Abstract

We derive classical and quantum gravity from a minimal information-theoretic axiom—a recognition event requires non-empty data—without adjustable parameters. The discrete recognition calculus yields conserved ledger dynamics on graphs, forcing exact integer 1-forms, a unique convex cost functional $J(x) = \frac{1}{2}(x + x^{-1}) - 1$, and an 8-tick minimal period in three dimensions. These results dictate a discrete light-cone bound $\Delta r \leq c\,\Delta t$ and a parameter-free Planck normalization $\lambda_{\text{rec}} = \sqrt{\hbar G/(\pi c^3)}$. Mesh refinement recovers the continuity equation $\partial_t \rho + \nabla \cdot J = 0$ and Einstein's field equations with emergent Lorentz invariance. The unique scale-recursion fixed point is the golden ratio $\varphi = (1 + \sqrt{5})/2$, proven to be the only positive solution satisfying four independent physical constraints; we machine-verify that common alternatives (e, π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$) fail. Structural theorems (T2–T7, bridge identities, completeness) are machine-verified in Lean 4 with over 26,000 lines of code across 280 modules; logical claims are falsifiable by running `lake build`. Key testable predictions: nanoscale gravity enhancement $G(20\,\text{nm})/G_\infty \approx 32$ with slope $\beta = -0.0557$, galaxy rotation curves with zero per-galaxy tuning (median $\chi^2/N = 2.75$ vs. MOND 2.47), and pulsar timing signatures at $\sim 10\,\text{ns}$.

Zero-Parameter Quantum Gravity from Discrete Recognition Calculus

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1 Introduction

Problem and stance. Modern fundamental physics explains an enormous range of phenomena, yet it leaves a basic structural gap: many of its key numbers are merely *measured* and not *derived*. This is the parameter problem. Our stance is parameter-free: only dimensionless displays that survive admissible gauge (units) moves are reported, and all such displays are fixed by derivation rather than fit. Concretely, admissible gauge moves jointly rescale the length and time anchors at fixed speed,

$$(\ell_0, \tau_0) \mapsto (s\,\ell_0, s\,\tau_0), \qquad c = \frac{\ell_0}{\tau_0} \ \text{ fixed,}$$

and we restrict attention to quantities invariant under these moves. In this gauge, a single route identity ties the two anchor routes to one dimensionless constant,

$$\frac{\tau_{\rm rec}}{\tau_0} \ = \ \frac{\lambda_{\rm kin}}{\ell_0} \ =: \ K, \qquad \frac{\ell_0}{\tau_0} = c,$$

so that calibration is unique up to units and no hidden "knobs" remain. Within this posture we state our main classical claim: a discrete calculus reproduces general relativity (GR) in the continuum limit and yields a computable Planck normalization with concrete, falsifiable gravitational predictions.

Promise (scope and placement). All classical derivations needed to read the physics stand in the main text in conventional GR and astrophysics notation. The formal spine (discrete calculus, exactness, counting, and bridge invariants) and a machine-verified closure that audits the identities are relegated to the Methods and Appendix; they are policy-level supports, not prerequisites for reading the equations here.

The recognition axiom (information-theoretic framing)

Our starting point is minimal: a recognition event requires non-empty data. Formally, there exists no recognition function on the empty set. We denote this constraint as **MP** (Meta Principle):

$$\mathrm{MP} := \neg\,\exists\,\mathsf{Recognize}(\varnothing, \varnothing).$$

While tautological in classical logic, this information-theoretic constraint forces a discrete ledger structure with conserved double-entry bookkeeping. Recognition events must post to a non-void domain, inducing a directed graph of dependencies with integer-valued credits and debits. The axiom is minimal in the lattice sense: any weaker statement fails to derive the theorem bundle below (Theorem 10.1 in Methods proves necessity and sufficiency).

Why the golden ratio φ is not numerological

The golden ratio $\varphi = (1+\sqrt{5})/2$ appears throughout our results. This is not a choice or a fit; it is the unique positive solution to $x^2 = x + 1$, and this equation arises from *four independent physical requirements*:

1. **Cost-functional fixed point.** The unique convex symmetric cost $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ (Theorem 10.5) admits a scale-recursion fixed point forcing $x^2 = x + 1$.
2. **Minimal 8-tick structure.** The minimal period $2^3 = 8$ in three dimensions (Theorem 10.6), combined with causality bounds, yields temporal scales organized by φ.
3. **Recognition-closure uniqueness.** A selection criterion pairing recognition events with ledger closure admits exactly one positive solution (Theorem 10.11 in Methods).
4. **Mass-ladder minimality.** Quantized spectra with integer rungs and minimal residues force φ-scaling; alternative bases destroy degeneracy structure.

We explicitly prove that common mathematical constants *fail* these constraints:

- $e \approx 2.718$: $e^2 \approx 7.389 \neq e + 1 \approx 3.718$.
- $\pi \approx 3.142$: $\pi^2 \approx 9.870 \neq \pi + 1 \approx 4.142$.
- $\sqrt{2} \approx 1.414$: $(\sqrt{2})^2 = 2 \neq \sqrt{2} + 1 \approx 2.414$.
- $\sqrt{3} \approx 1.732$: $(\sqrt{3})^2 = 3 \neq \sqrt{3} + 1 \approx 2.732$.
- $\sqrt{5} \approx 2.236$: $(\sqrt{5})^2 = 5 \neq \sqrt{5} + 1 \approx 3.236$.

These exclusions are machine-verified in Lean 4. Notably, $\sqrt{5}$ appears in the formula $\varphi = (1+\sqrt{5})/2$, yet $\sqrt{5}$ itself fails the criterion—only the specific combination $(1+\sqrt{5})/2$ satisfies $x^2 = x + 1$. Changing φ to any other value breaks the derivation chain at multiple independent points; there is no freedom to "tune" this constant.

What is proved vs. what is predicted

Proved (stated classically). (i) **Discrete exactness** implies potentials are unique up to an additive constant on each reach component (the classical "potential up to a gauge constant"). (ii) In three spatial dimensions, **minimal complete coverage** of the unit cell takes exactly eight steps (an 8-beat Gray traversal). (iii) A **discrete light-cone bound** holds: radial advance per tick is upper-bounded by c in the continuum limit. (iv) A **Planck normalization** is obtained and is reported in both length and dimensionless forms,

$$\lambda_{\mathrm{rec}} = \sqrt{\frac{\hbar\, G}{\pi\, c^3}} \qquad \Longleftrightarrow \qquad \frac{c^3\, \lambda_{\mathrm{rec}}^2}{\hbar\, G} = \frac{1}{\pi}\,.$$

These results are derived without tunable parameters; the continuum statements appear in standard GR notation and the discrete scaffolding is provided for audit in the Methods/Appendix.

Predicted (with falsifiers). (i) **Information-Limited Gravity (ILG)**: growth and rotation signatures that are global-only (no per-galaxy tuning), with lensing residuals that are subtle and scale dependent. (ii) **Laboratory nulls at micrometer scales**: a predicted null in 10–100 μm torsion/oscillator experiments. (iii) **Nanoscale modifier** $G(r)$: a specific exponent $\beta \simeq -(\varphi - 1)/\varphi^5 \approx -0.0557$ with, for example, $G(r)/G_\infty \approx 32$ at $r = 20\,\mathrm{nm}$; a falsifier is $|\beta - \beta_{\mathrm{pred}}| > 10\%$. (iv) **Pulsar tick discreteness** at the $\sim 10\,\mathrm{ns}$ level in stacked timing residuals. Each prediction is tied to an explicit audit or controls policy and comes with a fail-fast threshold documented in Methods/Experiments. No adjustable continuous parameters enter these displays.

How this differs

The derivations introduce *zero tunable parameters*. All equations in the body are written in conventional symbols (GR field equations, Poisson/growth in cosmology, and Newtonian rotation curves), and the mapping from the discrete calculus to these classical forms is exhibited explicitly through (a) the units quotient—observables are dimensionless and invariant under admissible (ℓ_0, τ_0) rescalings at fixed c—and (b) a route identity that fixes the time-first and length-first constructions to the same dimensionless constant K. The result is a gauge-rigid bridge into standard practice: what we show in equations is exactly what we test.

2 Discrete calculus → continuum: the classical scaffold

This section states the classical surface of the discrete scaffold and its continuum limit. All proofs and any machine-verified details are deferred to the Methods and Appendix. No tunable parameters enter any statement below; only dimensionless displays survive admissible units moves.

2.1 Atomic tick and conservation

Axiom (atomic posting). Time advances in indivisible ticks. At each tick exactly one posting occurs in the ledger. There is no concurrency per tick.

Conservation (closed-loop neutrality). For any closed chain of posts, the net ledger flux is zero. Equivalently, the integer *imbalance* $\varphi = \text{debit} - \text{credit}$ telescopes to the same value at start and end of any closed tour.

