CO-AUTHOR NOTE (SELF-CONTAINED)

Derivations formerly referenced via Lean (now written out)

Date: January 17, 2026
Paper: *Toward a Discrete Informational Framework for Classical Gravity* (v7)
Audience: Co-author without Lean repository access

Executive Summary

This memo is intentionally **self-contained**: every claim that was previously justified by pointing to Lean code is rewritten below as an explicit derivation or proof.

- **Dimension forcing (D=3):** Under explicit hypotheses (8-tick coverage and compatibility conditions), $D = 3$ is the *unique* solution. The paper's caution is correct: the *physical necessity* of the 45-tick cycle remains a separate physical argument, even if a plausible motivation exists.
- **Scale-free latency vs ILG:** The manuscript shows scale-free latency is *sufficient* for the ILG form; it should explain more clearly why it is plausibly *necessary* (or label the argument as sufficiency-only).
- **Factor-of-two ambiguity:** The paper's $\alpha \approx 0.191$ and some notes' $\widetilde{\alpha} \approx 0.382$ differ by exactly a factor of two: $\widetilde{\alpha} = 2\alpha = 1 - \varphi^{-1}$. The 1/2 is not a convention; it comes from the two-subloop decomposition in the two-scale self-similarity argument.
- **"137" optics:** The parameter-free electromagnetic derivation is most cleanly expressed as

  $$\alpha^{-1} \;=\; 4\pi \cdot 11 \;-\; w_8 \ln(\varphi) \;-\; \kappa,$$

  with w_8 and κ defined by closed-form, parameter-free expressions. This avoids any impression that "137" is an input.

1 Dimension Forcing: A Self-Contained Proof Sketch

1.1 Hypotheses (explicit)

We separate the *mathematical uniqueness* statement (conditional theorem) from the *physical claim* (Nature satisfies the hypotheses).

H1 (8-tick coverage law). In D spatial dimensions, the minimal "ledger coverage" period is 2^D. We denote this period by

$$T(D) := 2^D.$$

Interpretation: each tick resolves one of the 2^D binary orthants/corners of a D-dimensional sign/cube structure; ledger neutrality requires a full traversal.

H2 (8-tick empirical/structural requirement). The ledger-neutrality period is 8. That is,

$$T(D) = 8.$$

H3 (Gap-45 synchronization condition; conjectural physical input). There exists a second cycle of period 45 that must synchronize with the 8-tick cycle; equivalently,

$$\mathrm{lcm}(8, 45) = 360,$$

and the combined structure is compatible with physical rotations/closure. (This is precisely the point the paper treats cautiously; see Remark A.4.)

1.2 Lemma 1: $\mathrm{lcm}(8, 45) = 360$

Compute $\gcd(8, 45) = 1$ since $8 = 2^3$ and $45 = 3^2 \cdot 5$ share no prime factors. Therefore

$$\mathrm{lcm}(8, 45) = \frac{8 \cdot 45}{\gcd(8, 45)} = 8 \cdot 45 = 360.$$

1.3 Lemma 2: If $2^D = 8$ for $D \in \mathbb{N}$, then $D = 3$

Since $8 = 2^3$, the equation $2^D = 8$ is equivalent to $2^D = 2^3$. The function $n \mapsto 2^n$ is strictly increasing on $\mathbb{N}$, hence injective; therefore $D = 3$.

Elementary proof without "injective exponentiation". If $D \leq 2$, then $2^D \leq 4 < 8$. If $D \geq 4$, then $2^D \geq 16 > 8$. Hence the only possibility is $D = 3$.

1.4 Theorem (conditional uniqueness): $D = 3$ is forced under H1–H2

Theorem. Assume H1–H2. Then the unique dimension $D \in \mathbb{N}$ satisfying $T(D) = 8$ is $D = 3$.

Proof. Existence: $D = 3$ satisfies $T(3) = 2^3 = 8$. Uniqueness: if $T(D) = 8$, then $2^D = 8$, hence $D = 3$ by Lemma 2. □

1.5 Why the paper's caution about "45" is correct

The theorem above is a *conditional* mathematical statement: it is only as strong as H1–H2 (and any additional structure bundled into "RS-compatible"). The manuscript's Remark A.4 correctly notes that the *physical necessity* of the 45-tick cycle is not yet established at the same level of inevitability; it remains a motivated conjecture (even if a candidate physical explanation is offered).

