Project 2- Solar Panels and Optimization

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1 Introduction

Solar Power Inc., the massive energy conglomerate based out of Boulder, Co, has chosen us to investigate how the efficiency of a solar panel depends on the season and its orientation, and make design recommendations for their new fields in the pacific island of Suluclac, which lies directly on the equator.

To tackle this massive problem, we had to break it up into parts. The solar radiation falling on an angled plane, such as a solar panel, depends on many factors including the orientation of the plane, the time of day, the season, and the weather.

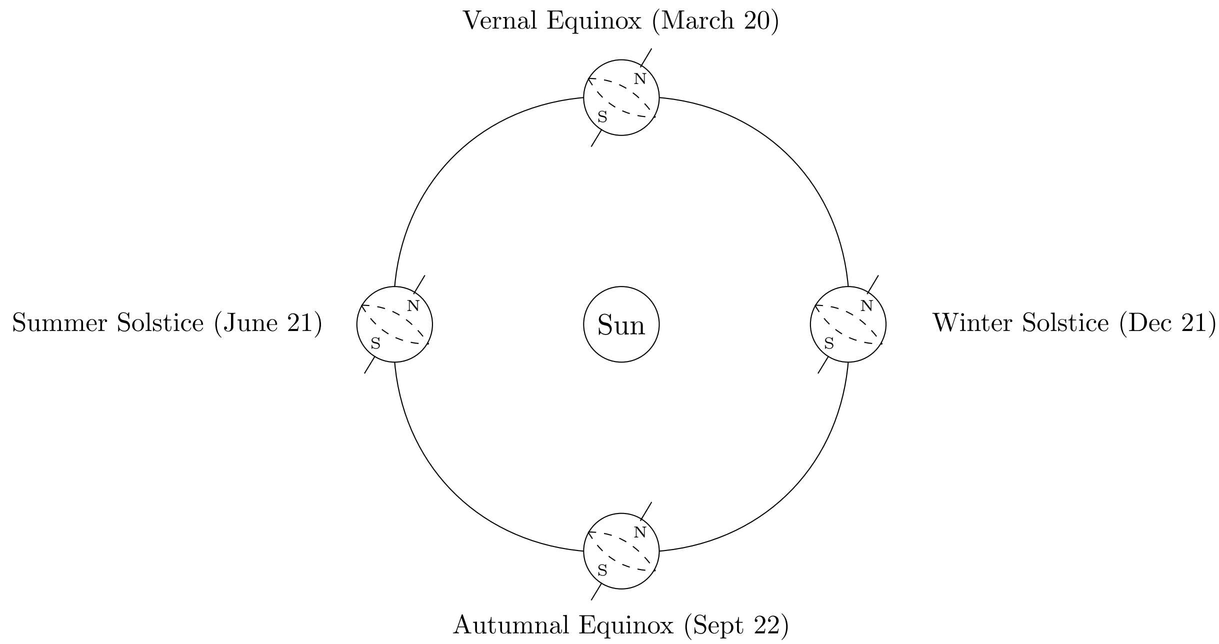


Figure 1: Earth orbiting the Sun and the relative axis tilt of Earth to the Sun. Notice that the north pole points away from the Sun during the northern hemisphere winter, and points toward the Sun during the northern hemisphere summer.

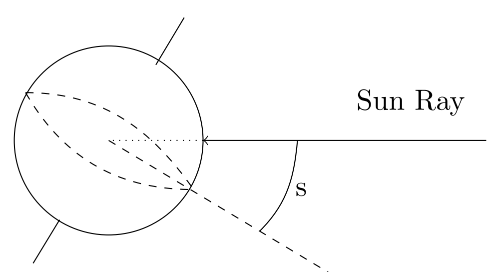


Figure 2: S is the angle between the plane of the equator and the sun ray at noon. By noon, we mean the time of the day when the Sun is at its highest position in the sky. At the summer solstice s = 23° and the vernal/autumnal equinox s = 0°.

