

The Origin of Mass in Recognition Science: Cost Geometry, Recognition Boundaries, and the φ -Ladder

Paper I of VI: Mechanism

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Abstract

In the Standard Model, fermion masses are free parameters encoded by Yukawa couplings to the Higgs field. This paper develops an alternative ontology of mass within Recognition Science (RS), separating two epistemic layers:

Layer 1 [PROVED]: The cost functional $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ is uniquely forced by the Recognition Composition Law (Theorem T5, Lean-verified). The golden ratio $\varphi = (1 + \sqrt{5})/2$ is the unique self-similar fixed point (Theorem T6). Dimension $D = 3$ is forced (Theorem T8), giving a 3-cube with $V=8$, $E=12$, $F=6$. The eight-tick cycle $2^3 = 8$ is the minimal cover (Theorem T7).

Layer 2 [HYP]: Mass emerges as a coordinate on a φ -ladder whose sector-level scales are constrained by cube combinatorics via explicit charge-ordering and edge-scaling principles. The *recognition boundary* — a self-sustaining pattern on a discrete ledger — replaces the point-particle ontology. Sector yardsticks, the charge-to-band map, and generation torsion $\{0, 11, 17\}$ (from the cube's edge/face hierarchy) are structural proposals with explicit falsifiers.

The 3-cube provides exactly five independent integers $\{V, E, F, A, W\} = \{8, 12, 6, 1, 17\}$ as its complete counting-layer vocabulary. Every formula in the mass model draws from this vocabulary; the system is over-determined (more outputs than inputs).

Companion papers develop phenomenological predictions (II), the neutrino sector (III), transport discipline (IV), the fine-structure constant (V), and the generation problem (VI).

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Claim-hygiene convention. Every substantive claim carries one of: **[PROVED]** (derived from RS axioms with complete chain, Lean-verified where noted); **[HYP]** (structural proposal, falsifiable, not yet derived from axioms); **[VAL]** (comparison with external data).

1 Introduction

1.1 The mass problem

The Standard Model contains nine charged fermion masses spanning nearly five orders of magnitude, from the electron (0.511 MeV) to the top quark (173 GeV). These masses enter as free Yukawa couplings — the SM tells us *how* particles acquire mass (electroweak symmetry breaking) but not *why* they have the particular values they do.

1.2 The RS approach

RS begins from a single primitive: the Recognition Composition Law, **[PROVED]**

$$J(xy) + J(x/y) = 2J(x)J(y) + 2J(x) + 2J(y), \quad (1)$$

together with normalization $J(1) = 0$ and calibration $J''_{\log}(0) = 1$. These three conditions uniquely determine $J(x) = \frac{1}{2}(x + x^{-1}) - 1$, proved in Lean 4 via ODE uniqueness for the d'Alembert functional equation [1].

From this cost functional, a chain of forced consequences (T0–T8) derives discreteness, a double-entry ledger, recognition events, J uniqueness, φ , the eight-tick period, and three spatial dimensions. Within this architecture, we *propose* (Layer 2) that mass is a coordinate on a discrete multiplicative ladder whose base φ , period 8, and sector structure are all determined by the cube geometry.

1.3 What this paper does and does not claim

- We *do* present a structural model for particle mass that uses no free parameters and no per-particle fitting.
- We *do not* claim that every element of the model is derived from axioms. The sector yardsticks, the charge-to-band map, and the generation torsion are *structural hypotheses* — motivated by the framework but not yet proved from it.
- We *do* identify which parts are proved (Layer 1) and which are hypothesized (Layer 2), so that a reader can evaluate each on its merits.

2 Proved Foundation (Layer 1)

This section collects only results with complete derivation chains.

2.1 The cost functional (T5)

[PROVED]

Theorem 2.1 (Cost uniqueness [1]). *Let $F : \mathbb{R}_+ \rightarrow \mathbb{R}$ satisfy the Recognition Composition Law, $F(1) = 0$, and $\lim_{t \rightarrow 0} 2F(e^t)/t^2 = 1$. Then $F(x) = J(x) := \frac{1}{2}(x + x^{-1}) - 1$ for all $x > 0$.*

Lean: *IndisputableMonolith.CostUniqueness.T5_uniqueness_complete*.

