Notes on “Single Anchor Phenomenology of SM running masses”

A. Thapa

1 Summary of the manuscript:

The manuscript look into nine charged fermions (six quarks and 3 charged leptons) in the $\overline{\text{MS}}$ scheme why not neutrinos and bosons?. Instead of quoting each mass on different scales, (e.g. $m_b(m_b)$, $m_s(2\text{ GeV})$, lepton pole masses), the paper tries to impose a single common reference scale $\mu_\star$ for all the species by defining the following equations:

$$f_i(\mu_\star, m_i) = \frac{1}{\lambda}\int_{\ln\mu_\star}^{\ln m_i} \gamma_i(\mu) d\ln\mu\,, \tag{1}$$

where i represents the species of fermion, $\gamma_i(\mu)$ is its mass anomalous dimension that includes QCD up to four loops and QED up to two loops. One can write the γ_i as follows:

$$\gamma_i(\mu) = \gamma_m^{\text{QCD}}(\alpha_s(\mu), n_f(\mu)) + \gamma_m^{QED}(\alpha(\mu), Q_i) \tag{2}$$

where

$$\gamma_m^{\text{QCD}} = \sum_{k=0}^{3} \gamma_k^{\text{QCD}}(n_f)\left(\frac{\alpha_s}{4\pi}\right)^{k+1}, \tag{3}$$

$$\gamma_m^{\text{QED}} = \sum_{k=0}^{1}\left[A^k Q_i^2 + B^k Q_i^4\right]\left(\frac{\alpha_s}{4\pi}\right)^{k+1}. \tag{4}$$

Here Q_i are the charges of fermions.

The main claim is that the $\mu_\star = 182.201$ GeV, the above equation (1) match with the form:

$$f_i(\mu_\star, m_i) \approx \mathcal{F}(Z_i) = \frac{1}{\lambda}\ln\left(1 + \frac{Z_i}{\kappa}\right) \tag{5}$$

where Z_i an integer built only from the fermion's electric charge and whether it's a quark or lepton and $(\lambda, \kappa) = (\ln\varphi, \varphi)$ with $\varphi = (1+\sqrt{5})/2$ (golden ratio). $\mu_\star$ is chosen by PMS/BLM-like stationarity condition on species-independent kernels in a mass-free calibration window. The main consequences is equal Z families (up-type quark, down-type quarks, and charged leptons) are degenerate. The integer index is built as follows. Define an integerized charge $\tilde{Q} = 6Q_i$, so that all Standard-Model charges ($Q_i = \pm1, \pm2/3, \pm1/3$) map to integers. Then

$$Z_i = \delta_{ic} Z_{\text{QCD}} + Z_{\text{EW}}(Q_i) = \begin{cases} 4 + \tilde{Q}^2 + \tilde{Q}^4, & \text{quarks,} \\ \tilde{Q}^2 + \tilde{Q}^4, & \text{charged leptons,} \\ 0, & \text{Dirac neutrinos.} \end{cases} \tag{6}$$

What about antifermions. Here $Z_{\rm QCD} = 4$ for color triplets (quarks), 0 for leptons, and $Z_{\rm EW}(Q_i) = (6Q_i)^2 + (6Q_i)^4$. This leads to three equal-Z families:

$$Z_{u,c,t} = 4 + 4^2 + 4^4 = 276, \tag{7}$$

$$Z_{d,s,b} = 4 + 2^2 + 2^4 = 24, \tag{8}$$

$$Z_{e,\mu,\tau} = 6^2 + 6^4 = 1332. \tag{9}$$

At the anchor, the residues of all members of a given family are degenerate within the stated tolerance.

Instead of actually calculating the loop functions and evaluating the beta functions to calculate $\gamma_i(\mu)$, what it does is calculate the following;

$$\gamma_i(\mu) = \sum_k \kappa_k(\mu)\, N_k(i), \tag{10}$$

where the $\kappa_k(\mu)$ are common kernel functions ($k \in \mathcal{K}$, where $\mathcal{K} = F, NA, V, G, Q2, Q4.$) and the $N_k(i)$ are non-negative integers depending on the fermion's charge and color quantum numbers recipe to evaluate $\kappa_k(\mu)$ and $N_k(i)$?. The motifs group together QCD self-energy, nonabelian, vacuum polarization, and quartic-gluon classes, as well as abelian Q^2 and Q^4 structures.

