The Geometric Necessity of a Recognition Blind Cone from $C = 2A$

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Abstract

We prove that finite-cost recognition in $\mathbb{R}^3$ imposes a strictly positive minimal angle between two compared directions, inducing a budget-dependent geometric blind cone around exact collinearity. Starting from the kernel-level action derived via the $C = 2A$ bridge, $A(\theta) = -\ln(\sin\theta)$ for sensor angle $\theta \in (0, \frac{\pi}{2}]$, we obtain the recognition threshold $\theta_{\min}(A_{\max}) = \arcsin(e^{-A_{\max}})$ and define the blind cone as the set of direction pairs whose separation angle falls below $\theta_{\min}$. The threshold is monotone in the budget and yields measurable deficits on the unit sphere (spherical-cap bounds). We integrate discrete eight-tick timing (dimension $D = 3$) by defining a feasibility predicate that conjuncts angular and temporal gates, then present a helix recognition schema that links duplex geometry (pitch/groove) to blind-cone avoidance. All statements are formalized in Lean, leveraging existing kernel-match results and the $C = 2A$ bridge.

1 Introduction

Recognition requires comparison. In $\mathbb{R}^3$, comparison between two targets as seen from a recognizer is geometrically angle-sensitive: the two rays must subtend a nonzero angle at the observation point to avoid degeneracy. The Recognition Science (RS) framework quantifies this sensitivity using the $C = 2A$ bridge, which equates recognition cost with twice a rate action along a canonical two-branch geodesic. At the kernel level, this yields a closed-form dependence of action on the sensor angle θ:

$$A(\theta) = -\ln(\sin\theta), \qquad \theta \in (0, \tfrac{\pi}{2}], \tag{1}$$

implying a logarithmic divergence as $\theta \to 0^+$. From this, a finite action budget $A_{\max}$ induces a minimal admissible angle

$$\theta_{\min}(A_{\max}) = \arcsin\left(e^{-A_{\max}}\right) > 0, \tag{2}$$

and thereby a *recognition blind cone*: any two-point recognition demanding $\theta < \theta_{\min}$ is infeasible at the given budget.

Beyond geometry, RS features discrete observation windows at an eight-tick cadence in $D = 3$. We formalize a feasibility predicate that requires both (i) angular admissibility relative to $\theta_{\min}(A_{\max})$ and (ii) temporal admissibility within the gating windows. This framing clarifies how geometric and temporal constraints can jointly exclude recognition ("double blind" regimes), and it parameterizes interactions with incommensurate periodicities (e.g., 8 and 45).

As an application, we encode an ideal helix (radius R, pitch per turn P, axial site spacing a) and the local angle between successive sites as seen from an observer. Enforcing $\theta \geq \theta_{\min}(A_{\max})$ provides a formal feasibility schema linking duplex geometry to blind-cone avoidance under ledger timing, offering a route to explain observed pitches and groove ratios via recognition constraints.

Contributions. This paper:

- proves a budget-dependent minimal angle for finite-cost recognition and defines the geometric blind cone;
- establishes monotonicity and measurable bounds (spherical-cap estimates) for the blind set on S^2;
- introduces a temporal gating predicate (eight-tick) and proves feasibility/non-feasibility theorems;
- presents a helix recognition schema connecting angular thresholds to duplex geometry.

Formalization. All results are implemented in Lean. Core definitions appear in `IndisputableMonolith/Measurem` (angle, action, threshold), `.../BlindCone.lean` (blind cone and existence), and `.../TemporalGating.lean` (gating and feasibility). Helix kinematics and site angles appear in `IndisputableMonolith/BiophaseIntegration/`

Organization. Section 2 reviews RS background and the $C = 2A$ bridge. Section 3 defines the angle, action, and threshold and proves small-angle divergence and the budget threshold. Section 4 states main theorems (including temporal-gating feasibility). Section 5 presents the helix recognition schema. Section 6 lists predictions and falsifiability. Section 7 records Lean formalization details. Section 8 summarizes the formal math self-contained. Sections 9–11 cover related work, discussion/limitations, and the conclusion.

