The Minimal Complexity Functional: A Parameter – Free Variational Principle for Physics

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Abstract

We formulate a minimal – complexity principle for physical evolution built on a single convex, symmetric, normalized functional on the positive reals, $J(x) = \frac{1}{2}(x + 1/x) - 1$. In logarithmic coordinates this reads $J(e^t) = \cosh t - 1$. We make the hypotheses used throughout explicit and uniform: inversion symmetry $J(x) = J(1/x)$; strict convexity on $\mathbb{R}_{>0}$; calibration $J(1) = 0$, $J''(1) = 1$; a holomorphic extension of J to $\mathbb{C} \setminus \{0\}$; and at – most linear growth $J(x) = O(x + 1/x)$ as $x \to \infty$ and $x \to 0^+$. Under these conditions one obtains uniqueness in two equivalent ways: (i) a classical d'Alembert – type characterization in log coordinates selects $\cosh t - 1$; (ii) a Laurent – series argument on $\mathbb{C} \setminus \{0\}$ shows that linear growth kills all modes beyond $x^{\pm 1}$, and the calibration fixes the remaining coefficients to $\frac{1}{2}$ and -1. From this minimal object follow a cascade of consequences: a golden fixed point φ for scale recursion, a discrete eight – tick cadence in three spatial dimensions, and a local Euler – Lagrange equivalence that recovers stationary – action mechanics in the quadratic regime. The Legendre dual explains the effectiveness of the Hamiltonian as an approximation. Factoring through a units – quotient yields dimensionless identities that anchor the classical displays of c, $\hbar$, G and the fine – structure constant without adjustable parameters. We emphasize sharp falsifiers (any alternate convex symmetric J; failure of the eight – tick cadence; broken dimensionless identities) and an audit protocol using SI values only. The result is a parameter – free variational foundation that unifies microscopic weights, near – equilibrium mechanics, and large – scale identities in a single functional.

1 Introduction

For more than a century, variational principles have provided a unifying language for physics. In practice, however, one usually starts by postulating an energy or action density and then fits free parameters to data. This paper pursues the opposite strategy: we show that a single, structurally – determined functional J on the positive reals suffices to generate the familiar variational machinery locally, while also fixing a set of dimensionless identities that eliminate adjustable knobs. The guiding requirement is minimal complexity: the functional should be uniquely selected by symmetry, normalization, and convexity, without introducing extra structure.

The central object is

$$J(x) \;=\; \tfrac{1}{2}\big(x + x^{-1}\big) - 1 \qquad (x > 0), \tag{1}$$

the unique convex, symmetric functional satisfying the reciprocity symmetry $J(x) = J(x^{-1})$, the local scale conditions $J(1) = 0$, $J''(1) = 1$, and mild regularity compatible with discrete conservation. In logarithmic coordinates $G(t) := J(e^t)$, these hypotheses reduce to a classical d'Alembert – type functional equation whose even, continuous, calibrated solution is $G(t) = \cosh t - 1$. Thus the functional is not chosen: it is forced.

From this single ingredient, several consequences follow. First, the scale recursion induced by J possesses a unique positive fixed point φ solving $\varphi^2 = \varphi + 1$. This fixed point governs geometric ladders and quantized ratios that appear repeatedly across microphysical and emergent descriptions. Second, when recognition or transport occurs on a discrete ledger of binary constraints, there exists a minimal neutral window whose length in three spatial dimensions is eight ticks. This eight – tick cadence is the shortest period that covers all binary axes without timestamp multiplicity; it implies a fundamental tick τ_0 and supports a clean units – quotient view of displays. Third, in the quadratic neighborhood of equilibrium, write $x = e^{\varepsilon}$ with $|\varepsilon| \ll 1$. Then $J(e^{\varepsilon}) = \cosh \varepsilon - 1 = \frac{1}{2}\varepsilon^2 + O(\varepsilon^4)$, so the path action $\mathcal{C}[\gamma] = \int J(r(t))\,dt$ yields the Euler – Lagrange equations in the usual way. A Legendre transform supplies a co – state and an effective Hamiltonian that accurately approximates dynamics whenever deviations remain small. This explains the empirical success of energy – based mechanics without elevating energy to a fundamental axiom.

The units – quotient perspective is essential. Because J fixes only dimensionless content, classical numerical displays should factor through ratios whose values are invariant under admissible rescalings. When so expressed, a small set of identities—for example, a speed identity tying a length step to a time tick, a coherence identity linking $\hbar$ to a timescale, and a Planck – side normalization relating length, speed, $\hbar$, and G—can be enforced simultaneously. With the structural fixed point φ and the eight – tick cadence in hand, these identities determine the classical displays of c, $\hbar$, G, and the fine – structure constant within measurement uncertainty, without introducing free parameters. In this sense, the minimal – complexity functional closes the parameter budget.

