Charged Fermion Masses with Single-Anchor Phenomenological Validation

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The Fermion Mass Hierarchy Problem

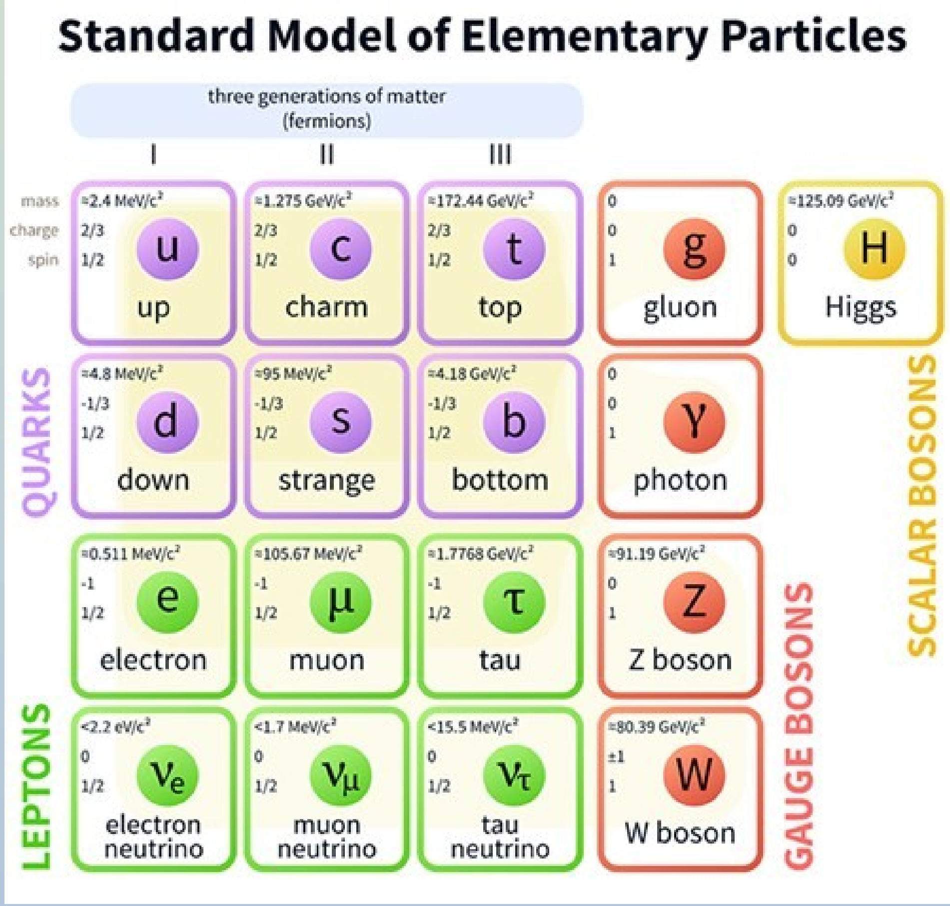
1. Standard Model (SM): 9 charged fermion masses include 9 independent parameters.
2. No explanation for mass hierarchies spanning 5 orders of magnitude
   Electron: 0.511 MeV → Top quark: 172,000 MeV

Current approaches:

- Froggatt-Nielsen: horizontal symmetries + many parameters
- Koide formula: empirical relations for leptons only
- GUT theories: predict specific ratios, often falsified
- Yukawa couplings: unexplained hierarchy remains

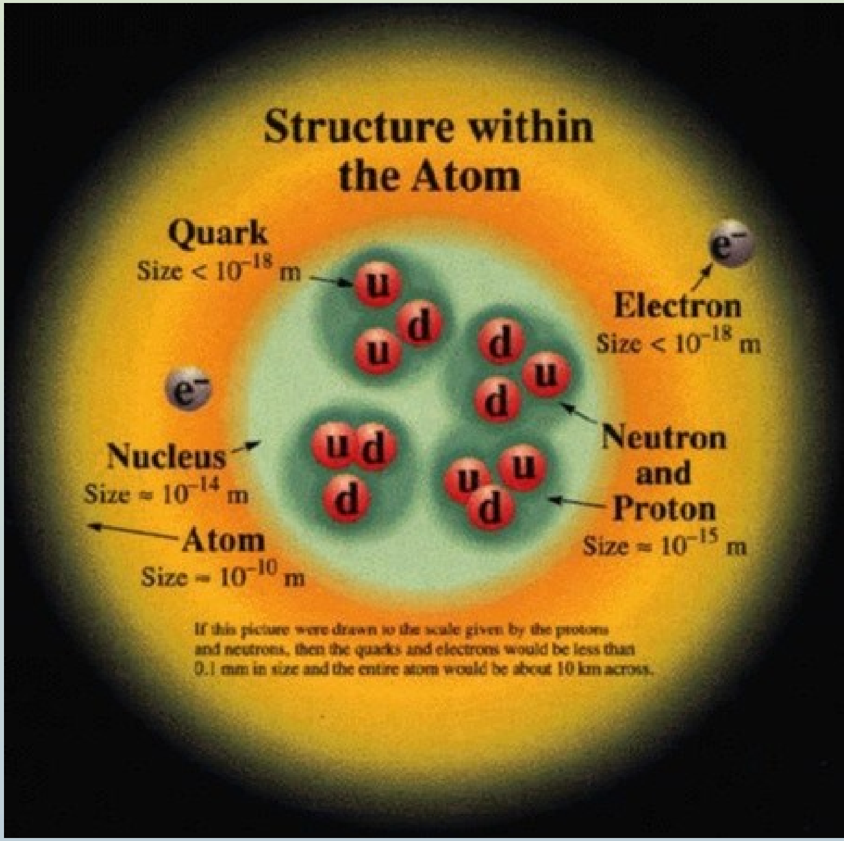
Missing: Why three generations?

