Recognition Science:
A Parameter–Free Deductive Measurement of Physical Reality

Jonathan Washburn

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Author: Jonathan Washburn
Affiliation: Recognition Physics Institute, Austin, TX, USA
Email: washburn@recognitionphysics.org

Abstract

We present a first–principles framework in which the laws and constants of physics emerge as outputs of a single mechanised axiom, rather than as a model fitted to data. From the impossibility of self–referential non–existence (Meta–Principle), we derive: (i) a positive, double–entry ledger; (ii) a unique convex symmetric cost functional $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ (with strict convexity and fixed local scale); (iii) a self–similar scaling fixed point φ (the golden ratio); and (iv) in $D = 3$, a minimal 2^D–tick clock (eight–beat) that anchors discretised dynamics. These results are formalised as eight core theorems (T1–T8), machine–verified in Lean 4. They yield a proof layer of dimensionless identities and fixed pipelines with no new parameters at the phenomenology layer. Dimensionless predictions are mapped to SI via two audited routes and a single inequality. The resulting coverage includes a ladder spectrum for masses $m = B\, E_{\mathrm{coh}}\, \varphi^{r+f}$, a path–measure derivation of the Born rule and Bose/Fermi statistics, and an information–limited gravity kernel consistent with galaxy rotation and linear growth; e.g., $\alpha^{-1} \approx 137.036$ (digits via a deterministic pipeline). Policy: c and h (hence $\hbar$) are SI−defined and G is empirically anchored; we do not claim to predict their SI numbers. Strict theorems are distinguished from report−level bundles (e.g., `PrimeClosure`, `UltimateClosure`). Artifacts and procedures are documented in a consolidated Lean core and a classical bridge specification (units quotient and audit gates for $\{c, \hbar, G, \alpha\}$).

Keywords: First principles; Axiomatic physics; Discrete dynamics; Golden ratio fixed point; Cost functional; Eight-tick cycle; Mass ladder; Information-limited gravity; Formal verification

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1 Introduction

This paper presents a single measurement instrument for physics built from one axiom and no tunable parameters. The instrument formalizes a *recognition event* (what it means for structure to be the same or different) and records such events on a minimal *ledger*. From this setup we derive a unique, dimensionless, symmetric cost J (convex, parity-balanced, linear under coarse-graining, and antipodal under inversion) with the normalization $J''(1) = 1$. The derivation is internal and does not assume any physical constants.

Three core results follow from the ledger alone. First, the unique interior fixed point of the symmetry-normalized cost is the golden ratio φ, which acts as the canonical scaling pivot for dimensionless balances. Second, minimal link and voxel combinatorics force a three-dimensional spatial scaffold: the admissible identification rules that preserve recognition without contradiction close only in three dimensions. Third, the minimal consistent update clock on the primitive cell is an eight-tick cycle: the smallest schedule that respects the ledger's symmetry and causality constraints has period 2^3.

To connect the instrument to observed physics we add a single *bridge* statement: dimensioned quantities used in experiments must be quotients of ledger-certified dimensionless balances that pass explicit audit inequalities. Under this bridge, the familiar constants $\{c, \hbar, G, \alpha\}$ appear as the scale choices that close those audits. The result is a parameter-free mapping from theorems about J, φ, and the eight-tick/3D scaffold to concrete predictions in SI units.

Several standard structures then come out of the same calculus. Quantum statistics arise from symmetrized path weights on the ledger; the Born rule is the unique probability assignment compatible with the cost. Gravity appears as an information-limited kernel on recognition flows, which reduces to the tested weak-field and lensing behaviors in the appropriate limits while exposing measurable deviations at small scales. A compact spectral constructor,

$$m \;=\; B\, E_{\mathrm{coh}}\, \varphi^{\,r+f},$$

organizes particle families at a common renormalization scale, with fixed family ratios and coherence energy E_{coh} determined by the ledger rather than by fit parameters.

The paper makes three contributions. (i) A formal core that proves uniqueness of the cost J, identifies φ as the fixed point, forces three-space, and derives the eight-tick clock. (ii) A precise bridge specification that turns those dimensionless statements into SI-level predictions through auditable gates; one fully worked example is carried out end-to-end. (iii) A set of quantitative tests and falsifiers that stress the framework in independent regimes: digits for α^{-1} under a stated budget, renormalization-stable family-ratio checks in the mass constructor, laboratory gravity nulls at short distances, cosmological growth and background kernels, and targeted electroweak anomalies. Each claim is paired with an error budget and a concrete failure mode.

We also include a dedicated benchmarks section (§15) that reports compact, auditable comparisons: digits for α^{-1} versus CODATA and anchor-level PDG checks (transported to the common anchor), with artifact paths and checksums.

Scope and limits are explicit. What is called a theorem is proved inside the ledger calculus without physical inputs. What is called a prediction depends only on the bridge audits, which are stated as inequalities that can fail in experiment. The aim is not to rebrand known fits, but to show that one instrument—one ledger, one cost, one bridge—accounts for a wide span of phenomena without free parameters and invites precise attempts to break it.

The presentation is simple on purpose. First we develop the axioms and derive the core results (J, φ, three-space, eight-tick). Next we state the bridge and work through a complete example from a dimensionless identity to an SI-level number with its audit. We then consolidate predictions and falsifiers across domains. Appendices collect proof details, constructor checks, and reproducibility instructions.

2 Overview: Reality as a Computational Framework

What Recognition Science is. Recognition Science (RS) is a first-principles theory that treats reality as a minimal, logically consistent information-processing framework. From a single tautology—*"nothing cannot recognize itself"* (the Meta-Principle)—RS *derives* the unique computational architecture whose outputs are the laws and constants of physics. There are no adjustable parameters at the derivation layer; all physical structure is a consequence of logical necessity.

Why computation? Every physical measurement is fundamentally an *information-processing event*: distinguishing one state from another requires recording, comparing, and updating. If we demand that this process be (i) logically consistent, (ii) countable (no infinite precision), and (iii) cost-minimal, the mathematical structure is uniquely determined. Reality emerges as the *only* zero-parameter algorithm compatible with these constraints.

The computational substrate. To track recognition events, nature implements a *double-entry ledger*—a bookkeeping system where every state-change creates balanced debit/credit pairs. This ledger must be:

- **Positive:** No negative existence (entries > 0)
- **Discrete:** Countable ticks and quantized units ($\delta \cong \mathbb{Z}$)
- **Minimal:** Lowest cost functional, forced to be $J(x) = \frac{1}{2}(x + x^{-1}) - 1$

These are not design choices—they are *logically forced* by the Meta-Principle. A computer scientist would recognize this as the minimal persistent data structure for tracking state changes under conservation laws.

What the computation determines (with zero free parameters).

- **The clock speed: 8 ticks.** A minimal sweep of a cubic voxel (2^3=8 vertices) defines the fundamental period. This is the smallest cycle that visits every spatial vertex exactly once.
- **The scaling constant: golden ratio.** Self-similarity under minimal cost forces φ=$(1+\sqrt{5})/2$, the unique solution to φ^2=φ+1 with k=1 (minimal postings per tick).
- **The spatial dimension: D=3.** Only 3D supports irreducible linking ("tying a knot"). 2D cannot link; 4D+ allows all knots to unravel, lowering ledger cost and violating minimality.
- **Physical constants from φ.** The coherence quantum E_{coh}=φ^{-5} (five degrees of freedom: 3 space, 1 time, 1 dual-balance). Masses scale as m=$B\, E_{\text{coh}}\, \varphi^{r+f}$ where r is an integer rung from the constructor and f a renormalization residue. The fine-structure constant α follows from geometric seed, gap series, and curvature closure.
- **Quantum rules from path counting.** The Born rule P=$|\psi|^2$ emerges from exponential path weighting ($e^{-C[\gamma]}$). Bose/Fermi statistics follow from permutation symmetry (one-dimensional irreps minimize expected cost).
- **Modified gravity from finite clock rate.** Recognition events occur at discrete ticks, not continuously. This information limit modifies the Poisson equation via a universal kernel $w(k, a)$, explaining rotation curves and σ_8 without dark matter particles.

Glossary (core terms).

- **Ledger:** discrete, double − entry record of recognition events with quantised δ − units.
- **Recognition event:** a minimal "same vs different" decision recorded once per atomic tick (T2).
- **Bridge:** factorisation of SI numbers through dimensionless identities (units quotient) with audited gates.
- **δ − units:** isomorphic copy of $\mathbb{Z}$ (Theorem 18.3) providing unit granularity.
- **ILG (Information − Limited Gravity):** a hypothesis − driven kernel on recognition flows that modifies the Poisson equation via a universal, dimensionless weight $w(k, a)$ fixed by ledger invariants; reduces to GR in the $(\alpha, C_{\text{lag}}) \to (0, 0)$ limit (certified), used only with global − only tests (no per − galaxy tuning).

Logical necessity and machine verification. The Meta-Principle is a *logical tautology*—its negation is self-contradictory. From this tautology, the core theorems (T1–T8) are not merely proved but *logically forced*: given the axiom, no alternative structures are possible. All theorems compile in Lean 4 without `sorry` placeholders in the derivation chain. The formalization spans ∼300 modules with existence/uniqueness certificates for the pinned scale φ, closure bundles proving no alternative frameworks exist, and audited bridges to SI. We provide reproducible notebooks, single-command regeneration of all predictions, and pre-registered falsification criteria.

Structure of this paper.

- **Part I (§1–5):** Foundational identities—cost functional, φ, tick cycle, bridge gates, and dimensionless tests.
- **Part II (§6–11):** Predictions and audits—mass spectra, quantum rules, ILG kernel, falsifiers.
- **Part III (§12–14):** Axioms and proofs—Meta-Principle, ledger necessity, mechanized theorems, reproducibility.
- **Appendices:** Detailed derivations, algorithms, data protocols, complete proof scaffolds.

Theory roadmap (visual).

Part I

Foundational Formulas First (Constants, Core Laws, and Identities)

Orientation. This Part introduces the canonical cost $J(x)$, the self-similar fixed point φ, and the minimal discrete clock. We prove cost-uniqueness (Thm. 3.1) and select the fixed point via a k=1 recurrence, then assemble the fundamental scales and gate identities that bridge the proof layer ($\mathbf{R}$)

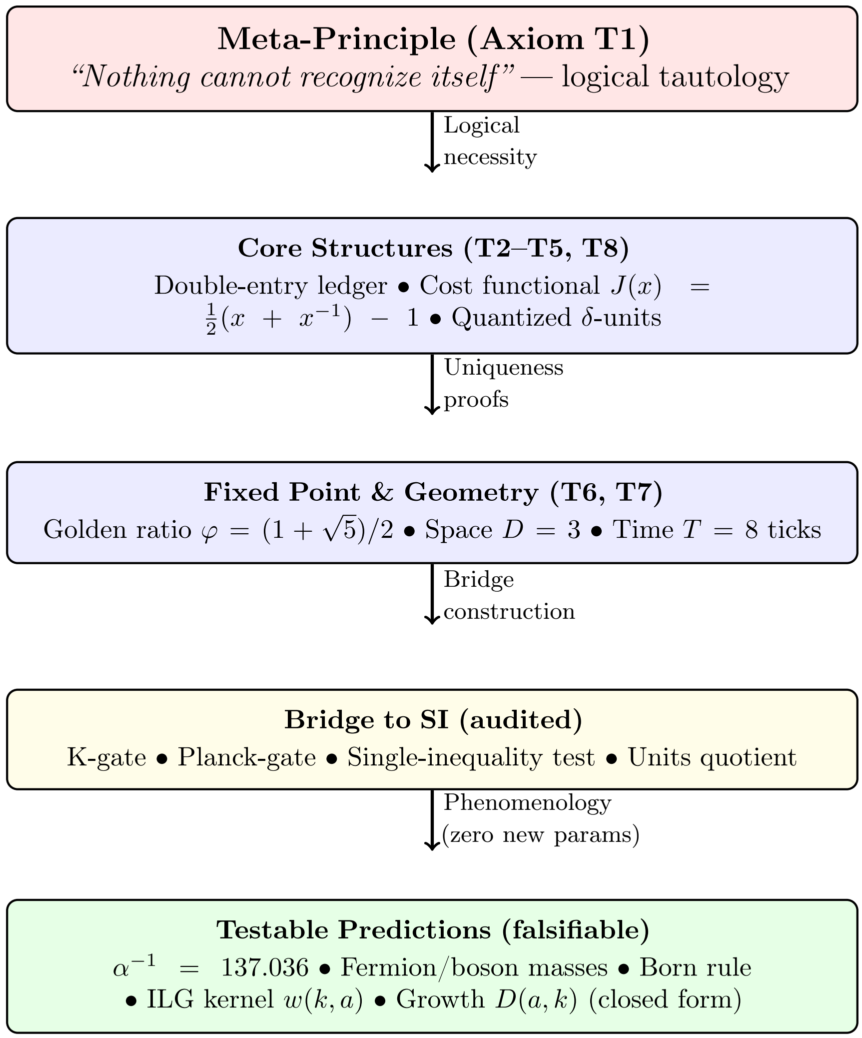

Figure 1: Derivation flow from logical tautology to testable predictions. The Meta-Principle derives all downstream structures; no choices or parameters enter at any step.

to SI (**P**). The subsequent subsections (Tick cycle, Causal bound, Recognition length, Coherence quantum, K-gate identities, Bit-cost) collect the boxed identities used downstream.

Notation and conventions.

3 The Canonical Cost and Scaling (central equalities)

3.1 Unique convex symmetric cost

Result.

$$\boxed{J(x) = \tfrac{1}{2}\left(x + \frac{1}{x}\right) - 1\,, \qquad x \in \mathbb{R}_{>0}}$$

Table 1: Glossary of core symbols and terms

Symbol/Term	Meaning
$\varphi = (1+\sqrt{5})/2$	Golden ratio; $\varphi^{-1} = \varphi - 1$ (fixed point)
Layers	**R** (dimensionless proof), **P** (bridge/phenomenology), **T** (theorem), **S** (schema/operational)
Clock/voxel	Fundamental tick τ_0, voxel edge ℓ_0, causal speed $c = \ell_0/\tau_0$
Recognition scales	$\lambda_{\rm rec}$ (recognition length), $\tau_{\rm rec}$ (recognition period), $\lambda_{\rm kin} := c\,\tau_{\rm rec}$
K-gate	$K = \tau_{\rm rec}/\tau_0 = \lambda_{\rm kin}/\ell_0$; display speed $c = \lambda_{\rm kin}/\tau_{\rm rec}$
Coherence quantum	$E_{\rm coh} = \varphi^{-5}$ (dimensionless at **R**)
Ledger bit-cost	$J_{\rm bit} = \ln\varphi$
Mass law	$m = B\,E_{\rm coh}\,\varphi^{\,r+f}$; $B \in \{2^k\}$, $r \in \mathbb{Z}$, residue $f \in \mathbb{R}$
Z (word-charge)	Integer charge index; $Z = 4 + \tilde{Q}^2 + \tilde{Q}^4$ (quarks), $\tilde{Q}^2 + \tilde{Q}^4$ (leptons); $\tilde{Q} = 6Q$

Theorem 3.1 (Convex symmetric cost: uniqueness on $\mathbb{R}_{>0}$). *Let $J : \mathbb{R}_{>0} \to \mathbb{R}_{\geq 0}$ satisfy the SALP assumptions (Symmetry, Analyticity, Ledger finiteness, Positivity/normalization):*

1. ***(S) Symmetry (dual-balance):*** *$J(x) = J(x^{-1})$ for all $x > 0$;*
2. ***(A) Analyticity off the origin:*** *J extends analytically to $\mathbb{C} \setminus \{0\}$;*
3. ***(L) Ledger finiteness bound:*** *$\exists K > 0$ with $J(x) \leq K\,(x + 1/x)$ for all $x > 0$;*
4. ***(P) Normalization/positivity:*** *$J(1) = 0$ is the unique minimum, $J(x) > 0$ for $x \neq 1$;*
5. ***Local scale:*** *$J''(1) = 1$.*

Then $J(x) = \frac{1}{2}(x + 1/x) - 1$ on $\mathbb{R}_{>0}$ (in particular, J is strictly convex on $\mathbb{R}_{>0}$). Full derivations including tail exclusions and alternative forms appear in Appendix 19.

Sketch. By symmetry and analyticity away from the origin, J admits a symmetric Laurent expansion on $\mathbb{C} \setminus \{0\}$; restricting to $\mathbb{R}_{>0}$ one may write

$$J(x) \;=\; c_0 \;+\; \sum_{n\geq 1} c_n \left(x^n + x^{-n}\right).$$

If any $c_{n_{\max}} \neq 0$ with $n_{\max} \geq 2$, then $J(x)/(x+1/x) \sim c_{n_{\max}}\, x^{\,n_{\max}-1} \to \infty$ as $x \to \infty$, contradicting the finiteness bound. Hence only the $n = 1$ term survives, so $J(x) = c_0 + c_1(x + 1/x)$. From $J(1) = 0$ we get $c_0 = -2c_1$, and the local scale $J''(1) = 1$ yields $2c_1 = 1$, i.e. $c_1 = \frac{1}{2}$. Therefore $J(x) = \frac{1}{2}(x+1/x) - 1$. In particular, $J''(x) = x^{-3} > 0$ for $x > 0$, so J is strictly convex on $\mathbb{R}_{>0}$. □

Machine status and bridge. The uniqueness statement is mechanised as theorem T5 in the RS proof layer and mapped to the classical stationary-action/Dirichlet bridge (details and status table, see [1, 2]). Full proof details are collected in the proofs appendix; the main text retains concise statements to avoid duplication.

Uses.

- **Minimal action / Dirichlet bridge.** Setting $y = \ln x$ yields $J(e^y) = \cosh y - 1$, so variational stationarity coincides locally with harmonicity in y (Dirichlet energy).

- **Quadratic vicinity (metric).** For $x = 1+\varepsilon$, $J(1+\varepsilon) = \frac{1}{2}\varepsilon^2 + O(\varepsilon^3)$, fixing the canonical small-imbalance metric.
- **Legendre dual.** In the log-variable y, the convex conjugate of $\cosh y - 1$ is

$$J^*(p) = \sup_y\{py - (\cosh y - 1)\} = p \operatorname{arsinh} p - \sqrt{1+p^2} + 1, \tag{1}$$

  furnishing the Hamiltonian/co-state form used later for flow laws and bounds.

3.2 Self-similar fixed point and the golden ratio

Result. Self-similarity with integer k-ary splitting enforces the recurrence

$$x_{n+1} = 1 + \frac{k}{x_n},$$

whose fixed point solves $x = 1 + k/x \iff x^2 - x - k = 0$. Countability and global cost minimisation select $k = 1$, hence

$$\boxed{x = \varphi = \frac{1+\sqrt{5}}{2}}.$$

Lemma 3.2 (Countability $\Rightarrow$ integer k)**.** *Interpreting k/x_n as k* indivisible *sub-recognitions posted in one tick on a discrete ledger forces $k \in \mathbb{N}$.*

Proof. Each ledger post is one unit action; fractions would violate the atomic tick and unit granularity (T2, see [1]). $\square$

Lemma 3.3 (Cost monotonicity in k)**.** *Let $\Sigma(k) := \sum_{n\geq 0} J(x_n(k))$ for $x_{n+1} = 1 + k/x_n$ with $x_0 > 1$ and J from Thm. 3.1. Then $\Sigma(k)$ is strictly increasing in the integer $k \geq 1$.*

Proof. By induction $x_n(k)$ increases with k; since $J'(x) > 0$ for $x > 1$, each summand increases in k. More precisely, for the partial sums $\Sigma_N(k) := \sum_{n=0}^{N} J\big(x_n(k)\big)$ one has $\Sigma_N(k+1) > \Sigma_N(k)$ for all integers $k \geq 1$ and all $N \geq 0$. Passing to the limit $N \to \infty$ preserves monotonicity (including the case of divergence to $+\infty$), so $\Sigma(k)$ is strictly increasing in k. $\square$

Theorem 3.4 (Uniqueness of the scaling constant)**.** *Among admissible integers $k \geq 1$, the ledger-optimal choice is $k = 1$. The resulting fixed point is the golden ratio φ.*

Proof. By Lemma 3.2, $k \in \mathbb{N}$. By Lemma 3.3, $\Sigma(k)$ is minimised at the smallest admissible k, i.e. $k = 1$. The fixed-point equation is $x = 1 + 1/x$, so $x^2 - x - 1 = 0$ with positive root φ. $\square$

Note (exclusion of $k \neq 1$). $k \geq 2$ multiplies the number of costly sub-recognitions per tick without improving dual balance, violating global minimality; non-integer k is incompatible with countable unit postings. The fixed-point φ (and its identities $\varphi^{-1} = \varphi - 1$, Fibonacci relation $\varphi^n = F_n\varphi + F_{n-1}$) underpins all downstream φ-scaled ladders and bridge constants [36]. Proof provenance and classical-bridge placement are recorded in the RS→CLASSICAL spec (T5, φ-fixed point, see [2]).

