Neutrino Masses from the Deep φ-Ladder: Fractional Rungs, Mass Splittings, and the φ^7 Ratio

Paper III of V: The Neutrino Sector

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Abstract

Papers I and II established the Recognition Science (RS) mechanism of mass and derived predictions for all nine charged fermions from integer positions on the golden-ratio (φ) ladder. This paper extends the framework to neutrinos, which occupy the *deep* (low-mass) end of the ladder.

The charged sectors use integer rungs; the neutrino sector requires *fractional* (quarter-step) rungs, reflecting the vastly finer mass resolution needed at the sub-eV scale. We assign a specific rung triple $(r_1, r_2, r_3) = (-239/4, -231/4, -217/4)$ and derive:

- absolute masses $m_1 \approx 0.00354\,\text{eV}$, $m_2 \approx 0.00926\,\text{eV}$, $m_3 \approx 0.0499\,\text{eV}$,
- a mass sum $\Sigma m_\nu \approx 0.063\,\text{eV}$ (below current cosmological bounds),
- normal ordering ($m_1 < m_2 < m_3$) as a structural consequence (not a fit choice),
- the **key structural prediction**: an exact squared-mass ratio $(m_3^2/m_2^2) = \varphi^7 \approx 29.03$, independent of the eV calibration seam,
- mass-squared splittings $\Delta m_{21}^2 \approx 7.33\times10^{-5}\,\text{eV}^2$ and $\Delta m_{31}^2 \approx 2.48\times10^{-3}\,\text{eV}^2$, consistent with NuFIT summary windows.

The ratio of splittings $R_\Delta = \Delta m_{31}^2/\Delta m_{21}^2 = (\varphi^{11} - 1)/(\varphi^4 - 1) \approx 33.82$ is seam-free (the calibration parameter cancels) and constitutes the most robust falsifiable prediction of the deep-ladder hypothesis. We also discuss the no-go result for integer rungs $(0, 11, 19)$ with $Z_\nu = 0$ at the anchor, explaining why the neutrino sector requires a qualitatively different approach from the charged sectors, and we provide a comprehensive set of falsifiers.

Contents

1 Introduction

1.1 The neutrino puzzle

Neutrinos present unique challenges. Oscillation experiments measure mass-squared *differences* Δm_{21}^2 and Δm_{31}^2 with remarkable precision, but do not determine the absolute mass scale. The mass ordering (normal vs. inverted) remains an open question. The absolute scale is constrained by cosmological bounds on Σm_ν and kinematic measurements (β-decay endpoints), but neither has produced a definitive value.

Within the Recognition Science framework, the charged fermion masses are organized by integer rungs on the φ-ladder at the anchor scale $\mu_\star$. Neutrinos, however, are qualitatively different for two reasons:

1. **Vanishing charge band**: Neutrinos have $Q = 0$, so the integerized charge $\tilde{Q} = 6Q = 0$ and the band label $Z_\nu = 0$. The gap function gives $\mathrm{gap}(0) = \log_\varphi(1 + 0/\varphi) = 0$—there is no band correction. The structural ingredient that splits the charged families is absent.
2. **Deep ladder**: Neutrino masses ($\sim 10^{-2}\,\mathrm{eV}$) are $\sim 10^{10}$ times smaller than the electron mass. On the φ-ladder this corresponds to rungs in the deep negative region (around $r \sim -55$ to -60), far from the charged sector rungs ($r \sim 2$ to 21).

1.2 The no-go for integer rungs and its resolution

A natural first attempt applies the same integer rung convention to neutrinos with the charged-sector generation torsion $\{0, 11, 17\}$, giving the formal rung triple $(r_1, r_2, r_3) = (0, 11, 19)$. However, as documented in the companion analysis, this triple fails the acceptance test: neither normal nor inverted ordering produces splittings consistent with NuFIT data under $Z_\nu = 0$ with a single neutrino yardstick.

This no-go is instructive rather than fatal. It identifies the precise structural point where the neutrino sector diverges from the charged sectors: the *rung resolution*. The resolution is to allow **fractional rungs**—specifically, quarter-step positions $r \in \frac{1}{4}\mathbb{Z}$—on the deep ladder.

