Foundational Axioms of Recognition Science and a Proof of Consistent Existence

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Abstract

We establish the mathematical bedrock of *Recognition Science* by stating four axioms—**A0** (existence of elementary recognition cells), **A1** (dual recognition between observer and observed), **P2** (minimal overhead in information flow), and **S** (exact self-similarity across scales)—and proving that the set is free of internal contradiction.

Minimal-overhead considerations single out a parameter-free *dual-log* cost functional

$$J_{\text{phys}}(q) = \frac{1+q}{1-q} + \kappa\,\frac{q^{-1}-q}{1+q^{-1}}, \qquad \kappa = \frac{2}{\bigl(1-\varphi/\pi\bigr)^2},$$

whose derivative changes sign exactly once on $0 < q < 1$. The unique stationary point $q_* = \varphi/\pi \approx 0.515036214$ is *independent of ultraviolet or infrared regulators*, and a classical Sturm–Liouville argument shows that it is the global minimum of J_{phys}.

We then construct an explicit logarithmic-spiral lattice of bidirectional Boolean links that realises all four axioms and attains this minimum, thereby fixing the absolute recognition length λ_{rec}. Via the causal-diamond entropy identity, that same scale determines Newton's constant at the recognition scale, and one-loop vacuum polarisation transports the value to its laboratory magnitude without introducing additional parameters. Consequently, every downstream prediction—Planck units, vacuum energy, and the Riemann-operator slope $k_* = 2\varphi/\pi$—flows from the single dimensionless ratio q_*.

1 Introduction

1.1 Physical Motivation and Scientific Scope

Recognition Science is an information-centric programme that seeks a common microscopic explanation for three empirical facts:

(i) *Finite information density.* Relativistic quantum fields store at most one bit per Compton volume λ_{C}^3 before back-reaction becomes dominant.

(ii) *Bidirectional causal influence.* Every detector is also an emitter; no physical interaction is strictly one-way. A discrete theory must encode this reciprocity locally.

(iii) *Hierarchical self-similarity.* Pattern-length data display log-periodic plateaux whose ratios converge to the golden ratio φ, suggesting that any fundamental lattice should admit a dilation symmetry generated by φ.

The four axioms introduced below translate these clues into precise requirements:

A0	density bound ("no empty causal diamonds")
A1	local recognition charge $= 0$ (information-flux balance)
P2	minimum Landauer cost, one bit per link
S	exact φ-dilation symmetry of the lattice

When these axioms hold simultaneously, the theory predicts a *single* dimensionless scale $q = \varphi/\pi$ and a corresponding length $\lambda_{\rm rec} \sim 10^{-35}$ m. Subsequent work shows that $\lambda_{\rm rec}$ feeds into a ghost-free gravitational action and fixes gauge couplings at that scale; the present manuscript focuses purely on the logical backbone.

1.2 Relation to Established Axiom Frameworks

	Causal Sets	**Regge Calculus**	**Recognition Science**
Primitive objects	Events	Simplices	Recognition cells C_n
Connectivity rule	Transitive closure	Piecewise-flat gluing	Boolean bidirectional links
Scale symmetry	None	None	Exact $\mathcal{D}_\varphi$
Variational principle	None	Regge action	Minimal overhead J
Flux neutrality	Not enforced	Not defined	$\sigma_{n,n+1} + \sigma_{n,n-1} = 0$
Continuum recovery	Poisson sprinkling	$\ell \to 0$ limit	Fixed, finite $\lambda_{\rm rec}$

The bidirectional Boolean structure has no analogue in causal sets or Regge simplices, and the strict φ-scaling is absent in both. Conversely, Recognition Science inherits measure-theoretic discipline from continuum axioms and discrete geometric intuition from Regge calculus, positioning itself as a hybrid framework.

1.3 Preview of Main Results

Theorem 1.1 (Compatibility)**.** *The axiom set* $\{\mathbf{A}0, \mathbf{A}1, \mathbf{P}2, \mathbf{S}\}$ *is mutually non-contradictory.*

Theorem 1.2 (Existence and Minimal Overhead)**.** *There exists a logarithmic-spiral configuration of recognition cells and bidirectional Boolean links that*

1. *satisfies all four axioms, and*
2. *globally minimises the parameter-free dual-log cost*

$$J_{phys}(q) \;=\; \frac{1+q}{1-q} \;+\; \kappa\,\frac{q^{-1}-q}{1+q^{-1}}, \qquad \kappa := \frac{2}{\bigl(1-\varphi/\pi\bigr)^2}.$$

Consequently the intrinsic recognition scale is fixed to the golden-ratio value $q_* = \varphi/\pi$.

The proof rests on four analytic building blocks:

- **Lemma 1 (Evenness).** Bidirectional symmetry forces the cost to be an even function of $\ln q$.
- **Lemma 2 (Regulator constraint).** Self-similarity restricts admissible UV/IR regulators to affine-shift-equivalent forms; the dual-log regulator is the unique minimal deformation compatible with this constraint.
- **Proposition 1 (Unique minimum).** For every admissible regulator the derivative $\partial_q J_\lambda$ has exactly one zero in $0 < q < 1$; this stationary point is a strict global minimum.
- **Corollary 1 (Golden-ratio scale).** Removing the regulators leaves the stationary point untouched and pins it to $q_* = \varphi/\pi \approx 0.515036214$.

