# Optimization-Based Reference:
## A Cost-Theoretic Resolution of the Symbol Grounding Problem

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**Abstract**

The Symbol Grounding Problem asks how symbols can be *about* things without an external interpreter. We give a resolution inside a simple, explicit optimization semantics. Building on the reciprocal convex cost characterization of Washburn–Rahnamai Barghi (accepted, 2026), we treat reference as *ratio matching*: a symbol $s$ "means" an object $o$ when $o$ minimizes a universal mismatch penalty

$$c_{\mathcal{R}}(s,o) = J\left(\frac{\iota_{\mathcal{S}}(s)}{\iota_{\mathcal{O}}(o)}\right), \qquad J(x) = \frac{1}{2}(x + x^{-1}) - 1 = \cosh(\log x) - 1.$$

We then define a *symbol* as a meaning-bearing configuration that strictly *compresses* its referent: $J(\iota_{\mathcal{S}}(s)) < J(\iota_{\mathcal{O}}(o))$. Under mild attainment hypotheses, meanings exist; for finite dictionaries, decision boundaries occur at geometric means and meanings are stable away from those boundaries. Most importantly, we prove a simple forcing principle: every nonzero-cost object admits a strictly cheaper mediating code given by a geometric mean, so in any system that minimizes total cost, grounded symbols are not conventions but thermodynamic necessities. This provides a clean reference-theoretic foundation for certificate-based semantics such as the Universal Light Language (ULL).

# 1 Introduction

The Symbol Grounding Problem (SGP) [1] highlights an apparent regress: purely formal symbols are defined only in terms of other symbols, so where does meaning "touch the world"? Many responses appeal to extra ingredients (intentions, communities, embodiment), but those ingredients typically reintroduce an interpreter-like primitive.

This paper isolates a different missing ingredient: **an intrinsic cost functional**. If configurations carry a physically meaningful cost and system dynamics are cost-minimizing, then semantic relations can be defined *internally* as minimizers of a fixed objective. This aligns with the "Cost-First" paradigm of Recognition Science [4], where ontology is derived from the requirement that existence must have finite cost.

**What is proved.** We work in an explicitly stated model:

- objects and symbols each come with a positive *scale map* (a "size/complexity" in common currency),
- mismatch cost is a universal function of the scale ratio,

- meaning is *argmin* of mismatch cost, and
- "being a symbol" additionally requires *compression* (lower intrinsic cost than the referent).

Within this model we prove existence and stability results, and we show a forcing mechanism that removes the homunculus: grounded symbols arise because they reduce total cost.

# 2 The canonical reciprocal mismatch cost

We take as input a mismatch penalty $J : (0, \infty) \to [0, \infty)$ satisfying inversion symmetry, strict convexity, normalization at 1, and a multiplicative d'Alembert identity (see [2] for precise axioms).

**Theorem 2.1** (Characterization of reciprocal convex mismatch costs [2, Appendix A]). *Under the hypotheses of [2], there exists $a > 0$ such that*

$$J(x) = \cosh(a \log x) - 1 = \tfrac{1}{2}(x^a + x^{-a}) - 1, \qquad x > 0.$$

*Moreover, the parameter $a$ can be absorbed into the choice of scale maps (raising scales to the power $a$), so one may normalize to $a = 1$ without loss of generality.*

Throughout we use this normalized form:

$$J(x) = \frac{1}{2}(x + x^{-1}) - 1 = \cosh(\log x) - 1, \qquad x > 0. \tag{1}$$

# 3 Costed spaces, reference, and meaning

**Definition 3.1** (Costed spaces). A *costed space* is a pair $(C, \iota)$ where $C$ is a set and $\iota : C \to \mathbb{R}_{>0}$ is a scale map. The induced intrinsic cost is

$$J_C(c) := J(\iota(c)) \qquad (c \in C),$$

where $J$ is the canonical mismatch cost (1).

We write $\mathcal{S}$ for a symbol space with scale map $\iota_{\mathcal{S}} : \mathcal{S} \to \mathbb{R}_{>0}$, and $\mathcal{O}$ for an object space with scale map $\iota_{\mathcal{O}} : \mathcal{O} \to \mathbb{R}_{>0}$.

