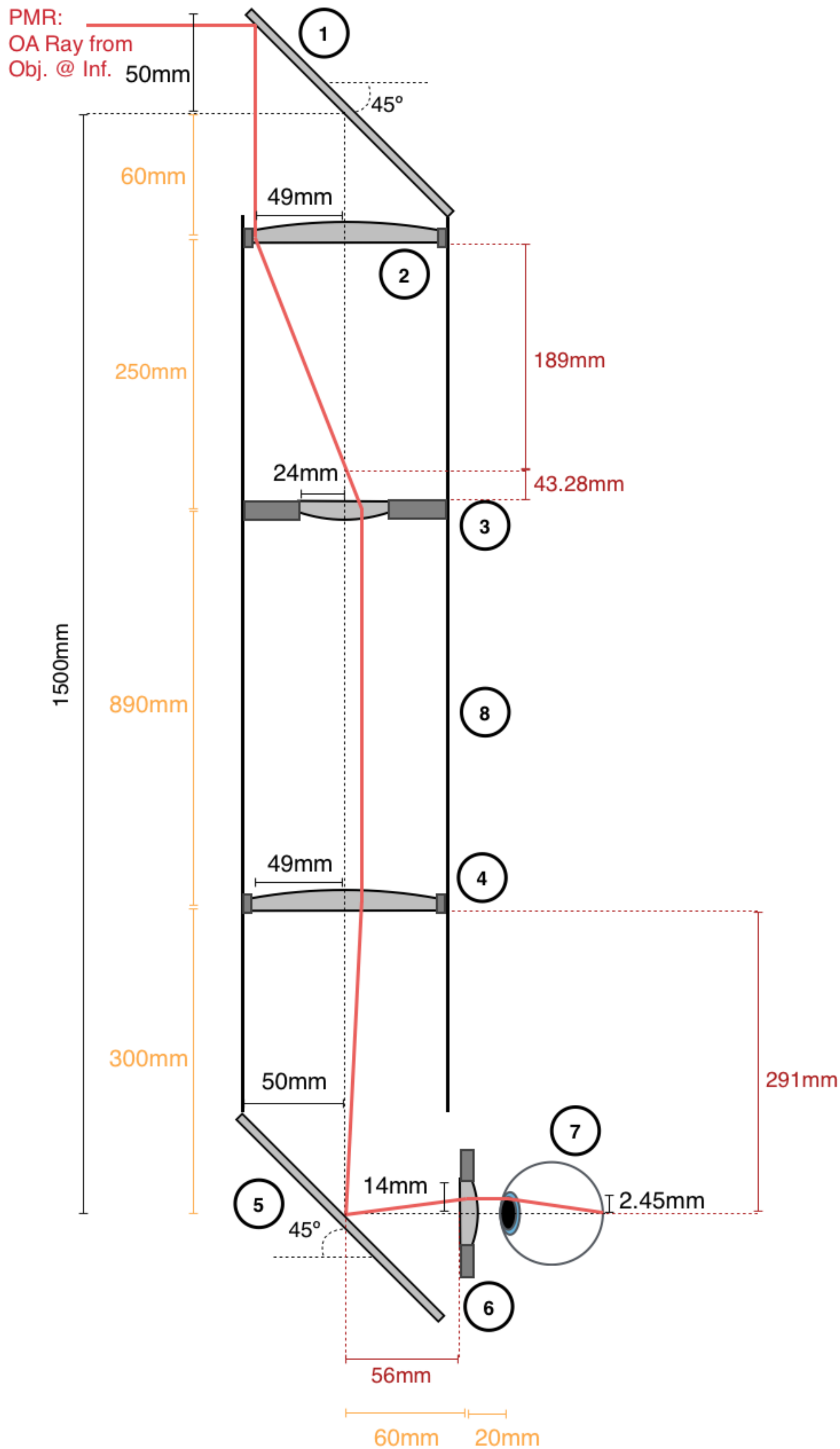
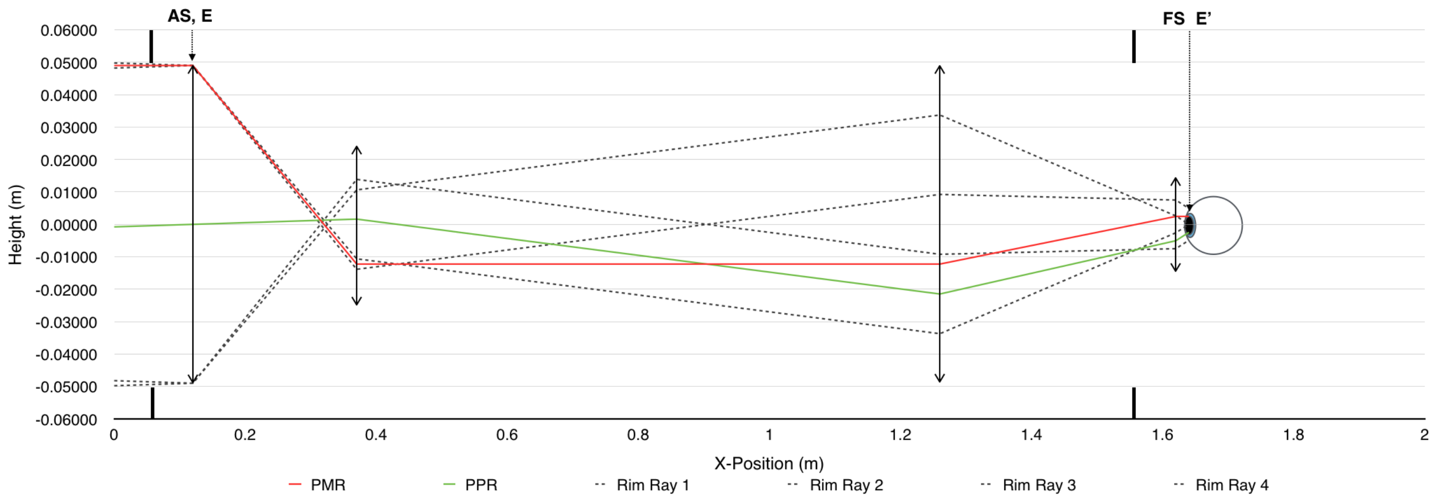