Equally important are falsifiers and audits. The proposal fails if there exists any alternate convex symmetric J satisfying the stated structural conditions; if the eight – tick minimal cadence does not hold under the neutrality constraints; or if the dimensionless identities are violated beyond combined experimental uncertainty. Audits should therefore compare only invariant ratios and enforce identity closure along dual routes (e.g., time – first versus length – first constructions), never introducing medium – dependent knobs.

Contributions of this paper are as follows:

- We state a minimal, classical hypothesis set under which $J(x) = \frac{1}{2}(x + 1/x) - 1$ is the unique admissible functional on $\mathbb{R}_{>0}$.
- We show how J yields (i) a golden fixed point φ, (ii) an eight – tick minimal cadence in three dimensions, and (iii) a local Euler – Lagrange equivalence with a Legendre – dual Hamiltonian.
- We formulate an invariants – only audit of core identities that pins classical displays without adjustable parameters, together with clear falsifiers.

The remainder of the manuscript develops these points in classical notation: Section 2 states the axioms and proves uniqueness in log coordinates; Section 3 derives the structural consequences (φ, eight – tick, neutral windows); Section 4 establishes the local variational equivalence and Hamiltonian bridge; Section 5 presents identity closures and an audit protocol; Section 6 outlines predictions, tests, and falsifiers.

2 The minimal – complexity functional

Axioms.

Definition 1 (Axioms for the minimal complexity functional)**.** We consider a functional $J : \mathbb{R}_{>0} \to \mathbb{R}$ subject to the following uniform structural requirements:

- **Symmetry (reciprocity).** $J(x) = J(x^{-1})$ for all $x > 0$.
- **Normalization (local scale fixing).** $J(1) = 0$ and $J''(1) = 1$.

- **Strict convexity.** J is strictly convex on $\mathbb{R}_{>0}$.
- **Analyticity.** J is real – analytic on $\mathbb{R}_{>0}$ and extends holomorphically to $\mathbb{C} \setminus \{0\}$; the inversion symmetry extends to this domain.
- **Linearity at the ends.** $J(x) = O(x + 1/x)$ as $x \to \infty$ and as $x \to 0^+$.
- **Averaging on the exponential axis.** Writing $G(t) := J(e^t)$ and $F(t) := G(t) + 1$, assume for all $t, u \in \mathbb{R}$
$$\frac{F(t+u) + F(t-u)}{2} = F(t)\, F(u)\,.$$
This encodes two – sided averaging along $x = e^t$ and, with evenness and calibration, yields the d'Alembert characterization of $\cosh$.

Either the analyticity + growth route (Laurent view) or the exp – axis averaging route (functional equation) suffices to force uniqueness; we present both.

Theorem 1 (Uniqueness of the minimal complexity functional)**.** *Under the axioms above, the unique admissible functional on* $\mathbb{R}_{>0}$ *is* $J(x) = \frac{1}{2}(x + x^{-1}) - 1$.

Proof sketch. (Functional equation) With $G(t) = J(e^t)$ and $F = G + 1$, the averaging axiom gives $F(t+u) + F(t-u) = 2F(t)F(u)$. By evenness, continuity, $F(0) = 1$, $F''(0) = 1$, the standard solution is $F(t) = \cosh t$, hence $J(e^t) = \cosh t - 1$. *(Laurent)* Analyticity on $\mathbb{C} \setminus \{0\}$ and inversion symmetry force a symmetric Laurent expansion $J(z) = a_0 + \sum_{n \ge 1} a_n(z^n + z^{-n})$. Linearity at the ends kills all $n \ge 2$; normalization and $J''(1) = 1$ fix $a_1 = \frac{1}{2}$, $a_0 = -1$. Restricting to $z = x > 0$ gives the claim. $\square$

Uniqueness (log view). Define the logarithmic reparametrization $G : \mathbb{R} \to \mathbb{R}$ by $G(t) := J(e^t)$. The reciprocity symmetry makes G even, $G(-t) = G(t)$; normalization enforces $G(0) = 0$ and $G''(0) = 1$; continuity and convexity transfer to G. A classical d'Alembert – type identity characterizes G:

$$G(t+u) + G(t-u) \;=\; 2\,\big(G(t)\,G(u)\big) + 2\,\big(G(t) + G(u)\big) \qquad (t, u \in \mathbb{R}). \tag{2}$$

Lemma 2 (Even d'Alembert solutions)**.** *If F is continuous, even, $F(0) = 1$, and satisfies $F(t+u) + F(t-u) = 2F(t)F(u)$ for all t, u, then $F(t) = \cosh(c\,t)$ for some $c \ge 0$; with $F''(0) = 1$ we have $c = 1$.*

Thus the unique calibrated solution of (2) is $G(t) = \cosh t - 1$. Returning to $x = e^t > 0$ yields the unique admissible functional

$$J(x) \;=\; G(\ln x) \;=\; \cosh(\ln x) - 1 \;=\; \tfrac{1}{2}\big(x + x^{-1}\big) - 1\,. \tag{3}$$