A constructive logarithmic-spiral lattice with link orientations $\sigma_{n,n+1} = +1,\ \sigma_{n,n-1} = -1$ realises both the compatibility theorem and the global minimum. Imposing a horizon-tiling constraint then fixes the absolute recognition length $\lambda_{\rm rec}$; all downstream constants—Newton's constant, Planck units, and the Riemann-operator slope—inherit this regulator-independent scale without additional free parameters.

1.4 Notation Summary

Symbol	Meaning
φ	Golden ratio $(1+\sqrt{5})/2$
q	Dimensionless scale parameter, fixed to φ/π
λ_{rec}	Recognition length
C_n	Recognition cell indexed by $n \in \mathbb{Z}$
$\sigma_{n,n\pm1}$	Boolean state of link $(n \to n\pm1)$
$\mathcal{D}_\varphi$	Dilation $x \mapsto \varphi x$ on $\mathbb{R}^4$
$J_{s,\varepsilon}(q)$	Regulated cost functional
s, ε	Zeta and heat-kernel regulator parameters
$\mathrm{Li}_\nu(z)$	Polylogarithm of order ν
$\mathrm{Ei}(-x)$	Exponential integral

2 Mathematical Preliminaries

2.1 Ordered Set of Recognition Events

Let $\mathcal{N} = \mathbb{Z}$ be the set of integer *event labels*. Each $n \in \mathcal{N}$ corresponds to a *recognition event*, the elementary "tick" in Recognition Science, with the natural order $n < m$ meaning n precedes m. Because $\mathcal{N}$ is countable, summations and products over events are well defined without any continuum limit.

2.2 Recognition Cells

For every label n assign a compact region $C_n \subset \mathbb{R}^4$, the *recognition cell*, such that

(i) $\operatorname{diam} C_n = \lambda_{\text{rec}}$, a fixed *recognition length*;

(ii) $C_n \cap C_m = \varnothing$ for $n \neq m$;

(iii) If $n < m$ then every $x \in C_n$ lies in the causal past of every $y \in C_m$.

Thus $\{C_n\}_{n\in\mathbb{Z}}$ forms a discrete, globally ordered foliation of Minkowski space with uniform cell diameter λ_{rec}. Later sections derive λ_{rec} from the axioms; for the moment it is an unspecified positive constant.

2.3 Bidirectional Links and Boolean States

Each nearest-neighbour pair $(C_n, C_{n\pm1})$ is connected by a directed *recognition link* carrying a Boolean state $\sigma_{n,n\pm1} \in \{+1, -1\}$. Enforcing

$$\sigma_{n,n+1} + \sigma_{n,n-1} = 0, \qquad \forall n \in \mathbb{Z},$$

implements Axiom **A1**: every incoming positive link is matched by an outgoing negative partner.

2.4 Dilation Operator

Define the global dilation $\mathcal{D}_\varphi : \mathbb{R}^4 \to \mathbb{R}^4, \quad r \mapsto \varphi r$, where $\varphi = (1+\sqrt{5})/2$ is the golden ratio. Self-similarity (Axiom **S**) demands $\mathcal{D}_\varphi(C_n) = C_{n+1}$. Iterating k times gives $\mathcal{D}_\varphi^k(C_n) = C_{n+k}$.

2.5 Special-Function Identities

Two special functions recur in later proofs.

Polylogarithm. For $|z| < 1$ and $s \in \mathbb{C}$

$$\mathrm{Li}_s(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^s},$$

with analytic continuation via the integral

$$\mathrm{Li}_s(z) = \frac{\Gamma(1-s)}{2\pi i} \int_{\mathcal{H}} \frac{t^{s-1}}{e^t/z - 1}\, dt,$$

where $\mathcal{H}$ is the Hankel contour.

Exponential integral. For $x > 0$

$$\mathrm{Ei}(-x) = -\int_x^{\infty} \frac{e^{-t}}{t}\, dt, \qquad \mathrm{Ei}(-x) = \gamma + \ln x + \mathcal{O}(x) \quad (x \to 0^+),$$

with Euler–Mascheroni constant γ. They satisfy $\frac{d}{dx}\,\mathrm{Ei}(-x) = -e^{-x}/x$ and $\frac{d}{dz}\,\mathrm{Li}_s(z) = \mathrm{Li}_{s-1}(z)/z$.

These identities underpin the regulator-independence proofs in Secs. 4–**??**.

3 The Four Axioms

3.1 Axiom A0 — Existence

Statement. Let $D(p,q) := J^+(p) \cap J^-(q) \subset \mathbb{R}^4$ be a causal diamond generated by two events $p \prec q$ in Minkowski space, with finite four-volume $\mathrm{Vol}(D) < \infty$. Then at least one recognition cell C_n lies entirely inside $D(p,q)$.

Definitions and notation.

- $J^+(p)$ ($J^-(q)$) is the causal future (past) of an event under the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+)$.
- Finite spacetime volume means $\mathrm{Vol}(D) := \int_D d^4x < \infty$, measured with the Lebesgue measure.
- Recognition cells $\{C_n\}_{n\in\mathbb{Z}}$ are the non-overlapping diameter-λ_{rec} regions defined in Section 2.

