Corner-Exit Slivers for Calibrated Sheet Constructions: Deterministic Face Incidence and Uniform Boundary Control

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Abstract

We introduce corner-exit slivers: local calibrated template pieces inside a cube whose footprint is a uniformly fat simplex meeting only a prescribed set of boundary faces. The corner-exit geometry provides deterministic control of where sheet boundaries can occur, a key requirement in mesh-based gluing of many small calibrated pieces.

In Euclidean $\mathbb{C}^n$, we construct a $(2p-1)$-parameter translation family of complex $(n-p)$-plane templates whose intersection with a cube has identical corner-exit footprint geometry. We prove two robust properties: (i) face incidence is stable under sufficiently small C^1 graph perturbations, so a realized sheet intersects a cube face if and only if that face is designated by the template; (ii) the boundary mass on each designated face is comparable to a fixed scale $v^{(k-1)/k}$, where v is the interior k-volume of the sliver.

Combining this geometry with Bergman-scale holomorphic manufacturing yields holomorphic corner-exit slivers in projective Kähler manifolds with an L^1-type interface estimate controlling the total boundary-face mass by $O(v^{(k-1)/k})$. We further build robust corner-exit template families for finite nets of calibrated directions, providing a uniform dictionary of geometric building blocks for later global coherence and gluing arguments.

1 Introduction

When one assembles many local sheet pieces on a cubical mesh, the only place the assembled object can fail to be closed is along mesh interfaces: codimension-one faces of cubes. If the sheet pieces are arbitrary, their boundary traces on cube faces can be geometrically complicated and hard to match. Corner-exit slivers are designed to remove that difficulty.

A corner-exit sliver is built from a local *template* whose footprint inside a cube is a uniformly fat simplex placed near a chosen cube vertex. The defining inequalities force the footprint to touch only a prescribed set of "near" faces (those incident to that vertex) and to stay a definite distance from all other faces. This has two consequences:

(G1) Deterministic face incidence. Any sufficiently small C^1 graph perturbation of the template intersects exactly the same set of cube faces.

(G2) Uniform boundary control. Each designated face slice has $(k-1)$-mass comparable to the k-volume of the footprint to the power $(k-1)/k$.

These two outputs are exactly what one needs later for global matching and gluing: (G1) makes the boundary *combinatorial*, and (G2) makes it *quantitatively summable.*

Remark 1 (Recognition Geometry framing (optional))**.** Let $\mathcal{C}$ be a configuration space of templates (for example, translation parameters t in a fixed admissible box), and let $\mathcal{E}$ be the finite event space of face-incidence patterns (subsets of cube faces). The map

$$R : \mathcal{C} \to \mathcal{E}, \qquad R(t) = \{\text{faces hit by the footprint } E(t)\}$$

is a recognizer in the sense of Recognition Geometry. A corner-exit construction produces a large region of $\mathcal{C}$ on which R is constant (one resolution cell), and the stability theorem below says that small C^1 perturbations remain in the same cell. In short: corner-exit slivers turn "which faces are hit" into a robust finite-resolution observable.

2 Corner-exit templates and slivers in cubes

We begin in Euclidean space. Let $d \geq 2$ and let

$$Q = [0, h]^d \subset \mathbb{R}^d$$

be the standard cube of side length $h > 0$. A *codimension-one face* of Q is any set of the form $\{x_i = 0\} \cap Q$ or $\{x_i = h\} \cap Q$.

Definition 1 (Corner-exit simplex footprint)**.** Fix an integer $1 \le k < d$ and a vertex v of Q. Let $P \subset \mathbb{R}^d$ be an affine k-plane and set

$$E := P \cap Q.$$

We say that E is a *corner-exit simplex footprint at v* if:

1. E is a (nondegenerate) k-simplex with one vertex at v;
2. there exist *distinct* codimension-one faces $F_0, \ldots, F_k$ of Q, each incident to v, such that the $k+1$ facets of E are exactly the sets $E \cap F_i$ $(i = 0, \ldots, k)$;
3. E meets no other codimension-one faces of Q.

The faces $F_0, \ldots, F_k$ are called the *designated exit faces.*

Definition 2 (Quantitative fatness for simplices)**.** Let $E \subset \mathbb{R}^d$ be a k-simplex, and let $\Pi = \mathrm{aff}(E)$ be its affine span. Fix $\Lambda \ge 1$. We say E is Λ-*fat* if there exists an affine isomorphism $A : \Pi \to \mathbb{R}^k$ such that

$$\|DA\| \le \Lambda, \qquad \|(DA)^{-1}\| \le \Lambda,$$

and $A(E)$ is a standard k-simplex of some scale $s > 0$.

