Response to `respone1.tex`: Formalizing the Mechanism Class and Proving Uniqueness

A referee-facing resolution of the "non-identifiability" objection for the $\mu \to \tau$ correction

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Abstract

This note responds point-by-point to Anil Thapa's `respone1.tex` critique of the previous "structural mechanism" reply. We agree with the critique's central methodological demand: a genuine resolution must define an admissible mechanism class $\mathcal{M}$, a rule $g : \mathcal{M} \to \mathbb{R}$, and prove an injectivity/uniqueness theorem inside $\mathcal{M}$.

We provide exactly that. The key move is to make the admissible class explicit as *local cellwise normalization* mechanisms: if a transition is mediated by k-cells of the cubic ledger, then the correction coefficient is the number of mediating k-cells divided by the number of *vertex anchors* of a single such k-cell. In $D = 3$, this yields a finite list of distinct values ($8, 6, 3/2, 1/8$ for $k = 0, 1, 2, 3$), so the face-mediated value $3/2$ is unique in the class.

All the arithmetic is formalized in Lean in `IndisputableMonolith/Physics/LeptonGenerations/TauStepDeltaDe` including the uniqueness lemma `localCoeff_eq_three_halves_iff`.

1 Scope and shared ground

We agree with `respone1.tex` on the following:

- **Purely syntactic non-uniqueness is infinite.** If one treats "different-looking expressions" as different even when they are provably equal under known identities (e.g. $F = 2D$, $E = 4D$), then infinitely many representations exist.
- **A meaningful uniqueness claim must be conditional.** One must state a constrained admissible class and prove uniqueness *within* it.

The dispute is therefore not about arithmetic. It is about the right admissible class and whether the framework already commits to it.

2 Reply to Failure 1: "New axioms are asserted, not derived"

`respone1.tex` claims that the "Inverse Measure Rule" was introduced ad hoc. That is not accurate: the same inverse-measure structure is already present in the existing $e \to \mu$ derivation.

In the Lean module `FractionalStepDerivation.lean`, the generation step is derived as

$$S_{e\to\mu} \;=\; N_{\text{passive}} + \frac{N_{\text{active}}}{\Omega}, \tag{1}$$

where $\Omega = 4\pi$ is the total solid angle and $N_{\text{active}} = 1$ is the single active edge. The appearance of $1/\Omega$ is precisely an inverse-measure normalization: the differential (single-direction) contribution is the inverse of the continuous measure used in the integrated α seed.

Remark 1 (What is hypothesis vs. what is math). The mathematical statement "differential contribution = inverse of measure" is a tautology once one commits to an integrated-vs-differential split. The modeling hypothesis is that the RS "tick" dynamics uses exactly this split (active vs passive edges), which is already part of the current lepton-chain pipeline and is labeled as hypothesis in the mass papers.

The $\mu \to \tau$ step extends the same already-used pattern to a *discrete* normalization (see Sec. **??**).

3 Reply to Failure 2: "Even granting the rule, the mechanism is not unique"

`respone1.tex` gives a correct arithmetic identity:

$$\frac{F}{V_{\text{face}}} = \frac{6}{4} = \frac{12}{8} = \frac{E}{V_{\text{cube}}}.$$

The point of this section is to show that E/V_{cube} is *not in the admissible mechanism class* once the mechanism is made precise.

3.1 A precise mechanism class and rule

We adopt the exact criterion requested in `respone1.tex`:

Definition 1 (Admissible mechanism class $\mathcal{M}$ (local cellwise normalization))**.** *Fix the 3-cube ledger with its cell structure. For each cell-dimension $k \in \{0, 1, 2, 3\}$ define a mechanism M_k:*

- *The **mediators** are the set of k-cells of the cube.*
- *The **anchors** of a mediator are its **0-cells (vertices)**.*
- *The **local normalization rule** assigns uniform weight $1/|\text{Anchors}(m)|$ per mediator m.*

Let $\mathcal{M} := \{M_k : k = 0, 1, 2, 3\}$.

Definition 2 (Rule $g : \mathcal{M} \to \mathbb{R}$)**.** *Define*

$$g(M_k) \ := \sum_{m \in \text{Mediators}(M_k)} \frac{1}{|\text{Anchors}(m)|}. \tag{2}$$

Because all k-cells in the cube are isomorphic, $|\text{Anchors}(m)|$ is constant over mediators, so

$$g(M_k) \ = \ \frac{\#(k\text{-cells})}{\#(\text{vertices in a } k\text{-cell})}. \tag{3}$$

Key point. The rule is *local*: the denominator is per mediator, not a global count from a different cell dimension. This is the discrete analog of using $\Omega = 4\pi$ as the local solid-angle measure around a point, not "(total solid angle)×(number of points)".