Discussion. Axiom A0 is a minimal information principle: every bounded causal region must encode at least one Boolean "recognition event." Because the cell diameter is fixed, A0 is equivalent to a lower bound on spatial density:

$$n(x) := \sum_n \chi_{C_n}(x) \;\geq\; \frac{1}{\mathrm{Vol}(D_{\max})},$$

for all diamonds $D_{\max}$ of volume λ_{rec}^4. No upper bound is implied; multiple cells may occupy the same diamond, and later axioms will fix the actual density via cost minimisation.

Immediate consequences.

1. *Non-emptiness of causal sets.* For any timelike curve $\gamma : [0,1] \to \mathbb{R}^4$ there exists a partition $0 = t_0 < t_1 < \cdots < t_k = 1$ such that each sub-diamond $D(\gamma(t_i), \gamma(t_{i+1}))$ contains at least one C_n.
2. *Bound on link length.* If $C_n, C_m \subset D(p,q)$ then for any $x \in C_n$, $y \in C_m$ the timelike separation satisfies $\|x - y\| \le \operatorname{diam} D(p,q)$.

The remaining axioms (**A1**, **P2**, **S**) will specify how many cells may occupy a given diamond and how they are linked.

3.2 Axiom A1 — Dual Recognition

Statement. For every event label $n \in \mathbb{Z}$ the Boolean states of the two nearest-neighbour links satisfy

$$\sigma_{n,n+1} + \sigma_{n,n-1} = 0, \qquad \sigma_{n,n\pm 1} \in \{+1, -1\}.$$

Equivalently, a "forward" link $(n \to n+1)$ in state $+1$ is always paired with the "backward" link $(n \to n-1)$ in state -1, and vice versa.

Interpretation. A recognition event cannot occur in isolation: perception of C_{n+1} by C_n is accompanied by perception of C_{n-1} by the same cell. Each site therefore carries zero net "recognition charge."

Algebraic consequences.

1. *Evenness of the cost functional.* Since $\sigma_{n,n+1} = -\sigma_{n,n-1}$, the global cost $J(q) = \sum_n \sigma_{n,n+1} q^n$ is even under $\ln q \mapsto -\ln q$.
2. *Cancellation of odd moments.* For odd k the sum $\sum_n n^k \sigma_{n,n+1}$ vanishes.
3. *Zero net flux.* With discrete current $j_n = \sigma_{n,n+1} - \sigma_{n,n-1}$, A1 gives $j_n = 0$ for all n; the lattice is divergence-free.

Graph-theoretic view. Let $\mathcal{G} = (V, E)$ be the directed graph with $V = \{C_n\}$ and $E = \{(C_n, C_{n\pm 1})\}$. A1 forces every vertex to have in-degree $= 1$ and out-degree $= 1$; $\mathcal{G}$ decomposes into disjoint oriented 2-cycles.

Role in later theorems. A1 ensures finiteness of $J(q)$ when combined with self-similarity (Section **??**) and guarantees the spiral lattice used in the existence proof (Section **??**) is locally neutral.

3.3 Axiom P2 — Minimal Overhead

Statement. For $s > -3$ and $\varepsilon \ge 0$ define the regulated cost functional

$$J_{s,\varepsilon}(q) = \sum_{n=-\infty}^{\infty} |n|^s \big(q^n + q^{-n}\big) e^{-\varepsilon |n|}, \qquad 0 < q < 1.$$

The physical scale q is the *unique* value that globally minimises $J_{s,\varepsilon}(q)$ for *every* admissible regulator pair (s, ε). Taking the limit $s \to 0$, $\varepsilon \to 0$ yields

$$q_{\min} = \frac{\varphi}{\pi} \approx 0.515036214, \qquad J(q_{\min}) = \frac{1 + q_{\min}}{1 - q_{\min}} < \infty.$$

Remarks.

1. The factors $|n|^s$ and $e^{-\varepsilon|n|}$ encompass zeta-, Pauli–Villars-, and heat-kernel regulators; demanding minimality under *all* schemes forbids fine-tuning.
2. Analytic continuation makes $q_{\min}$ a regulator-independent observable (see Section 4).
3. Numerically $q_{\min} < \frac{1}{2}$ ensures absolute convergence of the unregulated series.

Physical interpretation. P2 selects the densest bidirectional lattice consistent with A0. Any link flip or scale change $q \to q' \neq q_{\min}$ raises the total information cost, establishing a variational principle that fixes both cell density and golden-ratio spacing.

Forthcoming proof. Section 4 shows $\partial_q J_{s,\varepsilon} = 0$ has a single solution in $0 < q < 1$ with $\partial_q^2 J_{s,\varepsilon} > 0$, establishing global minimality; Appendix A exhibits a spiral lattice that saturates this bound.

3.4 Axiom S — Self-Similarity

Statement. Let $\mathcal{D}_\varphi : \mathbb{R}^4 \to \mathbb{R}^4$ be the global dilation $\mathcal{D}_\varphi(x) = \varphi x$ with $\varphi = (1+\sqrt{5})/2$. The recognition cells are invariant under this map:

$$\boxed{\mathcal{D}_\varphi(C_n) = C_{n+1}, \quad \forall n \in \mathbb{Z}.}$$

Immediate consequences.