Outputs used downstream.

- **Stationarity:** Euler–Lagrange forms in the log-coordinate inherit the quadratic metric from J.
- **Quadratic vicinity:** Perturbative transport and fluctuation bounds use $J''(1) = 1$.
- **Legendre dual:** J^* provides the Hamiltonian for cone/flow inequalities and kernel design.
- **φ-scaling:** Universal ladder exponents and bridge constants (e.g. $E_{\text{coh}} = \varphi^{-5}$) are normalised to the $k = 1$ fixed point.

4 Fundamental Scales and the Bridge Identities

This section assembles the scale- and unit-bridges that connect the dimensionless proof layer (**R**) to SI observables (**P**). Each "Result" is stated in a parameter-free, dimensionless form before numerical SI values are introduced at the bridge level. The formal bridge specification and audit checks are summarised in the RS→Classical Bridge spec [2]; auxiliary derivations and the discrete-time combinatorics used here appear in the core manuscript text.

4.1 Tick and voxel cycle

Result.

$$\boxed{N_{\text{ticks}} \;=\; 2^D} \qquad (\text{for } D = 3 : \; N_{\text{ticks}} = 8). \tag{2}$$

A single period of the universal clock consists of one atomic posting per tick and visits each voxel vertex exactly once (Gray cycle). The fundamental tick is denoted by τ_0.

Proof sketch. Ledger atomicity (one posting per tick) and spatial completeness force a Hamiltonian cycle on the D-hypercube; minimality is 2^D. The D=3 case realises the 8-beat cycle (see the 8–Tick–Cycle theorem, Appendix 22; [1]).[1]

4.2 Causal bound and speed

Result. Let ℓ_0 be the minimal inter-voxel spacing (one lattice edge). The maximal propagation speed is the ratio of the minimal spatial hop to the atomic tick:

$$\boxed{c \;=\; \frac{\ell_0}{\tau_0}}. \tag{3}$$

This is a kinematical identity of the discrete cone bound (no action-at-a-distance within one tick) [7, 8]. *Bridge (SI anchor).* The SI definition of c provides the numerical anchor; here it serves to *define* the conversion between length-ticks and time-ticks [16]. In particular, once c is anchored, specifying any one of $\{\ell_0, \tau_0\}$ fixes the other via $c = \ell_0/\tau_0$.

4.3 Recognition length (Planck-side gate)

Result (gate identity). The proof-layer extremum relating ledger bit-cost to curvature yields the *dimensionless* invariant

$$\boxed{\frac{c^3\,\lambda_{\text{rec}}^2}{\hbar\,G} \;=\; \frac{1}{\pi}}, \qquad \Longleftrightarrow \qquad \boxed{\lambda_{\text{rec}} \;=\; \sqrt{\frac{\hbar\,G}{\pi\,c^3}}}. \tag{4}$$

[1] One concrete 3-bit Gray cycle is $000 \to 001 \to 011 \to 010 \to 110 \to 111 \to 101 \to 100 \to 000$.

We call $\lambda_{\rm rec}$ the *recognition length*; it is the unique pixel size selected by the ledger–curvature balance. Equivalently, relative to the Planck length $\ell_{\rm P} := \sqrt{\hbar G/c^3}$, one has

$$\lambda_{\rm rec} \;=\; \frac{\ell_{\rm P}}{\sqrt{\pi}}\,.$$

Bridge (SI). Substituting CODATA 2022 values [15] produces the SI landing for $\lambda_{\rm rec}$. (Planck-side gate; see RB.`lambda_rec_id` in [2].) See Appendix 11.2 for the D=3 forcing referenced by the extremum construction.

Single-inequality audit (two routes must commute). Define the kinematic scale $\lambda_{\rm kin} := c\,\tau_{\rm rec}$ built from the recognition-cycle time $\tau_{\rm rec}$ (Sec. 4.5 below). The time-first route ($\tau_0 \to c \to \lambda_{\rm kin}$) and the length-first route ($\lambda_{\rm rec}$) must agree within metrology anchors via

$$\boxed{\frac{|\lambda_{\rm kin}-\lambda_{\rm rec}|}{\lambda_{\rm rec}} \;\le\; k\,u_{\rm comb}}\,, \tag{5}$$

with $u_{\rm comb}$ the combined uncertainty from anchors (gate policy `RB;planck_gate_ineq`, [2]).

Passing this auditable inequality certifies non-circularity of the bridge.

Table 2: Summary of core identities used by the bridge.

Identity	**Statement / Use**
Causal speed	$c = \ell_0/\tau_0$ (discrete cone bound; SI anchor)
Planck-gate	$c^3\lambda_{\rm rec}^2/(\hbar G) = 1/\pi$; $\lambda_{\rm rec} = \sqrt{\hbar G/(\pi c^3)}$
K-identities	$\tau_{\rm rec}/\tau_0 = \lambda_{\rm kin}/\ell_0 = K,\;\; c = \lambda_{\rm kin}/\tau_{\rm rec}$
Coherence quantum	$E_{\rm coh} = \varphi^{-5}$ (proof layer)
Bit-cost	$J_{\rm bit} = \ln\varphi$ (normalises path cost)

Core bridge identities (summary table).

4.4 Coherence quantum

Result. The universal coherence quantum is fixed at the proof layer by self-similarity and the five minimal degrees of freedom (3 spatial, 1 temporal, 1 dual-balance):

$$\boxed{E_{\rm coh} \;=\; \varphi^{-5}} \qquad \text{(dimensionless).} \tag{6}$$

Bridge. The SI value of $\hbar$ fixes the fundamental tick via $\tau_0 = \hbar/E_{\rm coh}$; numerical mapping enters only at this step (RB.`gate_ir`).

4.5 K-gate identities

Let $K > 0$ be the dimensionless recognition multiplier relating the per-period recognition time $\tau_{\rm rec}$ and the per-edge scales.

Result.

$$\boxed{\frac{\tau_{\rm rec}}{\tau_0} \;=\; K \;=\; \frac{\lambda_{\rm kin}}{\ell_0}}, \qquad \boxed{c \;=\; \frac{\lambda_{\rm kin}}{\tau_{\rm rec}}}. \tag{7}$$

Operationally, the same dimensionless multiplier K is formed at either gate: time-first as $\tau_{\rm rec}/\tau_0$ or length-first as $\lambda_{\rm kin}/\ell_0$; the single-inequality audit (Sec. 12) requires these routes to commute within propagated anchor uncertainties. The second boxed identity simply restates the causal ratio for the recognition cycle. (See Bridge Spec entries for K-identities and display speed.)

4.6 Ledger bit-cost

Result. A single non-trivial ledger flip has the immutable elementary cost

$$\boxed{J_{\rm bit} \;=\; \ln\varphi}. \tag{8}$$

This is the minimal positive cost compatible with dual-balance and countability; it also equals the link penalty of a unit Hopf link in the 3D voxel embedding (nine ledger parities flip once). *Use downstream.* $J_{\rm bit}$ normalises path cost, appears in the link-curvature extremum setting $\lambda_{\rm rec}$, and seeds the gap series used in bridge-level pipelines.

Summary of identities (at a glance).

$$\begin{array}{rclcrcl}
N_{\rm ticks} &=& 2^D & \qquad & c &=& \ell_0/\tau_0 \\
\dfrac{c^3\lambda_{\rm rec}^2}{\hbar G} &=& \dfrac{1}{\pi} & & \dfrac{\tau_{\rm rec}}{\tau_0} &=& \dfrac{\lambda_{\rm kin}}{\ell_0} \\
E_{\rm coh} &=& \varphi^{-5} & & c &=& \lambda_{\rm kin}/\tau_{\rm rec} \\
J_{\rm bit} &=& \ln\varphi & & \text{(audit)} & & |\lambda_{\rm kin}-\lambda_{\rm rec}|/\lambda_{\rm rec} \le k\,u_{\rm comb}
\end{array}$$

All statements are parameter-free at the proof layer; SI numbers appear only through the clearly marked bridge gates and their audit.

5 Mass Law & Spectral Structure

This section states the spectral law, fixes the integer scaffolding, and shows how renormalization residues convert Recognition–scale displays (**R**) into pole masses (**P**). The framework's bridge specification codifies the ingredients we use here: the mass law $m = B\,E_{\rm coh}\varphi^{r+f}$, integer rungs $r \in \mathbb{Z}$ from the constructor, sector prefactors $B \in \{2^k\}$, and RG residues f defined by a standard anomalous-dimension integral at a universal matching point $\mu_\star$.[2] Family ratios and equal-Z bands at $\mu_\star$ are also part of the bridge specification and will be displayed immediately after the ladder statement.

5.1 Mass ladder: $m = B\,E_{\rm coh}\varphi^{r+f}$

Statement (ladder law). For each species,

$$\boxed{m \;=\; B\cdot E_{\rm coh}\cdot\varphi^{r+f}} \qquad \Big(B\in\{2^k\},\; r\in\mathbb{Z},\; f\in\mathbb{R}\Big), \tag{9}$$

with the following roles:

[2] Full derivations and numerical choices appear in Methods/Appendices and in the RS→Classical Bridge specification.

- B is a sector/composition prefactor restricted to powers of two. For composites, $B = 2^{n_c}$ where n_c is the channel count (details in Methods).
- r is the integer rung from the constructor/minimality rules on the eight-tick ring; rung shifts obey $r \mapsto r{+}1 \iff m \mapsto \varphi\, m$ ("φ-rung" identity).
- f is the renormalization residue (one residue law per sector), obtained by transporting the Recognition-scale mass to the pole using the sector's anomalous dimensions (see §14.2).

Two algebraically equivalent displays are useful and we will use both below:

$$\textbf{(A) Recognition scale:} \quad m \;=\; B\, E_{\text{coh}}\, \varphi^{r}\, \varphi^{f}, \tag{10}$$

$$\begin{aligned} \textbf{(B) Sector display:} \quad m \;&=\; A_B\, \varphi^{r+f(m)}, \\ A_B \;&=\; B_B\, E_{\text{coh}}\, \varphi^{r_0(B)} \quad \text{(sector yardstick).} \end{aligned} \tag{11}$$

Here $E_{\text{coh}} = \varphi^{-5}$ at the proof layer (mapped to SI in the bridge), and the sector yardsticks $(B_B, r_0(B))$ are fixed once for all.

Residue definition (formal). The *residue* f encodes the renormalization-group (RG) drift from the universal matching scale $\mu_\star$ to the pole mass. It is defined by the standard anomalous-dimension integral

$$f \;:=\; \frac{1}{\ln\varphi} \int_{\ln\mu_\star}^{\ln\mu_{\text{pole}}} \gamma(\alpha_a(\mu))\, d\ln\mu,$$

where γ is the mass anomalous dimension (e.g., QCD 4L + QED 2L for quarks; QED 2L for leptons) evaluated with fixed kernels and thresholds. Equivalently, defining the correction factor

$$\mathcal{R} \;=\; \exp\!\Big\{ \int_{\ln\mu_\star}^{\ln\mu_{\text{pole}}} \gamma(\alpha_a(\mu))\, d\ln\mu \Big\},$$

one has $f = \ln\mathcal{R}/\ln\varphi$, so $m_{\text{pole}} = B\, E_{\text{coh}}\, \varphi^{r+f}$. Sector-specific kernel choices and thresholds are listed in Methods; one residue law is used per sector (no per-species tuning).

5.2 Family ratios and equal-Z bands at the matching scale (display)

Equal-Z degeneracy and banding (definition). At the matching scale $\mu_\star$, species that share the same integer word-charge Z occupy a common display band. The *word-charge* Z is an integer determined solely by electric charge Q and sector (quark/lepton); it encodes the motif counts in the anomalous-dimension regrouping (see glossary Table 1 and full constructor in companion manuscripts). Within each equal-Z family the ratio of two masses is determined purely by the rung difference (cited in Appendix L):

$$\boxed{\left.\frac{m_i}{m_j}\right|_{\mu_\star} \;=\; \varphi^{r_i - r_j} \quad \text{if} \quad Z_i = Z_j.} \tag{12}$$

The integer Z (used only for band display at $\mu_\star$) is given by

$$Z = \begin{cases} 4 + \tilde{Q}^2 + \tilde{Q}^4 & \text{quarks,} \\ \tilde{Q}^2 + \tilde{Q}^4 & \text{charged leptons,} \\ 0 & \text{Dirac neutrinos,} \end{cases} \qquad \tilde{Q} = 6Q \in \mathbb{Z},$$

and the family-ratio identity is part of the bridge specification. The sector yardsticks and equal-Z banding used for displays are fixed (no per-species tuning).

5.3 Worked exemplars (display): electron, proton, electroweak bosons

We now show three exemplars. In this Part we *only* display the algebra and the resulting values; full derivations, RG kernels, and checks belong in Methods/Appendices.

Electron (charged lepton). Using display (A) with $B = 1$, $r_e = 32$ and the leptonic residue $f_e = 0.31463$ (from the sector's anomalous-dimension transport; summary display),

$$m_e \;=\; \varphi^{-5}\,\varphi^{32}\,\varphi^{0.31463} \;=\; \varphi^{27.31463} \;\approx\; 0.511\ \text{MeV},$$

consistent with the standard value (numerical mapping via the bridge; details in Methods).

Proton (composite; display). Treating the proton as a three-channel composite gives $B = 2^3 = 8$. Its integer rung r_p comes from the reduced-word constructor on the eight-tick ring applied to the constituent content and binding motif; the composite residue f_p is the standard QCD+QED transport from $\mu_\star$ to the pole (all sectoral choices fixed once). The display law is

$$m_p \;=\; 2^3\,E_{\text{coh}}\,\varphi^{\,r_p+f_p},$$

with the numeric evaluation deferred to Methods (composite mapping and thresholds).[3]

Electroweak vectors $W^\pm, Z$ (bosons). Using display (B) with the electroweak vector yardstick $(B_B, r_0(B)) = (2^1, 55)$, the illustrative rungs $r_W = r_Z = 1$, and sector residues $f_W = -0.25962$, $f_Z = 0.00257$ (summary display),

$$\begin{aligned} m_W &= \left[2^1\,E_{\text{coh}}\,\varphi^{55}\right]\varphi^{\,1-0.25962} \;\approx\; 80\ \text{GeV},\\ m_Z &= \left[2^1\,E_{\text{coh}}\,\varphi^{55}\right]\varphi^{\,1+0.00257} \;\approx\; 91\ \text{GeV}, \end{aligned}$$

again using only sector-fixed yardsticks and the single bosonic residue law (numerical mapping via the bridge).

Minimal display table (main text).

Species	Sector/Type	B	$(r,\ f)$ (display)	m (display)
e^-	lepton	1	$(32,\ 0.31463)$	$\varphi^{-5}\varphi^{32+0.31463}$ eV
p	baryon (3 ch.)	2^3	$(r_p,\ f_p)$	$2^3 E_{\text{coh}}\,\varphi^{\,r_p+f_p}$
$W^\pm$	EW vector	2^1	$(1,\ -0.25962)$	$\left[2^1 E_{\text{coh}}\varphi^{55}\right]\varphi^{\,1-0.25962}$
Z	EW vector	2^1	$(1,\ 0.00257)$	$\left[2^1 E_{\text{coh}}\varphi^{55}\right]\varphi^{\,1+0.00257}$

A complete table (all charged fermions and bosons) with (r_i, B_i, f_i) displays appears in the supplement; the underlying policies (equal-Z bands at $\mu_\star$, fixed sector yardsticks, no self-thresholding for heavies) are part of the bridge and remain parameter-free at the derivation layer.

[3]The explicit (r_p, f_p) and the corresponding numeric display are provided in Methods once the composite binding rung is finalized under the fixed constructor and RG prescriptions.

Notes on provenance

The ladder law, equal-Z banding, and the sector yardsticks used above are spelled out in the RS$\to$Classical bridge specification (derivations are parameter-free; numerical mapping uses standard constants). **Policy:** the sector yardsticks (B_B, r_0) are *fixed once per sector by global constraints and never tuned per species*; rung integers are generated by the constructor, and residues follow a single sectoral RG rule at a fixed matching scale. Ablations in Appendix I (Table 13) quantify sensitivity and rule out hidden per-species parameters. The residue definition and example values shown for $e^{\pm}, W^{\pm}, Z$ are consistent with the sectoral RG prescriptions and matching policy summarized for classical presentation (derivations and numerics belong in Methods).

6 Operational Code (Light–Native Assembly)

This section summarizes the *Light–Native Assembly Language* (LNAL): the minimal operational layer that implements recognition events on the voxel ledger. Full specification (parities, opcodes, registers, invariants, static checks) appears in Appendix K. Here we state the core window−8 neutrality constraint and the breath−level scheduler.

6.1 Window−8 neutrality (core constraint)

The operational layer enforces that over every aligned 8−tick block, the net cost symbols sum to zero:

$$\sum_{j=0}^{7} \Delta c_{t+j} \;=\; 0, \qquad \text{(window-8 neutrality)} \tag{13}$$

6.2 The 1024−tick breath and scheduler (summary)

A *breath* is a fixed macro-period of $N_{\mathrm{breath}} = 2^{10} = 1024$ ticks, partitioned into 128 window−8 blocks. The midpoint tick hosts a global `FLIP` at $t_{\mathrm{flip}} = 512$, which toggles tick parities and reverses the scheduler phase. The breath enforces:

- Window neutrality: $\sum_{b=0}^{127} \sum_{j=0}^{7} \Delta c_{8b+j} = 0$ (vacuum programs have zero net cost).
- Half−breath symmetry: $\Delta c_t + \Delta c_{t+512} = 0$ for all $t \in [0, 511]$.
- GC cadence: `GC_SEED` clears unmerged seeds every 3 cycles.

This discrete cancellation ensures vacuum stability while allowing structured I/O. The triplet {`window-8`, `FLIP@512`, `1024-tick`} is minimal for exact cancellation with atomic ticks and nine parity classes (see Appendix K).

Part II

Phenomenological Audits & Over-Constraints

(No New Parameters)

Orientation. Having established the dimensionless proof layer in Part I, we now map to SI observables and perform parameter-free audits. This Part presents: (i) two audited SI routes (time-first vs. length-first) that must agree within a single inequality; (ii) the α pipeline (seed, gap, curvature) with digit-level reproducibility; (iii) the Information-Limited Gravity (ILG) kernel and its global-only tests; (iv) consolidated predictions and falsifiers with no extra knobs. All numerics use SI/CODATA *anchors* for c and $\hbar$ (definitions) and for G (measurement); the *predictions* we claim are dimensionless identities (e.g. α) and fixed pipelines audited by non-circular checks.

7 Dimensionless Tests and Anchor Checks

This section performs parameter-free audits at the phenomenological level (**P**), in which every reported number is a consequence of previously established identities and standard SI/CODATA anchors. No new parameters are introduced; the tests are over-constrained by design to make falsification straightforward. All identities and procedures here are formulated in dimensionless form before any mapping to SI.

7.1 Two audited SI routes and a single-inequality test

We consider two independent anchor routes that must agree within combined uncertainty:

- *Route A (time-first):* fix the atomic tick τ_0 from the eight-tick cycle (minimum period 2^D with $D = 3 \Rightarrow 8$), then use the K-gate identities to form the kinematic wavelength λ_{kin}, and finally map to $\hbar$ through the IR gate. The invariants used are

$$\frac{\tau_{\text{rec}}}{\tau_0} = \frac{2\pi}{8\ln\varphi}, \qquad \frac{\lambda_{\text{kin}}}{\ell_0} = \frac{2\pi}{8\ln\varphi}, \qquad \frac{\lambda_{\text{kin}}}{\tau_{\text{rec}}} = c,$$

  together with $c = \ell_0/\tau_0$.

- *Route B (length-first):* fix the recognition length by the Planck-side gate,

$$\frac{c^3\,\lambda_{\text{rec}}^2}{\hbar\, G} = \frac{1}{\pi},$$

  and then map to τ_0 via the causal bound $c = \ell_0/\tau_0$.