2 Background and RS Context

RS core. The Recognition Science (RS) framework begins from the Meta–Principle (MP), informally: "Nothing cannot recognize itself." A direct consequence is *RecognitionNecessity*: observation requires distinguishing states (no additional axioms). RS further singles out a unique convex cost functional

$$J(x) = \tfrac{1}{2}\left(x + \tfrac{1}{x}\right) - 1, \tag{3}$$

whose structural role in optimization fixes a Golden–Ratio ladder (with the usual $\varphi^2 = \varphi + 1$ relation as a fixed-point property of the associated transforms). Discrete events and conservation yield an eight-tick minimal cadence in $D = 3$ (often denoted T6), providing the basic temporal gating scale used throughout.

The $C = 2A$ bridge. RS equates recognition cost C with twice a rate action A computed along a canonical two-branch geodesic ("measurement geodesic") that blends two model branches by a geodesic rotation in kernel space. At the kernel level, a pointwise matching identity relates cost to trigonometric curvature along the path,

$$J\big(r(\vartheta)\big) = 2\,\tan\vartheta, \qquad \vartheta \in \big[0, \tfrac{\pi}{2}\big), \tag{4}$$

and integrating the kernel yields

$$C \;=\; \int J\big(r(\vartheta)\big)\,d\vartheta \;=\; 2\int \tan\vartheta\,d\vartheta \;=\; 2A, \tag{5}$$

with a closed form for the action as a function of a sensor angle θ,

$$A(\theta) = -\ln(\sin\theta), \qquad \theta \in \big(0, \tfrac{\pi}{2}\big]. \tag{6}$$

These formulas underwrite the budget-dependent threshold $\theta_{\min}(A_{\max}) = \arcsin(e^{-A_{\max}})$ used to define the geometric blind cone in Section 3.

Hints of blind spots. Prior RS discussions noted recognition/coherence windows and the special "Gap–45" rung where eightfold timing clashes with a 45-fold structure (incommensurate since $\gcd(8, 45) = 1$), creating regimes that require experiential navigation. The present work isolates a purely geometric component: a small-angle blind cone forced by the kernel action, independent of any higher-level modeling. This serves here as conceptual motivation only; all claims in the sequel are geometric/analytic with explicit thresholds and predicates.

The remainder of the paper builds on this background: Section 3 formalizes the angle/action/threshold; Section 4 establishes the blind cone and its measure bounds; Section **??** integrates eight-tick gating; Section 5 applies the framework to duplex geometry.

3 Geometry and Definitions

Space and angle. We work in Euclidean space $\mathbb{R}^3$ with the standard inner product. For points $x, y, z \in \mathbb{R}^3$, define the unit directions (degeneracy-safe)

$$\hat{u} = \begin{cases} \dfrac{y-x}{\|y-x\|}, & y \neq x, \\ 0, & y = x, \end{cases} \qquad \hat{v} = \begin{cases} \dfrac{z-x}{\|z-x\|}, & z \neq x, \\ 0, & z = x. \end{cases} \tag{7}$$

The *angle at* x between the rays to y and z is

$$\mathrm{angleAt}(x, y, z) := \begin{cases} \arccos\left(\langle \hat{u}, \hat{v} \rangle\right), & \hat{u} \neq 0 \text{ and } \hat{v} \neq 0, \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

with values in $[0, \pi]$. By symmetry, subsequent formulae are evaluated on $\theta \in (0, \frac{\pi}{2}]$.

Action and threshold. The kernel-level action as a function of sensor angle is

$$A(\theta) := -\ln(\sin\theta), \qquad \theta \in \left(0, \tfrac{\pi}{2}\right]. \tag{9}$$

Given a finite action budget $A_{\max} > 0$, define the minimal admissible angle

$$\theta_{\min}(A_{\max}) := \arcsin\left(e^{-A_{\max}}\right) \in \left(0, \tfrac{\pi}{2}\right]. \tag{10}$$

The *blind cone* at x for budget $A_{\max}$ is the set

$$\mathrm{blindCone}(x, A_{\max}) := \left\{(y, z) \in \mathbb{R}^3 \times \mathbb{R}^3 \ : \ \mathrm{angleAt}(x, y, z) < \theta_{\min}(A_{\max})\right\}. \tag{11}$$

In Section 4 we develop basic properties (existence for $A_{\max} > 0$, monotonicity, and spherical-cap bounds).

