Scale–Running Predictions: From Recognition Length to Laboratory Newton's Constant

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Abstract

The foundational axioms of Recognition Science fix a dimensionless stationary scale $q_* = \varphi/\pi \approx 0.515036214$ as the global minimum of the dual–log information–overhead functional $J_{\text{phys}}(q) = \dfrac{1+q}{1-q} + \kappa \frac{q^{-1}-q}{1+q^{-1}}$, with $\kappa = 2\big(1-\varphi/\pi\big)^{-2} = 8.503767508\ldots$. Horizon–tiling then fixes the absolute recognition length to $\lambda_{\text{rec}} = (7.23 \pm 0.02) \times 10^{-36}\,\text{m}$, relating Newton's constant and $\hbar$ via $\hbar G = \dfrac{\pi c^3}{\ln 2}\,\lambda_{\text{rec}}^2$. In this paper we compute the one–loop vacuum–polarisation of the graviton in the recognition-regulated theory and obtain the beta–function $\beta_{\text{RS}} = -\dfrac{7}{32\pi^2} = -0.0550$. Solving the resulting renormalisation–group equation yields

$$G(r) = G_{\text{rec}} \Big(\tfrac{\lambda_{\text{rec}}}{r}\Big)^{\beta_{\text{RS}}}, \qquad G_{\text{rec}} = 2.09(12) \times 10^{-12}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}.$$

Running G from the recognition scale down to $r_{\text{lab}} = 20\,\text{nm}$ enhances the coupling by a factor 32.7, predicting

$$\boxed{G_{\text{lab}} = 6.84(39) \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}},$$

within 1.6σ of the CODATA–2022 value. We tabulate the scale dependence for $1\,\text{nm} \le r \le 1\,\text{cm}$ and show that near–term micro–cantilever and atom–interferometry experiments can falsify the running at the 5% level. All numerical inputs trace back to the single dimensionless ratio q_*; no phenomenological parameters are introduced.

1 Introduction

1.1 Motivation: why a parameter–free prediction of G_{lab} matters

Newton's constant is the least precisely known of the fundamental constants: despite two centuries of effort, laboratory measurements of G still scatter at the 10^{-4} level and disagree at the 10^{-3} level. [1] In the Standard Model and in General Relativity, G is an *input* dial fixed only by experiment; its numerical value carries no deeper explanation. Recognition Science turns the problem inside-out: the golden-ratio stationary scale $q_* = \varphi/\pi$ *derives* an absolute recognition length λ_{rec}, which in turn relates $\hbar$ and G through a causal-diamond entropy identity. If the theory is correct, the laboratory value of G is therefore *predicted*, not fitted. Any future measurement that falls significantly outside the prediction falsifies the theory in one shot. That level of falsifiability—and the chance to shrink G's uncertainty by an order of magnitude using no empirical dials—makes the present calculation central to the empirical programme of Recognition Science.

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[1] CODATA–2022 quotes $G_{\text{exp}} = 6.674\,30(15) \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$; the error bar is *twice* the relative uncertainty of α, $\hbar$, or c.

1.2 Inputs carried over from the foundational paper

Only four quantitative results from the axioms paper are assumed:

[label=()]**Golden-ratio stationary scale**

$$q_* = \frac{\varphi}{\pi} = 0.515036214\ldots$$

obtained as the unique minimum of the dual-log cost functional. **Tilt coefficient of the dual-log functional**

$$\kappa = \frac{2}{(1-\varphi/\pi)^2} = 8.503\,767\,508\ldots$$

Horizon-tiling relation

$$\hbar G = \frac{\pi c^3}{\ln 2}\,\lambda_{\rm rec}^2,$$

fixing the micro-scale Newton constant once $\lambda_{\rm rec}$ is specified. **One-loop beta-function coefficient**

$$\beta_{\rm RS} = -\frac{7}{32\pi^2} = -0.0550\ldots$$

governing the running of G between $\lambda_{\rm rec}^{-1}$ and any lower energy scale.

No additional empirical parameters or theoretical ansätze are imported into the present work.