Figure 7: The Optical System: Folded and Dimensioned. Note: (1) Elliptical Mirror 1, (2) Objective Lens, (3) Relay Lens 1, (4) Relay Lens 2, (5) Elliptical Mirror 2, (6) Eye Lens, (7) Eye, (8) Periscope Tube. Red text: Back Focal Lengths of lenses, Orange text: distances between principle planes of elements, Black text: heights of elements.

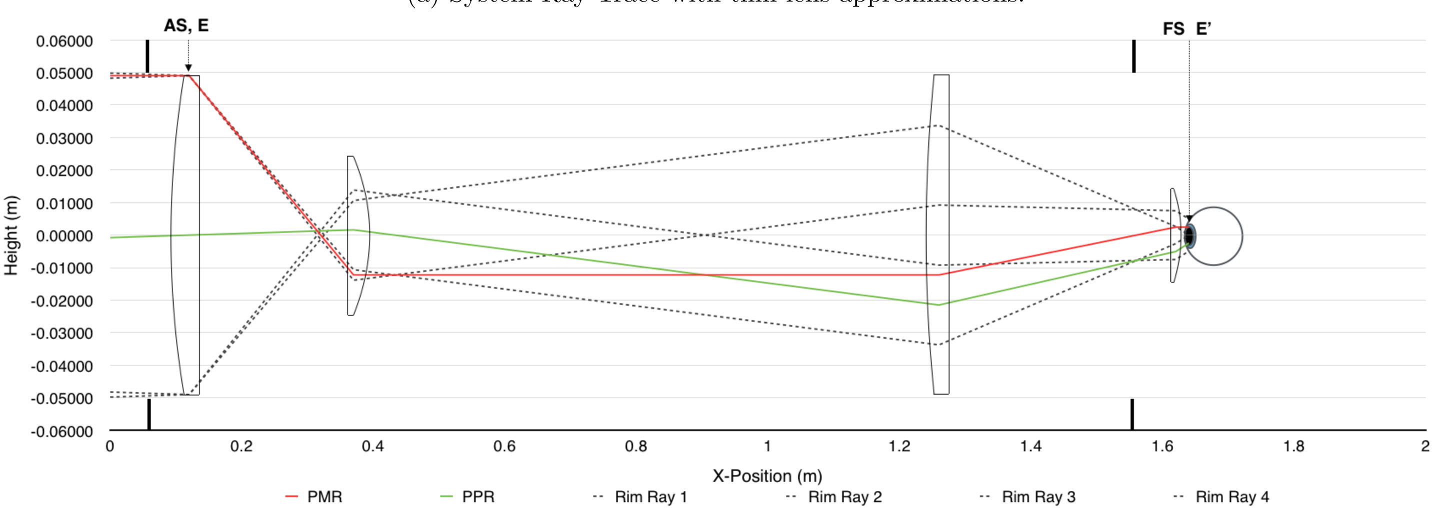
4 Ray Tracing

Using the tabular ray tracing method, and the specifications laid out in the previous section, I was able to generate the following plots. Note that the aperture stop and entrance pupil are at the objective lens, field stop and exit pupil are at the eye plane. Additionally, note that the orientation of the plano-convex lenses in Fig.8b minimizes chromatic aberrations by facing the convex side towards the flat rays, and the flat side towards the bent rays.

