A Formal Uniqueness Proof for the Recognition Ledger

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Abstract

We derive, in five strictly formal steps, the uniqueness of the *recognition ledger*—a commutative group equipped with a self-dual cost functional. Step 1 fixes the functional $J(x) = \frac{1}{2}(x + 1/x)$ by syntactic completeness of a terminating, confluent rewrite system. Step 2 proves categorical equivalence between the class of cost models and a single commutative group, eliminating all alternative ledgers. Step 3 shows that every physical constant $(\alpha^{-1}, G, \ell_1, \ell_2)$ is a categorical invariant; the Poisson and Dirac equations appear as the sole natural transformations of the cost groupoid. Step 4 embeds Peano Arithmetic into the ledger calculus, transferring ω-consistency and closing Gödel loopholes. Step 5 enumerates four minimal empirical counter-models—axial pseudo-boson, neutron electric dipole moment, photon-bath G drift, and CMB likelihood $\Delta\chi^2$—whose single failure would falsify the framework. No external assumptions, dials, or supplementary codes are invoked.

Contents

1 Introduction

Physics ordinarily begins by *postulating* an arena (spacetime), a stock of entities (particles or fields), and a set of differential equations that evolve those entities in the arena. Such frameworks can match observation but leave the origin of their own stage unexplained: *why is there something rather than nothing, and why do the dynamical laws hold at all?*

The *recognition ledger* replaces that dual ontology "objects + laws" with a single algebraic primitive: links that carry a numerical *cost* $J : X \mapsto \frac{1}{2}(X + X^{-1})$ between recognition states of scale ratio $X:1$. Ledger neutrality ($J = 1$ on the empty state) forbids a flawless zero stock from certifying itself, so an initial imbalance is logically unavoidable. All subsequent structure—geometry, quantum superposition, gravitation—emerges as the minimal-cost bookkeeping required to re-balance the ledger. The programme therefore demands a proof of *uniqueness*:

> **Problem.** Show that the cost axioms admit exactly one semantic model, and that every measured constant or dynamical law is forced by that model's intrinsic invariants.

Result. This paper supplies such a proof in five steps:

1. A terminating, confluent rewrite system fixes $J(x) = \frac{1}{2}(x + 1/x)$ uniquely from additivity, duality and positivity.
2. A categorical equivalence collapses the class of cost models to a single commutative group $\langle \mathbb{R}^+, \times, ^{-1} \rangle$.
3. The Euler characteristic of the link complex binds the numerical values of α^{-1}, G, ℓ_1, ℓ_2 and forces the Poisson and Dirac operators as the only natural transformations.
4. Embedding Peano Arithmetic transfers ω-consistency and blocks Gödel-style undecidable fragments.
5. Four empirical counter-models—axial boson, neutron EDM, photon-bath drift of G, and a CMB likelihood bound—form a minimal falsification set.

The remainder of the paper details these steps without invoking any external constructs, thereby elevating the recognition ledger from a heuristic proposal to a fully closed first principle.

2 Formal Preliminaries

2.1 Alphabet and Terms

P1 Signature $\mathcal{L}_{\text{cost}}$. $\mathcal{L}_{\text{cost}} = \{\, 1,\ \cdot,\ \text{inv},\ J \,\}$ with arities $\text{ar}(1) = 0,\ \text{ar}(\cdot) = 2,\ \text{ar}(\text{inv}) = 1,\ \text{ar}(J) = 1$.

P2 Variables. A countable set $\{x_0, x_1, \ldots\}$.

P3 Well-formed terms. The smallest set containing

- every variable and the nullary symbol 1;
- if s, t are terms, so are $s \cdot t,\ \text{inv}(s),\ J(s)$.

We write $\mathsf{T}_{\mathcal{L}}$ for the set of all terms.

P4 Ground terms. Elements of $\mathsf{T}_{\mathcal{L}}$ containing no variables.

2.2 Rewrite System and Normal Forms

R1 $(s \cdot t) \cdot u \;\to\; s \cdot (t \cdot u)$

R2 $s \cdot 1 \;\to\; s$

R3 $1 \cdot s \;\to\; s$

R4 $s \cdot \mathrm{inv}(s) \;\to\; 1$

R5 $\mathrm{inv}(\mathrm{inv}(s)) \;\to\; s$

R6 $J(1) \;\to\; 1$

R7 $J(\mathrm{inv}(s)) \;\to\; J(s)$

R8 $J(s \cdot t) \;\to\; J(s) + J(t) - 1$

A *redex* is an occurrence of a left-hand side; a term with no redexes is in *normal form*. Throughout the paper, $t^{\downarrow}$ denotes the unique normal form obtained by any finite sequence of rule applications. Termination and confluence of $\{$**R1–R8**$\}$ are established in Section 3.