Date	s	Increasing/Decreasing
Dec. 21, 2009	s = $-23°$	
Dec. 22, 2009 - March 19, 2010	$-23° < s \leq 0°$	Increasing with time
March 20, 2010 - June 20, 2010	$0° < s \leq 23°$	Increasing with time
June 21, 2010 - Sept. 21, 2010	$23° < s \leq 0°$	Decreasing with time
Sept. 22, 2010 - Dec. 20, 2010	$0° < s \leq -23°$	Decreasing with time

Table 1: Angle s for based on times of the year.

In investigating the most efficient use of solar panels on the remote pacific island Suluclac, we needed to know how solar radiation incident to a plane, I_P tilted at an angle u at time of day and year corresponding to t and s respectively is given by the function:

$$\boxed{\textbf{Intensity of Radiation: } = I_p(s,t,u) = I_0(cos(s)cos(u)cos(t) - sin(s)sin(u))}$$

$\mathbf{I_0}$: intensity of solar radiation **s**: angle between the sun at noon and the equator plane
t: an angle proportional to the time of day such that:
$t = -90° = -\frac{\pi}{2}$ radians at dawn
$t = 0°$ at noon
$t = 90° = \frac{\pi}{2}$ radians at dusk.
u: angle between the solar panel and the ground

The amount of energy collected from a solar panel is dependent on more than the angle at which sun rays strike the panel. It is also dependent on the absorption factor, which depends on the distance the sun's rays have traveled through the atmosphere before reaching the ground. Beer's law describes this with a function $\boxed{A(s,t)}$ such that $0 < A(s,t) \leq 1$. $A(s,t) = 1$ means all sunlight gets through atmosphere, or no absorption, and $A(s,t) = 0$ means no sunlight gets through, or complete absorption.

Another factor that varies the amount of radiation to reach the earth's surface is the relative cloudiness. We will describe the cloudiness in a way similar to Beer's Law, $\boxed{C(s,t)}$, where $0 < C(s,t) \leq 1$. $C(s,t) = 1$ means all sunlight gets through clouds, or no absorption, and $C(s,t) = 0$ means no sunlight gets through, or complete absorption.

In the case of Suluclac, the cloudiness is given by the equation:

$$\boxed{C(s,t) = \frac{3 - (1 + (s - 0.2)^2)cos^2 t}{3}}$$

From our function for cloudiness, $C(s,t)$, and Beer's law, $A(s,t)$, we can conclude that the total amount of energy received per square meter and per day is given by the equation:

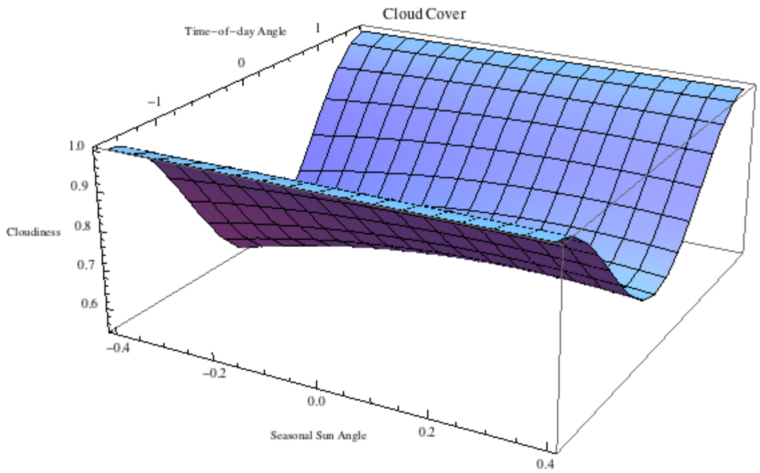
$$W(s,u) = \int_{t_{min}}^{t_{max}} A(s,t)C(s,t)I_p(s,u,t)dt \qquad t_{min} = -\frac{\pi}{2} \qquad t_{max} = \frac{\pi}{2}$$