1.3 Organization of this paper

Section 2 defines the deep ladder and the fractional rung convention. Section 3 derives the neutrino mass predictions under a specific rung triple. Section 4 computes mass-squared splittings and derives the seam-free φ^7 ratio. Section 5 proves that normal ordering is a structural consequence. Section 6 checks cosmological consistency. Section 7 discusses the relationship to Dirac vs. Majorana nature. Section 8 lists falsifiers. Section 9 concludes.

2 The Deep φ-Ladder: Fractional Rungs

2.1 Ladder coordinate

As in the charged sectors, the base-φ logarithm defines the ladder coordinate: [PROVED]

$$r(x) := \log_\varphi(x) = \frac{\ln x}{\ln \varphi}. \tag{1}$$

For two masses $m_a, m_b > 0$ separated by rung offset Δr: [PROVED]

$$\frac{m_a}{m_b} = \varphi^{\Delta r}, \qquad \frac{m_a^2}{m_b^2} = \varphi^{2\Delta r}. \tag{2}$$

2.2 Quarter-step convention

For the neutrino sector, we extend the rung lattice: [HYP]

$$r \in \tfrac{1}{4}\mathbb{Z}. \tag{3}$$

This extension is motivated by:

- **Resolution**: neutrino splittings are extremely small compared to charged sectors, requiring finer exponent increments than integer steps provide,
- **Octave compatibility**: quarter steps are the simplest refinement compatible with the eight-tick period ($8 \times \frac{1}{4} = 2$, an integer).

2.3 Rung assignment

The specific deep-ladder rung triple is: [HYP]

$$(r_1, r_2, r_3) = \left(-\frac{239}{4},\ -\frac{231}{4},\ -\frac{217}{4}\right). \tag{4}$$

The rung differences are:

$$r_2 - r_1 = \frac{-231 - (-239)}{4} = 2, \tag{5}$$

$$r_3 - r_2 = \frac{-217 - (-231)}{4} = \frac{7}{2}, \tag{6}$$

$$r_3 - r_1 = \frac{-217 - (-239)}{4} = \frac{11}{2}. \tag{7}$$

The appearance of $11/2$ for $r_3 - r_1$ and $7/2$ for $r_3 - r_2$ reflects the deep-ladder signature: the "11" of the charged sector generation torsion appears halved, while the "7" carries echoes of the φ^7 structural identity that governs the atmospheric-to-solar hierarchy.

3 Neutrino Mass Predictions

3.1 The eV reporting seam

Absolute neutrino masses in eV require a global calibration seam: [CERT]

$$\kappa_{\mathrm{eV}} := \frac{\hbar}{\tau_0 \cdot (1\,\mathrm{eV})} \approx 1.086 \times 10^{10}\,\mathrm{eV}, \tag{8}$$

where τ_0 is the fundamental tick (from the eight-tick closure). This seam is fixed once for the entire framework and is *not* adjusted per neutrino.

Equivalently, pinning the seam algebraically: [CERT]

$$\kappa_{\mathrm{eV}} = 2^{-22}\,\varphi^{51} \times 10^{6}\,\mathrm{eV}. \tag{9}$$

3.2 Mass law for neutrinos

The deep-ladder mass hypothesis is: [HYP]

$$m_i^{\mathrm{pred}} = \kappa_{\mathrm{eV}} \cdot \varphi^{r_i}, \qquad i \in \{1, 2, 3\}. \tag{10}$$

Note the absence of a gap function ($Z_\nu = 0 \Rightarrow \mathrm{gap}(0) = 0$).

3.3 Predicted absolute masses

Evaluating (10) with the rung triple (4): [CERT]

$$m_1^{\text{pred}} \approx 0.00354\,\text{eV}, \tag{11}$$
$$m_2^{\text{pred}} \approx 0.00926\,\text{eV}, \tag{12}$$
$$m_3^{\text{pred}} \approx 0.0499\,\text{eV}. \tag{13}$$

The mass sum: [CERT]

$$\Sigma m_\nu^{\text{pred}} \approx 0.063\,\text{eV}. \tag{14}$$

4 Mass-Squared Splittings and the φ^7 Ratio

4.1 Splitting definitions

Standard definitions: [PROVED]

$$\Delta m_{21}^2 := m_2^2 - m_1^2, \qquad \Delta m_{31}^2 := m_3^2 - m_1^2. \tag{15}$$

4.2 Predicted splittings

From the mass law (10): [PROVED]

$$\Delta m_{ij}^2 = \kappa_{\text{eV}}^2 \left(\varphi^{2r_i} - \varphi^{2r_j}\right). \tag{16}$$

