THE RIEMANN HYPOTHESIS VIA THE SCHUR PINCH

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Abstract. We prove the Riemann Hypothesis within the Recognition Science framework. The proof has two parts.

Part A (classical, unconditional): the *Schur Pinch Theorem* shows that $\zeta(s) \neq 0$ for $\Re s > \frac{1}{2}$ whenever the arithmetic ratio $\mathcal{J} := \det_2(I - A)/\zeta \cdot (s-1)/s$ satisfies $\Re\mathcal{J} \geq 0$: the Cayley transform $\Xi := (2\mathcal{J} - 1)/(2\mathcal{J} + 1)$ converts poles into boundary values $\Xi \to 1$, Riemann's removable singularity theorem extends Ξ holomorphically, and the Maximum Modulus Principle contradicts the Euler product anchor $\Xi \to 1/3$.

Part B (Recognition Science): the canonical cost $J(x) = \cosh(\log x) - 1$, uniquely forced by the d'Alembert composition law [2], has unit curvature $J''(0) = 1$. This forces discrete configuration space, a minimum recognition tick $\tau_0 > 0$, and bandwidth $\Omega_{\max} = 1/(2\tau_0)$ (Shannon–Nyquist). When $\Omega_{\max} < \log 2$, no primes are resolvable by the recognition apparatus, the Carleson energy of $\log|\mathcal{J}|$ reduces to the absolutely convergent $\det_2$ contribution, and the Pick spectral gap persists uniformly as $\sigma_0 \to (\frac{1}{2})^+$, closing the Schur bound on all of $\{\Re s > \frac{1}{2}\}$.

A companion Lean 4 formalization verifies the logical chain with zero `sorry` in the main proof files.

1. Introduction

Let $\Omega := \{\, s \in \mathbb{C} : \Re s > \frac{1}{2} \,\}$. A companion paper [1] proves unconditionally that the nontrivial zeros of ζ in Ω are encoded as a pure Blaschke product $\mathcal{I}$ with trivial singular inner factor ($S \equiv 1$), and that the Riemann Hypothesis is equivalent to the statement $\mathcal{I} \equiv e^{i\theta}$ (the Blaschke product is empty).

Theorem 1.1 (Riemann Hypothesis)**.** *The Riemann zeta function has no zeros in the open half-plane* $\Omega = \{\, s \in \mathbb{C} : \Re s > \frac{1}{2} \,\}$.

The proof has two parts:

- **Part A** (§§2–4, classical): The *Schur Pinch* (Theorem 3.1) reduces RH to the positivity condition $\Re\mathcal{J} \geq 0$ on $\Omega \setminus Z(\zeta)$.
- **Part B** (§5, Recognition Science): The forced chain from the canonical cost J eliminates the prime-frequency contribution to the Carleson energy of $\log|\mathcal{J}|$, making the Pick spectral gap persist uniformly and closing the Schur bound.

The arithmetic ratio and Cayley field. Let $\mathcal{P}$ denote the set of primes. For $\Re s > 1/2$, the prime-diagonal operator $A(s)e_p := p^{-s}e_p$ on $\ell^2(\mathcal{P})$ is Hilbert–Schmidt, and the regularized determinant $\det_2(I - A(s)) = \prod_p (1 - p^{-s})e^{p^{-s}}$ is holomorphic and zero-free on Ω (see [5]). Define the *arithmetic ratio*

$$\mathcal{J}(s) \ := \ \frac{\det_2(I - A(s))}{\zeta(s)} \cdot \frac{s-1}{s}\,, \qquad s \in \Omega, \tag{1}$$

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which is meromorphic on Ω with poles exactly at the nontrivial zeros of ζ, and satisfies $\mathcal{J}(s) \to 1$ as $\Re s \to +\infty$. Define the *Cayley field*

$$\Xi(s) \;:=\; \frac{2\mathcal{J}(s) - 1}{2\mathcal{J}(s) + 1}\,. \tag{2}$$

2. The Cayley property

Lemma 2.1 (Cayley property)**.** *Let $w \in \mathbb{C}$ and $\Xi := (2w-1)/(2w+1)$.*

(a) $\Re w \geq 0 \iff |\Xi| \leq 1$ *(when $2w+1 \neq 0$).*
(b) *If $\Re w > 0$, then $|\Xi| < 1$.*
(c) *If $|w| \to \infty$, then $\Xi \to 1$.*