In the case of Suluclac, the energy collected by the solar panel each day is given by the equation:

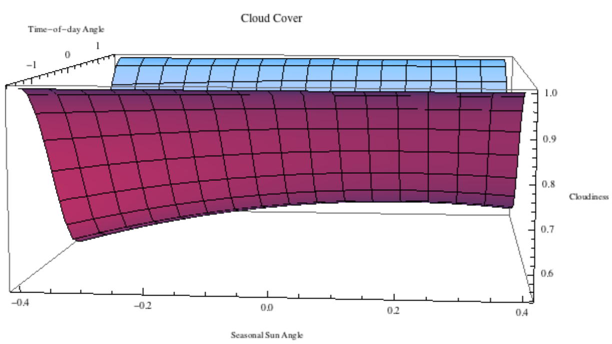
$$\boxed{W(s,u) = 1 + (1 + 0.65s - 1.2s^2 - 0.4s^3 + 0.35s^4)cos(u) + (1.4s - 0.4s^2 - 1.5s^3 - 0.35s^4)sin(u)}$$

2 Meteorology

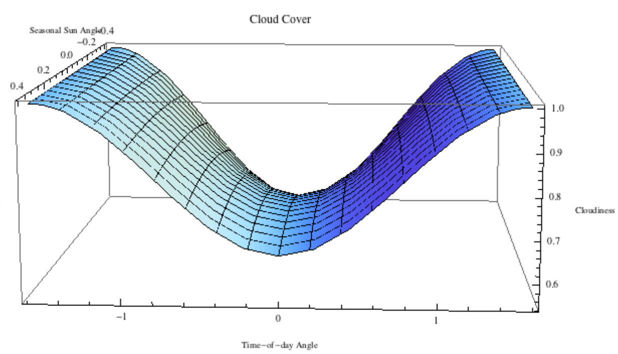
First, we did a study on the cloud cover of Suluclac, varying both the time of day angle and the seasonal sun angle.



(a) 3D representation of cloud cover as a function of time-of-day angle and seasonal sun angle.



(b) In early seasons, the cloudiness is very low. Cloud cover peaks around 2/3 the way through a year, then makes a small downturn during the end of the year.



(c) During the course of a day, the cloud cover peaks at night time and is lowest around the middle of the day.

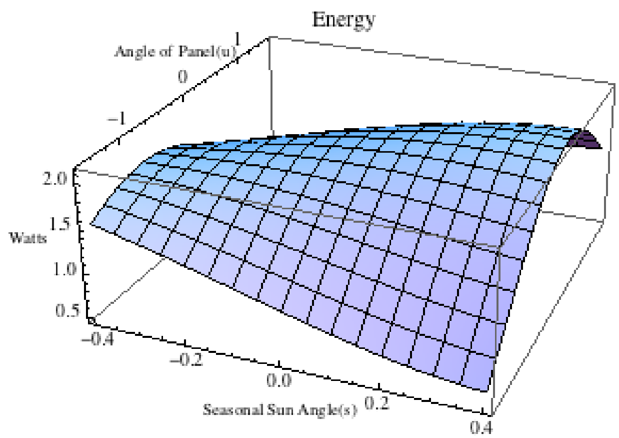
Figure 3

3 Energy Preliminary Study

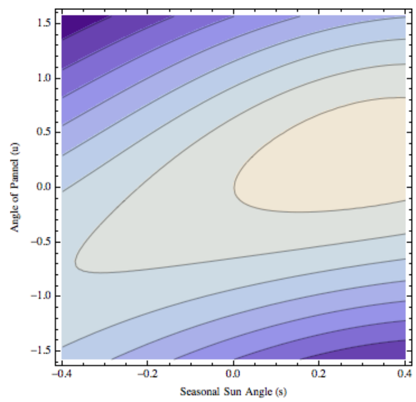
$$W(s,u) = \int_{t_{min}}^{t_{max}} A(s,t)C(s,t)I_p(s,u,t)dt \qquad t_{max} = \frac{\pi}{2} \qquad t_{min} = -\frac{\pi}{2}$$

$$\boxed{W(s,u) = 1 + (1 + 0.65s - 1.2s^2 - 0.4s^3 + 0.35s^4)cos(u) + (1.4s - 0.4s^2 - 1.5s^3 - 0.35s^4)sin(u)}$$