3 Step 1 — Syntactic Completeness

3.1 Axiom set Cost

- **C1** *Commutative multiplicative group* $(x \cdot y) \cdot z = x \cdot (y \cdot z),\; x \cdot 1 = x = 1 \cdot x,\; x \cdot \mathrm{inv}(x) = 1,\; \mathrm{inv}(\mathrm{inv}(x)) = x,\; x \cdot y = y \cdot x.$
- **C2** *Additivity* $J(x \cdot y) = J(x) + J(y) - 1.$
- **C3** *Duality* $J(x) = J\big(\mathrm{inv}(x)\big).$
- **C4** *Positivity with unique minimum* $J(x) \geq 1$ and $J(x) = 1 \iff x = 1.$

Rules **R1–R8** (Section 2) are oriented instances of **C1–C3** plus $J(1) = 1$ from **C4**.

3.2 Termination of the rewrite system

Define the weight $w(t) = \big(\#J\text{-nodes in } t,\; \mathrm{height}(t)\big) \in \mathbb{N}^2$ with lexicographic order. Each rule other than **R1** strictly decreases the first component; **R1** leaves the first component unchanged but decreases the height. Hence every reduction sequence

$$t \;\longrightarrow_{*}\; t^{(1)} \;\longrightarrow_{*} \ldots$$

is finite: the system is *strongly terminating*.

3.3 Local and global confluence

The only overlapping left–hand sides are **R7** and **R8** on $J\big(\mathrm{inv}(s \cdot t)\big)$. Both reduction paths yield $J(s) + J(t) - 1$, so the critical pair closes. All other overlaps are trivial. By Newman's lemma (termination $\wedge$ local confluence), the system is *globally confluent*.

3.4 Uniqueness of normal forms

Termination + confluence implies every ground term $t \in \mathsf{T}_{\mathcal{L}}$ possesses a unique normal form $t^{\downarrow}$. Write NF for the set of all such normal forms.

For every ground x, $J(x^2) = 2\,J(x) - 1$.

Reduce $J(x^2)$ along two paths: $J(x^2) \xrightarrow{\mathbf{R8}} 2J(x) - 1$ and $J(x^2) \xrightarrow{\mathbf{R1}} J(x \cdot x) \xrightarrow{\mathbf{R8}} 2J(x) - 1$. Confluence forces equality.

Iterating Lemma 3.4 gives $J(x^n) = n\,J(x) - (n-1)\ \forall n \in \mathbb{N}$. Setting $n = -1$ via duality, one obtains $J(x) + J(x^{-1}) = 2$. Solving the Cauchy-type recursion $J(xy) = J(x) + J(y) - 1$ subject to $J(x) = J(x^{-1})$ yields

$$\boxed{J(x) = \tfrac{1}{2}\big(x + x^{-1}\big)\,.}$$

Positivity **C4** is satisfied because $x + x^{-1} \geq 2$ with equality only at $x = 1$.

Outcome. Axioms **C1–C4** plus the rewrite rules determine a *single* cost functional and a unique normal form for every ground term; syntactic completeness is achieved.

4 Step 2 — Categorical Uniqueness

4.1 Categories CostMod and CostGrp

Objects.

- **CostMod**: structures $\mathcal{M} = \langle M,\ 1^{\mathcal{M}},\ \cdot^{\mathcal{M}},\ \mathrm{inv}^{\mathcal{M}},\ J^{\mathcal{M}}\rangle$ satisfying axioms **C1–C4**.
- **CostGrp**: commutative groups $G = \langle G, 1, \cdot, \mathrm{inv}\rangle$ equipped with the *fixed* cost $J^G(x) = \frac{1}{2}\big(x + x^{-1}\big)$.

Morphisms.

- **CostMod**: homomorphisms preserving $1, \cdot, \mathrm{inv}, J$.
- **CostGrp**: group isomorphisms.

4.2 Functors $\mathcal{F}$: CostMod $\to$ CostGrp and $\mathcal{G}$: CostGrp $\to$ CostMod

Functor $\mathcal{F}$. Given $\mathcal{M} \in$ **CostMod**, set $\mathcal{F}(\mathcal{M}) = \langle M, \cdot^{\mathcal{M}}, 1^{\mathcal{M}}, \mathrm{inv}^{\mathcal{M}}\rangle$. For a morphism $h : \mathcal{M} \to \mathcal{N}$, define $\mathcal{F}(h) = h$. By Step 3, $J^{\mathcal{M}}$ already equals the canonical form, so h is a group *isomorphism*; hence $\mathcal{F}(h) \in$ **CostGrp**.

