

Lecture 2: Image Classification pipeline

Administrative: Piazza

For questions about midterm, poster session, projects, etc, use Piazza!

SCPD students: Use your @stanford.edu address to register for Piazza; contact scpd-customerservice@stanford.edu for help.

Administrative: Assignment 1

Out yesterday, due 4/17 11:59pm

- K-Nearest Neighbor
- Linear classifiers: SVM, Softmax
- Two-layer neural network
- Image features

Administrative: Friday Discussion Sections

(Some) Fridays 12:30pm - 1:20pm in Gates B03

Hands-on tutorials, with more practical detail than main lecture

We may not have discussion sections every Friday, check syllabus:

<http://cs231n.stanford.edu/syllabus.html>

This Friday: Python / numpy / Google Cloud setup

Administrative: Course Project

Project proposal due 4/24

Administrative: Python + Numpy

CS231n Convolutional Neural Networks for Visual Recognition

Python Numpy Tutorial

This tutorial was contributed by [Justin Johnson](#).

We will use the Python programming language for all assignments in this course. Python is a great general-purpose programming language on its own, but with the help of a few popular libraries (numpy, scipy, matplotlib) it becomes a powerful environment for scientific computing.

We expect that many of you will have some experience with Python and numpy; for the rest of you, this section will serve as a quick crash course both on the Python programming language and on the use of Python for scientific computing.

<http://cs231n.github.io/python-numpy-tutorial/>

Administrative: Google Cloud

We will be using Google Cloud in this class

We will be distributing coupons to all enrolled students

See our tutorial here for walking through Google Cloud setup:

<https://github.com/cs231n/gcloud>

Image Classification: A core task in Computer Vision



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← Input Image ·

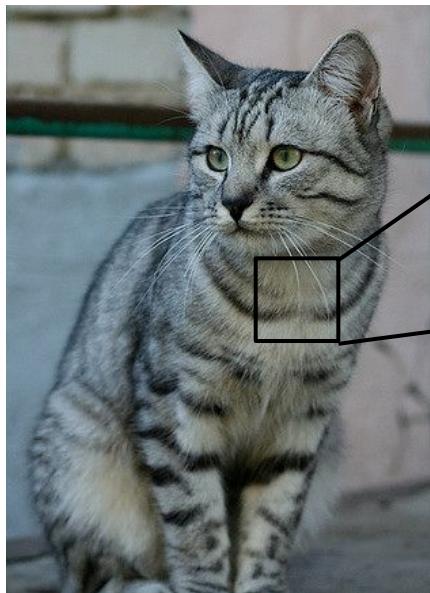
(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

cat

category
label.

t
fixed.

The Problem: Semantic Gap



This image by [Nikita](#) is
licensed under [CC-BY 2.0](#).

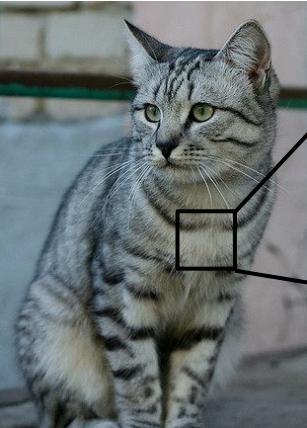
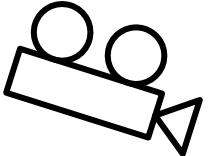
[105 112 108 111 104 99 106 99 96 103 112 119 104 97 93 87]
[91 98 102 106 104 79 98 103 99 105 123 136 110 105 94 85]
[76 85 90 105 128 105 87 96 95 99 115 112 106 103 99 85]
[99 81 93 120 131 127 100 95 98 102 99 96 93 101 94]
[106 91 61 64 69 91 88 85 101 107 109 98 75 84 96 95]
[114 108 85 55 55 69 64 54 64 87 112 129 98 74 84 91]
[133 137 147 103 65 81 80 65 52 54 74 84 102 93 85 82]
[128 137 144 140 109 95 86 70 62 65 63 63 60 73 86 101]
[125 133 148 137 119 121 117 94 65 79 80 65 54 64 72 98]
[127 125 131 147 133 127 126 131 111 96 89 75 61 64 72 84]
[115 114 109 123 150 148 131 118 113 109 100 92 74 65 65 72 78]
[89 93 90 97 108 147 131 118 113 114 113 109 106 95 77 80]
[63 77 86 81 77 79 102 123 117 115 117 125 125 130 115 87]
[62 65 82 89 78 71 80 101 124 126 119 101 107 114 131 119]
[63 65 75 88 89 71 62 81 120 138 135 105 81 98 110 118]
[87 65 71 87 106 95 69 45 76 130 126 107 92 94 105 112]
[118 97 82 86 117 123 116 66 41 51 95 93 89 95 102 107]
[164 146 112 88 82 120 124 104 76 48 45 66 88 101 102 109]
[157 170 157 128 93 86 114 132 112 97 69 55 70 62 99 94]
[130 128 134 161 139 100 109 118 121 134 114 87 65 53 69 86]
[128 112 96 117 150 144 128 115 104 107 102 93 87 81 72 79]
[123 107 96 86 83 112 153 149 122 109 104 75 80 107 112 99]
[122 121 102 88 82 86 94 117 145 148 153 102 58 78 92 107]
[122 164 148 103 71 56 78 83 93 103 119 139 102 61 69 84]]

What the computer sees

An image is just a big grid of numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Challenges: Viewpoint variation



```
[1185 112 188 111 184 99 186 99 96 183 112 119 184 97 93 87]  
[ 91 98 182 106 104 79 98 103 99 105 123 136 118 105 94 85]  
[ 76 85 98 105 128 105 87 96 95 99 115 112 106 103 99 85]  
[ 99 81 98 105 128 105 127 108 98 99 115 112 106 103 99 85]  
[104 91 86 84 69 91 68 85 101 102 109 98 75 84 96 95]  
[114 108 85 55 55 69 64 54 64 87 112 129 98 74 84 94 91]  
[133 137 147 103 65 81 80 65 52 54 74 84 102 93 85 82]  
[128 137 144 148 105 95 86 78 62 65 63 63 68 73 86 101]  
[102 125 131 147 133 127 116 131 111 98 89 75 61 64 72 84]  
[127 125 131 147 133 127 116 131 111 98 89 75 61 64 72 84]  
[115 115 189 123 150 148 131 118 113 109 108 92 74 65 72 78]  
[ 89 93 98 97 108 147 131 118 113 114 113 108 106 95 77 80]  
[ 63 77 86 81 77 79 182 123 137 115 111 125 125 130 115 87]  
[ 62 85 88 89 73 62 81 128 138 135 105 81 98 118 118]  
[ 63 65 75 88 89 73 62 81 128 138 135 105 81 98 118 118]  
[ 87 65 71 87 100 95 69 45 76 138 126 107 92 94 105 112]  
[118 97 82 86 117 123 116 66 41 51 95 93 89 95 102 107]  
[164 149 112 88 100 108 128 184 78 48 66 66 66 101 102 108]  
[157 98 100 118 93 86 104 124 120 115 109 89 75 70 70 84]  
[138 128 134 161 139 180 109 118 121 134 114 87 65 53 69 86]  
[128 112 96 117 150 144 120 115 104 107 102 93 87 81 72 79]  
[123 107 96 86 83 112 153 149 122 189 104 75 88 107 112 99]  
[122 121 102 80 82 86 94 117 145 148 153 105 58 78 92 107]  
[122 164 148 103 71 56 78 83 93 103 119 139 102 61 69 84]
```

All pixels change when
the camera moves!

[This image](#) by [Nikita](#) is
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Challenges: Background Clutter



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[This image is CC0 1.0 public domain](#)

Challenges: Illumination



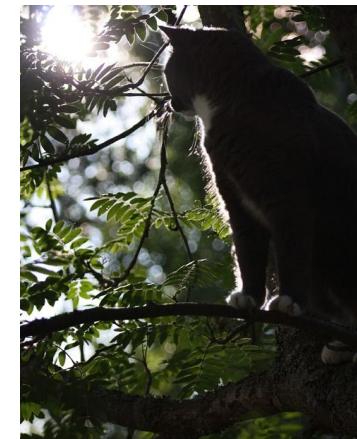
This image is [CC0 1.0 public domain](#)



This image is [CC0 1.0 public domain](#)

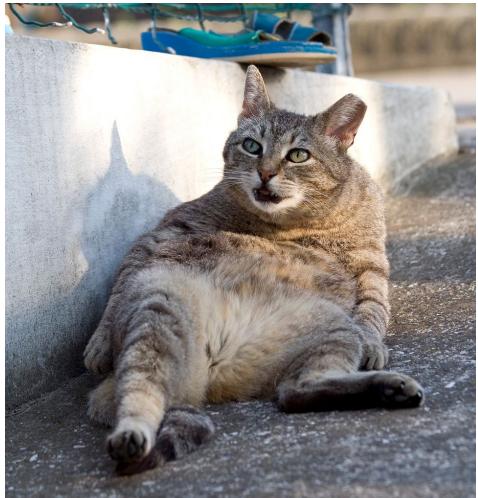


This image is [CC0 1.0 public domain](#)



This image is [CC0 1.0 public domain](#)

Challenges: Deformation



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[This image by sare bear is](#)
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[This image by Tom Thai is](#)
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Challenges: Occlusion



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[This image is CC0 1.0 public domain](#)



[This image by jonsson is licensed under CC-BY 2.0](#)

Challenges: Intraclass variation



[This image](#) is CC0 1.0 public domain

An image classifier

```
def classify_image(image):  
    # Some magic here?  
    return class_label
```

Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for
recognizing a cat, or other classes.