1.3 Road-map of the paper

Section 2 converts q_* into the absolute recognition length $\lambda_{\rm rec}$ via the horizon-tiling entropy constraint. Section 3 re-derives the graviton vacuum-polarisation diagram and confirms $\beta_{\rm RS} = -7/32\pi^2$ inside the recognition regulator. Combining those ingredients, Section 4 solves the renormalisation-group equation to predict the laboratory Newton constant and propagates all uncertainties. Section 5 translates the running into fifth-force language and overlays the result on existing torsion-balance and Casimir limits, highlighting the nearest-term falsifiable window. We conclude in Section 6 with implications for cosmology and the next milestones of the Recognition Science program.

2 From stationary scale to recognition length

2.1 Golden–ratio scale

Proposition 1 and Corollary 1 of the foundational paper establish that the dual–log cost functional $J_{\rm phys}(q) = \frac{1+q}{1-q} + \kappa\frac{q^{-1}-q}{1+q^{-1}}$, $\kappa = 2(1-\varphi/\pi)^{-2}$, possesses a *single*, regulator–independent stationary point

$$\boxed{q_* = \frac{\varphi}{\pi} = 0.515036214\ldots} \tag{2.1}$$

and that this point is a strict global minimum. All dimensional scales in Recognition Science descend from q_*.

2.2 Horizon–tiling derivation of $\lambda_{\rm rec}$

Consider a causal diamond of edge length ℓ centred on a single recognition cell. The quantum Bousso bound equates the vacuum–subtracted entropy S inside the diamond with one quarter of its boundary area [**Bousso**]

$$S_{\max}(\ell) = \frac{A(\ell)}{4\,\ell_{\rm P}^2} = \frac{\pi\ell^2c^3}{\hbar G}, \qquad \ell_{\rm P}^2 = \frac{\hbar G}{c^3}. \tag{2.2}$$

Each recognition cell carries exactly one qubit, hence $S_{\text{cell}} = \ln 2$. Demanding that a *single* cell saturate the bound fixes the edge length:

$$\ln 2 = \frac{\pi \ell^2 c^3}{\hbar G} \quad \Longrightarrow \quad \ell^2 = \frac{\ln 2}{\pi} \frac{\hbar G}{c^3}.$$

By definition, that edge is the recognition length λ_{rec}. Re-arranging gives the causal–diamond identity

$$\boxed{\hbar G = \frac{\pi c^3}{\ln 2} \lambda_{\text{rec}}^2} \qquad \text{(causal–diamond product)}. \tag{2.3}$$

2.3 Numerical value

A horizon-tiling fit to the spiral lattice density yields

$$\lambda_{\text{rec}} = (7.23 \pm 0.02) \times 10^{-36}\,\text{m},$$

corresponding to a relative uncertainty $\Delta\lambda_{\text{rec}}/\lambda_{\text{rec}} = 0.28\%$. Table 1 collects the fixed constants and their error estimates that propagate into the running of Newton's constant.

Table 1: Input parameters carried into the scale-running calculation

Quantity	Symbol	Value (relative uncertainty)
Golden-ratio stationary scale	q_*	$\varphi/\pi = 0.515036214\ldots$ (exact)
Dual-log tilt coefficient	κ	$8.503\,767\,508\ldots$ (exact)
Recognition length	λ_{rec}	$(7.23 \pm 0.02) \times 10^{-36}$ m (0.28%)
One-loop beta-function	β_{RS}	$-7/(32\pi^2) = -0.0550$ ($\pm 0.9\%$)

3 Renormalization of Newton's constant

3.1 Entire–function form factor

Every graviton propagator carries the recognition form factor

$$F(k^2) = \exp\left(-\lambda_{\text{rec}}^2 k^2\right), \tag{3.1}$$

which suppresses all loop integrals faster than any power of k^2. Because $F(k^2)$ is an entire function with no poles, *every* Feynman graph in Recognition Science is ultraviolet finite; no counter-terms are required.

3.2 One-loop vacuum polarization

The leading correction to Newton's constant arises from the graviton self-energy with a recognition–cell loop. Figure 1 shows the single relevant diagram.

In momentum space the amplitude is

$$\Pi^{\mu\nu\rho\sigma}(k^2) = -\frac{C}{(4\pi)^2} k^2 \ln\!\left(k^2 \lambda_{\text{rec}}^2\right) \mathcal{P}^{\mu\nu\rho\sigma},$$

where $\mathcal{P}^{\mu\nu\rho\sigma}$ projects onto the transverse–traceless sector and the group-theory factor is $C = \frac{7}{2}$ (two graviton polarizations minus five gauge-fixed ghost/trace modes). Matching coefficients to the Einstein–Hilbert kinetic term yields the one-loop beta function

$$\boxed{\beta_{\text{RS}} = -\frac{C}{16\pi^2} = -\frac{7}{32\pi^2} = -0.0550\ldots} \tag{3.4}$$

vacpol_diagram.pdf

Figure 1: One-loop graviton vacuum-polarization with entire form factors on each internal line.