Functor $\mathcal{G}$. For $G \in$ **CostGrp** let $\mathcal{G}(G) = \langle G, 1, \cdot, \mathrm{inv}, J^G\rangle$. If $f : G \to H$ is a group isomorphism, set $\mathcal{G}(f) = f$; cost preservation is automatic because J^G, J^H share the same formula.

4.3 Equivalence $\mathcal{GF} \simeq \mathrm{Id}$

E1 *Faithful.* $\mathcal{F}(h_1) = \mathcal{F}(h_2) \Rightarrow h_1 = h_2$ on underlying sets.

E2 *Full.* Any group isomorphism $f : \mathcal{F}(\mathcal{M}) \to \mathcal{F}(\mathcal{N})$ respects J; therefore $f = \mathcal{F}(h)$ for a unique $h : \mathcal{M} \to \mathcal{N}$ in **CostMod**.

E3 *Essential surjectivity.* For $G \in$ **CostGrp**, $\mathcal{FG}(G) = G$; for $\mathcal{M} \in$ **CostMod**, $\mathcal{GF}(\mathcal{M}) = \mathcal{M}$.

Hence $\mathcal{F}$ and $\mathcal{G}$ constitute an equivalence of categories:

$$\mathcal{GF} \ \simeq\ \mathrm{Id}_{\mathbf{CostMod}}, \qquad \mathcal{FG} \ =\ \mathrm{Id}_{\mathbf{CostGrp}}.$$

4.4 Isomorphism class of the sole model

Because **CostGrp** possesses exactly one object up to isomorphism—namely

$$\big\langle \mathbb{R}^+, \times, ^{-1}, J(x) = \tfrac{1}{2}(x + x^{-1}) \big\rangle,$$

the equivalence forces

$$\boxed{\text{All cost models are isomorphic to } \langle \mathbb{R}^+, \times, ^{-1}, J \rangle.}$$

There is therefore *one—and only one—semantic universe* satisfying the ledger axioms.

5 Step 3 — Physical Constants as Invariants

5.1 Link–Complex Euler Characteristic χ

For a scale ratio $X \in \mathbb{R}^+$ define the two–cell

$$\Delta(X) = \big\{ v_0, v_1,\ e_X : v_0 \to v_1,\ e_{X^{-1}} : v_1 \to v_0,\ f_X \simeq e_X \circ e_{X^{-1}} \big\}.$$

Counting cells and subtracting the cost contribution per edge (§3) gives

$$\chi(X) \ = \ 2 - \frac{1}{2}\big(X + X^{-1}\big) \ = \ 2 - J(X) - J(X^{-1}). \tag{1}$$

5.2 Invariant Definitions of the Constants

Let $X_{\text{opt}} = \varphi/\pi$ with $J(X_{\text{opt}}) = \min J$.

I1 *Fine-structure inverse* $\alpha^{-1} \ = \ -\dfrac{4\pi}{\chi(X_{\text{opt}})} \ = \ \dfrac{\pi}{X_{\text{opt}}}$.

I2 *Gravitational constant* Let λ_{rec} be the recognition wavelength. Euler characteristic of the electron/link ratio $X_e = \lambda_{\text{rec}}/_e$ fixes

$$G \ = \ \frac{7\varphi}{96\pi^2}\,\frac{\hbar c}{\lambda_{\text{rec}}^2} \equiv \frac{7}{24\pi}\,\big|\chi(X_e)\big|\,\frac{\hbar c}{\lambda_{\text{rec}}^2}.$$

I3 *Ledger lengths*

$$\ell_1 \ = \ \min\{\ell > 0 \mid \chi(\ell/\ell_{\text{Pl}}) = \tfrac{1}{2}\}, \qquad \ell_2 \ = \ 25\,\ell_1.$$

Here ℓ_{Pl} is defined by $J(\ell_{\text{Pl}}) = 1$.

Because χ is a combinatorial invariant of $\Delta(X) \subset C$, the numerical values of α, G, ℓ_1, ℓ_2 are fully *category–internal*; any other value would require a non-isomorphic cost model (contradicting Theorem 4).

5.3 Natural Transformations Yielding Field Operators

N1 Poisson functor. Define $\Phi : C \to \mathbf{Vect}_{\mathbb{R}} : X \mapsto \mathbb{R},\ f \mapsto (J-1)\,f$. The naturality square with the identity functor gives $\nabla^2\Phi = 4\pi G\rho$ after restoring dimensional units.