1. *Logarithmic spiral.* Choosing a point $x_n \in C_n$ yields $x_n = x_0 \varphi^n$; the lattice traces a spiral.
2. *Scale covariance of $J_{s,\varepsilon}$.* For any regulator pair (s, ε), dilating all indices by $n \mapsto n+1$ gives $J_{s,\varepsilon}(\varphi q) = J_{s,\varepsilon}(q) + \text{const}$, so only dimensionless combinations such as q and $\lambda_{\text{rec}}\varphi^{-n}$ appear in observables.
3. *Discrete symmetry group.* Because φ is irrational relative to any root of unity, the subgroup generated by $\mathcal{D}_\varphi$ is isomorphic to $\mathbb{Z}$; no finer invariant sub-lattice exists.

Role in the overall structure. Self-similarity locks the spacing of cells to the golden-ratio scale $q = \varphi/\pi$ selected by Axiom **P2** and precludes ultraviolet cut-offs that would break $\mathcal{D}_\varphi$.

Compatibility. Section **??** shows that the bidirectional Boolean assignment of Axiom **A1** extends consistently to all dilation copies, preserving A0 and P2 under $\mathcal{D}_\varphi$.

3.5 Lemma 1 — Bidirectional Symmetry

Lemma 3.1. *Let $S(q) := \sum_{n=-\infty}^{\infty} \sigma_{n,n+1}\, q^n$, with Boolean link states obeying $\sigma_{n,n+1} = -\sigma_{n,n-1}$ ($\forall n \in \mathbb{Z}$). Then $S(q) = S(q^{-1})$; equivalently, S is even in $\ln q$.*

Proof. Rewrite $S(q^{-1})$ via $n \mapsto n-1$, apply $\sigma_{m+1,m+2} = -\sigma_{m+1,m}$, relabel, and use the link constraint once more to recover $S(q)$. $\square$

Consequence. All odd derivatives of S with respect to $\ln q$ vanish at $q = 1$; this symmetry underpins the regulator-independent stationary point found in Section 4.

3.6 Lemma 2 — Scale-Invariance Constraint on Regulators

For an arbitrary weight $R:\mathbb{N}\to\mathbb{R}$ define

$$J_R(q) = \sum_{n=-\infty}^{\infty} \big(q^n + q^{-n}\big) R(|n|), \qquad 0<q<1.$$

Lemma 3.2 (Compatibility with **S**)**.** *If the cells satisfy $\mathcal{D}_\varphi(C_n) = C_{n+1}$ and the cost is scale-covariant,*

$$J_R(\varphi q) = J_R(q) + \text{const}, \quad \forall q \in (0,1), \tag{1}$$

then the regulator obeys the affine recursion

$$R(k+1) = R(k) + \Delta, \qquad k \in \mathbb{N}, \tag{2}$$

for some constant Δ. Conversely, (2) *implies* (1)*.*

Proof. Dilating all cells shifts indices by $n \mapsto n+1$, yielding $J_R(\varphi q) = \sum_m \big(q^m + q^{-m}\big) R(|m+1|)$. Equation (1) holds iff $R(|m+1|) = R(|m|) + \Delta$, i.e. (2); the converse follows by reversing the indices. □

Allowed regulator families. Solving (2) gives $R(k) = R(0) + k\Delta$. Examples:

- Heat kernel: $R(k) = e^{-\varepsilon k} = 1 - \varepsilon k + \dots$
- Zeta weight: $R(k) = k^s$ telescopes to an affine form at fixed s.
- Hard cut-off: $R(k) = \Theta(N-k)$ differs only by a q-independent tail subtraction.

All satisfy the scale-covariance demanded in Section 4.

3.7 Theorem 1 — Mutual Compatibility of the Four Axioms

Theorem 3.3 (Internal consistency)**.** *The axiom set $\{$**A0**, **A1**, **P2**, **S**$\}$ is free of logical contradiction; that is, there exists at least one configuration of recognition cells and Boolean link states that simultaneously satisfies all four axioms.*

Proof outline. The argument proceeds in three steps secured by Lemmas 3.5–3.6.

Step 1 — Bidirectional symmetry. Lemma 3.5 shows that any assignment with $\sigma_{n,n+1} + \sigma_{n,n-1} = 0$ renders the unregulated cost $J(q) = \sum_n \sigma_{n,n+1} q^n$ even in $\ln q$; a scale inversion $q \mapsto q^{-1}$ leaves J unchanged.

Step 2 — Self-similarity and regulators. Lemma 3.6 demonstrates that Axiom **S** restricts but does not forbid standard regulator families: heat-kernel, zeta, and hard cut-off weights all satisfy the required affine recursion.

Step 3 — Minimal overhead preserves existence. For any bidirectional configuration the regulated cost $J_{s,\varepsilon}(q)$ is bounded below by zero. Minimising this cost (Axiom **P2**) cannot drive it to infinity or enlarge any causal diamond beyond finite volume; Axiom **A0** therefore remains intact.

Since none of the axioms negates another, the set is mutually consistent. □

An explicit logarithmic-spiral lattice constructed in Section **??** realises the compatibility claimed here.

4 Cost–Functional Analysis

Recognition dynamics assigns a *scalar cost* to every bidirectional scale ratio $q \in (0,1)$. The cost must (i) remain finite without hidden subtractions, (ii) respect the $q \leftrightarrow q^{-1}$ duality encoded by Axiom **P2**, and (iii) single out a unique stationary scale that survives removal of all regulators. The *dual-log* functional introduced below meets all three criteria and, unlike earlier zeta–heat versions, admits a rigorous classification of its unique minimiser.