Attempts have been made

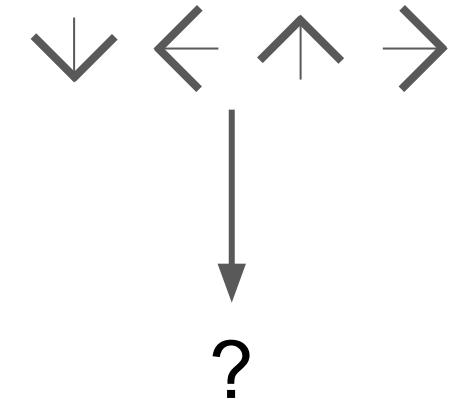


Find edges
Hubel & Wiesel



Find corners

* Not to work well
Super brain the (MRI)
 σ_x^2 obj ~ 3
How to work?



John Canny, "A Computational Approach to Edge Detection", IEEE TPAMI 1986

Machine Learning: Data-Driven Approach

1. Collect a dataset of images and labels
2. Use Machine Learning to train a classifier
3. Evaluate the classifier on new images

Example training set

```
def train(images, labels):  
    # Machine learning!  
    return model
```

```
def predict(model, test_images):  
    # Use model to predict labels  
    return test_labels
```

airplane



automobile



bird



cat



deer



First classifier: Nearest Neighbor

```
def train(images, labels):  
    # Machine learning!  
    return model
```



Memorize all
data and labels

```
def predict(model, test_images):  
    # Use model to predict labels  
    return test_labels
```



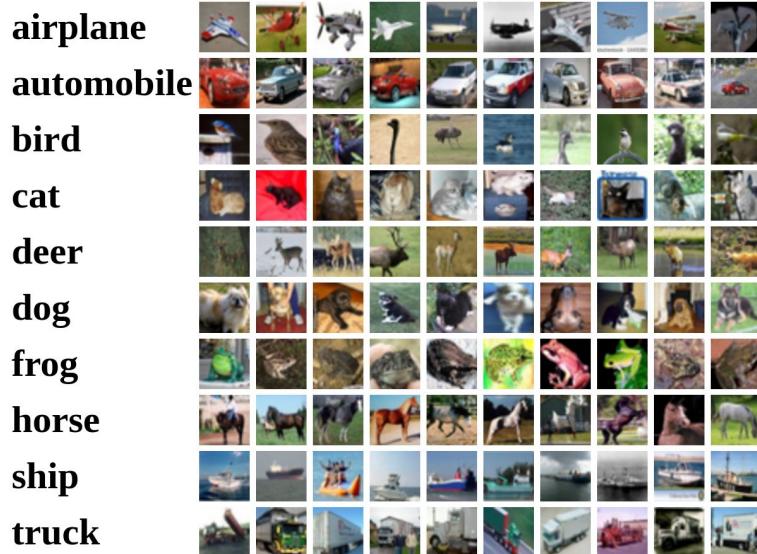
Predict the label
of the most similar
training image

Example Dataset: CIFAR10

10 classes

50,000 training images

10,000 testing images



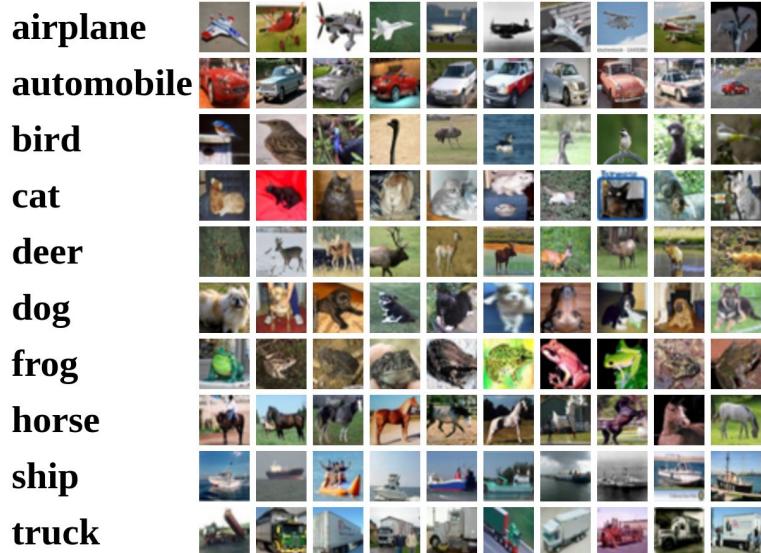
Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

Example Dataset: CIFAR10

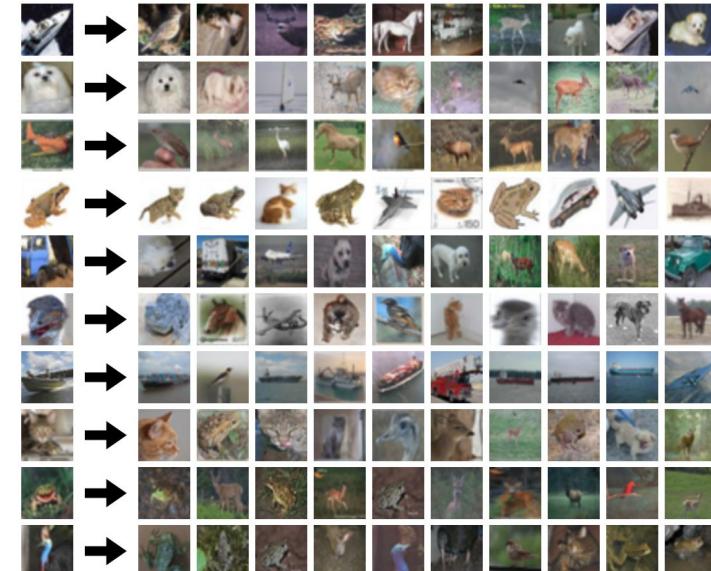
10 classes

50,000 training images

10,000 testing images



Test images and nearest neighbors



Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

Distance Metric to compare images *Manhattan dist.*

L1 distance:

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

test image			
56	32	10	18
90	23	128	133
24	26	178	200
2	0	255	220

training image			
10	20	24	17
8	10	89	100
12	16	178	170
4	32	233	112

pixel-wise absolute value differences

46	12	14	1
82	13	39	33
12	10	0	30
2	32	22	108

add
→ 456

```

import numpy as np

class NearestNeighbor:
    def __init__(self):
        pass

    def train(self, X, y):
        """ X is N x D where each row is an example. Y is 1-dimension of size N """
        # the nearest neighbor classifier simply remembers all the training data
        self.Xtr = X
        self.ytr = y

    def predict(self, X):
        """ X is N x D where each row is an example we wish to predict label for """
        num_test = X.shape[0]
        # lets make sure that the output type matches the input type
        Ypred = np.zeros(num_test, dtype = self.ytr.dtype)

        # loop over all test rows
        for i in xrange(num_test):
            # find the nearest training image to the i'th test image
            # using the L1 distance (sum of absolute value differences)
            distances = np.sum(np.abs(self.Xtr - X[i,:]), axis = 1)
            min_index = np.argmin(distances) # get the index with smallest distance
            Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

        return Ypred

```

Nearest Neighbor classifier

```

import numpy as np

class NearestNeighbor:
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```

Nearest Neighbor classifier

Memorize training data

```

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```

Nearest Neighbor classifier

For each test image:
 Find closest train image
 Predict label of nearest image

```

import numpy as np

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```

Nearest Neighbor classifier

Q: With N examples,
how fast are training
and prediction?

```

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```

Nearest Neighbor classifier

Q: With N examples,
how fast are training
and prediction?

A: Train O(1),
predict O(N)

```

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    def __init__(self):
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```

Nearest Neighbor classifier

Q: With N examples,
how fast are training
and prediction?

$O(N^2)$

A: Train $O(1)$,
predict $O(N)$

$O(N^2)$

$O(N^2)$
 $\leq O(N^2)$

This is bad: we want
classifiers that are **fast** point
at prediction; **slow** for + rich
training is ok

$O(N^2)$ but $O(N^2)$ CNN
 $O(N^2)$ but $O(N^2)$ 2D

```

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```

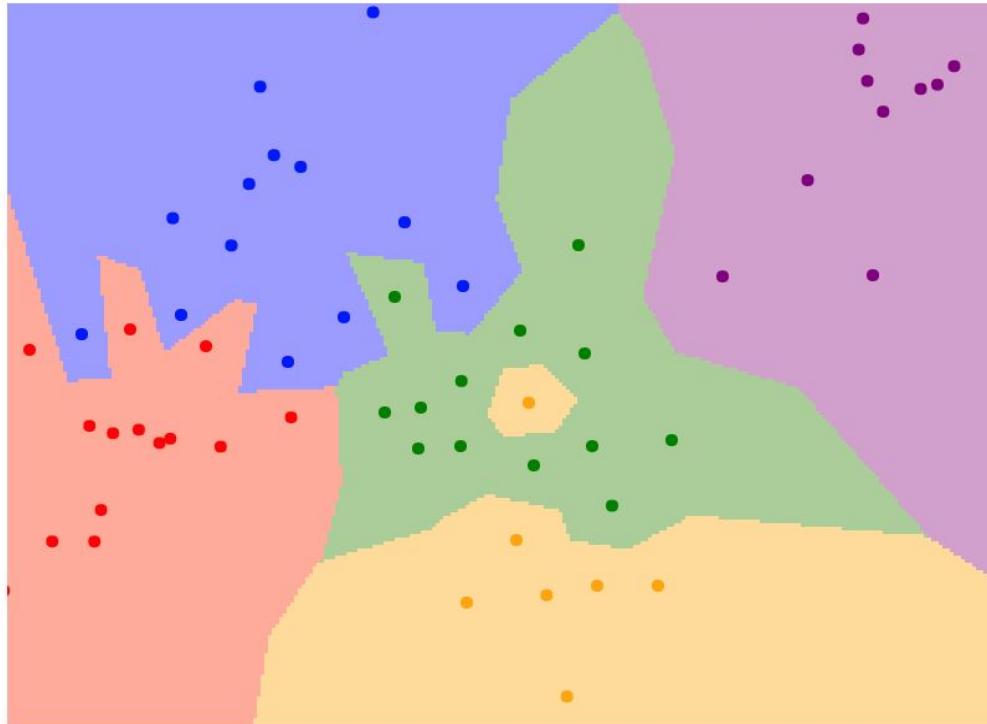
Nearest Neighbor classifier

Many methods exist for fast / approximate nearest neighbor (beyond the scope of 231N!)