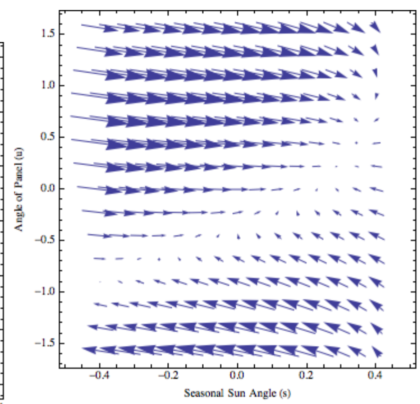
$$\begin{aligned}\triangledown W &= \langle \frac{\partial W}{\partial s}, \frac{\partial W}{\partial u} \rangle \\ &= \langle (0.65 - 2.4s - 1.2s^2 + 1.4s^3)cos(u) + (1.4 - 0.8s - 4.5s^2 - 1.4s^3)sin(u), \\ &\quad (1.4s - 0.4s^2 - 1.5s^3 - 0.35s^4)cos(u) - (1 + 0.65s - 1.2s^2 - 0.4s^3 + 0.35s^4)sin(u) \rangle\end{aligned}$$



(a) 3D representation of energy output as a function of angle of solar panel and seasonal sun angle.



(b) From this contour plot, we can see that there seems to be a maxima around (0.3,0.2).



(c) Using the $\bigtriangledown W$, we created a vector plot, where we can estimate the maxima closer than the contour plot, to around (0.35,0.5).

Figure 4

If we use Mathematica's FindRoot function, we are able to determine where $\bigtriangledown W$ was equal to the zero vector.

$$\boxed{\text{Possible Critical Point}: \ (0.324, 0.320)}$$

What we are still unsure about is the type of point is is. From the 3D, contour, and vector plot, we could hypothesize that the critical point is a maximum. But, for the sake of showing work, we will classify the critical point using the second derivative test.

$$D(s,u) = W_{ss}W_{uu} - (W_{su})^2$$

$$W_{ss} = \frac{\partial^2 W}{\partial s^2} \qquad W_{uu} = \frac{\partial^2 W}{\partial u^2} \qquad W_{su} = \frac{\partial}{\partial u}\left(\frac{\partial W}{\partial s}\right)$$

If we test the discriminant $D(s,u)$ at the possible critical point (0.324,0.320) we achieve:

$$D(0.324, 0.320) = W_{ss}|_{(0.324,0.320)}W_{uu}|_{(0.324,0.320)} - (W_{su}|_{(0.324,0.320)})^2 > 0$$

Since the value of the discriminant was positive, we can then test for a minima or maxima by plugging the critical point into $W_{ss}|_{(0.324,0.320)} < 0$ so $\boxed{\text{the critical point (0.324, 0.320) is a local maximum.}}$

$$\boxed{\text{Energy at CP}: \; W(0.324, 0.320) = 2.133 \; Watts}$$

4 Energy at Edges of Domain

While we have found a critical point inside the domain $D = 0.4 \leq s \leq 0.4, \pi \leq t \leq \pi$, we have yet to check the edges to see if they contribute any other critical points. To do this, we check each boundary individually.