Please, allow me to deliver this talk without interruptions, the material is dense, and covers fundamental partcle physics



The Fermion Mass Hierarchy Problem

In the Standard Model of particle physics, the Higgs field permeates all space, and fundamental particles, such as electrons and quarks, acquire their mass by interacting with this field



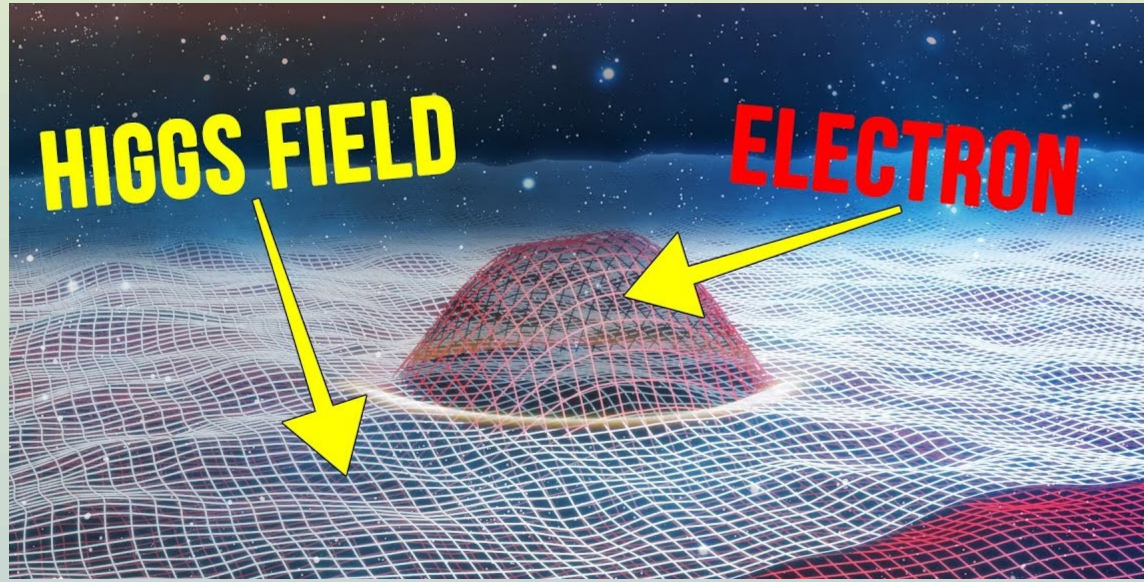
Mass Generation (The Higgs Mechanism): Without the Higgs field, electrons would have no mass and would travel at the speed of light, making it impossible for atoms to form. The coupling to the Higgs field acts as a drag on the electron, which manifests as inertial mass.

Yukawa Coupling: The Higgs field couples to fermions (like electrons) through a specific mechanism called "Yukawa interaction". The strength of this coupling determines the particle's mass. Because the electron has a non-zero mass, it must, by definition, interact with the Higgs field.

Electroweak Symmetry Breaking: The Higgs field has a non-zero value in empty space (vacuum expectation value). This nonzero value breaks the electroweak symmetry, allowing leptons (including electrons) and quarks to acquire mass.

Interaction with the Vacuum: Electrons do not necessarily interact with the "Higgs boson" particle directly in daily life, but rather with the background Higgs field that pervades the vacuum.

The Fermion Mass Hierarchy Problem



mass anomalous dimension

$$\gamma_i^{\mathrm{SM}} := \gamma_m^{\mathrm{QCD}} + \gamma_m^{\mathrm{QED}} + \gamma_m^{\mathrm{Yuk}}$$

TABLE I. Comparison of approaches to the charged-fermion mass hierarchy.