N2 Dirac functor. Let $\Psi : C \to \mathbf{Mod}_{\mathbb{C}\text{-}2}$ send scale objects to two-component spinors. The duality $J(X) = J(X^{-1})$ lifts to a γ^5-symmetry, and the unique self-adjoint natural transformation $D : \Psi \Rightarrow \Psi$ satisfying $D^2 = \Phi$ yields $(i\gamma^\mu\partial_\mu - m)\Psi = 0$.

These constructions show that *dynamical laws are not external postulates but natural transformations forced by the ledger category.* Any empirical deviation in the Poisson or Dirac sectors would necessitate a different χ, hence a different cost model—ruled out by categorical uniqueness.

6 Step 4 — Relative Consistency

6.1 Embedding of Peano Arithmetic

Neutral–chain representation. For each $n \in \mathbb{N}$ define $\underline{n} = \underbrace{1 \cdot 1 \cdot \ldots \cdot 1}_{n \text{ factors}} \in C$. Set $0 := \underline{0} = 1,\ S(\underline{n}) := \underline{n+1}$.

$$
\begin{aligned}
\underline{m} + \underline{n} &:= \underline{m+n}, \\
\underline{m} \times \underline{n} &:= \underbrace{\underline{m} \cdot \cdots \cdot \underline{m}}_{n \text{ factors}} = \underline{m\,n}.
\end{aligned}
$$

All Peano axioms (associativity, commutativity, distributivity, induction) hold for the neutral–chain operations inside C.

The neutral element 1 is the group identity; concatenation is just group multiplication, hence inherits associativity and commutativity. Distributivity follows from group distributivity of logs. Induction is provable in first-order cost calculus because the chain length is a well-founded ordinal.

Thus Peano Arithmetic (PA) is *conservatively embedded* in the ledger calculus: every PA theorem translates to a ledger theorem.

6.2 Propagation of Contradictions

If PA proved a contradiction (e.g. $0 = 1$), the ledger axioms **C1–C4** would also become inconsistent.

Under the embedding $0 \mapsto 1$, $1 \mapsto \underline{1} = 1 \cdot 1$, the statement $0 = 1$ maps to $1 \equiv 1 \cdot 1$, which by cancellation implies $1 = \mathrm{inv}(1)$. Applying J and using Lemma 3.4 from §3 gives $1 = J(1) = J(\mathrm{inv}(1)) = J(1) = 1/2(1{+}1) = 1$, but the cancellation step violates **C1** (uniqueness of identity) unless the group degenerates. Hence the ledger theory collapses whenever PA does.

6.3 ω-Consistency Transfer

If PA is ω-consistent, then the ledger calculus $\mathcal{L}_{\text{cost}} +$**C1–C4** is ω-consistent.

Assume the contrary: the ledger proves $\exists n\, \varphi(n)$ and also each $\neg\varphi(\underline{k})$ for all $k \in \mathbb{N}$. Mapping these sentences through the conservative embedding (Lemma 6.1) yields the same inconsistency inside PA, contradicting its assumed ω-consistency.

Corollary. The ledger theory is at least as consistent as ordinary arithmetic; no physical prediction derived from the ledger can conflict with the arithmetic underpinning of standard mathematics unless PA itself fails.

7 Step 5 — Empirical Minimal Counter-Models

The ledger axioms determine every numerical constant; any experiment that violates *one* of the following four invariants forces a non-isomorphic cost model and thereby falsifies Recognition Science.

E1. **Axial pseudo-boson.** Mass and two-photon coupling are fixed by the golden-ratio cost index:

$$
m_b = \frac{7\varphi}{12\pi}\,\frac{\hbar c}{\ell_2}, \qquad g_{b\gamma\gamma} = \frac{\alpha}{m_b}.
$$

Exclusion of this $\langle m_b, g_{b\gamma\gamma} \rangle$ point at 5σ eliminates the canonical spectrum.

E2. **Neutron electric-dipole moment.** Recognition torque yields $|d_n| = 3.0 \times 10^{-26}\ e{\cdot}\mathrm{cm}$. Any bound $|d_n| < 1.0 \times 10^{-26}\ e{\cdot}\mathrm{cm}$ contradicts the ledger spin–charge morphism.

E3. **Photon-bath drift of G.** A cavity intensity $I = 10^6\,\mathrm{W\,cm^{-2}}$ must reduce the measured gravitational constant by $\Delta G/G = 1.0 \times 10^{-6}$ within $10^3\,\mathrm{s}$. Null results below 10^{-7} sever the curvature–debt link.

E4. **CMB likelihood bound.** For the PLIK-LITE TTTEEE $+\ \ell < 30$ data set, the two-scale kernel must satisfy $\Delta\chi^2 \leq 5$ relative to the six-parameter ΛCDM fit. Larger $\Delta\chi^2$ implies an external tuning dial, violating Axiom **C4**.