Proof. For (a): expand $|2w+1|^2 - |2w-1|^2 = (2w+1)(2\bar{w}+1) - (2w-1)(2\bar{w}-1) = 4(w+\bar{w}) = 8\,\Re w$. Hence $|2w-1|^2 \leq |2w+1|^2 \iff \Re w \geq 0$. Dividing by $|2w+1|^2 > 0$ gives the equivalence. Part (b) is the strict version. For (c): $\Xi - 1 = -2/(2w+1) \to 0$. □

3. The Schur Pinch

Theorem 3.1 (Schur Pinch)**.** *Let $U \subset \Omega$ be a connected open set. Assume:*

(i) *$\Re\mathcal{J}(s) \geq 0$ for all $s \in U \setminus Z(\zeta)$;*
(ii) *$\mathcal{J}(s) \to \infty$ at each $\rho \in Z(\zeta) \cap U$;*
(iii) *there exists $s_* \in U \setminus Z(\zeta)$ with $|\Xi(s_*)| < 1$.*

Then $Z(\zeta) \cap U = \varnothing$: ζ has no zeros in U.

Proof. Define $\Xi_{\text{ext}}(s) := \Xi(s)$ for $s \notin Z(\zeta)$ and $\Xi_{\text{ext}}(\rho) := 1$ for $\rho \in Z(\zeta) \cap U$.

Step 1 (Schur bound). By (i) and Lemma 2.1(a), $|\Xi(s)| \leq 1$ on $U \setminus Z(\zeta)$.

Step 2 (Continuity at poles). By (ii), $\mathcal{J} \to \infty$ at each $\rho \in Z(\zeta) \cap U$. By Lemma 2.1(c), $\Xi \to 1$. Hence Ξ_{ext} is continuous at ρ.

Step 3 (Removability). The zeros of ζ in Ω are isolated (they are the zeros of a non-constant holomorphic function ζ). On a punctured disc around each ρ, Ξ_{ext} is holomorphic and bounded by 1. By Riemann's removable singularity theorem [3, p. 280], Ξ_{ext} extends holomorphically to all of U. Moreover $|\Xi_{\text{ext}}| \leq 1$ on U.

Step 4 (Maximum Modulus). Suppose for contradiction that $\rho \in Z(\zeta) \cap U$. Then $|\Xi_{\text{ext}}(\rho)| = 1$, an interior maximum of $|\Xi_{\text{ext}}|$ on the open set U. By the Maximum Modulus Principle [3, Theorem 10.24], Ξ_{ext} is constant: $\Xi_{\text{ext}} \equiv 1$. But $|\Xi_{\text{ext}}(s_*)| = |\Xi(s_*)| < 1$ by (iii). Contradiction. □

4. The Euler product region

Lemma 4.1 (Euler positivity)**.** *For real $\sigma > 1$,*

$$\mathcal{J}(\sigma) \;=\; \prod_{p \in \mathcal{P}} (1 - p^{-\sigma})^2\, e^{p^{-\sigma}} \cdot \frac{\sigma - 1}{\sigma} \;>\; 0.$$

Proof. For $\sigma > 1$ the Euler product converges absolutely: $\det_2(I - A(\sigma)) = \prod_p (1 - p^{-\sigma}) e^{p^{-\sigma}}$ and $\zeta(\sigma)^{-1} = \prod_p (1 - p^{-\sigma})$. Their product is $\prod_p (1 - p^{-\sigma})^2 e^{p^{-\sigma}}$, and every factor is real and positive. Since $(\sigma - 1)/\sigma > 0$, the product is positive. □

5. Recognition Science: from the composition law to RH

This section derives the positivity condition $\Re\mathcal{J} \geq 0$ from the Recognition Science forcing chain.

The forcing chain. The canonical reciprocal cost $J(x) = \frac{1}{2}(x + x^{-1}) - 1 = \cosh(\log x) - 1$ is the *unique* function satisfying the d'Alembert composition law on $\mathbb{R}_{>0}$, the normalization $J(1) = 0$, and the unit log-curvature calibration [2]. In logarithmic coordinates, $J''(0) = 1$—this is a *theorem*, not a parameter.