Approach	Free Params	Scope	Falsifiable Test	Type
SM Yukawas	9 continuous	All 9	None	Baseline
Froggatt–Nielsen [11]	$\sim$ 6–9 (charges)	All 9	Texture fits	Fit
Koide formula [13]	3	Leptons only	Mass relations	Empirical
RG fixed points [15, 16]	Derived	Top only	Running to IR	Dynamical
GUTs [17, 18]	GUT scale	All 9	High-scale relations	Unification
This work (RS)	1 constant (φ) + 9 integers (r_i) + 1 tolerance	9 (gauge-only) leptons absolute	ppm clustering at anchor	Discrete geometry

The complete Standard Model mass anomalous dimension includes a Yukawa contribution:

$$\gamma_i(\mu) := \frac{d\ln m_i(\mu)}{d\ln\mu}$$

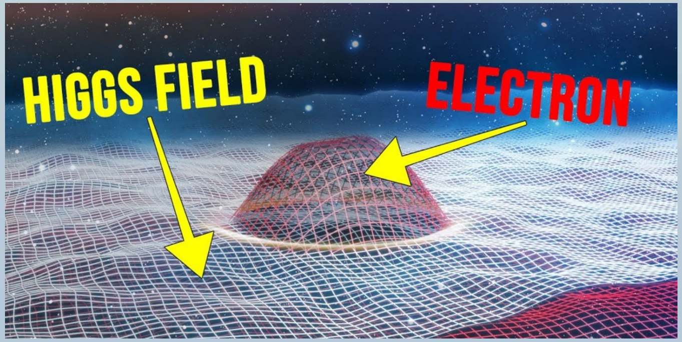
directly encodes how the running mass changes with energy scale.

$\mu\, dm/d\mu = -\gamma_m m$ in much of the QCD literature.

$$\gamma_i^{\mathrm{SM}}(\mu) \;=\; \gamma_m^{\mathrm{QCD}}(\alpha_s(\mu), n_f(\mu)) \;+\; \gamma_m^{\mathrm{QED}}(\alpha(\mu), Q_i) \;+\; \gamma_m^{\mathrm{Yuk}}(\mu)$$

$$\gamma_i(\mu) \;=\; \gamma_m^{\mathrm{QCD}}(\alpha_s(\mu), n_f(\mu)) \;+\; \gamma_m^{\mathrm{QED}}(\alpha(\mu), Q_i)$$

the running Yukawa coupling



$$\gamma_m^{\mathrm{Yuk}}(\mu) \;=\; -\frac{3}{2}\,\frac{y_i^2(\mu)}{16\pi^2} \;+\; \mathcal{O}(y_i^4, y_i^2\alpha_s, y_i^2\alpha)$$

$$y_i(\mu) = \sqrt{2}\,m_i(\mu)/v$$

$v = 246.22\,\mathrm{GeV}$ the Higgs VEV

The Fermion Mass Hierarchy Problem

TABLE II. **Particles analyzed in this work and representative PDG mass inputs.** *What this table shows:* The nine charged fermions (three generations each of leptons, up quarks, down quarks) with their PDG-recommended mass values, mass schemes, and Yukawa correction estimates. *PDG scheme & scale column:* Leptons are quoted as pole masses (on-shell, scale-independent); quarks are quoted as $\overline{\mathrm{MS}}$ running masses at the stated scale ($\mu = 2\,\mathrm{GeV}$ for light quarks, $\mu = m_{\mathrm{pole}}$ for heavy quarks). *Transport method column:* "Gauge-only" means this manuscript uses ONLY gauge interactions (QCD+QED) to transport PDG masses to the anchor $\mu_\star = 182.201\,\mathrm{GeV}$; Yukawa contributions to RG running are **deliberately omitted** in the baseline analysis (Sec. VI). For leptons, this means QED-only transport (color singlets have no QCD). For quarks, this means QCD+QED transport. In both cases, Yukawa RG contributions are omitted. *Yukawa coupling column:* $y_i^{\mathrm{SM}}(\mu_\star) := \sqrt{2}m_i(\mu_\star)/v$ is the effective SM Yukawa at the anchor, where $v = 246.22\,\mathrm{GeV}$ is the Higgs vev. *Yukawa correction columns:* $\Delta^{\mathrm{RS}}_{\mathrm{Yuk}}$ is the one-loop constant-y estimate of Yukawa-induced percentage shift; $\Delta f_{\mathrm{Yuk}}/\delta f_{\mathrm{tol}}$ is the ratio to clustering tolerance $\delta f_{\mathrm{tol}} = 5\times10^{-6}$. Ratios $\ll 1$ (leptons, light quarks) mean gauge-only transport is accurate; ratios $\gg 1$ (top: ~ 400) mean absolute predictions require Yukawa-inclusive transport (future work). The neutron is listed only as a composite reference mass scale.