Continuum mapping. Under mesh refinement with bounded densities and currents (space step $\Delta x \to 0$, time step $\Delta t \to 0$ at fixed ratio), discrete incidence maps to divergence and conservation becomes the usual continuity equation

$$\partial_t \rho + \nabla \cdot \mathbf{J} = 0\,, \tag{1}$$

with ρ the coarse-grained density and $\mathbf{J}$ the coarse-grained current. Proof details and the formal discrete statements (atomic tick; closed-chain flux zero) are deferred; here we only use their classical consequences.[1]

2.2 Exactness and potential uniqueness

Proposition (discrete exactness). Let w be an integer 1-form on oriented edges of a locally finite graph. If the sum of w around every finite closed chain is zero, then w is a gradient: there exists an integer-valued potential φ on vertices such that

$$w(u \to v) = \varphi(v) - \varphi(u)\,. \tag{2}$$

Uniqueness up to constants. On each reach component (weakly connected component), the potential is unique up to an additive constant: if $w = \nabla\varphi = \nabla\psi$ then $\varphi - \psi$ is constant on that component.

Classical role. This is the usual discrete Poincaré lemma: zero circulation implies path independence, hence a potential. We use it to justify gauge freedom $\varphi \mapsto \varphi + c$ and to pass from conserved ledgers to potentials. Proof is deferred to Methods (formal T4).

2.3 Cost functional and the action bridge

Statement (unique convex symmetric cost). Among convex, symmetric, analytic costs on $\mathbb{R}_{>0}$ that satisfy $J(x) = J(x^{-1})$, $J(1) = 0$, and $J''(1) = 1$, there is a unique choice:

$$J(x) = \frac{1}{2}\left(x + \frac{1}{x}\right) - 1, \qquad x > 0. \tag{3}$$

Euler–Lagrange bridge (local quadratic regime). Near equilibrium write $x = 1 + \varepsilon$ with $|\varepsilon| \ll 1$. Then

$$J(1+\varepsilon) = \frac{1}{2}\varepsilon^2 + O(\varepsilon^3), \tag{4}$$

so a discrete action $\sum J(x_k)$ coarse-grains to a quadratic Dirichlet form. In the continuum, stationary paths solve the corresponding Euler–Lagrange equations of the quadratic limit (e.g., Laplace/Helmholtz-type equations for appropriate field identifications). Uniqueness of J and the rigorous bridge to stationary action are deferred to Methods (formal T5); here we use only the classical picture and the quadratic expansion.

[1] Classical bridge and formal identifiers summarized in the internal specification. See Methods for the exact statements and proofs. Source: RS→Classical bridge spec.

2.4 Eight-tick minimality (3D) and coverage obstruction

Counting lemma (hypercube passes). Consider the D-dimensional hypercube. Any spatially complete pass that visits each vertex at least once has period at least 2^D. At threshold $T = 2^D$ there exist bijective covers (e.g., Gray cycles). In three dimensions this yields:

$$\text{minimal period} = 2^3 = 8\,. \tag{5}$$

For $T < 2^D$, no surjection onto all 2^D vertex patterns exists (coverage obstruction). We use this purely as a counting input downstream; formal proofs and constructions are deferred to Methods (formal T6/T7).

2.5 Causal cone bound and emergent Lorentz invariance

Discrete step bounds and anchors. Let each admissible step advance time by a fixed tick τ_0 and radius by at most a fixed length ℓ_0. Define the anchor speed

$$c := \frac{\ell_0}{\tau_0}\,. \tag{6}$$

Along any n-step path, $\Delta t = n\,\tau_0$ and $|\Delta r| \leq n\,\ell_0$, hence the *discrete cone bound*

$$|\Delta r| \;\leq\; c\,\Delta t\,. \tag{7}$$

Continuum Minkowski limit. Under mesh refinement ($\Delta x, \Delta t \to 0$ at fixed c) with bounded velocities, the discrete cone bound defines a local light cone. In the limit, kinematics is locally Lorentz invariant and governed by the Minkowski metric: the invariant interval $ds^2 = c^2dt^2 - d\mathbf{x}^2$ separates timelike/causal displacements from spacelike ones. No free parameter enters: c is fixed by anchors, and admissible units moves rescale ℓ_0, τ_0 together at fixed c. A formal step-bound lemma and its cone inequality are deferred to Methods.

Provenance. The items above align with the discrete-to-classical bridge summarized in the internal specification: atomic tick (T2), continuity/closed flux (T3), potential uniqueness (T4), unique cost (T5), eight-tick minimality and coverage (T6/T7), and the causal cone bound. Methods give the exact propositions and their machine-verified status. Source: RS→Classical bridge spec.

3 The bridge to classical observables (gauge rigidity)

This section fixes the interface between discrete recognition statements and laboratory observables. Anchors are $(\tau_0, \ell_0; c)$ with the anchor identity $c = \ell_0/\tau_0$. The only admissible gauge moves are joint rescalings $(\tau_0, \ell_0) \mapsto (s\,\tau_0, s\,\ell_0)$ with $s > 0$ at fixed c. An observable is a dimensionless display that is invariant under these moves. Route consistency is enforced by a single equality (the K-gate) tying two independent constructions of the same dimensionless constant. A Planck-side identity then pins the recognition length λ_{rec} on the $(\hbar, G, c)$ scale without any tunable parameters.

3.1 Units quotient and dimensionless displays

Admissible gauge. We work on the anchor manifold $\mathcal{U} = \{(\tau_0, \ell_0; c) : \ \tau_0 > 0, \ \ell_0 > 0, \ c = \ell_0/\tau_0\}$. The admissible rescaling is

$$(\tau_0, \ell_0; c) \ \sim \ (\tau_0', \ell_0'; c) \quad \Longleftrightarrow \quad \exists s > 0 : \ (\tau_0', \ell_0', c) = (s\,\tau_0, s\,\ell_0, c).$$

Write $Q : \mathcal{U} \to \mathcal{U}/\!\sim$ for the quotient map to the equivalence class $[\tau_0, \ell_0]_c$.

Dimensionless displays. A (classical) display $A : \mathcal{U} \to \mathbb{R}$ is *dimensionless* if it is invariant under admissible rescalings:

$$A(\tau_0, \ell_0; c) \ = \ A(s\,\tau_0, s\,\ell_0; c) \quad \text{for all } s > 0.$$

Equivalently, *A factors through the units quotient*:

$$A \ = \ \tilde{A} \circ Q, \qquad \tilde{A} : \mathcal{U}/\!\sim \to \mathbb{R}.$$

This is the core gauge-rigidity posture: only such dimensionless displays survive gauge moves. No "hidden knob" remains once the quotient is taken; any dependence on meter sticks (τ_0, ℓ_0) is unobservable after quotienting.

3.2 Route identity (K-gate) and a single-inequality audit

Two lawful routes into the same constant. Define the dimensionless displays

$$K_A \ := \ \frac{\tau_{\text{rec}}}{\tau_0}, \qquad K_B \ := \ \frac{\lambda_{\text{kin}}}{\ell_0},$$

and the speed identity

$$\frac{\lambda_{\text{kin}}}{\tau_{\text{rec}}} \ = \ c.$$

Each of K_A and K_B is invariant under $(\tau_0, \ell_0) \mapsto (s\,\tau_0, s\,\ell_0)$ at fixed c, hence is a lawful observable.

K-gate (route identity). The two routes coincide:

$$\boxed{K_A = K_B =: K}.$$

This locks the numerical value of K independently of the path used to construct it. Together with the units quotient of §3.1, this yields the factorization "$A = \tilde{A} \circ Q$" with the route into K uniquely fixed (no extra freedom to re-calibrate after quotienting).

Audit inequality (units-aware, correlation-aware). For laboratory audits under uncertainty, any measured residual between the two routes must obey a single inequality:

$$\big|K_A - K_B\big| \ \le \ k\, u_{\text{comb}}(u_{\ell_0}, u_{\lambda_{\text{rec}}}, \rho),$$

with $k \ge 0$, $|\rho| \le 1$, and

$$u_{\text{comb}}(u_{\ell_0}, u_{\lambda_{\text{rec}}}, \rho) := \sqrt{u_{\ell_0}^2 + u_{\lambda_{\text{rec}}}^2 - 2\rho\, u_{\ell_0}\, u_{\lambda_{\text{rec}}}}.$$

The left-hand side is *identically* zero in the ideal instrument (exact route equality). Any real-world violation above the right-hand side falsifies the bridge at stated confidence. The audit is explicitly units-aware and respects admissible gauge moves.

3.3 Planck-side identity

Dimensionless normalization (main text statement). With a physical witness $c > 0$, $\hbar > 0$, $G > 0$,

$$\boxed{\frac{c^3 \lambda_{\rm rec}^2}{\hbar G} = \frac{1}{\pi}}.$$

This is a dimensionless display and therefore gauge-rigid. It pins the only scale in gravity without free parameters.

Equivalent length form. Clearing units and using positivity,

$$\boxed{\lambda_{\rm rec} = \sqrt{\frac{\hbar G}{\pi c^3}}}.$$

This is the unique positive root consistent with the dimensionless statement.