A good implementation:
<https://github.com/facebookresearch/faiss>

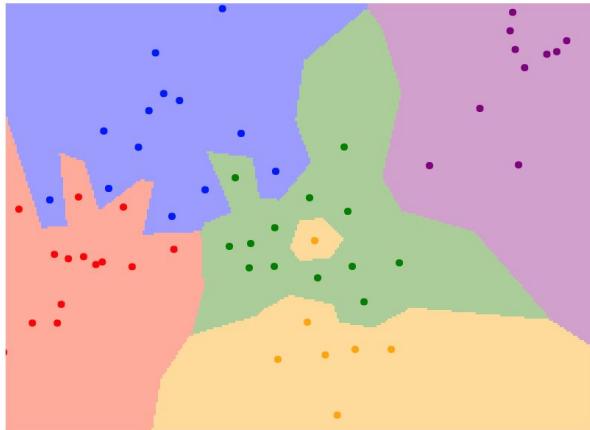
Johnson et al, “Billion-scale similarity search with GPUs”, arXiv 2017

What does this look like?

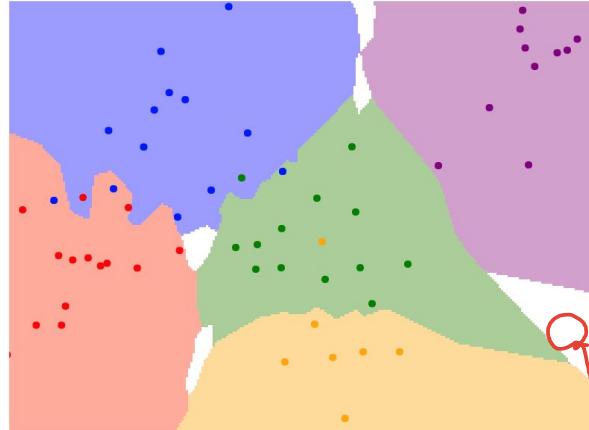


K-Nearest Neighbors

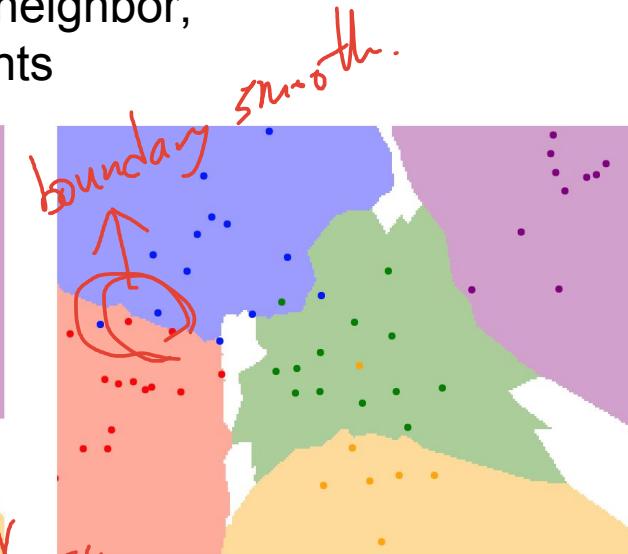
Instead of copying label from nearest neighbor,
take **majority vote** from K closest points



$K = 1$



$K = 3$



$K = 5$

What does this look like?



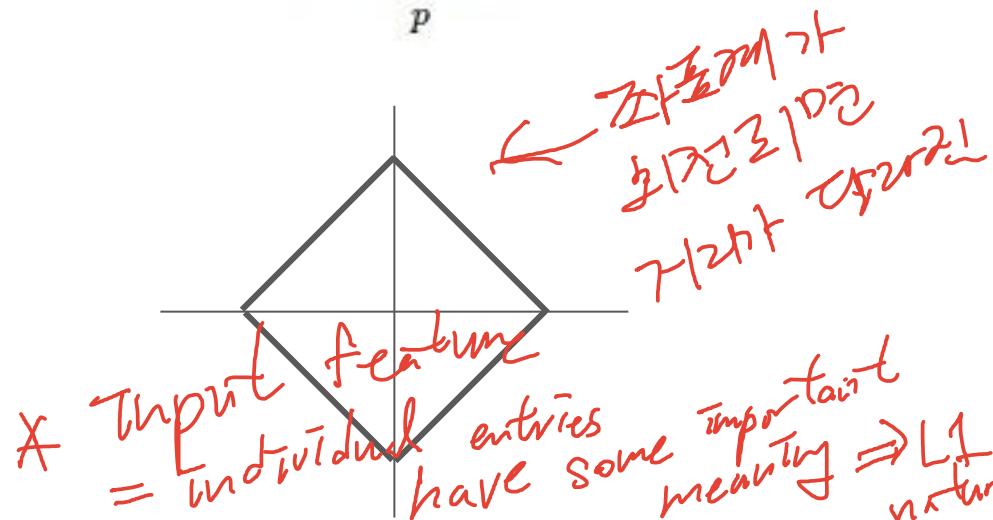
What does this look like?



K-Nearest Neighbors: Distance Metric

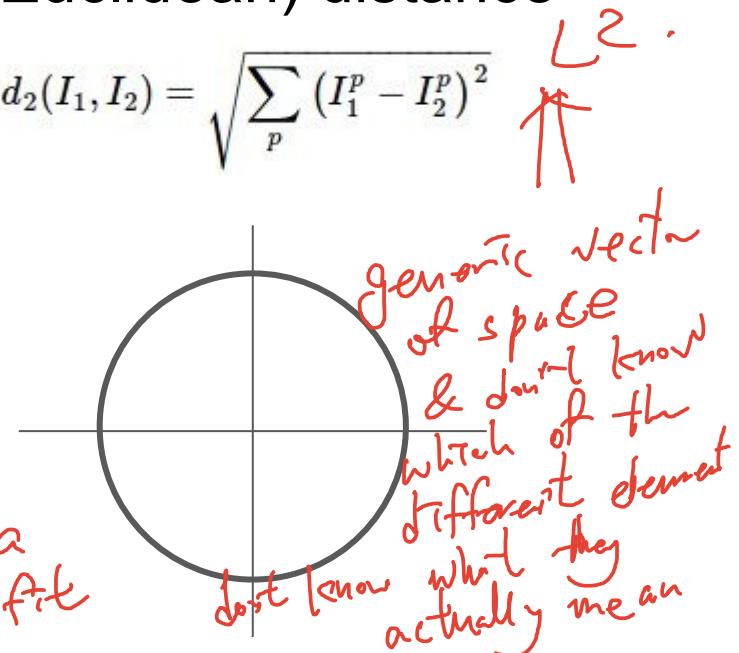
L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$



L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p (I_1^p - I_2^p)^2}$$



K-Nearest Neighbors: Distance Metric

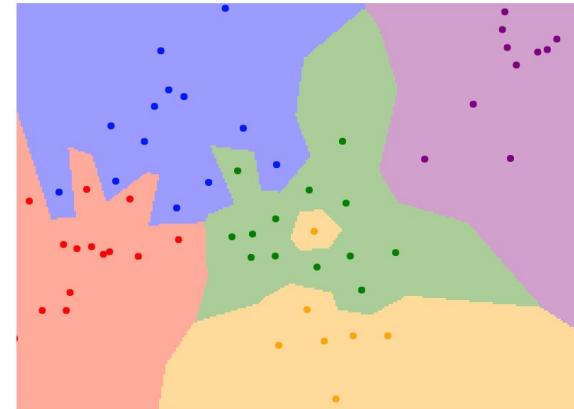
L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

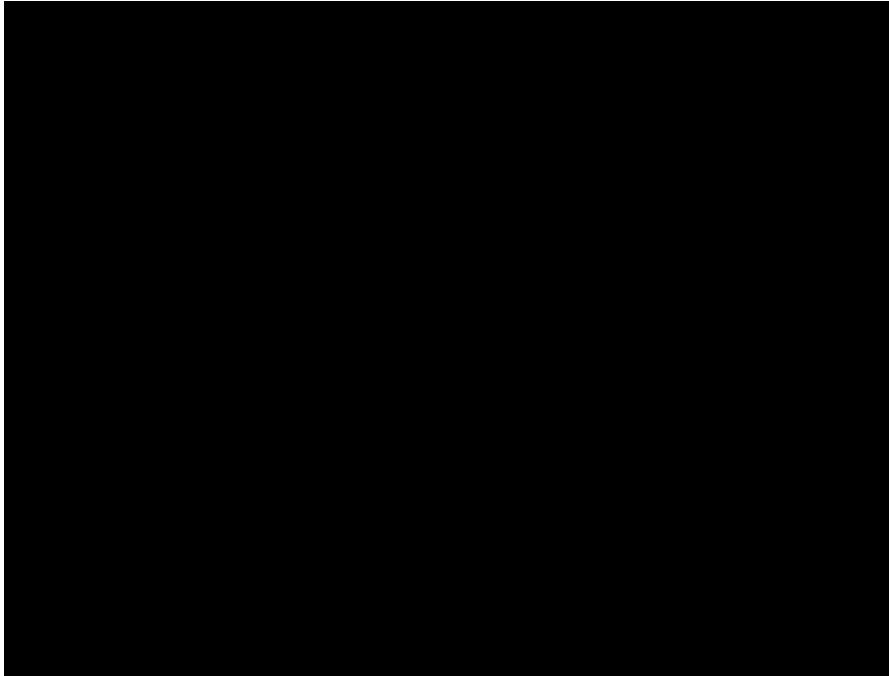


L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p (I_1^p - I_2^p)^2}$$



K-Nearest Neighbors: Demo Time



<http://vision.stanford.edu/teaching/cs231n-demos/knn/>

Hyperparameters

What is the best value of k to use?