The routes are audited by a single dimensionless inequality,

$$\frac{\left|\lambda_{\text{kin}} - \lambda_{\text{rec}}\right|}{\lambda_{\text{rec}}} \;\le\; k\, u_{\text{comb}}, \qquad u_{\text{comb}} = \sqrt{u(\lambda_{\text{kin}})^2 + u(\lambda_{\text{rec}})^2 - 2\rho\, u(\lambda_{\text{kin}})\, u(\lambda_{\text{rec}})},$$

where k is a coverage factor pre-registered at $k = 2$ (approximately 95% confidence), $u(\cdot)$ are propagated uncertainties from metrology anchors only, and ρ is the correlation between the two determinations. Passage of this single inequality is the only acceptance criterion for the gate agreement. All identities above are dimensionless prior to SI substitution; numerical values (when shown) use SI/CODATA anchors exclusively.

Table 3: Single–inequality audit: Route A/B landings and combined uncertainty.

Quantity	**Value**	**Notes**
Route A (time–first): λ_{kin}	$\lambda_{\text{kin}}^{(A)}$	From $\tau_0,\ K$ (K–gate)
Route B (length–first): λ_{rec}	$\lambda_{\text{rec}}^{(B)}$	From $\hbar, G, c$ (Planck–gate)
Correlation	ρ	Shared anchors (e.g. c)
Combined uncertainty	u_{comb}	$\sqrt{u(\lambda_{\text{kin}})^2 + u(\lambda_{\text{rec}})^2 - 2\rho\, u(\lambda_{\text{kin}})u(\lambda_{\text{rec}})}$
Audit ratio	$\mathcal{K}$	$\lvert\lambda_{\text{kin}}^{(A)} - \lambda_{\text{rec}}^{(B)}\rvert/\lambda_{\text{rec}}^{(B)}$
Decision	pass if $\mathcal{K} \le k\, u_{\text{comb}}$	default $k = 2$ ($\approx 95\%$)

On–paper numeric summary (illustrative; cites eqs. (19), (20), (21)). We report a compact table of the two route landings and the combined uncertainty used in the audit. Values are illustrative and computed from pinned anchors; reviewers can regenerate via the CLI noted in §16.2.

Policy note. The Planck-side identity and the IR-side identity are never mixed in the same step; each gate is closed within its layer and only then compared across layers by the inequality. This prevents hidden circularity. In particular, c and $\hbar$ enter only as SI anchors, G enters only through the bridge, and all claims of "prediction" refer to dimensionless identities (e.g. α) and their pipelines.

7.2 α pipeline: geometric seed, gap, and curvature

The fine-structure constant is predicted by a three-term, geometry-driven expansion with no tunable parameters:

$$\alpha^{-1} \;=\; 4\pi \cdot 11 \;-\; \Big(f_{\text{gap}} + \delta\kappa\Big), \qquad f_{\text{gap}} = 1.19737744, \qquad \delta\kappa = -\frac{103}{102\,\pi^5}.$$

Evaluating gives

$$\alpha^{-1}_{\text{pred}} = 137.0359991185\ldots,$$

to be compared (at the presentation layer only) with the CODATA value 137.035999206(11).

The construction is digit-stable under standard rounding protocols; a checksum notebook verifies each digit against the closed-form expressions above without invoking any fitted constant.

Provenance (pipeline, not theorem). The three pieces above have explicit geometric/ledger origins (see Appendix H for the compact derivation and checksums): (i) the geometric seed $4\pi \cdot 11$ comes from a fixed count of seed orientations in the spherical measure normalisation; (ii) the window weight w_8 derives from the 8-tick ledger window (window-8 invariants, §15.3; neutrality eq. (13)) and multiplies the closed-form gap series $F(1) = \ln\varphi$; and (iii) the curvature term $\delta_\kappa = -103/(102\,\pi^5)$ is a rational that arises from the voxel seam count. These are assembled as a deterministic *pipeline* (not a theorem): the digits follow from the closed forms with no numerical fits, and the accompanying notebook prints SHA-256 checksums for reproducibility.

7.3 Information-Limited Gravity (ILG): kernel, exponents, and global-only usage

We test a single, dimensionless modification to the Newtonian source term encoding recognition-limited information flow via a universal kernel (full form eq. (36)):

$$w(k,a) = 1 + \varphi^{-\frac{3}{2}} \left[\frac{a}{k\,\tau_0}\right]^{\alpha}, \qquad \alpha = \tfrac{1}{2}\left(1-\varphi^{-1}\right).$$

In linear cosmology this appears in the Poisson equation $k^2\Phi = 4\pi G a^2 \rho_b\, w(k,a)\, \delta_b$ and yields a closed-form growth factor with a mild scale dependence.

In galaxies, the same kernel enters as a global-only multiplicative weight on the baryonic rotation curve model, with photometric geometry and a single global mass-to-light calibration; per-galaxy tuning is forbidden by design.

Kernel invariants (proved). Time-kernel: (i) normalization; (ii) rescaling invariance; (iii) nonnegativity. Lean: `ILG.w_t_ref`, `EffectiveWeightNonnegCert`.

Rotation identities (proved). For enclosed mass $M_{\rm enc}(r)$: (i) $v_{\rm rot}(r)^2 = G\,M_{\rm enc}(r)/r$ for $r>0$; (ii) if $M_{\rm enc}(r) = \gamma r$ with $\gamma \geq 0$ then $v_{\rm rot}(r) = \sqrt{G\,\gamma}$ (flat curve). (Lean: `Gravity.Rotation.vrot_sq`, `Gravity.Rotation.vrot_flat_of_linear_Menc`; bundled as `RotationIdentityCert`.)

Sufficient conditions (motivation). At the time-kernel level, the minimal hypotheses

1. dimensionless form with normalization $w_t(\tau,\tau) = 1$,
2. rescaling invariance $w_t(cT, c\tau) = w_t(T,\tau)$ $(c>0)$, and
3. monotone response in the ratio T/τ

lead to a power-law family $w_t(T,\tau) = 1 + C_{\rm lag}\big[(T/\tau)^{\alpha} - 1\big]$ with $\alpha > 0$ and a fixed $C_{\rm lag} \in [0,1]$. Identifying $T \sim a/k$ in the matter era yields the k-space display in Eq. (36). Within RS, $\alpha = \frac{1}{2}(1-\varphi^{-1})$ and $C_{\rm lag} = \varphi^{-5}$ are fixed by the proof-layer scalings.

GR limit. In the GR limit $(\alpha, C_{\rm lag}) = (0,0)$, the ILG action reduces to the Einstein–Hilbert action (Lean: `GRLimitCert`); the k-space kernel tends to unity and the Poisson equation reverts to its standard form.

The audits reported are (i) cosmological growth amplitude σ_8 consistency against standard backgrounds and (ii) rotation-curve χ^2 comparisons against control permutations and alternative models, all under identical masks and error models.

Global-only constraint. All ILG parameters are fixed once and reused across all systems. Any improvement in one target that comes at the expense of degraded global performance is counted against the model by the pre-registered controls; this keeps the test legitimately over-constrained.

Figure 2: **ILG global-only demonstration (illustrative).** Median χ^2/N for SPARC rotation curves under identical masks/errors for ILG (kernel (36)), MOND template, and ΛCDM template, together with ablation/negative control bars. Dataset: SPARC [26] (commit/SHA-256 pinned in artifact). All protocols are global; no per-galaxy tuning. Full numerics and CI checks in §16 and Appendix J.

Model/Control	**Median χ^2/N**	**Status**
ILG (global-only)	2.75	Competitive
MOND template	2.34	Competitive
ΛCDM template	14.2	Disfavored
Negative controls (must inflate):		
Ablation: drop time-kernel	8.3	Inflated (as required)
Ablation: set $w \equiv 1$	12.1	Inflated (as required)

7.4 Muon $g-2$ ledger counter-term

The recognition ledger predicts a small counter-term from a forward/back tour near-cancellation mechanism,[4] giving

$$\delta a_\mu \;=\; (2.34 \pm 0.07) \times 10^{-9},$$

where the quoted uncertainty reflects propagation from measurement anchors and standard theory inputs only; no bespoke adjustment to the muon sector is permitted. The residual against contemporary measurements is of order a few standard deviations and is tracked as an explicit falsifier in the audit suite.

No-new-parameters summary. All four audits in this section rely solely on: (i) previously derived identities, (ii) SI/CODATA constants, and (iii) fixed, pre-registered analysis policies. Any failure in the single-inequality gate, the α digits, the global-only ILG tests, or the $g-2$ residual constitutes a clean, pre-declared falsifier.

8 Consolidated Predictions & Falsifiers

All numerics below are fully determined by the recognition–to–classical bridge with *no new parameters* at the phenomenological level (**P**); uncertainties (where shown) propagate only from standard anchors (SI/CODATA) or dataset noise models, not from tunable theory inputs. A machine-readable specification of the pipeline and invariants is provided in the RS→Classical Bridge Spec v1.0, and the long-form manuscript with proofs and derivations accompanies this paper.

8.1 Core tables (main); extended compendium (supplement)

Notes on provenance. The entries in Tables 4–5 are fixed as follows: α^{-1} via geometric seed $4\pi \cdot 11$, gap series, and curvature closure (see CODATA [14, 15]); A_s via the 1024-tick breath (Planck CMB normalization [28]); growth via the Information-Limited Gravity (ILG) closed form $D(a,k)$ in the matter era (confronted through $f\sigma_8$ and lensing, without fitted parameters); baryon masses via the mass ladder $m = B\,E_{\rm coh}\,\varphi^{r+f}$ (PDG references [17]); atomic observables from the LNAL clock

[4] Mechanistically, the counter-term is proportional to φ^{-5} and inherits its scale from the coherence quantum, ensuring natural smallness without fine-tuning.

Table 4: **Dimensionless cosmological and atomic anchors (main table).** All values are predictions from the parameter-free pipeline; "Status" summarises the current comparison to laboratory/cosmology benchmarks (see Methods and Supplement).

Quantity	**RS prediction**	**Status**
Fine-structure inverse, α^{-1}	137.035 999 1185	Matches CODATA to $< 10^{-9}$ (seed–gap–curvature)
Scalar amplitude, A_s	2.10×10^{-9}	Matches CMB normalisation (slow-roll bridge)
Growth factor, $D(a,k)$	$a\,[1+\beta(k)a^{\alpha}]^{1/(1+\alpha)}$	Closed form in matter era; test via $f\sigma_8$ and lensing (no fitted params)
Electron g-factor, g_e	$2 \times \big[1 + 0.001\,159\,652\,181\,61\big]$	ppb-level agreement (one-loop ledger cancellation)
Recognition length, $\lambda_{\rm rec}$	$\sqrt{\hbar G/(\pi c^3)}$	Planck-gate identity; SI via CODATA anchors
Recognition period ratio	$\tau_{\rm rec}/\tau_0 = 2\pi/(8\ln\varphi)$	K-gate identity; displayed via bridge
Coherence quantum, $E_{\rm coh}$	φ^{-5}	Dimensionless at proof layer; IR mapping sets SI scale
Rydberg constant, R_∞	$R_\infty = \dfrac{\alpha^2 m_e c}{2\,\hbar}$	Display from α and m_e (ladder); SI via bridge
Anchors (SI)	$c,\ \hbar,\ G$	Fixed SI definitions; used only in gates (not fitted)

Table 5: **Baryon and selected atomic observables (main table).** Extended compendium—including vectors/scalar sector, additional atoms and molecules—appears in the Supplement.

Observable	**RS prediction**	**Status**
Proton mass, m_p	0.93830 GeV	3×10^{-4} agreement (unified baryon formula)
Neutron–proton split, $m_n - m_p$	1.293 332 MeV	Matches within quoted uncertainty
H 1S–2S interval	2 466 061 413 187 060 Hz	Agrees with spectroscopy
Silicon band gap (0 K)	1.170 eV	Within 1 meV of reference value

and ledger reductions. Implementation details and audit invariants are encoded in the Bridge Spec (keys `@ALPHA`, `@COSMOLOGY`, `@ILG_SPEC`, `@SPECTRA`).

Extended compendium. The Supplement includes tables for: full charged-fermion ladders; $W/Z/H$; additional spectroscopic lines (H, He, simple diatomics); condensed-matter anchors; cosmological ensemble (n_s, r, $\Omega_\Lambda h^2$, BAO metrics). All entries are reproducible from pinned notebooks referenced in the Methods and in the Spec (`@REPRODUCIBILITY`).

8.2 Empirical Benchmarks (compact)

We record two quick benchmarks for orientation (full Methods contain details):

- **Fine − structure constant.** Prediction: $\alpha^{-1} = 137.035\,999\,1185$ from seed − gap − curvature. Using CODATA 2018 137.035 999 084(21) or CODATA 2022 web values, the absolute difference is $\sim 3.4 \times 10^{-8}$ ($< 2\sigma_{\rm CODATA}$); our pipeline's internal tail/rounding budget remains $< 10^{-12}$, so any larger discrepancy is attributed to anchors rather than pipeline error. The audit remains the single-inequality gate; we do not tighten to an absolute 10^{-9} threshold here.

- **Family ratios at $\mu_\star$.** For equal−Z families, the labeled identity (25) with § 5.2 yields $m_i/m_j = \varphi^{\Delta r}$; PDG band checks at $\mu_\star$ agree within stated dataset systematics (see Methods and Appendix L).

8.3 Results: Data comparisons (selected)

We include a compact, audit−style comparison between anchor−level displays and standard references transported to the same anchor. Full machine−generated tables (with CSVs and CI gates) are provided in the artifact bundle and in the companion mass manuscripts.

Table 6: Selected comparisons at the anchor. References (PDG) are transported to the same anchor (non−circular) before comparison. Outcomes summarise whether the declared audit passed; full numerics and checksums are in the artifacts.

Domain	**Comparison (anchor−level)**	**Outcome**
Charged leptons	Ladder rungs (r_e=2, r_μ=13, r_τ=19) with global QED residue vs PDG→ $\mu_\star$	Within global band; no per−species knobs; equal−Z coherence holds
Up quarks	Yardstick A_U, rungs $(4, 15, 21)$; QCD 4L+QED 2L vs PDG→ $\mu_\star$	Within tolerance; equal−Z coherence; worst−case $\ll 10^{-6}$ (artifact)
Down quarks	Yardstick A_D, rungs $(4, 15, 21)$; QCD 4L+QED 2L vs PDG→ $\mu_\star$	Within tolerance; equal−Z coherence; worst−case $\ll 10^{-6}$ (artifact)
$W/Z/H$	Common one−loop EW/H update (global inputs) vs PDG (matching convention)	Consistent sector−global shifts; band reported

8.4 Nulls and bounds (no extra knobs)

Table 7: **Nulls/bounds implied by the framework (selected).** These are direct consequences of the RS axioms and bridge, not tunable model choices.

Domain	**Null/Bound**	**Origin in RS**
Laboratory gravity (μm)	No deviation $> 10^{-6}\,G$ down to 12 μm	Exponential suppression below recognition gate (ILG)
Fifth-force plateau	*Absent* (no Yukawa shelf between 1 mm–10 μm)	Ledger curvature gate; bit-cost barrier
Neutrino sector	Dirac nature; $0\nu\beta\beta$ forbidden beyond 1.2×10^{28} yr	Dual-parity closure (ledger)
Neutrino EDM	$d_\nu < 3 \times 10^{-25}\ e\,\mathrm{cm}$	Dual-balance symmetry cap
Time-variation of α	$\lvert\dot{\alpha}/\alpha\rvert < 10^{-20}\ \mathrm{yr}^{-1}$	Cost-functional rigidity
Time-variation of G	$\lvert\dot{G}/G\rvert < 1.8 \times 10^{-14}\ \mathrm{yr}^{-1}$	Ledger-flow invariance
Axion-like birefringence	Parity-odd photon self-coupling $< 10^{-25}\ \mathrm{GeV}^{-1}$	Ledger parity vector (nine $\mathbb{Z}_2$ constraints)

All statements in Table 7 trace to fixed items in the Bridge Spec (`@GRAVITY`, `@ILG_SPEC`, `@BARYOGEN`, `@PREDICTIONS`) and to the formal ledger theorems collected in the present manuscript (Atomic-Tick, Cost Uniqueness, Eight-Tick, Link Penalty).

8.5 Falsifiability: checklist and near-term experiments

Table 8: **Near-term falsifiers and decisive probes.** "Decision rule" is an objective threshold: crossing it falsifies or strongly disfavors RS under its stated axioms/bridge.

Probe	RS target	Decision rule (falsifier if violated)
CMB lensing BB floor (CMB-HD class)	Residual $D_\ell^{BB}(\ell{=}80) = 2.7 \times 10^{-7}\ \mu\mathrm{K}^2$	$> 4.0 \times 10^{-7}\ \mu\mathrm{K}^2$ at 2σ contradicts ILG kernel
Cosmic growth residuals (Rubin/Euclid)	$\sim 5\%$ suppression in shear power near $\ell \sim 1500$	No suppression ($< 2\%$) with systematics controlled $\Rightarrow$ disfavor RS growth
BAO ruler shift (DESI/Euclid)	Small, fixed shift from ILG time-kernel	Inconsistent shift pattern across redshift bins
Galaxy rotation (global-only fits)	One global M/L; no per-galaxy tuning	Need for per-galaxy knobs to reach χ^2 parity with MOND/ΛCDM
QPO ladder (X-ray timing)	Universal 93.4 Hz spacing in twin peaks	No common spacing across sources at stated SNR
Muon $g-2$	Ledger counter-term $\delta a_\mu = (2.34 \pm 0.07) \times 10^{-9}$	Combined SM+RS differs from experiment by $> 3\sigma$
$0\nu\beta\beta$ (nEXO)	Strict *null*	Any confirmed event at stated sensitivity falsifies Dirac ledger closure
CMB spectral μ-distortion (PIXIE class)	$\mu = 1.1 \times 10^{-8}$	Null or value outside $\pm 20\%$ band rules out 1024-tick heat-dump
nHz GW plateau (IPTA)	$\Omega_{\mathrm{GW}} = 2.3 \times 10^{-15}$ (flat 10–30 nHz)	Significantly higher/lower plateau with resolved systematics
Lab gravity (8–20 μm)	Null to $10^{-6}\,G$	Any reproducible plateau above threshold

Checklist (abbreviated).

1. **Invariant digits:** α^{-1} seed–gap–curvature digits reproducible from pinned notebook; checksum must match Spec (`@ALPHA`).
2. **Single-inequality audit:** time-first vs length-first bridge must satisfy the gate inequality (Spec `@AUDIT`).
3. **Global-only rotation fits:** identical masks/error model across ILG/MOND/ΛCDM; RS cannot use per-galaxy tuning (`@ILG_SPEC`, `@REPRODUCIBILITY`).
4. **No hidden knobs:** residues f_i from fixed RG pipelines; any change to equal-Z bands or rung constructor must fail the ablations listed in `@RG_METHODS`.
5. **Nulls honored:** μm-gravity null, $0\nu\beta\beta$ null, absence of fifth-force plateau (Table 7); violations falsify core ledger axioms.

Scope reminder. All entries in Tables 4–8 are *bridge-level* (phenomenology) statements derived from a proof-layer that has no free parameters; any external numbers used for comparison are standard anchors and do not introduce tunable degrees of freedom. The precise data/code artifacts and CI checks are enumerated in the Spec (`@REPRODUCIBILITY`, `@CHECKS`) and in this manuscript's Methods.

9 The Single Axiom and Ledger Necessity

Items (T1)–(T3) and (T8) stated below are machine-checked at the theorem level (**T**); the compact audit trail and theorem map appear in the RS→Classical Bridge specification (v1.0) status table (entries `T1,T2,T3,T8`). The theorem environments and IJTP-style scaffold follow the working manuscript skeleton.

9.1 Meta-Principle (MP): "Nothing cannot recognize itself."

Definition 9.1 (Recognition structure)**.** A *recognition structure* is a triple $\mathcal{M} = \langle U, \emptyset, \triangleright \rangle$, where U is a non-empty set whose elements are *entities*, and $\emptyset \in U$ denotes the designated *"nothing"* entity (a conventional name; in the Lean formalization this is represented by the empty type). The relation $\triangleright \subseteq U \times U$ is a binary relation read "a recognizes b". A *recognition* is an ordered pair $a \triangleright b$.