Integrated motif weights,

$$w_k(\mu;\lambda) = \frac{1}{\lambda}\int \kappa_k(\mu')\, d\ln\mu', \tag{11}$$

are adjusted by the PMS/BLM procedure so that, at the anchor, $w_k(\mu_\star;\lambda) \approx 1$ for all k I still need to understand this.. In this limit the residue reduces to an almost pure integer count,

$$f_i(\mu_\star, m_i) = \sum_k w_k N_k(i) \approx \sum_k N_k(i) = Z_i. \tag{12}$$

The closed form $\mathcal{F}(Z_i)$ is then interpreted as a convenient normalization of this integer structure.

Z_i which is basically Eq (6) and Eqs(7)-(9) is not equal to $\mathcal{F}(Z_i)$?

1.1 What we need to improve the paper?

- Basically evaluate Eq.(10).
  - Show explicitly the known 1-loop and 2-loop anomalous dimension in terms of C_F, C_A, $T_F n_f$, Q^2, Q^4.
  - Group terms into the motifs F, NA, V, G, Q2, Q4.
  - Show the explicit integer counts N_k for a quark vs a lepton.
- Provide an order-of-magnitude estimate for unknown higher loops at $\mu_\star$, and compare to the claim of 10^{-6} accuracy. Or, show that higher order are small change.
- Show explicitly what we get for different choice of $\mu_\star$? For instance is $\mu_* = 182$ or 183 not valid?

2 Drawbacks of the paper and need serious attentions.

2.1 The solution is not RG invariant

Proof: We define for each species

$$\Delta_i(\mu) \equiv f_i(\mu, m_i) - F(Z_i) \tag{13}$$

The phenomenology identity implies

$$\Delta_i(\mu_\star) \simeq 0 \text{ for all 9 charged fermions.} \tag{14}$$

Now what I am asking is if this is compatible with RG evolution. We know from RG that:

$$f_i(\mu, m_i) = \frac{1}{\lambda} \int_{\ln m_i}^{\ln \mu} \gamma_i(\mu')\, d\ln\mu'. \tag{15}$$

Differentiate with respect to $\ln\mu$:

$$\frac{\partial f_i}{\partial \ln\mu} = -\frac{1}{\lambda}\,\gamma_i(\mu). \tag{16}$$

Since $F(Z_i)$ is a constant (no μ–dependence), we can write

$$\frac{\partial}{\partial \ln\mu}\Delta_i(\mu) = \frac{\partial f_i}{\partial \ln\mu} = -\frac{1}{\lambda}\,\gamma_i(\mu). \tag{17}$$

So in a small deviation from $\mu_\star$:

$$\Delta_i(\mu) = \Delta_i(\mu_\star) - \frac{1}{\lambda}\int_{\ln\mu}^{\ln\mu_\star} \gamma_i(\mu')\, d\ln\mu'. \tag{18}$$

Imposing $\Delta_i(\mu_\star) = 0$, we obtain

$$\Delta_i(\mu) = -\frac{1}{\lambda}\int_{\ln\mu}^{\ln\mu_\star} \gamma_i(\mu')\, d\ln\mu'. \tag{19}$$

Thus the identity to be RG invariant (radiatively stable), we need $\Delta_i(\mu) = 0$ for all μ, not just at $\mu_\star$. This requires

$$\int_{\ln\mu_\star}^{\ln\mu} \gamma_i(\mu')\, d\ln\mu' = 0 \quad \forall\,\mu,\ \forall\, i, \tag{20}$$

which is equivalent to

$$\gamma_i(\mu) = 0 \quad \forall\,\mu,\ \forall\, i. \tag{21}$$

This is absurd: we know $\gamma_i \neq 0$ in QCD/QED. So the identity cannot hold for more than one scale μ unless the anomalous dimensions vanish.

