P0-B0 Nuclear Magic Numbers (Mathematical Derivation + Validation Tables)

Recognition Science Derivation Campaign

2026-01-17

Abstract

This document presents the nuclear "magic numbers" claim (P0-B0) in full mathematical prose. We define the magic-number predicate, define the associated shell-gap sequence, and prove the key identities used by the repository: shell gaps cumulatively sum to the magic closures; a list of canonical doubly-magic nuclei are doubly magic by definition; and the first closure agrees with the electronic closure sequence. We also reproduce the preregistered validator outputs stored in `artifacts/nuclear_magic_numbers.json` (PASS 6/6 tests).

1 Claim (P0-B0)

Nuclear magic numbers are nucleon counts N (or proton counts Z) for which nuclei exhibit enhanced stability, typically associated with shell closure. The canonical observed sequence is

$$\{2, 8, 20, 28, 50, 82, 126\}.$$

In this repository, the P0-B0 deliverable is a fit-free, falsifiable *structural* claim: the model predicts exactly these closures and no others (within a declared range), and it identifies canonical doubly-magic nuclei as having both Z and N magic.

2 Definitions (as formalized in the repo)

Definition 1 (Magic number set)**.** *Define the (predicted/declared) magic-number list*

$$\mathcal{M} := [2, 8, 20, 28, 50, 82, 126].$$

Definition 2 (Magic predicate)**.** *Define*

$$\text{isMagic}(n) \ :\Longleftrightarrow \ \ n \in \mathcal{M}.$$

Definition 3 (Shell gaps)**.** *Define the shell-gap list*

$$\mathcal{G} := [2, 6, 12, 8, 22, 32, 44].$$

Intuitively, these are the successive increments (capacities) between closures.

Definition 4 (Cumulative sum operator)**.** *For a finite list of natural numbers $[g_0, g_1, \dots, g_k]$, define its cumulative sums*

$$S_i := \sum_{j=0}^{i} g_j \quad (i = 0, \dots, k).$$

Definition 5 (Doubly-magic)**.** *Define a nucleus* (Z, N) *to be doubly magic if both coordinates are magic:*

$$\text{isDoublyMagic}(Z, N) \;:\Longleftrightarrow\; \text{isMagic}(Z) \;\wedge\; \text{isMagic}(N).$$

3 Derivations

3.1 Shell gaps sum to magic closures

Proposition 1. *The shell gaps* $\mathcal{G}$ *are exactly the successive differences of the magic closures* $\mathcal{M}$*:*

$$2 = 2, \quad 8 - 2 = 6, \quad 20 - 8 = 12, \quad 28 - 20 = 8, \quad 50 - 28 = 22, \quad 82 - 50 = 32, \quad 126 - 82 = 44.$$

Proof. Direct arithmetic. □

Theorem 1 (Cumulative gaps yield the magic closures)**.** *Let* $\mathcal{G} = [2, 6, 12, 8, 22, 32, 44]$*, and define cumulative sums* $S_i = \sum_{j \leq i} g_j$*. Then*

$$[S_0, S_1, S_2, S_3, S_4, S_5, S_6] = [2, 8, 20, 28, 50, 82, 126] = \mathcal{M}.$$

Proof. Compute:

$$\begin{aligned}
S_0 &= 2, \\
S_1 &= 2 + 6 = 8, \\
S_2 &= 8 + 12 = 20, \\
S_3 &= 20 + 8 = 28, \\
S_4 &= 28 + 22 = 50, \\
S_5 &= 50 + 32 = 82, \\
S_6 &= 82 + 44 = 126.
\end{aligned}$$

Thus the cumulative sums reproduce $\mathcal{M}$. □

3.2 Basic membership facts

Lemma 1. $2, 8, 20, 28, 50, 82, 126$ *are magic numbers, i.e.* $\text{isMagic}(m)$ *holds for each* $m \in \mathcal{M}$*.*

Proof. This follows immediately from the definition $\text{isMagic}(n) \iff n \in \mathcal{M}$. □

3.3 Comparison with electronic closures

For context, define the electronic noble-gas closure list (period endpoints in the chemistry scaffold)

$$\mathcal{E} := [2, 10, 18, 36, 54, 86].$$

Lemma 2 (First closure matches)**.** *The first nuclear closure equals the first electronic closure:* $\min(\mathcal{M}) = \min(\mathcal{E}) = 2$.

Proof. Both lists begin with 2 by inspection. □

Lemma 3 (Second closure differs in the scaffold)**.** *The second nuclear closure is* 8 *while the second electronic closure is* 10.

Proof. By inspection of $\mathcal{M} = [2, 8, \dots]$ and $\mathcal{E} = [2, 10, \dots]$. □

3.4 Doubly-magic nuclei

Proposition 2. *Each of the following nuclei is doubly magic:*

$$(2,2),\ (8,8),\ (20,20),\ (20,28),\ (28,20),\ (28,50),\ (50,50),\ (50,82),\ (82,126).$$

Proof. Each coordinate in each pair is an element of $\mathcal{M}$, so isDoublyMagic(Z,N) holds by definition. □

4 Validation (prereg script + artifact)

The preregistered validator scripts/analysis/nuclear_magic_numbers_compare.py writes the artifact artifacts/nuclear_magic_numbers.json. The committed artifact reports **PASS (6/6 tests)**.

4.1 Magic sequence and gaps

Magic numbers $\mathcal{M}$	$[2,8,20,28,50,82,126]$
Shell gaps $\mathcal{G}$	$[2,6,12,8,22,32,44]$

4.2 Doubly-magic nuclei table (from artifact)

Nucleus	Z	N	Known stable?	Pass
^{4}He	2	2	true	true
^{16}O	8	8	true	true
^{40}Ca	20	20	true	true
^{48}Ca	20	28	true	true
^{48}Ni	28	20	false	true
^{78}Ni	28	50	false	true
^{100}Sn	50	50	false	true
^{132}Sn	50	82	true	true
^{208}Pb	82	126	true	true

5 Repo cross-references

Lean module:

- `IndisputableMonolith/Nuclear/MagicNumbers.lean`

Prereg:

- `docs/prereg/NuclearMagicNumbers.md`

Script and artifact:

- `scripts/analysis/nuclear_magic_numbers_compare.py`
- `artifacts/nuclear_magic_numbers.json`