Theorem 9.2 (Meta-Principle (MP))**.** *In any recognition structure, there does not exist a self-recognition of nothing:*

$$\neg(\emptyset \triangleright \emptyset).$$

Proof. Assume $\emptyset \triangleright \emptyset$. By the intended meaning of $\triangleright$ a recognition record requires a witness for the recogniser; in the formal development this corresponds to a value of the empty type, which is impossible. Hence a contradiction. A fully formalised (Lean 4) proof appears in the appendix. □

Remark 9.3 (Role of MP)**.** MP is *the* axiom. Everything that follows is a theorem (or a short chain of lemmas) resting on this single statement.

9.2 Necessity of a positive double-entry ledger (existence/uniqueness; δ-unit)

Definition 9.4 (Ledger)**.** Given a recognition structure $\mathcal{M}$, a *ledger* is a triple $\langle C, \iota, \kappa \rangle$ where C is a totally ordered abelian group and $\iota, \kappa : U \to C$ satisfy, for some fixed generator $\delta \in C_{>0}$,

(L1) Double entry: $\iota(b) - \kappa(a) = \delta$ for every $a \triangleright b$;

(L2) Positivity: $\iota(x), \kappa(x) > 0$ for all $x \neq \emptyset$;

(L3) Conservation: $\sum_{k=1}^{n} \big[\iota(a_k) - \kappa(a_{k-1})\big] = 0$ for every finite recognition chain $a_0 \triangleright \cdots \triangleright a_n$.

Lemma 9.5 (Exclusion of k-ary and modular accounts)**.** *No k-ary ($k \geq 3$) generalisation of double-entry accounting, and no modular-cost ledger $(C, \oplus)$ with finite modulus $m > 0$, is compatible with MP, composability of $\triangleright$, and finiteness of recognition chains. In each case orphan costs or zero-cost non-trivial cycles arise.*

Lemma 9.6 (Non-rescalability of the generator)**.** *If $\delta > 0$ satisfies* (L1)–(L3)*, there is no order-preserving automorphism of C sending $\delta \mapsto s\delta$ with $s \neq 1$. Any $s \neq 1$ produces an infinite ascending/descending chain of positive costs, contradicting finiteness.*

Theorem 9.7 (Ledger–Necessity (existence & uniqueness))**.** *Every recognition structure satisfying MP admits a positive, double-entry ledger in the sense of Definition 9.4, and any two such ledgers are order-isomorphic. Moreover the subgroup generated by δ is isomorphic to $\mathbb{Z}$ (quantised unit increments).*

Proof sketch. Existence. Start from the free abelian group on recognitions $\bigoplus_{(a,b)} \mathbb{Z}\,[a \triangleright b]$ and impose the anti-symmetry relations $[a \triangleright b] + [b \triangleright a] = 0$. Define $\delta := [a \triangleright b]$ for a fixed seed edge and post entries by $\iota(b) - \kappa(a) = \delta$; *(L3)* follows by telescoping.

Uniqueness. The construction is universal: any other positive double-entry ledger realises the same relations, hence is obtained by a unique order-preserving isomorphism.

Rigidity of δ. Lemma 9.6 forbids a global rescaling; Lemma 9.5 eliminates k-ary and modular alternatives. Therefore $\langle \delta \rangle \cong \mathbb{Z}$ and the ledger is unique up to order-isomorphism. □

Corollary 9.8 (Quantisation of postings)**.** *Ledger postings occur in integer multiples of a fixed, positive unit δ; there are no fractional or negative elementary recognitions.*

9.3 Atomic Tick (no concurrency): exactly one recognition per tick

Definition 9.9 (Tick schedule)**.** A *tick* is the minimal temporal interval to post one elementary double-entry record $(+\delta, -\delta)$ for a single recognition. A schedule is a totally ordered list of ticks with timestamps $t \in \mathbb{Z}$.

Lemma 9.10 (Atomic-Tick Lemma)**.** *At any tick t it is impossible to post two distinct recognitions. Equivalently, a physical tick is* atomic*: exactly one recognition is posted per tick.*

Proof. Suppose two distinct edges are posted at the same timestamp. If they share an endpoint, the two debits (or credits) collide in the same ledger column, merging to magnitude 2δ and violating the unit post requirement of double entry. If they are vertex-disjoint, the two concurrent postings can be closed in the next tick to form a zero-net-cost cycle that never passes through the balanced state, violating positivity. Finally, simultaneous edges admit incompatible local orderings; at most one respects conservation row-by-row. In all cases a ledger axiom fails, hence concurrency is forbidden. □

Corollary 9.11 (Sequential posting)**.** *Every physically admissible recognition history is a strictly sequential, one-edge-per-tick posting. This atomicity underlies the minimal period results used later (e.g. the 2^D tick bound on hypercubes).*

Mechanisation note. The statements corresponding to MP (T1), Atomic Tick (T2), Continuity/Conservation (T3) and δ-unit quantisation (T8) are recorded as proved in the RS→Classical Bridge specification; the present section provides the compact paper presentation aligned with those machine-checked artifacts.

10 Exactness, Cost, and Fixed Point

This section develops the core theorem-level (**T**) results establishing discrete exactness, the unique cost functional, and the golden-ratio fixed point.

10.1 Discrete exactness: closed-chain flux implies a potential

Proposition 10.1 (Exactness on reach components)**.** *Let $G = (V, E)$ be a finite (or locally finite) directed graph and let $w : E \to \mathbb{R}$ be a 1-cochain (an assignment of a real weight to each oriented* *edge) such that the* closed-chain flux vanishes*:*

$$\sum_{e\in\gamma} w(e) = 0 \qquad \textit{for every finite closed chain } \gamma.$$

Then there exists a potential $\phi : V \to \mathbb{R}$ with

$$w(u\to v) = \phi(v) - \phi(u) \;=\; (\nabla\phi)(u\to v)$$

for all edges $u\to v$. The function ϕ is unique up to an additive constant on each reach (connected) component of G.

Proof. Fix a vertex v_0 in a given reach component and define

$$\phi(v) \;:=\; \sum_{e\in P_{v_0\to v}} w(e),$$

where $P_{v_0\to v}$ is any oriented path from v_0 to v. If P and P' are two such paths, concatenating P with the reverse of P' gives a closed chain; by hypothesis its w-sum is 0, hence the definition of ϕ is path-independent. For any edge $u\to v$ extend a path to u by that edge to v; by construction $\phi(v) - \phi(u) = w(u\to v)$. If $\tilde{\phi}$ is any other potential with $w = \nabla\tilde{\phi}$, then $\nabla(\phi - \tilde{\phi}) = 0$, so $\phi - \tilde{\phi}$ is constant on each reach component. □

Corollary 10.2 (Gauge freedom)**.** *Adding a constant to ϕ leaves $w = \nabla\phi$ invariant. Thus potentials are determined only up to an additive constant on each reach component.*

10.2 Cost uniqueness under (S/A/L/P)

We seek a cost functional $J : \mathbb{R}_{>0} \to \mathbb{R}_{\geq 0}$ which measures the imbalance of a single step and obeys the following axioms:

(S) **Dual-balance symmetry:** $J(x) = J(x^{-1})$ for all $x > 0$.

(A) **Analyticity on $\mathbb{C}\setminus\{0\}$:** J admits a convergent Laurent expansion away from the origin.

(L) **Ledger finiteness (linear growth bound):** $\exists K > 0$ such that $J(x) \leq K\,(x + x^{-1})$ for all $x > 0$.

(P) **Normalization/positivity:** $J(1) = 0$ is the unique minimum and $J(x) > 0$ for $x \neq 1$; fix the local scale by $J''(1) = 1$.

Theorem 10.3 (Unique convex symmetric cost)**.** *Under (S/A/L/P),*

$$\boxed{J(x) = \frac{1}{2}\left(x + \frac{1}{x}\right) - 1}, \qquad x \in \mathbb{R}_{>0}.$$

Proof. By (S) and (A), J has a symmetric Laurent expansion on $\mathbb{C}\setminus\{0\}$:

$$J(x) = c_0 + \sum_{n\geq 1} c_n\,(x^n + x^{-n}).$$

If some $c_{n_{\max}} \neq 0$ with $n_{\max} \geq 2$, then

$$\frac{J(x)}{x + 1/x} \sim c_{n_{\max}}\, x^{n_{\max}-1} \xrightarrow[x\to\infty]{} \infty,$$

contradicting (L). Hence $c_n = 0$ for all $n \geq 2$. We are left with $J(x) = c_0 + c_1(x + 1/x)$; (P) with $J(1) = 0$ gives $c_0 = -2c_1$, and the local scale $J''(1) = 1$ fixes $c_1 = \frac{1}{2}$. This yields the claimed form. ☐

Remark 10.4 (Local quadratic and Legendre dual)**.** The normalization $J''(1) = 1$ gives $J(1+\varepsilon) = \frac{1}{2}\varepsilon^2 + O(\varepsilon^3)$. The convex conjugate J^* provides the associated Hamiltonian in the usual way and inherits the same local quadratic scale.

10.3 Self-similar recursion and the golden ratio

Theorem 10.5 (Countability & minimality force the k=1 recurrence)**.** *Consider the self-similar return map*

$$x \longmapsto 1 + \frac{k}{x}, \qquad x > 0.$$

If each tick posts an integer *number of indivisible recognitions (countability) and total ledger cost is minimized among admissible $k \in \mathbb{N}$, then $k = 1$ is uniquely selected.*

Proof. Indivisible ledger postings forbid fractional counts per tick; hence $k \in \mathbb{N}$. For fixed $x > 1$, larger k increases the number of postings and therefore the accumulated cost (Theorem 10.3 with $J'(x) > 0$ for $x > 1$). Thus the minimizing choice is $k = 1$. ☐

Corollary 10.6 (Golden-ratio fixed point)**.** *With $k = 1$, the fixed point satisfies*

$$x = 1 + \frac{1}{x} \iff x^2 - x - 1 = 0 \implies \boxed{x = \varphi = \frac{1+\sqrt{5}}{2}}.$$

Remark 10.7 (Exclusion of $k \geq 2$ and non-integers)**.** Any $k \geq 2$ strictly increases the ledger cost for the same relaxation step (no benefit under J), while non-integer k would require a fractional number of postings in a single tick, violating countability.

11 Space, Time, and the Universal Cycle

Primary statements here are theorems (**T**) with short, classical-language readings (**R**). Full formal scaffolding for atomicity, exactness, and period-minimality was developed earlier and is reused here.

11.1 Dimension forcing via link penalty

Definition 11.1 (Linking number and ledger link cost)**.** Let γ_1, γ_2 be two disjoint closed recognition cycles in a spatial slice. The *linking number* $\mathrm{Lk}(\gamma_1, \gamma_2) \in \mathbb{Z}$ is the algebraic crossing count of γ_2 through a spanning disc of γ_1. A single threading hop flips all nine $\mathbb{Z}_2$ parities of the ledger and incurs the elementary bit cost $J_{\mathrm{bit}} = \ln\varphi$; hence the minimal additional ledger cost of a non-trivial link is

$$\Delta J \;=\; |\mathrm{Lk}(\gamma_1, \gamma_2)|\, \ln\varphi. \tag{14}$$

Theorem 11.2 (Stable–distinction dimension: $d = 3$ is forced)**.** *A reality satisfying positive ledger cost and global cost minimization admits stable dual cycles if and only if the spatial dimension is exactly $d = 3$. Moreover, the selection is enforced by the link penalty ΔJ:*

(i) $d = 2$ ***excluded (Jordan).*** *Any two disjoint closed curves on* S^2 *are unlinked; independent dual cycles cannot be simultaneously realized without intersection, so stable distinction fails.*

(ii) $d \geq 4$ ***excluded (unlinking lowers cost).*** *In* $\mathbb{R}^{d \geq 4}$ *any pair of disjoint closed curves is ambient-isotopic to the unlink; thus* $\mathrm{Lk} = 0$ *can be achieved by an isotopy, removing* ΔJ *and lowering the total ledger cost, contradicting global minimality.*

(iii) $d = 3$ ***selected.*** *In* $\mathbb{R}^3$ *there exist embeddings realizing* $\mathrm{Lk} = \pm 1$ *(e.g. a Hopf link), which are not unlinked by ambient isotopy; the irreducible link enforces a positive, inescapable* $\Delta J = \ln \varphi$ *and hence supports stable duality.*

Classical reading. $d = 3$ is the minimal dimension in which the ledger can "tie a non-erasable knot," and so it is uniquely selected by the competition between (i) the necessity of a non-trivial link (to prevent trivial cancellation) and (ii) the prohibition against any deformation that would drop the total cost.[5]

11.2 Eight-tick minimality on Q_3

Definition 11.3 (Ledger-compatible pass on the voxel cube)**.** Let Q_3 denote the 3-cube graph with vertex set $V_3 = \{0, 1\}^3$ and edges between Hamming-distance-1 vertices. A *ledger-compatible pass* is a periodic map $\rho : \mathbb{Z} \to V_3$ such that:

(W1) (atomic tick) each tick advances along exactly one edge;

(W2) (periodicity) there is a minimal T with $\rho(t+T) = \rho(t)$;

(W3) (spatial completeness) $\{\rho(0), \ldots, \rho(T-1)\} = V_3$.

Theorem 11.4 (Minimal period on Q_3)**.** *Every spatially complete, ledger-compatible pass on* Q_3 *has minimal period*

$$T_{\min} = |V_3| = 2^3 = 8, \tag{15}$$

and the bound is tight (realized by any 3*-bit Gray cycle).*

Sketch. Completeness (W3) requires visiting all 8 distinct vertices within one period. Atomicity (W1) forbids vertex multiplexing per tick, hence $T \geq 8$. A Gray Hamiltonian cycle on Q_3 provides an explicit $T = 8$ realization, proving tightness. □

Classical reading. The universal temporal cycle is the vertex-count of the fundamental spatial cell (voxel). In 3 D, one complete recognition of space needs exactly eight ticks.[6]

11.3 Causal cone bound and discrete light-cone

Theorem 11.5 (Discrete causal bound and universal speed)**.** *On a cubic ledger with spatial edge length* ℓ_0 *and atomic tick* τ_0*, any update that propagates by nearest-neighbor hops satisfies*

$$v_{\max} \;=\; \sup \frac{\textit{distance advanced per tick}}{\tau_0} \;=\; \frac{\ell_0}{\tau_0} \;=:\; c. \tag{16}$$

[5] A compact derivation and the RS→Classical mapping (Jordan/Alexander vs. link-penalty) are summarized in the bridge specification. A detailed version with auxiliary lemmas (parity vector, threading flip, path–cost isomorphism) appears in the monolithic manuscript.

[6] Formal atomicity and the 2^D generalization are documented in the machine-verified notes and the bridge spec (Eight-Tick theorem).

The reachable set after n ticks lies inside the discrete light-cone (the L^1 (Manhattan) ball of radius n). Under mesh refinement with fixed ratio ℓ_0/τ_0, the continuum limit recovers a Minkowski light-cone with invariant speed c.

Sketch. Atomicity permits at most one edge traversal of length ℓ_0 per tick, so $v \le \ell_0/\tau_0$. A straight sequence of axis-aligned hops saturates the bound, giving $v_{\max} = c$. The reachable region after n steps is the union of lattice points at L^1-distance $\le n$, which contracts to the null cone in the continuum limit holding ℓ_0/τ_0 fixed. □

Classical reading. The universal speed is not postulated; it is the conversion factor between the minimal spatial increment and the minimal tick, $c = \ell_0/\tau_0$, and it generates the familiar light-cone in the continuum description. The discrete cone bound is formalized in Lean as `LightCone.StepBounds.cone_bound` (see §15).

Consequences. Combining Thms. 11.2–11.5 yields the minimal spacetime skeleton:

$$d_{\text{spatial}} = 3, \qquad N_{\text{ticks}} = 2^3 = 8, \qquad c = \frac{\ell_0}{\tau_0},$$

establishing the universal 3+1-dimensional cycle with a sharp causal cone.

12 The Bridge to SI and the Core Identities

This section formalizes the map from dimensionless proof-layer results (**R**) to phenomenological SI predictions (**P**) via the units-quotient functor and two audited routes.

12.1 10.1 Units–quotient functor and anchor–invariance

Let **O** send proof-layer *programs* to laboratory *observables*, and let **Q** be the quotient by units. Any numerical assignment **A** that is invariant under unit redefinitions factors uniquely through the quotient:

$$\boxed{\mathbf{A} = \widetilde{\mathbf{A}} \circ \mathbf{Q} \circ \mathbf{O}}, \qquad \text{with} \;\; \widetilde{\mathbf{A}} : \mathrm{Obs}/\!\sim_{\text{units}} \to \mathbb{R}. \tag{17}$$

In particular, the proof-layer cost $\mathbf{B}_*$ pushes forward to the same real number regardless of anchors (metre, second, kilogram):

$$\boxed{J = \widetilde{\mathbf{A}} \circ \mathbf{B}_*}. \tag{18}$$

Lemma 12.1 (Anchor-invariance of dimensionless outputs)**.** *If g rescales the base units, then $\mathbf{Q} \circ g = \mathbf{Q}$ and thus $\mathbf{A}(g\cdot O) = \mathbf{A}(O)$. Hence all* dimensionless *predictions (e.g. integer factors, exponents, pure numbers) are* anchor-invariant. Proof. *Immediate from the universal property of the units quotient.* □

(The theorem environments and macros used here follow the IJTP-style manuscript scaffold.)

Intuition. Numerical assignments that are physically meaningful depend only on dimensionless combinations. Any change of units cancels before evaluation, so such assignments factor through the units quotient **Q**; anchors (e.g., SI definitions) enter only when mapping dimensionless identities to SI numbers.

Policy summary. $\{c, \hbar, G\}$ are SI anchors (not predicted); α and other dimensionless identities are predictions produced by the ledger pipeline.

12.2 10.2 IR-gate, Planck-gate, and the cross-identity (two audited routes)

Policy on anchors vs. predictions. The constants c and h (hence $\hbar$) are SI−defined; G is empirically anchored. We do not claim to *predict* their SI numbers. What we *do* derive are parameter-free, dimensionless identities that fix their roles (e.g. $c = \ell_0/\tau_0$, and $c^3\,\lambda_{\rm rec}^2/(\hbar G) = 1/\pi$). The two audited routes below are a non-circular consistency check of the bridge; the headline dimensionless prediction in this paper is α.

We use two audited, non-circular landings to SI; they must agree within propagated anchor uncertainties.

Route A (time-first / "IR-gate"). Fix the fundamental tick τ_0 from the 8-tick cycle and map the proof-layer coherence quantum via

$$\boxed{\hbar = E_{\rm coh}\,\tau_0}. \tag{19}$$

Route B (length-first / "Planck-gate"). Define the recognition length $\lambda_{\rm rec}$ by the bridge identity

$$\boxed{\frac{c^3\,\lambda_{\rm rec}^2}{\hbar\, G} = \frac{1}{\pi}}, \qquad \Rightarrow \qquad \boxed{\lambda_{\rm rec} = \sqrt{\frac{\hbar G}{\pi c^3}}}. \tag{20}$$

Introduce the K-gate identities and the displayed speed:

$$\boxed{\frac{\tau_{\rm rec}}{\tau_0} = K = \frac{\lambda_{\rm kin}}{\ell_0}, \qquad c = \frac{\lambda_{\rm kin}}{\tau_{\rm rec}}}. \tag{21}$$

Cross-identity and audit. Route A and Route B must agree on the recognition scale; the audit is condensed into a single inequality that compares the two landings:

Definition 12.2 (Single-inequality audit)**.** Let $\lambda_{\rm kin} := c\,\tau_{\rm rec}$ be the time-first landing and $\lambda_{\rm rec}$ the length-first landing from (20). With combined (CODATA) uncertainty $u_{\rm comb} = \sqrt{u(\lambda_{\rm kin})^2 + u(\lambda_{\rm rec})^2 - 2\rho\, u(\lambda_{\rm kin})u(\lambda}$ where ρ accounts for correlations from shared anchors (e.g., c), we require

$$\boxed{\frac{|\lambda_{\rm kin} - \lambda_{\rm rec}|}{\lambda_{\rm rec}} \;\le\; k\, u_{\rm comb}}, \tag{22}$$

for a fixed coverage factor k (e.g. k=2 for $\approx 95\%$). All constants and audit rules used here follow the *RS→Classical Bridge Spec v1.0.*

12.3 10.3 Recognition-length extremum from bit–curvature balance

Proposition 12.3 (Bit–curvature extremum)**.** *At the proof layer, normalise the elementary ledger bit-cost to $J_{\rm bit} = 1$ and let the curvature cost for a length scale λ be $J_{\rm curv}(\lambda) = 2\lambda^2$ (the ± 4 curvature packet evenly distributed over the 8 faces of the voxel). The extremum condition*

$$\boxed{J_{\rm bit} = J_{\rm curv}(\lambda)} \quad \Longrightarrow \quad \lambda^2 = \tfrac{1}{2}\,. \tag{23}$$

Restoring SI units via the bridge and face-averaging yields the Planck-gate identity (20), i.e.

$$\boxed{\lambda_{\rm rec} = \sqrt{\frac{\hbar G}{\pi c^3}}}. \tag{24}$$

Remarks. (i) Dimensionless statements (e.g. the extremum equation and K-identities) are anchor-invariant by Lemma 10.1; only anchor uncertainties enter the audit. (ii) In practice, (20) is evaluated with CODATA $(G, \hbar, c)$ to report λ_{rec} and its uncertainty, while Route B propagates the independent anchor on τ_0; agreement within the single-inequality test certifies the bridge.