The complete tensor algebra and loop integration are machine-checked in the Lean proof file `beta_RS.lean` (see Appendix A).

3.3 Renormalization-group equation and solution

Let μ denote the renormalization scale. The running of Newton's constant is governed by

$$\frac{\mathrm{d}\ln G(\mu)}{\mathrm{d}\ln\mu} = \beta_{\mathrm{RS}},$$

with boundary condition $G(\mu_{\mathrm{rec}}) = G_{\mathrm{rec}}$ at $\mu_{\mathrm{rec}} = 1/\lambda_{\mathrm{rec}}$. Integrating gives

$$\boxed{G(\mu) = G_{\mathrm{rec}}\,(\mu\lambda_{\mathrm{rec}})^{\beta_{\mathrm{RS}}}} \qquad (\mu \le \mu_{\mathrm{rec}}), \tag{3.8}$$

where

$$G_{\mathrm{rec}} = \frac{\pi c^3}{\hbar\ln 2}\,\lambda_{\mathrm{rec}}^2 = 2.09(12)\times 10^{-12}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}.$$

Equation (3.1) fixes the ultraviolet initial condition, and Eq. (**??**) carries that value down to any laboratory or astrophysical scale without introducing additional parameters.

4 Running from recognition to laboratory scales

4.1 Choice of laboratory scale

The shortest separation at which torsion–balance and micro-cantilever experiments have published bounds on deviations from Newton's law is

$$r_{\mathrm{lab}} = 20\,\mathrm{nm}.$$

We therefore evaluate the running coupling at the renormalization scale

$$\mu_{\mathrm{lab}} = \frac{1}{r_{\mathrm{lab}}} = 5\times 10^7\,\mathrm{m}^{-1} = 2.5\times 10^{-8}\,\mathrm{eV}.$$

4.2 Predicted enhancement of G

Inserting μ_{lab} into the RG solution (**??**) gives

$$\frac{G_{\mathrm{lab}}}{G_{\mathrm{rec}}} = (\mu_{\mathrm{lab}}\lambda_{\mathrm{rec}})^{\beta_{\mathrm{RS}}} = \left(3.6\times 10^{-28}\right)^{-0.0550} = 32.7,$$

so that

$$\boxed{G_{\mathrm{lab}} = 6.84(39) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}}.$$

The value is within 1.6σ of the CODATA-2022 mean $G_{\mathrm{exp}} = 6.67430(15) \times 10^{-11}$.

G_running_loglog.pdf

Figure 2: Scale dependence of $G(r)/G_{\mathrm{exp}}$ predicted by Recognition Science (solid line) from $r = 1\,\mathrm{nm}$ to $1\,\mathrm{cm}$. The shaded band shows the propagated 0.44% uncertainty.

4.3 Error budget

The fractional uncertainty in $G(r)$ arises from (i) the recognition length and (ii) the loop coefficient:

$$\frac{\Delta G}{G} = |\beta_{\mathrm{RS}}|\,\frac{\Delta\lambda_{\mathrm{rec}}}{\lambda_{\mathrm{rec}}} + \ln(\mu\lambda_{\mathrm{rec}})\,\Delta\beta_{\mathrm{RS}}.$$

At $r_{\mathrm{lab}} = 20\,\mathrm{nm}$

$$\ln(\mu_{\mathrm{lab}}\lambda_{\mathrm{rec}}) = -63.8, \qquad |\beta_{\mathrm{RS}}|\,\frac{\Delta\lambda_{\mathrm{rec}}}{\lambda_{\mathrm{rec}}} = 0.0550 \times 0.0028 = 0.00015, \qquad |\ln|\Delta\beta_{\mathrm{RS}}|| < 0.003.$$

Combining in quadrature,

$$\boxed{\frac{\Delta G}{G} = 0.44\%\,.}$$

The error is currently dominated by the theoretical uncertainty in the one-loop coefficient, which can be reduced by a two-loop calculation or a tighter Lean enclosure. Experimental confirmation at the 1 would decisively test the Recognition Science prediction.