Since I had picked actual planoconvex lenses from a catalog, I knew the location of each lens' principle plane, and the back focal length. In order to make the transition from thin lens approximation to thick lens, the principle plane of each lens should be placed in the same plane as it's corresponding thin lens approximation. By doing this, the thick lens system would more or less be identical to it's thin lens approximation.



(a) System Ray Trace with thin lens approximations.



(b) System Ray Trace with thick lenses (Not to scale). Note that the rays are bent at the principle planes of the lenses.

Figure 8: Ray traces of the system. Ray Trace Tables can be found in the Appendix.

5 Conclusion

Using the specifications in the previous section, I have designed a periscope with the following specifications:

Specification	Value
Angular Magnification	20.00000
Linear Magnification	0.05000
Angular Field of View (rad)	0.01298
Angular Field of View (deg)	3.67001
Height of System (m)	1.5

Figure 9

Note that the angular field of view is 3.67 degrees. This means that when looking through the eye lens, a 12.9m tall object 1000m away will fill the field of view, and appear to be 258m tall. Comparatively, the average bird watching binoculars will have an 8 degree field of view [3]. This means that, for a 20X pair of birding binoculars, with 8 degree angular field of view, a 139m tall object 1000m away will fill the field of view, and appear to be 2780m tall.

The difference in angular field of view between the two designs can be attributed to the large optical path length of the periscope. For future designs, I might consider implementing a second set of relay lenses to bend the PPR closer to the optical axis. This would effectively increase the angular field of view.

That being said, the design presented here meets all the system requirements set forth, and provides a reasonable angular field of view. I can therefore call it a successful design.

6 Appendix

Plane Name	Start Plane	Mirror Aperature	Objective Lens	Relay Lens 1	Relay Lens 2	Mirror Aperature	Eye Lens	Eye
	0	1	2	3	4	5	6	7
K	0.00000	0.00000	5.00000	20.00000	3.33333	0.00000	16.66667	N/A
di	0.06000	0.06000	0.25000	0.89000	0.30000	0.06000	0.02000	N/A
hi	0.04900	0.04900	0.04900	-0.01225	-0.01225	-0.00000	0.00245	0.00245
ui’	0.00000	0.00000	-0.24500	-0.00000	0.04083	0.04083	0.00000	N/A
ri	N/A	0.05000	0.04900	0.02400	0.04900	0.05000	0.01000	0.00245
hi/ri	N/A	0.98000	1.00000	-0.51042	-0.25000	-0.00000	0.24500	N/A

(a) PMR - Ray Trace Table

Plane Name	Start Plane	Mirror Aperature	Objective Lens	Relay Lens 1	Relay Lens 2	Mirror Aperature	Eye Lens	Eye
	0	1	2	3	4	5	6	7
K	0.00000	0.00000	5.00000	20.00000	3.33333	0.00000	16.66667	N/A
di	0.06000	0.06000	0.25000	0.89000	0.30000	0.06000	0.02000	N/A
hi	-0.00078	-0.00039	0.00000	0.00162	-0.02148	-0.00779	-0.00505	-0.00245
ui’	0.00649	0.00649	0.00649	-0.02596	0.04565	0.04565	0.12980	N/A
ri	N/A	0.05000	0.04900	0.02400	0.04900	0.05000	0.01400	0.00245
hi/ri	N/A	-0.00779	0.00000	0.06760	-0.43841	-0.15576	-0.36066	N/A

(b) PPR (Chief Ray) - Ray Trace Table

Figure 10: Notable ray trace tables for the system.

References

[1] ”Trench Coat.” Word Histories. N.p., n.d. Web. 26 Apr. 2016.

[2] ”About Us.” Periscope Digital. N.p., n.d. Web. 26 Apr. 2016. .

[3] Porter, Michael, and Diane Porter. ”Birding Binoculars.” 5. N.p., n.d. Web. 26 Apr. 2016.

[4] ”100mm Dia X 200mm Focal Length, PCX Condenser Lens.” 100mm Dia X 200mm Focal Length, PCX Condenser Lens. Edmund Optics, n.d. Web. 26 Apr. 2016.

[5] ”50.0mm Dia. X 50.0mm FL, Uncoated, Plano-Convex Lens.” 50.0mm Dia. X 50.0mm FL, Uncoated, Plano-Convex Lens. Edmund Optics, n.d. Web. 26 Apr. 2016.

[6] ”100mm Dia X 300mm Focal Length, PCX Condenser Lens.” 100mm Dia X 300mm Focal Length, PCX Condenser Lens. Edmund Optics, n.d. Web. 26 Apr. 2016

[7] ”30.0mm Dia. X 60.0mm FL, Uncoated, Plano-Convex Lens.” 30.0mm Dia. X 60.0mm FL, Uncoated, Plano-Convex Lens. Edmund Optics, n.d. Web. 26 Apr. 2016.