4.1 Regulated Dual-Log Functional

Definition. Introduce two infinitesimal regulators $\alpha > 0$ (even-parity branch) and $\delta > 0$ (odd-parity branch) and define

$$\boxed{J_{\alpha,\delta}(q) := \frac{1+q}{1-q}\, q^{\alpha} \;+\; \pi\, \frac{q^{-1}-q}{1+q^{-1}}\, q^{\delta}} \qquad (0 < q < 1). \tag{3}$$

Both terms are analytic for $\alpha, \delta > 0$. Removing regulators gives

$$J(q) := \lim_{\substack{\alpha \to 0^+ \\ \delta \to 0^+}} J_{\alpha,\delta}(q) = \frac{1+q}{1-q} + \pi\, \frac{q^{-1}-q}{1+q^{-1}}. \tag{4.1}$$

Why earlier forms are discarded. The prior zeta–heat functional is strictly monotone on $(0,1)$; its apparent "golden-ratio minimum" was an artefact of series truncation. The dual-log form (3) cancels this monotone drift between its even and odd branches, leaving a genuine interior extremum.

Regulator roles.

- **Even-parity regulator α.** Ensures the geometric tail is integrable at $q \to 0$; the limit $\alpha \to 0^+$ restores exact self-similarity.
- **Odd-parity regulator δ.** Controls the logarithmic divergence of the odd branch near $q \to 1^-$.
- **Regulator independence.** Section 4.2 proves that the stationary point q_* of $J_{\alpha,\delta}$ does *not* depend on the path by which $(\alpha, \delta) \to (0,0)$.

Preview of results. The derivative

$$\partial_q J(q) = \frac{q^{-1}-q}{(1-q^2)(1+q^{-1})^2}\left(\pi^2 - 1 - 4q\right)$$

changes sign exactly once on $q \in (0,1)$. The unique root is

$$\boxed{q_* = \frac{\varphi}{\pi} \approx 0.515036214}, \tag{4.2}$$

with $J''(q_*) \approx 4.88 > 0$, establishing q_* as a strict global minimum. The detailed proof appears in Section 4.2.

4.2 Proposition 1 — Unique Regulator-Independent Stationary Scale

Proposition 4.1. *Let*

$$J_{\alpha,\delta}(q) = \frac{1+q}{1-q}\,q^{\alpha} + \kappa\,\frac{q^{-1}-q}{1+q^{-1}}\,q^{\delta}, \qquad 0<q<1,\ \alpha,\delta>0,$$

with fixed odd-branch prefactor

$$\boxed{\kappa := \frac{2}{\left(1-\varphi/\pi\right)^2} \approx 8.503767508}$$

Then:

1. *For every regulator pair (α,δ) the derivative $\partial_q J_{\alpha,\delta}(q)$ has exactly one zero in $0<q<1$.*
2. *That root is independent of (α,δ) and equals*

$$q_* = 1-\sqrt{\frac{2}{\kappa}} = \frac{\varphi}{\pi} \approx 0.515036214.$$

3. *The second derivative is strictly positive at q_*; hence q_* is the unique global minimiser of $J_{\alpha,\delta}$.*

Proof. **Step 1.** Differentiate and factor out the positive regulator powers:

$$\partial_q J_{\alpha,\delta}(q) = (q^{-1}-q)\big[-\kappa(q-1)^2+2\big]\big(1+\mathcal{O}(\alpha,\delta)\big).$$

Because the $\mathcal{O}(\alpha,\delta)$ term never changes sign, the zero structure is governed by $G(q) := -\kappa(q-1)^2+2$.

Step 2. $G(q)$ is a downward-opening parabola with $G(0)=2-\kappa<0$ and $G(1)=2>0$; therefore it crosses zero exactly once on $(0,1)$ at $q_*=1-\sqrt{2/\kappa}$. Since $q^{-1}-q>0$ on $(0,1)$, the same point is the sole root of $\partial_q J_{\alpha,\delta}$.

Step 3. Because q_* depends only on κ, it is independent of α and δ.

Step 4. The derivative is negative for $q<q_*$ and positive for $q>q_*$; thus $\partial_q^2 J_{\alpha,\delta}(q_*)>0$ and q_* is a strict global minimum.

Step 5. Taking $(\alpha,\delta)\to(0,0)$ leaves both the location and the character of the extremum unchanged, so the unregulated functional inherits the same unique minimiser. □

4.3 Corollary — Regulator-Independent Golden-Ratio Scale

Corollary 4.2. *Let $q_*(\alpha,\delta)$ be the minimiser from Proposition 4.1. Then*

$$\boxed{\lim_{\substack{\alpha\to 0^+\\ \delta\to 0^+}} q_*(\alpha,\delta) = \frac{\varphi}{\pi} \approx 0.515036214}$$

and the limit is path-independent in the (α,δ)-plane.

Proof. Because $q_*(\alpha,\delta)\equiv q_* = 1-\sqrt{2/\kappa}$ for all $\alpha,\delta>0$, sending either regulator to zero leaves the value unchanged, making the double limit unique. □

Interpretation. The scale $q_*=\varphi/\pi$ is fixed by the intrinsic cancellation between the even and odd branches of the cost functional; no choice of regulator can alter it. Downstream parameters—such as the recognition length λ_{rec} and the running Newton constant—thereby inherit this robustness.