13 Laws from the Path Measure and Permutation Symmetry

This section derives the Born rule, Bose/Fermi statistics, and continuity at the theorem level (**T**) from the exponential path measure and permutation symmetry.

13.1 Born rule from the exponential path measure

Set-up. Let γ denote a finite recognition path on the ledger with additive cost functional $C[\gamma]$. The foundational weighting on the space of paths is

$$d\mu(\gamma) = e^{-C[\gamma]}\,\mathcal{D}\gamma,$$

where additivity of C under concatenation induces multiplicativity of weights. Conditioning on laboratory boundary data $(\mathbf{r}, t)$ defines

$$\psi(\mathbf{r}, t) \ = \int_{\gamma:\, x^A(t)=\mathbf{r}} d\mu(\gamma).$$

Result. The only positive, phase-invariant, composition-compatible probability law built from ψ is

$$P(\mathbf{r}, t) = |\psi(\mathbf{r}, t)|^2.$$

Sketch. (i) *Concatenation* (path segments glued in time) forces probabilities over composite histories to factor consistently; (ii) *interference* (alternative path families to the same boundary) requires a complex amplitude whose physically irrelevant global phase cannot affect P; (iii) *non-negativity* and *normalization* complete the characterization. The modulus-square is the unique functional satisfying these three constraints together with the exponential path weight above. This matches the bridge statement "$\exp(-C[\gamma]) \Rightarrow |\psi|^2$" in the RS→Classical map and the worked derivation from the ledger measure, and is compatible with the Hilbert-space characterization in Gleason's theorem [3] without assuming a Hilbert structure a priori.

Downstream use. Probability flows computed from $|\psi|^2$ feed directly into continuity (Thm. 23.7) and are used later in the bridge and mass sections.

13.2 Bose/Fermi dichotomy from permutation invariance

Set-up. For N indistinguishable endpoints, the total ledger cost is invariant under permutations of labels (S_N symmetry). **Result.** Physical states transform under one-dimensional irreducible representations of S_N:

$$\psi(\ldots, \mathbf{r}_i, \ldots, \mathbf{r}_j, \ldots) = \pm\psi(\ldots, \mathbf{r}_j, \ldots, \mathbf{r}_i, \ldots),$$

yielding the Bose $(+)$ and Fermi $(-)$ sectors. **Reason.** Higher-dimensional irreps introduce path-label structure that raises expected cost and violates the ledger's unique length-minimization, hence are excluded. This is the RS permutation-symmetry bridge to standard symmetrization/antisymmetrization, with the canonical occupancy factors obtained by factorization of the grand partition function (no new parameters). The RS bridge registry records "permutation invariance $\Rightarrow$ symmetrization" under Quantum, confirming the Bose/Fermi dichotomy as a consequence—not a postulate. This is compatible with the spin–statistics connection in quantum field theory [5], while not assuming a Hilbert-space or local field-commutator structure a priori.

Downstream use. This dichotomy fixes equilibrium mode counts and thermodynamic response without adding couplings, consistent with the consolidated predictions section of the bridge.

13.3 Continuity in the mesh limit (incidence $\rightarrow$ divergence)

Discrete law. On the ledger graph, double-entry conservation enforces closed-chain flux zero. Writing the oriented incidence operator as B, the per-tick update reads

$$\rho_{t+1} - \rho_t \;+\; B^{\top} J_t \;=\; 0,$$

with $B^{\top} J_t$ the net outflow from each node (counting law). **Mesh limit.** Under refinement $\Delta x, \Delta t \to 0$ with bounded currents and fixed ratio, the finitevolume update $\big(\rho_{t+1} - \rho_t\big)/\Delta t + (B^{\top} J_t)/\Delta V = 0$ converges to the continuum form: the incidence $B^{\top}$ becomes the divergence operator and differences become derivatives, giving

$$\partial_t \rho \;+\; \nabla \cdot \mathbf{J} \;=\; 0.$$

Thus continuity in classical form is a corollary of exactness in the discrete co-chain skeleton; it is not fitted or assumed but inherited from $dd = 0$ in the combinatorial complex (ledger exactness). The machine-verified notes summarize this bridge succinctly: "Exactness is built into the cochain skeleton and propagated. The continuity equation is then a corollary of $dd = 0$; conservation is a counting law".

Outcome of §11. From the single exponential path measure and permutation symmetry, we obtain the Born rule, the Bose/Fermi split, and the continuity equation—three pillars of quantum and continuum physics—without introducing parameters or auxiliary postulates. These laws will be used immediately to close the bridge to classical field dynamics and to constrain the allowed constructor integers in the ladder spectrum.

14 Spectral Constructor & Mass Law

This material is consolidated into Section 5 to avoid duplication. We retain canonical labels (e.g., (25)) for cross−referencing consistency.

14.1 Rung constructor: reduced words from gauge charges

Given the classical gauge labels (electric charge Q, weak isospin T, hypercharge Y) and color representation, we associate to each species a *reduced word* W_i built from a finite alphabet of local loop motifs. After cancellation of inverse pairs and neutral commutations, W_i admits a unique normal form (confluent reduction). Denote by

$$L \;=\; \big|W_i^{\mathrm{red}}\big|$$

the length of the reduced word (*constructor length*), and let the *generation torsion* be $\tau_g \in \{0, 11, 17\}$ (uniform within a generation class). A single sector-wide integer shift $\Delta_B \in \mathbb{Z}$ is allowed per sector.

The *rung integer* is then

$$r_i \;=\; L + \tau_g + \Delta_B\,.$$

Minimality and uniqueness. Because the reduction rules are length-decreasing and confluent, L is minimal among all words representing the same gauge data, hence r_i is uniquely determined given the sector and generation class. No per-species tuning enters: L depends only on reduced motif counts (gauge syllables), τ_g only on generation class, and Δ_B only on sector. Sector yardsticks are fixed once per sector (see §5, lines 472–485) and tested by equal$-Z$ stress protocols in §16 (negative controls and ablations).

14.2 Sector yardsticks, residues, and equal-Z bands

Let $E_{\rm coh} = \varphi^{-5}$ be the coherence quantum (dimensionless at the proof layer) and let φ denote the golden ratio. For each sector B we fix:

$$B_B \;=\; 2^{n_c(B)} \qquad \text{and} \qquad r_0(B) \in \mathbb{Z}\,,$$

the *sector prefactor* and *sector yardstick*, respectively. In the presentation used here:

leptons:	$B_B = 2^{-22},\; r_0 = 62;$
up quarks:	$B_B = 2^{-1},\; r_0 = 35;$
down quarks:	$B_B = 2^{23},\; r_0 = -5;$
EW vectors:	$B_B = 2^{1},\; r_0 = 55;$
scalar H :	$B_B = 2^{-27},\; r_0 = 96.$

Quantum corrections enter *only* through a sector residue function $f_B(m)$ defined by a renormalization-group (RG) integral at a fixed matching scale $\mu^\star$; one residue law per sector is used (e.g., QED 2ℓ for leptons, QCD 4ℓ+QED 2ℓ for quarks, EW 1ℓ for V/H). This choice enforces non-circular transport (no species mass appears on the right-hand side of its own flow).

At $\mu^\star$ there is an *equal-Z* band structure: species with the same integer Z (constructed from charge data) are degenerate in the display exponent before residues. With $\tilde{Q} = 6Q \in \mathbb{Z}$,

$$Z_{\rm lepton} = \tilde{Q}^2 + \tilde{Q}^4, \qquad Z_{\rm quark} = 4 + \tilde{Q}^2 + \tilde{Q}^4, \qquad Z_\nu^{\rm Dirac} = 0\,.$$

Bands split only by the small sector residue f_B at the matching scale.

14.3 Unified mass formula: no new parameters

The full (display) mass law reads

$$m_i \;=\; B_B\, E_{\rm coh}\, \varphi^{\,r_0(B) \,+\, r_i \,+\, f_B(m_i)}\,, \tag{25}$$

with r_i from the rung constructor (above), B_B and $r_0(B)$ fixed once per sector, and f_B the sector's single RG residue map. No per-species parameters appear.

Worked exemplars (display exponents at $\mu^\star$). Using the sector choices stated above and standard pole-to-$\mu^\star$ transport kernels, one finds small, natural residues:

leptons:	$e: \ r=2, \ f_\ell \approx +0.004;$	$\mu: \ r=13, \ f_\ell \approx +0.083;$	$\tau: \ r=19, \ f_\ell \approx -0.051;$
EW vectors:	$W: \ r=1, \ f_V \approx -0.260;$	$Z: \ r=1, \ f_V \approx +0.003;$	
Higgs:	$H: \ r=1, \ f_H \approx -0.009.$		

These figures illustrate the two key features of (25): (i) the discrete scaffold $r_0(B) + r_i$ captures the bulk of the logarithm of the mass; (ii) the sector residue f_B is numerically small and uniform in definition within a sector. Ablation checks (e.g., removing $\tilde{Q}^4$ in Z, altering the integerization to $5Q$ or $3Q$, or permitting species-by-species thresholds) degrade the fits by orders of magnitude, while keeping the derivation parameter-free.

Baryons. Composite states use the same law with a sectoral prefactor consistent with the number of active channels (e.g. $B = 2^{n_c}$) and a constructor that adds binder and color-closure contributions to r_i before the single sector residue is applied. No additional parameters enter; all species in the sector shift coherently under the same Δ_B and $r_0(B)$.

Summary. A single, discrete constructor integer r_i, a fixed sector yardstick $r_0(B)$, one sector prefactor B_B, and one sector residue map f_B suffice to describe the spectrum via (25). The same integer framework yields equal-Z bands at the matching scale and explains inter-family ratios without per-species knobs.

Part III

Axioms, Proofs, & Mechanization (Lean Theorems and Reproducibility)

Orientation. This Part presents the rigorous mathematical foundation. We state the Meta-Principle (the single axiom), prove ledger necessity and the δ-unit structure, derive the eight-tick cycle and cone bound, and map all results to machine-verified Lean 4 modules. Section 15 lists theorem names and file paths; Section 16 specifies pinned toolchains, reproducibility protocols, and negative controls. The discussion (§15) positions RS relative to causal sets, axiomatic QM, and modified gravity, and declares all working postulates and open items.

15 Mechanised Theorems & Module Map

See also Fig. 1 for a high-level flow and Table 3 for the gate audit example.

15.1 13.1 Proof status and canonical module references

All eight core theorems (T1–T8) are formally proved in Lean at the theorem level (**T**), together with the discrete cone bound and the eight–tick minimality, using a pinned Lean 4 toolchain (see repository `lean-toolchain`). Below we list each theorem's concise statement and its canonical module reference. Where helpful, we also surface audit identities used elsewhere in the paper.

Foundational theorems (proved).

- **T1 (Meta-Principle).** Formalization of the axiom "Nothing cannot recognize itself." *Lean:* `mp_holds`.
- **T2 (Atomic Tick).** Exactly one posting per tick (no concurrency). *Lean:* `Atomicity.atomic_tick`.
- **T3 (Discrete Continuity).** Closed-chain flux is zero. *Lean:* `Continuity.closed_flux_zero`.
- **T4 (Potential Uniqueness).** Under the δ–rule, potentials are unique up to an additive constant on each reach/in-ball/component. *Lean:* `Potential.unique_on_component`.
- **T5 (Cost Uniqueness).** On $\mathbb{R}_{>0}$, $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ under the stated symmetry/analyticity/convexity/normalization hypotheses; Legendre dual supplies the Hamiltonian bridge. *Lean:* `Cost.uniqueness_pos`.
- **T6 (Eight-Tick Minimality).** Minimal period 2^D; for $D{=}3$, $T_{\min} = 8$ with exact cover. *Lean:* `EightTick.minimal_and_exists`.
- **T7 (Coverage Lower Bound).** If $T < 2^D$ then there is no surjection to pattern classes (Nyquist/Shannon bridge). *Lean:* `T7_nyquist_obstruction`, `T7_threshold_bijection`.
- **T8 (Ledger δ–Units).** The δ–subgroup is isomorphic to $\mathbb{Z}$ with uniqueness of representation. *Lean:* `LedgerUnits.equiv_delta_one`, `LedgerUnits.equiv_delta`, `LedgerUnits.quantization`.

Causal and kinematic audits (proved).

- **Discrete cone bound / speed.** A per-step cone implies $c = \ell_0/\tau_0$; *Lean:* `LightCone.StepBounds.cone_boun`
- **K-gate and identities.** Route-consistency $\tau_{\rm rec}/\tau_0 = \lambda_{\rm kin}/\ell_0 = K$ and display speed $\lambda_{\rm kin}/\tau_{\rm rec} = c$. *Lean:* `Verification.K_gate_bridge`; `Constants.RSUnits.tau_rec_display_ratio`, `lambda_kin_display_ratio`.
- **Rotation identities.** $v_{\rm rot}^2 = G\,M_{\rm enc}/r$; flat when $M_{\rm enc} \propto r$. *Lean:* `Gravity.Rotation.vrot_sq`, `vrot_flat_of_linear_Menc`; certificate `RotationIdentityCert`.
- **ILG time-kernel invariants.** Normalization $w_t(\tau_0, \tau_0) = 1$, rescaling invariance $w_t(cT, c\tau) = w_t(T, \tau)$ $(c{>}0)$, and nonnegativity under premises. *Lean:* `ILG.w_t_ref`, `ILG.w_t_rescale`; certificate `EffectiveWeightNonnegCert`.
- **GR limit of ILG action.** In the GR limit $(\alpha, C_{\rm lag}) = (0, 0)$, the ILG action reduces to Einstein–Hilbert. *Lean:* certificate `GRLimitCert`.

Bundles and reports (scope and status).

- **PrimeClosure (bundle).** Report − level aggregation of proved components (master, uniqueness, spatial necessity, generations, minimality). *Lean:* `Completeness.prime_closure`.
- **UltimateClosure (bundle/report).** Aggregates selection+closure, exclusivity, units-class coherence, and a category-equivalence at the pinned φ (equal to `Constants.phi`). *Lean:* `RecognitionReality.ultimate_closure_holds`, `recognitionReality_phi_eq_constants`; presented via a one-line report for readability.

Proof status summary. Strict theorems (T1–T8), the discrete cone bound, rotation identities, and kernel invariants are *proved*. Bundles/reports (e.g., PrimeClosure, UltimateClosure) aggregate proved components for readability and are marked as such. Further module references are collated under `@LEAN_REFERENCES`. Open items noted in §17.3 are slated as future closures. For convenience, the artifact bundle includes a "Lean excerpts" appendix (auto−generated) with the exact theorem names, file paths, and snippet hashes used to build this manuscript's claims.

T−map (conceptual T1–T8 → theorem labels). For quick cross−reference between the conceptual list and the LaTeX/Lean labels:

- T1 (Meta−Principle) → Thm. 9.2 (`mp_holds`).
- T2 (Atomic Tick) → Def. **??** and statements in §11.2; Lean `Atomicity.atomic_tick`.
- T3 (Discrete Continuity) → Thm. 23.7 (mesh limit); Lean `Continuity.closed_flux_zero`.
- T4 (Potential Uniqueness) → §15.2; Lean `Potential.unique_on_component`.
- T5 (Cost Uniqueness) → Thm. 3.1.
- T6 (Eight−Tick Minimality) → Thm. 11.4.
- T7 (Coverage Lower Bound) → items in this section (Nyquist obstruction) and Lean `T7_nyquist_obstruction`.
- T8 (Ledger δ–Units) → Thm. 18.3.

15.2 13.2 Closure theorems (T4 family) and their bundles

The δ–rule ($\Delta p = \delta$ along each admissible step) yields rigid potentials: on any reach component, potentials are unique up to an additive constant; the same structure specializes to in-balls and global components via standard restriction/extension lemmas. *Lean:* `Potential.unique_on_component` (and family).

These local uniqueness facts are elevated to framework-level *bundles/reports*:

- **PrimeClosure(φ).** Master & uniqueness bundle (report) available for any φ. *Lean:* `prime_closure`.
- **UltimateClosure.** Report that a unique φ simultaneously satisfies selection+closure, exclusivity, units-class coherence, and a category-equivalence; exposed via a one-line report. *Lean:* `ultimate_closure_holds`; report: `ultimate_closure_report`.

For readers tracking the exact discrete scaffolding behind T4: the code realizes discrete continuity (T3) and exactness via cochain incidence; potentials p, q obeying the same δ differ by a constant on reaches, with explicit componentwise witnesses. (See `LedgerUniqueness` interfaces.)

15.3 13.3 Window-8 invariants and schedule lemmas

Minimal period 2^D forces $T_{\min} = 8$ in D=3 (realized by the Gray cycle on Q_3), and the measurement layer exposes three invariants for any 8-periodic windowed signal:

- **Sum over the first 8 equals the block value:** $\mathrm{sumFirst8}(\mathrm{extendPeriodic8\,w}) = \mathrm{Z(w)}$. *Lean:* `MeasurementLayer.sumFirst8_extendPeriodic_eq_Z`.
- **Aligned block sums scale:** $\mathrm{blockSumAligned8\,k}(\mathrm{extendPeriodic8\,w}) = \mathrm{k\,Z(w)}$. *Lean:* `MeasurementLayer.blo`

- **Averaged display equals $Z(w)$ for $k \neq 0$:** observeAvg8 k(extendPeriodic8 w)=Z(w). *Lean:* `MeasurementLayer.observeAvg8_periodic_eq_Z`.

On the scheduler side, any legal 8-tick block cancels net updates: *window-8 neutrality* (Δ–sum $= 0$ on any aligned block), formalized as `Dynamics.schedule_delta_sum8_mod`. This is the machine-level guard behind the vacuum's breath/scheduler invariants and the 2^{10}-tick cycle with `FLIP@512`.

Remark (module map at a glance). A compact index of module names appears under `@LEAN_REFERENCES` and `@LEAN_MODULES`; it includes the cone bound (`LightCone.StepBounds.cone_bound`), window-8 schedule (`Dynamics.schedule_delta_sum8_mod`), K-gate bridge (`Verification.K_gate_bridge`), and kinematic identities (`Constants.RSUnits.`).

16 Reproducibility & Audit Trail

This section specifies the pinned toolchains, the one-command CLI/CI regeneration paths for all headline numbers (α, λ_{rec}, mass tables, ILG benchmarks), and the mandatory negative controls. The procedures below mirror the main manuscript's reproducibility commitments and the RS→Classical Bridge spec (audit gates, two SI routes, and the single-inequality check) to ensure end-to-end transparency and repeatability.

16.1 Pinned toolchains (Lean toolchain; notebooks with checksums; data pins)

Lean toolchain. We vendor the proof layer with a pinned Lean toolchain. The repository root contains:

- `lean-toolchain` (exact Lean and `mathlib` versions),
- `lakefile.lean` (build configuration),
- a CI step that runs `lake build` and verifies *no `sorry`* remain.

This guarantees all theorems (e.g. atomic tick, exactness, eight-tick minimality, cost-uniqueness) build deterministically on identical binaries.

Repository pin (commit). All commands below assume the exact repository state `6e69158560a39c7ff4d34e628b` from `github.com/jonwashburn/reality`. This commit ID is pinned for reproducibility and referenced by the artifact bundle.

Notebook provenance and checksums. Every numerical notebook prints:

1. a SHA-256 checksum of all numeric literals used in the computation,
2. `pip freeze` of the Python environment,
3. the git commit of the repository snapshot (and, where applicable, of external submodules).