Group	Particle	Gen.	Q	PDG mass	PDG scheme & scale	Transport method	$y_i^{\mathrm{SM}}(\mu_\star)$ (def.)	$\Delta^{\mathrm{RS}}_{\mathrm{Yuk}}$ (%)	Clustering tolerance	Ratio $\Delta f_{\mathrm{Yuk}}/\delta f_{\mathrm{tol}}$
Charged leptons	e	1	-1	0.510999 MeV	Pole mass	gauge-only	2.87×10^{-6}	$\sim10^{-10}$	5×10^{-6}	$\sim4\times10^{-7}$
	μ	2	-1	105.658 MeV	Pole mass	gauge-only	5.99×10^{-4}	$\sim3\times10^{-6}$	5×10^{-6}	$\sim10^{-2}$
	τ	3	-1	1776.86 MeV	Pole mass	gauge-only	1.01×10^{-2}	$\sim5\times10^{-4}$	5×10^{-6}	~2
Up-type quarks	u	1	$+2/3$	2.2 MeV	$\overline{\mathrm{MS}}(\mu = 2\,\mathrm{GeV})$	gauge-only	1.00×10^{-5}	$\sim10^{-9}$	5×10^{-6}	$\sim3\times10^{-6}$
	c	2	$+2/3$	1270 MeV	$\overline{\mathrm{MS}}(m_c)$	gauge-only	5.61×10^{-3}	$\sim3\times10^{-4}$	5×10^{-6}	~1
	t	3	$+2/3$	162,500 MeV	$\overline{\mathrm{MS}}(m_t)$	gauge-only	9.29×10^{-1}	~0.1	5×10^{-6}	~400
Down-type quarks	d	1	$-1/3$	4.7 MeV	$\overline{\mathrm{MS}}(\mu = 2\,\mathrm{GeV})$	gauge-only	2.15×10^{-5}	$\sim10^{-9}$	5×10^{-6}	$\sim10^{-5}$
	s	2	$-1/3$	93 MeV	$\overline{\mathrm{MS}}(\mu = 2\,\mathrm{GeV})$	gauge-only	4.25×10^{-4}	$\sim10^{-6}$	5×10^{-6}	$\sim5\times10^{-3}$
	b	3	$-1/3$	4180 MeV	$\overline{\mathrm{MS}}(m_b)$	gauge-only	2.00×10^{-2}	$\sim2\times10^{-3}$	5×10^{-6}	~9
Hadron (ref.)	n	–	0	939.565 MeV	Pole mass	(composite)	N/A	N/A	N/A	N/A

Pole mass is independent of the energy scale of measurement and represents the real-world mass.
Electron has 100% Yukawa contribution, 0.1-1% QED, and 0% QCD.

RS Framework for Charged Fermion Masses: 1. Energy Yardsticks

Recognition Science refers to a discrete, structural description of mass organization.
Standard Model uses continuous, dynamical description of masses via 9 independent Yukawa couplings.

sector yardstick

$$A_B := 2^{B_{\text{pow}}(B)} E_{\text{coh}}\, \varphi^{r_0(B)}$$

encodes the actual hierarchical structure

$B \in \{\text{Lepton}, \text{UpQuark}, \text{DownQuark}\}$

$B_{\text{pow}}(B)$ and $r_0(B)$ integer exponents

E_{coh} is a sector-independent coherence

$E_{\text{coh}} := 1\,\text{GeV}$

Sector	B_{pow}	r_0	Notes
Charged leptons (ℓ)	-22	62	shared within sector
Up-type quarks (u)	-1	35	shared within sector
Down-type quarks (d)	$+23$	-5	shared within sector

Negative B_{pow} (leptons: -22) means small masses
positive B_{pow} (down quarks: $+23$) means large masses

For the exponential coefficients we use the cube counts and the crystallographic constant:

$$V := 8, \qquad E_{\text{total}} := 12, \qquad F := 6, \qquad W := 17,$$
$$A_z := 1, \qquad E_{\text{passive}} := E_{\text{total}} - A_z = 11$$

$$\begin{aligned} B_{\text{pow}}(\text{Lepton}) &= -2E_{\text{passive}} = -22, \\ B_{\text{pow}}(\text{UpQuark}) &= -A_z = -1, \\ B_{\text{pow}}(\text{DownQuark}) &= 2E_{\text{total}} - 1 = 23, \end{aligned}$$

$$\begin{aligned} r_0(\text{Lepton}) &= 4W - 6 = 62, \\ r_0(\text{UpQuark}) &= 2W + A_z = 35, \\ r_0(\text{DownQuark}) &= E_{\text{total}} - W = -5. \end{aligned}$$

RS Framework for Charged Fermion Masses: 2. Gap Function

$\tilde{Q} := 6Q$

$$Z(Q,\text{sector}) := \begin{cases} 4+\tilde{Q}^2+\tilde{Q}^4, & \text{quarks (color fundamental)},\\ \tilde{Q}^2+\tilde{Q}^4, & \text{leptons (color singlet)}. \end{cases}$$