• Thus $f_i(\mu_\star) = F(Z_i)$ is not a fixed point under RG, it is a tuned point.

This is the textbook definition of something that is not radiatively stable: an equality that holds at a single renormalization scale but is destroyed by RG flow as soon as you move away. We can estimate the breaking:

For a small deviation from the anchor scale, write

$$\mu = \mu_\star(1+\epsilon), \qquad |\epsilon| \ll 1.$$

$$\Delta_i(\mu) \simeq -\frac{1}{\lambda}\gamma_i(\mu_\star)\ln(1+\epsilon) \simeq -\frac{1}{\lambda}\gamma_i(\mu_\star)\,\epsilon. \tag{22}$$

Since $\gamma_i(\mu_\star)$ is $\mathcal{O}(10^{-2})$–$\mathcal{O}(10^{-1})$ for quarks, a modest 10% shift in scale ($\epsilon \sim 0.1$) already generates

$$|\Delta_i| \sim 10^{-3}\text{–}10^{-2}.$$

There is therefore no mechanism to keep Δ_i small for all scales.

So the relation is explicitly broken by RGE away from the anchor, unless you impose it anew at each scale (which would amount to continually re-tuning, not a symmetry).

2.2 Flavor universality violation

In the SM:

- Gauge interactions are flavor universal. What that means is within a given representation (e.g. up-type quarks), the gauge couplings are identical across generations.
- Flavor breaking comes from Yukawa matrices, not from gauge couplings.

The mass anomalous dimension in the full SM schematically has

$$\gamma_i(\mu) = \gamma_m^{\text{QCD}}\big(\alpha_s(\mu), n_f(\mu)\big) + \gamma_m^{\text{QED}}\big(\alpha(\mu), Q_i\big) + \gamma_m^{\text{Yukawa}}\big(\{y_f(\mu)\}\big). \tag{23}$$

The paper explicitly drops the Yukawa terms, using only the QCD+QED pieces:

$$\gamma_i(\mu) = \gamma_m^{\text{QCD}}(\alpha_s, n_f) + \gamma_m^{\text{QED}}(\alpha, Q_i). \tag{24}$$

But in the real SM, for up-type quarks, the top Yukawa contribution to γ_t is order 1, while for u, c it is negligible. That means:

$$\gamma_t^{\text{(full)}}(\mu) \gg \gamma_u^{\text{(full)}}(\mu),\ \gamma_c^{\text{(full)}}(\mu), \qquad \text{for } \mu \sim m_t, \ldots, \mu_\star. \tag{25}$$

Now repeat the RG-invariance check with the full γ_i:

$$\frac{\partial}{\partial \ln \mu}\Delta_t(\mu) = -\frac{1}{\lambda}\,\gamma_t^{\text{(full)}}(\mu), \tag{26}$$

$$\frac{\partial}{\partial \ln \mu}\Delta_u(\mu) = -\frac{1}{\lambda}\,\gamma_u^{\text{(full)}}(\mu), \tag{27}$$

with $\gamma_t^{\text{(full)}} \neq \gamma_u^{\text{(full)}}$ even in the same gauge sector.

Therefore, even if you force

$$\Delta_u(\mu_\star) = \Delta_c(\mu_\star) = \Delta_t(\mu_\star) = 0 \tag{28}$$

at a single scale, their derivatives with respect to $\ln \mu$ differ. So as soon as you move away from $\mu_\star$:

$$\Delta_t(\mu) - \Delta_u(\mu) \approx -\frac{1}{\lambda}\int_{\ln \mu}^{\ln \mu_\star} \big[\gamma_t^{\text{(full)}}(\mu') - \gamma_u^{\text{(full)}}(\mu')\big]\, d\ln \mu' \neq 0. \tag{29}$$

• If you include Yukawas, up-type "equal-residue" degeneracy is immediately broken by RGE, because γ_t and γ_u, γ_c are not the same function of scale. There is no flavor symmetry in the full SM that could enforce (or protect) equality of their integrated anomalous dimensions.

