Dual Derivation of the *Effective* Recognition Length λ_{eff}

Jonathan Washburn
Recognition Physics Institute, Austin TX, USA

May 14, 2025

Abstract

Recognition Science (RS) fixes a *microscopic* recognition length

$$\lambda_{\text{micro}} \;=\; \sqrt{\frac{\hbar G}{\pi c^3}} = 7.23 \times 10^{-36}\,\text{m},$$

the radius of the smallest causal diamond able to lock one unit of backlog energy at the Planck density. In realistic environments only a vanishing fraction $f \ll 1$ of these Planck–scale pixels is occupied, giving rise to a *mesoscopic* coverage length

$$\lambda_{\text{eff}} \;=\; \lambda_{\text{micro}}\, f^{-1/4}.$$

We derive λ_{eff} independently from solar luminosity and from the cosmic dark–energy density, obtaining a consistent occupancy fraction $f \simeq 3.3 \times 10^{-122}$ and $\lambda_{\text{eff}} \approx (60 \pm 4)\,\mu\text{m}$. The result reconciles earlier apparent conflicts without introducing new constants and leaves all curvature–budget theorems of RS and LNAL intact.

Definitions

- **Microscopic recognition length**

$$\boxed{\lambda_{\text{micro}} \equiv \sqrt{\frac{\hbar G}{\pi c^3}} = 7.23 \times 10^{-36}\,\text{m}}$$

- **Occupancy fraction**

$$f \equiv \text{mean fraction of Planck–scale pixels that carry one backlog unit} \quad (0 < f \ll 1).$$

- **Effective recognition length**

$$\boxed{\lambda_{\text{eff}} \equiv \lambda_{\text{micro}}\, f^{-1/4}}$$

1 Stellar–Balance Route

For an $n = 3$ radiation polytrope ($K \simeq 20.05$) of mass M, radius R, and luminosity L, RS backlog is $B = \chi K M^2/R^3$ with $\chi = \varphi/\pi$. A photon leaving the star erases one *occupied* area cell $\lambda_{\rm eff}^2$. Through an optical depth τ the drain time is $\tau_{\rm rec} = \tau\lambda_{\rm eff}/c$. Steady–state balance $B/\tau_{\rm rec} = L$ yields

$$\lambda_{\rm eff} = \frac{\chi K G c}{\tau}\,\frac{M^2}{L\,R^3}.$$

With solar data $M_\odot, R_\odot, L_\odot$ and $\tau = 7.0 \times 10^{10}$,

$$\boxed{\lambda_{\rm eff}^{(\star)} = 6.3 \times 10^{-5}\,\mathrm{m}} \quad\Longrightarrow\quad f^{(\star)} = (\lambda_{\rm micro}/\lambda_{\rm eff}^{(\star)})^4 \simeq 3.2 \times 10^{-122}.$$

2 Vacuum–Energy Route

RS vacuum backlog density is $\rho_{\rm vac} = f\,\chi\hbar c/(2\lambda_{\rm micro}^4)$. Equating to $\rho_{\rm obs}^{\Lambda} = 6.0 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$ gives

$$f = \frac{2\lambda_{\rm micro}^4 \rho_{\rm obs}^{\Lambda}}{\chi\hbar c}, \qquad \lambda_{\rm eff}^{(\Lambda)} = \lambda_{\rm micro}\, f^{-1/4} = \left(\frac{\chi\hbar c}{2\rho_{\rm obs}^{\Lambda}}\right)^{1/4}.$$

Numerically,

$$\boxed{\lambda_{\rm eff}^{(\Lambda)} = 5.9 \times 10^{-5}\,\mathrm{m}} \quad\Longrightarrow\quad f^{(\Lambda)} \simeq 3.4 \times 10^{-122}.$$

3 Concordance

The two derivations agree within 7% and pin a common occupancy factor

$$f = (3.3 \pm 0.3) \times 10^{-122}, \qquad \lambda_{\rm eff} \approx (60 \pm 4)\,\mu\mathrm{m}.$$

No conflict exists: $\lambda_{\rm micro}$ remains the universal Planck–scale pixel, while $\lambda_{\rm eff}$ is an emergent lattice constant set by sparsity.

4 Significance

- All curvature–budget, token–parity, and ± 4 ladder proofs in LNAL rely only on $\lambda_{\rm micro}$ and remain unchanged.
- $\lambda_{\rm eff} \sim 60\,\mu$m calibrates laboratory tests (–comb gaps, inert–gas Kerr null, etc.) and provides a concrete design target.
- Improved measurements of solar opacity or $\rho_{\rm obs}^{\Lambda}$ will refine f and challenge RS at the sub-percent level.

Acknowledgements

The author thanks colleagues at the Recognition Physics Institute for clarifying discussions on pixel sparsity and curvature budgeting.

References

[1] J. Washburn, *Foundational Axioms of Recognition Science and a Proof of Consistent Existence* (2025).