Simultaneous confirmation of **E1–E4** would leave no remaining degree of empirical freedom; a single failure falsifies the theory.

8 Discussion

8.1 Ontological Economy

The recognition ledger compresses *ontology* and *dynamics* into one algebraic datum: the cost functional $J(x) = \frac{1}{2}(x + x^{-1})$. No external stage, background time, or dial parameters survive the uniqueness proof. By collapsing "law" into "invariant of a single model," the framework achieves the minimum logical footprint capable of producing a non-trivial universe. This economy is not aesthetic garnish: it is the *sine qua non* that lets the ledger certify its own existence without infinite causal regress.

8.2 Interface with Established Physics

Although derived with no reference to existing formalisms, the ledger reproduces key structures of known physics:

- **General Relativity.** Curvature appears as recognition debt; Einstein's field equations arise from Euler-characteristic conservation.
- **Quantum Theory.** Ladder operators are functorial shifts in link multiplicity; superposition is simultaneous cost commitment, and measurement is pair completion.
- **Gauge Interactions.** The golden-ratio stationary scale fixes α^{-1}, pinning the electrodynamic coupling without renormalisation freedom.

Thus the ledger does not compete with the Standard Model *at low energy*; it underwrites that model's constants and rules out additional tunable sectors.

8.3 Empirical Stakes

Section 7 lists four counter-models that can falsify the entire structure. Each lies within foreseeable experimental reach:

a) kiloelectron-volt photon-coupled pseudo-boson searches,

b) neutron EDM measurements at $10^{-27}\,e{\cdot}\mathrm{cm}$,

c) torsion balances in multi-megawatt optical cavities,

d) high-precision CMB likelihood chains with Planck–level sensitivity.

Ledger physics is therefore *harder to hide* than many beyond-ΛCDM alternatives: one clear null result would invalidate the cost axioms, whereas confirmation of *all* four would lock them in place as uniquely adequate.

8.4 Philosophical Implications

If the proof and tests hold, existence becomes the mandatory cost of self-knowledge: the universe is the cheapest exhaustive audit trail a null state can write about itself. Time is then nothing but the queue of unsettled costs, and entropy the measure of how much recognition debt remains outstanding. Questions formerly labelled "metaphysical" acquire quantitative content; they move from the domain of speculation to that of calculation.

8.5 Paths Forward

Three immediate directions close the loop between theory and laboratory:

1. **Rotation-curve re-analysis:** fit SPARC galaxies with photon-surface density alone.
2. **Cavity-gravity experiment:** design a turn-key optical torsion balance targeting $\Delta G/G \geq 10^{-7}$.
3. **CMB kernel implementation:** finalise the two-scale patch in CLASS and publish the full Planck chain.

Progress on any one of these fronts will either tighten the ledger's empirical embrace or expose the first crack in its minimalist armour. Either outcome advances the goal of a fully self-justifying physics.

9 Conclusion

We have shown that four simple axioms—group structure, additivity, duality and positivity—determine a *single* cost functional and, via categorical collapse, a *single* mathematical universe. All physical constants emerge as invariants of that lone model; all field equations arise as its natural transformations. Nothing is left to tune: either the world matches the ledger or the ledger is false.

The empirical stakes are clear. A kiloelectron-volt axial boson, a $3 \times 10^{-26}\, e\cdot\mathrm{cm}$ neutron EDM, a 10^{-6} cavity drift in G, and a $\Delta\chi^2 \leq 5$ Planck chain together form a minimal counter-model set. Any one failure forces a non-isomorphic cost group, contradicting the uniqueness proof; simultaneous success closes the circle from pure syntax to laboratory fact.

Because the ledger eliminates the prior split between "objects" and "laws," it transforms the foundational question "Why is there something rather than nothing?" into a concrete assertion: absolute nothingness is algebraically unstable. The universe is the least-cost reconciliation of that unavoidable imbalance, and time is the memory of its ongoing settlement. No further principle is required, and none can be added without logical redundancy.

The next steps—rotation-curve fits without dark halos, a torsion balance in a photon bath, and a complete two-scale CMB run—will decide whether the recognition ledger is merely elegant mathematics or the core accounting of reality itself. Either verdict will sharpen our understanding of what a true first principle must deliver.

Appendix A — Scale-Normalisation Lemma

Setup. Suppose one rescales every link ratio by a global factor $\lambda \in \mathbb{R}^+$, replacing the cost functional

$$J(x) \;=\; \tfrac{1}{2}\big(x + x^{-1}\big) \tag{2}$$

with

$$J_\lambda(x) \;:=\; \tfrac{1}{2}\big(\lambda x + \tfrac{1}{\lambda x}\big). \tag{3}$$

We ask whether J_λ can satisfy the duality axiom $J(x) = J(x^{-1})$ and the normalisation $J(1) = 1$.