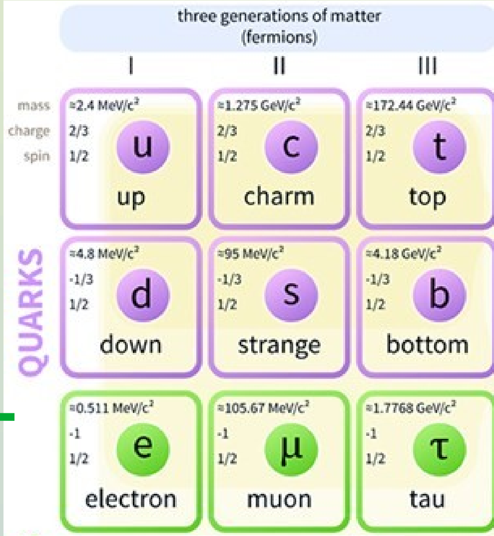
$$\mathscr{F}(Z) := \log_{\varphi}\left(1+\frac{Z}{\varphi}\right)$$ **gap function**

$$A_B := 2^{B_{\text{pow}}(B)}\, E_{\text{coh}}\, \varphi^{r_0(B)}$$ **sector yardstick**

AIM: to construct structural mass frame

? ?

$$m_i^{(\text{struct})}(\mu_\star) = \underbrace{A_B\,\varphi^{r_i-8}}_{\text{skeleton: sector + rung}} \times \underbrace{\varphi^{\mathscr{F}(Z_i)}}_{\text{band: charge family}}$$



Species	Z_i
Down-type Quarks	
Down (d)	24
Strange (s)	24
Bottom (b)	24
Up-type Quarks	
Up (u)	276
Charm (c)	276
Top (t)	276
Charged Leptons	
Electron (e)	1332
Muon (μ)	1332
Tau (τ)	1332

RS Framework for Charged Fermion Masses: 3. Single Anchor

To connect the transport kernels to the integer map Z, we decompose the anomalous dimension into six gauge-invariant motifs. The gauge-only anomalous dimension has natural building blocks—QCD color factors (fundamental, adjoint, vector, gluon) and QED charge powers (Q^2 , Q^4). Decomposing γ_i into these motifs reveals the integer structure: quarks carry four QCD motifs, leptons carry none, explaining the +4 offset in the Z map.

$$\gamma_i(\mu) \;=\; \gamma_m^{\mathrm{QCD}}(\alpha_s(\mu), n_f(\mu)) \;+\; \gamma_m^{\mathrm{QED}}(\alpha(\mu), Q_i)$$

$$\gamma_i(\mu) = \sum_{k\in\{F,NA,V,G,Q2,Q4\}} \kappa_k(\mu)\, N_k(i)$$

$\kappa_k(\mu)$ are universal kernel functions

$N_k(i)$ are integer counts

quarks have $N_F = N_{NA} = N_V = N_G = 1$ (sum=4)

gauge motif $k \in \mathcal{K}_{\mathrm{gauge}} := \{F, NA, V, G, Q2, Q4\}$

$$\hat{w}_k(\mu;\Delta) = \lambda^{-1} \int_{\ln\mu}^{\ln\mu+\Delta} \kappa_k(t)\, dt$$

$$w_k(\mu;\Delta) := \hat{w}_k(\mu;\Delta)/\bar{\hat{w}}(\mu;\Delta)$$

$$\tfrac{1}{6} \sum_k w_k(\mu;\Delta) = 1$$

We minimize the variance of these normalized weights over Δ = 1.0:

$$\mathrm{Var}_k[w_k](\mu) \;:=\; \frac{1}{6} \sum_{k\in\mathcal{K}_{\mathrm{gauge}}} \left(w_k(\mu;\Delta) - 1\right)^2$$

Minimization yields the anchor

$$\mu_\star = 182.201\,\mathrm{GeV}, \quad \lambda = \ln\varphi \approx 0.4812$$

RS Framework for Charged Fermion Masses: 4. Sector Rungs

The transport residue

$$f_i^{\mathrm{RG}}(\mu_1,\mu_2) := \frac{1}{\lambda}\int_{\ln\mu_1}^{\ln\mu_2} \gamma_i(\mu)\, d\ln\mu$$

$$\lambda = \ln\varphi$$

$\gamma_i(\mu)$ is the gauge-only $\overline{\mathrm{MS}}$ mass anomalous dimension (QCD+QED)

The transport mass is defined by

$$m_i(\mu_2) = m_i(\mu_1)\,\varphi^{f_i^{\mathrm{RG}}(\mu_1,\mu_2)}$$

Fermion	μ_{ref}	$f^{\mathrm{RG}}(\mu_\star,\mu_{\mathrm{ref}})$
Electron (e)	m_e^{pole}	0.049426
Muon (μ)	m_μ^{pole}	0.028791
Tau (τ)	m_τ^{pole}	0.017876
Up (u)	2 GeV	0.482193
Down (d)	2 GeV	0.476388
Strange (s)	2 GeV	0.476388
Charm (c)	m_c	0.547013
Bottom (b)	m_b	0.380746
Top (t)	m_t	0.009797