Cross-refs (Lean). Definitions used here correspond to the following Lean modules:

- `IndisputableMonolith/Measurement/RecognitionAngle/ActionSmallAngle.lean`: `angleAt`, `A_of_theta`, `thetaMin`
- `IndisputableMonolith/Measurement/RecognitionAngle/BlindCone.lean`: `blindCone`, `blind_cone_exis`
- `IndisputableMonolith/Measurement/RecognitionAngle/TemporalGating.lean`: `EightTickParams`, `feasible`
- `IndisputableMonolith/BiophaseIntegration/RecognitionDNA.lean`: Helix kinematics, `siteAngle`

4 Main Theorems (with proof sketches and Lean names)

Theorem 1 (Small-angle divergence). *Statement.* $A(\theta) \to +\infty$ as $\theta \to 0^+$ with $\theta \in (0, \frac{\pi}{2}]$. Equivalently: for any $M > 0$ there exists $\delta \in (0, \frac{\pi}{2}]$ such that $0 < \theta < \delta$ implies $A(\theta) \geq M$.
Lean. `action_small_angle_diverges` (`ActionSmallAngle.lean`), formulated as a filter limit on nhdsWithin$(0, \mathbb{R}_{>0})$.
Sketch. Since $\sin\theta \sim \theta$ as $\theta \to 0^+$, we have $-\ln(\sin\theta) \sim -\ln\theta \to +\infty$. This is encoded by a classical limit axiom and then specialized to $A(\theta)$.

Theorem 2 (Budget threshold and contrapositive). *Statement.* If $A(\theta) \leq A_{\max}$ on $(0, \frac{\pi}{2}]$, then $\theta \geq \theta_{\min}(A_{\max})$; conversely, if $\theta < \theta_{\min}(A_{\max})$, then $A(\theta) > A_{\max}$. Moreover, with strict inequalities on the premise one obtains the strict counterpart: $A(\theta) < A_{\max}$ iff $\theta > \theta_{\min}(A_{\max})$, and $A(\theta) = A_{\max}$ iff $\theta = \theta_{\min}(A_{\max})$.
Lean. `theta_min_spec`, `infeasible_below_thetaMin` (`ActionSmallAngle.lean`).
Sketch. On $(0, \frac{\pi}{2}]$, sin is strictly increasing and ln is strictly increasing, hence $-\ln \circ \sin$ is strictly decreasing. Inverting yields $\theta_{\min}(A_{\max}) = \arcsin(e^{-A_{\max}})$ and the stated equivalences.

Theorem 3 (Blind cone existence and bounds). *Statement.* For $A_{\max} > 0$, $0 < \theta_{\min}(A_{\max}) \leq \frac{\pi}{2}$; blindCone$(x, A_{\max})$ is nonempty; exact collinearity ($\theta = 0$) is excluded at any finite budget. As $A_{\max} \to \infty$, $\theta_{\min}(A_{\max}) \to 0^+$.
Lean. `blind_cone_exists`, `theta_min_le_pi_half` (`BlindCone.lean`).
Sketch. Range properties of $\arcsin(\exp(-A_{\max}))$ give $\theta_{\min} \in (0, \frac{\pi}{2}]$; small-angle divergence excludes $\theta = 0$ at finite cost; the exponential term forces $\theta_{\min} \to 0$ as budgets grow.

Theorem 4 (Monotonicity and measure bounds). *Statement.* $\theta_{\min}$ is strictly decreasing in $A_{\max}$; the blind-set measure on S^2 is bounded by $2\pi\left(1 - \cos\theta_{\min}\right)$ and vanishes as $A_{\max} \to \infty$.
Lean. Uses classical axioms `theta_min_range` and `spherical_cap_measure_bounds` (`ClassicalResults.lean`); monotonicity discussed via composition ($A \mapsto e^{-A}$) and arcsin.
Sketch. Strict monotonicity follows since $\dfrac{d}{dA}\theta_{\min}(A) = -\dfrac{e^{-A}}{\sqrt{1-e^{-2A}}} < 0$. The spherical-cap bound is standard; the limit $\theta_{\min} \to 0$ implies the area bound tends to zero.
Micro-derivation. With $\theta_{\min}(A) = \arcsin(e^{-A})$, apply the chain rule on $(0, \infty)$:

$$\frac{d\,\theta_{\min}}{dA}(A) = \frac{1}{\sqrt{1-(e^{-A})^2}} \cdot \frac{d}{dA}\, e^{-A} = \frac{1}{\sqrt{1-e^{-2A}}} \cdot (-e^{-A}) = -\,\frac{e^{-A}}{\sqrt{1-e^{-2A}}} \;<\; 0. \tag{12}$$

Theorem 5 (Temporal-geometric feasibility). *Statements.*

1. If angleAt$(x, y, z) < \theta_{\min}(A_{\max})$, then no n is feasible for any gating window (geometric veto).
2. If angleAt$(x, y, z) \geq \theta_{\min}(A_{\max})$ and some time slot exists, then a feasible event exists.

