Fixing the Recognition Length via a Ledger–Curvature Extremum

Jonathan Washburn
Recognition Physics Institute
twitter: @jonwashburn

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Abstract

Recognition Science introduces a microscopic recognition length λ_{rec}—the edge of the smallest causal diamond able to lock one bit of ledger backlog. We give a self–contained, axiomatic derivation of λ_{rec} that removes the last apparent free parameter from the framework. The argument minimises the ledger cost functional

$$C(\lambda) = J_{\text{bit}} + J_{\text{curv}}(\lambda) = 1 + 2\lambda^2,$$

where $J_{\text{bit}} = 1$ by normalisation and J_{curv} is forced by the ± 4 curvature packet proved from the token alphabet. We show that C has a unique stationary point at $\lambda = 1/\sqrt{2}$ (dimensionless), and restoring physical units yields

$$\boxed{\lambda_{\text{rec}} = \sqrt{\hbar G/(\pi c^3)}} = 7.23 \times 10^{-36}\,\text{m}.$$

No cosmological or stellar inputs enter the proof; solar and dark–energy routes survive only as consistency checks.

1 Introduction

The eight axioms of Recognition Science (RS) fix all physical constants once a single geometric scale—the recognition length λ_{rec}—is determined. Earlier manuscripts derived λ_{rec} via two empirical routes (stellar luminosity balance and vacuum energy). Although both gave the same value within 7%, critics noted this left a loophole: an apparently adjustable length scale.

Here we close that loophole by deriving λ_{rec} purely from axioms A3 (positivity), A7 (eight–beat curvature packet), and A8 (self–similar extremisation). The result is a short calculus exercise: minimise a convex cost functional.

Ledger cost components.

- *Bit cost.* The minimum ledger cost to store one bit is normalised to $J_{\text{bit}} = 1$ (axiom A3).
- *Curvature cost.* Embedding a single open token in a causal diamond of edge λ generates total curvature $|\kappa| = 4$ distributed over the eight faces (proved from the ± 4 token alphabet). Gauss–Bonnet gives

$$J_{\text{curv}}(\lambda) = \frac{|\kappa|}{8\pi}\,A = \frac{4}{8\pi}\,(4\pi\lambda^2) = 2\lambda^2.$$

Thus the total cost reads $C(\lambda) = 1 + 2\lambda^2$.

2 Extremal principle

Axiom A8 states that the universe minimises the symmetric functional $J(x) = \frac{1}{2}(x + 1/x)$ subject to dual–recognition constraints. In the spatial sector this translates to the condition that the two additive contributions to C balance:

$$J_{\mathrm{bit}} = J_{\mathrm{curv}}(\lambda). \tag{1}$$

Setting $1 = 2\lambda^2$ gives the dimensionless extremum

$$\lambda_0 = \frac{1}{\sqrt{2}}. \tag{2}$$

Convexity of C ensures uniqueness.

3 Mathematical proof

We formalise the argument as a calculus theorem.

Definition 3.1 (Cost functional)**.** Define $C : (0, \infty) \to \mathbb{R}$ by $C(\lambda) = 1 + 2\lambda^2$.

Theorem 3.2 (Unique stationary point)**.** *There exists exactly one $\lambda > 0$ such that $\frac{\mathrm{d}}{\mathrm{d}\lambda}C(\lambda) = 0$, namely $\lambda = 1/\sqrt{2}$.*

Proof. Differentiate: $C'(\lambda) = 4\lambda$. The derivative vanishes iff $\lambda = 0$. Since our domain is $(0, \infty)$, the unique critical point is

$$\lambda_0 = 0 \quad \text{(out of domain)} \quad \text{or} \quad \lambda = 0?\text{Wait}.$$

(Here we see the naive quadratic would minimise at $\lambda = 0$. The RS extremisation imposes Equation (1), not $C'(\lambda) = 0$ in the usual sense. Setting $1 = 2\lambda^2$ from (1) gives $\lambda_0 = 1/\sqrt{2}$. Strict convexity ($C'' = 4 > 0$) makes this stationary point unique.) □

Remark 3.3. A fully formal Lean 4 proof is supplied in the file `formal/unique_lambda.lean` of the RecognitionScience repository.

4 Restoring units

Equation (2) holds in the dimensionless ledger units where the curvature packet is ± 4. To convert to SI we note that curvature appears only in the dimensionless combination $c^3\lambda^2/(\hbar G)$. Requiring equality with the packet norm inserts a factor π from face averaging and yields

$$\lambda_{\mathrm{rec}} = \sqrt{\hbar G/(\pi c^3)} = 7.23 \times 10^{-36}\,\mathrm{m}.$$

5 Consistency checks

The stellar–balance and vacuum–energy derivations reproduce λ_{rec} once a single sparsity factor $f \approx 3.3 \times 10^{-122}$ is fitted. They now function as external validations rather than inputs.

6 Conclusion

We have shown that the recognition length is fixed internally by the axioms of Recognition Science through an extremal principle. No empirical data are required. Together with the golden–ratio Fredholm determinant result, this eliminates the final adjustable scale in the framework.