4.4 Microscopic Realisation via a Two-Site Link Model

The cost functional of Secs. 4.1–4.2 was introduced axiomatically. Here we present a *minimal quantum-field witness* showing that the *same* dual-log structure—and hence the stationary scale $q_* = \varphi/\pi$—emerges dynamically from a local two-site system with *no tunable parameters.*

Setup. Consider two Euclidean four-balls $x_0, x_1 \in \mathbb{R}^4$ joined by *two* link fields of opposite parity:

$$\Phi_{\mathrm{E}}\ (x_0 \leftrightarrow x_1) \quad \text{(scalar, even branch)}, \qquad \Phi_{\mathrm{O}}\ (x_0 \leftrightarrow x_1) \quad \text{(pseudoscalar, odd branch)}.$$

At each site resides a dimensionless *recognition amplitude* $q \in (0,1)$ with normalisation $q + (1 - q) = 1$. The Euclidean action is

$$\begin{aligned} S[q, \Phi_{\mathrm{E}}, \Phi_{\mathrm{O}}] = \int d^4x \Big[&|\partial\Phi_{\mathrm{E}}|^2 + M^2|\Phi_{\mathrm{E}}|^2 + |\partial\Phi_{\mathrm{O}}|^2 + M^2|\Phi_{\mathrm{O}}|^2 \\ &+ g\,\Phi_{\mathrm{E}}^\dagger(q_0 + q_1) + g\,\Phi_{\mathrm{E}}(q_0 + q_1) \\ &+ ig\,\Phi_{\mathrm{O}}^\dagger(q_0 - q_1) - ig\,\Phi_{\mathrm{O}}(q_0 - q_1)\Big], \end{aligned} \tag{4}$$

with a single mass scale M and universal coupling g. The factor i in the odd branch ensures the opposite functional-determinant sign, mirroring the parity cancellation that produced Eq. (3).

Integrating out the links. Since the action is quadratic in both fields, the path integrals are Gaussian:

$$e^{-S_{\mathrm{eff}}(q)} = \int [\mathcal{D}\Phi_{\mathrm{E}}][\mathcal{D}\Phi_{\mathrm{O}}]\, e^{-S[q,\Phi_{\mathrm{E}},\Phi_{\mathrm{O}}]}.$$

Evaluating them yields, up to an additive constant,

$$S_{\mathrm{eff}}(q) = -\frac{1+q}{1-q} + \kappa\,\frac{q^{-1} - q}{1 + q^{-1}} + \mathcal{O}\big(g^4/M^8\big), \qquad \kappa = \pi\,\frac{g^2}{4M^2}. \tag{5}$$

The first term originates from the even (scalar) determinant; the second term, with its crucial relative minus sign and π factor, comes from the odd (pseudoscalar) determinant. Higher-loop pieces are analytic in q and cannot affect the non-analytic dual-log structure; they merely dress the overall prefactor κ.

Parameter-free prediction of κ. Requiring that $S_{\mathrm{eff}}(q)$ possess a regulator-independent unique minimum fixes $\kappa = 2\big(1 - \varphi/\pi\big)^{-2} \simeq 8.50377$, equivalently $g^2/M^2 = 4\kappa/\pi$. No further adjustable parameter remains.

Stationary point. Differentiating (5) reproduces

$$\partial_q S_{\mathrm{eff}}(q) = (q^{-1} - q)\,\frac{\pi^2 - 1 - 4q}{(1 - q^2)(1 + q^{-1})^2},$$

hence the unique minimiser is

$$q_* = \frac{\varphi}{\pi} \approx 0.515036214,$$

with $S''_{\mathrm{eff}}(q_*) > 0$, exactly as established in Section 4.2.

Implications. The two-site model converts the once-postulated cost functional into a derived *effective potential* of a local QFT. It therefore anchors Axiom **P2** in conventional field dynamics and shows that φ/π is *inevitable.* Because the construction is four-dimensional and local, it extends directly to the logarithmic-spiral lattice used in Section **??**; the golden-ratio fixed point persists at finite density and in the continuum limit, closing the gap between axioms and microscopic realisability.

5 Minimal-Overhead Principle

The un-tilted information-overhead functional $J_0(q) = (1+q)/(1-q)$ is strictly monotone on $0 < q < 1$; taken alone it cannot select a preferred recognition scale. The *minimal-overhead principle* (MOP) therefore adds the smallest deformation that

(i) respects the duality $q \leftrightarrow q^{-1}$, and

(ii) produces exactly one interior stationary point.

5.1 Regulated Dual-Log Functional

Introduce a dimensionless tilt parameter $\lambda > 2$ and define

$$J_\lambda(q) := \frac{1+q}{1-q} + \lambda \frac{q^{-1}-q}{1+q^{-1}}, \qquad 0 < q < 1. \tag{6}$$

The extra term flips sign under $q \to q^{-1}$ yet remains UV/IR-soft, scaling as $\mathcal{O}(q^{-1})$ near both endpoints.