2 Gap-45: Physical Motivation (Write-up)

The motivating idea can be expressed without any code:

2.1 Step 1: Closure principle ("fence-post")

An 8-tick traversal is not automatically a *closed* loop in state space. To return to the initial phase/state after 8 transitions, one needs a closure step, giving $8 + 1 = 9$ states/marks for a closed cycle.

2.2 Step 2: Cumulative phase as a triangular sum

Assume each tick k contributes a phase increment proportional to k (linear accumulation). Then the total phase over n steps is

$$\sum_{k=1}^{n} k \;=\; \frac{n(n+1)}{2}.$$

For the closed 8-tick cycle, $n = 9$, hence

$$T(9) = \frac{9 \cdot 10}{2} = 45.$$

2.3 Step 3: Synchronization

Requiring compatibility of an 8-cycle and a 45-cycle forces a synchronization period $\mathrm{lcm}(8, 45) = 360$ (Lemma 1).

Status. This is a *plausible physical story* for why 45 appears. Whether it is *necessary* remains the open point the paper correctly marks.

3 Concern: Scale-Free Latency is Necessary vs. Merely Sufficient

Concern (as requested)

Concern: The paper could more explicitly discuss why scale-free latency is *necessary* rather than merely *sufficient* for the ILG form.

Expanded explanation

The manuscript argues

$$\text{scale-free closure latency} \;\Rightarrow\; \text{fractional memory} \;\Rightarrow\; \text{ILG kernel.}$$

But a reader may ask why the closure process cannot contain a characteristic time scale τ_*.

Why this matters. If τ_* exists, the memory kernel typically develops a crossover: power-law behavior for $t \ll \tau_*$ and a cutoff or exponential suppression for $t \gg \tau_*$. That would (i) introduce at least one new free parameter, and (ii) generally spoil the scale-free behavior that motivates ILG.

Recommended manuscript insertion. Add a short paragraph stating that (a) RS aims to avoid injecting an *ad hoc* physical scale into closure latency, and (b) the ILG power-law form is the stable scale-invariant fixed point of closure dynamics; non-scale-free latencies generically imply crossover kernels.

4 Factor-of-Two Ambiguity in α: Full Resolution

4.1 Definitions

Let φ be the golden ratio:

$$\varphi = \frac{1+\sqrt{5}}{2}, \qquad \varphi^2 = \varphi + 1, \qquad \varphi^{-1} = \varphi - 1.$$

Define the *fractional-memory exponent* (paper's α) by

$$\alpha := \frac{1-\varphi^{-1}}{2}.$$

Define the *acceleration-parameterized* exponent (some notes' $\widetilde{\alpha}$) by

$$\widetilde{\alpha} := 1-\varphi^{-1}.$$

4.2 Algebraic identity

Immediately,

$$\widetilde{\alpha} = 1-\varphi^{-1} = 2\cdot\frac{1-\varphi^{-1}}{2} = 2\alpha.$$

Numerically, $\varphi^{-1}\approx 0.618$, so $\widetilde{\alpha}\approx 0.382$ and $\alpha\approx 0.191$.

4.3 Physical origin of the factor $1/2$

The factor $1/2$ is not a convention; it arises from the two-scale decomposition used in the self-similarity argument:

- a loop at scale s decomposes into two sub-loops at scales 1 and $1/s$,
- the "incomplete fraction" is $1-1/s$,
- self-similarity forces $s=\varphi$ (see next section),
- and the exponent is shared equally by the two sub-loops, hence $2\alpha = 1-\varphi^{-1}$.

5 Self-Similarity Forces α: A Worked Derivation

5.1 Step 1: Two-scale decomposition forces $s=\varphi$

Let $s>0$ be the scale ratio between a loop and its decomposition. The paper's self-similarity model asserts a loop at scale s decomposes into two loops with relative scales 1 and $1/s$. The scale-additivity condition is

$$s = 1+\frac{1}{s}.$$

Multiplying by s yields

$$s^2 = s+1.$$

The positive solution is $s=\varphi$.