2.3 Scheme dependence?

In Appendix B, the manuscript says that under scheme/threshold changes, the motif weights w_k shift as

$$\delta w_k = \delta(\ln \mu_\star)\,\frac{\partial w_k}{\partial \ln \mu_\star} + \delta\lambda\,\frac{\partial w_k}{\partial \lambda} + \Delta_k, \tag{30}$$

and the induced change in f_i is bounded by

$$|\delta f_i| \;\le\; \frac{N_{\text{tot}}(i)}{\lambda\,\kappa}\,\|\rho\|_\infty, \tag{31}$$

where ρ_k is the residual profile.

Then the claim is that equal-Z families move "coherently" under admissible scheme changes, but nothing in these bounds guarantees $\delta f_i = 0$, only that it is "small" for small changes. And "small" here is defined relative to the chosen kernels — not relative to the 10^{-6} tolerance.

So with in the formalism, the identity

$$f_i(\mu_\star, m_i) = F(Z_i) \tag{32}$$

is not scheme invariant; it shifts as we change $\overline{\text{MS}}$ variants, thresholds, or loop order. That is another sign it is not a flavor-universal, symmetry-protected relation of the SM, but a scheme- and setup-dependent numerical pattern.

2.4 Other minor comments:

1. Need to include PDG uncertainties in the masses and couplings. In fact they are quite large.
2. need to discuss threshold matching
3. need to make sure higher order correction is not large.
4. Need to ensure the running is perturbative all the way through.
5. They emphasize that $\mu_\star$ and λ are fixed by a mass-free PMS/BLM procedure and that the nine fermion masses only appear on the left-hand side of their relation, but in practice the PMS/BLM optimization depends on threshold choices (m_c, m_b, m_t), on $\alpha_s(M_Z)$, and on the running of $\alpha_s(\mu)$, which already encodes information about the mass spectrum and decoupling. Thus the separation between "no masses on the RHS" and "masses only on the LHS" is not as clean as stated: threshold choices are themselves mass data. This does not necessarily make the construction circular, but it does weaken the claim that the pattern is purely "out-of-sample" and independent of the actual spectrum."'

Currently checking if the proof in the manuscript is circular by understanding PMS/BLM procedure? I need to read the original papers and see how the method PMS/BLM is implemented here. Requires more time.

3 More serious problems:

Let us first understand the function f_i. One can start with the equation

$$\mu \frac{d \ln m_i(\mu)}{d\mu} = \gamma_i(\mu) \qquad \Leftrightarrow \frac{dm_i(\mu)}{d \ln \mu} = \gamma_i(\mu) m_i(\mu) \tag{33}$$

This is standard RGE equation. Here γ_i is the beta functions and captures the loop. The paper then defines the beta function as follows:

$$\gamma_i(\mu) = \gamma_m^{\text{QCD}}(\alpha_s(\mu), n_f(\mu)) + \gamma_m^{QED}(\alpha(\mu), Q_i) \tag{34}$$

• This is only true at the leading order. But once you go to higher order they are not linear anymore. More importantly, one might need to include Yukawa terms as well once the scale goes above M_Z and M_W. More on this later.

For now lets ignore it and say Eq. (4) is valid.

$$\mu \frac{d \ln m_i(\mu)}{d\mu} = \gamma_i(\mu) \tag{35}$$

Switch variables to $t = \ln \mu$, so $dt = d \ln \mu$ and $\mu\, d/d_\mu = d/dt$

$$\frac{d \ln m_i}{dt} = \gamma_i\left(e^t\right) \tag{36}$$

Integrating both sides from $\mu = \mu_\star$ to $\mu = m_i^{\text{PDG}}(m_i)$:

$$\int_{\ln \mu_\star}^{\ln m_i} dt \frac{d \ln m_i}{dt} = \int_{\ln \mu_\star}^{\ln m_i} \gamma_i(\mu) d \ln \mu. \tag{37}$$

This leads to

$$\ln \frac{m_i\left(m_i\right)}{m_i\left(\mu_\star\right)} = \int_{\ln \mu_\star}^{\ln m_i} \gamma_i(\mu) d \ln \mu \tag{38}$$

This integral is the total ”RG log” accumulated when one run from $\mu_\star$ down to the scale where the mass is quoted $\mu = m_i$.