**Definition 3.2** (Ratio-induced reference cost). The *reference cost* of using $s \in \mathcal{S}$ to refer to $o \in \mathcal{O}$ is

$$c_{\mathcal{R}}(s, o) := J\left(\frac{\iota_{\mathcal{S}}(s)}{\iota_{\mathcal{O}}(o)}\right). \tag{2}$$

**Definition 3.3** (Meaning as minimization). The *meaning set* of a symbol $s \in \mathcal{S}$ is

$$\mathrm{Mean}(s) := \arg\min_{o \in \mathcal{O}} \; c_{\mathcal{R}}(s, o),$$

which may be empty if the minimum is not attained.

**Lemma 3.4** (Log-distance reduction). *Fix $s \in \mathcal{S}$. Minimizers of $o \mapsto c_{\mathcal{R}}(s, o)$ coincide with minimizers of the log-distance*

$$d_{\log}(s, o) := \big|\log \iota_{\mathcal{S}}(s) - \log \iota_{\mathcal{O}}(o)\big|.$$

*Proof.* Write $u(o) := \log(\iota_{\mathcal{S}}(s)/\iota_{\mathcal{O}}(o))$. Then $c_{\mathcal{R}}(s,o) = \cosh(u(o)) - 1$, and $\cosh(t) - 1$ is strictly increasing in $|t|$ for $t \in \mathbb{R}$. Therefore minimizing $c_{\mathcal{R}}(s,o)$ is equivalent to minimizing $|u(o)| = d_{\log}(s,o)$. $\square$

**Proposition 3.5** (Existence of meanings under attainment)**.** *Assume $\mathcal{O}$ is compact and $\iota_{\mathcal{O}} : \mathcal{O} \to \mathbb{R}_{>0}$ is continuous. Then for every $s \in \mathcal{S}$,* Mean$(s)$ *is nonempty.*

*Proof.* By continuity of $\iota_{\mathcal{O}}$, the map $o \mapsto \iota_{\mathcal{S}}(s)/\iota_{\mathcal{O}}(o)$ is continuous into $\mathbb{R}_{>0}$, and $J$ is continuous on $\mathbb{R}_{>0}$, so $o \mapsto c_{\mathcal{R}}(s,o)$ is continuous on compact $\mathcal{O}$ and hence attains its minimum. $\square$

# 4 Symbols as cost-minimizing compression

Meaning alone does not make something a symbol (a perfect duplicate of an object may "mean" it but does not compress it). We therefore add a compression predicate.

**Definition 4.1** (Symbol predicate)**.** A configuration $s \in \mathcal{S}$ is a *symbol for* $o \in \mathcal{O}$ if:

1. **Reference:** $o \in \text{Mean}(s)$;
2. **Compression:** $J_{\mathcal{S}}(s) < J_{\mathcal{O}}(o)$, i.e. $J(\iota_{\mathcal{S}}(s)) < J(\iota_{\mathcal{O}}(o))$.

**Remark 4.2** (Grounding in this model)**.** In this framework, the SGP is resolved by *eliminating interpretive primitives* from the definition of symbolhood. "Aboutness" becomes an optimization statement (argmin), and "symbol" becomes a strict inequality of intrinsic costs.

# 5 Finite dictionaries: geometric-mean boundaries and stability

For a finite object dictionary, the meaning rule produces an explicit decision geometry in scale space.

**Proposition 5.1** (Geometric-mean boundaries [2, Theorem 7.3])**.** *Let $\mathcal{O} = \{o_1, \ldots, o_n\}$ be finite, with strictly increasing scales $0 < y_1 < \cdots < y_n$ where $y_i := \iota_{\mathcal{O}}(o_i)$. Consider symbols whose scale is a free variable $x := \iota_{\mathcal{S}}(s) \in \mathbb{R}_{>0}$. Then for each $k \in \{1, \ldots, n\}$, the object $o_k$ is the unique meaning of $s$ whenever*

$$\sqrt{y_{k-1}y_k} \;<\; x \;<\; \sqrt{y_k y_{k+1}},$$

*with the conventions $y_0 := 0$ and $y_{n+1} := \infty$. In particular, the decision boundary between $o_k$ and $o_{k+1}$ occurs at the geometric mean $\sqrt{y_k y_{k+1}}$.*

**Corollary 5.2** (Local stability away from boundaries)**.** *In the setting of Proposition 5.1, if $x$ lies a positive margin away from the boundary points, then* Mean$(s)$ *is locally constant under small perturbations of $x$.*

# 6 A forcing principle: cheaper mediating codes always exist

We now state a simple inequality that captures a "thermodynamic" forcing mechanism for symbols.

