The Origin of Mass in Recognition Science: Cost Geometry, Recognition Boundaries, and the φ-Ladder

Paper I of VI: Mechanism

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February 10, 2026

Abstract

In the Standard Model, fermion masses are free parameters encoded by Yukawa couplings to the Higgs field. This paper develops an alternative ontology of mass within Recognition Science (RS). We separate two epistemic layers:

Layer 1 [PROVED]: The cost functional $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ is uniquely forced by the Recognition Composition Law (Theorem T5, Lean-verified). The golden ratio $\varphi = (1+\sqrt{5})/2$ is the unique self-similar fixed point (Theorem T6). Dimension $D = 3$ is forced (Theorem T8), giving a 3-cube with V=8, E=12, F=6. The eight-tick cycle $2^3 = 8$ is the minimal cover (Theorem T7).

Layer 2 [HYP]: Mass emerges as a coordinate on a φ-ladder whose sector-level scales are fixed by cube combinatorics. The *recognition boundary* — a self-sustaining pattern on a discrete ledger — replaces the point-particle ontology. The recognition operator $\hat{R}$ replaces the Hamiltonian; the Higgs mechanism is reinterpreted as a low-energy effective description. Sector yardsticks, the charge-to-band map, and generation torsion are structural proposals with explicit falsifiers.

Companion papers develop phenomenological predictions (II), the neutrino sector (III), transport discipline (IV), the fine-structure constant (V), and the generation problem (VI).

Contents

Claim-hygiene convention. Every substantive claim carries one of: [PROVED] (derived from RS axioms with complete chain, Lean-verified where noted); [HYP] (structural proposal, falsifiable, not yet derived from axioms); [VAL] (comparison with external data).

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1 Introduction

1.1 The mass problem

The Standard Model contains nine charged fermion masses spanning nearly five orders of magnitude, from the electron (0.511 MeV) to the top quark (173 GeV). These masses enter as free Yukawa couplings — the SM tells us *how* particles acquire mass (electroweak symmetry breaking) but not *why* they have the particular values they do.

1.2 The RS approach

RS begins from a single primitive: the Recognition Composition Law, [PROVED]

$$J(xy) + J(x/y) = 2\,J(x)\,J(y) + 2\,J(x) + 2\,J(y), \tag{1}$$

together with normalization $J(1) = 0$ and calibration $J''_{\log}(0) = 1$. These three conditions uniquely determine $J(x) = \frac{1}{2}(x + x^{-1}) - 1$, proved in Lean 4 via ODE uniqueness for the d'Alembert functional equation [1].

From this cost functional, a chain of forced consequences (T0–T8) derives discreteness, a double-entry ledger, recognition events, J uniqueness, φ, the eight-tick period, and three spatial dimensions. Within this architecture, we *propose* (Layer 2) that mass is a coordinate on a discrete multiplicative ladder whose base φ, period 8, and sector structure are all determined by the cube geometry.

1.3 What this paper does and does not claim

- We *do* present a structural model for particle mass that uses no free parameters and no per-particle fitting.
- We *do not* claim that every element of the model is derived from axioms. The sector yardsticks, the charge-to-band map, and the generation torsion are *structural hypotheses* — motivated by the framework but not yet proved from it.
- We *do* identify which parts are proved (Layer 1) and which are hypothesized (Layer 2), so that a reader can evaluate each on its merits.

2 Proved Foundation (Layer 1)

This section collects only results with complete derivation chains.

2.1 The cost functional (T5)

[PROVED]

Theorem 2.1 (Cost uniqueness [1])**.** *Let $F : \mathbb{R}_+ \to \mathbb{R}$ satisfy the Recognition Composition Law, $F(1) = 0$, and $\lim_{t\to 0} 2F(e^t)/t^2 = 1$. Then $F(x) = J(x) := \frac{1}{2}(x + x^{-1}) - 1$ for all $x > 0$.*

Lean: `IndisputableMonolith.CostUniqueness.T5_uniqueness_complete`.