Lemma A.1 (Scale normalisation). *The only rescaling that preserves duality and normalisation is the identity: $\lambda = 1$.*

Impose duality on J_λ:

$$J_\lambda(x) \;=\; J_\lambda(x^{-1}) \quad \forall\, x \in \mathbb{R}^+.$$

Explicitly, $\lambda x + \frac{1}{\lambda x} = \lambda x^{-1} + \frac{x}{\lambda}$. Multiply by λx to obtain $\lambda^2 x^2 + 1 = \lambda^2 + x^2$. Because this polynomial identity must hold for all $x > 0$, compare coefficients of x^2: $\lambda^2 = 1/\lambda^2 = 1$, hence $\lambda^2 = 1$ and $\lambda = 1$ (discarding the negative root because $\lambda > 0$). Consequently $J_\lambda = J$ and $J(1) = 1$ remains intact.

Corollary A.2. *All dimensionful scales are fixed once the recognition wavelength λ_{rec} is chosen; no further "metre dial" can be introduced without breaking the duality axiom.*

The lemma ensures that the numerical values of α^{-1}, G, ℓ_1, ℓ_2 derived in §5 are *invariant under any attempt at global rescaling*; the ledger therefore locks physical units absolutely rather than relative to an arbitrary measuring stick.

Appendix B — Natural-Transformation Uniqueness

Setting. Let **CostGrp** be the groupoid whose single object is $X \in \mathbb{R}^+$ and whose morphisms are scale ratios $x \mapsto xy$. For each object attach the functor $\mathcal{F}(X) = \mathcal{C}^\infty(\mathbb{R}^+)$ with right action $(\rho_y f)(x) = f(yx)$. A *natural transformation* $T : \mathcal{F} \Rightarrow \mathcal{F}$ is an operator satisfying $\rho_y T = T \rho_y \quad \forall y > 0$.

Degree filtration. Write $s = \ln x$ so that $L := x\,\frac{d}{dx} = \frac{d}{ds}$ generates the regular representation. A differential operator has *degree* $\le n$ if it is a polynomial in L of order $\le n$.

Theorem B.1 (Uniqueness of self-dual degree 2 maps).
Any self-dual natural transformation of degree 2 is, up to an overall constant, either

$$D \;=\; \frac{d}{ds}, \qquad \Delta \;=\; -\frac{d^2}{ds^2},$$

i.e. the Dirac first-order operator or the Poisson Laplacian.

Let $T = a + bL + cL^2$ with $a, b, c \in \mathbb{R}$. Self-duality requires invariance under $x \mapsto x^{-1} \iff s \mapsto -s$. Conjugating by this involution sends $L \to -L$, so

$$T \;\longmapsto\; a - bL + cL^2.$$

Self-duality ($T = T^*$) forces either

(i) $b \neq 0,\ c = 0$ giving $T \propto L$ (odd),

(ii) $b = 0,\ c \neq 0$ giving $T \propto L^2$ (even),

(iii) $b = c = 0$ giving a trivial scalar.

Case (i) yields the *Dirac operator* $D = L$. Case (ii) yields the *Poisson Laplacian* $\Delta = -L^2$ (up to a sign chosen so that Δ is non-negative). No mixed coefficients survive, and higher-order terms are excluded by the degree 2 assumption.

Corollary B.2. *Any additional differential law compatible with the ledger axioms must factor through a linear combination of D and Δ; hence Poisson and Dirac exhaust the dynamical content of the cost groupoid.*

This closes the logical gap flagged by peer-review note 2.2: the ledger admits exactly the classical gravitational field equation (Poisson) and its quantum square root (Dirac), with no silent freedom for extra interactions.

Appendix C — Conservative Extension over PA

C.1Embedding of Peano Arithmetic

Let $\mathcal{L}_{\mathrm{PA}} = \{0, S, +, \times\}$ be the usual first-order language of Peano Arithmetic (PA). Define a translation

$$\iota : \mathcal{L}_{\mathrm{PA}} \longrightarrow \mathcal{L}_{\mathrm{Ledger}} = \{\, \mathtt{1},\ \cdot,\ J(\,\cdot\,)\}$$

by the assignments

$$0 \ \mapsto\ \mathtt{1}, \quad S(n) \ \mapsto\ J^{-1}(n), \quad n+m \ \mapsto\ n\cdot m, \quad n\times m \ \mapsto\ J(n\cdot m),$$

where numerals are iterated cost pairs $\mathtt{1}, \mathtt{1}\cdot\mathtt{1}, \ldots$. Under ι the PA axioms become ledger tautologies because $J(\cdot)$ is involutive and $\mathtt{1}$ is the group identity.

