XI-SENSOR V3.2: RESOLUTION OF THE M4–M5 ENGINEERING TARGETS

JONATHAN WASHBURN

Abstract. We resolve the two engineering targets introduced in v3.1. First, we prove that the originally stated M4 inequality

$$\left|\int \psi_L\, \partial_\sigma U\right| \le C\, L\, \sqrt{E_D(U)}, \qquad E_D(U) = \iint_Q |\nabla U|^2\, \sigma\, d\sigma\, dt,$$

is false in general (even for a single Blaschke potential source). Second, we prove the M5 Whitney-box energy estimate unconditionally for the corrected boundary-neutralized field:

$$E_D(U_D) \le C_{M5}\, \log^2\langle t_0\rangle\, |I|,$$

with C_{M5} independent of the Whitney parameter c. We then state the corrected next target M4* and quantify why M4*+M5 does not yet close RH.

1. Setup

For an off-line zero

$$\rho = \frac{1}{2} + \delta + i\gamma, \qquad \delta > 0, \qquad \rho^{\#} = 1 - \overline{\rho} = \frac{1}{2} - \delta + i\gamma,$$

define the half-plane Blaschke potential

$$G_\rho(s) := \log\left|\frac{s-\rho^{\#}}{s-\rho}\right|.$$

Then G_ρ is harmonic on $\{\mathrm{Re}\, s > 1/2\} \setminus \{\rho\}$, $G_\rho(1/2+it) = 0$, and

$$\partial_\sigma G_\rho(1/2+it) = \frac{2\delta}{\delta^2 + (t-\gamma)^2} \ge 0.$$

Fix $t_0 \in \mathbb{R}$ and

$$L = \frac{c}{\log^2\langle t_0\rangle}, \qquad I = [t_0 - L, t_0 + L], \qquad Q(\alpha' I) = \{1/2 + \sigma + it : 0 < \sigma \le \alpha' L,\ t \in \alpha' I\}.$$

2. M4 (as stated in v3.1) is false

Theorem 2.1 (Counterexample to M4)**.** *The inequality*

$$(1) \qquad \left|\int_{\mathbb{R}} \psi_{L,t_0}(t)\, \partial_\sigma U(1/2+it)\, dt\right| \le C\, L\, \sqrt{E_D(U)}, \qquad E_D(U) = \iint_{Q(\alpha' I)} |\nabla U|^2\, \sigma\, d\sigma\, dt,$$

cannot hold with C independent of L for all harmonic U with zero boundary trace on $\sigma = 0$.

Proof. Take

$$U(s) = G_{\rho_0}(s), \qquad \rho_0 = \frac{1}{2} + \delta_0 + it_0, \quad \delta_0 > 0,$$

and assume $L \le \delta_0/(4\alpha')$.

Date: February 12, 2026.

Lower bound:

$$\int \psi_{L,t_0}\,\partial_\sigma U(1/2+it)\,dt \ge \int_{t_0-L}^{t_0+L} \frac{2\delta_0}{\delta_0^2+(t-t_0)^2}\,dt = 4\arctan(L/\delta_0) \asymp \frac{L}{\delta_0}.$$

Energy bound: on $Q(\alpha' I)$, $|\nabla U| \ll 1/\delta_0$, hence

$$E_D(U) \ll \frac{1}{\delta_0^2}\iint_{Q(\alpha' I)} \sigma\,d\sigma\,dt \ll \frac{L^3}{\delta_0^2}.$$

Therefore

$$L\sqrt{E_D(U)} \ll \frac{L^{5/2}}{\delta_0}.$$

As $L \to 0$, $L^{5/2}/\delta_0 = o(L/\delta_0)$, contradicting (1) for any L-independent constant C. $\square$

Remark 2.2. So the v3.1 target M4 is not merely unproved; it is false. This is the decisive scaling obstruction.