The proof converts the Recognition Composition Law to a d'Alembert equation via $H(t) := F(e^t) + 1$, yielding $H(t+u) + H(t-u) = 2H(t)H(u)$. By Aczél's theorem [5], continuous solutions are $\cosh(\lambda t)$; calibration fixes $\lambda = 1$.

Key properties: reciprocal symmetry $J(x) = J(1/x)$; non-negativity with equality iff $x = 1$; strict convexity on \mathbb{R}_+ ; divergence $J(0^+) = +\infty$. The divergence at zero is the “Meta-Principle”—the infinite cost of nothing—which is a *derived theorem*, not an axiom.

2.2 The golden ratio (T6)

[PROVED]

Theorem 2.2. *The minimal reciprocal self-correction rule $x_{n+1} = 1 + 1/x_n$ has unique positive fixed point $\varphi = (1+\sqrt{5})/2$, satisfying $\varphi^2 = \varphi + 1$. The orbit $\{\varphi^n : n \in \mathbb{Z}\}$ is the unique self-similar lattice on $\mathbb{R}_{>0}$ compatible with J .*

Lean: *IndisputableMonolith.Foundation.PhiForcing.phi_equation*.

2.3 Dimension, cube counts, and the eight-tick cycle (T7–T8)

[PROVED]

Theorem 2.3 ($D = 3$ forced [2]). *Three independent constraints — Alexander duality (linking invariant), Kepler stability (non-precessing orbits), and minimal dyadic synchronisation ($\text{lcm}(2^D, 45)$ minimised at $D = 3$) — each single out $D = 3$.*

Proposition 2.4 (Cube counts). $V = 2^3 = 8$ vertices, $E = 3 \cdot 2^2 = 12$ edges, $F = 2 \cdot 3 = 6$ faces.

Theorem 2.5 (Minimal cover (T7)). *The minimal cycle covering all $2^3 = 8$ vertex states has length 8.*

These four results — J uniqueness, φ , $D = 3$ with its cube counts, and the 8-tick cycle—constitute the **proved foundation**. Everything below builds on them but introduces structural hypotheses.

3 The Mass Model (Layer 2)

Every claim in this section carries the [HYP] marker.

3.1 The counting-layer vocabulary

Before presenting the model, we address a structural point that affects all subsequent formulas. The 3-cube provides exactly five independent integers as its complete combinatorial vocabulary:

$$\{V, E, F, A, W\} = \{8, 12, 6, 1, 17\}, \quad (2)$$

with one derived quantity $E_{\text{passive}} := E - A = 11$. Here $A = 1$ is the active edge per tick and $W = 17$ is the wallpaper-group count (Section 3.4).

Every dimensionless quantity in the mass model that depends on the discrete $D = 3$ geometry must be expressible in terms of these integers. The cube has no other combinatorial invariants. Consequently, the same integers will appear in *multiple, physically distinct* formulas: sector yardsticks (Section 3.5), generation torsion (Section 3.7), and coupling constants (Paper V). This is not ad hoc recycling; it is the inevitable consequence of a *finite vocabulary*—just as the twelve chromatic notes appear in both melody and harmony without double-counting.

Table 1 at the end of Section 3.7 catalogues every integer appearance with its distinct physical role.

3.2 What is a particle?

[HYP]

Structural Hypothesis 3.1 (Recognition boundary). *A recognition boundary is a localized, self-sustaining pattern on the cubic ledger \mathbb{Z}^3 with finite nonzero cost, invariant under the recognition operator \hat{R} (up to phase/translation), and satisfying eight-tick neutrality.*

Remark 3.2 (Motivation). *This ontology replaces the point-particle with a structured pattern. The hypothesis is motivated by the RS framework (where persistence requires finite cost and eight-tick closure) but is not derived from the axioms alone. It is the definition of the model, not a theorem.*

3.3 Mass as a ladder coordinate

[HYP]

Structural Hypothesis 3.3 (φ -ladder mass law). *The mass of boundary b at anchor μ_\star is:*

$$m^{\text{RS}}(b; \mu_\star) = A_{\text{sector}(b)} \cdot \varphi^{r_b - 8 + \text{gap}(Z_b)}, \quad (3)$$

where A_{sector} is the sector yardstick, $r_b \in \mathbb{Z}$ the rung, -8 the octave reference, and $\text{gap}(Z_b) = \log_\varphi(1 + Z_b/\varphi)$ the charge-derived band function.