Definition 3 (Corner-exit sliver (geometric form))**.** Let $E = P \cap Q$ be a corner-exit simplex footprint with designated exit faces $F_0, \ldots, F_k$ and gap

$$\delta := \min\{\mathrm{dist}(E, F) : \ F \text{ a codimension-one face of } Q, \ F \notin \{F_0, \ldots, F_k\}\}.$$

A smooth oriented k-submanifold $Y \subset \mathbb{R}^d$ is a *corner-exit sliver over E in Q* if $Y \cap Q$ is the image of a C^1 embedding $\Phi : E \to \mathbb{R}^d$ such that:

1. (*small slope*) Φ is a C^1 graph over E with slope $\le \varepsilon$, meaning $\|D\Phi - \mathrm{Id}\|_{C^0(E)} \le C\,\varepsilon$ in the coordinates of P;
2. (*small displacement*) $\sup_{x \in E} |\Phi(x) - x| < \delta/2$.

The purpose of these definitions is that they isolate what later gluing arguments actually use: the face incidence pattern and the size of boundary traces.

3 An explicit complex corner-exit translation template in $\mathbb{C}^n$

We now give an explicit construction of corner-exit templates in $\mathbb{C}^n$ with a $(2p-1)$-parameter translation family and identical footprints.

Fix integers $n \geq 1$ and $1 \leq p \leq n$, and set

$$k := 2(n-p), \qquad d := 2n.$$

Identify $\mathbb{C}^n \cong \mathbb{R}^{2n}$ and write $\mathbb{C}^n = \mathbb{C}^{n-p} \times \mathbb{C}^p$ with coordinates $z = (u, w)$, where $u = (u_1, \ldots, u_{n-p})$ and $w = (w_1, \ldots, w_p)$.

Lemma 1 (A concrete complex corner-exit translation template in a cube)**.** *Let $Q = [0,h]^{2n} \subset \mathbb{R}^{2n} \cong \mathbb{C}^n$ be the coordinate cube with vertex 0. Fix a constant $0 < c_0 < 1$ and choose a scale $s > 0$ with $s \leq c_0 h/100$.*

Define a complex $(n-p)$-plane $P \subset \mathbb{C}^n$ as the graph of the complex-linear map $A : \mathbb{C}^{n-p} \to \mathbb{C}^p$ given by

$$w_1 = -(1-i)\sum_{j=1}^{n-p} u_j, \qquad w_2 = \cdots = w_p = 0.$$

For translation parameters $t = (t_1, \ldots, t_p) \in \mathbb{C}^p$, write

$$P_t := \{(u, Au + t) : u \in \mathbb{C}^{n-p}\}.$$

Assume t satisfies the interior-margin bounds

$$\Re t_1 = s, \qquad 2s \leq \Im t_1 \leq 3s, \qquad 2s \leq \Re t_j, \Im t_j \leq 3s \quad (2 \leq j \leq p).$$

Then $E(t) := P_t \cap Q$ has the following properties:

1. *(*Corner-exit simplex footprint*) $E(t)$ is a k-simplex contained in the ball $B(0, c_0 h)$.*
2. *(*Fixed designated exit faces*) The $k+1$ facets of $E(t)$ lie on the $k+1$ coordinate faces*

   $$F_{\Re u_j = 0}, \quad F_{\Im u_j = 0} \ (1 \leq j \leq n-p), \qquad \text{and} \qquad F_{\Re w_1 = 0},$$

   and $E(t)$ meets no other codimension-one faces of Q.
3. *(*Uniform fatness and equal footprint geometry*) Throughout the admissible parameter box (with fixed $\Re t_1 = s$), the footprints are identical up to translation in transverse directions. In particular, $\mathcal{H}^k(E(t))$ and each facet measure $\mathcal{H}^{k-1}(E(t) \cap F)$ are independent of t. Moreover, $E(t)$ is Λ-fat for a constant Λ depending only on (n, p).*

Finally, the admissible parameter box has real dimension $2p-1$, so for any separation scale $\eta > 0$ one can choose an ordered η-separated list $(t_a)_{a\ge 1}$ in that box. All footprints $E(t_a)$ then have exactly equal k-volume and exactly equal facet measures.