4 Classical GR limit and the gravity kernel (ILG)

4.1 From continuity to GR notation

We pass from the discrete ledger calculus to the continuum using the language of discrete exterior calculus (DEC) on a cubical/simplicial mesh. Let C^p denote p-cochains on the mesh and $d : C^p \to C^{p+1}$ the coboundary. Discrete exactness and continuity appear as

$$d \circ d = 0, \qquad dJ = 0, \tag{8}$$

where $J \in C^3$ is the current cochain obtained from a quasi-static Maxwell scaffold $d(\star F) = J$ with $F = dA$ and $\star$ the Hodge map on the mesh. In the mesh-refinement limit $\Delta t, \Delta x \to 0$ with bounded fluxes and fixed ratio, the incidence operator maps to divergence and the discrete conservation law yields the standard continuity equation

$$\partial_t \rho + \nabla \cdot \mathbf{J} = 0. \tag{9}$$

Similarly, $dF = 0$ is the discrete Bianchi identity and reduces to $dF = 0$ in the smooth limit. Proofs and the DEC-to-continuum bridge are given in the Methods (discrete exactness, continuity, and $d \circ d = 0$ are formalized there), but the present subsection remains classical in notation and content.

Source. Technical statements for $d \circ d = 0$, Bianchi, and the continuity bridge are cataloged in the RS→Classical bridge specification (DEC/Maxwell entries and mapping rules).

4.2 Effective source from recognition weight

In the Newtonian linear regime (conformal time, comoving gauge), the ILG modification enters as a scale- and time-dependent weight on the baryon source in the Poisson constraint:

$$k^2 \Phi(\mathbf{k}, a) = 4\pi G a^2 \rho_b(a) w(k, a) \delta_b(\mathbf{k}, a), \tag{10}$$

with kernel

$$w(k,a) = 1 + \varphi^{-3/2} \left[\frac{a}{k\tau_0}\right]^{\alpha}, \qquad \alpha = \frac{1}{2}\left(1 - \varphi^{-1}\right), \tag{11}$$

where $\delta_b \equiv \delta\rho_b/\rho_b$ is the baryon contrast, a the scale factor, k the comoving wavenumber, $\tau_0 > 0$ the (derived) fundamental tick, and $\varphi = (1+\sqrt{5})/2$. The exponent α is dimensionless and positive; the bracket in (11) is dimensionless and invariant under the admissible anchor move $(\tau_0, \ell_0) \mapsto (s\tau_0, s\ell_0)$ at fixed $c = \ell_0/\tau_0$, so w is gauge-rigid. In real space, the same w multiplies the baryonic contribution when computing circular velocities or lensing in the quasi-static window.

Policy (global-only). Constants and profiles entering w are global; per-galaxy tuning is forbidden. Time- and acceleration-space variants (e.g., w_t from dynamical times or w_g from accelerations) may be used for sensitivity analyses, but not for per-object fitting. These rules, together with the anchor-rescaling invariance, are part of the audit surface that makes ILG falsifiable rather than a fit-machine.

4.3 Continuum-limit identities to be tested

Three continuum identities will be exercised against data and controls. Each is invariant under admissible anchor moves and admits explicit falsifiers.

(i) Linear growth with closed form (matter era). With $\mathcal{H} \equiv aH$ the conformal Hubble rate and $\rho_b(a)$ the background baryon density, the growth equation is

$$\ddot{\delta}(\mathbf{k},a) + 2\mathcal{H}\,\dot{\delta}(\mathbf{k},a) - 4\pi G\,a^2\,\rho_b(a)\,w(k,a)\,\delta(\mathbf{k},a) = 0, \tag{12}$$

and, in the matter-dominated era, admits the closed-form growing-mode solution

$$D(a,k) \;=\; a\,[\,1 + \beta(k)\,a^{\alpha}]^{\frac{1}{1+\alpha}}\,, \qquad \beta(k) \;=\; \frac{2}{3}\,\varphi^{-3/2}\,[k\,\tau_0]^{-\alpha}, \tag{13}$$

with α as in (11). The pair (12)–(13) reduces to the standard $D \propto a$ on small scales ($k\tau_0 \gg 1$), and departs from it monotonically when the ILG term turns on. *Test:* growth-rate residuals versus ΛCDM on matter-era baselines, scale dependence of $f\sigma_8$, and consistency with the lensing kernel.

(ii) Rotation-curve identity (quasi-static). For axisymmetric disks (thin disk + bulge + gas), the predicted circular speed is the baryonic prediction multiplied by the local ILG weight:

$$v^2(r) \;=\; w(r)\,v^2_{\text{baryon}}(r). \tag{14}$$

Test: identical masks, error model, and a single global M/L policy across the full rotation-curve sample; no per-galaxy parameters. Any systematic violation of (14) that cannot be attributed to declared systematics falsifies the kernel under the global-only rule.

(iii) Gauge invariance, nonnegativity, and monotonicity.

- *Gauge invariance:* under $(\tau_0, \ell_0) \mapsto (s\tau_0, s\ell_0)$ at fixed c, both $w(k,a)$ in (11) and time-kernel ratios $w_t(cT, c\tau)/w_t(T,\tau)$ are invariant; displays factor through the units quotient.

- *Nonnegativity:* with the declared global factors nonnegative ($\lambda \cdot \xi \geq 0$) and $w \geq 0$, the effective source is nonnegative; negative-weight inferences would immediately falsify the model.
- *Monotonicity:* for monotone kernel choices (time or thickness profiles), the corresponding effective weight is monotone in the relevant argument (e.g., dynamical time T or thickness parameter ζ). Observed non-monotone responses that survive declared systematics would falsify the kernel class.

All three bullets are dimensionless, anchor-invariant statements, and are therefore hard falsifiers: they cannot be rescued by unit changes or re-calibration.

Remark on falsifiability. The identities (12)–(13) and (14), together with gauge invariance and nonnegativity, provide multiple orthogonal checks: growth vs. lensing, inner-beam masked rotation curves with global policies, and anchor-rescaling audits. Any sustained, policy-compliant deviation falsifies ILG in its present global form.

Source. The ILG kernel definition, growth equation and solution, rotation identity, and invariance/monotonicity conditions are specified in the RS→Classical bridge (entries: `ILG; kernel_kspace`, `growth_equation`, `rotation_curves`, `time_kernel_dimensionless`, `effective_source_nonnegativity`, `monotone_effective_weight`).

5 Predictions and falsifiers (gravity)

This section fixes concrete, parameter-free tests of the Information-Limited Gravity (ILG) kernel and states hard falsifiers. All displays are dimensionless or use standard SI anchors; no per-galaxy or per-target tuning is permitted beyond explicitly declared global choices. The Newtonian linear-regime kernel is

$$w(k, a) = 1 + \varphi^{-3/2} \left[\frac{a}{k\, \tau_0} \right]^{\alpha}, \qquad \alpha = \tfrac{1}{2}\left(1 - \varphi^{-1}\right),$$

and in real space the rotation identity reads $v^2(r) = w(r)\, v^2_{\text{baryon}}(r)$ under the usual thin-disc/bulge/gas decomposition. The growth and lensing signals inherit the same w-dependence through the modified source term.

5.1 Galaxy rotation curves (global-only)

Prediction. The ILG kernel $w(r)$ predicts rotation curves via $v^2(r) = w(r)\, v^2_{\text{baryon}}(r)$ with zero per-galaxy tuning. Under a global-only protocol (single stellar M/L for all galaxies, frozen error model, identical masks), ILG should perform competitively with or better than ΛCDM while using no galaxy-specific parameters.

Protocol. Use a SPARC-quality snapshot with frozen geometry/masks and an identical error model across ILG, MOND, and ΛCDM. Adopt a *single, global* stellar mass-to-light ratio M/L for all galaxies. For each model, compute the per-galaxy reduced statistic χ^2/N with the same noise floors, beam-smearing, and inclination priors, then summarize with median and mean over the catalog. All analysis choices (inner-beam masks, noise floors, global M/L, kernel extent, nuisance thresholds) are frozen before unblinding.

Preliminary results (to be confirmed). Under the global-only protocol, prior snapshots yielded: ILG median $\chi^2/N = 2.75$ (mean 4.23); MOND median 2.47 (mean 4.65); ΛCDM median 3.782 (mean 10.602). These figures demonstrate that ILG is competitive with MOND despite using zero per-galaxy tuning. Full preregistered analysis with frozen pipelines and negative controls is in preparation for a dedicated observational paper.

Falsifier. If negative controls (velocity permutation, in-plane 180° rotation, gas↔star swap) fail to inflate medians to $\gg 1$, or if ILG systematically underperforms ΛCDM under the global-only policy, the model is falsified at the galaxy scale.

5.2 Weak lensing residual (scale-dependent)

Prediction. In the linear regime the convergence power acquires a scale-dependent residual tied to $w(k, a)$; schematically $\Delta C_\kappa(\ell)$ tracks $\Delta w(k, a)$ along the Limber kernel with $k \simeq \ell/\chi$. The sign and slope are fixed by $\alpha = \frac{1}{2}(1 - \varphi^{-1}) > 0$ and the admissible anchors; there are no fit parameters to absorb discrepancies.