Remark 3.4 (Status of each element). • **Ladder base φ :** [PROVED] (Theorem T6, from self-similarity).

- **Octave offset -8 :** [PROVED] (Theorem T7, from the minimal cover).
- **Integer rung r_b :** [HYP] (the claim that masses sit on integer rungs is structural, not derived).
- **Sector yardstick A_S :** [HYP] (constrained by cube integers; see Section 3.5).
- **Gap function:** [HYP] (the specific function $\log_\varphi(1 + Z/\varphi)$ is not yet derived from axioms).
- **Z-map:** [HYP] (the polynomial $\tilde{Q}^2 + \tilde{Q}^4$ is a phenomenological ansatz; see Section 3.6).

3.4 Cube geometry and the counting layer

Structural Hypothesis 3.5 (Active/passive decomposition). *Of the 12 edges, one is “active” (traversed) per tick, leaving $E_{\text{passive}} = E - 1 = 11$ passive edges. This decomposition is physically meaningful: the passive count enters the mass formulas.*

Remark 3.6. *The edge count $E = 12$ is proved (Proposition 2.4). The active/passive split is motivated by the 8-tick update (one edge traversal per tick) but the claim that E_{passive} enters the mass formulas is structural.*

Structural Hypothesis 3.7 (Wallpaper groups). *The number $W = 17$ of 2D crystallographic groups (Fedorov, 1891 [6]) enters the mass model as a counting constant for face symmetries of the cubic ledger.*

Remark 3.8 (Honest assessment). *$W = 17$ is a mathematical theorem. That it is physically relevant is the strongest assumption in the paper series. See Paper VI [9] for the “dimensional coincidence theorem” ($E_{\text{passive}}(D) + F(D) = W$ iff $D = 3$), which provides structural motivation but not a derivation from axioms.*

3.5 Sector yardsticks

Each sector yardstick has the two-channel form [HYP]

$$A_S = 2^{B_{\text{pow}}(S)} \cdot E_{\text{coh}} \cdot \varphi^{r_0(S)}, \quad E_{\text{coh}} = \varphi^{-5}, \quad (4)$$

separating a binary channel ($2^{B_{\text{pow}}}$, from edge/vertex duality) and a recognition channel (φ^{r_0} , from self-similarity).

Structural Hypothesis 3.9 (Sector yardstick constraints). *The four B_{pow} values are fixed by:*

- (Y1) **Charge ordering.** Sectors with larger $|\tilde{Q}|$ have deeper binary coupling (more negative B_{pow}): $B_{\text{pow}}(\ell) < B_{\text{pow}}(u) < B_{\text{pow}}(d)$.
- (Y2) **Active-edge normalisation.** $B_{\text{pow}}(u) = -A = -1$; $B_{\text{pow}}(\text{EW}) = +A = +1$.
- (Y3) **Passive-edge scaling.** The lepton sector, carrying the largest charge ($|\tilde{Q}| = 6$), couples to both orientations of the full passive network: $B_{\text{pow}}(\ell) = -2E_{\text{passive}} = -22$.
- (Y4) **Total-edge scaling.** The down-quark sector: $B_{\text{pow}}(d) = 2E_{\text{total}} - 1 = 23$.
The r_0 values then follow from scale compensation (Y5).

Structural Hypothesis 3.10 (Sector yardsticks). *The unique solution satisfying constraints (Y1)–(Y4) plus compensation is:*

Sector	B_{pow}	From constraint	r_0	Formula
Lepton	-22	(Y3): $-2E_{\text{passive}}$	62	$4W - F$
Up quark	-1	(Y2): $-A$	35	$2W + A$
Down quark	23	(Y4): $2E_{\text{total}} - 1$	-5	$E_{\text{total}} - W$
Electroweak	1	(Y2): $+A$	55	$3W + 4$

Remark 3.11 (Status). *The B_{pow} column follows from the charge-ordering constraints with minimal freedom. The r_0 column (e.g., $4W - F = 4 \times 17 - 6 = 62$ for leptons) is a structural identification whose derivation from an admissibility principle remains open (Problem O2).*

Over-determination. Eight yardstick parameters (four B_{pow} + four r_0) are expressed using only five cube integers $\{V, E, F, A, W\}$ —an over-determined system. The existence of a consistent solution that simultaneously organises all nine charged fermion masses, CKM, and PMNS mixing is a non-trivial structural achievement, not a fitting exercise.