Proof. Write $u_j = x_j + iy_j$ with $x_j = \Re u_j$ and $y_j = \Im u_j$. On P_t one computes

$$\Re w_1 = \Re t_1 + \Re\Big(-(1-i)\sum_{j=1}^{n-p} u_j\Big) = s - \sum_{j=1}^{n-p}(x_j + y_j),$$

and

$$\Im w_1 = \Im t_1 + \Im\Big(-(1-i)\sum_{j=1}^{n-p} u_j\Big) = \Im t_1 + \sum_{j=1}^{n-p}(x_j - y_j).$$

The cube constraints on $w_2, \dots, w_p$ are automatic because $w_j \equiv t_j$ and the margin assumptions place each t_j strictly inside $(0,h) + i(0,h)$.

Consider the region cut out by $x_j \ge 0$, $y_j \ge 0$, and $\sum_j (x_j + y_j) \le s$. On this region one has $\big|\sum_j (x_j - y_j)\big| \le \sum_j (x_j + y_j) \le s$, hence

$$\Im w_1 \in [\Im t_1 - s,\ \Im t_1 + s] \subset [s, 4s] \subset (0,h),$$

so the faces $\{\Im w_1 = 0\}$ and $\{\Im w_1 = h\}$ are avoided. Also $\Re w_1 \in [0,s] \subset (0,h)$ avoids the face $\{\Re w_1 = h\}$. Finally $x_j, y_j \le s \ll h$ avoids the far faces $\{\Re u_j = h\}$ and $\{\Im u_j = h\}$.

Consequently, $E(t) = P_t \cap Q$ is cut out on P_t exactly by the inequalities

$$x_j \ge 0, \quad y_j \ge 0 \quad (1 \le j \le n-p), \qquad \text{and} \qquad \Re w_1 \ge 0,$$

which is equivalent (on P_t) to $\sum_j (x_j + y_j) \le s$ together with the nonnegativity of the $k = 2(n-p)$ real coordinates $(x_1, y_1, \dots, x_{n-p}, y_{n-p})$. This is the standard k-simplex of scale s embedded affinely in $\mathbb{R}^{2n}$, so $E(t)$ is a k-simplex and its facets lie exactly on the faces listed, proving (1) and (2). Since $s \le c_0 h/100$, the whole simplex lies in a ball $B(0, c_0 h)$.

The defining inequalities on (x_j, y_j) do not depend on t once $\Re t_1 = s$ is fixed, so $\mathcal{H}^k(E(t))$ and all facet measures are independent of t. Fatness follows because $E(t)$ is the image of the standard simplex under the fixed affine embedding $u \mapsto (u, Au + t)$; the distortion is controlled by the condition number of the fixed linear map $(, A)$, hence by a constant depending only on (n, p).

The dimension count is immediate: $\Re t_1$ is fixed, $\Im t_1$ varies in an interval (1 real parameter), and for each $j = 2, \dots, p$ both $\Re t_j$ and $\Im t_j$ vary (2 real parameters each), giving $1 + 2(p-1) = 2p-1$. $\square$

4 Two quantitative lemmas: graph distortion and simplex facet scaling

We now isolate two quantitative facts used repeatedly.

Lemma 2 (Small-slope graph distortion for volumes and facet measures)**.** *Let $k \geq 1$ and let $E \subset \mathbb{R}^k$ be measurable. Let $G : E \to \mathbb{R}^q$ be C^1 with*

$$\|DG\|_{L^\infty(E)} \leq \varepsilon, \qquad 0 < \varepsilon \leq 1.$$

Define the graph map $\Gamma : E \to \mathbb{R}^{k+q}$ by $\Gamma(x) = (x, G(x))$. Then

$$1 \;\leq\; J_k\Gamma(x) = \sqrt{\det\bigl(I_k + (DG(x))^T DG(x)\bigr)} \;\leq\; (1+\varepsilon^2)^{k/2} \;\leq\; 1 + k\varepsilon^2,$$

and consequently

$$\mathcal{H}^k(E) \;\leq\; \mathcal{H}^k(\Gamma(E)) \;\leq\; (1 + k\varepsilon^2)\,\mathcal{H}^k(E).$$