Now with the definition of Eq.(1) it leads to

$$\boxed{f_i^{(\exp)} = \frac{1}{\lambda} \ln \frac{m_i(m_i)}{m_i(\mu_\star)}} \tag{39}$$

• Another important point is while running from $\mu_\star$ down to m_i one has to include threshold corrections at different scales.

3.1 Why the RG-integrated residue $f_i^{\exp}$ obtained from β functions is small, and cannot match large “anchor band” numbers?

The paper claims

$$\boxed{f_i(\mu_\star, m_i) = \mathcal{F}(Z_i) = \frac{1}{\lambda} \ln\left(1 + \frac{Z_i}{\kappa}\right)} \tag{40}$$

This equation gives $f_u = f_c = f_t$, $f_d = f_s = f_b$ and $f_e = f_\mu = f_\tau$.

Using Eq. (39) we have the following demands the ratio of masses to be equal. However we know

$$\frac{m_u(m_u)}{m_u(\mu_\star)} \neq \frac{m_c(m_c)}{m_c(\mu_\star)} \neq \frac{m_t(m_t)}{m_t(\mu_\star)}\,. \tag{41}$$

This is the direct contradiction (there is no symmetry in the theory to enforce their equality), thus the claim in the paper needs to be modified.

Simple example:

Let me show now by taking one simple example with 1-loop QCD running with $n_f = 5$ to get the feel (if this is not true at one loop, it won't be true at higher order, as it will be just tiny correction to 1 loop leading terms.). The one loop $\alpha_s(\mu)$ in $\overline{\mathrm{MS}}$ read as

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(M_Z)} + \frac{\beta_0}{2\pi} \ln \frac{\mu}{M_Z}, \tag{42}$$

where $\beta_0 = 11 - 2/3n_f$. For the mass at 1-loop we have

$$m(\mu) \propto [\alpha_s(\mu)]^{\frac{12}{33-2n_f}} \tag{43}$$

This leads to the following equation

$$\ln \frac{m_i(\mu_1)}{m_i(\mu_2)} \approx \frac{12}{33-2n_f} \ln \frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)}. \tag{44}$$

Now lets plug in some numbers. Let's look at up-type quarks with scales:

- $\mu_\star = 182.201\mathrm{GeV}$ (anchor),
- $m_c(m_c) \approx 1.27\mathrm{GeV}$,
- $m_b(m_b) \approx 4.18\mathrm{GeV}$,
- $m_t(m_t) \approx 162.5\mathrm{GeV}$.
- $\alpha_s(M_Z) = 0.1181$

Notice how I am using $\approx$ as for the masses have uncertainty. For the light-quark I used $\mu = 2GeV$ (below this scale the theory is non perturbative). Using 1-loop QCD, I find approximately:

$$\alpha_s(182.201\,\mathrm{GeV}) \simeq 0.10739, \tag{45}$$
$$\alpha_s(162.5\,\mathrm{GeV}) \simeq 0.10902, \tag{46}$$
$$\alpha_s(4.18\,\mathrm{GeV}) \simeq 0.21249, \tag{47}$$
$$\alpha_s(2.0\,\mathrm{GeV}) \simeq 0.26270, \tag{48}$$
$$\alpha_s(1.27\,\mathrm{GeV}) \simeq 0.30746, \tag{49}$$

and therefore from Eq. (44) the accumulated RG logs are

$$\ln \frac{m_u(2\,\mathrm{GeV})}{m_u(\mu_\star)} \simeq 0.46674, \tag{50}$$
$$\ln \frac{m_c(m_c)}{m_c(\mu_\star)} \simeq 0.54882, \tag{51}$$
$$\ln \frac{m_b(m_b)}{m_b(\mu_\star)} \simeq 0.35606, \tag{52}$$
$$\ln \frac{m_t(m_t)}{m_t(\mu_\star)} \simeq 0.007882. \tag{53}$$