Transport of the PDG mass from reference scale to the anchor scale is defined by

$$m_i^{(\mathrm{data})}(\mu_\star) := m_i^{(\mathrm{PDG})}(\mu_{\mathrm{ref}})\,\varphi^{-f_i^{\mathrm{RG}}(\mu_\star,\mu_{\mathrm{ref}})}$$

We form empirical residue by normalizing the transported mass by its skeleton

$$r_i = \operatorname*{argmin}_{k\in\mathbb{Z}} \left| \log_\varphi\left(\frac{m_i^{(\mathrm{data})}(\mu_\star)}{A_B\,\varphi^{-8}}\right) - (k+\mathscr{F}(Z_i)) \right|$$

$r_i \in \mathbb{Z}$ is chosen to minimize the distance to the structural prediction:

k is a dummy integer index that runs over all integers k∈Z. It labels the candidate rung values ri which minimizes the absolute mismatch.

Sector	Particle	Gen.	r_i
Charged leptons	e	1	2
	μ	2	13
	τ	3	19
Up-type quarks	u	1	4
	c	2	15
	t	3	21
Down-type quarks	d	1	4
	s	2	15
	b	3	21

We form empirical residue by normalizing the transported mass by its skeleton

$$f_i^{(\mathrm{exp})}(\mu_\star) := \log_\varphi\left(\frac{m_i^{(\mathrm{data})}(\mu_\star)}{m_i^{(\mathrm{skel})}(\mu_\star)}\right)$$

$$m_i^{(\mathrm{data})}(\mu_\star) := m_i^{(\mathrm{PDG})}(\mu_{\mathrm{ref}})\,\varphi^{-f_i^{\mathrm{RG}}(\mu_\star,\mu_{\mathrm{ref}})}$$

$$m_i^{(\mathrm{struct})}(\mu_\star) \;=\; \underbrace{A_B\,\varphi^{r_i-8}}_{\text{skeleton: sector + rung}}$$

RS Framework for Charged Fermion Masses: 5. Empirical Residue

Comparison view of the single-anchor protocol, showing the Recognition Science (structural) and Standard Model (transport) branches and where they meet to form the **empirical residue** and perform the equal-Z clustering test.

Fixed structural inputs
charge Q_i, sector/color B

$Z_i = Z(Q_i, B)$
(Eq. 10)

Structural outputs at $\mu_\star$:
$f^{\mathrm{Rec}}(Z_i) = \mathscr{F}(Z_i)$ (Eq. 13)
yardstick A_B (Eq. 3)

Experimental input (PDG)
$m_i^{(\mathrm{PDG})}(\mu_{\mathrm{ref}})$

SM transport (gauge-only QCD+QED)
$\gamma_i(\mu)$ and $f_i^{\mathrm{RG}}(\mu_\star, \mu_{\mathrm{ref}})$
(Eq. 14)

Transported "data" at the anchor
$m_i^{(\mathrm{data})}(\mu_\star)$
(Eq. 17)

Construct the empirical residue at $\mu_\star$:

- assign r_i (Eq. 18) using $m_i^{(\mathrm{data})}(\mu_\star)$ and $(A_B, \mathscr{F}(Z_i))$
- $m_i^{(\mathrm{skel})}(\mu_\star) = A_B\, \varphi^{r_i - 8}$
- $f_i^{(\mathrm{exp})}(\mu_\star) = \log_\varphi\!\left(m_i^{(\mathrm{data})}(\mu_\star)/m_i^{(\mathrm{skel})}(\mu_\star)\right)$ (Eq. 19)

Comparison at the anchor (bottom line):

- test equal-Z clustering within δf_{tol} (Eq. 21)
- compare within-family means $\bar{f}_Z(\mu_\star)$ to $\mathscr{F}(Z)$ (Table V)

$$f_i^{(\mathrm{exp})}(\mu_\star) := \log_\varphi\left(\frac{m_i^{(\mathrm{data})}(\mu_\star)}{m_i^{(\mathrm{skel})}(\mu_\star)}\right)$$

Within-family mean and tolerance: $\bar{f}_Z(\mu_\star)$

$$\bar{f}_Z(\mu_\star) := \frac{1}{|\{j : Z_j = Z\}|}\sum_{j:Z_j=Z} f_j^{(\mathrm{exp})}(\mu_\star)$$

The single-anchor clustering statement is

$$\max_i \left| f_i^{(\mathrm{exp})}(\mu_\star) - \bar{f}_{Z_i}(\mu_\star) \right| \le \delta f_{\mathrm{tol}}.$$