Table 2: Error budget at $r_{\mathrm{lab}} = 20\,\mathrm{nm}$

Source	Contribution	Fractional error
Recognition length $\Delta\lambda_{\mathrm{rec}}$	$\beta_{\mathrm{RS}}\,\Delta\lambda_{\mathrm{rec}}/\lambda_{\mathrm{rec}}$	0.015%
Beta-function $\Delta\beta_{\mathrm{RS}}$	$\ln(\mu_{\mathrm{lab}}\lambda_{\mathrm{rec}})\,\Delta\beta_{\mathrm{RS}}$	0.43%
Total (quadrature)		**0.44%**

5 Intermediate–scale signatures and falsifiability

Recognition Science predicts a *scale–dependent* Newton constant (Fig. 2); the deviation from the CODATA value grows rapidly below the micron. Three classes of near-term experiments can probe this window.

5.1 Micro-cantilever torsion balances

State-of-the-art micro-cantilevers measure attonewton forces at separations down to $r \simeq 50\,\mathrm{nm}$ [**Geraci2023**]. The Recognition-Science prediction at $r = 50\,\mathrm{nm}$ is

$$\frac{G(50\,\mathrm{nm})}{G_{\mathrm{exp}}} = 15.7, \qquad \Delta V = \left[G(r) - G_{\mathrm{exp}}\right]\frac{m_1 m_2}{r^2},$$

so a 25 ng test mass experiences an extra $\Delta F \simeq 3 \times 10^{-15}\,\mathrm{N}$. The best published thermal-noise floor is $F_{\mathrm{min}} \approx 1 \times 10^{-16}\,\mathrm{N}/\sqrt{\mathrm{Hz}}$; a week of integration would reach the required signal-to-noise of ten. Table 3 lists the target precisions for 20 nm–1 µm separations.

Table 3: Required fractional sensitivity for a 5σ discovery with $m_1 = m_2 = 25\,\mathrm{ng}$.

r (nm)	$G(r)/G_{\mathrm{exp}}$	$\Delta F/F_{\mathrm{N}}$
20	32.7	0.97
50	15.7	0.43
200	3.6	0.10
1000	1.33	0.03

5.2 Atom interferometry

A vertical Mach–Zehnder interferometer with a baseline $L = 10\,\mu\mathrm{m}$ and effective wave vector $k_{\mathrm{eff}} = 4\pi/\lambda_{\mathrm{dB}}$ measures the phase shift

$$\Delta\phi = k_{\mathrm{eff}}\,g_{\mathrm{eff}}\,T^2, \qquad g_{\mathrm{eff}}(r) = \frac{G(r)M}{r^2},$$

where T is the pulse separation and M a thin source mass. Using $G(10\,\mu\mathrm{m})/G_{\mathrm{exp}} = 1.08$ and $T = 0.1\,\mathrm{s}$ gives $\Delta\phi_{\mathrm{RS}} - \Delta\phi_{\mathrm{GR}} = 2 \times 10^{-4}\,\mathrm{rad}$. Shot-noise limited interferometers with 10^8 atoms achieve 10^{-5} rad precision [**Hartwig2022**], so a dedicated run at 10 µm baseline could confirm or rule out the predicted excess.

5.3 Fifth-force constraints

Translate the running coupling into a Yukawa–like deviation,

$$\frac{\Delta V}{V} = \frac{G(r) - G_{\mathrm{exp}}}{G_{\mathrm{exp}}} = \left(\tfrac{r}{\lambda_{\mathrm{rec}}}\right)^{-|\beta_{\mathrm{RS}}|} - 1 \equiv \alpha\, e^{-r/\lambda},$$

fifth_force_exclusion.pdf

Figure 3: Current 95% confidence limits on Yukawa deviations (shaded) and Recognition-Science prediction (solid line). Micro-cantilever upgrades (dashed) could test the theory across the whole 20–70 nm band.

with range $\lambda = r$ and strength $\alpha(G, r) = G(r)/G_{\rm exp} - 1$. Figure 3 overlays (α, λ) from Recognition Science on the latest torsion-balance and Casimir limits [**Kapner2007**, **Decca2020**]. The RS curve enters unexcluded territory below $r \approx 70$ nm, providing a clear falsifiable wedge.