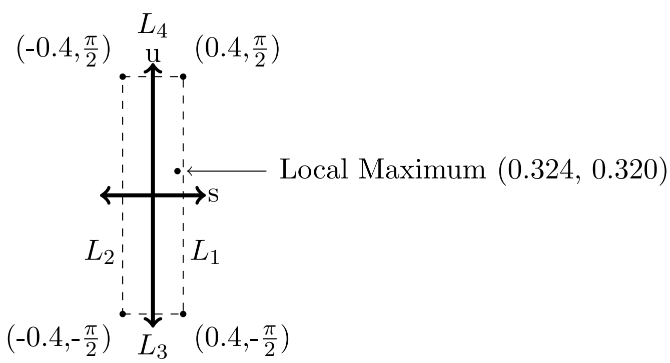


Figure 5

4.1 Checking the boundary L_1

First we check the boundary of L_1 from Figure 10. To do this, we set $s = 0.4$, which resulted in

$$W(0.4, u) = 1 + 1.05136cos(u) + 0.39104sin(u)$$

To find the critical points of L_1, we take the derivative with respect to u and solve for its roots.

$$\frac{d}{du}\Big[W(0.4, u)\Big] = 0.391cos(u) - 1.051sin(u)$$

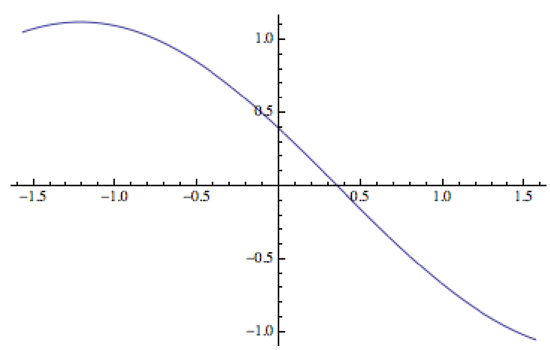


Figure 6: **Root at u = 0.356**

To determine if this $\boxed{\text{critical point at (0.4, 0.356)}}$ is a maximum or minimum, we take the second derivative and plug in the C.P.

$$\frac{d^2}{du^2}\Big[W(0.4,u)\Big]\Big|_{u=0.356} = (-1.05136cos(u) - 0.39104sin(u))\Big|_{u=0.356} = -1.12173$$

Since the second derivative results in a negative value, we know this $\boxed{\text{C.P. is the maximum}}$ of the boundary.

$$\boxed{W(0.4, 0.356) = 2.122\ Watts}$$

4.2 Checking the boundary L_2

First we check the boundary of L_2 from Figure 10. To do this, we set $s = -0.4$, which resulted in

$$W(-0.4, u) = 1 + 0.58256cos(u) - 0.53696sin(u)$$

To find the critical points of L_2, we take the derivative with respect to u and solve for its roots.

$$\frac{d}{du}\Big[W(-0.4,u)\Big] = -0.53696cos(u) - 0.58256sin(u)$$

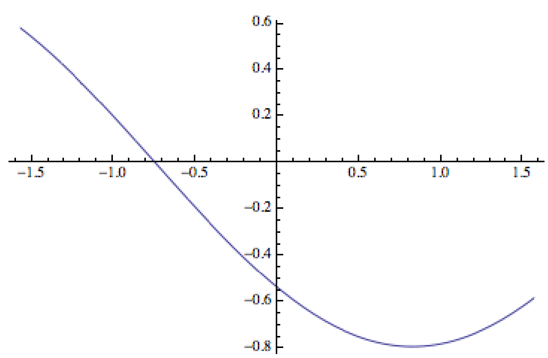


Figure 7: **Root at u = -0.745**

To determine if this $\boxed{\text{critical point at (-0.4, -0.745)}}$ is a maximum or minimum, we take the second derivative and plug in the C.P.

$$\frac{d^2}{du^2}\Big[W(-0.4,u)\Big]\Big|_{u=-0.745} = (-0.58256cos(u) + 0.53696sin(u))\Big|_{u=-0.745} = -0.792277$$

Since the second derivative results in a negative value, we know this $\boxed{\text{C.P. is the maximum}}$ of the boundary.

$$\boxed{W(-0.4, -0.745) = 1.792\ Watts}$$

4.3 Checking the boundary L_3

First we check the boundary of L_3 from Figure 10. To do this, we set $u = -\frac{\pi}{2}$, which resulted in

$$W(s, -\frac{\pi}{2}) = 1 - 1.4s + 0.4s^2 + 1.5s^3 + 0.35s^4$$

To find the critical points of L_3, we take the derivative with respect to s and solve for its roots.