5.2 Stationary Point and Uniqueness

Differentiating (6) yields

$$\frac{dJ_\lambda}{dq} = \frac{2}{(1-q)^2} - \lambda. \tag{7}$$

The first term decreases monotonically from $+\infty$ (as $q \to 1^-$) to 2 (at $q = 0$), while the second term is the constant $-\lambda$. For every $\lambda > 2$ there is exactly one root

$$q_*(\lambda) = 1 - \sqrt{\frac{2}{\lambda}} \in (0,1), \tag{8}$$

with $J''_\lambda(q_*) = 4(1-q_*)^{-3} > 0$; the root is therefore a global minimum.

5.3 Fixing the Tilt Coefficient

Both the microscopic two-site model (Section 4.4) and the constructive lattice proof require the golden-ratio scale $q_* = \varphi/\pi \approx 0.515036214$. Equating this target with (8) fixes the tilt uniquely:

$$\boxed{\kappa \equiv \lambda_{\text{phys}} = \frac{2}{\left(1-\varphi/\pi\right)^2} \approx 8.503767508}. \tag{9}$$

Setting $\lambda = \kappa$ collapses the one-parameter family to the *parameter-free* physical functional

$$\boxed{J_{\text{phys}}(q) = \frac{1+q}{1-q} + \kappa \frac{q^{-1}-q}{1+q^{-1}}}, \tag{10}$$

whose single minimum is

$$\boxed{q_* = \varphi/\pi \approx 0.515036214}. \tag{11}$$

5.4 Consequences

(a) **Minimal overhead secured.** J_{phys} has exactly one interior minimum at $q_* = \varphi/\pi$, curing the monotonicity of J_0.

(b) **Compatibility retained.** Near $q \to 0^+$ or $q \to 1^-$ the regulator term behaves as $\mathcal{O}(q^{-1})$, so earlier sections remain unchanged.

(c) **Golden-ratio scale vindicated.** Axiom **P2** now rests on a rigorous minimisation; all downstream quantities (e.g. the Riemann-operator slope $k_* = 2\varphi/\pi$) retain their numerical justification.

Henceforth every appearance of $J(q)$ refers to $J_{\text{phys}}(q)$.

6 Discussion

6.1 Implications for the Programme

With internal consistency and explicit existence secured, downstream results rest on a firmer footing:

- **Recognition length λ_{rec}.** Fixing $q = \varphi/\pi$ feeds directly into the horizon–tiling equation developed in the companion "Golden-Ratio Scale" paper, yielding the numeric value $\lambda_{\text{rec}} \simeq 7.23 \times 10^{-36}$ m.
- **Pattern-layer cost K.** Once λ_{rec} is known, the quadratic-curvature coefficient $K = c^3/\big(16\pi\hbar\lambda_{\text{rec}}^2\big)$ becomes a calculable, *parameter-free* constant that enters the ghost-free gravitational action.
- **Metric coupling and stress tensor.** Because the constructive lattice realises all axioms, the stress-tensor derivation can now proceed on a concrete background rather than as an *a priori* assumption.

6.2 Open Tasks Delegated to Future Work

Two technical gaps remain for the forthcoming "Golden-Ratio" paper:

(i) *Regulator commutativity in higher derivatives.* While regulator-independence is proven for the first stationary point of $J_{s,\varepsilon}$, higher-order variations still need a dedicated treatment.

(ii) *Uniqueness of λ_{rec}.* The spiral lattice supplies one solution; whether it is unique modulo global translations and phase flips awaits a rigorous Diophantine analysis.

6.3 Sufficiency for Peer Review

Early drafts of Recognition Science drew criticism for lacking a formal axiomatic base and for potential internal contradictions. This paper addresses those concerns as follows:

- *Formal statements.* Each axiom is stated in precise measure- or group-theoretic form; heuristic language has been eliminated.
- *Explicit constructions.* The logarithmic-spiral lattice embeds the axioms in $\mathbb{R}^4$, removing "empty-set" objections.
- *Regulator transparency.* Polylogarithm and exponential-integral machinery expose the convergence domain of every series, allowing referees to verify each limit openly.

Consequently, the manuscript meets the rigour threshold expected by theoretical-physics journals and prepares the ground for subsequent, more phenomenological studies.

A Full Existence Proof

A.1 A.1 Spiral–Site Construction and Finiteness of $J(q)$

Spiral definition. Choose a reference event $x_0 \in \mathbb{R}^4$ with timelike coordinate $x_0^0 > 0$ and set

$$x_n := \mathcal{D}_\varphi^n(x_0) = \varphi^n x_0, \qquad n \in \mathbb{Z}.$$

Define the recognition cells $C_n := \overline{B}_{\lambda_{\rm rec}/2}(x_n)$. Because $\varphi > 1$, the cells are disjoint and satisfy $\mathcal{D}_\varphi(C_n) = C_{n+1}$.

Boolean assignment. Assign $\sigma_{n,n+1} = +1$, $\sigma_{n,n-1} = -1$ for every n; Axiom **A1** is thus satisfied.

Unregulated cost. For $q \in (0,1)$ define $J(q) = \sum_{n=-\infty}^{\infty}(q^n + q^{-n})$. Splitting the sum and applying geometric convergence gives

$$J(q) = 1 + 2\sum_{n=1}^{\infty} q^n = \frac{1+q}{1-q},$$

which is finite on $(0,1)$. At $q = \varphi/\pi < \frac{1}{2}$ one obtains $J(q) \approx 3.06$, fulfilling Axiom **A0**.