Moreover, if $F \subset E$ is contained in a C^1 hypersurface in $\mathbb{R}^k$, then the same estimate holds in dimension $(k-1)$:

$$\mathcal{H}^{k-1}(F) \;\leq\; \mathcal{H}^{k-1}(\Gamma(F)) \;\leq\; (1 + (k-1)\varepsilon^2)\,\mathcal{H}^{k-1}(F).$$

Proof. The matrix $A := (DG)^T DG$ is positive semidefinite and every eigenvalue of A is at most $\|DG\|^2 \leq \varepsilon^2$. Hence $\det(I_k + A) \leq (1+\varepsilon^2)^k$ and so $J_k\Gamma \leq (1+\varepsilon^2)^{k/2}$. The inequality $(1+\varepsilon^2)^{k/2} \leq 1+k\varepsilon^2$ holds for $0 \leq \varepsilon \leq 1$ by convexity of $t \mapsto (1+t)^{k/2}$. The volume inequalities follow from integrating the Jacobian over E.

For the $(k-1)$-dimensional bound, restrict Γ to a $(k-1)$-dimensional tangent subspace: the corresponding Jacobian is controlled by the same eigenvalue bound, with k replaced by $k-1$. □

Lemma 3 (Facet mass comparable to $v^{(k-1)/k}$ for fat simplices)**.** *Fix $d \geq 2$, $1 \leq k < d$, and $\Lambda \geq 1$. Let $E \subset \mathbb{R}^d$ be a Λ-fat k-simplex and write $v_E := \mathcal{H}^k(E)$. Let $\sigma_0, \ldots, \sigma_k$ be the $(k-1)$-facets of E, and set $a_i := \mathcal{H}^{k-1}(\sigma_i)$. Then there exist constants $0 < c_\star \leq C_\star < \infty$ depending only on (k, Λ) such that for every $i = 0, \ldots, k$,*

$$c_\star\, v_E^{(k-1)/k} \;\leq\; a_i \;\leq\; C_\star\, v_E^{(k-1)/k}.$$

Proof. By Λ-fatness there is an affine isomorphism $A : \mathrm{aff}(E) \to \mathbb{R}^k$ with operator norms of DA and $(DA)^{-1}$ bounded by Λ, mapping E to a standard simplex Δ_s of some scale $s > 0$. Since A is affine, its k-Jacobian and $(k-1)$-Jacobian are constant on $\mathrm{aff}(E)$ and are bounded above and below by constants depending only on (k, Λ). Therefore

$$v_E = \mathcal{H}^k(E) \simeq_{k,\Lambda} \mathcal{H}^k(\Delta_s) \simeq_k s^k, \qquad a_i = \mathcal{H}^{k-1}(\sigma_i) \simeq_{k,\Lambda} \mathcal{H}^{k-1}(\text{facet of } \Delta_s) \simeq_k s^{k-1}.$$

Eliminating s yields $a_i \simeq_{k,\Lambda} v_E^{(k-1)/k}$ uniformly in i. $\square$

5 Stability under C^1 perturbations: deterministic face incidence and boundary control

We now prove the two main outputs (G1)–(G2) in a general Euclidean form.

Proposition 1 (Corner-exit footprint geometry for small-slope graphs)**.** *Fix $d \geq 2$ and $1 \leq k < d$. Let $Q = [0, h]^d \subset \mathbb{R}^d$ and let $E = P \cap Q$ be a corner-exit simplex footprint with designated exit faces $F_0, \ldots, F_k$ and gap $\delta > 0$ to all non-designated faces. Assume E is Λ-fat.*

Let $Y \subset \mathbb{R}^d$ be a smooth oriented k-submanifold such that $Y \cap Q = \Phi(E)$ where $\Phi : E \to \mathbb{R}^d$ is a C^1 graph over E with slope at most ε and displacement $\sup_{x \in E} |\Phi(x) - x| < \delta/2$.

Then:

(G1) *(*Face incidence is deterministic.*) For any codimension-one face F of Q,*

$$Y \cap F \neq \emptyset \iff F \in \{F_0, \ldots, F_k\}.$$

(G2) *(*Per-face boundary mass is uniformly controlled.*) For each $i = 0, \ldots, k$, the intersection $Y \cap F_i$ is a smooth oriented $(k-1)$-submanifold and*

$$\mathcal{H}^{k-1}(Y \cap F_i) = \big(1 + O_k(\varepsilon^2)\big)\, \mathcal{H}^{k-1}(E \cap F_i) \ \simeq_{k,\Lambda}\ v_E^{(k-1)/k},$$

where $v_E = \mathcal{H}^k(E)$ and the implied constants depend only on (k, Λ).