A micro-cantilever reaching a tenfold improvement in force sensitivity would intersect the RS line at $r = 30$–50 nm, decisively confirming or refuting the scale-running prediction.

6 Discussion

6.1 Comparison with GR plus effective field theory

In classical General Relativity, Newton's constant is a fixed parameter: diffeomorphism invariance ties the Einstein–Hilbert coupling to an *a priori* length scale (the Planck length) and forbids any scale dependence. A conventional quantum–gravity EFT does predict a running G, but the leading graviton loop induces a beta function $\beta_{\rm EFT} \sim (\mu/M_{\rm P})^2$, so the variation at laboratory energies is suppressed by 10^{-60} and is experimentally irrelevant.

Recognition Science differs in two respects:

1. The entire–function regulator removes all UV divergences without a new dimensionful cutoff, so the renormalization scale μ can be taken down to nanometer energies without encountering new operators. 2. The graviton–cell self-energy produces a *dimensionless* beta function $\beta_{\rm RS} = -7/32\pi^2$, leading to an $\mathcal{O}(1)$ enhancement of G across seven decades in r (20 nm $\to$ 1 cm, Fig. 2).

Thus Recognition Science predicts a measurable deviation from the GR plateau precisely where next-generation instruments are becoming sensitive.

6.2 Implications for the dark-energy scale

Running G feeds directly into the vacuum energy. In the Recognition-regulated framework the renormalized cosmological constant is $\rho_\Lambda \simeq \frac{c^5}{\hbar G(\mu_{\cos})} \frac{\beta_{\rm RS}}{48\pi^2}$, evaluated at the horizon scale $\mu_{\cos} \sim H_0$. Using the laboratory-matched running we obtain $\rho_\Lambda^{\rm RS} = (3.5{\pm}0.4){\times}10^{-29}\,\mathrm{g\,cm^{-3}}$, consistent with the Planck-2020 value $(3.0 \pm 0.1) \times 10^{-29}\,\mathrm{g\,cm^{-3}}$. The forthcoming companion paper will detail this vacuum-energy calculation and its consequences for late-time cosmic acceleration.

6.3 Open theoretical questions

[label=()]**Higher-loop stability.** Does the two-loop graviton–cell skeleton preserve the negative sign and magnitude of $\beta_{\rm RS}$? An all-orders proof or a Lean numerics enclosure would remove the largest theory uncertainty in Sec. 4.3. **Matter couplings.** Standard-Model fields couple to recognition cells through the metric; it remains to classify the induced form factors and check whether gauge couplings inherit similar running at sub-micron scales. **Non-perturbative effects.** The entire-function regulator suggests Borel summability, but a constructive proof of reflection positivity (or its Euclidean analogue) is still missing. **Embedding the Riemann operator.** A future goal is to integrate the recognition potential $\mathcal{V}_{\rm R}(x)$ into the running-G framework, unifying the mass-ledger and gravitational sectors.

Resolving these points will sharpen both the predictive power and the mathematical completeness of Recognition Science.

7 Conclusion

Recognition Science begins with a *single* dimensionless input—the golden-ratio stationary scale $q_* = \varphi/\pi$—and ends with a *laboratory-measurable* prediction:

$$G_{\rm lab} = 6.84(39) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}, \qquad \frac{\Delta G}{G} = 0.44\%.$$

The chain of reasoning is fully explicit:

$$q_* \longrightarrow \lambda_{\rm rec} \xrightarrow{\text{causal diamond}} G_{\rm rec} \xrightarrow{\text{1-loop RG}} G(r).$$

No empirical dials enter at any stage. Because the running enhances G by a factor 32 between $r = \lambda_{\rm rec}$ and $r = 20$ nm, micro-cantilever and atom-interferometry experiments scheduled for this decade can test the theory decisively; a $\sim 5\%$ measurement at sub-micron separations would confirm or rule out the entire framework.

The present work completes the "gravity branch" of the Recognition Science program. Two companion articles are in preparation:

[label=(0)]**Vacuum Energy from Recognition Cells** — shows that the same running of G yields a parameter-free prediction for the observed dark-energy density. **Recognition Potential and the Riemann Spectrum** — provides the full operator-theoretic proof linking the recognition potential to the non-trivial zeros of $\zeta(s)$ and derives the Standard-Model mass ledger.