Species	Z_i	$f_i^{(\mathrm{exp})}$	$\Delta\,(\times 10^{-6})$
Down-type Quarks ($Z = 24$)			
Down (d)	24	5.738112	-3
Strange (s)	24	5.738118	$+3$
Bottom (b)	24	5.738114	-1
Up-type Quarks ($Z = 276$)			
Up (u)	276	10.695341	-4
Charm (c)	276	10.695349	$+4$
Top (t)	276	10.695346	$+1$
Charged Leptons ($Z = 1332$)			
Electron (e)	1332	13.951821	-3
Muon (μ)	1332	13.951829	$+5$
Tau (τ)	1332	13.951823	-1

LEPTON MASSES

$$\delta_e := 2W + \frac{W + E_{\text{total}}}{4E_{\text{passive}}} + \alpha^2 + E_{\text{total}}\,\alpha^3$$

Using $W = 17, E_{\text{total}} = 12, E_{\text{passive}} = 11$

$\delta_e \approx 34.659$

$$m_e^{\text{pred}} := \underbrace{A_{\text{Lepton}}\,\varphi^{r_e - 8}}_{m_{\text{skel}}(e;\mu_\star)}\,\varphi^{\mathscr{F}(1332) - \delta_e}$$

The Fermion Mass Hierarchy Problem

$$S_{e\to\mu} := E_{\text{passive}} + \frac{1}{4\pi} - \alpha^2 \approx 11.0796,$$

$$S_{\mu\to\tau} := F - \frac{2W+D}{2}\,\alpha \approx 5.8651.$$

$$m_\mu^{\text{pred}} := m_e^{\text{pred}}\,\varphi^{S_{e\to\mu}},$$

$$m_\tau^{\text{pred}} := m_\mu^{\text{pred}}\,\varphi^{S_{\mu\to\tau}}.$$

Species	Predicted (MeV)	PDG (MeV)	Rel. dev. (%)
e	0.5110	0.5109989	+0.0002
μ	105.66	105.6584	+0.0015
τ	1776.8	1776.86	−0.0034

The $\overline{\text{MS}}$ QCD mass anomalous dimension is

$$\gamma_m^{\rm QCD}(\alpha_s, n_f) = -\sum_{k=0}^{3} \gamma_{\rm QCD}^{(k)}(n_f)\, a_s^{k+1}, \qquad a_s := \frac{\alpha_s}{4\pi}.$$

Coefficients for SU(3) ($C_F = 4/3$, $C_A = 3$, $T_F = 1/2$):

$$\begin{aligned}
\gamma_{\rm QCD}^{(0)} &= 3C_F, \\
\gamma_{\rm QCD}^{(1)}(n_f) &= \frac{3}{2}C_F^2 + \frac{97}{6}C_F C_A - \frac{10}{3}C_F T_F n_f, \\
\gamma_{\rm QCD}^{(2)}(n_f) &= (\text{known, 18-term expression}), \\
\gamma_{\rm QCD}^{(3)}(n_f) &= (\text{known, 78-term expression}).
\end{aligned}$$