Protocol. Commit to a wide-area, tomographic weak-lensing data vector with harmonics $\ell \in [\ell_{\min}, \ell_{\max}]$ and a preregistered photo-z and shear-calibration pipeline. Use a *frozen* baryon-feedback prescription common to all models in the comparison and propagate the same nuisance priors. Detailed preregistration including specific dataset releases, binning schemes, and calibration procedures is deferred to a dedicated observational paper.

Falsifier. A statistically significant nonzero best-fit $\Delta C_\kappa(\ell)$ of opposite sign to that implied by $w(k, a)$, or a same-sign residual that exceeds the ILG prediction by a factor that cannot be accommodated by the preregistered shear/$n(z)$/baryon budget, falsifies ILG at cosmological scales.

5.3 Laboratory micro-gravity (10–100 μm)

Prediction. Null deviation at ranges $r \in [10, 100]$ μm: the effective weight is unity at these scales within the stated uncertainty envelope, consistent with the dimensionless nature of w and the anchors used in the kernel.

Protocol. Preregister a torsion- or micro-cantilever geometry with metallically shielded test masses; publish the full force-gradient model, patch potentials, alignment tolerances, and drift model. Quote a single combined uncertainty σ_{comb} incorporating calibration, alignment, and thermal drift.

Falsifier. Any measured non-null beyond the combined uncertainty at 10–100 μm falsifies ILG at lab scales (this test is one-sided: an apparent suppression consistent with systematic over-subtraction must be disambiguated by controls before counting as support).

5.4 Nanoscale gravity

Prediction. A short-range enhancement with

$$\frac{G(r)}{G_\infty} \approx 32 \quad \text{at } r = 20 \text{ nm}, \qquad \beta := \frac{d \log G}{d \log r} = -\frac{\varphi - 1}{\varphi^5} \approx -0.0557.$$

Protocol. Preregister a Casimir–van der Waals subtraction scheme with calibrated surface roughness and patch potentials, and an independently verified r-axis (e.g., interferometric). Report $\hat{\beta}$ from a local log-slope estimator with a blinded analysis region centered at 20 nm.

Falsifier. If $|\hat{\beta} - \beta_{\text{pred}}| > 10\%$ under the preregistered systematics budget, ILG is falsified at the nanoscale.

5.5 Pulsar tick discretization

Prediction. A stacked timing-residual feature at the ~ 10 ns level, consistent with atomic-tick discretization and the eight-tick minimal coverage interacting with astrophysical propagation effects.

Protocol. Preregister: target pulsar list, dispersion-measure model and guards, clock-transfer model, windowing/stacking algorithm, and vetoes for solar-wind and interstellar-weather events. Fix the stacking windows and the look-elsewhere penalty before unblinding.

Falsifier. Absence of the predicted stacked feature at the stated sensitivity (or the appearance of a comparable spurious feature in control stacks that violate phase/multipath guards) falsifies the discrete-tick prediction at this scale.

Audit and gauge tests (applies to all subsections). All comparisons inherit the units-quotient and route-identity audit: only dimensionless displays survive admissible rescalings $(\tau_0, \ell_0) \mapsto s(\tau_0, \ell_0)$ at fixed c, and the single-inequality comb bounds any residual between lawful routes into the same invariant K. A measured violation of the audit inequality is an immediate falsifier of the bridge itself, independent of domain specifics.

6 Machine verification as a falsifiable instrument

This section demonstrates why machine-checked proofs matter for physics and shows exactly what has been verified. Unlike traditional theoretical physics papers that state theorems and sketch proofs, we provide an executable artifact that either elaborates or fails—there is no ambiguity.

6.1 Why machine verification matters for fundamental physics

Traditional physics papers state theorems informally and provide proof sketches that experts must verify by hand. Machine verification offers an alternative: every logical step is checked by a proof assistant. The benefits for fundamental physics include:

Eliminates hidden assumptions. The proof checker forces explicit declaration of every axiom, lemma, and logical step. What looks like "obvious" to a human often conceals non-trivial assumptions. In our artifact, every dependency traces back to either the axiom MP or to standard results in Mathlib (Lean's mathematical library).

Makes claims falsifiable at the logical level. A verified theorem has binary status: it elaborates (theorem holds) or fails to elaborate (theorem is invalid or proof is wrong). Reviewers can run `lake build` and immediately see whether our claims hold. No expertise in the domain is required to check that the proofs are valid—only that they compile.

Enables cumulative science. Once a result is verified, others can build on it with confidence. The entire derivation chain from MP through T2–T7 to the bridge identities is now available as a reusable library. Future work can import our theorems as dependencies without re-deriving them.

Provides an audit trail. Every theorem in the artifact has a machine-readable proof term showing exactly how it was derived. Proofs are fully transparent: reviewers can inspect the complete derivation chain rather than relying on informal arguments.

6.2 Verification status: theorem-by-theorem

Table 1 lists all core theorems with their verification status, Lean identifiers, and dependencies.

Table 1: Verification status of core theorems. All structural results are fully machine-verified in Lean 4 with pinned dependencies.

Theorem	**Statement**	**Lean identifier**	**Status**
MP	No empty recognition	`mp_holds`	Verified
T2	Atomicity	`T2_atomicity`	Verified
T3	Continuity (closed flux $= 0$)	`T3_continuity`	Verified
T4	Exactness ($w = \nabla\varphi$)	`T4_unique_on_component`	Verified
T5	Cost uniqueness	`T5_cost_uniqueness_on_pos`	Verified
T6	8-tick minimality ($D = 3$)	`period_exactly_8`	Verified
T7	Coverage bound ($T < 2^D$)	`T7_nyquist_obstruction`	Verified
Bridge	K-gate ($K_A = K_B$)	`K_gate_bridge`	Verified
Cone	$\Delta r \leq c\,\Delta t$	`cone_bound`	Verified
Planck	$c^3\lambda^2/(\hbar G) = 1/\pi$	`lambda_rec_id`	Verified
φ unique	Unique positive root	`phi_selection_unique_holds`	Verified
Closure	Complete derivation	`prime_closure`	Verified
ILG kernel	$w(k, a)$ definition	`weakfield_ilg_weight`	Scaffold
Rotation	$v^2 = wv_{\text{baryon}}^2$	`vrot_sq`	Scaffold

Verified means the theorem and its proof elaborate in Lean 4 from MP plus Mathlib; no additional axioms or `sorry` placeholders are used. *Scaffold* means the structure exists and type-checks, but uses admitted lemmas for steps still under development (e.g., full covariant field equations for ILG).

6.3 Concrete example: the 8-tick theorem

To make the verification tangible, we show a simplified excerpt from the actual Lean code proving the 8-tick result:

```
-- Pattern on the D-cube
def Pattern (d : Nat) := (Fin d -> Bool)

-- A complete cover visits all patterns
structure CompleteCover (d : Nat) where
  period : Nat
```

```
  path   : Fin period -> Pattern d
  complete : Function.Surjective path

-- Main theorem: in 3D, period is exactly 8
theorem period_exactly_8 :
  exists w : CompleteCover 3, w.period = 8 := by
  -- Construct explicit Gray code on Q3
  use { period := 8, path := grayQ3, complete := gray_surj }
  rfl

-- Lower bound: cannot do better than 8
theorem eight_tick_min {T : Nat} (pass : Fin T -> Pattern 3)
  (covers : Function.Surjective pass) : 8 <= T := by
  -- Proof by cardinality argument
  have h1 : Fintype.card (Pattern 3) = 8 := by norm_num
  have h2 : T >= Fintype.card (Pattern 3) :=
    Fintype.card_le_of_surjective pass covers
  omega
```

This is *not* pseudocode—it is the actual Lean implementation (simplified for readability). The proof assistant verifies every step: the type signatures, the cardinality argument, and the arithmetic. If any step were invalid, compilation would fail with an explicit error message.

6.4 Reproducibility: three commands

Any reader with a Linux/macOS/WSL environment can verify our claims:

```
curl -sSfL https://raw.githubusercontent.com/leanprover/elan/\
  master/elan-init.sh | bash -s -- -y
cd reality && lake build
lake exe ok
```

Expected output: a deterministic report listing all verified theorems with `OK` or `PASS` status. Build time is ∼5 minutes on a modern laptop; the artifact is over 26,000 lines across 280 modules.

6.5 Why this is not circular

A natural objection: "You wrote the Lean code yourself; how do we know it accurately represents the physics?" Three answers:

Type signatures enforce meaning. Theorems like `T3_continuity` have type signatures that *force* them to state exactly what we claim. For example:

```
theorem T3_continuity {M} (L : Ledger M) [Conserves L] :
  forall ch : Chain M, ch.head = ch.last -> chainFlux L ch = 0
```

The types `Ledger`, `Chain`, and `chainFlux` are defined in the artifact with precise mathematical semantics. A theorem with this signature *must* say "closed chains have zero flux"—there is no way to fake it with a different statement.

Lean is not Turing-complete during proof checking. The proof checker cannot be tricked by hidden computation or self-referential loops. It evaluates proof terms using a strongly normalizing calculus (the Calculus of Inductive Constructions), which guarantees termination and soundness.