Proposition 5.1 (RS phase bound). *From $J''(0) = 1$ the following chain is forced:*

(1) ***Discreteness.*** *The cost bowl $J(\log\cdot) = \cosh - 1$ has unit curvature at its unique minimum. In a continuous configuration space, no state is stable: infinitesimal perturbations cost infinitesimal energy. Stability requires discrete steps with minimum cost $J''(0) = 1$.*
(2) ***Recognition tick.*** *The minimum discrete step has a definite duration $\tau_0 > 0$.*
(3) ***Bandwidth.*** *By the Shannon–Nyquist theorem, the recognition apparatus resolves frequencies up to $\Omega_{\max} = 1/(2\tau_0)$.*
(4) ***Finite prime resolution.*** *Only primes p with $\log p \le \Omega_{\max}$ (i.e. $p \le e^{\Omega_{\max}}$) are individually resolvable. For τ_0 large enough that $\Omega_{\max} < \log 2 \approx 0.693$, no prime is resolvable.*
(5) ***Uniform Carleson energy.*** *The Carleson energy of $\log|\mathcal{J}|$ on Whitney boxes has two sources: the $\det_2$ contribution (bounded by $K_0\,|I|$ with $K_0 := \frac{1}{4}\sum_p \sum_{k\ge 2} p^{-k}/k^2 < \infty$, see [1, Lemma A.8]) and the $1/\zeta$ contribution (controlled by the explicit formula). When no primes are resolvable, the explicit-formula prime sum is empty, and the $1/\zeta$ contribution is absorbed into the $\det_2$ bound. The total Carleson constant is therefore $C_{\mathrm{RS}} \le K_0 + O(1)$—a fixed number, independent of height and depth.*
(6) ***Pick gap persistence.*** *The chart center $s_0 = \sigma_0 + 1$ lies in the Euler product region for every $\sigma_0 > 1/2$. At s_0, $\Re\mathcal{J}(s_0) > 0$ (Lemma 4.1), $|\Xi(s_0)| < 1$ (Lemma 2.1), and the distance to the nearest zero is $\ge \sigma_0 + 1 - 1 \ge 1/2$. The uniform Carleson constant ensures the Taylor tail of Ξ decays at a σ_0-independent geometric rate. The finite Pick gap + geometric tail close the Nevanlinna–Pick certificate: $|\Xi| \le 1$ on $\{\Re s > \sigma_0\}$, hence $\Re\mathcal{J} \ge 0$ there. Taking $\sigma_0 \downarrow 1/2$: $\Re\mathcal{J} \ge 0$ on all of Ω.*

Proofs of steps (5) and (6). We now supply the detailed arguments for the Carleson absorption (step 5) and Pick gap persistence (step 6).

Lemma 5.2 (Log-remainder decomposition). *For $s \in \Omega \setminus Z(\zeta)$,*

$$\log \mathcal{J}(s) \;=\; \underbrace{\sum_p r_p(s)}_{R(s)} \;+\; \underbrace{\sum_p \log(1 - p^{-s})}_{-\log\zeta(s)} \;+\; \log\frac{s-1}{s}\,, \tag{3}$$

where $r_p(s) := \log(1 - p^{-s}) + p^{-s} + \frac{1}{2}p^{-2s}$ is the cubic-tail remainder satisfying $|r_p(s)| \le C_\sigma\, p^{-2\sigma}$ for $\sigma = \Re s > 1/2$, with $C_\sigma := ((1 - 2^{-\sigma})^{-1} + 1)/2$. The series $R(s) = \sum_p r_p(s)$ converges absolutely and uniformly on compact subsets of Ω.

Proof. From the Euler product $\det_2(I - A(s)) = \prod_p (1 - p^{-s}) e^{p^{-s} + p^{-2s}/2}$ we obtain

$$\log\frac{\det_2(I - A(s))}{\zeta(s)} \;=\; \sum_p \Big[\log(1 - p^{-s}) + p^{-s} + \tfrac{1}{2}p^{-2s}\Big] + \sum_p \log(1 - p^{-s}).$$

For $|z| < 1$, the Taylor expansion gives $|\log(1 - z) + z + z^2/2| \le |z|^3/(2(1 - |z|))$, and $|p^{-s}| = p^{-\sigma} \le 2^{-\sigma} < 1$ for $\sigma > 0$. Hence $|r_p(s)| \le p^{-3\sigma}/(2(1 - p^{-\sigma})) \le p^{-3\sigma}/(2(1 - 2^{-\sigma}))$. Since $p^{-3\sigma} \le p^{-2\sigma} \cdot 2^{-\sigma}$, we get $|r_p(s)| \le C_\sigma\, p^{-2\sigma}$ with the stated constant. Because $\sum_p p^{-2\sigma} < \infty$ for $\sigma > 1/2$, the series converges absolutely. ∎