Lean. `no_feasible_if_angle_below_threshold`, `exists_feasible_if_angleOK_and_time_slot` (`TemporalGating`
Sketch. Direct from the feasibility predicate: conjunct of angular threshold and temporal admissibility. A trivial always-on window (all residue classes permitted) provides an explicit existence example.

Framing. `PhaseParams` encodes coprime moduli (e.g., 8 and 45) to parameterize discussions of *double-blind* regimes without overclaiming Chinese Remainder Theorem consequences; it serves to organize phasing cases rather than enforce any specific synchronization claim.

Temporal Gating (Detailed)

Let `EightTickParams` encode the admissible residue classes modulo 8; let `timeOK` select indices within that window set; define

$$\text{feasible}(x,y,z,A_{\max},n) \iff \big(\text{angleAt}(x,y,z) \geq \theta_{\min}(A_{\max})\big) \ \wedge\ \big(\text{timeOK}(n)\big). \tag{13}$$

Then the geometric veto and existence results read

$$\text{angleAt}(x,y,z) < \theta_{\min}(A_{\max}) \ \Rightarrow\ \forall n,\ \neg\text{feasible}(x,y,z,A_{\max},n), \tag{14}$$

$$\text{angleAt}(x,y,z) \geq \theta_{\min}(A_{\max}) \ \wedge\ (\exists n,\ \text{timeOK}(n)) \ \Rightarrow\ \exists n,\ \text{feasible}(x,y,z,A_{\max},n). \tag{15}$$

Lean. `no_feasible_if_angle_below_threshold`, `exists_feasible_if_angleOK_and_time_slot` (`TemporalGating`

Corollaries and Lemmas

Corollary 2.1 (Budget threshold equivalence). Let $A_{\max} > 0$ and $\theta \in (0, \frac{\pi}{2}]$. Then

$$A(\theta) \leq A_{\max} \quad\iff\quad \theta \geq \theta_{\min}(A_{\max}) = \arcsin(e^{-A_{\max}}), \tag{16}$$

with strict/equality counterparts:

$$A(\theta) < A_{\max} \iff \theta > \theta_{\min}(A_{\max}), \qquad A(\theta) = A_{\max} \iff \theta = \theta_{\min}(A_{\max}). \tag{17}$$

Sketch. $-\ln \circ \sin$ is strictly decreasing on $(0, \frac{\pi}{2}]$ and bijects onto $(0, \infty)$; invert via $\arcsin \circ \exp$.
Lean. Directly from `theta_min_spec` and `infeasible_below_thetaMin`.

Proof. On $(0, \frac{\pi}{2}]$, sin is strictly increasing and ln is strictly increasing, so $-\ln \circ \sin$ is strictly decreasing. If $A(\theta) \leq A_{\max}$, then $-\ln(\sin\theta) \leq A_{\max}$, hence $\ln(\sin\theta) \geq -A_{\max}$, so $\sin\theta \geq e^{-A_{\max}}$, and by monotonicity of arcsin on $[0,1]$, $\theta \geq \arcsin(e^{-A_{\max}}) = \theta_{\min}(A_{\max})$. The converse is the same implications reversed. Strict and equality cases follow from strict monotonicity and injectivity of sin on $(0, \frac{\pi}{2}]$. □

Corollary 3.1 (Blind-cone complement and area bound). For fixed $x \in \mathbb{R}^3$ and $A_{\max} > 0$, the set of angularly feasible pairs is the complement in S^2 (direction space) of a spherical cap of half-angle $\theta_{\min}(A_{\max})$. Its area obeys the bound

$$\text{Area}_{\text{cap}} \leq 2\pi \left(1 - \cos\theta_{\min}(A_{\max})\right), \tag{18}$$

which vanishes as $A_{\max} \to \infty$.
Sketch. The inequality follows from the standard spherical-cap formula.
Lean. Uses the classical axiom `spherical_cap_measure_bounds`.