Numerically: [CERT]

$$\Delta m_{21}^2 \approx 7.33 \times 10^{-5}\,\text{eV}^2, \tag{17}$$
$$\Delta m_{31}^2 \approx 2.48 \times 10^{-3}\,\text{eV}^2. \tag{18}$$

Both fall within NuFIT summary windows for normal ordering. [VAL]

4.3 The exact φ^7 squared-mass ratio

The seam κ_{eV} cancels in the squared-mass ratio: [PROVED]

$$\frac{(m_3^{\text{pred}})^2}{(m_2^{\text{pred}})^2} = \varphi^{2(r_3-r_2)} = \varphi^{2\times 7/2} = \varphi^7. \tag{19}$$

This is the single most important structural prediction of the neutrino sector: [HYP]

$$\boxed{\frac{m_3^2}{m_2^2} = \varphi^7 \approx 29.03.} \tag{20}$$

It is *seam-free* (independent of κ_{eV}) and testable with oscillation data alone once absolute mass information becomes available.

4.4 Seam-free splitting ratio

The ratio of mass-squared splittings is also seam-free: [PROVED]

$$R_\Delta := \frac{\Delta m_{31}^2}{\Delta m_{21}^2} = \frac{\varphi^{2(r_3-r_1)} - 1}{\varphi^{2(r_2-r_1)} - 1} = \frac{\varphi^{11} - 1}{\varphi^4 - 1} \approx 33.82. \tag{21}$$

This depends only on φ and the rung differences, not on any calibration convention.

5 Normal Ordering as a Structural Consequence

5.1 Monotonicity of the ladder map

Since $\varphi > 1$, the map $r \mapsto \kappa_{\mathrm{eV}} \cdot \varphi^r$ is strictly increasing in r for any $\kappa_{\mathrm{eV}} > 0$. [PROVED]

5.2 Rung ordering implies mass ordering

The rung triple satisfies $r_1 < r_2 < r_3$: [HYP]

$$-\frac{239}{4} < -\frac{231}{4} < -\frac{217}{4}. \tag{22}$$

By monotonicity: [PROVED]

$$m_1^{\mathrm{pred}} < m_2^{\mathrm{pred}} < m_3^{\mathrm{pred}}. \tag{23}$$

Normal ordering is not a choice in RS; it is forced by the discrete rung assignment. If future experiments decisively establish inverted ordering, the rung triple (4) is refuted.

6 Cosmological Consistency

6.1 The mass sum constraint

Current cosmological analyses within ΛCDM-like frameworks constrain: [VAL]

$$\Sigma m_\nu \lesssim 0.12\,\mathrm{eV} \quad \text{(representative bound)}. \tag{24}$$

The predicted sum $\Sigma m_\nu^{\mathrm{pred}} \approx 0.063\,\mathrm{eV}$ is comfortably within this bound. [VAL]

6.2 Near-future sensitivity

Next-generation surveys (e.g., DESI, Euclid, CMB-S4) may tighten the bound toward $\Sigma m_\nu \lesssim 0.06\,\mathrm{eV}$. If the bound crosses below $0.063\,\mathrm{eV}$, the deep-ladder mass scale is directly pressured. [VAL]

6.3 Kinematic endpoint

The KATRIN experiment constrains $m_\beta < 0.45\,\mathrm{eV}$ (90% CL) from tritium β-decay. The predicted effective mass $m_\beta^{\mathrm{pred}} \approx \sqrt{\sum |U_{ei}|^2 m_i^2} \sim 0.01\,\mathrm{eV}$ is far below current sensitivity but within reach of proposed future experiments. [VAL]

7 Dirac vs. Majorana Nature

The deep-ladder framework treats neutrinos as Dirac fermions with $Z_\nu = 0$ at the anchor. Under this assignment:

- Lepton number is conserved,
- The effective Majorana mass $m_{\beta\beta}$ for neutrinoless double-beta decay is zero,
- The mass hierarchy is governed purely by the rung triple and the single neutrino yardstick.

The Majorana alternative would require $Z_\nu \neq 0$ (a nonzero anchor residue) or additional discrete structure (writhe parity of the neutral braid triple). The current framework does not exclude the Majorana possibility in principle, but the simplest realization ($Z_\nu = 0$, Dirac) is the one tested here.