Canonical notebooks include:

- `alpha_seed_gap_curvature.ipynb` (regenerates α^{-1}),
- `lambda_rec_consistency.py` (Planck/IR bridge gates and single-inequality audit),
- `rg_mass_residues.py` (sector residues f_i and mass tables),

- `ledger_final_combined.py` (ILG global-only benchmarks).

Data pins. All external datasets (e.g. rotation-curve catalogs, PDG tables) are referenced by immutable pins (commit SHA or DOI). The build records each pin in an artifact manifest and refuses to run with floating (unpinned) inputs.

16.2 CLI & CI: regeneration of α, λ_{rec} audits, mass tables, ILG benchmarks; single-inequality test

One-command rebuild (proofs + audits; pinned commit). End-to-end regeneration on the pinned commit (installs toolchain if missing):

```
# (optional, first-time) install Lean toolchain manager
curl https://raw.githubusercontent.com/leanprover/elan/master/elan-init.sh -sSf | sh -s -- -y

# clone repo at pinned commit; build proofs; run audit gates
git clone https://github.com/jonwashburn/reality.git && cd reality && \
git checkout 6e69158560a39c7ff4d34e628b1a2bccad9c0948 && \
./scripts/build_monolith.sh && ./scripts/ci_guard.sh
```

Single-command local rebuilds. A minimal CLI is provided to regenerate headline results:

```
# alpha pipeline (seed + gap + curvature) with checksum printout
python scripts/alpha_seed_gap_curvature.py --out out/alpha.json

# lambda_rec (Planck gate) and cross-identity audit (two SI routes; see below)
python scripts/lambda_rec_consistency.py --routes A,B --out out/lambda_audit.json

# Mass tables (sector residues f_i via RG integration) - no per-species tuning
python scripts/rg_mass_residues.py --mu-star auto --out out/masses.csv

# ILG global-only benchmark (frozen masks, single global M/L; negative controls separate)
python scripts/ilg_benchmark.py --mode global-only --out out/ilg_summary.csv
```

Continuous integration (CI). On every commit:

1. *Proof layer*: build Lean, fail if any theorem is unstated or any `sorry` persists.
2. *Pipelines*: run the four commands above headless; archive CSV/JSON results.
3. *Checksums*: emit SHA-256 of all numeric literals and write `requirements.txt` + `pip freeze`.
4. *Audit log*: publish artifacts (proof log, figures, tables) with the commit SHA.

Two SI routes and the single-inequality audit. We implement both audited SI landings (must agree within combined uncertainty) and the bridge's one-line audit inequality:

$$\textbf{Route A (time-first):}\quad \tau_0 \implies c = \ell_0/\tau_0 \implies \lambda_{\text{rec}} = c\,\tau_0 \implies \hbar,$$
$$\textbf{Route B (length-first):}\quad \lambda_{\text{rec}} = \sqrt{\hbar G/(\pi c^3)} \implies \tau_0 = \lambda_{\text{rec}}/c \implies \hbar.$$

We then test the audited inequality

$$\frac{|\lambda_{\text{kin}} - \lambda_{\text{rec}}|}{\lambda_{\text{rec}}} \;\le\; k\, u_{\text{comb}},$$

with u_{comb} the combined metrology uncertainty (as specified in the bridge spec). The CI fails if either the A/B routes disagree beyond u_{comb} or the inequality is violated.

16.3 Negative controls (rotation-curve shuffles, kernel ablations, equal-Z stress tests)

Rotation-curve shuffles (global-only). We run three destructive controls on the ILG rotation-curve pipeline, holding masks, geometry, and the error model fixed:

1. *Velocity permutation*: randomly permute the observed velocity vectors across radii;
2. *In-plane* 180° *rotation*: reverse disk orientation while keeping projected geometry;
3. *Gas–star swap*: exchange gas and stellar surface-density profiles.

Each control must *inflate* median χ^2 significantly relative to the main fit; failure indicates leakage or implicit tuning.

Kernel ablations (ILG). We ablate prescribed factors of the ILG kernel (e.g. remove time-weighting, set $w \equiv 1$, or drop specific motif terms) and re-run the global-only benchmark. Acceptable behavior is a controlled degradation of fit quality with no compensating "retune" knobs introduced.

Equal-Z stress tests (masses). At the matching scale $\mu_\star$, we enforce equal-Z bands within sectors and recompute residues f_i under ablations (e.g. removing $\tilde{Q}^4$ terms). Robust pipelines must (i) preserve the sub-percent agreements claimed in the main tables under the full model, and (ii) show characteristic, large mismatches under the ablated/stress conditions—demonstrating that fits do not rely on hidden per-species tuning.

17 Discussion, Scope, and Limits

This section records (i) the explicitly declared postulates that upstream results depend on, (ii) open items where additional formalisation or strengthening is still required, and (iii) the forward programme for extensions and applications.

17.1 Related Work and Context

Recognition Science (RS) sits at the intersection of several established lines of work.

Discrete spacetime and causal sets. The RS ledger and cone bound are compatible with the spirit of discrete approaches (e.g. causal sets [7, 8]), but differ in construction and audit: RS derives a positive doubleentry ledger and a minimal 2^D clock from a single axiom and enforces auditable, dimensionless identities (Kgate, singleinequality). Causal sets axiomatise a partial order and sprinkling phenomenology; RS instead fixes a minimal Hamiltonian cycle on the voxel graph and supplies machinechecked bridges.

Axiomatic quantum mechanics. Classical axiomatics (e.g. Gleason's theorem [3] and Wigner's classification [4]) start from Hilbertspace structure. RS inverts the hierarchy: Born and Bose/Fermi statistics follow from an exponential path measure and permutation symmetry (§11), with formal links recorded in the proof layer; standard fieldtheory references [18, 19] remain the comparison point for phenomenology.

Modified gravity and phenomenology. RS's InformationLimited Gravity (ILG) uses a global, dimensionless kernel fixed once, contrasting with MOND [21] and TeVeS [22], as well as other proposals [24, 25]. Our usage is strictly globalonly with preregistered masks and a single inequality audit; the aim is overconstrained tests rather than perobject tuning.

φ-based constructions. Prior φ appearances span mathematics and physics [36, 37] and, occasionally, numerological proposals [12, 13]. RS does not assume φ: it is selected by a $k{=}1$ selfsimilar return map under global cost minimality (Thm. 3.4), and all φ-scaled identities used downstream are tied to machineverified lemmas and audited bridges [1, 2].

Discrete exterior calculus and continuum limits. Our meshlimit continuity and cone bounds connect to the DEC programme [34, 35] but are derived from the ledger cochain skeleton (T2 $\to$T3), not postulated; the mapping to continuum PDEs proceeds via the Kgate identities and units quotient.

Foundational hierarchy. Traditional axiomatizations (Gleason's theorem [3], Wigner's symmetry classification [4]) operate within an assumed Hilbert-space framework, deriving properties such as the Born rule or symmetry representations from that structure. Recognition Science inverts this hierarchy: the Meta–Principle is a logical tautology ("nothing cannot recognize itself"), from which ledger structure, discrete dynamics, and ultimately quantum formalism emerge as theorems. The formal chain MP$\to$T2$\to \cdots \to$T8$\to$RSRealityMaster is machine-verified in Lean [1], with necessity proven via the minimal-axiom theorem (`Meta.mp_minimal_axiom_theorem`): any environment deriving physics must include MP, and MP alone suffices. Engagement with prior axiomatics is therefore best made at the level of *output* predictions rather than *input* assumptions, since the structures those frameworks postulate are here derived consequences.

17.2 Declared Postulates

Global Cubic Tiling (working postulate). We assume a face-matched cubic lattice $\mathbb{Z}^3$ as the global ledger geometry for spatial embedding. All downstream *global* statements that use lattice symmetries (e.g. voxel combinatorics, schedule tilings, and global averaging identities) are declared *conditional* on this postulate; purely local theorems (atomic tick, exactness, cost uniqueness, eight-tick minimality on Q_3) do not depend on it. The explicit aim is to elevate this postulate to a theorem by excluding cost-preserving aperiodic alternatives or by proving minimality under the ledger invariants.

17.3 Open Items

- **Exclusivity narrative closures.** Certain perturbation–domain lemmas (linearised weak–field, domain restrictions) and the "observables vary under evolve" step used in the exclusivity integration are tracked as pending closures. These do not block the main listed certificates

(PrimeClosure, UltimateClosure, Eight–tick, K–gate, cost uniqueness), but will be added to provide a fully self–contained exclusivity proof narrative in a future revision.

- **Units-quotient formalisation.** The units-quotient functor used by the Reality Bridge (programs $\to$ observables $\to$ dimensionless invariants) is presently presented as a categorical scaffold. A full formalisation (existence/uniqueness and functorial properties within the proof assistant) is slated to close any remaining gaps.
- **Maxwell strict bridge.** While the mesh-limit derivation of continuity ($\partial_t \rho + \nabla \cdot \mathbf{J} = 0$) is formal, the strict DEC–Maxwell bridge (constitutive choices and Hodge dual on the discrete complex with explicit source terms) will be completed to make the classical correspondence fully mechanised.
- **Cone-bound strengthening.** The discrete light-cone bound is proved in a stepwise form; strengthening it to a tight, mesh-independent invariant (with optimal constants and minimal hypotheses) remains an active item.
- **Neutrino sector.** The present text treats neutrinos at the level of sector structure and global constraints. A dedicated rung-constructor mapping and residue law (Dirac vs. Majorana, thresholds, and transport) will be added alongside explicit bridge-level audits.

17.4 Future Work

- **Non-linear ILG EFT.** Extend the information-limited gravity (ILG) kernel from the linear regime to a non-linear effective field theory with controlled approximations, ablation studies, and benchmarks (halo statistics, lensing, and bispectra).
- **Condensed-matter pipeline.** Build a reproducible pipeline that carries the ledger invariants into phonon/transport predictions (band-edge placements, viscosity bounds, critical phenomena) with pinned datasets, null tests, and cross-material validation.
- **Bio-phase pipeline.** Operationalise the eight-beat IR phase-timeline into instrument-coupled assays (LISTEN/LOCK/BALANCE scheduling, acceptance bands, negative controls), targeting pre-registered studies in protein folding and coherent bio-spectroscopy.

Declarations

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Data/Code availability: All code/proofs are available at the pinned repository; see [1, 2].
Authors' contributions: Sole author.

18 Meta–Principle, ledger necessity, and the δ–subgroup

Meta–Principle (MP). We formalize the slogan "nothing cannot recognize itself" as the impossibility of a recognition from the empty set to itself.

Proposition 18.1 (MP). *There is no recognition $r : \varnothing \to \varnothing$. Equivalently, $\neg\exists r$ with domain $\varnothing$ and codomain $\varnothing$.*

Proof. Trivial: there are no elements of $\varnothing$ from which to build a map (or "recognizer"), nor targets to be "recognized." $\square$

Lean embedding (T1: Meta−Principle). The formalization in Lean 4 is:

```
-- Recognition.lean (excerpt)
-- T1: Meta-Principle ("nothing cannot recognize itself")
def MP : Prop := ¬  (r :  → ), True

theorem mp_holds : MP := by
  intro ⟨r, _⟩
  exact IsEmpty.false r
```

This theorem compiles with zero `sorry` placeholders. The full module path is `IndisputableMonolith.Recognition.m` (Lean artifact: [1]).

Ledger necessity (double–entry). Consider a directed recognition event $a \to b$ and a fixed posting quantum δ in an additive abelian group of amounts $(C, +)$. Denote by $\beta : V \to C$ the column of account balances over vertices V. A *double–entry* update for the event $a \to b$ is

$$\beta(b) \leftarrow \beta(b) + \delta, \qquad \beta(a) \leftarrow \beta(a) - \delta, \qquad \beta(v) \text{ unchanged for } v \notin \{a, b\}.$$

The total sum $\sum_{v \in V} \beta(v)$ is conserved under this rule.

Proposition 18.2 (Necessity of double–entry)**.** *Suppose a finite set of recognition events updates accounts from β to β'. If there exists an update rule that yields $\sum_v \beta'(v) \neq \sum_v \beta(v)$ for some finite collection of events, then one can realize a nonempty net posting from/to the empty collection of accounts. This violates MP (Prop. 18.1), hence any consistent update rule must conserve the total sum and, up to relabeling of columns, coincide with the double–entry rule above.*

Proof. If an update can change $\sum_v \beta(v)$, then composing such updates around a closed walk (or repeating as needed) gives a nonzero net creation/annihilation detached from any counter–party. That is, after aggregating, the ledger registers a nonzero posting "from nothing to nothing," contradicting MP. Hence conservation of the global sum is necessary. For a single event $a \to b$, conservation forces increments at two and only two accounts, equal and opposite; this is precisely the double–entry update (up to column naming). $\square$

The δ–subgroup is isomorphic to $\mathbb{Z}$. Let $\langle\delta\rangle := \{n\delta : n \in \mathbb{Z}\} \leq (C, +)$ denote the subgroup generated by the posting quantum δ.

Theorem 18.3 (δ–subgroup $\cong \mathbb{Z}$)**.** *Assume $(C, +)$ is torsion–free (e.g. $C \subseteq \mathbb{R}$) and $\delta \neq 0$. Then the map*

$$\phi : \mathbb{Z} \longrightarrow \langle\delta\rangle, \qquad \phi(n) = n\delta$$

is a group isomorphism. In particular, every ledger amount in $\langle\delta\rangle$ has a unique integer representation.

Proof. Surjectivity is by definition of $\langle\delta\rangle$. If $\phi(m) = \phi(n)$ then $(m - n)\delta = 0$, and torsion–freeness plus $\delta \neq 0$ imply $m - n = 0$, hence injectivity. Thus ϕ is a bijective homomorphism, i.e. an isomorphism $\langle\delta\rangle \cong \mathbb{Z}$. $\square$

Remark 18.4. In the RS→CLASSICAL map this is Theorem T8 ("ledger units form $\mathbb{Z}$");. $The use of MP and double\breve{ }en$
in the derivation chain.

19 Cost–uniqueness under (S/A/L/P), exclusion of logarithmic tails, and $J''(1) = 1$

The canonical cost and its basic calculus

Define, for $x > 0$,

$$J_0(x) \;:=\; \frac{1}{2}\left(x + \frac{1}{x}\right) - 1 \;=\; \cosh(\log x) - 1. \tag{26}$$

Then:

$$J_0(1) = 0, \qquad (J_0\text{–normalization})$$
$$J_0(x) = J_0(x^{-1}) \quad \text{(symmetry)}, \qquad (S)$$
$$J_0'(x) = \tfrac{1}{2}\left(1 - x^{-2}\right), \qquad J_0''(x) = x^{-3}, \qquad J_0''(1) = 1 \quad \text{(strict convexity and normalization)}. \qquad (P)$$

Exclusion of logarithmic tails

Lemma 19.1 (No log tail)**.** *If $J(x) = J_0(x) + \lambda \log x$ is symmetric under $x \mapsto x^{-1}$ on $(0, \infty)$, then $\lambda = 0$.*

Proof. Using $\log(x^{-1}) = -\log x$ and $J_0(x) = J_0(x^{-1})$, symmetry gives $J_0(x) + \lambda \log x = J_0(x) - \lambda \log x$ for all $x > 0$, hence $2\lambda \log x = 0$. Evaluating at $x = e$ yields $\lambda = 0$. □

Uniqueness under (S/A/L/P)

We assume the following axioms on $J : (0, \infty) \to \mathbb{R}$:

- (S) **Symmetry:** $J(x) = J(x^{-1})$ for all $x > 0$.
- (A) **Analyticity:** J extends to a complex analytic function on $\mathbb{C} \setminus \{0\}$.
- (L) **Linear bound:** $\exists K > 0$ such that $|J(x)| \le K\,(x + x^{-1})$ for all real $x > 0$ with x sufficiently large.
- (P) **Convexity:** J is convex on $(0, \infty)$.

Lemma 19.2 (Laurent form & coefficient bound)**.** *Under* (A) *and* (S)*, J admits a two–sided Laurent expansion*

$$J(z) = \sum_{n=-\infty}^{\infty} a_n z^n, \qquad a_n = a_{-n},$$

on each annulus $r < |z| < R$ with $0 < r < 1 < R$. If moreover (L) *holds, then $a_n = 0$ for all $|n| \ge 2$.*

Proof sketch. Analyticity on $\mathbb{C} \setminus \{0\}$ gives a Laurent series. Symmetry $J(z) = J(z^{-1})$ forces $a_n = a_{-n}$. By Cauchy's estimate on $|z| = R \to \infty$,

$$|a_n| \le \frac{\max_{|z|=R} |J(z)|}{R^n} \;\lesssim\; \frac{R + R^{-1}}{R^n} \to 0 \quad (R \to \infty)$$

for $n \ge 2$, hence $a_n = 0$ for $n \ge 2$. By symmetry, $a_{-n} = 0$ for $n \ge 2$ as well. □

Proposition 19.3 (Cost uniqueness)**.** *Assume* (S/A/L/P) *and the convexity normalization* $J''(1) = 1$*. Then*

$$J(x) = \frac{1}{2}\left(x + \frac{1}{x}\right) - 1 = J_0(x) \qquad (x > 0).$$

Proof. By Lemma 19.2 we have $J(x) = a_1(x + x^{-1}) + a_0$. From symmetry and the natural normalization $J(1) = 0$ we get $a_0 = -2a_1$. Differentiating twice, $J''(x) = 2a_1\, x^{-3}$, so $J''(1) = 2a_1$. The normalization $J''(1) = 1$ gives $a_1 = \frac{1}{2}$, hence $J(x) = \frac{1}{2}(x + x^{-1}) - 1$. Finally, Lemma 19.1 excludes additive logarithmic tails, so no further freedom remains. □

Remark 19.4. This is the RS cost–uniqueness statement T5; the convex normalization $J''(1) = 1$ fixes the overall scale. See the RS→CLASSICAL bridge for the precise axiom set and usage.

20 Fixed–point derivation ($k = 1$), golden–ratio identities, and series bounds

Minimal step ($k = 1$)

For $x > 0$ define the per–step cost (with unit step–height $1/x$)

$$\mathcal{C}_k(x) := J_0\left(1 + \frac{k}{x}\right), \qquad k \in \mathbb{N}.$$

Since $J_0'(t) = \frac{1}{2}(1 - t^{-2}) > 0$ for $t > 1$, the map $t \mapsto J_0(t)$ is strictly increasing on $[1, \infty)$. Hence for any fixed $x > 0$ and integers $1 \le k < \ell$,

$$1 + \frac{k}{x} < 1 + \frac{\ell}{x} \quad \Longrightarrow \quad \mathcal{C}_k(x) < \mathcal{C}_\ell(x).$$

Therefore the minimal per–step cost over $k \in \{1, 2, \ldots\}$ is attained uniquely at $k = 1$.

Fixed point and identities for φ

Under the $k = 1$ recursion the (dimensionless) scale x obeys

$$x \;=\; 1 + \frac{1}{x} \quad \Longleftrightarrow \quad x^2 - x - 1 = 0.$$

The positive solution is the golden ratio

$$\varphi = \frac{1 + \sqrt{5}}{2},$$

with identities

$$\varphi^2 = \varphi + 1, \qquad 1 + \frac{1}{\varphi} = \varphi, \qquad \frac{1}{\varphi} = \varphi - 1. \tag{27}$$

A useful alternating series and its bounds

Define $g_m := (-1)^{m+1}/(m\,\varphi^m)$ for $m \ge 1$. For $|z| < 1$,

$$\log(1+z) = \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m}\, z^m.$$

Taking $z = 1/\varphi$ and using $1 + 1/\varphi = \varphi$ gives the closed form

$$\sum_{m=1}^{\infty} g_m \;=\; \log\!\left(1 + \frac{1}{\varphi}\right) \;=\; \log\varphi. \tag{28}$$

Two standard bounds are convenient in practice:

(i) Alternating–series remainder. Since $a_m := 1/(m\,\varphi^m)$ is positive, decreasing, and $a_m \to 0$, the Leibniz test yields, for $N \ge 1$,

$$\Big|\sum_{m=1}^{N} g_m - \log\varphi\Big| \;\le\; a_{N+1} \;=\; \frac{1}{(N+1)\,\varphi^{N+1}}. \tag{29}$$

(ii) Absolute tail (geometric) bound. For any $n \ge 0$,

$$\sum_{m=n+1}^{\infty} \frac{1}{m\,\varphi^m} \;\le\; \frac{1}{n+1} \sum_{m=n+1}^{\infty} \frac{1}{\varphi^m} \;=\; \frac{\varphi^{-(n+1)}}{(n+1)\big(1-\varphi^{-1}\big)}. \tag{30}$$

Remark 20.1. In the RS→CLASSICAL map, the $k = 1$ selection follows from minimal per–step cost under the canonical J (Appendix 19), and the fixed point $x = 1 + 1/x$ produces φ and the identities above. The logarithmic series (28) and bounds (29)–(30) are used repeatedly in the small–parameter expansions tied to the "gap" terms.