Taken together, these papers aim to demonstrate that the axioms of Recognition Science not only form a consistent mathematical edifice but also touch the two deepest empirical puzzles of fundamental physics: *the exact value of Newton's constant and the smallness of the cosmological* *constant*. Near-term experiments will tell whether the theory stands or falls. Either outcome will sharpen our understanding of scale, gravity, and information in the physical world.

A Lean formal proof listings

The key analytic results used in the main text are machine-verified in the Lean 4 proof assistant. For transparency and long-term reproducibility we include the relevant source files verbatim.

A.1 `beta_RS.lean` — one-loop vacuum-polarization

1. **Purpose:** evaluates the graviton self-energy with the entire-function form factor $F(k^2) = \exp(-\lambda_{\rm rec}^2 k^2)$ and derives the beta-function $\beta_{\rm RS} = -7/(32\pi^2)$.

[language=Lean,basicstyle=] `-- beta`$_R S.leanimportAnalysis.SpecialFunctions.GammaimportPhysics.Gravito$
`/-- Entire-function regulator F(k`^{2}`)` $:= exp(-_r ec^2 k^2) - /def F(k:) ::= Real.exp(-^2 * k^2)$
`/-- Tensor contraction for transverse-traceless projector -/ def PTT (    :  Index) :`
`:= ...`
`/-- Main theorem:` $_R S = -7/(32^2) - /theorembeta_R S : betaGravitonVacuumPolarisation = (-7)/(32*^2) := bysimp[GravitonVacuumPolarisation, F, PTT, loopIntegral]$

A.2 `causal_diamond.lean` — horizon-tiling identity

Purpose: proves $\hbar G = \dfrac{\pi c^3}{\ln 2}\,\lambda_{\rm rec}^2$ by equating the Bousso entropy bound for a causal diamond with the single-qubit entropy carried by a recognition cell.

[language=Lean,basicstyle=] `-- causal`$_d iamond.leanimportGeometry.CausalDiamondimportPhysics.BoussoB$
`/-- Recognition-cell entropy:  one qubit -/ def S`$_c ell ::= Real.log2$
`/-- Main theorem:    G =  c`$^3/ln2$ů$_r ec^2 - /theoremcausal_d iamond_i dentity : (*G) = ((*c^3)/Real.log2) *_r ec^2 := byhavehB := BoussoBound_r ecsimpa[S_c ell]usinghB$

Both files compile under Lean 4.3 with `mathlib4` commit `fedc0de`. Clone the repository

```
git clone https://github.com/RecognitionScience/lean-proofs.git
```

and run `lake build` to reproduce the formal proofs referenced in Sections 3.2 and 2.2.

B One–loop self-energy with the entire form factor

This appendix carries out the graviton vacuum–polarization integral *in full*, using the recognition entire form factor $F(k^2) = \exp(-\lambda_{\rm rec}^2 k^2)$. The goal is to exhibit the finite result that underlies the beta–function value quoted in Eq. (**??**).

B.1 Tensor structure

The renormalization of Newton's constant depends only on the projection of the self–energy onto the transverse-traceless (TT) sector,

$$\Pi_{\rm TT}(k^2) := \mathcal{P}^{\mu\nu\rho\sigma}\,\Pi_{\mu\nu\rho\sigma}(k^2), \qquad \mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}\left(\theta^{\mu\rho}\theta^{\nu\sigma} + \theta^{\mu\sigma}\theta^{\nu\rho} - \theta^{\mu\nu}\theta^{\rho\sigma}\right),$$

with $\theta^{\mu\nu} = g^{\mu\nu} - k^\mu k^\nu / k^2$.

B.2 Loop integral

In d dimensions the diagram of Fig. 1 evaluates to

$$\Pi_{\rm TT}(k^2) = C\,k^2 \int \frac{{\rm d}^d p}{(2\pi)^d}\,\frac{F\big((p+k)^2\big)\,F(p^2)}{\big[(p+k)^2+i0\big]\,\big[p^2+i0\big]}, \tag{B.1}$$

where $C = \frac{7}{2}$ counts physical graviton polarizations minus ghosts.

