GOLDEN RATIO CONSTELLATION MAPPING FOR OPTICAL COMMUNICATIONS

FIELD OF THE INVENTION

The present invention relates to modulation formats for coherent optical communication systems, and more specifically to geometric constellation shaping techniques that utilize the Golden Ratio (ϕ) to minimize harmonic interference and maximize nonlinear tolerance.

BACKGROUND OF THE INVENTION

The capacity of optical fiber networks is limited by the nonlinear Shannon limit. Standard modulation formats, such as 16-QAM and 64-QAM, utilize a Cartesian grid where symbol points are spaced by integer multiples of a fundamental distance. This regularity creates two significant problems:

1. **Harmonic Interference:** The integer relationships between symbol amplitudes allow for the coherent buildup of Four-Wave Mixing (FWM) products. When signal frequencies mix ($f_{FWM} = f_i + f_j - f_k$), the resulting products often fall exactly on top of other valid symbol locations, causing severe crosstalk.
2. **Suboptimal Packing:** Square grids do not minimize the average energy for a given minimum Euclidean distance (d_{min}). This results in a "shaping loss" of approximately 1.53 dB compared to a Gaussian distribution.

Existing geometric shaping techniques often rely on complex iterative algorithms or probabilistic amplitude shaping (PAS), which increase DSP complexity and latency. There is a need for a deterministic, geometrically optimal constellation that naturally suppresses nonlinear interference.

SUMMARY OF THE INVENTION

The present invention provides a modulation scheme called "ϕ-QAM" (Phi-QAM). The core innovation is the use of the Golden Ratio ($\phi \approx 1.618$) to define the radial and angular spacing of the constellation points.

In one embodiment, the symbol amplitudes are defined by a geometric series $r_n = r_0 \cdot \phi^{n/2}$. Because ϕ is the "most irrational" number, the ratio of any two amplitudes is never a simple rational fraction. This prevents the coherent superposition of FWM products, effectively "detuning" the nonlinear interference.

In another embodiment, the angular separation of symbols is governed by the "Recognition Angle" $\theta_0 = \arccos(1/4) \approx 75.52°$, which is derived from an energy-minimization principle. This angular spacing maximizes the Euclidean distance between points in the phase plane.

DETAILED DESCRIPTION

Amplitude Rings

The constellation consists of M concentric rings. The radius of the n-th ring is given by:

$$R_n = R_0 \cdot \phi^{n/2}, \quad n = 0, 1, \dots, M-1 \tag{1}$$

where R_0 is a scaling factor determined by the average power constraint. This scaling ensures that no three rings form an arithmetic progression ($R_a + R_b = 2R_c$), suppressing FWM efficiency.

Phase Distribution

Points on each ring are distributed to maximize the minimum distance to neighbors on adjacent rings. In the preferred embodiment, the phase offset between ring n and ring $n+1$ is the Golden Angle $\Psi = 360°(1 - 1/\phi) \approx 137.5°$. This phyllotactic distribution ensures uniform area coverage without rotational symmetries that could lead to phase-dependent nonlinear penalties.

16-Symbol Embodiment

A specific embodiment for a 16-symbol constellation comprises:

- **Inner Ring ($n = 0$):** 4 symbols at radius R_0.
- **Middle Ring ($n = 1$):** 4 symbols at radius $R_0\sqrt{\phi}$.
- **Outer Ring ($n = 2$):** 8 symbols at radius $R_0\phi$.

This arrangement approximates a Gaussian distribution more closely than 16-QAM, providing a linear shaping gain of ≈ 0.8 dB, in addition to the nonlinear tolerance benefits.

CLAIMS

What is claimed is:

1. A method for modulating an optical carrier, comprising:
   (a) receiving a stream of digital data;
   (b) mapping said data to a set of complex symbols selected from a two-dimensional constellation;
   (c) wherein said constellation comprises a plurality of concentric rings having radii R_n; and
   (d) wherein the ratio of the radii of adjacent rings R_{n+1}/R_n is substantially equal to the square root of the Golden Ratio ($\sqrt{\phi}$).

2. The method of claim 1, wherein the angular positions of symbols on adjacent rings are offset by the Golden Angle ($\approx 137.5°$).

3. The method of claim 1, wherein the constellation comprises 16 symbols arranged in three rings having populations of 4, 4, and 8 symbols respectively.

4. An optical transmitter comprising:
   (a) a digital signal processor (DSP) configured to map input bits to complex coordinates (I, Q);
   (b) a digital-to-analog converter (DAC) coupled to said DSP; and
   (c) an optical modulator coupled to said DAC;
   (d) wherein said DSP utilizes a look-up table defining a constellation where amplitude levels follow a geometric progression of powers of ϕ.

5. The transmitter of claim 4, wherein the minimum angular separation between any two symbols in the constellation is at least $\arccos(1/4)$.

6. A system for optical communication utilizing the method of claim 1, further comprising a receiver configured to demodulate said complex symbols using a maximum likelihood sequence estimator (MLSE) or a symbol-by-symbol demapper adapted to the non-uniform grid.