$$\frac{d}{ds}\Big[W(s,-\frac{\pi}{2})\Big] = -1.4 + 0.8s + 4.5s^2 + 1.4s^3$$

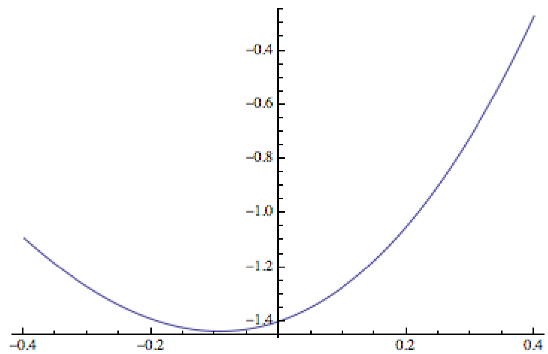


Figure 8: **Root at s = -0.766**

To determine if this $\boxed{\text{critical point at }(-0.766,-\frac{\pi}{2})}$ is a maximum or minimum, we take the second derivative and plug in the C.P.

$$\frac{d^2}{ds^2}\Big[W(s,-\frac{\pi}{2})\Big]\Big|_{s=-0.766} = (0.8 + 9.s + 4.2s^2)\Big|_{s=-0.766} = 11.500$$

Since the second derivative results in a positive value, we know this $\boxed{\text{C.P. is the minimum}}$ of the boundary.

$$\boxed{W(-0.766,-\frac{\pi}{2}) = 1.753\ Watts}$$

Because this C.P isn't is our domain D, we won't include it in our boundary plot.

4.4 Checking the boundary L_4

First we check the boundary of L_4 from Figure 10. To do this, we set $u = \frac{\pi}{2}$, which resulted in

$$W(s,\frac{\pi}{2}) = 1 - 1.4s + 0.4s^2 + 1.5s^3 + 0.35s^4$$

To find the critical points of L_4, we take the derivative with respect to u and solve for its roots.

$$\frac{d}{ds}\Big[W(s,\frac{\pi}{2})\Big] = 1.4 - 0.8s - 4.5s^2 - 1.4s^3$$

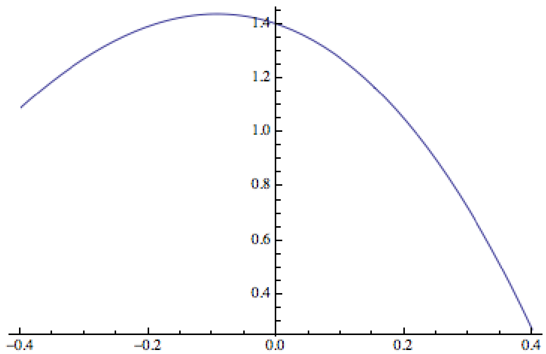


Figure 9: **Root at s = 0.450**

To determine if this $\boxed{\text{critical point at } (0.450, \frac{\pi}{2})}$ is a maximum or minimum, we take the second derivative and plug in the C.P.

$$\frac{d^2}{ds^2}\left[W(s, \frac{\pi}{2})\right]\Big|_{s=0.450} = (-0.8 - 9.s - 4.2s^2)\Big|_{s=0.450} = -5.70313$$

Since the second derivative results in a negative value, we know this $\boxed{\text{C.P. is the maximum}}$ of the boundary.

$$\boxed{W(0.450, \frac{\pi}{2}) = 1.398}$$

Because this C.P isn't is our domain D, we won't include it in our boundary plot.