What is the best **distance** to use?

These are **hyperparameters**: choices about the algorithm that we set rather than learn

→ *data size (number of training examples)*
batch size (number of training examples)
epoch number

Hyperparameters

$L_1 > L_2$. - problem-dependant.
- L_1 has coordinate dependency,
 \Rightarrow depends on the coordinate system
If you know that you have a vector

What is the best value of k to use?

What is the best **distance** to use?

These are **hyperparameters**: choices about the algorithm that we set rather than learn

Very problem-dependent.

and individual elements of the vector have a meaning
e.g. salary, that year ...

Must try them all out and see what works best.

Setting Hyperparameters

Idea #1: Choose hyperparameters
that work best on the data

Your Dataset

Setting Hyperparameters

Idea #1: Choose hyperparameters
that work best on the data

BAD: $K = 1$ always works
perfectly on training data

Your Dataset

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: $K = 1$ always works perfectly on training data

Your Dataset

Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

train

test

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: $K = 1$ always works perfectly on training data

Your Dataset

Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

BAD: No idea how algorithm will perform on new data

train

test

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: $K = 1$ always works perfectly on training data

Your Dataset

Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

BAD: No idea how algorithm will perform on new data

train

test

Idea #3: Split data into **train**, **val**, and **test**; choose hyperparameters on val and evaluate on test

Better!

train

validation

test

Setting Hyperparameters

Your Dataset

Idea #4: Cross-Validation: Split data into **folds**,
try each fold as validation and average the results

*small dataset.
not deep learning -*

fold 1	fold 2	fold 3	fold 4	fold 5	test
--------	--------	--------	--------	--------	------

fold 1	fold 2	fold 3	fold 4	fold 5	test
--------	--------	--------	--------	--------	------

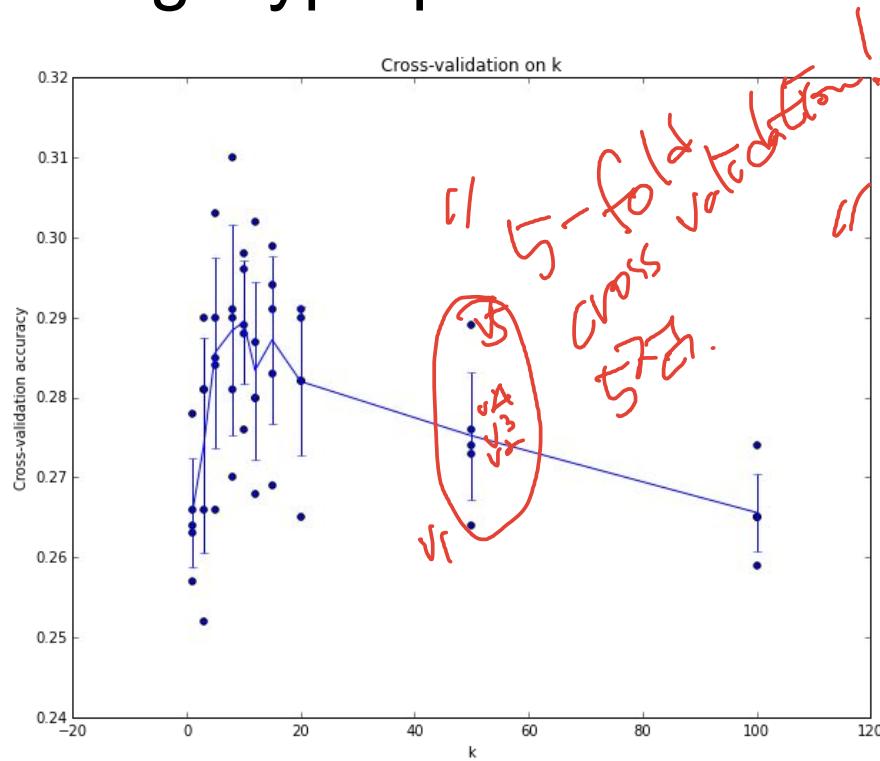
fold 1	fold 2	fold 3	fold 4	fold 5	test
--------	--------	--------	--------	--------	------

Useful for small datasets, but not used too frequently in deep learning

Identical folds

Hyperparameter

Setting Hyperparameters



Example of
5-fold cross-validation
for the value of k .

Each point: single
outcome.

The line goes
through the mean, bars
indicated standard
deviation

(Seems that $k \approx 7$ works best
for this data)

k-Nearest Neighbor on images **never used**.

-  Very slow at test time
- Distance metrics on pixels are not informative

L_1 / L_2 허가 및 사용하지 않은 이미지에 대한 거리 측정은 정보가 부족합니다.

Original



Boxed



Shifted



Tinted



(all 3 images have same L2 distance to the one on the left)

Original image is
CC0 public domain

k-Nearest Neighbor on images never used.

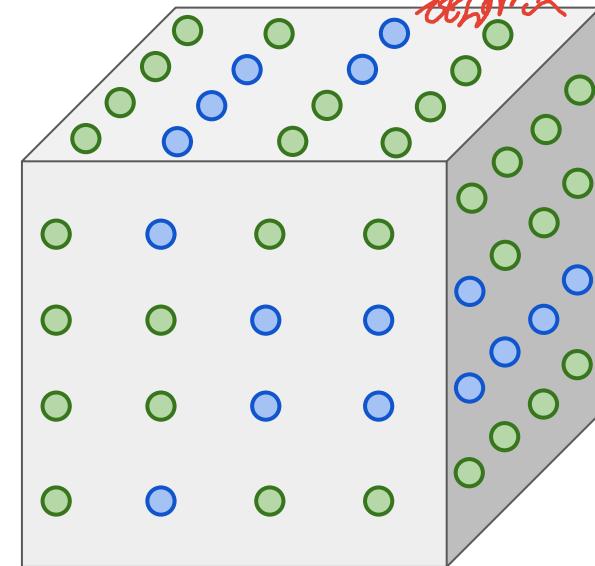
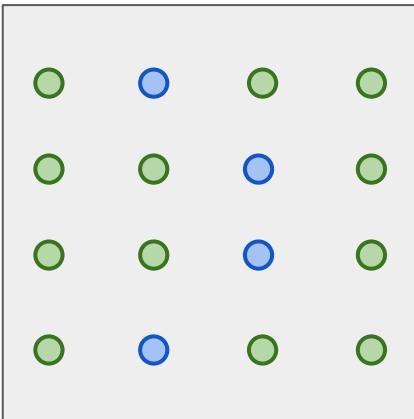
- Curse of dimensionality

2³ < 2¹²

Dimensions = 1
Points = 4



Dimensions = 2
Points = 4²



KNN densely 2¹² vs 2³
The 7 factors
otherwise fail to work
Dimensions = 3
Points = 4³ 2¹² vs 2³

K-Nearest Neighbors: Summary

In **Image classification** we start with a **training set** of images and labels, and must predict labels on the **test set**

The **K-Nearest Neighbors** classifier predicts labels based on nearest training examples

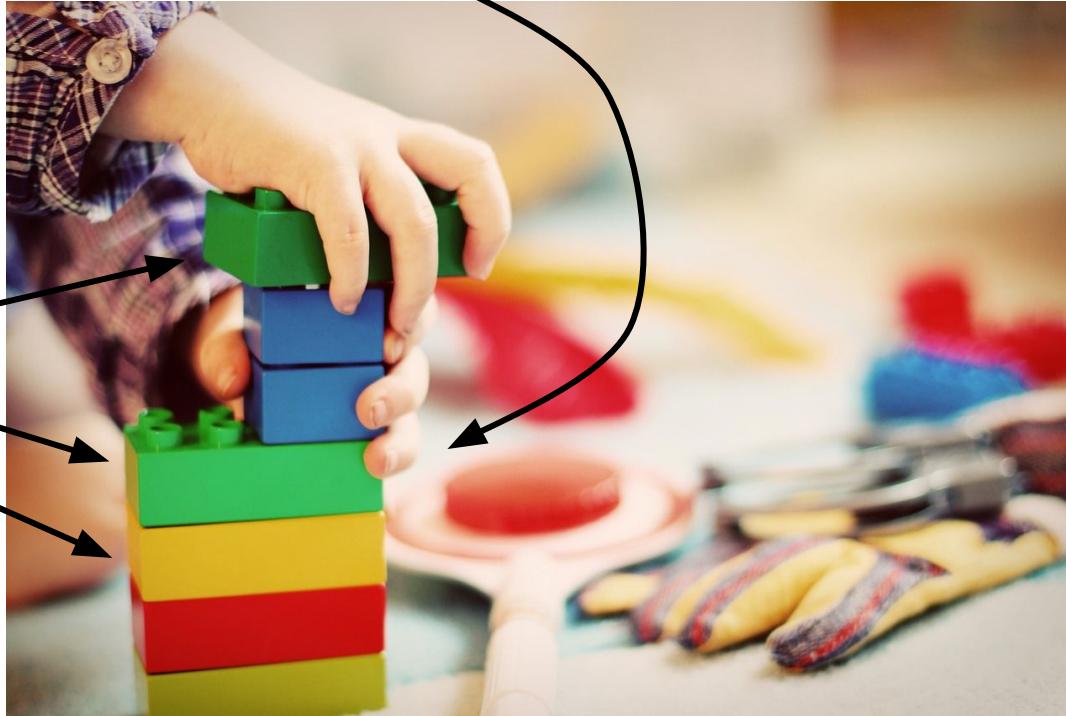
Distance metric and K are **hyperparameters**

Choose hyperparameters using the **validation set**;
only run on the test set once at the very end!