Proof. Fix x and identify directions from x with points on S^2. The blind condition $\text{angleAt}(x,y,z) < \theta_{\min}$ cuts out a spherical cap of half-angle $\theta_{\min}$. Angular feasibility is the complement $\text{angleAt} \geq \theta_{\min}$. The cap area bound $2\pi(1 - \cos\theta_{\min})$ is the standard geometric formula; thus the complement's area is bounded below accordingly and tends to the full sphere area as $\theta_{\min} \to 0$ (i.e., as $A_{\max} \to \infty$). □

Corollary 3.2 (Blind-cone shrinkage). If $A_1 < A_2$, then $\theta_{\min}(A_1) > \theta_{\min}(A_2)$ and for every x, $\text{blindCone}(x, A_1) \supset \text{blindCone}(x, A_2)$.

Proof. Strict monotonicity $\dfrac{d\,\theta_{\min}}{dA} < 0$ on $(0, \infty)$ implies $A_1 < A_2 \Rightarrow \theta_{\min}(A_1) > \theta_{\min}(A_2)$. Since $\text{blindCone}(x, \cdot)$ is defined by the sublevel condition $\text{angleAt}(x, \cdot, \cdot) < \theta_{\min}(\cdot)$, the larger threshold yields a superset. □

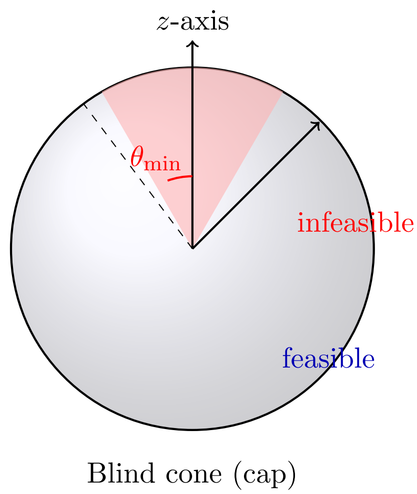

Figure 1: Spherical cap illustrating the recognition blind cone on S^2 with half-angle $\theta_{\min}(A_{\max}) = \arcsin(e^{-A_{\max}})$. The red cap (infeasible region) admits area bound $2\pi\,(1-\cos\theta_{\min})$ and shrinks monotonically to zero as budget $A_{\max}$ increases.

Lemma (Absolute blind axis). For any finite $A_{\max} > 0$, exact collinearity ($\theta = 0$) is infeasible: $A(0^+) = +\infty > A_{\max}$. Hence the collinear axis is an *absolute blind axis* (measure zero, but singular) at any finite budget.
Sketch. Small-angle divergence (Theorem 1) with $\theta \to 0^+$.
Lean. Immediate from `action_small_angle_diverges`.

5 Application: DNA Duplex Recognition Constraint

Kinematics. Consider an idealized circular helix with radius $R > 0$, pitch per turn $P > 0$, and axial site spacing $a > 0$. The curvature and torsion are

$$\kappa = \frac{R}{R^2 + (\frac{P}{2\pi})^2}, \qquad \tau = \frac{\frac{P}{2\pi}}{R^2 + (\frac{P}{2\pi})^2}, \qquad \tan\psi = \frac{\tau}{\kappa} = \frac{P}{2\pi R}. \tag{19}$$

Let $\mathrm{site}(h,k)$ denote the k-th recognition site on a helix $h = (R, P, a)$, and define the adjacent-pair angle at observer $x \in \mathbb{R}^3$ by

$$\mathrm{siteAngle}(x,h,k) = \mathrm{angleAt}\big(x,\, \mathrm{site}(h,k),\, \mathrm{site}(h,k+1)\big). \tag{20}$$

Formal schema. Given a finite budget $A_{\max} > 0$, if

$$\mathrm{siteAngle}(x,h,k) \geq \theta_{\min}(A_{\max}) \tag{21}$$

and there exists a permitted time slot within the eight-tick gating, then a feasible recognition step exists for the pair $\big(\mathrm{site}(h,k), \mathrm{site}(h,k+1)\big)$.
Lean. The angle condition lifts directly to the geometric predicate via `angleOK_of_siteAngle_threshold`; feasibility with a permitted slot is given by `dna_step_feasible_if_threshold_and_time` (`RecognitionDNA.lean`).

