Reply: Why "Structural Mechanism" Does Not Resolve Non-Identifiability

The response under critique concedes the purely algebraic point that many expressions evaluate to the same numerical constant (e.g. 1.5 can be written as $F/4$, $E/8$, $D/2$, etc.). It then claims this objection is irrelevant because a *mechanism* uniquely forces one form:

$$\Delta(3) = \frac{F}{V} = \frac{6}{4} = 1.5,$$

where $F$ is the number of cube faces and $V$ is the number of vertices of a face, interpreted as a "discrete solid angle" normalization. The response asserts that alternative expressions are ruled out because they allegedly correspond to the wrong "mechanism" (edge-mediated vs face-mediated, etc.).

This reply fails as a resolution for three independent reasons:

1. It introduces new principles ("Inverse Measure Rule," "Discrete–Continuous Duality," "discrete solid angle = vertex count") without derivation and then uses them as if they were theorems. This is not a derivation; it is a new hypothesis layer.
2. Even if these principles are granted, the coefficient remains *non-identifiable*: multiple *distinct* count/measure mechanism pairs produce the same value 1.5 using the same counting-layer data. Therefore the mechanism-to-number map is not injective.
3. The proposed "mechanism" is not uniquely specified even internally: in the cube case the integer 4 is simultaneously (i) vertices-per-face and (ii) edges-per-face, so the chosen "discrete measure" is not uniquely determined.

## 1. Minimal criterion for a resolution?

**Definition 1** (Resolution of the "infinite formulas" objection)**.** *Let $\mathcal{M}$ be the set of admissible model mechanisms and let $g : \mathcal{M} \to \mathbb{R}$ map each mechanism to the predicted coefficient (here, the tau-step correction coefficient or sub-coefficient). A genuine resolution must supply:*

1. *a* precise definition *of $\mathcal{M}$ (not narrative labels),*
2. *a* precise rule *$g$ (not analogy),*
3. *and a* uniqueness theorem *showing $g$ is injective on admissible mechanisms for the data used.*

*Absent injectivity, "uniqueness of derivation" is false in a literal mathematical sense.*

## 2. Failure 1: New axioms are asserted, not derived

The response introduces:

- "Inverse Measure Rule: Contribution = Count / Measure,"

- "Discrete–Continuous Duality Principle," and
- "discrete solid angle of a facet equals its vertex count $V$."

Postulating a selector is not a derivation: If the only reason a formula is "unique" is that one postulates an additional selector rule that excludes other formulas, then uniqueness is merely conditional on that selector and not a derived fact.

Proof: By definition, a derivation must show:

$$(\text{prior axioms}) \;\Rightarrow\; \Delta(3) = F/V.$$

But if instead one states new principles designed to make $\Delta(3)$ equal $F/V$, then the actual logical structure is:

$$(\text{prior axioms}) + (\text{new selector postulate}) \;\Rightarrow\; \Delta(3) = F/V.$$

This is hypothesis revision, not derivation. The critique was precisely about freedom in choosing the form of the correction; moving that freedom into a new selector postulate does not remove it.

*Remark* 1. A typical fallback is "the selector rule is physically motivated." That is not enough. One must still prove that the selector is *forced* by the framework, or provide an independent falsifier. Otherwise it remains an unconstrained degree of freedom.

## 3. Failure 2: Even granting the rule, the mechanism is not unique (explicit counterexample)

The response argues: "Only $F/V$ respects the face-mediated mechanism; alternatives like $E/8$ imply edge mediation and are excluded." This is false even on its own terms because the same numerical correction arises from distinct Count/Measure pairs using the same cube data.

Concrete degeneracy under Count/Measure Using only cube counts:

$$F = 6, \quad E = 12, \quad V_{\text{cube}} = 8, \quad V_{\text{face}} = 4,$$

the value 1.5 arises from at least two distinct Count/Measure pairs:

$$\frac{F}{V_{\text{face}}} = \frac{6}{4} = 1.5 \qquad \text{and} \qquad \frac{E}{V_{\text{cube}}} = \frac{12}{8} = 1.5.$$

Proof: Compute:

$$\frac{6}{4} = \frac{3}{2} = 1.5, \qquad \frac{12}{8} = \frac{3}{2} = 1.5.$$

These are distinct mechanism candidates if one takes the response's own narrative seriously: "faces distributed over face-anchors" versus "edges distributed over vertex-anchors." Both satisfy the same abstract template *Contribution = Count / Measure*.