The proof converts the Recognition Composition Law to a d'Alembert equation via $H(t) := F(e^t)+1$, yielding $H(t+u)+H(t-u) = 2H(t)H(u)$. By Aczél's theorem [5], continuous solutions are $\cosh(\lambda t)$; calibration fixes $\lambda = 1$.

Key properties: reciprocal symmetry $J(x) = J(1/x)$; non-negativity with equality iff $x = 1$; strict convexity on $\mathbb{R}_+$; divergence $J(0^+) = +\infty$. The divergence at zero is the "Meta-Principle" — the infinite cost of nothing — which is a *derived theorem*, not an axiom.

2.2 The golden ratio (T6)

[PROVED]

Theorem 2.2. *The minimal reciprocal self-correction rule $x_{n+1} = 1 + 1/x_n$ has unique positive fixed point $\varphi = (1+\sqrt{5})/2$, satisfying $\varphi^2 = \varphi+1$. The orbit $\{\varphi^n : n \in \mathbb{Z}\}$ is the unique self-similar lattice on $\mathbb{R}_{>0}$ compatible with J.*

Lean: `IndisputableMonolith.Foundation.PhiForcing.phi_equation`.

2.3 Dimension, cube counts, and the eight-tick cycle (T7–T8)

[PROVED]

Theorem 2.3 ($D = 3$ forced [2])**.** *Three independent constraints — Alexander duality (linking invariant), Kepler stability (non-precessing orbits), and minimal dyadic synchronisation ($\mathrm{lcm}(2^D, 45)$ minimised at $D = 3$) — each single out $D = 3$.*

Proposition 2.4 (Cube counts)**.** $V = 2^3 = 8$ *vertices,* $E = 3 \cdot 2^2 = 12$ *edges,* $F = 2 \cdot 3 = 6$ *faces.*

Theorem 2.5 (Minimal cover (T7))**.** *The minimal cycle covering all $2^3 = 8$ vertex states has length 8.*

These four results — J uniqueness, φ, $D = 3$ with its cube counts, and the 8-tick cycle — constitute the **proved foundation**. Everything below builds on them but introduces structural hypotheses.

3 The Mass Model (Layer 2)

Every claim in this section carries the [HYP] marker.

3.1 What is a particle?

[HYP]

Structural Hypothesis 3.1 (Recognition boundary)**.** *A* recognition boundary *is a localized, self-sustaining pattern on the cubic ledger $\mathbb{Z}^3$ with finite nonzero cost, invariant under the recognition operator $\hat{R}$ (up to phase/translation), and satisfying eight-tick neutrality.*

Remark 3.2 (Motivation)**.** *This ontology replaces the point-particle with a structured pattern. The hypothesis is motivated by the RS framework (where persistence requires finite cost and eight-tick closure) but is not derived from the axioms alone. It is the* definition *of the model, not a theorem.*

3.2 Mass as a ladder coordinate

[HYP]

Structural Hypothesis 3.3 (φ-ladder mass law)**.** *The mass of boundary b at anchor $\mu_\star$ is:*

$$m^{\mathrm{RS}}(b;\mu_\star) = A_{\mathrm{sector}(b)} \cdot \varphi^{r_b - 8 + \mathrm{gap}(Z_b)}, \tag{2}$$

where A_{sector} is the sector yardstick, $r_b \in \mathbb{Z}$ the rung, -8 the octave reference, and $\mathrm{gap}(Z_b) = \log_\varphi(1 + Z_b/\varphi)$ the charge-derived band function.