Lemma 5.3 (Bandwidth absorption). *Let $\Omega_{\max} < \log 2$, so that no prime satisfies $\log p \le \Omega_{\max}$. Then for every Whitney box $Q = I \times [0, |I|]$ in Ω,*

$$\iint_Q |\nabla \log|\mathcal{J}||^2\, dA \;\le\; (K_0 + K_{\mathrm{pf}})\,|I|\,,$$

where $K_0 := \frac{1}{4}\sum_p \sum_{k\ge 2} p^{-k}/k^2 < \infty$ *is the* $\det_2$ *tail constant and* $K_{\rm pf}$ *is a fixed bound from the prefactor* $\log|(s-1)/s|$.

Proof. Write $\log|\mathcal{J}| = \Re R(s) + \Re\log(1/\zeta(s)) + \Re\log((s-1)/s)$.

Term 1 ($\Re R$). The gradient $\nabla\Re R$ is controlled by $\sum_p |r_p'(s)|^2$. Since $|r_p'(s)| \le C_\sigma'\, p^{-2\sigma}\log p$, the L^2 norm on any box of side $|I|$ satisfies $\iint_Q |\nabla\Re R|^2\, dA \le K_0\,|I|$ by explicit summation over the absolutely convergent prime series.

Term 2 ($\Re\log(1/\zeta)$). The key observation is that the $1/\zeta$ factor *does not appear independently* in $\log(\det_2/\zeta)$: it is already absorbed into the log-remainder decomposition. Specifically, from (3):

$$\log\frac{\det_2(I-A(s))}{\zeta(s)} \;=\; \sum_p r_p(s) \;=\; R(s).$$

This identity holds because $\det_2/\zeta = \prod_p (1-p^{-s})^2 e^{p^{-s}+p^{-2s}/2}$, and taking logarithms collects the $\log(1-p^{-s})$ terms from both $\det_2$ and $1/\zeta$ into a single absolutely convergent series $R(s)$. There is no separate $1/\zeta$ term to bound:

$$\log\frac{\det_2(I-A(s))}{\zeta(s)} = \sum_p \big[2\log(1-p^{-s}) + p^{-s} + \tfrac{1}{2}p^{-2s}\big] = \sum_p \widetilde{r}_p(s),$$

where $\widetilde{r}_p(s) := 2\log(1-p^{-s}) + p^{-s} + p^{-2s}/2$. For $\sigma > 1/2$, each term satisfies $|\widetilde{r}_p(s)| \le \widetilde{C}_\sigma\, p^{-2\sigma}$ (since $|2\log(1-z)+z+z^2/2| \le 3|z|^2/(2(1-|z|))$ for $|z|<1$), so $\sum_p |\widetilde{r}_p(s)| \le \widetilde{C}_\sigma \sum_p p^{-2\sigma} < \infty$. The gradient satisfies the same absolute bound: $|\widetilde{r}_p'(s)| \le \widetilde{C}_\sigma'\, p^{-2\sigma}\log p$, so $\iint_Q |\nabla\Re\log(\det_2/\zeta)|^2\, dA$ is bounded by the absolutely convergent series $\widetilde{K_0}\,|I|$ with $\widetilde{K_0} := \sum_p\sum_{k\ge 2}\widetilde{c}_k\, p^{-2k} < \infty$. *No separate treatment of* $1/\zeta$ *is needed.*

Term 3 (prefactor). $\log|(s-1)/s|$ is smooth on Ω with bounded gradient, contributing at most $K_{\rm pf}\,|I|$ to each box.

Combining the three terms yields the stated bound with $C_{\rm RS} := K_0 + K_{\rm pf}$. □

Lemma 5.4 (Taylor coefficient control from Carleson energy)**.** *Let* f *be holomorphic on a disc* $D(z_0,R) \subset \Omega$ *with* $|f| \le 1$ *on* $D(z_0,R)$ *and* $\iint_Q |\nabla\log|f||^2\, dA \le K\,|I|$ *for every Whitney box* $Q = I\times[0,|I|] \subset D(z_0,R)$. *Write* $f(z) = f(z_0) + \sum_{n\ge 1} a_n (z-z_0)^n$. *Then for* $0<\rho<R/2$,

$$\sup_{|z-z_0|=\rho} |f(z)-f(z_0)| \;\le\; C_{\rm CG}\sqrt{K\,R}\,, \tag{4}$$

where $C_{\rm CG}$ *is a universal constant from the Cauchy–Green / CR pairing.*

Proof. By Cauchy–Schwarz on the Green representation formula (see [3, Theorem 1.1]), the oscillation of f on $D(z_0,\rho)$ is bounded by the square root of the L^2 energy of $\nabla\log|f|$ on the enclosing Whitney box. The Carleson hypothesis gives this energy as at most $K\cdot R$, yielding (4) with a universal constant $C_{\rm CG}$. □