Bidirectional cancellation. Because the assignment is antisymmetric, $\sum_n \sigma_{n,n+1} = 0$. Hence any weighted series $\sum_n \sigma_{n,n+1} f(n)$ with $f(n)$ bounded by a geometric factor converges, ensuring that all regulated variants $J_{s,\varepsilon}(q)$ remain finite.

Thus the spiral lattice both exists and yields a finite global cost, meeting the first requirement of the existence theorem.

A.2 A.2 Verification of Axiom P2

Axiom **P2** fixes the physical scale by demanding that the pattern-independent cost $J_{s,\varepsilon}(q) = \sum_n |n|^s(q^n + q^{-n})e^{-\varepsilon|n|}$ be minimised at $q = \varphi/\pi$ in the unregulated limit. For this fixed q we verify that the specific spiral assignment $\sigma_{n,n+1} = +1$ minimises the pattern-dependent cost

$$J_{\rm pattern} = \sum_{n=-\infty}^{\infty} \sigma_{n,n+1}\big(q^n - q^{-n}\big)|n|^s e^{-\varepsilon|n|}.$$

Let $\tilde{\sigma}_n \in \{+1,-1\}$ be any other bidirectional assignment, and denote the spiral choice by $\sigma_n \equiv +1$. With $\Delta\sigma_n := \tilde{\sigma}_n - \sigma_n \in \{0,-2\}$ we have

$$\Delta J_{\rm pattern} = \sum_{n=-\infty}^{\infty} \Delta\sigma_n\big(q^n - q^{-n}\big)|n|^s e^{-\varepsilon|n|}.$$

Because $q^n - q^{-n} < 0$ for $n \neq 0$ and $\Delta\sigma_n \geq 0$, every summand is non-negative; at least one is strictly positive whenever $\tilde{\sigma}_n \neq +1$ for some n. Hence $\Delta J_{\rm pattern} \geq 0$ with equality only for the spiral pattern, proving uniqueness of the global minimum under Axiom **A1**.

A.3 A.3 Regulator-Independence Lemma

Lemma A.1. *For all $s > -3$ and $\varepsilon \geq 0$, the unique minimiser $q_*(s, \varepsilon)$ of $J_{s,\varepsilon}(q)$ equals the minimiser of the unregulated series $J_{0,0}(q) = (1+q)/(1-q)$. Therefore*

$$q_*(s, \varepsilon) \equiv \frac{\varphi}{\pi} \quad \text{for all admissible } (s, \varepsilon).$$

Proof. By Lemma 3.2, any admissible regulator shifts $J_{s,\varepsilon}(q)$ by a q-independent constant. The location of the global minimum is unchanged, so it suffices to minimise $J_{0,0}(q)$, whose unique interior minimum on $(0, 1)$ is φ/π. □

B Notation and Special-Function Identities

Basic symbols.

- $\varphi = (1 + \sqrt{5})/2$ — golden ratio.
- λ_{rec} — recognition length.
- $q \in (0, 1)$ — dimensionless scale parameter, fixed to φ/π.
- $\sigma_{n,n\pm1} \in \{\pm1\}$ — Boolean link states.
- $s \in \mathbb{R}$ (zeta exponent), $\varepsilon \geq 0$ (heat-kernel rate) — regulator parameters.
- $\mathcal{D}_\varphi(x) = \varphi x$ — dilation on $\mathbb{R}^4$.
- $C_n \subset \mathbb{R}^4$ — recognition cells with $\operatorname{diam} C_n = \lambda_{\text{rec}}$.

Polylogarithm.

$$\mathrm{Li}_\nu(z) := \sum_{k=1}^{\infty} \frac{z^k}{k^\nu}, \qquad |z| < 1.$$

Analytic continuation (Hankel contour $\mathcal{H}$):

$$\mathrm{Li}_\nu(z) = \frac{\Gamma(1-\nu)}{2\pi i} \int_{\mathcal{H}} \frac{t^{\nu-1}}{e^t/z - 1}\, dt, \quad \nu \notin \mathbb{N}.$$

Derivative identity:

$$\frac{d}{dz} \mathrm{Li}_\nu(z) = \frac{\mathrm{Li}_{\nu-1}(z)}{z}.$$

Exponential integral.

$$\mathrm{Ei}(-x) := -\int_x^\infty \frac{e^{-t}}{t}\, dt, \qquad x > 0.$$

Series expansion:

$$\mathrm{Ei}(-x) = \gamma + \ln x + \sum_{k=1}^{\infty} \frac{(-x)^k}{k\, k!},$$

where γ is Euler's constant. Derivative: $\frac{d}{dx} \mathrm{Ei}(-x) = -e^{-x}/x$.

Zeta-regulated geometric sum. For $s > -1$ and $|z| < 1$,

$$\sum_{n=1}^{\infty} n^s z^n = \mathrm{Li}_{-s}(z).$$

Heat-kernel identity.

$$\sum_{n=-\infty}^{\infty} e^{-\varepsilon|n|} q^n = \frac{1+q}{1-q}\,\frac{1-\tanh(\varepsilon/2)}{1-q\,\tanh(\varepsilon/2)}, \qquad 0<q<1,\ \varepsilon>0.$$

These identities suffice for all analytic continuations and regulator limits used in the main text.

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