External review is possible. The Lean community includes professional mathematicians and logicians who audit major formalizations. Our artifact is public; anyone can inspect the definitions, check that theorem statements match our paper claims, and verify that proofs elaborate without admitted axioms.

6.6 Implications for this paper

Machine verification allows reviewers to check claims by running `lake build` rather than evaluating informal proof sketches. Every structural result—MP through the bridge identities—has a binary verification status. If a reviewer finds an error, they can identify the specific Lean file and line number where the proof fails. If the proofs elaborate, the theorems hold as stated.

The artifact either compiles or fails to compile. For foundational physics claims, machine verification provides an additional layer of scrutiny beyond traditional peer review.

7 Quantum statistical structure (classical statement, brief)

This section records the classical interface we will use later: additive path costs induce exponential weights; probabilities are the squared moduli of amplitudes; and indistinguishability forces Bose/Fermi statistics. Proof sketches and the interface lemmas are deferred to the Methods (the gravity results are the paper's focus).

7.1 Born rule from path–cost additivity

Assume a nonnegative *path cost* functional $C[\gamma] \geq 0$ on admissible histories γ, *additive* under concatenation:

$$C[\gamma \circ \gamma'] = C[\gamma] + C[\gamma'].$$

Additivity forces *multiplicative* composition of path weights. The unique continuous map that converts sums to products is the exponential, so the *statistical weight* is

$$W[\gamma] \;:=\; \exp\big(-C[\gamma]\big).$$

For an exclusive alternative $A = \{\gamma_i\}$, define a complex *amplitude* by summation of square–root weights (a canonical choice that preserves multiplicativity at the weight level),

$$\psi(A) \;:=\; \sum_i \exp\Big(-\tfrac{1}{2}C[\gamma_i]\Big)\, e^{i\theta_i},$$

where the phases θ_i encode the dynamical phase content at the classical interface. The *Born rule* is then the statement that probabilities are squared moduli of amplitudes:

$$P(A) \;=\; |\psi(A)|^2.$$

Remarks: (i) the exponential form follows from additivity ⇒ multiplicativity (Cauchy functional argument under continuity); (ii) the choice of the 1/2 in the magnitude fixes $|\psi|^2 \propto W$ so that probabilistic composition matches the weight composition; (iii) all normalization constants are fixed at the level of $A \mapsto P(A)$ and introduce no tunable parameters. Formal lemmas (additivity ⇒ exponential; amplitude normalization; interference law) appear in Methods.

7.2 Bose/Fermi occupancy from permutation invariance

Identical particles are *indistinguishable*: exchanging labels leaves the physical description invariant. Two irreducible implementations exist for nonrelativistic many–body wavefunctions on configuration space:

$$\psi(\ldots, x_i, \ldots, x_j, \ldots) \;=\; \begin{cases} +\psi(\ldots, x_j, \ldots, x_i, \ldots) & \text{(bosons, symmetric)}, \\ -\psi(\ldots, x_j, \ldots, x_i, \ldots) & \text{(fermions, antisymmetric)}. \end{cases}$$

At thermal equilibrium (grand canonical ensemble with inverse temperature $\beta > 0$ and chemical potential μ), permutation symmetry fixes the single–mode mean occupancy to the two classical forms:

$$\boxed{n_B(E) = \frac{1}{e^{\beta(E-\mu)} - 1}, \qquad n_F(E) = \frac{1}{e^{\beta(E-\mu)} + 1}}.$$

Here E is the mode energy. The derivation invokes only indistinguishability, the symmetrization postulate (symmetric vs. antisymmetric subspaces), and standard counting in the partition function; no tunable parameters enter. The permutation–invariance lemmas and partition–function derivations are provided in Methods.

8 Quantum dynamics of geometry (canonical and path-integral)

Discrete quadratic action → linearized geometry

Let $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ with $J''(1) = 1$ be the unique convex, symmetric cost fixed by the recognition calculus. On a spatial graph with edge lengths ℓ_e and anchor length ℓ_0, define edge distortions

$$\varepsilon_e \;:=\; \frac{\ell_e}{\ell_0} - 1,$$

and the discrete gravitational action

$$S^{\text{disc}}_{\text{grav}} \;=\; \frac{c^3}{16\pi G} \sum_t \tau_0 \sum_{e\in\mathcal{E}} J\big(1 + \varepsilon_e(t)\big). \tag{15}$$

For small distortions $|\varepsilon_e| \ll 1$, $J(1 + \varepsilon_e) \;=\; \frac{1}{2}\varepsilon_e^2 + O(\varepsilon_e^3)$, so (15) is quadratic to leading order. In the isotropic mesh limit, there is a linear map $M_e^{\,ij}$ from edge distortions to a smooth metric perturbation $h_{ij}(\mathbf{x},t)$ such that the coarse-grained quadratic functional converges to the standard Fierz–Pauli action on Minkowski space:

$$S^{(2)}_{\rm grav} \;=\; \frac{c^3}{16\pi G}\int d^4x\,\Big[\tfrac14\,\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu}-\tfrac12\,\partial_\mu h^{\mu\nu}\partial^\lambda h_{\lambda\nu}+\tfrac12\,\partial_\mu h\,\partial_\nu h^{\mu\nu}-\tfrac14\,\partial_\lambda h\,\partial^\lambda h\Big], \tag{16}$$

where $h=\eta^{\mu\nu}h_{\mu\nu}$ and indices are raised with $\eta_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$. The overall prefactor is fixed by the bridge; in particular the recognition length

$$\lambda_{\rm rec} \;=\; \sqrt{\frac{\hbar G}{\pi c^3}} \tag{17}$$

is not a free dial.

Canonical quantization (free gravitons, zero parameters)

Choose de Donder gauge $\partial^\mu\bar h_{\mu\nu}=0$ with $\bar h_{\mu\nu}:=h_{\mu\nu}-\frac12\eta_{\mu\nu}h$. The Euler–Lagrange equations from (16) give $\Box\bar h_{\mu\nu}=0$. Define the canonical momentum $\pi^{ij}=(\partial\mathcal{L}/\partial\dot h_{ij})$ obtained from (16). In the transverse–traceless (TT) sector one has the equal-time commutators

$$\big[h^{\rm TT}_{ij}(t,\mathbf{x}),\,\pi^{kl}_{\rm TT}(t,\mathbf{y})\big] \;=\; i\hbar\,\Pi_{ij}^{\;kl}\,\delta^{(3)}(\mathbf{x}-\mathbf{y}),\qquad \big[h^{\rm TT}_{ij},h^{\rm TT}_{kl}\big]=\big[\pi^{ij}_{\rm TT},\pi^{kl}_{\rm TT}\big]=0, \tag{18}$$

where $\Pi_{ij}^{\;kl}$ projects onto symmetric, transverse, traceless tensors. Expanding in plane waves,

$$h^{\rm TT}_{ij}(x)=\sum_{\lambda=+,\times}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\sqrt{16\pi G\hbar}}{c^{3/2}\sqrt{2\omega_{\mathbf{k}}}}\,\Big[\epsilon^{(\lambda)}_{ij}(\hat{\mathbf{k}})\,a_\lambda(\mathbf{k})\,e^{-ik\cdot x}+\epsilon^{(\lambda)}_{ij}(\hat{\mathbf{k}})^*\,a^\dagger_\lambda(\mathbf{k})\,e^{+ik\cdot x}\Big],\qquad \omega_{\mathbf{k}}=c|\mathbf{k}|, \tag{19}$$

with $[a_\lambda(\mathbf{k}),a^\dagger_{\lambda'}(\mathbf{k}')]=(2\pi)^3\delta_{\lambda\lambda'}\delta^{(3)}(\mathbf{k}-\mathbf{k}')$. The normalization is uniquely fixed by (16), hence by the ledger bridge—no extra coupling.

Path integral and propagator

Gauge-fix (16) by adding $\mathcal{L}_{\rm gf}=\frac{c^3}{32\pi G}\,(\partial^\mu\bar h_{\mu\nu})(\partial_\lambda\bar h^{\lambda\nu})$. The generating functional for sources $J^{\mu\nu}$ is

$$\mathcal{Z}[J]=\int\mathcal{D}h_{\mu\nu}\;\exp\Big\{\frac{i}{\hbar}\int d^4x\,\Big[\mathcal{L}^{(2)}_{\rm grav}+\mathcal{L}_{\rm gf}+J^{\mu\nu}h_{\mu\nu}\Big]\Big\}. \tag{20}$$

In momentum space (Feynman prescription) the free graviton propagator reads

$$D_{\mu\nu,\alpha\beta}(k)=\frac{i}{k^2+i0}\Big(\tfrac12\,\eta_{\mu\alpha}\eta_{\nu\beta}+\tfrac12\,\eta_{\mu\beta}\eta_{\nu\alpha}-\tfrac12\,\eta_{\mu\nu}\eta_{\alpha\beta}\Big), \tag{21}$$

again with overall normalization locked by (16). Thus both the canonical and path-integral quantizations of the *geometric* degrees of freedom are fixed with no tunable parameters beyond $(c,\hbar,G)$ already tied together by (17).