3. M5 is true unconditionally (corrected neutralization)

Definition 3.1 (Unit-strip neutralization (except target))**.** Fix a distinguished off-line zero $\rho_0 = \frac{1}{2} + \delta_0 + i\gamma_0$ and define

$$\mathcal{N} := \{\rho \ne \rho_0 :\ |\operatorname{Im}\rho - \gamma_0| \le 1,\ \operatorname{Re}\rho > 1/2\}.$$

Define

$$U_D(s) := \sum_{\rho \in Z_+ \setminus \mathcal{N}} m_\rho\, G_\rho(s).$$

Proposition 3.2 (M5)**.** *There exists C_{M5} independent of c such that*

$$E_D(U_D) := \iint_{Q(\alpha' I)} |\nabla U_D|^2\,\sigma\,d\sigma\,dt \le C_{M5}\,\log^2\langle t_0\rangle\,|I|.$$

Proof. Step 1: boundary bound. For $s \in \partial Q(\alpha'' I)$ and $\rho \in Z_+ \setminus \mathcal{N}$, either $\rho = \rho_0$ or $|\operatorname{Im}\rho - \gamma_0| > 1$. Assume $L \le 1/(4\alpha'')$. Then for $|\operatorname{Im}\rho - \gamma_0| > 1$:

$$|\operatorname{Im}s - \operatorname{Im}\rho| \ge |\operatorname{Im}\rho - \gamma_0| - \alpha'' L \ge \tfrac{1}{2}.$$

Using $\log(1+x) \le x$ and $\delta_\rho \le 1/2$:

$$G_\rho(s) = \frac{1}{2}\log\left(1 + \frac{4\sigma\delta_\rho}{(\sigma-\delta_\rho)^2 + (\operatorname{Im}s - \operatorname{Im}\rho)^2}\right) \ll \frac{\sigma}{(\operatorname{Im}s - \operatorname{Im}\rho)^2} \ll \frac{L}{(\operatorname{Im}s - \operatorname{Im}\rho)^2}.$$

Hence

$$\sum_{\substack{\rho \in Z_+ \setminus \mathcal{N} \\ \rho \ne \rho_0}} m_\rho G_\rho(s) \ll L \sum_\rho \frac{m_\rho}{|\operatorname{Im}\rho - \gamma_0|^2}.$$

By Riemann–von Mangoldt in unit shells:

$$\sum_\rho \frac{m_\rho}{|\operatorname{Im}\rho - \gamma_0|^2} \ll \sum_{n \ge 1} \frac{\log(\langle t_0\rangle + n)}{n^2} \ll \log\langle t_0\rangle.$$

So far part is $\ll L\log\langle t_0\rangle$.

For ρ_0:

$$G_{\rho_0}(s) \ll \frac{\sigma}{\delta_0} \ll \frac{L}{\delta_0}.$$

Therefore

$$M := \sup_{\partial Q(\alpha'' I)} |U_D| \ll L\log\langle t_0\rangle + \frac{L}{\delta_0} \ll \log\langle t_0\rangle,$$

since $L \le 1$.

Step 2: interior gradient estimate. U_D is harmonic on $Q(\alpha'' I)$ by construction. Standard interior gradient bounds for harmonic functions give

$$\sup_{Q(\alpha' I)} |\nabla U_D|^2 \ll \frac{M^2}{L^2}.$$

Integrating with weight σ:

$$E_D(U_D) \ll \frac{M^2}{L^2} \iint_{Q(\alpha' I)} \sigma \, d\sigma \, dt \ll \frac{M^2}{L^2} \cdot L^2 |I| \ll M^2 |I| \ll \log^2 \langle t_0 \rangle \, |I|.$$

This proves the claim. □

4. What is now solved, and the corrected next target

- M5 is solved unconditionally (Proposition 3.2).
- M4 in v3.1 form is false (Theorem 2.1).

The corrected candidate is:

$$\text{M4}^* : \quad \left| \int \psi_{L,t_0} \, \partial_\sigma U_D \right| \le C_{M4^*} \sqrt{\frac{E_D(U_D)}{L}}.$$

This scaling is dimensionally consistent and compatible with Theorem 2.1.

Remark 4.1. M4* + M5 gives an upper bound of order $\log\langle t_0 \rangle$, while the single-zero lower bound is order L. So M4*+M5 does *not* close RH. To close, one needs either:

- a stronger energy estimate than M5 (sub-logarithmic enough to beat the window scale), or
- a different boundary functional with a lower bound not decaying like L.

References

[1] H. Davenport, *Multiplicative Number Theory*, 3rd ed., Springer, 2000.
[2] E. C. Titchmarsh, *The Theory of the Riemann Zeta-Function*, 2nd ed., Oxford University Press, 1986.

Recognition Science Research Institute, Austin, TX, USA
Email address: jon@recognitionphysics.org