Falsifier: detection of neutrinoless double-beta decay at a rate inconsistent with zero $m_{\beta\beta}$ would require modification of the $Z_\nu = 0$ assignment.

8 The Integer-Rung No-Go and Its Structural Resolution

8.1 The formal rung triple $(0, 11, 19)$

If one applies the charged-sector generation torsion $\{0, 11, 17\}$ directly to neutrinos (with the same baseline rung conventions), the formal triple is $(r_1, r_2, r_3) = (0, 11, 19)$. The splitting ratio would be: [PROVED]

$$R_\Delta^{\text{integer}} = \frac{\varphi^{38} - 1}{\varphi^{22} - 1} \approx 2{,}207, \tag{25}$$

which is ~ 65 times larger than the observed $R_\Delta \approx 33.8$. Both normal and inverted orderings fail the acceptance test.

8.2 The structural explanation: confinement to the edge level

Paper VI derives the generation torsion $\{0, 11, 17\}$ from coupling levels of the 3-cube: generation 1 couples through the active edge only ($\tau_1 = 0$), generation 2 couples to all passive edges ($\tau_2 = E_{\text{passive}} = 11$), and generation 3 additionally couples to faces ($\tau_3 = E_{\text{passive}} + F = 17$).

For charged fermions ($Z \neq 0$), the nonzero charge band provides a **locking potential** that enables face-level coupling: the boundary "grips" the face structure through its charge. The full hierarchy spans $E_{\text{passive}} + F = 17 = W$ rungs.

For neutrinos ($Z = 0$), the charge band vanishes. Without it, **face coupling is blocked**: there is no charge-mediated mechanism to lock the neutral boundary to the 2-dimensional face structure. The neutrino hierarchy is therefore *confined to the edge level*, spanning only $E_{\text{passive}} = 11$ — but at **half resolution**:

$$\boxed{\text{Neutrino total span} = \frac{E_{\text{passive}}}{2} = \frac{11}{2}.} \tag{26}$$

The factor of $1/2$ arises because the neutral boundary, lacking a charge band, couples to the passive edge network with half the locking strength of a charged boundary. Integer steps require the charge band's discrete grip; without it, the coupling is "impedance-mismatched" and the effective step size halves.

8.3 The edge-level decomposition: $4 + 7 = 11$

The doubled rung differences of the deep-ladder neutrino triple are: [PROVED]

$$2 \times \Delta_{1\to2} = 4, \qquad 2 \times \Delta_{2\to3} = 7, \qquad 2 \times \Delta_{1\to3} = 11. \tag{27}$$

The sum $4 + 7 = 11 = E_{\text{passive}}$ is *exact*: the neutrino generation structure, in doubled coordinates, exhaustively partitions the passive edge count.

The sub-decomposition $4 + 7$ reflects the internal structure of the passive edge network of the 3-cube:

- $4 = 2^{D-1}$: the edges along a single spatial direction (one full "direction slot" of the cube).
- $7 = E_{\text{passive}} - 2^{D-1} = 11 - 4$: the remaining passive edges spanning the other two directions.

This is a **sub-partition** of the passive edge set—the same active/passive splitting principle (Paper VI) applied *within* the edge level:

	Charged sector	Neutrino sector	Neutrino (doubled)
Gen 1→2 step	$E_{\text{passive}} = 11$	2	$4 = 2^{D-1}$
Gen 2→3 step	$F = 6$	$7/2$	$7 = E_{\text{passive}} - 2^{D-1}$
Total span	$W = 17$	$11/2$	$11 = E_{\text{passive}}$

8.4 Why the φ^7 ratio and $R_\Delta \approx 33.8$ are now explained

The atmospheric-to-solar mass-squared ratio $m_3^2/m_2^2 = \varphi^7$ is no longer an unexplained numerical coincidence. It arises because: [HYP]

$$2(r_3 - r_2) = 2 \times \frac{7}{2} = 7 = E_{\text{passive}} - 2^{D-1} = 11 - 4. \tag{28}$$

The exponent 7 is the *remaining passive edge count after extracting one direction*: it is the second step of the sub-partition within the edge level.

The splitting ratio $R_\Delta = (\varphi^{11} - 1)/(\varphi^4 - 1) \approx 33.82$ now also has a structural interpretation:

- The numerator exponent $11 = 2 \times (11/2) = E_{\text{passive}}$ is the total passive edge count.
- The denominator exponent $4 = 2 \times 2 = 2^{D-1}$ is one direction's edge count.

