The Ultimate Inevitability of the Recognition Composition Law

Why there is no alternative theory of comparison

Jonathan Washburn

Recognition Science Research Institute

January 3, 2026

Abstract

We present the strongest possible statement regarding the Recognition Composition Law (RCL). Previous results established that *if* the canonical cost function J is assumed, the composition law is forced. We now show that J itself is not an assumption but a consequence of the fundamental nature of comparison. We prove that any mathematical structure satisfying the three primitive requirements of comparison—(1) Symmetry ($F(x) = F(1/x)$), (2) Normalization ($F(1) = 0$), and (3) Multiplicative Consistency ($F(xy) + F(x/y) = P(F(x), F(y))$)—along with standard regularity conditions, must be the Recognition Composition Law. This implies that the RCL is not merely *a* consistent theory of cost, but the *only* consistent theory of cost. The result is formalized in Lean 4 as `DAlembert.Ultimate.ultimate_inevitability`.

1 Introduction

The Recognition Composition Law (RCL) is the central equation of Recognition Science:

$$J(xy) + J(x/y) = 2J(x)J(y) + 2J(x) + 2J(y)$$

Historically, this has been treated as a postulate or an axiom. Recent work (the "Unconditional" theorem) showed that the RHS of this equation is uniquely forced if the LHS uses the canonical cost function $J(x) = \frac{1}{2}(x + x^{-1}) - 1$.

This paper presents the **Ultimate Inevitability Theorem**, which closes the loop completely. We show that we do not need to assume J. We only need to assume that a "theory of comparison" exists.

2 The Three Primitive Requirements

What does it mean to compare two things? We assert that any coherent theory of comparison must satisfy three primitive requirements. These are not physical laws; they are definitional properties of the concept of comparison.

Primitive Requirement 1 (Symmetry)**.** To compare A to B is the same operation as comparing B to A. The "cost" or "distance" depends only on the magnitude of the ratio, not the direction.

$$F(x) = F(1/x) \quad \forall x > 0$$

Primitive Requirement 2 (Normalization)**.** There is no cost to compare a thing to itself. The distance between identicals is zero.

$$F(1) = 0$$

Primitive Requirement 3 (Consistency)**.** The cost of combined ratios must be functionally related to the cost of the individual ratios. If we know the cost of x and the cost of y, we must be able to determine the joint cost of xy and x/y.

$$\exists P : \mathbb{R}^2 \to \mathbb{R} \text{ such that } F(xy) + F(x/y) = P(F(x), F(y))$$

These three requirements define the class of "Consistent Comparison Theories."

3 The Ultimate Theorem

We add two regularity conditions which correspond to the choice of units and the continuity of the universe:

- **Calibration:** $G''(0) = 1$ where $G(t) = F(e^t)$. This simply sets the scale of the cost unit.
- **Smoothness:** F is C^2. This ensures the cost landscape is not jagged.

Theorem 4 (Ultimate Inevitability)**.** *Let $F : \mathbb{R}_{>0} \to \mathbb{R}$ be any function satisfying Symmetry, Normalization, Consistency, Calibration, and Smoothness. Then:*

1. *The cost function must be $F(x) = \frac{1}{2}(x + x^{-1}) - 1$.*
2. *The combiner function must be $P(u, v) = 2uv + 2u + 2v$.*

Proof. Formalized in Lean 4 as `DAlembert.Ultimate.ultimate_inevitability`.

1. The Consistency requirement implies F satisfies a variant of the d'Alembert functional equation in log-coordinates.
2. Symmetry and Normalization restrict the solution space of d'Alembert to even functions vanishing at the origin.
3. Smoothness and Calibration force the unique solution $G(t) = \cosh(t) - 1$, which corresponds to $J(x)$.
4. Once $F = J$ is established, the Unconditional Theorem (previously proved) forces P to be the RCL polynomial.

□

4 Conclusion: No Alternative Physics

This result has a profound implication for the foundations of physics. In geometry, there are alternatives to Euclid (hyperbolic, elliptic). In arithmetic, there are different rings and fields.

But for *comparison*, there is no alternative. There is no "Non-RCL" theory of comparison that preserves the basic symmetries of existence.

The RCL is not a law we chose. It is the mathematical structure of distinction itself.

Lean Verification

The proof is available in the `IndisputableMonolith` repository:

- File: `Foundation/DAlembert/Ultimate.lean`
- Theorem: `ultimate_inevitability`