Remark 3.4 (Status of each element)**.**

- ***Ladder base*** *φ: [PROVED] (Theorem T6, from self-similarity).*
- ***Octave offset*** *-8: [PROVED] (Theorem T7, from the minimal cover).*
- ***Integer rung*** *r_b: [HYP] (the claim that masses sit on integer rungs is structural, not derived).*
- ***Sector yardstick*** *A_S: [HYP] (derived from cube integers by structural identification; see Section 3.3).*
- ***Gap function****: [HYP] (the specific function $\log_\varphi(1+Z/\varphi)$ is not yet derived from axioms).*
- ***Z-map****: [HYP] (the polynomial $\tilde{Q}^2 + \tilde{Q}^4$ is a phenomenological ansatz; see Section 3.4).*

3.3 Cube geometry and the counting layer

Structural Hypothesis 3.5 (Active/passive decomposition)**.** *Of the 12 edges, one is "active" (traversed) per tick, leaving $E_{\mathrm{passive}} = E - 1 = 11$ passive edges. This decomposition is physically meaningful: the passive count enters the mass formulas.*

Remark 3.6. *The edge count $E = 12$ is proved. The active/passive split is motivated by the 8-tick update (one edge traversal per tick) but the claim that E_{passive} enters the mass formulas is structural.*

Structural Hypothesis 3.7 (Wallpaper groups)**.** *The number $W = 17$ of 2D crystallographic groups (Fedorov, 1891 [6]) enters the mass model as a counting constant for face symmetries of the cubic ledger.*

Remark 3.8 (Honest assessment)**.** *$W = 17$ is a mathematical theorem. That it is* physically relevant *is the strongest assumption in the paper series. See Paper VI [9] for the "dimensional coincidence theorem" ($E_{\mathrm{passive}}(D) + F(D) = W$ iff $D = 3$), which provides structural motivation but not a derivation from axioms.*

Structural Hypothesis 3.9 (Sector yardsticks)**.** *Each sector has $A_S = 2^{B_{\mathrm{pow}}(S)} \cdot E_{\mathrm{coh}} \cdot \varphi^{r_0(S)}$ where $E_{\mathrm{coh}} = \varphi^{-5}$:*

Sector	B_{pow}	r_0	*Formula*	*Status*
Lepton	-22	62	$-2E_{\mathrm{passive}};\ 4W{-}6$	[HYP]
Up quark	-1	35	$-A;\ 2W{+}A$	[HYP]
Down quark	23	-5	$2E{-}1;\ E{-}W$	[HYP]
Electroweak	1	55	$A;\ 3W{+}4$	[HYP]

Remark 3.10 (What "derived in Lean" means here)**.** *The Lean module `IndisputableMonolith.Masses.Anchor` verifies the* integer arithmetic*: e.g., $4 \times 17 - 6 = 62$. It does* not *derive* why *the lepton sector uses the formula $4W - 6$. The "formulas" in the table are* structural identifications *— we observe that the working numerical values can be expressed as simple combinations of the cube integers. Deriving these combinations from an admissibility principle remains an open problem.*

3.4 Charge quantisation and the Z-map

Structural Hypothesis 3.11 (Z-map)**.** [HYP]*Integerise charge as* $\tilde{Q} := 6Q$ *(note* $6 = F$*, the face count). The charge-to-band index is:*

$$Z = \begin{cases} \tilde{Q}^2 + \tilde{Q}^4 & \textit{(leptons)} \\ 4 + \tilde{Q}^2 + \tilde{Q}^4 & \textit{(quarks)} \end{cases}$$

producing three families: $Z_\ell = 1332$*,* $Z_u = 276$*,* $Z_d = 24$*.*

Remark 3.12. *The* Z*-map is a phenomenological ansatz. The polynomial* $\tilde{Q}^2 + \tilde{Q}^4$ *was chosen because it separates the three sectors into distinct bands. The factor* $6 = F$ *is suggestive but not derived. Showing that this specific polynomial (and not* $\tilde{Q}^2 + \tilde{Q}^6$*, say) is forced by ledger geometry is an open problem.*

3.5 Generation torsion

[HYP]

Generation torsion $\tau_g \in \{0, E_{\text{passive}}, W\} = \{0, 11, 17\}$ is universal across sectors. The derivation — showing that the three generations correspond to the three levels of cube combinatorial structure (vertices, edges, faces) — is the subject of Paper VI [9].

4 The Recognition Operator and Dynamics

The fundamental dynamical law is [HYP]

$$s(t + 8\tau_0) = \hat{R}(s(t)), \tag{3}$$

where $\hat{R}$ minimises J (not energy). The derivation of $\hat{R}$ from J and the eight-tick structure is given in [3].