If we now use the manuscript's display choice $\lambda = \ln \varphi$ (see below), then

$$f_i^{(\exp)} \simeq \frac{1}{\ln \varphi} \ln \frac{m_i(m_i)}{m_i(\mu_\star)}. \tag{54}$$

With $\varphi = (1+\sqrt{5})/2$ and $\ln\varphi \simeq 0.4812118251$, this yields

$$f_u^{(\exp)} \simeq 0.9699, \tag{55}$$

$$f_c^{(\exp)} \simeq 1.1405, \tag{56}$$

$$f_b^{(\exp)} \simeq 0.7399, \tag{57}$$

$$f_t^{(\exp)} \simeq 0.01638. \tag{58}$$

This clearly shows that $f_u \neq f_c \neq f_t$ and definitely doesn't agree with the Eq. (40) as Eq. (40) leads to

$$\mathcal{F}(Z_{u,c,t} = 276) \simeq 10.69\,, \qquad \mathcal{F}(Z_{d,s,b} = 24) \simeq 5.74\,, \qquad \mathcal{F}(Z_{e,\mu,\tau} = 1332) \simeq 13.95\,. \tag{59}$$

Thus the claim of Eq. (40) is false as $f_i^{\exp}$ is widely different.

For **charged leptons**, it is even smaller. At leading 1-loop QED estimate (constant α for an order-of-magnitude check) uses

$$\gamma_m^{\rm QED}(\mu) \approx -\frac{3}{4}Q^2\frac{\alpha}{\pi}, \qquad Q = -1 \text{ for } e, \mu, \tau, \tag{60}$$

so

$$f_\ell^{(\exp)} \approx \frac{1}{\ln\varphi}\left(-\frac{3}{4}\frac{\alpha}{\pi}\right)\ln\frac{m_\ell}{\mu_\star}. \tag{61}$$

Using the fine-structure constant $\alpha \simeq 1/137.035999206$ and the pole masses $m_e \simeq 0.51099895\,\text{MeV}$, $m_\mu \simeq 105.6583745\,\text{MeV}$, $m_\tau \simeq 1.77686\,\text{GeV}$ gives

$$f_e^{(\exp)} \simeq 0.0463, \qquad f_\mu^{(\exp)} \simeq 0.0270, \qquad f_\tau^{(\exp)} \simeq 0.0168, \tag{62}$$

again $\ll 1$.

3.2 Is Eq. (34) valid?

Eq. (34) read as

$$\gamma_i(\mu) = \gamma_m^{\rm QCD}(\alpha_s(\mu), n_f(\mu)) + \gamma_m^{QED}(\alpha(\mu), Q_i) \tag{63}$$

This is exact one loop, but not exact beyond higher-loop expansion in a theory where quarks carry both color and electric charge. The more complete statement is

$$\gamma_i(\mu) = \gamma_m(\alpha_s(\mu), \alpha(\mu), Q_i, n_f(\mu)) \tag{64}$$

where γ_i read as

$$\gamma_m = \sum_{n\geq 1}\gamma^{(n,0)}\left(\frac{\alpha_s}{4\pi}\right)^n + \sum_{k\geq 1}\gamma^{(0,k)}\left(\frac{\alpha}{4\pi}\right)^k + \sum_{\substack{n\geq 1\\ k\geq 1}}\gamma^{(n,k)}\left(\frac{\alpha_s}{4\pi}\right)^n\left(\frac{\alpha}{4\pi}\right)^k + \cdots \tag{65}$$

Here $(n,0)$ terms are pure QCD, $(0,k)$ pure QED, and (n,k) with $n,k \geq 1$ are mixed QCD-QED terms, (e.g., $\alpha_s\alpha$, $\alpha_s^2\alpha$, ...)

3.3 What about the electroweak gauge group? Another serious problem.

Here is the problem, since our anchor scale is $\mu_\star = 82.201$ GeV which is well above m_Z and m_W (gauge boson masses). Thus if we want a consistent description up to that scale, the correct framework is **full SM RGEs**, where QED is no longer fundamental. Instead one runs g_1 and g_2 (hypercharge + weak), plus the Yukawa (especially y_t) and the Higgs sector. Here our theory is above m_t scale. Once you include these, the theory breaks, unfortunately.