R_Δ is a ratio of φ-polynomials built entirely from passive-edge sub-counts of the 3-cube.

8.5 Summary: the neutrino resolution

The neutrino no-go is resolved by three observations from the cube partition framework:

1. $Z = 0$ blocks face coupling $\Rightarrow$ the hierarchy is confined to the edge level ($E_{\text{passive}} = 11$, not $W = 17$).
2. Without charge-band locking, coupling operates at half resolution $\Rightarrow$ fractional ($\frac{1}{2}$-integer) rungs.
3. The edge level has internal structure ($4+7 = 11$) $\Rightarrow$ three generations are still accommodated within the reduced span.

This transforms the neutrino rung triple from a data-constrained hypothesis into a *structural consequence* of the same cube geometry that organizes the charged sectors.

9 Falsifiers

9.1 Seam-free falsifiers (depend only on φ and rung differences)

F1: Splitting-ratio mismatch. If $R_\Delta = \Delta m_{31}^2/\Delta m_{21}^2$ departs from the predicted value $(\varphi^{11} - 1)/(\varphi^4 - 1) \approx 33.82$ beyond experimental uncertainty, the rung triple is refuted. [VAL]

F2: Ordering mismatch. If inverted ordering is decisively established, the rung ordering $r_1 < r_2 < r_3$ is refuted. [VAL]

F3: Squared-mass ratio mismatch. If absolute mass information establishes $m_3^2/m_2^2 \neq \varphi^7$, the rung gap hypothesis is refuted. [VAL]

9.2 Scale falsifiers (test the eV reporting seam)

F4: Oscillation windows. If updated NuFIT windows exclude $\Delta m_{21}^{2,\text{pred}}$ or $\Delta m_{31}^{2,\text{pred}}$, either the rung triple or the seam is refuted. [VAL]

F5: Cosmological exclusion. If cosmological bounds establish $\Sigma m_\nu < 0.062\,\text{eV}$, the predicted mass scale is ruled out. [VAL]

F6: Direct mass detection. A kinematic measurement robustly implying a mass scale well above the predicted window refutes the deep-ladder assignment. [VAL]

F7: Neutrinoless double-beta decay. Detection of $0\nu\beta\beta$ at a level inconsistent with the Dirac $Z_\nu = 0$ assignment would require extending the framework. [VAL]

10 Conclusions

This paper has extended the Recognition Science mass framework to the neutrino sector via the deep φ-ladder with fractional (quarter-step) rungs.

10.1 What is structural

- The ladder mathematics: ratios are φ-powers of rung differences; the seam cancels from ratios. [PROVED]
- Normal ordering: forced by rung ordering plus $\varphi > 1$. [PROVED]
- The φ^7 squared-mass ratio: a seam-free structural prediction. [HYP]
- The seam-free splitting ratio $R_\Delta = (\varphi^{11} - 1)/(\varphi^4 - 1)$. [HYP]

10.2 What is hypothesized

- The quarter-step rung lattice $r \in \frac{1}{4}\mathbb{Z}$. [HYP]
- The specific rung triple $(-239/4, -231/4, -217/4)$. [HYP]
- The Dirac nature ($Z_\nu = 0$, $m_{\beta\beta} = 0$). [HYP]

10.3 What the validation indicates

Under the declared seam, Δm^2_{21} and Δm^2_{31} both fall within NuFIT windows. The mass sum $\Sigma m_\nu \approx 0.063\,\mathrm{eV}$ is consistent with current cosmological bounds. The splitting ratio $R_\Delta \approx 33.82$ is consistent with the experimental value $R_\Delta^{\exp} \approx 33.4$. [VAL]

10.4 The core falsifiers

The most robust tests are seam-free:

- The splitting ratio R_Δ (testable now),
- The mass ordering (testable with current and near-future experiments),
- The φ^7 ratio (testable when absolute mass information becomes available).

The neutrino sector represents the frontier of the RS mass program. The charged sectors exhibit remarkable agreement with data; the neutrino sector requires a structural extension (fractional rungs) that is natural within the framework but not yet derived from the same pure counting-layer arguments that fix the charged sector. Closing this gap—deriving the neutrino rung lattice from first principles—remains the primary open problem.

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