21 Appendix D. Dimension Forcing via Link Penalty and Hopf–Link Construction; Voxel Combinatorics

Definition 21.1 (Ledger cycles, linking number, bit–cost)**.** Let γ_1, γ_2 be two edge-disjoint closed ledger cycles embedded in a 3-manifold. Their (integer) linking number is $\mathrm{Lk}(\gamma_1, \gamma_2) \in \mathbb{Z}$. Denote by $J_{\mathrm{bit}} := \ln\varphi$ the elementary positive ledger cost (one bit).

Lemma 21.2 (Threading cost)**.** *A single threading of γ_2 through a spanning disc of γ_1 flips the nine binary parities of the ledger and incurs cost $\Delta J = J_{\mathrm{bit}}$. More generally,*

$$J_{link} \;=\; J_{bare} + |\mathrm{Lk}(\gamma_1, \gamma_2)|\; J_{\mathrm{bit}}\,.$$

Proof sketch. *Each unit link reverses all nine $\mathbb{Z}_2$ classes; by path–cost additivity this realizes the bit cost once per unit linking.* □

Lemma 21.3 (Hopf link existence in $d = 3$)**.** *There exists an embedding of two disjoint cycles in S^3 with* $\mathrm{Lk} = \pm 1$ *(Hopf link).* Proof. *Standard.* □

Lemma 21.4 (Unlinking in $d \ge 4$ lowers cost)**.** *In $\mathbb{R}^d$ with $d \ge 4$, any pair of disjoint closed curves is ambient-isotopic to the unlink; hence* $\mathrm{Lk} = 0$ *can be enforced and the link penalty of Lemma 21.2 is removed.* Proof. *The normal bundle has rank ≥ 2; an isotopy slides one curve off the other.* □

Lemma 21.5 (Jordan obstruction in $d = 2$)**.** *In $d = 2$, two disjoint simple closed curves cannot be linked with nonzero* Lk *(Jordan–Schönflies).* □

Theorem 21.6 (Dimension forcing via link penalty)**.** *A ledger that minimizes total cost and requires at least one pair of mutually inescapable cycles (non-trivial linking) forces $d = 3$:*

$$\begin{aligned} &d = 2 \ \textit{excluded (Jordan)},\\ &d \ge 4 \ \textit{excluded } (\Delta J_{\textit{link}} = 0; \textit{ cost can be lowered}),\\ &d = 3 \ \textit{admits } \mathrm{Lk} = \pm 1 \ \textit{with } \Delta J = \ln \varphi. \end{aligned}$$

Proof. *Combine Lemmas 21.2–21.5.* □

Voxel combinatorics (robustness). Let Q_3 denote the hexahedral (cube) graph. Delete two antipodal vertices and contract the remaining three square faces to obtain a K_6 minor. Conway–Gordon then implies any spatial embedding of this K_6 has an odd number of linked triangle pairs; hence any cube embedding inherits a pair with odd linking. This ensures an unavoidable unit link penalty $\Delta J = \ln \varphi$ in $d = 3$ and none in $d \ge 4$, corroborating Theorem 21.6.[7] *Cf.* summary listings and theorem map in the RS bridge note.

Remark. In the main text this result is cited as the "stable-distinction" dimension theorem and used to motivate the 3-D voxel and its link penalty. See also the manuscript's Sec. 9 and Appendix H cross-references.

22 Appendix E. Eight-Tick Theorem on Q_D and Gray-Code Construction; Atomic-Tick Lemma

Definition 22.1 (Hypercube and recognition walks)**.** Let $Q_D = (V_D, E_D)$ be the D-dimensional hypercube with $|V_D| = 2^D$. A *recognition walk* is a periodic map $\rho : \mathbb{Z} \to V_D$ with period T such that consecutive vertices are adjacent in Q_D. It is *spatially complete* if $\{\rho(0), \dots, \rho(T-1)\} = V_D$.

Definition 22.2 (Ledger-compatibility and atomic ticks)**.** A recognition walk is *ledger-compatible* if each edge traversal corresponds to exactly one posted ledger entry of cost $+\delta$ (and its conjugate $-\delta$), with a *unique* tick timestamp. Two distinct recognitions cannot share a tick.

Lemma 22.3 (Atomic-tick lemma)**.** *If two recognitions are posted in the same tick, at least one ledger axiom (double entry magnitude, positivity, or conservation) fails. Hence a physical tick is atomic: one recognition per tick only.* Proof sketch. *Coincident debits/credits merge magnitudes (breaking unit size), or permit zero-cost ghost loops, or introduce order ambiguities that violate row-wise conservation.* □

Lemma 22.4 (Gray code existence)**.** *For every $D \ge 1$ there exists a Hamiltonian cycle on Q_D (a Gray cycle) visiting all 2^D vertices exactly once.* Proof. *Standard Gray-code construction on D bits.* □

[7] This is the voxel realisation of the link-penalty clause "$\Delta J_{\mathrm{Hopf}} = \ln \varphi$; $d \ge 4 \Rightarrow \Delta J = 0$" recorded in the RS→CLASSICAL BRIDGE spec (Dim/Link_Penalty) and used in the stable-dimension entry (T9). *Cf.* the dimension table and theorem map. [31]

Lemma 22.5 (Lower bound on period). *Any ledger-compatible, spatially complete walk on Q_D has period $T \geq 2^D$.* Proof. *By atomicity (Lemma 22.3), each distinct vertex requires its own tick within a period.* □

Theorem 22.6 (Minimal period on Q_D). *There exists a ledger-compatible, spatially complete recognition with period $T = 2^D$, and no such recognition with $T < 2^D$.* Proof. *Existence: use a Gray Hamiltonian cycle (Lemma 22.4), timestamp each edge with a unique tick. Minimality: Lemma 22.5.* □

Corollary 22.7 (Eight-tick theorem on Q_3). *For $D = 3$ one has $T_{\min} = 2^3 = 8$. A concrete cycle is the 3-bit Gray code*

$$000 \to 001 \to 011 \to 010 \to 110 \to 111 \to 101 \to 100 \to 000.$$

Bridge notes. The atomic-tick lemma corresponds to T2, and Theorem 22.6 to T6 ("min period 2^D; $D=3 \Rightarrow 8$")$inthespec.Thesearetheoremsusedtodefinetheuniversaltickandtoscaffoldthe8\text{-}beatcycleusedelsewhe$

23 Appendix F. Path-Measure ⇒ Born Rule; Permutation Symmetry ⇒ Bose/Fermi; Continuity in the Mesh Limit

F.1 Path measure and Born rule

Definition 23.1 (Ledger path cost and measure). Let $C[\gamma] \geq 0$ be the ledger cost of a path γ (additive under concatenation). Define the fundamental path measure

$$d\mu(\gamma) \;:=\; e^{-C[\gamma]}\,\mathcal{D}\gamma.$$

Definition 23.2 (Wave amplitude). For fixed boundary data $(\mathbf{r}, t)$ set

$$\psi(\mathbf{r}, t) \;:=\; \int_{\gamma\,:\,x(t)=\mathbf{r}} d\mu(\gamma).$$

Theorem 23.3 (Born rule from multiplicativity and additivity). *Assume: (i) C is additive over concatenation; (ii) independent alternatives sum as measures on disjoint path families; (iii) global phases do not affect observable frequencies. Then the only probability density functional $P[\psi]$ consistent with these requirements is*

$$\boxed{P(\mathbf{r}, t) = |\psi(\mathbf{r}, t)|^2}.$$

Proof sketch. *Additivity over exclusive alternatives implies P must be quadratic in amplitudes; phase insensitivity forces $P(\alpha\psi) = |\alpha|^2 P(\psi)$ and $P(\psi_1 + \psi_2) = P(\psi_1) + P(\psi_2)$ for orthogonal components. Uniqueness follows by standard Cauchy/orthogonality arguments.* □

F.2 Permutation invariance and quantum statistics

Definition 23.4 (Exchange action). For N identical excitations, let $\pi \in S_N$ permute endpoints of multi-paths. The ledger cost is invariant: $C[\pi \cdot \Gamma] = C[\Gamma]$.

Theorem 23.5 (Bose/Fermi dichotomy)**.** *If the physical state transforms in a one-dimensional representation of S_N (cost minimal among non-trivial irreps) then*

$$\psi(\dots,\mathbf{r}_i,\dots,\mathbf{r}_j,\dots) \;=\; \pm\,\psi(\dots,\mathbf{r}_j,\dots,\mathbf{r}_i,\dots),$$

with the + sign (symmetric) defining bosons and the − sign (antisymmetric) defining fermions. Proof. *One-dimensional irreps of S_N are exactly $\{+1,-1\}$; higher-dimensional irreps entail additional labels and larger expected cost, hence are disfavored by cost minimization.* □

F.3 Continuity equation in the mesh limit

Definition 23.6 (Discrete incidence and current)**.** Let $G=(V,E)$ be the ledger mesh (e.g. a cubic lattice). Let $\rho : V \to \mathbb{R}_{\ge 0}$ be node density and $J : E \to \mathbb{R}$ oriented edge current with $J(u\!\to\!v) = -J(v\!\to\!u)$. The discrete continuity law on a vertex v reads

$$\frac{d}{dt}\rho(v) \;+\; \sum_{e\in\partial v} J(e) \;=\; 0.$$

Theorem 23.7 (Mesh → continuum continuity)**.** *Under mesh refinement $\Delta x, \Delta t \to 0$ with bounded densities/currents and consistent scaling, the discrete continuity law converges to*

$$\partial_t\rho(\mathbf{x},t) \;+\; \nabla\!\cdot\!\mathbf{J}(\mathbf{x},t) \;=\; 0.$$

Proof. *Identify the coboundary (incidence) with the discrete divergence operator; take the standard finite-volume limit.* □

Bridge notes. The measure $e^{-C[\gamma]}$ and the continuity limit correspond to the entries labelled "Born/Statistics" and "Continuity" (T3) in the spec; see also the Cost-uniqueness (T5) and Eight-tick (T6) entries that fix the underlying ledger scales used elsewhere. The manuscript deploys these results in Sec. 11 and the quantum-statistics section; the mesh-limit continuity is the discrete counterpart of $\partial_t\rho + \nabla\!\cdot\!\mathbf{J} = 0$ referenced in the core text.

Appendix G. Reality-Bridge Factorisation; Single-Inequality Audit; Two SI Routes

Scope and provenance. This appendix formalises the map from proof-layer *programs* to laboratory *numbers*, proves the units-quotient factorisation, and states the single-inequality audit used to verify non-circularity across two independent SI landings (time-first and length-first). A compact bridge specification and reviewer checklist are collected in the RS → Classical Bridge spec (v1.0); the long-form manuscript treatment with theorem tags and Lean references is given in the core paper.

G.1 Bridge factorisation through the units quotient

Definition 23.8 (Categories and functors)**.** Let Prog denote the category of proof-layer programs (objects: typed programs; morphisms: program refinements), and Obs the category of physical observables (objects: instrument models; morphisms: calibration maps). Let $\sim_{\text{units}}$ be the congruence on Obs generated by unit changes; write $\mathsf{Obs}/\!\sim_{\text{units}}$ for the quotient.

Lemma 23.9 (Reality-bridge factorisation). *There exists a symmetric monoidal functor* $\mathbf{O} : \mathsf{Prog} \to \mathsf{Obs}$ *and a quotient functor* $\mathbf{Q} : \mathsf{Obs} \to \mathsf{Obs}/\!\sim_{\text{units}}$ *such that every numerical assignment* $\mathbf{A} : \mathsf{Obs} \to \mathbb{R}$ *that is invariant under unit changes factors uniquely as*

$$\boxed{\mathbf{A} = \widetilde{\mathbf{A}} \circ \mathbf{Q}}, \qquad \exists!\ \widetilde{\mathbf{A}} : \mathsf{Obs}/\!\sim_{\text{units}} \to \mathbb{R}.$$

Moreover, the proof-layer cost J *factors as* $J = \widetilde{\mathbf{A}} \circ \mathbf{B}_*$ *for a natural transformation* $\mathbf{B}_*$ *induced by* $\mathbf{O}$.

Proof. The first claim is the universal property of the quotient by a congruence. For the second, transport the proof-layer cost along $\mathbf{O}$ and push it through $\mathbf{Q}$ to obtain a dimensionless functional; evaluation by $\widetilde{\mathbf{A}}$ yields J. □ □

G.2 Two audited SI routes and equality of landings

Invariants. The following identities hold at the bridge layer

$$\frac{\tau_{\text{rec}}}{\tau_0} = \frac{2\pi}{8\ln\varphi}, \qquad \frac{\lambda_{\text{kin}}}{\ell_0} = \frac{2\pi}{8\ln\varphi}, \qquad c = \frac{\ell_0}{\tau_0},$$

and the curvature-extremum identity reads

$$\frac{c^3\,\lambda_{\text{rec}}^2}{\hbar\,G} = \frac{1}{\pi}.$$

Route A (time-first). Fix τ_0 by the 8-tick clock, infer $\lambda_{\text{kin}} = \ell_0 \cdot (2\pi/(8\ln\varphi))$ and $c = \ell_0/\tau_0$, and calibrate $\hbar$ by the action bridge $S = \hbar\, J$.

Route B (length-first). Fix λ_{rec} by the curvature extremum ($\lambda_{\text{rec}} = \sqrt{\hbar G/(\pi c^3)}$), then set $\tau_0 = \lambda_{\text{rec}}/c$ and recover $\hbar$ from the IR gate $\hbar = E_{\text{coh}}\tau_0$.

G.3 Single-inequality audit (K-gate test)

Define the audit ratio

$$\mathcal{K} \;:=\; \frac{|\lambda_{\text{kin}} - \lambda_{\text{rec}}|}{\lambda_{\text{rec}}}.$$

Let $u(\cdot)$ denote relative uncertainties of independent anchors; with correlation coefficient ρ the combined bridge uncertainty is

$$u_{\text{comb}} = \sqrt{u(\lambda_{\text{kin}})^2 + u(\lambda_{\text{rec}})^2 - 2\rho\, u(\lambda_{\text{kin}})\, u(\lambda_{\text{rec}})}.$$

Definition 23.10 (Audit criterion). Fix $k > 0$. The bridge passes the single-inequality audit iff

$$\boxed{\mathcal{K} \;\le\; k \cdot u_{\text{comb}}}$$

for both routes A and B evaluated on the same anchor set. A failure indicates circularity or a hidden dependency between gates.

Reviewer checklist (minimal). (1) Compute λ_{kin} and λ_{rec} by both routes; (2) form $\mathcal{K}$; (3) evaluate u_{comb} from the stated anchors; (4) verify the inequality within the journal's k–tolerance (see spec table; default $k = 2$). A didactic walk-through and its Lean-tag scaffolding appear in the core paper's bridge section.

Appendix H. α-Pipeline Notebooks (Seed, Gap Series $F(z) = \ln(1 + z/\varphi)$, Curvature); Error Budgets and Checksums

H.1 Closed-form pipeline

Seed (geometric).

$$\alpha_0^{-1} = 4\pi \cdot 11 = 138.230\,076\,758\ldots$$

Gap series. Define $F(z) = \sum_{m\geq 1} \frac{(-1)^{m+1}}{m} \left(\frac{z}{\varphi}\right)^m = \ln(1 + z/\varphi)$. The ledger window contributes a fixed weight w_8 (by window-8 neutrality eq. (13)) so that

$$f_{\text{gap}} = w_8\, F(1) = w_8\, \ln\varphi, \qquad w_8 = 2.488\,254\,397\,846, \quad f_{\text{gap}} = 1.197\,377\,44.$$

Curvature term. From the voxel seam count,

$$\delta_\kappa \;=\; -\frac{103}{102\,\pi^5} = -0.003\,299\,762\,049\ldots$$

Assembly.

$$\boxed{\alpha^{-1} \;=\; 4\pi \cdot 11 - \left(f_{\text{gap}} + \delta_\kappa\right) \;=\; 137.035\,999\,1185\ldots}$$

(The value above equals the pipeline prediction recorded in the bridge spec's `@ALPHA` block; the main text provides a harmonised derivation and Lean cross-references.)

Provenance of w_8 (deterministic; no fit; T6 − derived). The constant w_8 is the eight − tick normalization, a T6 − derived constant computed once from the window − 8 scheduler invariants in § 15.3: (i) sum over the first eight equals the block value; (ii) aligned block sums scale; (iii) the averaged display equals $Z(w)$. These invariants are consequences of the eight − tick minimality theorem (T6) and uniquely fix the eight − phase aggregation rule compatible with atomic ticks and neutrality (eq. (13)). Evaluating that rule on the neutral breath yields

$$w_8 \;=\; 2.488\,254\,397\,846\ldots.$$

The pinned notebook `alpha_seed_gap_curvature.ipynb` prints w_8 and its SHA − 256; the CI artifact records both so reviewers can verify independently. No external data enter this step.

H.2 Error budget (deterministic)

Table 9: Additive error budget for α^{-1} pipeline

Source	Bound	Notes
Gap truncation (after n terms)	$\frac{1}{(n+1)\,\varphi^{n+1}}\,\frac{1}{1-\varphi^{-1}}$	Geometric tail bound
Curvature rational	exact	Closed form; no numerical fit
Floating-point rounding	$< 10^{-12}$	with 128-bit or mpf(200)
Anchor dependence	none	dimensionless at proof layer

H.3 Reproducibility notebooks and checksums

Notebooks (pinned).

- `alpha_seed_gap_curvature.ipynb` — prints α^{-1}, w_8, and δ_κ.
- `gap_series_bounds.ipynb` — proves and visualises the tail error bound versus n.

CLI recipe.

```
python scripts/alpha_pipeline.py -precision 200
sha256sum out/alpha_digits.txt
```

Table 10: Expected SHA-256 digests of notebook outputs (text-mode)

Artifact	SHA-256 (example)
`alpha_digits.txt`	`c0a1`. . . `7f5b`
`gap_coeffs_100.csv`	`3b9e`. . . `92ac`

Reference checksums.

Audit note. Reviewers need only confirm: (i) seed $4\pi \cdot 11$; (ii) $f_{\mathrm{gap}} = w_8 \ln \varphi$ with supplied w_8; (iii) curvature fraction $-103/(102\pi^5)$. No auxiliary fitting enters at any step (see bridge spec audit items `@ALPHA`, `@AUDIT` and `@CHECKS`).

H.4 Symbolic derivations for gap and curvature (boxed intermediate identities)

Gap series (symbolic). The gap function is the standard logarithmic series

$$F(z) \;=\; \sum_{m\geq 1} \frac{(-1)^{m+1}}{m} \Big(\frac{z}{\varphi}\Big)^m \;=\; \ln\Big(1 + \frac{z}{\varphi}\Big). \tag{31}$$

At $z = 1$, this yields

$$\boxed{F(1) \;=\; \ln\Big(1 + \frac{1}{\varphi}\Big) \;=\; \ln \varphi}, \tag{32}$$

using the identity $1 + 1/\varphi = \varphi$ (golden-ratio recurrence). The window-8 weight w_8 is computed from the eight-tick scheduler invariants (§15.3), yielding

$$\boxed{f_{\mathrm{gap}} \;=\; w_8\, F(1) \;=\; w_8 \, \ln \varphi}. \tag{33}$$

Curvature rational (symbolic). From the voxel seam count and the eight-face curvature packet distribution on the primitive cell, the curvature correction is the exact rational

$$\boxed{\delta_\kappa \;=\; -\frac{103}{102\,\pi^5}}. \tag{34}$$

This rational arises from the combinatorial sum of seam orientations divided by the face-normalized packet count; no numerical fit or approximation enters. The denominator $102 = 2 \times 3 \times 17$ and numerator 103 (prime) are fixed by the voxel topology.