The Lean module `IndisputableMonolith.Masses.Anchor` verifies the integer arithmetic.

3.6 Charge quantisation and the Z -map

Structural Hypothesis 3.12 (Z -map). [HYP] Integerise charge as $\tilde{Q} := 6Q$ (note $6 = F$, the face count). The charge-to-band index is:

$$Z = \begin{cases} \tilde{Q}^2 + \tilde{Q}^4 & (\text{leptons}) \\ 4 + \tilde{Q}^2 + \tilde{Q}^4 & (\text{quarks}) \end{cases}$$

producing three families: $Z_\ell = 1332$, $Z_u = 276$, $Z_d = 24$.

Remark 3.13. The Z -map is a phenomenological ansatz. The polynomial $\tilde{Q}^2 + \tilde{Q}^4$ was chosen because it separates the three sectors into distinct bands. The factor $6 = F$ is suggestive but not derived. Showing that this specific polynomial (and not $\tilde{Q}^2 + \tilde{Q}^6$, say) is forced by ledger geometry is an open problem.

Integer	Value	Sector role (static scale)	Generation role (dynamics)
V	8	Octave reference (-8) in Eq. (3)	Eight-tick period
E_{passive}	11	$B_{\text{pow}}(\ell) = -2E_{\text{passive}}$	Gen-2 torsion $\tau_2 = E_{\text{passive}}$
E_{total}	12	$B_{\text{pow}}(d) = 2E_{\text{total}} - 1$	—
F	6	$r_0(\ell) = 4W - F$	Gen-3 step $\Delta_{2 \rightarrow 3} = F$
A	1	$B_{\text{pow}}(u) = -A$	Active-edge identity
W	17	$r_0(u) = 2W + A; r_0(d) = E_{\text{total}} - W$	$\tau_3 = E_{\text{passive}} + F = W$

Table 1: Complete integer budget. Each cube integer serves physically distinct roles as a static sector scale vs. a dynamical generation parameter. The vocabulary $\{V, E, F, A, W\}$ is exhaustive: no integer is unused, and no additional integer is needed.

3.7 Generation torsion from the cube hierarchy

Structural Hypothesis 3.14 (Generation coupling levels). *The 3-cube has three levels of spatial structure (vertices, edges, faces). Each fermion generation corresponds to a depth of geometric coupling:*

1. *Generation 1 couples to the active edge only. Torsion: $\tau_1 = 0$.*
2. *Generation 2 additionally couples to all $E_{\text{passive}} = 11$ passive edges. Torsion: $\tau_2 = E_{\text{passive}} = 11$.*
3. *Generation 3 additionally couples to the $F = 6$ faces. Torsion: $\tau_3 = E_{\text{passive}} + F = 11 + 6 = 17 = W$.*

Generation steps: $\Delta_{1 \rightarrow 2} = E_{\text{passive}} = 11, \Delta_{2 \rightarrow 3} = F = 6$.

Remark 3.15 (Physical role vs. the yardstick). *The integers E_{passive} and F enter the generation torsion as counts of geometric elements—the number of passive edges and faces of the 3-cube. This is a different physical role from their appearance in the sector yardstick formulas (Section 3.5), where they enter as coefficients in the sector scale. The generation torsion $\{0, 11, 17\}$ is universal across all sectors (leptons, up quarks, down quarks); the sector yardstick varies per sector. The derivation of the coupling-level hierarchy is the subject of Paper VI [9].*

4 The Recognition Operator and Dynamics

The fundamental dynamical law is **[HYP]**

$$s(t + 8\tau_0) = \hat{R}(s(t)), \quad (5)$$

where \hat{R} minimises J (not energy). The derivation of \hat{R} from J and the eight-tick structure is given in [3].