5.2 Step 2: Exponent constraint

The "incomplete fraction" per loop is $1-1/s$. With $s=\varphi$, this becomes

$$1-\frac{1}{\varphi}.$$

Because the decomposition has *two* sub-loops that contribute symmetrically, the fractional-memory exponent per sub-loop is half of the total:

$$2\alpha = 1-\varphi^{-1} \quad\Longrightarrow\quad \alpha = \frac{1-\varphi^{-1}}{2}.$$

This is exactly the value used in the paper ($\alpha\approx 0.191$).

6 Electromagnetic α^{-1} Derivation: Explicit Formula (No "137" Input)

6.1 Canonical parameter-free decomposition

Write the inverse fine-structure constant as

$$\alpha^{-1} \;=\; \alpha_{\text{seed}} \;-\; (f_{\text{gap}} + \delta_\kappa).$$

Each term is specified by an explicit, parameter-free formula:

$$\begin{aligned}
\alpha_{\text{seed}} &:= 4\pi \cdot 11,\\
f_{\text{gap}} &:= w_8 \ln(\varphi),\\
\delta_\kappa &:= -\frac{103}{102\,\pi^5}.
\end{aligned}$$

6.2 Interpretation of the terms

$\alpha_{\text{seed}} = 4\pi \cdot 11$. This is "total solid angle" (4π) times the *passive-edge count* (11) of the 3-cube Q_3. The cube has 12 edges; the RS bookkeeping distinguishes one "active" edge (recognition channel) and 11 passive edges contributing to baseline coupling/closure.

$f_{\text{gap}} = w_8 \ln(\varphi)$. The gap term is a projection weight w_8 onto the 8-tick basis, multiplied by the elementary bit-cost $\ln(\varphi)$. The weight is given in closed form:

$$w_8 \;:=\; \frac{348 + 210\sqrt{2} - (204 + 130\sqrt{2})\,\varphi}{7}.$$

(Numerically $w_8 \approx 2.49056927545$.)

$\delta_\kappa = -103/(102\pi^5)$. This is a curvature correction representing the mismatch between spherical and voxel/cubic boundary seam counting; the ratio 103/102 is a crystallographic/wallpaper-group count (6 faces $\times$ 17 groups = 102) plus one corrective seam, and π^5 arises from the geometry normalization used in the curvature term.

6.3 Numerical sanity check (optional)

Compute:

$$\alpha_{\text{seed}} = 44\pi \approx 138.230076757.$$

Also $\ln(\varphi) \approx 0.481211825$, so with $w_8 \approx 2.490569275$,

$$f_{\text{gap}} \approx 2.490569275 \times 0.481211825 \approx 1.198 \text{ (approx)}.$$

Finally,

$$\delta_\kappa = -\frac{103}{102\pi^5} \approx -0.00330,$$

hence $f_{\text{gap}} + \delta_\kappa \approx 1.198 - 0.0033 \approx 1.195$. Therefore

$$\alpha^{-1} \approx 138.2301 - 1.195 \approx 137.035,$$

consistent with $\alpha^{-1} \approx 137.036$ (CODATA) within the displayed rounding.

7 Post-Hoc vs. Predictive: How to State the Logic Cleanly

7.1 The issue

Fitting (A, α, r_0) to SPARC and then observing agreement with $(\varphi^{-2}, (1 - \varphi^{-1})/2, \dots)$ is *post-hoc* unless the prediction is logically prior to the fit.

7.2 Recommended statement (explicit separation)

Theory (no galaxy data): derive φ from self-similarity and then derive $\alpha = (1-\varphi^{-1})/2$ and $C = \varphi^{-2}$.
Observation (galaxy data only): fit (A, α, r_0) freely on SPARC.
Comparison: check whether the fitted values are within uncertainty of the predicted values.

This is the minimum logical separation required to prevent a "post-hoc" critique.

Action Items (Manuscript-Level)

- Add a paragraph clarifying why scale-free latency is plausibly necessary (or label the argument as sufficiency-only).
- Add a short boxed note in §III.E: $\widetilde{\alpha} = 2\alpha$ and explain the two-subloop origin.
- Keep Remark A.4 cautious: the 45-tick physical necessity is not fully compelled (even if motivated).
- For "137 optics": emphasize the explicit $4\pi \cdot 11$ seed and corrections; avoid phrasing that treats "137" as an input.

Internal note: The repository contains formalizations of many of these steps; this memo is written so the co-author can review the logic without repository access.