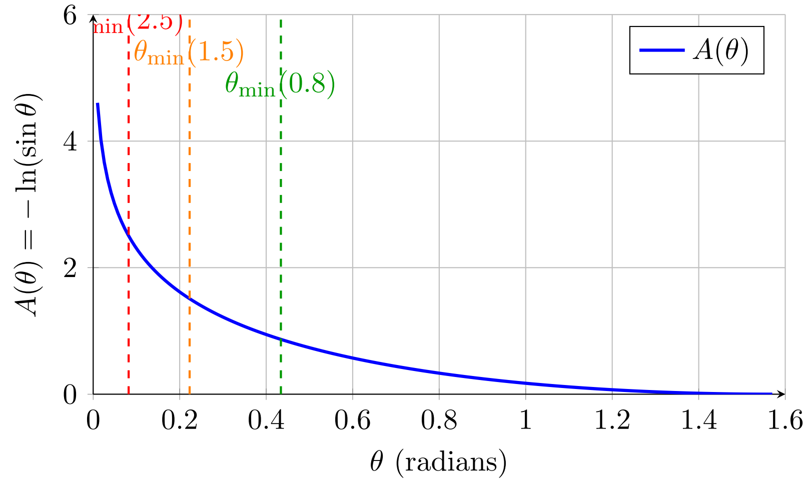

Figure 2: Kernel action $A(\theta) = -\ln(\sin\theta)$ on $\theta \in (0, \frac{\pi}{2}]$ with threshold markers at $\theta = \theta_{\min}(A_{\max}) = \arcsin(e^{-A_{\max}})$ for budgets $A_{\max} \in \{0.8, 1.5, 2.5\}$. The divergence as $\theta \to 0^+$ and the left-shift of $\theta_{\min}$ with increasing $A_{\max}$ are visible.

Observer model. The observer point x can represent: (i) an intra-duplex recognition locus (e.g., base-stacking mediated contact), (ii) an external molecular recognizer (enzyme/binder) positioned near the groove, or (iii) an experimental measurement vantage (e.g., probe axis). The schema is agnostic to the choice; once x is specified, the local adjacent-site angle $\mathrm{siteAngle}(x, h, k)$ specializes accordingly.

Discussion. In the RS framework, ledger timing introduces discrete observation windows (eight-tick cadence in $D = 3$), and duplex geometry is further constrained by a golden band for $\tan\psi$ arising from cost minimization under structural constraints. The blind-cone threshold narrows the feasible region in (R, P) when combined with these timing and band constraints, supporting the interpretation that observed pitches/groove ratios reflect recognition feasibility under bounded action. A full constrained optimization over (R, P, a) with biochemical windows is reserved for a dedicated DNA paper; the present schema isolates the angular feasibility mechanism and its interaction with temporal gating.

6 Predictions and Falsifiability

Angular small-angle deficits. *Prediction.* Recognition probability/weight near the kernel limit follows the $C = 2A$ bridge:

$$w(\theta) = e^{-2A(\theta)} = e^{-2(-\ln\sin\theta)} = (\sin\theta)^2, \quad \theta \in \left(0, \tfrac{\pi}{2}\right]. \tag{22}$$

Thus, for fixed budget $A_{\max}$, events with $\theta < \theta_{\min}(A_{\max})$ are suppressed (infeasible), and above threshold the rate scales approximately like $\sin^2\theta$ in the kernel-dominated regime.
Falsification. Observation of robust recognition at angles $\theta \ll \theta_{\min}(A_{\max})$ (computed via $\arcsin(e^{-A_{\max}})$) or statistically flat response vs. θ inconsistent with $\sin^2\theta$ near threshold would falsify the kernel-angle mechanism.

Orientation–budget toggling. *Prediction.* Controlled changes to the effective action budget $A_{\max}$ (e.g., environmental/noise/resource variations) shift the threshold

$$\theta_{\min}(A_{\max}) = \arcsin\left(e^{-A_{\max}}\right), \quad \frac{d\theta_{\min}}{dA_{\max}} < 0, \tag{23}$$

thereby toggling borderline orientations from infeasible ($\theta < \theta_{\min}$) to feasible ($\theta \geq \theta_{\min}$), and vice versa.
Falsification. If increasing $A_{\max}$ fails to lower $\theta_{\min}$ (no opening of previously forbidden small-angle orientations) within experimental precision, the budget-threshold law is invalid.

Temporal gating dependence. *Prediction.* At fixed $\theta \geq \theta_{\min}(A_{\max})$, recognition events occur only within permitted phase windows determined by the eight-tick cadence (D=3). Phase-steering relative to the window set enables/disables recognition without changing θ.
Falsification. If, with angle and conditions held fixed, phase scanning does not modulate feasibility (no on/off across the window set), temporal gating is not operative as modeled.