Linear Classification

Neural Network

Linear
classifiers



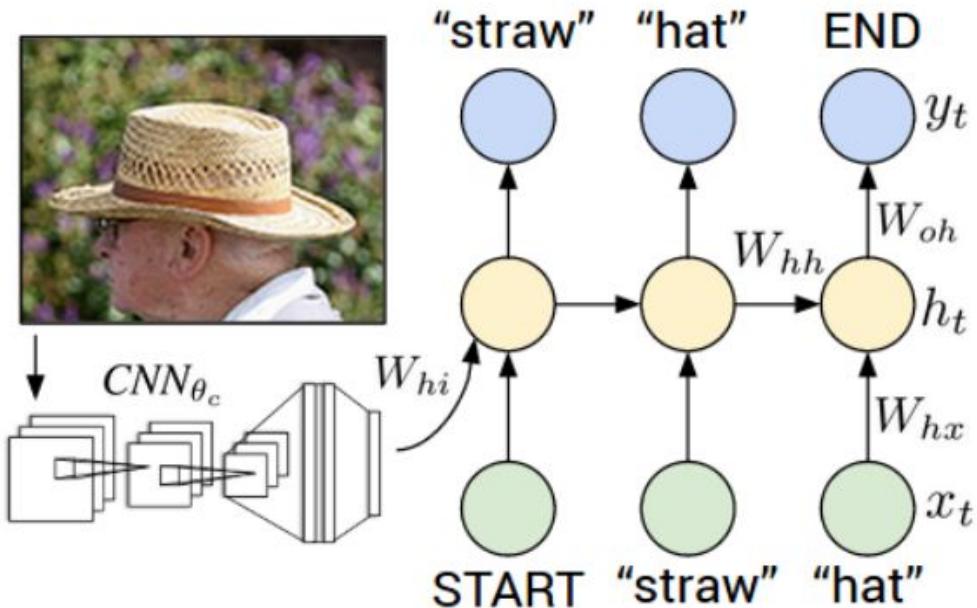
This image is CC0 1.0 public domain

Two young girls are playing with lego toy. *Boy is doing backflip on wakeboard*



Man in black shirt is playing guitar.

Construction worker in orange safety vest is working on road.



Karpathy and Fei-Fei, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015
Figures copyright IEEE, 2015. Reproduced for educational purposes.

Recall CIFAR10

airplane



automobile



bird



cat



deer



dog



frog



horse



ship



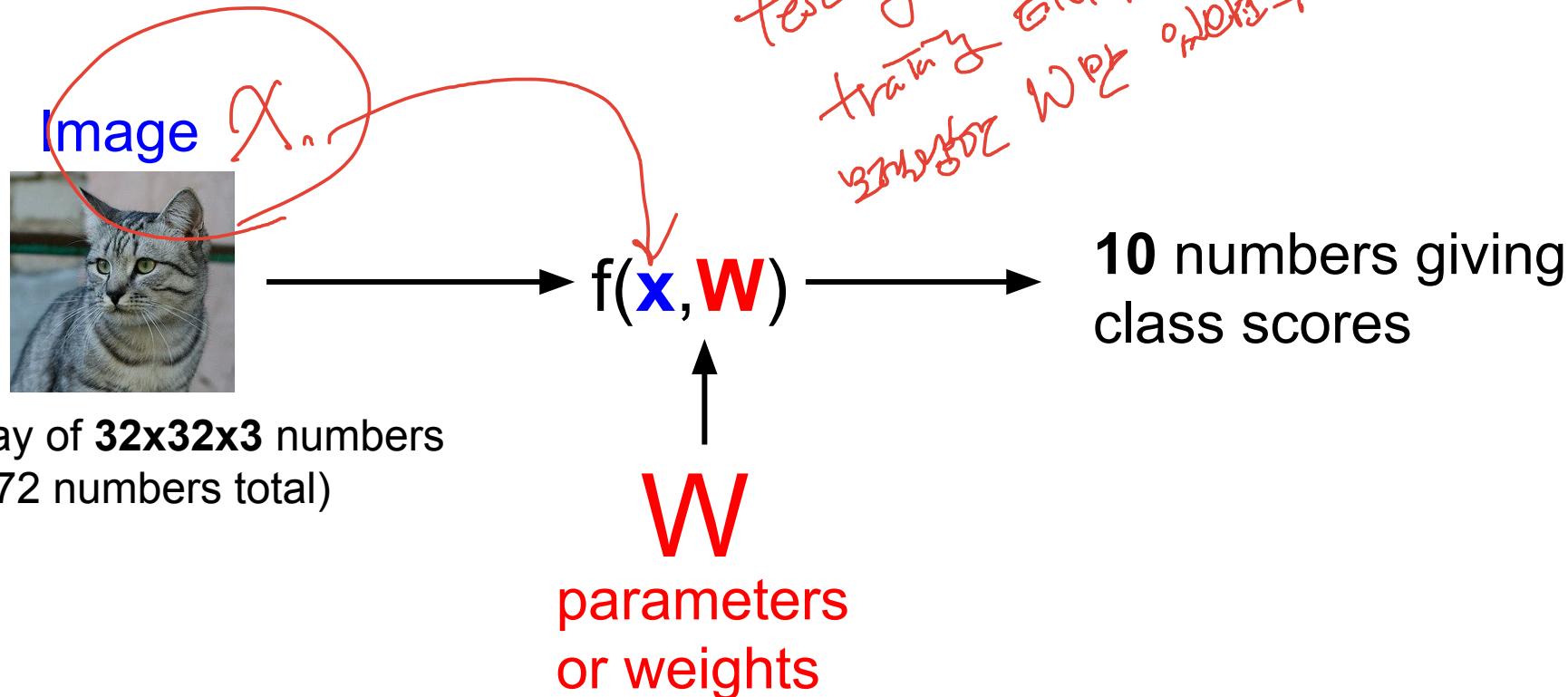
truck



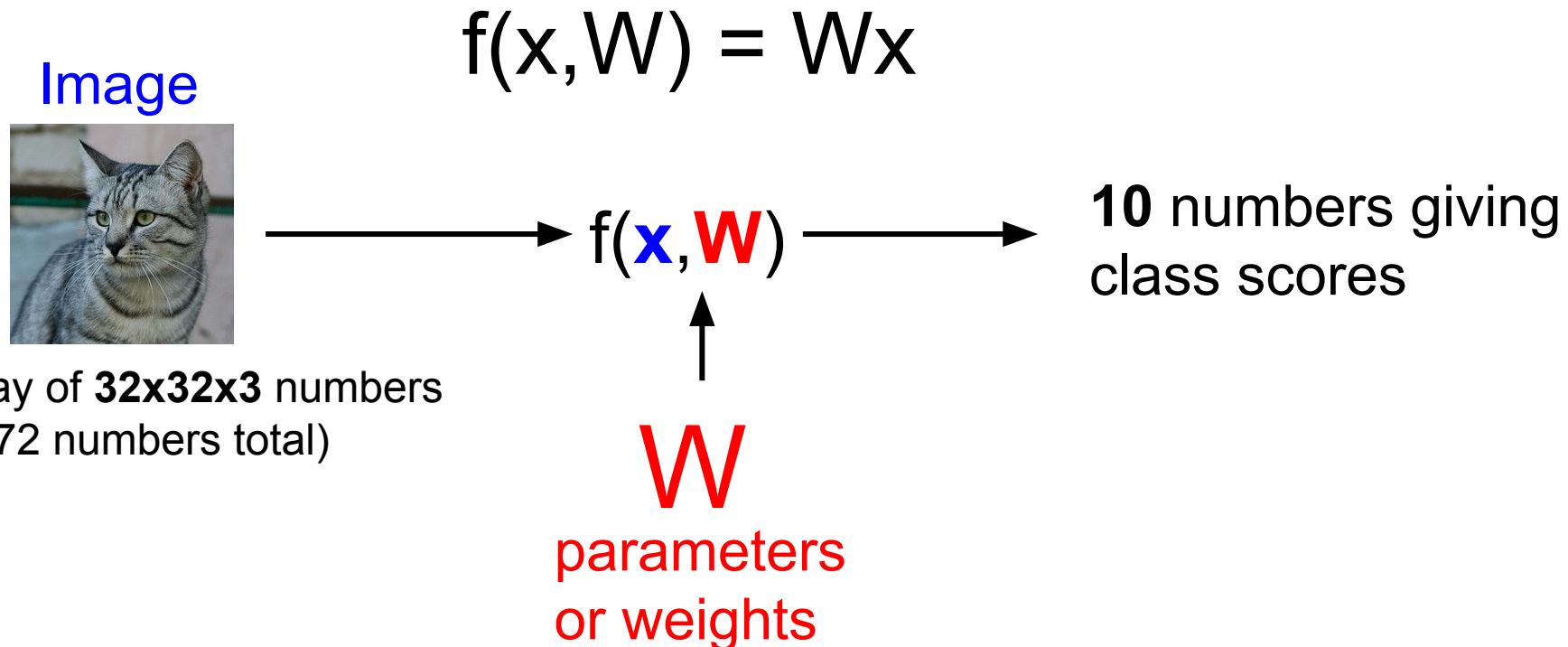
50,000 training images
each image is **32x32x3**

10,000 test images.