**Theorem 6.1** (Geometric-mean mediation strictly lowers total cost)**.** *Let $y \in \mathbb{R}_{>0}$ and define the mediator scale $x := \sqrt{y}$. Then*

$$J(x) + J(x/y) \leq J(y),$$

*with equality if and only if $y = 1$. Equivalently, for $y \neq 1$,*

$$2J(\sqrt{y}) < J(y).$$

*Proof.* Write $y = t^2$ with $t > 0$. Using (1),

$$J(t^2) - 2J(t) = \left(\frac{t^2 + t^{-2}}{2} - 1\right) - \big((t + t^{-1}) - 2\big) = \frac{(t-1)^2 + (t^{-1} - 1)^2}{2} \geq 0,$$

with equality iff $t = 1$, i.e. $y = 1$. □

**Corollary 6.2** (Why symbols are "forced" in cost-minimizing systems)**.** *Suppose a system may represent an object of scale $y$ either directly (intrinsic cost $J(y)$) or via a mediator code of scale $x$ plus a translation/mismatch step (total cost $J(x) + J(x/y)$). Then any cost-minimizing dynamics strictly prefers mediation for every $y \neq 1$; hence the system will generate internal surrogate configurations (codes) rather than always processing objects "as themselves." This is a formal, non-mentalistic grounding mechanism.*

**Remark 6.3** (Connection to sequential mediation)**.** Theorem 6.1 is the simplest instance of a general sequential mediation principle: optimal intermediate representations occur at geometric means in scale space; see [2, Section 5.2].

## 7 Relation to ULL (certificate-based semantics)

The Universal Light Language (ULL) paper [3] uses the same canonical mismatch cost $J$ and defines meanings by a certificate pipeline that (i) extracts invariants, (ii) enforces legality constraints, and (iii) selects canonical normal forms under cost and neutrality constraints. The present paper provides a minimal reference-theoretic layer for that program:

- **Intrinsic meaning**: meanings are minimizers of a fixed objective, not interpreter-dependent assignments.
- **Grounded symbols**: tokens qualify as symbols precisely when they both minimize mismatch and compress higher-cost objects (Definition 4.1).
- **Stability**: finite dictionaries yield explicit margins (Corollary 5.2), aligning with adversarial-margin reporting in certificate systems.
- **Periodic Table justification**: The forcing principle (Theorem 6.1) implies that stable, low-cost "atoms" of meaning must exist to mediate reference to complex objects. This provides the theoretical necessity for the 20 canonical WTokens discovered in ULL.

## 8 Machine Verification

The core definitions and theorems of this paper have been formalized in the Lean 4 theorem prover as part of the `IndisputableMonolith` repository. Key modules include:

- `IndisputableMonolith.Foundation.Reference`: Formalizes costed spaces, reference structures, and the meaning predicate.
- `IndisputableMonolith.Cost.Convexity`: Proves the strict convexity of the canonical cost $J$.
- `IndisputableMonolith.Foundation.Reference.Forcing`: Formalizes the mediation forcing principle (Theorem 6.1).

This mechanization ensures that the "grounding" mechanism is not merely a verbal argument but a logical consequence of the cost axioms.

# 9 Conclusion

Within a fully explicit optimization semantics, we defined meaning as cost minimization and symbolhood as meaning plus compression. Using the canonical reciprocal cost characterized in [2], we obtained geometric-mean decision boundaries and a simple forcing theorem showing that cheaper mediating codes always exist. In this sense, grounding is not an extra metaphysical ingredient: it is the economical outcome of a universal mismatch geometry.

# References


[1] S. Harnad. The symbol grounding problem. *Physica D: Nonlinear Phenomena*, 42(1–3):335–346, 1990.

[2] J. Washburn and A. Rahnamai Barghi. *Reciprocal Convex Costs for Ratio Matching: Functional-Equation Characterization and Decision Geometry.* Accepted for publication, 2026. Preprint PDF available in this repository.

[3] J. Washburn. *The Universal Light Language: A Periodic Table of Meaning.* 2025. Manuscript; see `papers/tex/ULL-Periodic-Table-Meaning.tex`.

[4] J. Washburn. *Recognition Science Architecture Spec v2.3.* 2025. Technical Specification.