4.5 Checking the Corners

Lastly, we have to check the corner of the domain to ensure we aren't neglecting any maxima or minima.

$$W(0.4, \pi/2) = 1.39104 \quad W(0.4, -\pi/2) = 0.60896 \quad W(-0.4, \pi/2) = 0.46304 \quad W(-0.4, -\pi/2) = 1.53696$$

4.6 Summary of Energy at Edges of Domain

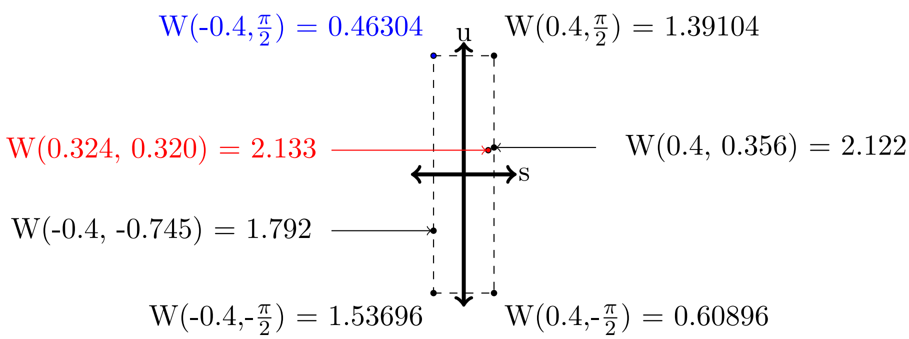


Figure 10: From this figure, we can see that the absolute maximum of the domain D is at (0.324, 0.320) and the absolute minimum of the domain is at (-0.4,$\frac{\pi}{2}$)

5 Energy by Season

Since u can vary, the season corresponds to the value of s that produces the greatest energy for any value of u. This would occur at the absolute maximum found above (0.324, 0.320). When $s = 0.324$ radians, it is either between March 20 and June 20, or June 21 and Sept 21. The first period is technically spring, while the second is summer. Technically, the same amount of energy is collected during the spring and summer.

6 Finding the Optimal Angle for Solar Panels

In order to find the optimal angle for solar panels at a fixed s* value, we simply solved for the maximum of the function W(s*,u). By plotting many points using varying s* values, we were able to create an optimal path, which follows the "ridge" of the 3D plot in Figure 4a.

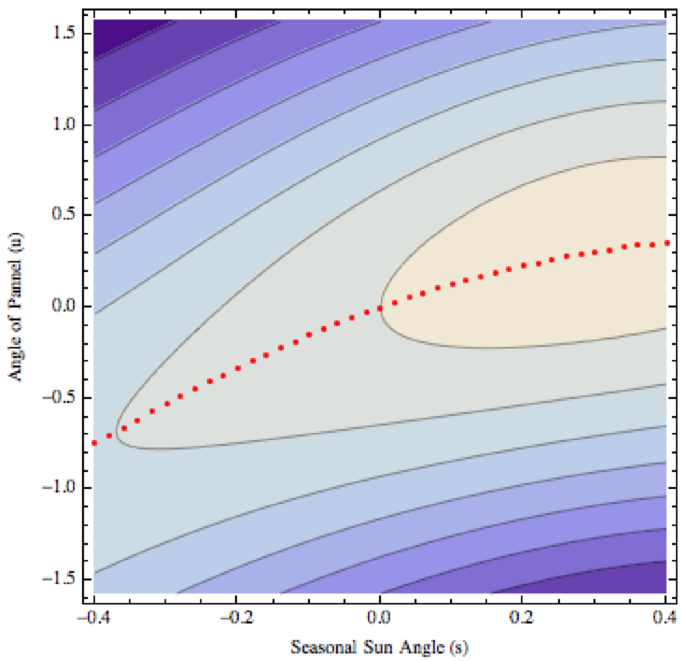


Figure 11: The optimal path plotted on the contour plot is a sensible one because it follows directly the "ridge" of the 3D W(s,u) in Figure 4a.

7 Maintenance of Solar Panels

Panels should be replaced when s is lowest, even at the most efficient value of u. Looking at Figure 11, this occurs at s=-0.4. The days or weeks needed to replace the panels should be chosen so they are centered around the winter solstice, Dec 21.