Non-identifiability persists under the proposed mechanism template: Under the response's own template class

$$\Delta = \frac{\text{Count}}{\text{Measure}},$$

the tau-step correction is *not identifiable* from cube data because there exist multiple distinct (Count, Measure) pairs producing the same value.

Proof: Identifiability would require uniqueness of the generating pair:

$$\frac{c_1}{m_1} = \frac{c_2}{m_2} \implies (c_1, m_1) = (c_2, m_2) \quad \text{for admissible pairs.}$$

But the lemma exhibits $(c_1, m_1) = (F, V_{\text{face}})$ and $(c_2, m_2) = (E, V_{\text{cube}})$ with equal ratios and unequal pairs. Therefore the mapping is not injective, hence not identifiable.

*Remark* 2. The response tries to rescue uniqueness by declaring that one pair is "wrong mechanism." But that is precisely the contested point: the framework has not provided a theorem that selects (faces, face-vertices) and forbids (edges, cube-vertices). Without such a theorem, the choice is discretionary.

## 4. Failure 3: The "discrete measure" is not uniquely defined even for faces

Even restricting to "face-mediated" reasoning, the denominator 4 is ambiguous for a square face: it is both the number of vertices per face and the number of edges per face.

<u>Internal ambiguity of the alleged discrete measure</u> For the cube, the rule "normalize by vertex count of the mediating object" does not uniquely fix the normalization because multiple inequivalent structural quantities coincide numerically:

$$V_{\text{face}} = E_{\text{face}} = 4.$$

Proof: A square face has 4 vertices and 4 edges. Therefore the proposed normalization by "vertex anchors" cannot be distinguished, at $D = 3$, from an equally plausible normalization by "edge anchors," while yielding the same numeric correction. The response does not provide a theorem that picks vertices rather than edges as the correct "discrete measure."

*Remark* 3. This is an important point: the response claims it has eliminated the arbitrariness of the integer 4. It has not. It has only renamed the 4 after selecting it.

## 5. "Structural" vs "numerical" is not a valid separation here

The response asserts that algebraic non-uniqueness applies only to "numerical representations," not to "structural derivations."

<u>If structure is not independently formalized, it collapses to numerical relabeling:</u> If a proposed "mechanism" is not encoded as a formal object with independent constraints and falsifiers, then any selection of a formula can be rebranded as a "mechanism," and the distinction from numerical representation evaporates.

Proof: Given a desired value $x$, one can always define a post-hoc mechanism label $\mathsf{Mech}_x$ whose rule is "output $x$." If mechanisms are not independently constrained, the mapping "mechanism $\to$ number" provides no explanatory restriction.

Therefore, for a mechanism claim to do work, it must be formalized in a way that makes alternative mechanisms *impossible* (or at least testably false). The response supplies neither a formal definition of admissible mechanisms nor a falsifier that would distinguish the proposed mechanism from rivals.

## 6. Fixing one coefficient cannot establish a fermion mass law

Even if one accepted the tau-step story, the lepton pipeline still contains additional hand-specified hypothesis formulas (e.g. the electron break $\delta_e$ and the $e \to \mu$ step correction terms), so the system remains underdetermined at the level required to claim a universal mass law.

**Conclusion.** The response does not solve the "infinite formulas" objection. It replaces it with a new unproven selector postulate and then asserts uniqueness by narrative mechanism labeling. Even granting its template, explicit degeneracy remains.

### What would actually close the loophole? (Non-negotiable checklist)

To legitimately claim the mechanism resolves the objection, one must provide:

1. Formal definitions of "edge-mediated" and "face-mediated" as distinct objects in the theory,
2. A theorem that the tau-step must be face-mediated (not stipulated),
3. A theorem that the discrete normalization for face mediation is *vertex count* (not edges-per-face, not total vertices, not any other coincident count),
4. A uniqueness theorem proving injectivity of the mechanism-to-coefficient map,
5. An empirical falsifier that would refute the rule if the universe were different.

Until then, the proposed resolution is not a derivation; it is an after-the-fact explanation.