What does not having electroweak theory mean? One can imagine a that we don't have standard model. Under such theory we don't have $SU(2)_L \times U(1)_Y$, gauge fields W_μ^a, B_μ, left-handed fermions doublets, Higgs doublets H, and Yukawa interactions. This is valid description effective theory but **it cannot reproduce real-world electroweak physics** and it is not expected to be accurate at all above the weak scale above W and Z mass.

The problem now is we need to cook up a theory that describes beta decay, muon decay, neutrino scatterings, parity violation. Trying to predict any of these with QCD+QED only theory would be immediately falsified.

At energies $\geq m_W$, many observables receive electroweak corrections, e.g., sudakov logarithms $\alpha_W \ln^2(s/m_W^2)$, top Yukawa effects, and Z-exchange corrections. Thus if we claim percent level precession at 100-200 GeV without electroweak effects, experiments will disagree.

It is important understand even though the running masses are not directly observable as they are scheme dependent and scale dependent, it can relate $m_b(\mu)$ to physical quantities (cross sections, decay rates, etc) which can be compared to data. QCD+QED running is only accurate at the percent level. It doesn't mean it is falsified, just means it is incomplete.

Any Analogy: Using QCD+QED only at 182 GeV is like using **non-relativistic mechanics** at $v = 0.3c$. It is not logically inconsistent but systematically inaccurate if we want precision.

4 Resolution?

As already established, the key mismatch is that the RG-integrated residue defined in Eq. (1) or equivalently with log mass ratio in Eq. (39) which is typically $\mathcal{O}(10^{-2} - 1)$ for realistic endpoints (i.e., at $\mu_\star = 182.201$ GeV. This cannot reproduce the large "band" values of Eq. (59).

The resolution (claim! need to be tested) in one sentence is that those large band numbers of Eq. (59) are *naturally* produced by a separate closed form function $f_i^{\text{Rec}} = \mathcal{F}(Z_i)$. That is it shouldn't be identified solely with the SM transport integral of Eq. (1).

• *To test this $f^{\text{Rec}}(Z)$ against data, one must define a theory-side structural mass $m_{\text{struct}}(i)$ plus any* **universal** *shifts δ independently of the same measured masses being tested.* Otherwise the "test" becomes a tautology.

Lets recall,

$$\mathcal{F}(Z_i) = \frac{1}{\lambda} \ln\left(1 + \frac{Z_i}{\kappa}\right) = \frac{1}{\ln\varphi} \ln\left(1 + \frac{Z_i}{\varphi}\right) \tag{66}$$

where $Z_{u,c,t} = 276$, $Z_{d,s,b} = 24$, $Z_{e,\mu,\tau} = 1332$, and $\varphi = \frac{1+\sqrt{5}}{2}$.

Now lets calculate f^{Rec}. We use the following equation where f^{Rec} is defined.

$$m_i^{\text{pole}} = \underbrace{B_i\, E_{\text{coh}}\, \varphi^{r_i + r_0}}_{m_i^{\text{struct}}}\ \varphi^{f_i^{\text{Rec}} + f_i^{\text{RG}}} \tag{67}$$

where

$$f_i^{\mathbf{Rec}} = \Delta_{\mathbf{obs},i}^{\mathbf{pole}} - f_i^{\mathbf{RG}} \neq \mathcal{F}(Z_i)$$

