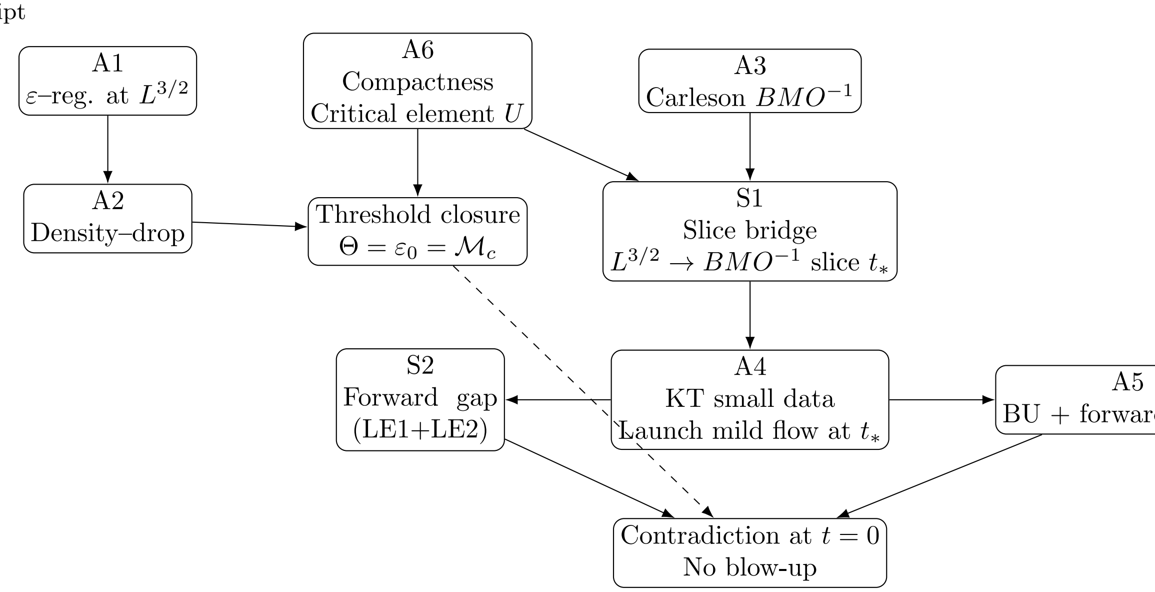
NS Global Regularity — One-Page Roadmap

A1–A6 = classical inputs; S1–S2 = novel statements proved in this manuscript

Statements (informal):

- A1 (*Critical ε–regularity*). If $(x_0, t_0; r_0) \le \varepsilon_A$, then $\sup_{Q_{r_0/2}} |\omega| \lesssim r_0^{-2\,2/3}$.
- A2 (*Density–drop*). If $(0, 0; 1) \le \varepsilon_0 + \eta$ (η small) then $(0, 0; \vartheta) \le \varepsilon_0 + c\eta$.
- A3 (*Carleson characterization of BMO^{-1}*). Heat–flow square function equivalence.
- A4 (*Koch–Tataru small data*). $\|u_0\|_{BMO^{-1}} \le \varepsilon_{\mathrm{SD}} \Rightarrow$ global mild solution, smooth for $t > 0$.
- A5 (*Backward/forward uniqueness*). Carleman backward uniqueness and forward energy uniqueness.
- A6 (*Compactness/critical element*). Suitable solutions are compact locally in L^3; extract an ancient critical element; semicontinuity.
- S1 (*Slice bridge*). If $\sup_{(x,t),r}(x, t; r) \le \varepsilon$ on a unit window, then $\exists\, t_* :\ \|u(\cdot, t_*)\|_{BMO^{-1}} \lesssim \varepsilon^{2/3}$.
- S2 (*Forward gap*). If $\|u(\cdot, t_*)\|_{BMO^{-1}} \le \varepsilon$, then on $[t_* + c,\ t_* + 1]$: $\sup_x(x, t; 1) \lesssim \varepsilon^{3/2}$ (via local $BMO^{-1} \to L^3$ and $L^3 \to L^{3/2}$ embeddings).

Legend:
A = classical input
S = novel in this manuscript



Flow of the proof (one line). Assume first blow-up $\Rightarrow$ extract ancient critical element U (A6); density–drop pins $\mathcal{M}_c = \varepsilon_0$ (A1,A2); slice bridge gives small BMO^{-1} at some t_* (S1,A3); launch KT flow (A4) and obtain a forward gap (S2); backward/forward uniqueness identifies U with the smooth flow on $[t_*, 0]$ and contradicts saturation at $(0, 0; 1)$ (A5).