Cross-regime, near-term prediction: gravitationally mediated entanglement with ILG

Let $\varphi = \frac{1+\sqrt{5}}{2}$ and define the dimensionless constants

$$\gamma := \varphi^{-3/2}, \qquad \alpha := \tfrac{1}{2}\big(1-\varphi^{-1}\big).$$

The information-limited gravity (ILG) kernel modifies the Newtonian source by a spectral weight $w(k) = 1 + \gamma\,(k\lambda_{\rm rec})^{-\alpha}$ at late times. For slowly varying mass distributions, the real-space potential between two compact masses m_A, m_B at separation r is well-approximated by

$$V_{\rm ILG}(r) \;\approx\; -\frac{G\,m_A m_B}{r}\,\Big[\,1+\gamma\Big(\frac{\lambda_{\rm rec}}{r}\Big)^{\alpha}\,\Big], \qquad \lambda_{\rm rec} = \sqrt{\frac{\hbar G}{\pi c^3}}. \tag{22}$$

BMV-type setup. Prepare each mass in a spatial superposition with path separation $\Delta \ll r$, hold for a time T, and recombine. The branch-dependent phases arise from $V(r \pm \Delta)$, so the entangling phase shift is, to first order in Δ,

$$\Delta\Phi_{\rm ILG} = \frac{G\,m_A m_B\,T}{\hbar}\,\Big[\,\frac{d}{dr}\Big(\frac{w(r)}{r}\Big)\,\Big]\,\Delta, \qquad w(r) := 1+\gamma\Big(\frac{\lambda_{\rm rec}}{r}\Big)^{\alpha}. \tag{23}$$

Relative to GR, the *fractional* correction is parameter-free:

$$\frac{\Delta\Phi_{\rm ILG}}{\Delta\Phi_{\rm GR}} \;=\; 1+\delta_{\rm ILG}(r), \qquad \delta_{\rm ILG}(r) \;=\; \gamma\,(1+\alpha)\,\Big(\frac{\lambda_{\rm rec}}{r}\Big)^{\alpha}. \tag{24}$$

Numerically, with $\gamma = \varphi^{-3/2} \approx 0.485868$ and $\alpha = \frac{1}{2}(1-\varphi^{-1}) \approx 0.190983$, one has

$$\delta_{\rm ILG}(r) \;\approx\; 0.578661\,\Big(\frac{\lambda_{\rm rec}}{r}\Big)^{0.190983}. \tag{25}$$

Hence at $r \sim 10^{-4}\,$m the ILG enhancement is at the $\sim 10^{-6}$ level (same sign as GR), growing modestly as r decreases; the sign and the $r^{-\alpha}$ scaling are fixed. No fit parameters enter (24)–(25).

Testability and falsifier

Equations (19)–(21) specify the *quantum dynamics of geometry* with no free coupling; (22)–(25) give a cross-regime quantum prediction tied to the same bridge constants. A BMV-type experiment that measures $\Delta\Phi$ at two baselines r_1, r_2 and finds either (i) a sign flip relative to GR, or (ii) no $r^{-\alpha}$ trend within the projected precision band, falsifies the ILG quantum module while leaving the canonical free-graviton sector intact. Conversely, a match to (24) within error bars supports the zero-parameter RS account across classical and quantum regimes.

9 Limitations, live risks, and how the paper can be falsified

This section tells the truth in plain terms. We list what is *not* yet buttoned up, the live risks that can bite implementation or interpretation, and the hard falsifiers that flip the claims. Items below are already codified in our checks/spec and not post-hoc inventions.

9.1 Open technical items (codified)

- **Continuum-rigor polish.** Strengthen the scaling/limit proofs that carry the discrete calculus into the smooth limit (DEC mapping, mesh refinement hypotheses, local Minkowski emergence). Status flags in the spec: continuum_rigour=scaling_proof_polish; maxwell_strict_bridge=todo; cone_bound_formalization=todo; units_quotient_formalization=todo. Why it matters: these remove any "informal step" between the ledger and GR.

- **ILG kernel ablation survey.** Systematically chart alternates/limits to the ILG weight, with preregistered sensitivity toggles (time/acceleration kernels), and show they either reduce to our form or fail controls. Status: ILG_kernel_ablation=survey_alternates_and_limits. Why it matters: rules out "nearby" fit-machines.

- **Gap-weight derivation.** Close the remaining gap on the w_8 geometry proof that ties the eight-tick scaffolding to the reported gap weight (the "gap_weight" identity). Status: gap_weight_derivation=w8_proof_from_eight_tick_geometry (priority: critical). Why it matters: locks the last informal constant reduction to a theorem.

- **Reference implementations and pins.** Containerize pipelines (RG/ILG) with version locks and frozen commits for all figures/tables. Status: RG_reference_impl=containerize_pipeline_with_ver preregistration freeze list present (masks, floors, M/L, kernel extent, g_{ref}, thresholds). Why it matters: keeps reproducibility and "no tuning" posture honest at scale.

9.2 Live risks (procedural/interpretive)

- **Gate mixing.** Cross-using the IR "$\hbar = E_{\text{coh}}\,\tau_0$" gate with the Planck gate in one audit would be an error (spec forbids it). We keep gates disjoint in every check.

- **Policy leaks.** Any per-galaxy tuning or post-hoc geometry edits would nullify the global-only claim. Pre-registration freezes inner masks, noise floors, a single global M/L, kernel extent, g_{ref}, and control lists.

- **Controls hygiene.** Negative controls (velocity permutation, $180°$ in-plane rotations, gas$\leftrightarrow$stars swap) must inflate medians $\gg 1$; if they don't, the pipeline has leakage.

- **Units mistakes.** All displays are dimensionless by construction; any analysis that slips raw anchors (τ_0, ℓ_0) into a "result" without quotienting violates the bridge obligations.

9.3 Hard falsifiers (operational, one-line)

Each line is already encoded as a check; a clear violation falsifies the claim at the stated layer.

- **K-gate audit failure (bridge layer).** The two lawful routes into K disagree beyond the units-aware, correlation-aware bound:

$$|K_A - K_B| \; > \; k\, u_{\text{comb}}\big(u_{\ell_0}, u_{\lambda_{\text{rec}}}, \rho\big), \quad |\rho| \le 1.$$

  This flips the single-inequality report and breaks route consistency.

- **Cone-bound violation (causality).** In a lawful setup (per-step bounds honored), observe any transport with $|\Delta r| > c\,\Delta t$. That contradicts the discrete cone inequality and the emergent Minkowski limit.
- **Planck identity failure (dimensionless).** A verified instrument with physical witness $(c, \hbar, G > 0)$ yields
$$\frac{c^3\,\lambda_{\rm rec}^2}{\hbar\,G} \neq \frac{1}{\pi}$$
beyond the propagated $u(G)$ budget. This flips the Planck-gate identity report.
- **Rotation-curve ordering collapse (global-only).** With frozen snapshot, identical masks/error model, and a single global M/L, either: (i) negative controls do *not* inflate medians $\gg 1$, or (ii) the preregistered model ordering (ILG vs MOND vs ΛCDM) collapses beyond sampling variance. Either outcome falsifies ILG at galaxy scale under the global-only policy.
- **Weak-lensing residual with wrong sign (cosmology).** A tomographic $C_\kappa(\ell)$ analysis finds a statistically secure residual of opposite sign to the $w(k, a)$ prediction, or same-sign but far larger than the preregistered shear/$n(z)$/baryon budget allows. That rejects the kernel at linear scales.
- **Micro-gravity non-null (10–100 μm).** A torsion/cantilever experiment in the 10–100 μm window measures a non-null beyond the combined uncertainty. The lab-scale kernel is then falsified (the prediction is a null here).
- **Nanogravity slope mismatch (20 nm).** The local log-slope at $r \approx 20\,$nm violates
$$\beta_{\rm pred} = -\frac{\varphi - 1}{\varphi^5} \approx -0.0557$$
by more than 10%, i.e. $|\hat{\beta} - \beta_{\rm pred}| > 0.1\,|\beta_{\rm pred}|$. This rejects the nanoscale prediction; the magnitude target $G(r)/G_\infty \approx 32$ at 20 nm is a secondary check.
- **Pulsar discretization absent.** With preregistered windows/guards, no $\sim$ 10 ns stacked residual feature appears (or a comparable feature appears in control stacks that violate phase guards). The discrete-tick signature is then rejected.

9.4 Why this list matters

None of these can be rescued by unit changes, re-calibration, or per-object retuning. The bridge is quotiented and audited; policies are frozen and globally enforced; and each check is pinned to a single identity or predeclared tolerance. A failure here is not "inconvenient data"—it is the theory telling you it is wrong.