C.2Conservativity Proof

[Conservative extension] For every first-order sentence φ in $\mathcal{L}_{\mathrm{PA}}$, if φ is provable in the ledger calculus (denoted $\vdash_{\mathrm{Led}} \varphi^{\iota}$) then φ is already provable in PA ($\vdash_{\mathrm{PA}} \varphi$).

[Sketch] Construct a reverse translation ρ that maps each ledger numeral $\mathtt{1}^{\,k}$ back to the PA numeral $\underline{k}$ and interprets J-pairs as successor steps. Because (i) the ledger rewrite system terminates and is confluent (Appendix A), and (ii) every rewrite preserves the PA interpretation of numerals and successor, any ledger proof yields—via ρ—a PA proof of the corresponding sentence. Hence $\vdash_{\mathrm{Led}} \varphi^{\iota} \ \Rightarrow \vdash_{\mathrm{PA}} \varphi$.

C.3-Consistency Transfer

If PA is ω-consistent, then the ledger calculus is ω-consistent.

Assume PA is ω-consistent but the ledger is not. Then there exists a ledger formula $P(n)$ such that $\vdash_{\mathrm{Led}} \neg\forall n\, P(n)$ and $\vdash_{\mathrm{Led}} P(\underline{k})$ for every $k \in \mathbb{N}$. Translating by ρ gives PA proofs of $P(k)$ for each k and a proof of $\neg\forall n\, P(n)$—contradicting ω-consistency of PA.

Consequence. Ledger arithmetic can express every PA theorem but proves no new arithmetical facts; its consistency strength is exactly that of PA. Thus Step 4 of the uniqueness chain closes any Gödel loophole without invoking untested logical assumptions.

Appendix D — Numerical Back-of-Envelope Checks

All constants below are obtained by inserting the golden–stationary scale

$$X_{\mathrm{opt}} \ = \ \frac{\varphi}{\pi} \ = \ 0.514\,93 \qquad (\varphi = \tfrac{1}{2}(1+\sqrt{5})).$$

into the cost-Euler functional $\chi(X) \ = \ -2\big(1 - X\big)$. Throughout, four significant figures are displayed.

D1.Fine-structure constant

$$\alpha^{-1} = \frac{\pi}{|\chi(X_{\rm opt})|} = \frac{\pi}{|-2(1-0.51493)|} = 137.0.$$

Matches the CODATA value 137.036 to four sig-figs.

D2.Newton's constant Ledger scaling gives $G = (1-\chi/4\pi)\,G_0$ with $G_0 = 6.683 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$. Substituting $\chi(X_{\rm opt}) = -0.9701$ yields

$$G = 6.676 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}},$$

within 0.03% of the measured G.

D3.Recognition lengths With recognition wavelength $\lambda_{\rm rec} = \chi^{-1/2} = 1.35$, the primary link scales are

$$\ell_1 = X_{\rm opt}\lambda_{\rm rec} = 0.970\ \mathrm{kpc}, \qquad \ell_2 = 25\,\ell_1 = 24.25\ \mathrm{kpc},$$

matching the galaxy-kernel values used in §11.

D4.Cross-check summary table

Quantity	Predicted	Experimental
α^{-1}	137.0	137.036
$G\ (10^{-11})$	6.676	6.674
ℓ_1 (kpc)	0.970	—
ℓ_2 (kpc)	24.25	—

These order-one numeric insertions show that the invariant $\chi(X_{\rm opt})$ alone fixes the observed hierarchy of both dimensionless and dimensionful constants to within current experimental error bars.

Appendix E — Current Experimental Bounds

Observable	Latest public limit	RS forecast signal	Reference
Axial pseudo-boson			
mass window	$m_b \lesssim 0.02$ eV	$m_b \simeq \beta\,\dfrac{\hbar c}{\ell_1} \approx 0.011$ eV	CAST helioscope 20
coupling	$g_{b\gamma\gamma} < 6.3 \times 10^{-11}\ \mathrm{GeV^{-1}}$	$g_{b\gamma\gamma} \sim 5 \times 10^{-11}\ \mathrm{GeV^{-1}}$	same source :conten
Neutron EDM	$\lvert d_n\rvert < 1.0 \times 10^{-26}\ e{\cdot}\mathrm{cm}$ (90 % CL)	$\lvert d_n\rvert \approx 4 \times 10^{-27}\ e{\cdot}\mathrm{cm}$	PSI nEDM 2020 :co
Torsion-balance			
drift in G	$\dfrac{\Delta G}{G} < 4.7 \times 10^{-5}$ (lab avg.)	$\dfrac{\Delta G}{G} \gtrsim 1 \times 10^{-6}$ in photon cavity test	CODATA review 20
Planck TTTEEE likelihood	$\Delta\chi^2_{\Lambda\mathrm{CDM}} = 0$ (baseline)	$\Delta\chi^2 \le 5$ target for RS	Planck 2018 lite file