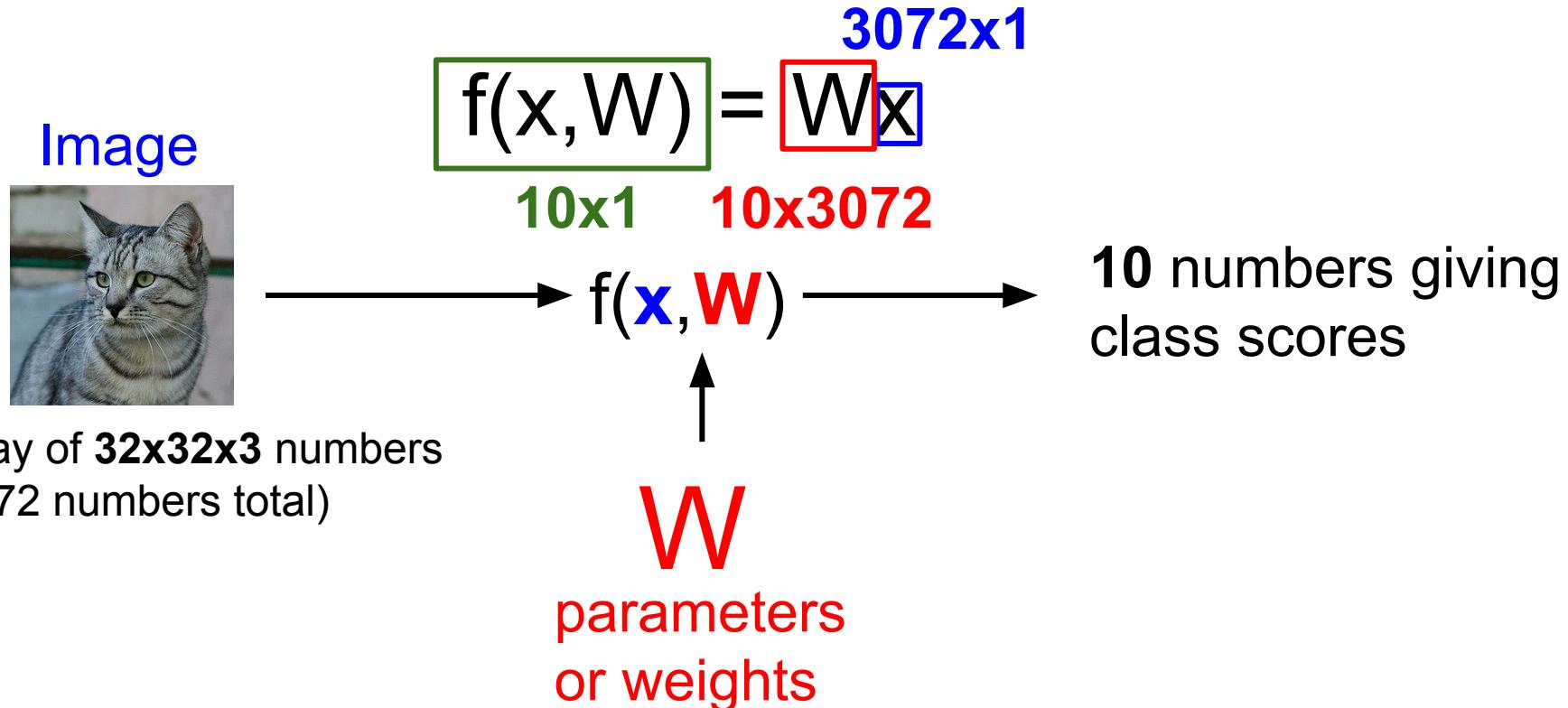
Parametric Approach



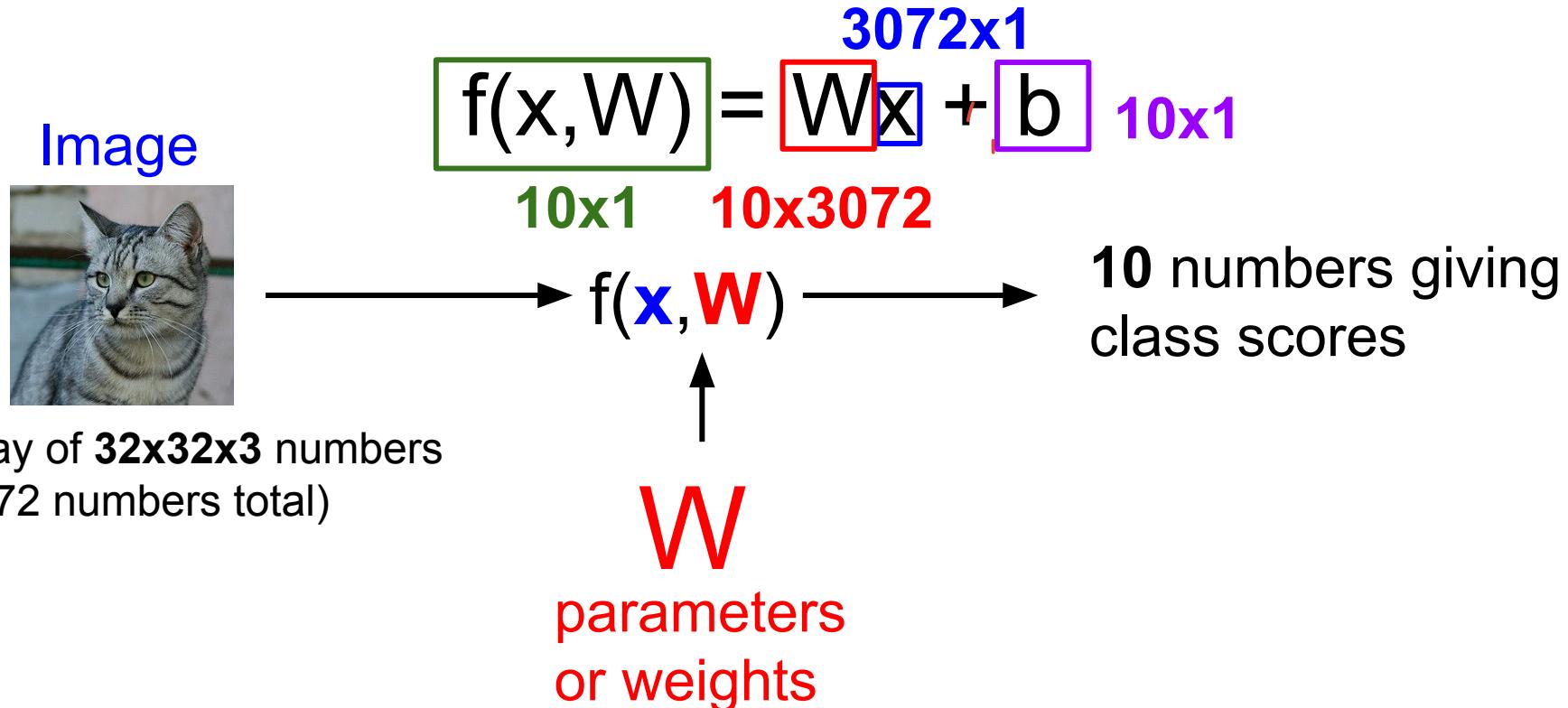
Parametric Approach: Linear Classifier



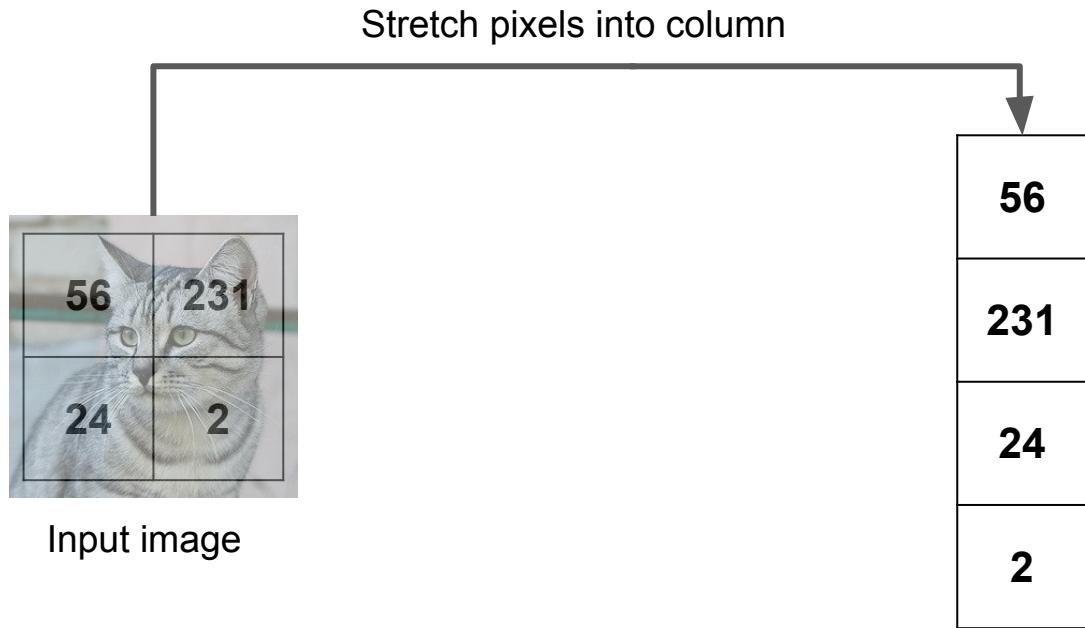
Parametric Approach: Linear Classifier



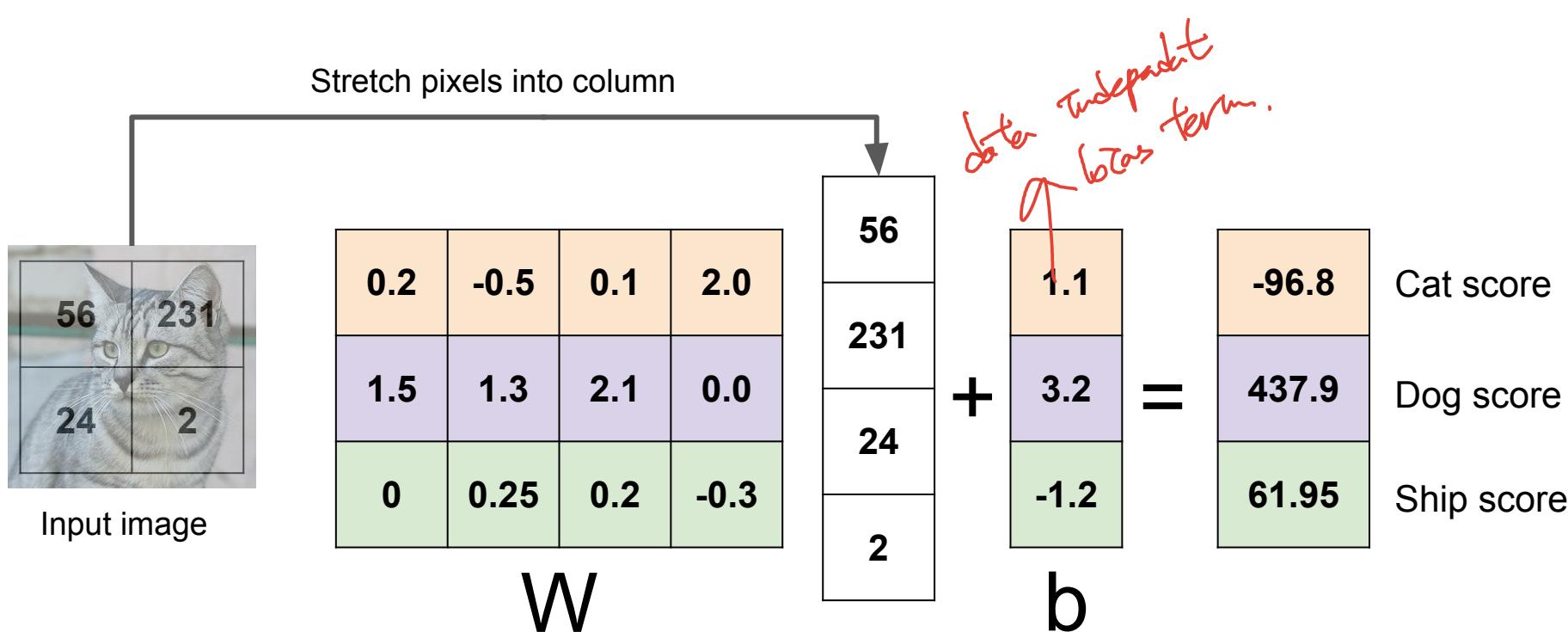
Parametric Approach: Linear Classifier



Example with an image with 4 pixels, and 3 classes (**cat/dog/ship**)



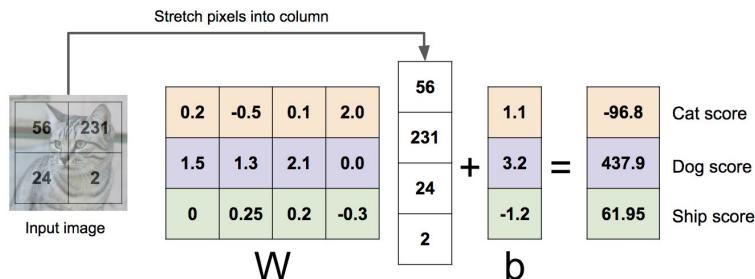
Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



Example with an image with 4 pixels, and 3 classes (**cat/dog/ship**)

Algebraic Viewpoint

$$f(x, W) = Wx$$



Example with an image with 4 pixels, and 3 classes (**cat/dog/ship**)

Algebraic Viewpoint

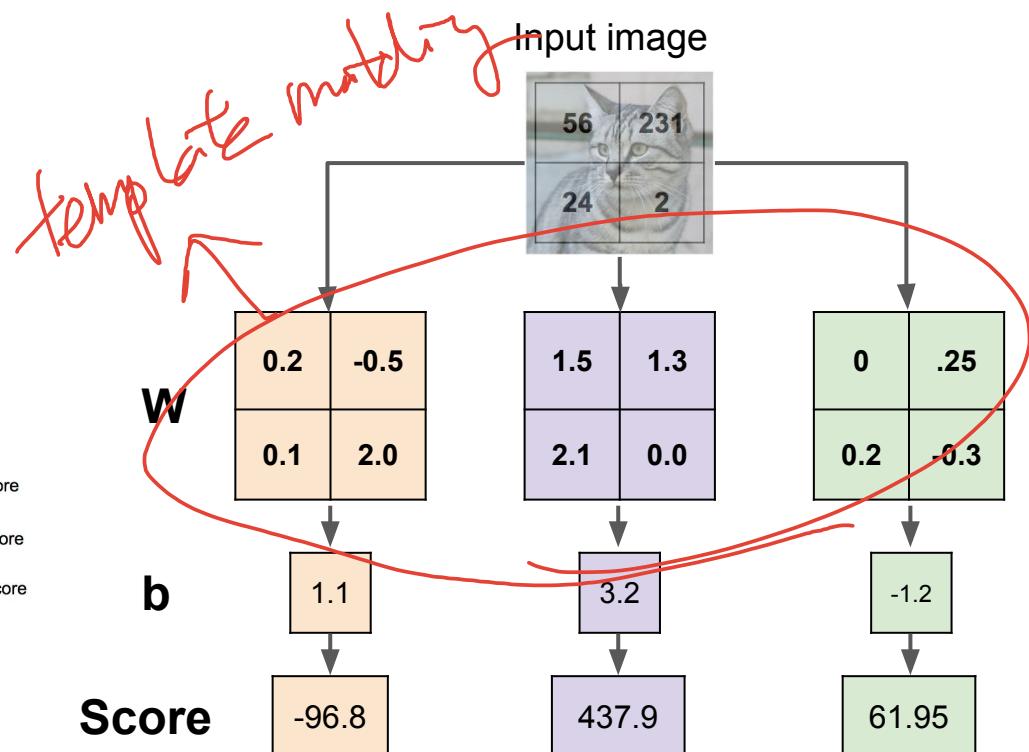
$$f(x, W) = Wx$$

Stretch pixels into column