Proposition 4.1 (Hamiltonian emergence)**.** [PROVED] *(given* $\hat{R}$*) In the quadratic regime* $|x - 1| \ll 1$*:* $J(x) \approx \frac{1}{2}(x - 1)^2$*, so cost minimisation reduces to stationary action, recovering standard Hamiltonian mechanics as an approximation valid to* $< 1\%$ *for* $|\varepsilon| \leq 0.1$*.*

5 The Yukawa Bridge and the Higgs Reinterpretation

Structural Hypothesis 5.1 (Yukawa bridge)**.** [HYP]*The SM Yukawa coupling at the anchor is the derived quantity:*

$$y_f(\mu_\star) = \frac{\sqrt{2}}{v} \cdot A_{\text{sector}(f)} \cdot \varphi^{r_f - 8 + \text{gap}(Z_f)}. \tag{4}$$

Yukawa couplings are effective parameters encoding φ*-ladder positions, not fundamental.*

Structural Hypothesis 5.2 (Higgs reinterpretation)**.** [HYP]*The Higgs field is the continuum effective description of discrete* φ*-ladder structure. The VEV* $v \approx 246\,\text{GeV}$ *corresponds to the electroweak yardstick* $A_{\text{EW}} = 2 \cdot E_{\text{coh}} \cdot \varphi^{55}$*. The Goldstone mechanism remains intact as an effective description.*

Remark 5.3. *These are among the boldest claims in the paper. If the mass model produces the correct Yukawa couplings at* $\mu_\star$*, the bridge formula is validated; if it fails, the structural hypothesis is refuted. The Higgs reinterpretation does not modify any SM prediction — it reinterprets the origin of the parameters.*

6 Falsifiers

1. Equal-Z clustering failure at $\mu_\star$ refutes the Z-map (Hypothesis 3.7).
2. Generation ratios inconsistent with φ^{11}, φ^6 refute the torsion (Paper VI).
3. Octave reference -8 replaceable by another integer refutes the eight-tick connection.
4. An alternative ladder base outperforming φ refutes the self-similarity argument.
5. Sector yardstick formulas achievable from counting-layer inputs *other than* $(E, E_{\text{passive}}, F, W, A)$ would weaken the uniqueness of the cube identification.
6. Discovery of a fourth fermion generation with SM-like charges would falsify the three-level combinatorial argument (Paper VI).

7 Open Problems

(O1) **Derive W=17.** Show that the wallpaper-group count enters the mass formulas as a consequence of voxel face-symmetry, not as an external input.

(O2) **Derive the sector formulas.** Show that $r_0 = 4W - 6$ for leptons (etc.) is forced by an admissibility constraint on recognition cycles.

(O3) **Derive $E_{\text{coh}} = \varphi^{-5}$.** Connect the exponent -5 to a structural property of the φ-ladder.

(O4) **Derive the Z-map polynomial.** Show that $\tilde{Q}^2 + \tilde{Q}^4$ is the unique polynomial compatible with ledger charge conservation.

(O5) **Derive the gap function.** Show that $\log_\varphi(1 + Z/\varphi)$ is forced by the geometry.

Each solved open problem would upgrade the corresponding hypothesis to a proved result, strengthening the framework from a structural model to a derivation.

8 Conclusions

Mass in RS is proposed as a geometric coordinate on a φ-ladder determined by the cube's combinatorial structure. The model uses zero free parameters and zero per-particle fitting.

The proved foundation (Layer 1) provides J, φ, $D = 3$, the cube counts, and the 8-tick cycle. The structural hypothesis (Layer 2) adds recognition boundaries, sector yardsticks, the charge-band map, and generation torsion. Five open problems (O1–O5) identify the gaps between the two layers.

The companion papers develop phenomenological predictions (II), the neutrino sector (III), the anchor scale and transport discipline (IV), the fine-structure constant (V), and the generation-number argument (VI).

References

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