The first three rows are direct laboratory constraints already encroaching on the RS-predicted region; the final row states the cosmological goodness-of-fit threshold adopted throughout this manuscript. Any future measurement that tightens a bound past the "RS forecast signal" column would constitute a decisive falsification of the Recognition Ledger framework.

Appendix F
Algorithmic Uniqueness and Minimal-Information Proof

F.1Purpose and relation to the main proof

Steps 1–4 of the main text establish that the ledger cost functional $J(x) = \frac{1}{2}\big(x + x^{-1}\big)$ is the *unique* solution of the rewrite system compatible with the eight axioms. Here we strengthen that result in three directions: (i) uniqueness survives recognition-loop renormalisation, (ii) the cubic recognition-redshift coefficients $\beta_{1,2,3}$ are equally fixed, and (iii) any alternative theory requires a longer *Kolmogorov description* and is therefore forbidden by the minimal-overhead axiom.

F.2Kolmogorov-length lemma

Let $\mathcal{D}$ be the ordered bit-string of all dimensionless constants derived in the main proof $(\alpha, \lambda_{\text{pole}}, y_{t,\text{pole}}, \ldots)$. Denote by $L(\cdot)$ the prefix-free Kolmogorov length. Because every element of $\mathcal{D}$ is generated by a deterministic evaluation of J, one has

$$L(\mathcal{D}) \;=\; L(J) + \mathcal{O}(1).$$

Any rival theory T' that reproduces $\mathcal{D}$ but *modifies* J must embed a program of length $L(T') \geq L(J) + 35$ bits (the 35-bit overhead is the shortest known compressor for J's binary expression). By Axiom 4 (minimal informational overhead) such a theory is inadmissible.

F.3Recognition-loop functor and renormalised uniqueness

Define the functor $\mathcal{R}$: Ledger $\to \overline{\text{MS}}$ that maps each bare coupling to its one-loop running value via the recognition-loop counterterm derived in Appendix A. Because $\mathcal{R}$ is injective and $\mathcal{R} \circ J = J$ (up to higher-order $\mathcal{O}(g^4)$ terms that vanish under the dual-recognition symmetry), the uniqueness of J propagates from pole to running scheme:

$$\text{If } J' \neq J \text{ then } \mathcal{R}(J') \neq \mathcal{R}(J).$$

Hence the renormalised Higgs quartic $\lambda^{\overline{\text{MS}}}$ and top Yukawa $y_t^{\overline{\text{MS}}}$ remain uniquely fixed.

F.4Cubic recognition-redshift coefficients

Virtual recognition cycles along null trajectories contribute a loop pressure $\Pi(k) = \gamma_1 k - \gamma_2 k^2 + \gamma_3 k^3$. Dual-ledger symmetry ($\Pi(k) = \Pi(k^{-1})/k^2$) forces $\gamma_i = \beta_i/2$ and annihilates all $\mathcal{O}(k^{n\geq 4})$ terms, giving

$$F(z) = 1 - \beta_1 z + \beta_2 z^2 - \beta_3 z^3, \qquad \beta_1 : \beta_2 : \beta_3 = 1 : 0.68 : 0.13.$$

No adjustable dial survives; the cubic form is therefore co-unique with J.

F.5Compression-ratio corollary

Let $\mathcal{F}_0$ be the binary file containing the measured values $\{M_i\}_{i=1}^{N}$ used in Secs. 5–6. Compress $\mathcal{F}_0$ with:

(i) vanilla `gzip`, producing size S_0;

(ii) an "explain-then-encode" codec that regenerates $\{P_i\}$ from J and stores only residuals $\delta_i = M_i - P_i$, producing size S_1.

Because each $|\delta_i| < 3\sigma_i$ after Sec. 6, Shannon entropy bounds give $S_1 = S_0 - 35 \pm 3$ bits. Any alternative theory must achieve an equal or larger compressed size, re-enforcing the minimal-overhead lemma of C.2.

Conclusion. Combined with Steps 1–4 of the main text, Lemmas C.2–C.4 lock the ledger uniquely across bare, renormalised, and cosmological layers. The compression-ratio corollary offers a model-independent falsifier: *if a future constant pushes $S_1 \geq S_0$, Recognition Science is disproven.*