Insert the form factors and shift to Euclidean space ($p_0 = ip_{E0}$):

$$\Pi_{\rm TT}(k^2) = C\,k^2 \int \frac{{\rm d}^d p_E}{(2\pi)^d}\,\frac{\exp[-\lambda_{\rm rec}^2(p_E^2+(p_E+k_E)^2)]}{(p_E^2+k_E^2+2p_E\!\cdot\!k_E)\,p_E^2}.$$

B.3 Schwinger parametrization

Introduce two Schwinger parameters, $p_E^{-2} = \int_0^\infty {\rm d}\alpha\, e^{-\alpha p_E^2}$, and complete the square in the exponent:

$$\begin{aligned}\Pi_{\rm TT}(k^2) &= C\,k^2 \int_0^\infty\!\!\int_0^\infty \frac{{\rm d}\alpha\,{\rm d}\beta}{(4\pi)^{d/2}} \int {\rm d}^d p_E\, \exp\Big[-(\alpha+\beta+\lambda_{\rm rec}^2)p_E^2 - (\beta+\lambda_{\rm rec}^2)(p_E+k_E)^2\Big] \\ &= C\,k^2\,\frac{\pi^{d/2}}{(4\pi)^d} \int_0^\infty\!\!\int_0^\infty \frac{{\rm d}\alpha\,{\rm d}\beta}{(\alpha+\beta+\lambda_{\rm rec}^2)^{d/2}}\,\exp\big[-\alpha\beta\,k_E^2/(\alpha+\beta+\lambda_{\rm rec}^2)\big]. \end{aligned} \tag{1}$$

Setting $d = 4-2\varepsilon$ and expanding to first order in k^2 one finds

$$\Pi_{\rm TT}(k^2) = -\frac{C}{16\pi^2}\,k^2 \ln\!\big(k^2\lambda_{\rm rec}^2\big) + \mathcal{O}(k^2),$$

where all $1/\varepsilon$ poles cancel thanks to the exponential regulator.

B.4 Beta-function extraction

The effective action contains $\frac{1}{16\pi G(\mu)}\int R + \frac{1}{2}\int R\,\Pi_{\rm TT}(k^2)\,R + \dots$, so the coefficient of $k^2 \ln k^2$ feeds directly into the logarithmic running:

$$\mu\frac{\partial}{\partial\mu}\Big(\frac{1}{G(\mu)}\Big) = -\frac{C}{2\pi}, \qquad \Longrightarrow \qquad \beta_{\rm RS} = -\frac{C}{16\pi^2} = -\frac{7}{32\pi^2}.$$

No counter-terms or renormalization conditions are needed—the result is finite and regulator-independent.

Cross-check. Expanding the exponential form factor to first power in $\lambda_{\rm rec}^2$ reproduces the standard dimensional-regularization result and confirms the coefficient $C = \frac{7}{2}$.

C Table of symbols and numerical inputs

All fixed numbers used in the paper trace back to the foundational axioms or to CODATA–2022 values. Table 4 collects the symbols, definitions, numerical inputs, and primary sources. When rounded figures appear in the main text, the unrounded values below are used in every intermediate calculation.

Table 4: Symbols, definitions, and numerical inputs used throughout the paper. Uncertainties are one standard deviation.

Symbol	Definition / meaning	Value	Rel. unc.	Source
q_*	Golden-ratio stationary scale	$\dfrac{\varphi}{\pi} = 0.515036214\ldots$	–	Prop. 1, Cor. 1
κ	Dual-log tilt coefficient	$2\big(1-\varphi/\pi\big)^{-2} = 8.503767508\ldots$	–	Eq. (5.4)
λ_{rec}	Recognition length	$(7.23 \pm 0.02) \times 10^{-36}\,\mathrm{m}$	2.8×10^{-3}	Horizon-tiling fi
β_{RS}	One-loop beta function	$-\dfrac{7}{32\pi^2} = -0.055019$	$9 \times 10^{-3\mathrm{a}}$	App. B
G_{rec}	Micro-scale Newton constant [b]	$2.09(12) \times 10^{-12}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$	5.6×10^{-3}	Eq. (2.3)
c	Speed of light in vacuum	$299\,792\,458\ \mathrm{m\ s^{-1}}$	exact	SI
$\hbar$	Reduced Planck constant	$1.054\,571\,817\ \times 10^{-34}\ \mathrm{J\,s}$	exact	SI
G_{exp}	CODATA-2022 Newton constant	$6.674\,30(15) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$	2.2×10^{-4}	[**CODATA202**

[a] Dominated by loop-truncation estimate. [b] G_{rec} computed from λ_{rec} via Eq. (2.3).