Response to Referee: Dimensional Rigidity / $D = 3$

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Referee comment (summary). The referee correctly notes that, as originally phrased, each of our three conditions (T),(K),(S) can read as a disguised restatement of $D = 3$ rather than three *convergently independent* constraints (in the set-theoretic sense that each condition allows multiple dimensions and only the intersection forces $D = 3$). In particular, (S) is monotone in D and therefore selects the smallest admissible D once a lower bound is assumed.

Our response (high level). We agree with this point and have revised the manuscript to (i) make the logic explicit and non-circular, and (ii) separate two different roles played by our constraints:

- **Weak (set-valued) constraints** that are each individually non-singleton and whose intersection is $\{3\}$ (a genuinely "convergent" pattern).
- **Sharp (characterizing) constraints** that, once adopted, already imply $D = 3$; these are now presented as strengthened cross-checks / sharpenings rather than as the sole basis for an "independence" claim.

Where the manuscript changed. We updated (a) the Abstract language, (b) the Introduction (added an "allowed-dimension set" clarification and the weak A/B/C triad), (c) the statement/interpretation of (S) (explicitly a tie-breaker on an admissible set), and (d) downstream wording in the "Main Result" synthesis, summary table, and Conclusion to consistently reflect this logical structure.

1. "Independent constraints" and allowed-dimension sets. We added a short paragraph defining the allowed-dimension set

$$\mathcal{A}_X := \{D \in \mathbb{N} : \text{constraint } (X) \text{ holds in dimension } D\}$$

and clarified that our use of "independent" was intended to mean "arising from distinct physical/mathematical sectors" (topology vs. dynamics vs. computation), not that each $\mathcal{A}_X$ is non-singleton. To avoid ambiguity, we now explicitly include a *set-theoretic* convergent triad (see item 3 below).

2. The role of (S) (synchronization). We agree that the minimization statement in (S) is an Occam/complexity tie-breaker once an admissible set of dimensions is specified. We revised the text around Theorem (S) to state this plainly:

- (S) does *not* derive the lower bound (capacity) by itself; it selects the minimal synchronization overhead *among admissible D*.

- We now treat (S) as a computational cost principle that is meaningful after other constraints have already ruled out low-dimensional cases (or after a capacity axiom fixes the admissible set).

3. New "convergent independence" triad (weak A/B/C). To address the referee's core logical concern, we now include (in the Introduction) a weaker triad of constraints whose allowed-dimension sets are non-singleton but whose intersection is $\{3\}$. Informally:

- **(A) Same-dimension linking** (topological): existence of a nontrivial $\mathbb{Z}$-valued linking invariant between two same-dimensional extended objects implies D is odd (so $\mathcal{A}_A = \{1, 3, 5, \dots\}$).
- **(B) Green-kernel orbital stability** (dynamical): stability of near-circular bound orbits under the Laplacian Green-kernel family forces $D < 4$ (so $\mathcal{A}_B \supseteq \{1, 2, 3\}$ or, under mild exclusions, $\{2, 3\}$).
- **(C) Minimal geometric capacity** (geometric/operational): a minimal capacity assumption excludes the $D = 1$ case (so $\mathcal{A}_C \subseteq \{2, 3, 4, \dots\}$).

With these definitions, $\mathcal{A}_A \cap \mathcal{A}_B \cap \mathcal{A}_C = \{3\}$, matching the referee's requested "nontrivial sets whose intersection is a singleton" pattern.

4. Status of the sharper (T/K/S) statements. We retain the sharper statements (loop–loop linking; non-precession; $N = 45$ synchronization minimality) but now label them as strengthened specializations and cross-checks, rather than the sole basis for an "independent constraints" claim.

5. Formalization note. We also tightened the Lean formalization of the 2^D–45 arithmetic forcing by removing an artificial boundedness hypothesis from the Gap-45 derivation; the lemma is now fully general in D (this does not change the paper's mathematical content, but improves the mechanized certificate).

We thank the referee for pushing us to sharpen the logical structure and presentation.