Proof. (G1) Let F be any codimension-one face not in $\{F_0, \ldots, F_k\}$. By definition of δ, $\mathrm{dist}(E, F) \geq \delta$. If $y \in Y \cap F$, then $y = \Phi(x)$ for some $x \in E$, hence

$$\delta \ \leq\ \mathrm{dist}(x, F) \ \leq\ |x - \Phi(x)| \ <\ \delta/2,$$

a contradiction. Therefore $Y \cap F = \emptyset$ for all non-designated faces.

Conversely, if $F = F_i$ is designated, then $E \cap F_i$ is a nonempty facet of E. Because Φ is C^1-close to the identity on E and maps E into Q, the image of this facet must meet F_i. (Geometrically: the facet is the locus where a defining inequality of E becomes equality; a small C^1 perturbation preserves that boundary contact.) Thus $Y \cap F_i \neq \emptyset$.

(G2) The intersection $Y \cap F_i$ is a small-slope graph over the facet $E \cap F_i$, so the $(k-1)$-dimensional graph distortion lemma gives

$$\mathcal{H}^{k-1}(Y \cap F_i) = \big(1 + O_k(\varepsilon^2)\big)\,\mathcal{H}^{k-1}(E \cap F_i).$$

Since E is Λ-fat, the facet scaling lemma yields $\mathcal{H}^{k-1}(E \cap F_i) \simeq_{k,\Lambda} v_E^{(k-1)/k}$, giving the final bound. $\square$

Remark 2 (Currents language (what later gluing uses))**.** If Y is a smooth oriented k-manifold without boundary, the integration current $[Y]$ satisfies $\partial([Y]\llcorner Q) = [Y]\llcorner\partial Q$. In that case, $\mathrm{Mass}(\partial([Y]\llcorner Q)\llcorner F_i) = \mathcal{H}^{k-1}(Y \cap F_i)$. Thus (G2) can be read as a quantitative per-face boundary mass estimate for the restricted current $[Y]\llcorner Q$.

6 Boundary-face L^1 interface control

We now package (G2) into an L^1-type estimate that is robust under taking many slivers.

Proposition 2 (L^1 interface mass control on boundary faces)**.** *Fix $d \geq 2$ and $1 \leq k < d$. Let $Q = [0,h]^d$ and let $Y^{(1)}, \ldots, Y^{(N)} \subset \mathbb{R}^d$ be smooth oriented k-submanifolds such that for each a the piece $Y^{(a)} \cap Q$ is a corner-exit sliver over a Λ-fat corner-exit simplex footprint E_a (with some designated exit faces depending on a), with slope at most $\varepsilon \leq 1$.*

Write $v_a := \mathcal{H}^k(E_a)$. Then there exists a constant $C = C(k,\Lambda)$ such that

$$\sum_{a=1}^{N} \sum_{F \subset \partial Q \ \text{codim-1}} \mathcal{H}^{k-1}\big(Y^{(a)} \cap F\big) \ \leq \ C\,(1 + O_k(\varepsilon^2)) \ \sum_{a=1}^{N} v_a^{(k-1)/k}.$$

In particular, if one measures boundary by the mass of $\partial([Y^{(a)}]\llcorner Q)$, then

$$\sum_{a=1}^{N} \mathrm{Mass}\big(\partial([Y^{(a)}]\llcorner Q)\big) \ \leq \ C\,(1 + O_k(\varepsilon^2)) \ \sum_{a=1}^{N} v_a^{(k-1)/k}.$$

Proof. Each footprint E_a has exactly $k+1$ designated exit faces, and meets no other codimension-one faces. By (G2), on each designated face the $(k-1)$-measure of the boundary slice is $\simeq_{k,\Lambda} v_a^{(k-1)/k}$ up to the common $1+$

$O_k(\varepsilon^2)$ factor coming from the graph distortion. Summing the $k+1$ face contributions per sliver and then summing over a yields the stated bound. $\square$

Corollary 1 (No-heavy-tail uniformity for the explicit complex template family)**.** *In the explicit $\mathbb{C}^n$ construction of the corner-exit translation template, all footprints $E(t)$ in the admissible parameter box have identical k-volume and identical facet measures. Therefore an ordered separated list (t_a) produces a family of corner-exit templates with exactly equal per-piece boundary scales (no heavy tails along the order). If a realized family $(Y^{(a)})$ consists of small-slope graphs over these footprints with a common slope bound ε, then all per-piece boundary masses differ only by a common $(1+O(\varepsilon^2))$ factor.*

7 Uniform corner-exit template dictionaries for direction nets

For later global constructions, one often fixes a finite net of calibrated directions and wants a uniform corner-exit template family associated to each direction label.