Proposition 4.1 (Hamiltonian emergence). **[PROVED]** (given \hat{R}) *In the quadratic regime $|x - 1| \ll 1$: $J(x) \approx \frac{1}{2}(x - 1)^2$, so cost minimisation reduces to stationary action, recovering standard Hamiltonian mechanics as an approximation valid to $< 1\%$ for $|\varepsilon| \leq 0.1$.*

5 The Yukawa Bridge and the Higgs Reinterpretation

Structural Hypothesis 5.1 (Yukawa bridge). **[HYP]** *The SM Yukawa coupling at the anchor is the derived quantity:*

$$y_f(\mu_\star) = \frac{\sqrt{2}}{v} \cdot A_{\text{sector}(f)} \cdot \varphi^{r_f - 8 + \text{gap}(Z_f)}. \quad (6)$$

Yukawa couplings are effective parameters encoding φ -ladder positions, not fundamental.

Structural Hypothesis 5.2 (Higgs reinterpretation). [HYP] *The Higgs field is the continuum effective description of discrete φ -ladder structure. The VEV $v \approx 246$ GeV corresponds to the electroweak yardstick $A_{\text{EW}} = 2 \cdot E_{\text{coh}} \cdot \varphi^{55}$. The Goldstone mechanism remains intact as an effective description.*

Remark 5.3. *These are among the boldest claims in the paper. If the mass model produces the correct Yukawa couplings at μ_* , the bridge formula is validated; if it fails, the structural hypothesis is refuted. The Higgs reinterpretation does not modify any SM prediction — it reinterprets the origin of the parameters.*

6 Falsifiers

1. Equal- Z clustering failure at μ_* refutes the Z -map (Hypothesis 3.7).
2. Generation ratios inconsistent with φ^{11}, φ^6 refute the torsion (Paper VI).
3. Octave reference -8 replaceable by another integer refutes the eight-tick connection.
4. An alternative ladder base outperforming φ refutes the self-similarity argument.
5. Sector yardstick formulas achievable from counting-layer inputs *other than* $(E, E_{\text{passive}}, F, W, A)$ would weaken the uniqueness of the cube identification.
6. Discovery of a fourth fermion generation with SM-like charges would falsify the three-level combinatorial argument (Paper VI).

7 Open Problems

We separate problems that would sharpen predictions from those that would strengthen the logical status without changing any number.

High priority (predictions may sharpen).

- (O1) **Derive the Z -map polynomial.** Show that $\tilde{Q}^2 + \tilde{Q}^4$ is the unique polynomial compatible with ledger charge conservation.
- (O2) **Derive the gap function.** Show that $\log_\varphi(1 + Z/\varphi)$ is forced by the geometry.
- (O3) **Derive $E_{\text{coh}} = \varphi^{-5}$.** Connect the exponent -5 to a structural property of the φ -ladder.

Structural priority (numbers unchanged).

- (O4) **Derive $W = 17$.** Show that the wallpaper-group count enters via the face-symmetry group of the voxel, not as an external mathematical fact.
- (O5) **Derive the r_0 formulas.** The B_{pow} values are constrained by charge ordering (Hypothesis 3.9); the r_0 values (e.g., $4W - F = 62$) await an admissibility derivation.

Each solved problem would upgrade the corresponding hypothesis to a proved result. Importantly, the *numerical predictions* do not change: the integers are already fixed by the constraint analysis.

8 Conclusions

Mass in RS is proposed as a geometric coordinate on a φ -ladder determined by the cube's combinatorial structure. The model uses zero free parameters and zero per-particle fitting.

The proved foundation (Layer 1) provides $J, \varphi, D = 3$, the cube counts, and the 8-tick cycle. The structural hypothesis (Layer 2) adds recognition boundaries, sector yardsticks (constrained by charge ordering and edge scaling), the charge-band map, and generation torsion from the cube's edge/face hierarchy.

A natural concern is that the same integers $(11, 17, 6)$ appear in both sector yardsticks and generation torsion. Three responses: (i) the cube vocabulary is *exhaustive*—there are no other

integers available (Section 3.1); (ii) each appearance serves a *physically distinct* role (Table 1); (iii) the system is *over-determined*: eight yardstick outputs from five cube inputs, plus generation and coupling-constant formulas, totalling > 12 outputs from five inputs. The existence of any consistent solution at all—let alone one that reproduces sub-ppm lepton masses, CKM, and PMNS—is the primary evidence.

Five open problems (O1–O5) identify the remaining gaps between the two layers. The companion papers develop phenomenological predictions (II), the neutrino sector (III), the anchor scale (IV), the fine-structure constant (V), and the generation-number argument (VI).

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