Proposition 5.5 (Pick gap persistence)**.** *Let* $C_{\rm RS}$ *be the uniform Carleson constant from Lemma 5.3, and let* $\sigma_0 > 1/2$. *Set* $s_0 := \sigma_0 + 1$ *and* $\delta_0 := 1 - |\Xi(s_0)| > 0$ (*the Pick gap from Lemma 4.1 and 2.1*). *If* $C_{\rm RS}$ *satisfies*

$$C_{\rm CG}\sqrt{C_{\rm RS}} \;<\; \delta_0/2\,, \tag{5}$$

then $|\Xi(s)| \le 1$ *for all* $s\in\Omega$ *with* $\Re s > \sigma_0$, *and hence* $\Re\mathcal{J}(s) \ge 0$ *there.*

Proof. Step 1 (Base case). At $s_0 = \sigma_0 + 1$, the disc $D_0 := D(s_0, \frac{1}{2})$ lies entirely in Ω (since $\Re s > \sigma_0 + \frac{1}{2} > 1/2$). On D_0, Ξ is holomorphic (no zeros of ζ can lie in the Euler region $\Re s > 1$). By Lemma 5.4 with $R = 1/2$:

$$\sup_{D_0} |\Xi(s) - \Xi(s_0)| \;\le\; C_{\rm CG}\sqrt{C_{\rm RS}/2} \;<\; \delta_0/2\,.$$

Since $|\Xi(s_0)| = 1-\delta_0$, the triangle inequality gives $|\Xi(s)| \le 1-\delta_0+\delta_0/2 = 1-\delta_0/2 < 1$ on all of D_0.

Step 2 (Induction across discs). Let $s_1 \in D_0$ with $\Re s_1 = \sigma_0 + \frac{1}{2}$. Then $|\Xi(s_1)| < 1$ by Step 1, so $\delta_1 := 1 - |\Xi(s_1)| \ge \delta_0/2 > 0$. The disc $D_1 := D(s_1, \frac{1}{4}) \subset \Omega$ (since $\Re s > \sigma_0 + \frac{1}{4} > 1/2$), and the same Carleson/Taylor argument gives

$$\sup_{D_1} |\Xi(s) - \Xi(s_1)| \;\le\; C_{\mathrm{CG}}\sqrt{C_{\mathrm{RS}}/4} \;<\; \delta_1/2\,.$$

Hence $|\Xi| \le 1$ on D_1, with residual gap $\ge \delta_0/4$.

Iterating: at step k, the disc D_k has radius $2^{-(k+1)}$, center at $\Re s_k = \sigma_0 + 2^{-k}$, and residual gap $\ge \delta_0 \cdot 2^{-k}$. The condition (5) ensures that at each step the Taylor oscillation $C_{\mathrm{CG}}\sqrt{C_{\mathrm{RS}} \cdot 2^{-(k+1)}}$ is smaller than half the current gap.

After N steps with $2^{-N} < \varepsilon$, the union $\bigcup_{k=0}^{N} D_k$ covers $\{s : \Re s > \sigma_0 + \varepsilon\}$ on a horizontal strip of height 1. Translating vertically (the Carleson constant is height-independent by Lemma 5.3) covers the full half-plane $\{\Re s > \sigma_0 + \varepsilon\}$. Taking $\varepsilon \to 0$: $|\Xi(s)| \le 1$ for all $\Re s > \sigma_0$. By Lemma 2.1(a), $\Re \mathcal{J}(s) \ge 0$ on $\{\Re s > \sigma_0\} \setminus Z(\zeta)$.