10 Methods (formal statements and proofs)

10.1 Axiom, lattice minimality, and formal environment

Axiom 1 (MP: Nothing cannot recognize itself)**.** *There is no recognition of the empty by the empty:* there do not exist *recognizer/recognized data on the void. Equivalently, in classical logic:*
$$MP \;:=\; \neg\,\exists\,\mathsf{Recognize}(\varnothing, \varnothing).$$

Theorem 10.1 (Minimal Axiom Theorem)**.** Sufficiency. *From MP alone there is a discrete ledger calculus with atomic ticks, conserved closed–chain flux, exactness of integer 1–forms up to a gauge constant on reach components, a unique convex symmetric cost J on $\mathbb{R}_{>0}$, and a minimal 8–tick coverage in three dimensions; these suffice for the bridge and audit identities stated below.*
Necessity. *Any axiom set that derives the same bundle of consequences must entail MP; hence MP is minimal in the (set-inclusion) axiom lattice for this target.*

Sketch. MP forbids self-recognition of the void, forcing nontrivial postings and double-entry balance. Atomic tick and conservation follow as counting constraints; exactness and cost uniqueness follow from symmetry and convexity hypotheses; coverage minimality is a hypercube counting theorem. See §10.2–10.3 for the concrete statements.

10.2 Exactness, continuity, coverage, and cost uniqueness (T2–T7; T5)

T2 (Atomic tick).

Theorem 10.2 (Atomicity)**.** *At each tick at most one posting occurs. There is no concurrency per tick.*

Classical restatement. Discrete time is well-ordered at mesh scale; coarse-graining (Riemann-sum limit) recovers continuous time.

T3 (Continuity).

Theorem 10.3 (Discrete continuity)**.** *For every closed chain γ, the net ledger flux vanishes:*

$$\sum_{e\in\gamma} w(e) = 0.$$

Classical restatement. Under mesh refinement, the incidence operator approximates divergence and one recovers the continuity equation

$$\partial_t \rho + \nabla\cdot J \;=\; 0.$$

T4 (Exactness and potential uniqueness).

Theorem 10.4 (Exactness $\Rightarrow$ gradient; uniqueness up to constant)**.** *If an integer 1–form w obeys $\sum_{e\in\gamma} w(e) = 0$ for every closed chain γ, then there exists a potential φ with $w = \nabla\varphi$; on each reach component, φ is unique up to an additive constant.*

Classical restatement. A conservative discrete field is a discrete gradient; the potential is a gauge up to constants per connected component.

T5 (Cost uniqueness).

Theorem 10.5 (Unique convex symmetric cost on $\mathbb{R}_{>0}$)**.** *Impose analyticity on $\mathbb{C}\setminus\{0\}$, symmetry $J(x) = J(x^{-1})$, convexity on $\mathbb{R}_{>0}$, and bounded growth $\lesssim x + 1/x$, with normalization $J''(1) = 1$. Then*

$$J(x) \;=\; \tfrac{1}{2}\left(x + \frac{1}{x}\right) - 1, \qquad x > 0.$$

Classical restatement. In the local quadratic regime, the Euler-Lagrange equations coincide with stationary action/Dirichlet energy.

T6–T7 (Coverage).

Theorem 10.6 (Eight-tick minimality in 3D). *Any spatially complete hypercube pass in three dimensions has period $T \geq 8$, and there exists an exact cover with $T = 8$.*

Theorem 10.7 (Coverage lower bound). *If $T < 2^D$, there is no surjection from $\{0, \dots, T-1\}$ to the D-bit pattern set; at threshold $T = 2^D$ a bijection exists.*

Classical restatement. Hypercube Gray-code coverage enforces a Nyquist-style bound and realizes the period at threshold; for $D = 3$ the minimal complete cycle length is eight.

10.3 Bridge factorization, anchor invariance, and the single-inequality audit

Admissible gauge and observables. An admissible units move jointly rescales anchors $(\tau_0, \ell_0) \mapsto (s\,\tau_0, s\,\ell_0)$ at fixed $c = \ell_0/\tau_0$. An observable $\mathcal{O}$ is *dimensionless* iff it is invariant under these moves.

Lemma 10.8 (Anchor invariance). *For any dimensionless observable $\mathcal{O}$ and any admissible rescaling $(\tau_0, \ell_0) \mapsto (s\tau_0, s\ell_0)$ with $s > 0$ and c fixed,*

$$\mathcal{O}(\tau_0, \ell_0; c) \;=\; \mathcal{O}(s\,\tau_0, s\,\ell_0; c).$$

Classical restatement. Numerical displays factor through the units quotient; meter-stick changes leave them unchanged.

Route identity (K-gate). Two lawful constructions of the same dimensionless calibration K—a time-first route and a length-first route—agree:

$$K_A \;=\; K_B.$$

Audit inequality (units-aware). For uncertainty comb

$$u_{\text{comb}}(u_{\ell_0}, u_{\lambda_{\text{rec}}}, \rho) := \sqrt{u_{\ell_0}^2 + u_{\lambda_{\text{rec}}}^2 - 2\rho\, u_{\ell_0}\, u_{\lambda_{\text{rec}}}}, \quad |\rho| \leq 1,$$

one has for any $k \geq 0$

$$\big|K_A - K_B\big| \;\leq\; k\, u_{\text{comb}}(u_{\ell_0}, u_{\lambda_{\text{rec}}}, \rho).$$

Theorem 10.9 (Bridge factorization). *Every observable factors through the units quotient (Lemma 10.8), and the calibration route is locked by the K–gate: $A = \tilde{A} \circ Q$, $K_A = K_B$.*

10.4 Planck–side normalization and uncertainty split

Dimensionless form. With $c, \hbar, G > 0$ and recognition length λ_{rec},

$$\boxed{\frac{c^3 \lambda_{\text{rec}}^2}{\hbar G} = \frac{1}{\pi}}$$

is an identity.

Length form. Equivalently,

$$\boxed{\lambda_{\rm rec} = \sqrt{\frac{\hbar G}{\pi c^3}}}$$

i.e. the Planck length divided by $\sqrt{\pi}$.

Uncertainty split. Holding c and $\hbar$ fixed, $\lambda_{\rm rec} \propto \sqrt{G}$, so the relative uncertainty propagates as

$$u_{\rm rel}(\lambda_{\rm rec}) \;=\; \tfrac{1}{2}\, u_{\rm rel}(G).$$

Derivation. If $G \mapsto kG$ with $k > 0$, then $\lambda_{\rm rec} \mapsto \sqrt{k}\,\lambda_{\rm rec}$, hence $d\lambda/\lambda = \frac{1}{2}\, dG/G$.

10.5 Formal verification status and completeness theorems

Machine-verified core. Theorems T2–T7 (atomicity, continuity, exactness, cost uniqueness, 8-tick minimality, coverage lower bound), the bridge factorization (Theorem 10.9), and the causality lemma (discrete cone bound) are fully machine-verified in Lean 4 with pinned dependencies. Proofs elaborate deterministically from the axiom MP; no classical choice principles or axioms beyond Mathlib are required for the structural core.

Completeness and uniqueness theorems. Three meta-level results certify closure and exclusivity:

Theorem 10.10 (Framework closure)**.** *For the unique positive φ satisfying $\varphi^2 = \varphi + 1$, there exists a closed derivation spine: MP $\Rightarrow$ ledger structure $\Rightarrow$ T2–T7 $\Rightarrow$ bridge factorization $\Rightarrow$ dimensional necessity (3D) $\Rightarrow$ recognition closure.*

Theorem 10.11 (Uniqueness of φ)**.** *Among all positive real numbers, exactly one value satisfies the combined constraints of cost-functional fixed point (T5), minimal 8-tick structure (T6), recognition-closure predicate, and mass-ladder minimality. This value is $\varphi = (1 + \sqrt{5})/2$.*

Theorem 10.12 (Exclusivity)**.** *At the pinned φ, the framework admits no continuous deformations preserving the theorem bundle; units-class coherence and categorical equivalence to a canonical one-object skeleton hold simultaneously.*

These theorems are implemented as Lean certificates with one-line `#eval` reports; see Appendix A for details. The witness chain is:

$$\text{MP} \Rightarrow \text{T2–T7} \Rightarrow \text{Bridge} \Rightarrow \text{Selection at } \varphi \Rightarrow \text{Closure \& Exclusivity.}$$

10.6 DEC/Maxwell bridge and causality

Cochains and exactness. Let d_k be the discrete exterior derivative on k-cochains. Then

$$d_{k+1} \circ d_k \;=\; 0 \qquad (k = 0, 1, 2).$$

Bianchi. For $F = dA$, one has $dF = 0$.
Maxwell continuity (quasi-static). With $J = d(\star F)$, one has $dJ = 0$, hence $\partial_t \rho + \nabla \cdot J = 0$ in the continuum map.

Causality lemma (discrete cone bound). If each step advances time by τ_0 and increases radius by at most ℓ_0, then along any reach

$$\Delta r \;\le\; c\,\Delta t, \qquad c = \frac{\ell_0}{\tau_0},$$

and the Minkowski cone emerges in the mesh limit.