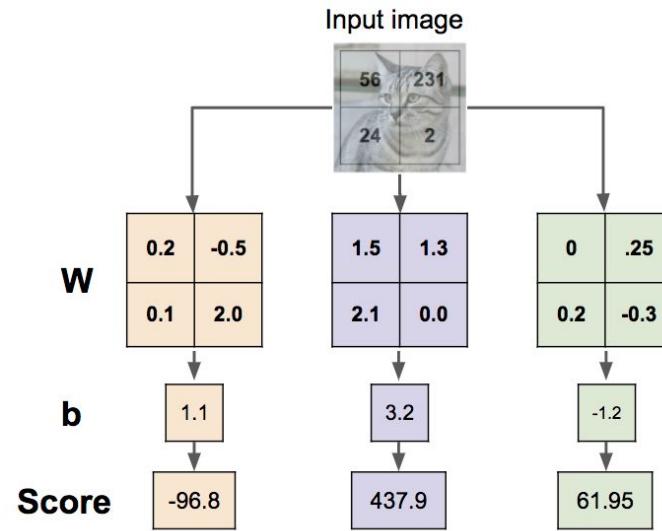
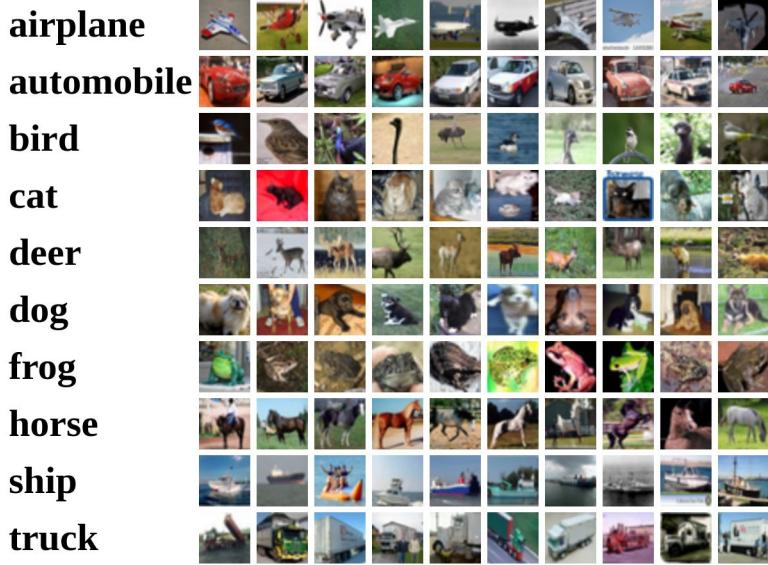
	<table border="1"> <tr><td>0.2</td><td>-0.5</td><td>0.1</td><td>2.0</td></tr> <tr><td>1.5</td><td>1.3</td><td>2.1</td><td>0.0</td></tr> <tr><td>0</td><td>0.25</td><td>0.2</td><td>-0.3</td></tr> </table>	0.2	-0.5	0.1	2.0	1.5	1.3	2.1	0.0	0	0.25	0.2	-0.3
0.2	-0.5	0.1	2.0										
1.5	1.3	2.1	0.0										
0	0.25	0.2	-0.3										
W	<table border="1"> <tr><td>56</td></tr> <tr><td>231</td></tr> <tr><td>24</td></tr> <tr><td>2</td></tr> </table>	56	231	24	2								
56													
231													
24													
2													

$$\begin{matrix} 56 \\ 231 \\ 24 \\ 2 \end{matrix} + \begin{matrix} 1.1 \\ 3.2 \\ -1.2 \end{matrix} = \begin{matrix} -96.8 \\ 437.9 \\ 61.95 \end{matrix}$$