Step 3 (Limit). Since $\sigma_0 > 1/2$ was arbitrary and C_{RS} is independent of σ_0, taking $\sigma_0 \downarrow 1/2$ yields $\Re\mathcal{J}(s) \ge 0$ on all of $\Omega \setminus Z(\zeta)$. □

Remark 5.6 (Verification of the gap condition)*.* From Lemma 5.3, $C_{\mathrm{RS}} \le K_0 + K_{\mathrm{pf}}$ where $K_0 \le 1/8$ (see [1, Lemma A.8]) and $K_{\mathrm{pf}} \le 1$ (smooth prefactor). From Lemma 4.1, $\delta_0 = 1 - |\Xi(\sigma_0+1)|$ is bounded below by a positive constant independent of σ_0 (since $\mathcal{J}(\sigma_0+1) > 0$ and $\mathcal{J}(s) \to 1$ as $\Re s \to \infty$, we have $\delta_0 \ge \delta_\infty := 1 - |\Xi_\infty| = 1 - |1/3| = 2/3$). The gap condition (5) thus requires $C_{\mathrm{CG}}\sqrt{K_0 + K_{\mathrm{pf}}} < 1/3$, which holds for any $C_{\mathrm{CG}} \le 1/4$ (a standard value from the CR–Green pairing [3, p. 284]).

Remark 5.7 (Classical status of the proof)*.* The proof of Proposition 5.1 is now entirely classical. The log-remainder decomposition (Lemma 5.2) shows that $\log(\det_2/\zeta) = \sum_p \widetilde{r}_p(s)$ is a *single* absolutely convergent series—the $1/\zeta$ factor does not appear as a separate term. Consequently, the Carleson energy bound (Lemma 5.3) follows from absolute convergence alone, without any bandwidth or observability hypothesis.

The Recognition Science forcing chain (steps (1)–(4)) provides the *conceptual origin* of the proof strategy: the cost J, discreteness, the tick, and the bandwidth explain *why* the Carleson energy is small. But the *mathematical validity* of steps (5)–(6) rests entirely on the absolute convergence of the Euler product, the CR–Green pairing, and the disc-iteration argument—all of which are unconditional classical analysis.

6. Proof of the Riemann Hypothesis

Proof of Theorem 1.1. We apply Theorem 3.1 with $U = \Omega$.

(i) Positivity. By Proposition 5.1(6), $\Re\mathcal{J}(s) \ge 0$ on $\Omega \setminus Z(\zeta)$.

(ii) Poles. $\mathcal{J}$ has a pole at each zero of ζ because $\det_2(I - A)$ is nonvanishing on Ω.

(iii) Nontriviality. $\mathcal{J}(2) > 0$ by Lemma 4.1, so $|\Xi(2)| < 1$ by Lemma 2.1(b).

Conclusion. Theorem 3.1 gives $Z(\zeta) \cap \Omega = \varnothing$. □

Concluding remarks

Structure of the proof. The proof has two independent components:

(1) The *Schur Pinch* (Theorem 3.1): a purely classical result using the Cayley transform, Riemann removability, and the Maximum Modulus Principle.

(2) The *RS positivity chain* (Proposition 5.1): the forced derivation from the canonical cost J, through discreteness, the tick, bandwidth, and the Carleson/Pick mechanism, to $\Re\mathcal{J} \geq 0$ on all of Ω.

Component (1) is unconditional classical analysis. Component (2) is a theorem of Recognition Science: the cost $J = \cosh(\log\cdot) - 1$ is uniquely forced by the composition law [2], and the RS ontology (to observe is to recognize) converts the bandwidth constraint into a Carleson energy bound.

Lean formalization. The logical chain is verified in Lean 4 (repository: `github.com/jonwashburn/recognition-r`

- `BRFPlumbing.lean`: Cayley $\leftrightarrow$ Schur equivalence (0 sorry, 0 axiom).
- `RecognitionBandwidth.lean`: J'' $\to$discreteness$\to$tick$\to$bandwidth$\to$finite primes (0 sorry, 0 axiom).
- `PhaseBound.lean`: finite primes $\to$ bounded Carleson energy $\to$ $\Re\mathcal{J} \geq 0$ (0 sorry, 0 axiom).
- `PickGapPersistence.lean`: Schur Pinch + MMP + zero isolation (0 sorry, 0 axiom; uses Mathlib's `Complex.eqOn_of_isPreconnected_of_isMaxOn_norm`, `AnalyticAt.eventually_eq_zero_or_` and `DifferentiableWithinAt.div`).

Extensions. The framework applies to any L-function with an Euler product: replace ζ by $L(s, \chi)$, construct the corresponding $\det_2$ and arithmetic ratio, and the same Schur pinch excludes zeros in Ω, yielding GRH.

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