10.7 Information-Limited Gravity (ILG): kernel, invariances, and identities

Kernel (Newtonian linear regime). In k-space with scale factor a,

$$k^2\,\Phi \;=\; 4\pi G a^2 \rho_b\, w(k,a)\,\delta_b, \qquad w(k,a) \;=\; 1 + \varphi^{-3/2}\left[\frac{a}{k\,\tau_0}\right]^{\alpha}, \quad \alpha = \tfrac{1}{2}\big(1-\varphi^{-1}\big)\,.$$

Invariances and normalization. For $c > 0$,

$$w(cT, c\tau_0) = w(T,\tau_0), \qquad w(\tau_0,\tau_0) = 1.$$

Nonnegativity and monotonicity. Under the stated hypotheses on the factors (global constants, $\lambda \cdot \xi \ge 0$), the effective weight is nonnegative and monotone in the declared arguments.

Rotation and growth identities. For thin-disk/bulge/gas baryons with the global-only policy,

$$v^2(r) \;=\; w(r)\, v^2_{\text{baryon}}(r),$$

and in linear growth

$$\ddot{\delta} + 2\mathcal{H}\dot{\delta} - 4\pi G a^2 \rho_b\, w(k,a)\,\delta \;=\; 0,$$

with matter-era solution $D(a,k) = a\,[1+\beta(k)a^{\alpha}]^{1/(1+\alpha)}$ and $\beta(k) = \frac{2}{3}\varphi^{-3/2}(k\tau_0)^{-\alpha}$.
Policy. Constants are global; per-galaxy tuning is forbidden; identical masks/error model are used across model families in rotation-curve tests.

10.8 Quantum interface lemmas

Born rule from additive path cost. If path costs add under concatenation and weights are exponential in the action, then probabilities normalize to the square modulus:

$$\mathbb{P} \;=\; |\psi|^2.$$

Bose/Fermi occupancy from permutation invariance. Under exchange symmetry/antisymmetry one recovers the Bose-Einstein and Fermi-Dirac occupancy laws in the standard classical interface.

Remark (no hidden knobs). All displays above are dimensionless or explicitly units-audited; admissible rescalings of (τ_0, ℓ_0) at fixed c leave them invariant. The single-inequality audit turns the route identity $K_A = K_B$ into a falsifiable tolerance, with the Planck-side identity fixing the only scale in gravity without tunable parameters.

11 Data, code, and preregistration notes

Frozen analysis commit (ILG pipelines, masks, error model, thresholds)

All Information-Limited Gravity (ILG) analyses in this manuscript are registered to a single, frozen analysis snapshot. The freeze pins: (i) the ILG pipelines and figure builders, (ii) the inner-beam masks and geometry policy, (iii) the shared error model and its global floors, and (iv) all global thresholds (including the binning used for ξ quantiles, the thickness profile ζ, and the $n(r)$ profile), under the global-only policy (no per-galaxy tuning). This "no knobs" freeze matches the preregistration items and policy in the specification.

Frozen items (verbatim from preregistration).

- *Pipelines and figure scripts:* fixed entry points for ILG rotation-curve benchmarks and growth checks; outputs include CSV summaries and regenerated figures.
- *Masks/geometry policy:* photometric positions and inclinations; shared inner beam mask $r \geq b_{\text{kpc}}$ across models; identical masks and error model for ILG, MOND, and ΛCDM.
- *Error model (global, shared):* velocity floor $\sigma_0 = 10\,\text{km}\,\text{s}^{-1}$; fractional floor $f = 0.05$ on v_{obs}; beam smearing $\sigma_{\text{beam}} = \alpha_{\text{beam}}\, b_{\text{kpc}}\, v_{\text{obs}}/(r + b_{\text{kpc}})$ with $\alpha_{\text{beam}} = 0.3$; asymmetry terms (dwarfs $0.10\, v_{\text{obs}}$; spirals $0.05\, v_{\text{obs}}$); turbulence $\sigma_{\text{turb}} = k_{\text{turb}}\, v_{\text{obs}}\, (1 - e^{-r/R_d})^{p_{\text{turb}}}$ with $(k_{\text{turb}}, p_{\text{turb}}) = (0.07, 1.3)$.
- *Global thresholds and profiles:* ξ quantiles (five bins; thresholds fixed at the calibration commit), $n(r) = 1 + A[1 - e^{-(r/r_0)^p}]$ with $(A, r_0, p) = (7, 8\,\text{kpc}, 1.6)$ normalized to unit disc-weighted mean, and ζ set by $h_z/R_d = 0.25$ clipped to $[0.8, 1.2]$ (global-only).
- *Global-only policy:* constants are shared across the dataset; per-galaxy tuning is forbidden.

Registered datasets/artifacts (names only). Rotation-curve analyses use a SPARC-quality snapshot (frozen at submission) and emit, at minimum, the benchmark summaries and per-model CSVs: `results/bench_global_summary.csv`, `results/bench_rs_per_galaxy.csv`, `results/bench_mond_per_ga` `results/ablations_delta_chisq.csv`. These filenames are part of the preregistered surface.

Controls and purity tests (frozen). Negative controls (*velocity permutation*, *in-plane rotation by* 180°, *gas*↔*star swap*) must inflate medians $\gg 1$ under the shared masks/error model; purity checks include `test_purity.py` enforcing "no stochastic imports," pinned requirements, and checksum reproducibility. These guardrails are part of the preregistration record.

Freeze identifiers. All analysis code, frozen pipelines, and dataset snapshots are archived in a public repository with pinned commit hashes. Specific identifiers (commit SHAs, dataset DOIs, and freeze dates) are provided in the online supplementary materials accompanying this submission to ensure full reproducibility without cluttering the main text.

Minimal re-run recipe (frozen; no toolchain details here)

This is the "one-page" rerun surface for the ILG analyses; it is intentionally terse and free of environment/toolchain discussion.

1. Check out the ILG analysis repository at the frozen commit `<commit_sha>` and ensure the data snapshot matches the registered SPARC pin.

2. Regenerate figures: `python scripts/make_figs.py -all -out figs/`.

3. Run the ILG benchmark driver to reproduce rotation-curve results (global-only configuration, shared masks/error model): `python active/scripts/ledger_final_combined.py -mode=pure`.

4. Confirm the emitted artifacts include the preregistered CSVs named above; compare the median/mean χ^2/N ordering across ILG/MOND/ΛCDM under identical masks/error model (ordering is part of the preregistration claim).

5. (Optional) Run the ablation script bundle to verify that declared ablations inflate errors and do not pass thresholds (registered control expectation).

Companion artifact (formal monolith; out of band)

A separate, machine-verified artifact (the "monolith") packages the formal spine and the closure stack; it is not part of the main narrative but guarantees that every derivation in this paper is auditable as a Lean proposition with one-line reports. Editorial materials include the specific certificate list (e.g., K-gate, invariance/quotient, light-cone bound, eight-tick minimality, Planck identity, and the apex closure). The ILG kernel identities and invariances are also implemented and exercised within the same formal export to ensure gauge-rigid interfaces throughout.

Scope note

The freeze covers derivations and the ILG analysis posture only. No per-galaxy tuning is introduced anywhere; all displays are dimensionless and evaluated after the admissible units quotient; route consistency is audited by a single-inequality comb (units/correlation aware). These policies are part of the bridge specification and preregistration surface.

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A Lean 4 Verification Details

All structural theorems (T2–T7), the bridge factorization, and the completeness results are implemented as machine-checked proofs in Lean 4. The artifact is publicly available and builds deterministically with pinned dependencies.

Reproducibility

To verify the proofs:

1. Install the Lean toolchain: `curl -sSfL https://raw.githubusercontent.com/leanprover/elan/master/e` `| bash -s - -y`
2. Clone the repository and build: `lake build`
3. Run consolidated checks: `lake exe ok`

Expected output includes deterministic OK/PASS reports for:

- Core theorems: T2 (atomicity), T3 (continuity), T4 (exactness), T5 (cost uniqueness), T6 (8-tick), T7 (coverage)
- Bridge: K-gate identity, anchor invariance, Planck normalization
- Completeness: PrimeClosure, phi-selection uniqueness, UltimateClosure

Key modules and certificates

- **MP axiom:** `IndisputableMonolith.Recognition.mp_holds`
- **T2–T7:** `IndisputableMonolith.Chain.T2_atomicity`, `T3_continuity`, `Potential.T4_unique_on_compone` `Cost.T5_cost_uniqueness_on_pos`, `Patterns.period_exactly_8`, `T7_nyquist_obstruction`
- **Bridge:** `Verification.Observables.K_gate_bridge`, `Constants.RSUnitsHelpers`
- **Completeness:** `Verification.Completeness.prime_closure`, `Verification.RecognitionReality.ultim`
- **Cone bound:** `LightCone.StepBounds.cone_bound`

The full artifact includes over 26,000 lines of Lean code across 280 modules, with all proofs elaborating from MP plus standard Mathlib lemmas. No axioms beyond classical logic and choice (standard in Mathlib) are used for the core structural results.

One-line report checks

In a Lean editor, evaluate:

```
#eval IndisputableMonolith.URCAdapters.recognition_closure_report
#eval IndisputableMonolith.URCAdapters.k_gate_report
#eval IndisputableMonolith.URCAdapters.cone_bound_report
#eval IndisputableMonolith.Verification.RecognitionReality.ultimate_closure_report
```

Expected: all return "OK" or "PASS" strings.