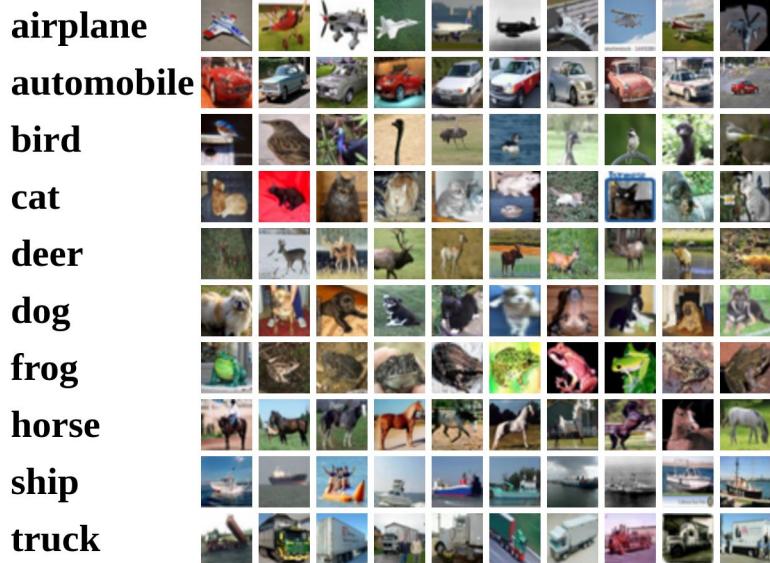
Cat score
Dog score
Ship score



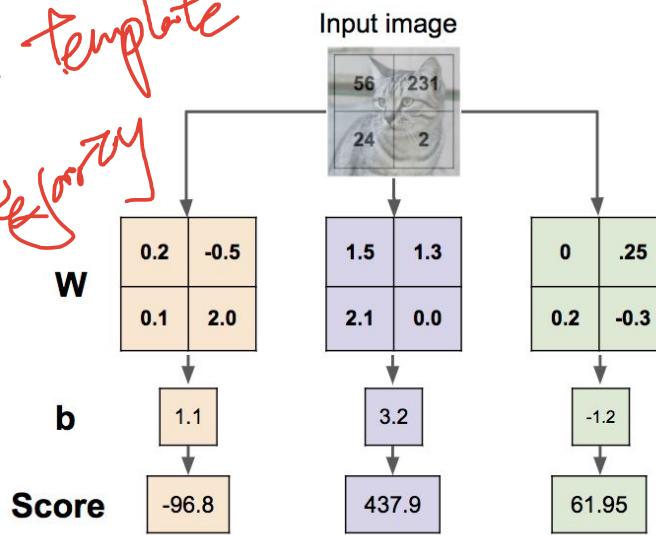
Interpreting a Linear Classifier



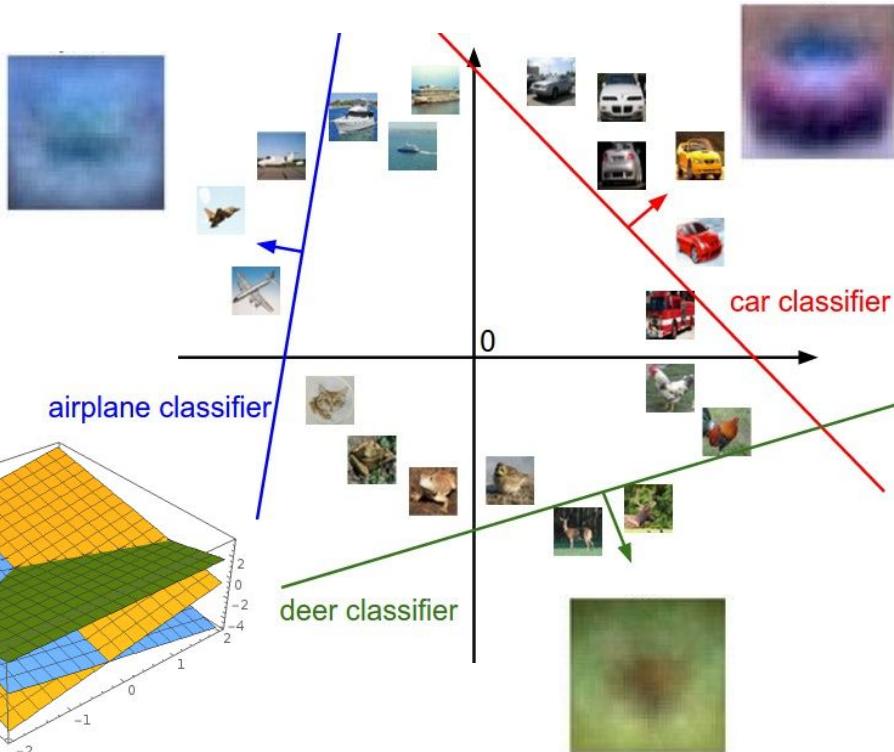
Interpreting a Linear Classifier: Visual Viewpoint



*Style template
for category*



Interpreting a Linear Classifier: Geometric Viewpoint



$$f(\mathbf{x}, \mathbf{W}) = \mathbf{W}\mathbf{x} + b$$



Array of **32x32x3** numbers
(3072 numbers total)

Plot created using [Wolfram Cloud](#)

[Cat image](#) by [Nikita](#) is licensed under [CC-BY 2.0](#)

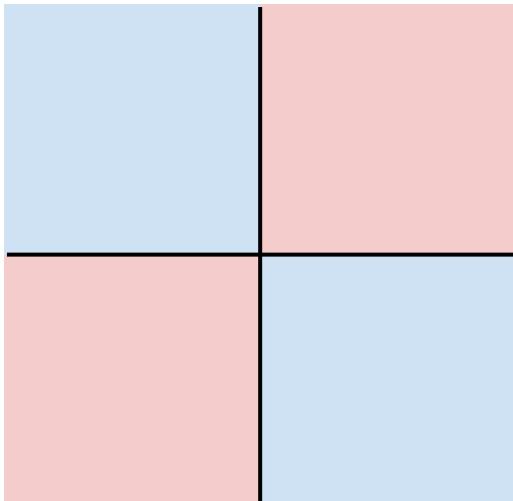
Hard cases for a linear classifier

Class 1:

First and third quadrants

Class 2:

Second and fourth quadrants

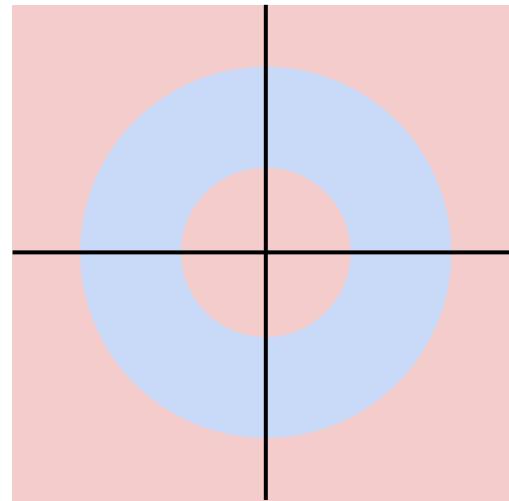


Class 1:

$1 \leq L_2 \text{ norm} \leq 2$

Class 2:

Everything else



Class 1:

Three modes

Class 2:

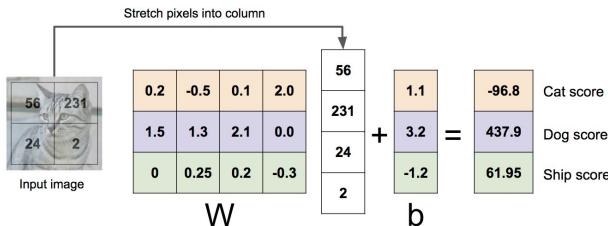
Everything else



Linear Classifier: Three Viewpoints

Algebraic Viewpoint

$$f(x, W) = Wx$$



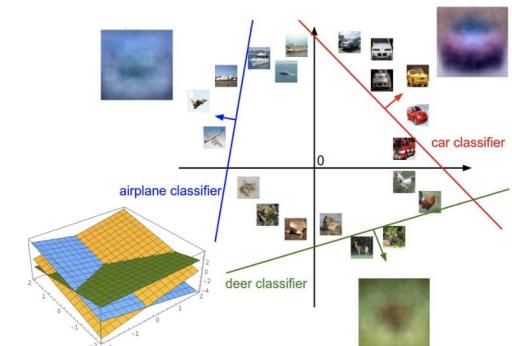
Visual Viewpoint

One template per class



Geometric Viewpoint

Hyperplanes cutting up space



So far: Defined a (linear) score function $f(x, W) = Wx + b$

Example class scores for 3 images for some W :

How can we tell whether this W is good or bad?



airplane	-3.45	-0.51	3.42
automobile	-8.87	6.04	4.64
bird	0.09	5.31	2.65
cat	2.9	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	-4.34
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

[Cat image](#) by [Nikita](#) is licensed under [CC-BY 2.0](#)

[Car image](#) is [CC0 1.0](#) public domain

[Frog image](#) is in the public domain

$$f(x, W) = Wx + b$$

Coming up:

- Loss function
- Optimization
- ConvNets!

(quantifying what it means to have a “good” W)

(start with random W and find a W^* that minimizes the loss)

(tweak the functional form of f)