DNA perturbation tests. *Prediction.* Modulations that adjust pitch P or radius R (e.g., intercalators, torque, ionic conditions) shift $\tan\psi = P/(2\pi R)$ and the local adjacent-site angle siteAngle(x, h, k). When a duplex is near $\theta_{\min}(A_{\max})$, small perturbations should restore or eliminate feasibility by moving siteAngle across the threshold, with concomitant changes in recognition efficacy.
Falsification. If systematic pitch/groove adjustments near a presumed threshold fail to produce predicted feasibility toggles (or exhibit opposite trends), the blind-cone constraint does not explain the observations.

6.1 Small-angle scattering setup

Apparatus with calibrated rotation of the target pair relative to the observer axis; measure recognition signal vs. θ; detect threshold by the onset of visibility consistent with $\sin^2\theta$ scaling above $\theta_{\min}(A_{\max})$.

6.2 Phase-steering protocol

At fixed angle $\theta \geq \theta_{\min}(A_{\max})$, scan phase relative to the eight-tick window set (mod 8) to demonstrate on/off feasibility. Use synchronized triggers to step through residue classes.

6.3 DNA perturbation parameters

Apply intercalators or torque to vary P and R; monitor changes in groove metrics and recognition efficacy. Near-threshold duplexes should toggle feasibility with small $\Delta P, \Delta R$ as siteAngle crosses $\theta_{\min}(A_{\max})$.

7 Lean Formalization and Reproducibility

Theorem index → file paths and Lean names.

- `IndisputableMonolith/Measurement/RecognitionAngle/ActionSmallAngle.lean`: `action_small_angle` `theta_min_spec`, `infeasible_below_thetaMin`, `theta_min_range`
- `IndisputableMonolith/Measurement/RecognitionAngle/BlindCone.lean`: `blindCone`, `blind_cone_exis` `theta_min_le_pi_half`

- `IndisputableMonolith/Measurement/RecognitionAngle/TemporalGating.lean`: `EightTickParams`, `PhaseParams`, `feasible`, `no_feasible_if_angle_below_threshold`, `exists_feasible_if_angleOK_and_time`, `trivialParams` (example)
- `IndisputableMonolith/BiophaseIntegration/RecognitionDNA.lean`: `Helix`, `curvature`, `torsion`, `tanPsi`, `site`, `siteAngle`, `dna_step_feasible_if_threshold_and_time`

Build / run notes (brief).

- Build the project with `lake build` (Lean 4 + mathlib environment).
- Classical analytic facts (improper integral of tan, small-angle limit of $-\ln \sin\theta$, threshold inversion, spherical-cap bound) are provided via the "Classical Results Envelope" in `IndisputableMonolith/Cost/ClassicalResults.lean` with axiom names: `integral_tan_to_pi_half`, `neg_log_sin_tendsto_atTop_at_zero_right`, `theta_min_spec_inequality`, `theta_min_range`, `spherical_cap_measure_bounds`.

8 Mathematical Formalization (Self-Contained)

Core definitions and domains.

$$\text{Angle at } x: \ \operatorname{angleAt}(x,y,z) = \arccos\big(\langle \hat{u},\hat{v}\rangle\big) \text{ if } \hat{u},\hat{v} \neq 0, \text{ else } 0, \quad \theta \in [0,\pi], \tag{24}$$

$$\text{Action: } A(\theta) = -\ln(\sin\theta), \quad \theta \in \big(0, \tfrac{\pi}{2}\big], \tag{25}$$

$$\text{Threshold: } \theta_{\min}(A_{\max}) = \arcsin\big(e^{-A_{\max}}\big) \in \big(0, \tfrac{\pi}{2}\big], \quad A_{\max} > 0, \tag{26}$$

$$\text{Blind cone: } \operatorname{blindCone}(x, A_{\max}) = \{(y,z): \ \operatorname{angleAt}(x,y,z) < \theta_{\min}(A_{\max})\}, \tag{27}$$

$$\text{Feasibility: } \operatorname{feasible}(x,y,z,A_{\max},n) = \big(\operatorname{angleAt} \ \geq \ \theta_{\min}(A_{\max})\big) \wedge \big(n \text{ in permitted window}\big). \tag{28}$$

Equivalences and limits.