Proposition 3 (Uniform corner-exit templates for a finite calibrated direction net)**.** *Fix (n,p) and work locally in holomorphic charts in a Kähler manifold so that cubes are defined in holomorphic normal coordinates. Let $\{\Pi_1,\dots,\Pi_M\}$ be a finite set of complex $(n-p)$-planes in $\mathbb{C}^n$ (a finite direction net). Then there exist constants $c_0 \in (0,1)$ and $\Lambda \geq 1$, depending only on (n,p), with the following property:*

For each $j \in \{1,\dots,M\}$ there exists a choice of unitary holomorphic coordinates in which Π_j is represented as a fixed reference plane, and in those coordinates the explicit complex corner-exit translation template produces a $(2p-1)$-parameter family of calibrated planes whose cube intersections are Λ-fat corner-exit simplex footprints with the same designated-face pattern and the same footprint scale. All constants (Λ, the slope-to-face-incidence tolerance, and the per-face mass comparability constants) are uniform in j.

Proof. All complex $(n-p)$-planes in $\mathbb{C}^n$ are related by unitary transformations, and unitary transformations preserve calibration, Euclidean norms, and the quantitative fatness constants of simplices. Because the net is finite, one may choose for each Π_j a unitary coordinate frame adapted to it, and then apply the explicit template construction in that frame with the same fixed linear map A. Uniformity of the constants follows because the explicit template constants depend only on (n, p) and the chosen scale separation $s \ll h$, not on j. $\square$

8 Interface with Bergman-scale holomorphic manufacturing

The Euclidean results above are purely geometric. To obtain *holomorphic* corner-exit slivers in a projective Kähler manifold, one needs a separate analytic input: a local manufacturing result guaranteeing that, on a Bergman-scale cube, a holomorphic complete intersection can be made into a small-slope graph over a prescribed affine complex plane template.

We record the interface as a clean corollary: if holomorphic manufacturing produces a small-slope graph over a corner-exit footprint, then the corner-exit conclusions follow automatically from the geometry already proved.

Corollary 2 (Holomorphic corner-exit slivers inherit deterministic face incidence and boundary control)**.** *Let X be a smooth complex projective manifold with Kähler form ω arising as the curvature of an ample line bundle. Fix p and let $k = 2(n - p)$.*

Fix a holomorphic chart identifying a neighborhood with a cube $Q = [0, h]^{2n} \subset \mathbb{C}^n$. Let P_t be any plane in the explicit complex corner-exit translation template family (with footprint scale $s \ll h$) and let $E(t) = P_t \cap Q$ be the corresponding corner-exit simplex footprint.

Suppose a holomorphic complete intersection $Y \subset X$ (cut out by p holomorphic sections of a large tensor power) satisfies that $Y \cap Q$ is a single C^1 graph over $E(t)$ with slope at most ε and displacement $< \delta/2$, where δ is the gap from $E(t)$ to all non-designated cube faces.

Then $Y \cap Q$ is a corner-exit sliver and satisfies (G1)–(G2) from the stability proposition: it intersects a cube face if and only if that face is designated by the template, and each designated face slice has boundary mass $\simeq v^{(k-1)/k}$ where $v = \mathcal{H}^k(E(t))$. Consequently, any finite family of such holomorphic corner-exit slivers in Q satisfies the L^1 interface estimate of the previous section.

Proof. The hypotheses are exactly those of the stability proposition (with $d = 2n$). Holomorphicity is used only to ensure smoothness of Y and the absence of boundary inside Q; the face-incidence and boundary-mass conclusions are geometric consequences of being a small-slope graph over a corner-exit footprint. □

9 Discussion

Corner-exit slivers are structurally stronger than generic "graph over a plane" pieces for one specific reason: they promote boundary behavior from geometry to combinatorics. A corner-exit footprint comes with a *finite* face-incidence pattern that is stable under perturbations, and with a uniform boundary scale that depends only on the footprint volume. This makes it possible to run later matching and gluing arguments on a mesh without needing to solve a geometric boundary-tracing problem anew in every cell.