The $\overline{\text{MS}}$ QED mass anomalous dimension is

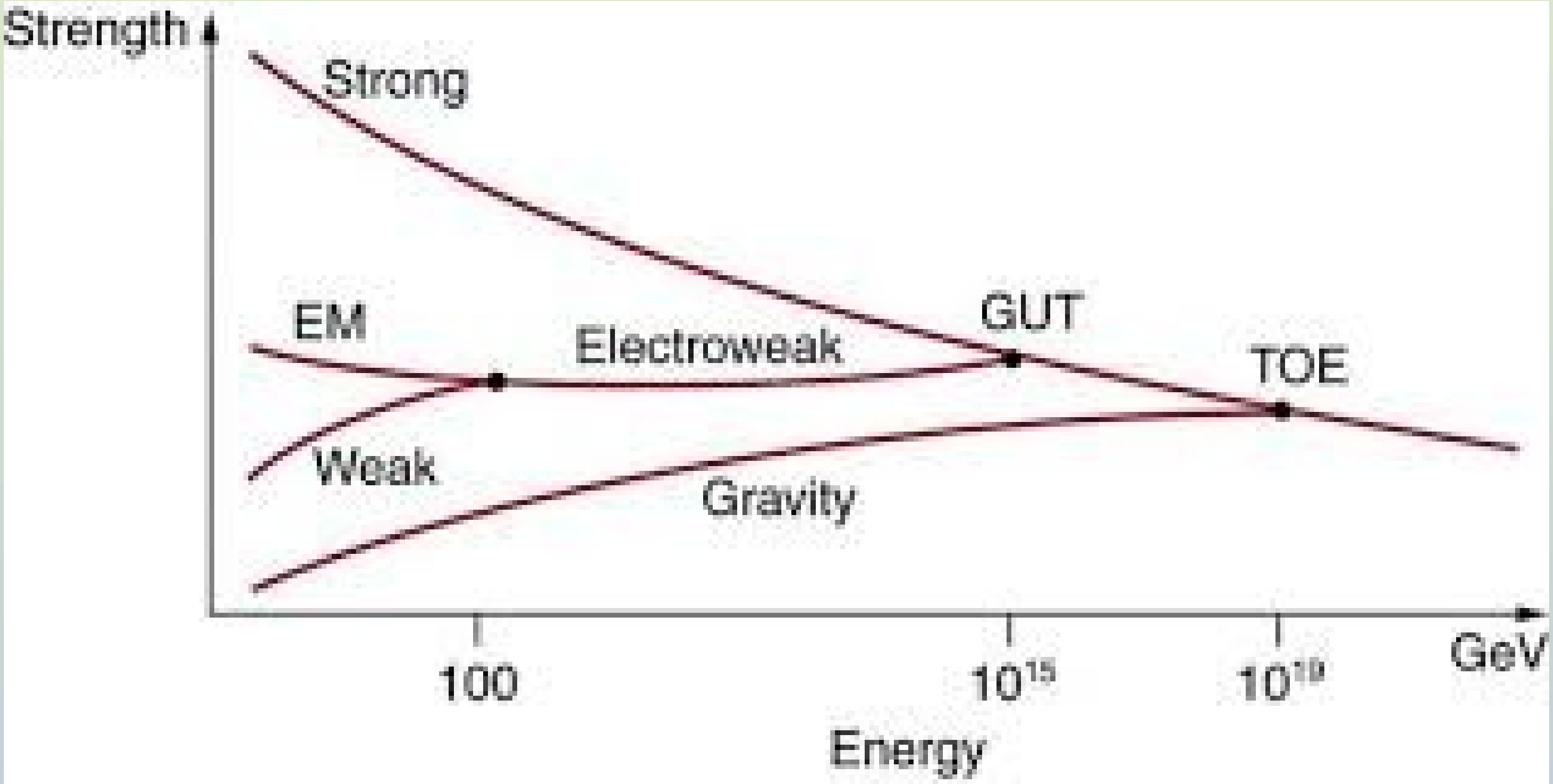
$$\gamma_m^{\rm QED}(\alpha, Q_i) = -\sum_{k=0}^{1} \left[A^{(k)} Q_i^2 + B^{(k)} Q_i^4\right] a_e^{k+1}, \qquad a_e := \frac{\alpha}{4\pi}.$$

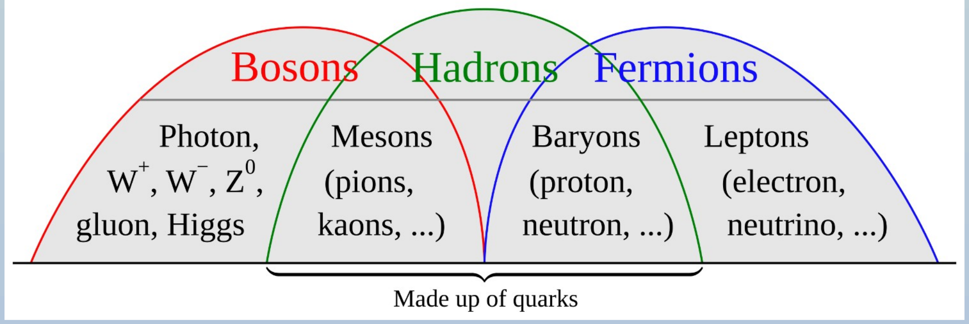
$$\begin{aligned}
A^{(0)} &= 3, \quad B^{(0)} = 0, \\
A^{(1)} &= -\frac{5}{2}S_2, \quad B^{(1)} = -\frac{3}{2}, \quad S_2 = \sum_f Q_f^2.
\end{aligned}$$

Additional Information II: QCD and QED motifs

Motif k	**Physical origin**	**Quarks**	**Leptons**
F	Fundamental self-energy (C_F terms)	1	0
NA	Non-abelian vertex ($C_F C_A$ terms)	1	0
V	Vacuum polarization ($C_F T_F n_f$ terms)	1	0
G	Quartic gluon (higher C_A structures)	1	0
$Q2$	Abelian Q^2 (QED 1-loop and mixed)	$(6Q_i)^2$	$(6Q_i)^2$
$Q4$	Abelian Q^4 (QED 2-loop self-energy)	$(6Q_i)^4$	$(6Q_i)^4$

Additional Information III: Grande Unified Theory (GUT)





CONCLUSIONS

1. We present a discrete-geometry framework organizing charged fermion masses.

2. At a single anchor $\mu_\star$=182.201 GeV, we derived empirical residues showing a clustering for equal-charge families.

3. For charged leptons, we managed to fit absolute PDG lepton masses.

4. Yukawa contributions are a known limitation for quarks, motivating future extensions.

Thank you for your attention.

Our RS Institute (Austin TX, USA) also thanks theor.dep. OIVTRAN for the provided opportunity to present our recent advances in fermion masses.