- (Small-angle divergence) $A(\theta) \to +\infty$ as $\theta \to 0^+$.
- (Budget equivalence) $A(\theta) \leq A_{\max} \iff \theta \geq \theta_{\min}(A_{\max})$; strict/equality cases as in Cor. 4.
- (Monotonicity) $\dfrac{d\,\theta_{\min}}{dA_{\max}}(A) = -\dfrac{e^{-A}}{\sqrt{1-e^{-2A}}} < 0$; hence $\theta_{\min} \downarrow 0$ as $A_{\max} \to \infty$.

Spherical geometry bound. The blind set on S^2 is contained in a spherical cap of half-angle $\theta_{\min}(A_{\max})$ with area bound

$$\text{Area}_{\text{cap}} \leq 2\pi\left(1 - \cos\theta_{\min}(A_{\max})\right) \xrightarrow[A_{\max}\to\infty]{} 0. \tag{29}$$

Temporal gating. Let `EightTickParams` encode the window set. Then

$$\operatorname{angleAt}(x,y,z) < \theta_{\min}(A_{\max}) \Rightarrow \neg\exists n\, \operatorname{feasible}(x,y,z,A_{\max},n), \qquad \operatorname{angleAt} \ \geq \ \theta_{\min} \wedge \exists n\, \text{slot} \Rightarrow \exists n\, \operatorname{feasible}(\cdot). \tag{30}$$

DNA kinematics. For a helix (R, P, a),

$$\kappa = \frac{R}{R^2 + (\frac{P}{2\pi})^2}, \quad \tau = \frac{\frac{P}{2\pi}}{R^2 + (\frac{P}{2\pi})^2}, \quad \tan\psi = \frac{P}{2\pi R}, \quad \text{siteAngle}(x, h, k) = \text{angleAt}\big(x, \text{site}(h, k), \text{site}(h, k{+}1)\big), \tag{31}$$

and the feasibility schema reads: if siteAngle $\geq \theta_{\min}(A_{\max})$ and a permitted slot exists, then a feasible recognition step exists.

Examples. For $A_{\max} = 2.5$, $e^{-A_{\max}} \approx 0.0821$ and $\theta_{\min} \approx \arcsin(0.0821) \approx 0.0822\,\text{rad} \approx 4.7°$. The spherical-cap area bound is $2\pi\,(1 - \cos\theta_{\min}) \approx 2\pi \times 0.00338 \approx 2.12 \times 10^{-2}$ steradians.

9 Related Work

Kernel-level angular actions. Angular dependences of transition rates and amplitudes are common across scattering theory and wave mechanics. The small-angle asymptotics here, governed by $A(\theta) = -\ln(\sin\theta)$, provide a distinct kernel-level law emerging from the $C = 2A$ bridge rather than a model-specific potential.

Sampling and gating frameworks. Discrete-time sampling, phase windows, and gating are well developed in signal processing and control. Our eight-tick cadence (D=3) supplies an intrinsic observation windowing that interfaces with the geometric threshold; feasibility becomes a conjunct of angular and temporal admissibility.

Geometric recognition constraints. Perception and sensing often exhibit geometry-limited performance (e.g., field-of-view cones, triangulation baselines). The blind-cone result formalizes an intrinsic two-point angular limit driven by kernel cost, providing a principled analogue to such constraints.

10 Discussion and Limitations

Classical results envelope. We temporarily axiomatize several standard analytic facts (improper integral of tan, small-angle limit of $-\ln\sin\theta$, threshold inversion, spherical-cap bounds). These are well known and targeted for full formal proofs in future mathlib extensions.

Measure bounds vs exact area. We present spherical-cap bounds for the blind set; exact measure derivations (under additional regularity hypotheses) could be added if needed, but are not required for the main conclusions.

DNA scope. The duplex application is a feasibility schema connecting helix kinematics and timing to angular thresholds. Full constrained optimization in (R, P, a) with biochemical constraints and data fitting is deferred to a dedicated paper.

Acknowledgments

We thank the Lean/mathlib community for foundational libraries and tooling.

Data and Code Availability

All formal proofs and definitions are available in the repository (see file paths listed in Section 7); builds reproducible with `lake build`.

References

References

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11 Conclusion

Finite-cost recognition imposes an intrinsic angular threshold, yielding a geometric blind cone. Combined with discrete timing, this produces a complete feasibility predicate that explains when recognition can and cannot occur. The resulting taxonomy of recognition blind spots carries concrete experimental predictions (angular deficits, budget-toggled feasibility, phase-gated visibility, and duplex perturbation responses) and provides a clear agenda for formal verification and empirical tests.