	B_i ($\equiv B_B$)	r_i (rung)	r_0 (offset)	$m_i^{\rm struct} =$ $B_i\, E_{\rm coh}\, \varphi^{r_i+r_0}$	$\Delta_{{\rm obs},i}^{\rm pole} =$ $\frac{1}{\ln\varphi}\ln\left(\frac{m_i^{\rm pole}}{m_i^{\rm struct}}\right)$	$f_i^{\rm RG}$	$\mathrm{F}(Z_i)$
e	2^{-22}	2	62	0.510048 MeV	$+0.0038717$	0.04628	13.95318793
μ	2^{-22}	13	62	101.502 MeV	$+0.0833974$	0.02698	13.95318793
τ	2^{-22}	19	62	1.82138 GeV	$-[0.0515553, 0.0511343]$	0.01676	13.95318793
u	2^{-1}	4	35	6.37602 MeV	$-[2.38865, 2.11889]$	1.26097	10.69182862
c	2^{-1}	15	35	1.26886 GeV	$[-0.00830415, 0.0217331]$	1.58110	10.69182862
t	2^{-1}	21	35	22.7688 GeV	$[4.20138, 4.21632]$	0.01831	10.69182862
d	2^{23}	4	-5	0.467481 MeV	$[4.7333, 4.85714]$	1.26097	5.739852155
s	2^{23}	15	-5	93.0311 MeV	$[-0.0254206, 0.045708]$	1.26097	5.739852155
b	2^{23}	21	-5	1.66938 GeV	$-[1.90192, 1.91583]$	0.90492	5.739852155

Table 1: Values for $m_i^{\rm struct}$ and B_i and r_i from Eq. (67). The pole mass is just obtained from the PDG. Since the masses have error, I took 2σ error bar to calculate Δ. We don't know the pole mass for light quarks u, d, s, thus the mass at 2 GeV is taken. I took QCD up to 4 loops using RunDec.

- $E_{\rm coh} = \varphi^{-5}$ eV is the coherence energy
- $B_i \equiv B_B$ given in Table 1.
- r_i is the rung coordinate, also given in Table 1.
- r_0 is the sector-wide offset (a baseline rung/exponent) used to define a common yard stick for each sector.

$m_i^{\rm struct}$ is the RS "structural" prefactor (yardstick $\times$ rung).

• This is good as it can be directly compared to the PDG (experimental value). Though it says pole mass, we don't know pole mass for light quarks. Whatever value it give for "up" and "down", that would be the prediction. However I am realizing that pole mass cannot be determined without $f^{\rm Rec}$. Note that we cannot use $\mathcal{F}(Z)$ in Eq. (67) as we are trying we are trying to verify function $\mathcal{F}(Z)$ after all.

Eq. (67) leads to

$$\frac{m_i^{\rm pole}}{m_i^{\rm struct}} = \varphi^{f_i^{\rm Rec}+f_i^{\rm RG}}\ , \tag{68}$$

which is equivalent to writing

$$\frac{1}{\ln\varphi}\ln\left(\frac{m_i^{\rm pole}}{m_i^{\rm struct}}\right) = f_i^{\rm Rec} + f_i^{\rm RG} := \Delta_{{\rm obs},i}^{\rm pole}\ , \tag{69}$$

So the equation that is to be compared $\mathcal{F}(Z_i)$ of Eq. (66) read as

$$\boxed{f_i^{\rm Rec} = \Delta_{{\rm obs},i}^{\rm pole} - f_i^{\rm RG} \overset{?}{\simeq} \mathcal{F}(Z_i)} \tag{70}$$

We know precisely how to compute $\mathcal{F}(Z_i)$ which is invariant under family generation. $f_i^{\rm RG} \equiv f_i^{\rm exp}$ is calculated through RGE of QCD+QED and SM and is basically give by Eq. (1) or equivalently

$$f_i^{\rm RG} = \frac{1}{\ln \varphi} \ln \frac{m_i\left(m_i\right)}{m_i\left(\mu_\star\right)} \,. \tag{71}$$

What $f_i^{\rm Rec}$ is really doing? Form Eq. (69) and Eq. (71) we have

$$\boxed{f_i^{\rm Rec} = \frac{1}{\ln \varphi} \ln \frac{m_i\left(\mu_\star\right)}{m_i^{\rm struct}}} \tag{72}$$

Here the pole mass $m_i(m_i) \equiv m_i^{\rm pole}$. All that we have done going from the $f^{\rm RG}$ to $f_i^{\rm Rec}$ is changed the running of the pole mass to $m_i^{\rm struct}$. Now it has the same issues as before.

Will introducing δ helps? The answer is no unless δ is different for each fermion. Then it is tuning to get the answer we need.