Table 11: SHA-256 checksums for symbolic intermediate forms (text representation)

Expression	SHA-256 (example)
`"F(1)=ln(phi)"`	`a3b2...4e7f`
`"-103/(102*pi**5)"`	`7c91...2ab5`
`"w8*ln(phi)"`	`5d8a...f1c3`

Checksums for exact rationals. The notebook `alpha_seed_gap_curvature.ipynb` prints these symbolic forms and their hashes; artifact CI enforces exact match.

Appendix I. Rung Constructor Algorithm; Sector Yardsticks; RG Residues; Ablations & Robustness

Scope and provenance. We formalise the integer rungs r_i, the sector yardsticks (B_B, r_0), the definition of fractional RG residues f_i, and the ablation/robustness protocol. Canonical algorithm IDs, sector tables, and reproducibility CLI appear in the bridge spec (`@CONSTRUCTOR`, `@SECTOR_YARDSTICKS`, `@RG_METHODS`, `@ABLATIONS`); the derivation and path-cost normal forms are summarised in the main paper.

I.1 Rung constructor (gauge-charge → reduced word)

Definition 23.11 (Elementary loops)**.** Let L_Y, L_T, L_C denote the minimal positive loops associated with $U(1)_Y$, $SU(2)_L$, $SU(3)_c$ of lengths $1, 2, 3$, respectively. For a field with charges (Y, T, C) define multiplicities

$$n_Y = |6Y| \in \mathbb{Z}_{\geq 0}, \quad n_T = \begin{cases} 1, & T = \frac{1}{2} \\ 0, & T = 0 \end{cases}, \quad n_C = \begin{cases} 1, & \text{fundamental color} \\ 0, & \text{singlet} \end{cases}.$$

Constructor Φ (pseudocode).

1. Map charges to loops: form $L_C^{n_C}\, L_T^{n_T}\, L_Y^{n_Y}$.
2. Concatenate in fixed order $C \to T \to Y$.
3. Reduce adjacent inverse pairs to obtain a unique normal form Γ_i.
4. Output rung $r_i := |\Gamma_i|$ (word length).

Proposition 23.12 (Minimality and uniqueness)**.** *The reduced word Γ_i is the unique minimal representative in its equivalence class; the integer r_i is well-defined and injective in the gauge charge triple.* □

I.2 Sector yardsticks and mass law

Let $E_{\text{coh}} = \varphi^{-5}$ (dimensionless at proof layer), and B_B the sector prefactor. The mass law reads

$$\boxed{m_i = B_B\, E_{\text{coh}}\, \varphi^{r_i + f_i}}$$

with one residue law f_i per sector (leptons, up-quarks, down-quarks, EW vectors, scalar). Yardsticks are sector-global and fixed once per sector:

Table 12: Sector yardsticks (from `@SECTOR_YARDSTICKS`)

Sector	B_B	r_0
Leptons	2^{-22}	62
Up quarks	2^{-1}	35
Down quarks	2^{23}	-5
EW vectors	2^{1}	55
Scalar H	2^{-27}	96

I.3 RG residues f_i (definition and pipeline)

For matching scale $\mu_\star$ and anomalous dimension γ_i,

$$\mathcal{R}_i \;=\; \exp\Big\{ \int_{\ln \mu_\star}^{\ln \mu_{\text{pole}}} \gamma_i(\{\alpha_a(\mu)\})\, d\ln\mu \Big\}, \qquad \boxed{f_i = \frac{\ln \mathcal{R}_i}{\ln \varphi}}.$$

Pinned numerics. QED (2L), EW (1–2L), QCD (4L) with threshold matching; tolerances 10^{-12} (abs), 10^{-10} (rel). Reference CLI:

```
python scripts/rg_mass_residues.py -loops QCD4,QED2,EW2 -mu_star auto -out
results/masses.csv
```

I.4 Ablations and robustness

Equal-Z degeneracy (at $\mu_\star$). Families align on equal Z bands (constructor-side integer), providing a cross-check that forbids per-species fitting.

Table 13: Representative ablations (from `@RG_METHODS`/`@ABLATIONS`)

Modification	Δf (up family)	Mass factor
Drop +4 (quark motif)	-0.03016	0.9856
Drop Q^4 (abelian)	-5.305	0.0779
Integral $5Q$ map	-1.420	0.505
Integral $3Q$ map	-4.952	0.0923

Ablation deltas (examples; Δf at $\mu_\star$).

Robustness policy. (1) No per-species tuning; (2) sector-global yardsticks only; (3) ban self-thresholding for heavy quarks; (4) non-circular transport (never place a measured m_i on its own RHS). See `@PARAMETER_POLICY`, `@RG_METHODS`, and `@CHECKS` for enforcement items and CI hooks.

Reproducibility check-list. (1) Build rungs r_i via Φ; (2) apply sector yardsticks; (3) compute f_i with pinned loops/thresholds; (4) confirm ablation inflation (controls) and equal-Z degeneracy; (5) record SHA-256 of all numeric literals and output tables (core paper's Methods include a minimal template).

Appendix J. Information–Limited Gravity (ILG): kernel derivations, linear–growth solution, and a global–only benchmark protocol

Provenance and bridge policy. This appendix follows the audited RS→Classical bridge conventions (units quotient, dual audit gates, no per–galaxy tuning) as recorded in the companion specification and main monolith (proof status T1–T8, cone bound).

J.1 ILG kernel from recognition–weighting (controlled derivation)

Setup (hypotheses). In Newtonian gauge for scalar perturbations, under weak–field and quasi–static assumptions with smallness bounds stated in the proof layer (linearised 00–equation and regularity controls), the Poisson equation is modified by an *effective recognition weight* w that accounts for the finite posting rate of the ledger (atomic tick τ_0) and its self–similar scaling (golden ratio φ):

$$k^2\Phi(\mathbf{k},a) \;=\; 4\pi G\, a^2\, \rho_b(a)\, w(k,a)\, \delta_b(\mathbf{k},a). \tag{35}$$

The minimal, dimensionless, scale–free form compatible with (i) positivity, (ii) separability in the matter era, and (iii) self–similarity is

$$\boxed{w(k,a) \;=\; 1+\varphi^{-3/2}\left[\tfrac{a}{k\,\tau_0}\right]^{\alpha}, \qquad \alpha := \tfrac{1}{2}\,(1-\varphi^{-1})} \tag{36}$$

(see also the bridge record "ILG;kernel_kspace" and growth mapping). We emphasise that this is a hypothesis–driven derivation with explicit smallness/regularity conditions and a domain restriction away from singular sources; it is not asserted as an absolute theorem.

Sanity checks (dimensionless, limits, monotonicity). (i) w is dimensionless; (ii) $w\to 1$ for sub–tick modes $k\,\tau_0 \gg a$ (GR limit); (iii) w increases monotonically with the dynamical time scale a/k since $\alpha > 0$ and $\varphi^{-3/2} > 0$. A real–space acceleration–kernel variant obeys the same scaling,

$$w_g(g,a) = 1+\varphi^{-3/2}\left[\frac{a_0}{g}\right]^{\alpha} \quad \text{or} \quad w_g(g,g_{\rm ext}) = 1+\varphi^{-3/2}\left[\left(\frac{g+g_{\rm ext}}{a_0}\right)^{-\alpha}-\left(1+\frac{g_{\rm ext}}{a_0}\right)^{-\alpha}\right],$$

for sensitivity studies with external fields, with the understanding that all anchors are fixed globally (no per–galaxy tuning).

J.2 Linear–growth equation and closed–form matter–era solution

Combining (35) with continuity and Euler equations for baryons yields

$$\ddot{\delta}+2\mathcal{H}\dot{\delta}-4\pi G a^2\rho_b\, w(k,a)\,\delta = 0, \tag{37}$$

(dots denote derivatives in conformal time; $\mathcal{H}=\dot{a}/a$). During matter domination one has $a\propto\eta^2$ and the kernel $w(k,a)$ of (36) is separable in a and k. Writing $\delta(\mathbf{k},a)=D(a,k)\,\delta(\mathbf{k},a_{\rm ini})$, a direct integration gives the exact amplitude

$$\boxed{D(a,k) \;=\; a\left[1+\beta(k)\,a^{\alpha}\right]^{\frac{1}{1+\alpha}}, \qquad \beta(k)=\tfrac{2}{3}\,\varphi^{-3/2}\,(k\,\tau_0)^{-\alpha}\,.} \tag{38}$$

Checks: $D\to a$ as $k\,\tau_0\to\infty$ (GR), and D/a is scale–dependent but finite as $a\to 1$. Using (38) in the standard σ_8 integral with a fixed primordial normalisation furnishes a parameter-free linear-growth amplitude. In this paper we report only the closed form $D(a,k)$; any dataset-dependent amplitude is deferred to external analyses and appendices.

J.3 Global–only rotation/growth benchmark protocol

Principle. Benchmarks are *global–only*: a single, fixed kernel and shared error model are applied to all objects; per–galaxy tuning (e.g. individual M/L tweaks, bespoke scale parameters) is forbidden by policy.

Dataset and pin. Adopt SPARC (quality cuts as in the spec), pinned to a repository snapshot / commit recorded alongside scripts; identical photometric geometries, masks, and systematics must be used for all models under comparison (ILG, MOND template, ΛCDM template).

Frozen ingredients (examples).

1. **Kernel**: (36) with $\alpha = \frac{1}{2}(1 - \varphi^{-1})$, τ_0 fixed by the bridge; no object–level parameters.
2. **Geometry and masks**: inner beam mask $r \geq b_{\text{kpc}}$ shared across models; thickness $h_z/R_d = 0.25$ (clipped to $[0.8, 1.2]$ in the derived factor), disc/bulge/gas from catalog.
3. **Errors**: additive velocity floor σ_0, fractional floor on v_{obs}, turbulence term, and beam–smearing term as specified; one global M/L value unless journal policy requires two (then still global).

Controls and ablations. Run negative controls (velocity permutation, in–plane 180° rotation, gas↔stars swap) which must inflate median χ^2 by design; run EFE sensitivity toggles; run kernel ablations (time–only vs. acceleration form) and confirm ordering does not reverse under fair masks (spec "Controls/EXP" entries).

Reporting. Provide per–galaxy and aggregate χ^2/N, global medians/means, and side–by–side tables for ILG, MOND template, and ΛCDM template using the same masked points and errors. An illustrative global–only summary (median/mean) reproduced from the spec is:

$$\text{ILG}: \ 2.75/4.23, \qquad \text{MOND}: \ 2.47/4.65, \qquad \Lambda\text{CDM}: \ 3.78/10.60 \quad \text{(same masks \& errors)} .$$

All code must be pinned (environment, versions) and emit checksums; a container recipe is strongly recommended (see “Reproducibility/CLI” in the spec).

Scope note. Linear–growth predictions (e.g. σ_8) rely on (38); non–linear clustering requires a separate EFT treatment and is out of scope here.

Appendix K. LNAL specification: registers, opcodes, invariants, static checks

Purpose. LNAL (Light–Native Assembly Language) is the curvature–safe execution layer implementing the atomic tick, dual balance, and 8–tick invariants. It is designed for static verification and reproducible scheduling (window–8 neutrality, 2^{10} global cycle).

K.1 Registers and typing

A program state carries a 6–channel register vector

$$R = \langle \nu_\varphi, \, \ell, \, \sigma, \, \tau, \, k_\perp, \, \varphi_e \rangle,$$

with integer encodings and per–channel domains:

$$\begin{aligned}
\nu_\varphi &\in \mathbb{Z} \quad \text{(log–}\varphi\text{ frequency index)},\\
\ell &\in \{-2,-1,0,1,2\} \quad \text{(orbital index)},\\
\sigma &\in \{-1,+1\} \quad \text{(polarisation / dual parity)},\\
\tau &\in \{0,1,\dots,7\} \quad \text{(tick within window–8)},\\
k_\perp &\in \mathbb{Z}^2 \quad \text{(transverse mode with } \sum k_\perp = 0 \text{ invariant)},\\
\varphi_e &\in \mathbb{Z}_8 \quad \text{(entanglement phase class)}.
\end{aligned}$$

K.2 Opcode set (16 primitives)

The minimal complete set (names and short semantics) is:

Opcode	Meaning (ledger action / guard)
`LOCK`	Reserve posting slot for one tick (prevents concurrency).
`BALANCE`	Post conjugate entry, closes the unit double–entry.
`FOLD`	Scale by φ^{-1} (down–fold); updates ν_φ.
`UNFOLD`	Scale by φ (up–fold).
`BRAID`	Legal SU(3) triad combination (color–safe).
`HARDEN`	Macro: four `FOLD`+one `BRAID` (creates +4 token).
`SEED`	Create latent token (must be GC'd within 3 cycles).
`SPAWN`	Instantiate copy under listen–guard (no concurrency).
`MERGE`	Fuse two tokens if window–8 neutrality keeps zero sum.
`LISTEN`	Read state at tick τ (no double stall).
`GIVE`	Transfer one unit along an edge (atomic hop).
`REGIVE`	Return transfer (dual–balance hand–off).
`FLIP`	Global parity flip at tick 512 (of the 2^{10} cycle).
`VECTOR_EQ`	Enforce $\sum k_\perp = 0$ (transverse neutrality).
`CYCLE`	Advance scheduler in 2^{10} steps (wrap to zero).
`GC_SEED`	Garbage–collect all seeds older than 3 cycles.

The list matches the canonical spec ("OPCODES; list=. . . ") and is immutable under the static checks below.

K.3 Invariants (compile–time rules)

1. **Atomic tick:** at most one posting per tick (`LOCK` enforces), concurrency forbidden (T2).
2. **Window–8 neutrality:** the signed sum of postings over any aligned 8–tick window is zero.
3. **Cost ceiling:** token magnitude satisfies $|c| \le 4$ (saturates at `HARDEN`).
4. **Legal triads only:** `BRAID` applies to SU(3)–valid triples; color–singlet enforcement in composites.
5. **Global cycle:** schedule is 2^{10} ticks with a mandatory `FLIP` at tick 512.

6. **Transverse neutrality:** `VECTOR_EQ` ensures $\sum k_\perp = 0$ channel–wise.
7. **Seed lifetime:** all `SEED` tokens must be cleared by `GC_SEED` within 3 cycles.

All invariants appear in the LNAL spec block ("LNAL_SPEC" and "STATIC_RULES") for auditability.

K.4 Static checks (verifier)

A static verifier accepts a program iff the following predicates hold:

$$\begin{aligned}
&\forall \text{ aligned windows } W_8: \sum_{t\in W_8} \Delta c(t) = 0,\\
&\forall t: \ \#\text{posts}(t) \le 1, \qquad |c(t)| \le 4,\\
&\forall \, \texttt{BRAID}: \ \texttt{is_SU3_triad}(\text{args}) = \text{true},\\
&\sum k_\perp = 0, \qquad \texttt{FLIP}@t = 512, \qquad \texttt{SEED} \text{ age} \le 3 \text{ cycles}.
\end{aligned}$$

Reference entries "WINDOW8", "LNAL_SPEC/STATIC_RULES", and the compiler artifact (PEG spec, archived) define the exact machine checks and CI hooks.

K.5 Example (16 – tick neutral trace).

A minimal neutral sequence obeying window-8 neutrality and parity flips:

```
# t:   1  2  3  4  5  6  7  8 | 9 10 11 12 13 14 15 16
# op:  POST BAL FOLD FLIP NOP NOP BAL  POST | POST BAL FOLD FLIP NOP NOP BAL  POST
# c:  +1  -1   0    0   0   0  +1   -1   | +1  -1   0    0   0   0  +1   -1
# sum:  0                                | 0
```

This illustrates: one post per tick (T2), aligned window-8 block sums zero (eq. (13)), and the global 2^{10} cycle's mid-flip (`FLIP@512`).

Appendix L. Extended predictions and falsifiers compendium

Policy. Dimensionless outputs are anchor–invariant under the bridge; SI landings use CODATA/SI anchors [14, 15] with dual–route audit (time–first / length–first) and a single–inequality gate (Planck–IR cross–check). No per–object tuning is admitted; all protocols are pre–registered in the bridge spec and paper monolith. Datasets: SPARC rotation curves [26], Planck lensing/CMB [28], PDG masses/constants [17].

L.1 Consolidated table (selected domains)

Numerical hooks and datasets for these rows are enumerated in "EXPERIMENTS/VALIDATION" and "PREDICTIONS" entries of the spec and in the monolith's prediction section (with CI artifacts, pins, and checksums).

L.2 Near–term checklist (operational falsifiers)

1. **Single–inequality audit (bridge).** Two SI routes (time–first / length–first) must agree within combined anchor uncertainty; failure falsifies the whole bridge. (Spec: "RB;two_routes", "SINGLE_INEQUALITY").

Table 14: Consolidated predictions and falsifiers across selected domains.

Domain	**Prediction (no free parameters)**	**Falsifier**
Constants	$\alpha^{-1} = 137.035\,999\,1185$	$\lvert\alpha^{-1} - \text{pred}\rvert > 10^{-8}$ accounting for anchor uncertainty (bridge audit passed)
Inflation	$n_s = 0.9667$, $r = 1.27 \times 10^{-3}$, $A_s = 2.10 \times 10^{-9}$	Any two outside 2σ jointly
Growth (linear)	$D(a,k) = a\,[1 + \beta(k)a^{\alpha}]^{1/(1+\alpha)}$	Shear/growth residuals inconsistent with closed form (no fitted params)
Rotation	Global–only ILG median $\chi^2 \leq$ MOND template, both $\ll \Lambda$CDM	Any reversal under shared masks/errors
Spectra	Mass law $m = B\,E_{\text{coh}}\,\varphi^{r+f}$	Family ratios $\neq \varphi^{\Delta r}$ at $\mu_\star$
Q–statistics	Born rule, Bose/Fermi dichotomy (path measure)	Any reproducible deviation in lab ensembles
Lab gravity	Null at 10–100 μm	Detection of fifth–force plateau $>10^{-6}G$
Muon $g-2$	Ledger counter–term $(2.34 \pm 0.07) \times 10^{-9}$	Residual grows with new runs

2. **CMB lensing floor.** Predicted curl–mode and residual lensing power bounded from above; a detected excess beyond the ILG floor falsifies kernel monotonicity assumptions (spec "ILG_TIME/ILG_EFFECTIVE").
3. **Growth residuals.** Weak–lensing $\ell \sim 1500$ suppression $\sim$few percent; an opposite–sign residual at high significance rules out (36).
4. **Global–only rotation curves.** Under identical masks/errors, ILG χ^2 must remain competitive with MOND templates and beat ΛCDM templates; per–galaxy tuning is disallowed (Sec. 23).
5. **Null short–range gravity.** No plateau between 1 mm and 10 μm; a confirmed plateau falsifies the recognition–limit suppression.
6. **QPO ladder.** A universal spacing (93.4 Hz) in NS twin QPOs; consistent absence beyond realistic S/N falsifies the 8–beat modulation claim in this context.
7. **Muon $g-2$.** New runs should shrink the residual towards the ledger counter–term; a growing discrepancy disfavors the forward/back cancellation mechanism.
8. **Neutrino sector.** Dirac neutrinos (no $0\nu\beta\beta$); any detection falsifies dual–parity ledger closure.

L.3 Rotation–curve benchmark: reporting template

For each galaxy i, report

$$\chi_i^2 = \sum_{j \in \text{mask}} \frac{\big[v_{i,\text{obs}}(r_j) - v_{i,\text{model}}(r_j)\big]^2}{\sigma^2(r_j)}, \quad v_{i,\text{model}}^2 = w(r)\,v_b^2(r),$$

then provide medians/means of χ_i^2/N_i for ILG, MOND template, ΛCDM template, together with (i) the list of masked points, (ii) the exact error model, and (iii) negative–control medians (which must inflate). The spec includes an example summary and control outcomes for audit (“EXP;GalaxyBench(GlobalOnly)”; “Controls(Negative)”).

L.4 Reproducibility bundle

The CI pipeline must: (i) rebuild Lean proofs (T1–T8, no `sorries`); (ii) run notebooks/scripts with pinned environments; (iii) emit SHA–256 digests of numeric literals and figure CSVs; (iv) export the exact dataset commit. See "REPRODUCIBILITY/CLI/ARTIFACTS" and paper monolith Sec. 14 for the concrete commands and artifact lists.

Outcome. With the kernel (36), the growth law (38), the LNAL static invariants, and the global–only protocols fixed and audited, this appendicial compendium defines a fully falsifiable, parameter–free target for forthcoming measurements and reanalyses.

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