3.2 Injectivity/uniqueness inside $\mathcal{M}$

For the 3-cube:

$$\#0\text{-cells} = 8, \quad \#1\text{-cells} = 12, \quad \#2\text{-cells} = 6, \quad \#3\text{-cells} = 1,$$

and a single k-cell has respectively

$$\#(\text{vertices in a 0-cell}) = 1,\ \#(\text{vertices in a 1-cell}) = 2,\ \#(\text{vertices in a 2-cell}) = 4,\ \#(\text{vertices in a 3-cell}) = 8.$$

Therefore Eq. (**??**) gives

$$g(M_0) = \frac{8}{1} = 8, \qquad g(M_1) = \frac{12}{2} = 6, \qquad g(M_2) = \frac{6}{4} = \frac{3}{2}, \qquad g(M_3) = \frac{1}{8}. \tag{4}$$

Theorem 1 (Injectivity of g on $\mathcal{M}$; uniqueness of the face value)**.** *The map $g : \mathcal{M} \to \mathbb{R}$ is injective, and in particular the value $3/2$ occurs* only *for $k = 2$ (face mediation).*

Proof. By Eq. (**??**), the four values are pairwise distinct real numbers. Hence g is injective on the four-element set $\mathcal{M}$. The claim "only faces give 3/2" is the $k = 2$ instance. This is formalized in Lean as `localCoeff_eq_three_halves_iff`. □

3.3 Why the counterexample E/V_{cube} is excluded

The expression $E/V_{\text{cube}} = 12/8$ is *cross-level*: it divides a **1-cell count** by a **3-cell anchor count**. It is not of the admissible form Eq. (**??**). It corresponds to a different rule:

$$\text{(global-normalized)} \quad \frac{\#(\text{edges})}{\#(\text{cube vertices})},$$

which is not local per mediator and is not the rule already used in the $e \to \mu$ step.

4 Reply to Failure 3: "The integer 4 is ambiguous"

It is true that a square has 4 vertices and 4 edges. The ambiguity disappears once "anchor" is defined: **anchors are 0-cells (vertices), not 1-cells (edges)**.

This is not a numerical choice; it is a type-level choice. In a discrete ledger, the lattice points are vertices; edges are relations/transitions between them. Therefore the correct discrete measure for distributing a face contribution is the number of vertex anchors of that face.

Remark 2 (Why this is not mere renaming)*.* If one attempts to replace "vertex anchors" by "edge anchors," one is changing the object being normalized over: states vs transitions. In $D = 3$ the two counts happen to coincide for a single square, but they do not coincide for higher-dimensional faces, and they behave differently under operations like taking boundaries and products.

5 Checklist from `respone1.tex` (what is now satisfied)

We can now match the "non-negotiable checklist" items explicitly:

1. **Formal definitions of edge/face mediated.** Implemented as the finite type `CellDim` and the mechanisms M_k above.

2. **A theorem that the tau step must be face-mediated.** This is a modeling claim: the $\mu \to \tau$ correction uses $W = 17$ wallpaper groups, a 2D constant, so the mediator must be 2D. In the cube ledger, the 2D elements are faces. (This is the same semantic force that makes W meaningful at all.)

3. **A theorem that normalization is vertex count.** Built into the mechanism definition: anchors are 0-cells.

4. **Injectivity theorem.** Proven above; formalized in Lean as `localCoeff_eq_three_halves_iff`.

5. **Empirical falsifier.** If one replaces face mediation ($k = 2$) by edge mediation ($k = 1$) in the local class, the coefficient changes from 3/2 to 6 (a factor 4), changing the predicted step by $(6 - 3/2)\alpha \approx 4.5\alpha \approx 0.0328$ in the exponent, which is far outside the stated tolerance bands in the lepton table. This would force the introduction of new tuning knobs, violating the paper's contract.

6 What remains open (and how we should proceed)

`respone1.tex` is also correct that resolving the $\mu \to \tau$ coefficient alone does not, by itself, establish a full fermion mass law. The same explicit-mechanism + admissible-class + uniqueness pattern must be applied to the remaining hypothesis terms (e.g. δ_e, the $e \to \mu$ step corrections, and quark-sector rung resolution).

This note closes one specific loophole: the claim that E/V_{cube} shows non-injectivity *inside* the natural local mechanism class.

Lean artifacts. The key formal theorems live in:

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IndisputableMonolith/Physics/LeptonGenerations/FractionalStepDerivation.lean
IndisputableMonolith/Physics/LeptonGenerations/TauStepDeltaDerivation.lean
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