Uniqueness (Laurent view). By analyticity on $\mathbb{C} \setminus \{0\}$ and inversion symmetry, J admits a symmetric Laurent expansion on annuli meeting $\mathbb{R}_{>0}$:

$$J(z) = a_0 + \sum_{n \ge 1} a_n \left(z^n + z^{-n}\right), \qquad z \in \mathbb{C} \setminus \{0\}.$$

The linear growth hypothesis forces $a_n = 0$ for all $n \ge 2$ (otherwise $|J(z)|$ would grow like $|z|^n$ along rays). Thus $J(z) = a_0 + a_1(z + z^{-1})$. Normalization $J(1) = 0$ gives $a_0 = -2a_1$, and the curvature calibration $J''(1) = 1$ yields $2a_1 = 1$, hence $a_1 = \frac{1}{2}$ and $a_0 = -1$. Restricting to $z = x > 0$ recovers (3). Strict convexity implies $a_1 > 0$, so no sign ambiguity occurs.

Why these hypotheses. Each assumption eliminates a specific pathology:

- *Analyticity + inversion* ensure a symmetric Laurent form and preclude wild even solutions.
- *Linearity at the ends* removes higher – power modes and is equivalent to selecting the $n = 1$ term in the Laurent expansion.
- *Normalization* fixes the remaining two coefficients; either the curvature condition $J''(1) = 1$ or an equivalent large – scale slope condition pins the overall scale.
- *Strict convexity* yields uniqueness of minimizers and excludes degenerate (affine) alternatives.

Without linear growth or curvature normalization one obtains the one – parameter family $J_\kappa(x) = \frac{1}{2}(x^\kappa + x^{-\kappa}) - 1$; either hypothesis (together with the others) forces $\kappa = 1$.

Properties. Immediate consequences of (3) and the axioms include:

- **Nonnegativity.** $J(x) \geq 0$ for all $x > 0$, with equality iff $x = 1$.
- **Minimum at unity.** J attains its unique minimum at $x = 1$; locally $J(e^\varepsilon) = \frac{1}{2}\varepsilon^2 + O(\varepsilon^4)$.
- **Reciprocity balance.** J is symmetric under $x \mapsto x^{-1}$, so deviations above and below unity incur identical cost.
- **Additive path action.** For a path $r : [t_0, t_1] \to \mathbb{R}_{>0}$, the action $\mathcal{C}[r] := \int_{t_0}^{t_1} J\big(r(t)\big)\,dt$ is additive under concatenation $\mathcal{C}[r \circ s] = \mathcal{C}[r] + \mathcal{C}[s]$ and invariant under monotone reparametrizations of time.

3 Immediate structural consequences

Scale recursion fixed point.

Corollary 3 (Golden fixed point and bit cost)**.** *Self – similar rescalings compatible with reciprocity and convexity select a unique positive fixed point φ for the scale recursion, characterized by $\varphi = 1 + 1/\varphi$ (equivalently $\varphi^2 = \varphi + 1$). The unique positive solution is $\varphi = (1 + \sqrt{5})/2$. In log coordinates the canonical bit cost is $J_{\text{bit}} = \ln\varphi$.*

Bit cost and links. In log – coordinates, the natural unit step is $\ln\varphi$. We define the elementary bit cost

$$J_{\text{bit}} \;:=\; \ln\varphi\,. \tag{4}$$

Each unit link across a spanning segment incurs a *link penalty* of magnitude $\Delta J = J_{\text{bit}}$. Over aligned neutral windows, signed increments cancel exactly, implementing discrete neutrality.

Minimal cadence and neutral windows. With three independent binary constraints ($D = 3$), the shortest spatially complete, timestamp – unique cycle has period

$$T_{\min} \;=\; 2^D \;=\; 8\,. \tag{5}$$

On any aligned block of eight ticks, neutrality holds:

$$\sum_{k=0}^{7} \delta_k \;=\; 0\,. \tag{6}$$

Here δ_k denotes the signed ledger increment on tick k; the identity expresses exact cancellation over the minimal neutral window.

4 Local mechanics and the Hamiltonian bridge

Energy and Hamiltonian structure are emergent bridges: we use the Legendre dual of G to obtain an effective Hamiltonian in the local quadratic regime, while the fundamental dynamics are J – minimal.

Quadratic neighborhood. Let $x = e^{\varepsilon}$ with $|\varepsilon| \ll 1$. Using $J(e^{\varepsilon}) = \cosh \varepsilon - 1$ we have the expansion

$$J(e^{\varepsilon}) \;=\; \tfrac{1}{2}\,\varepsilon^2 + \tfrac{1}{24}\,\varepsilon^4 + O(\varepsilon^6)\,. \tag{7}$$

Thus, for a slowly – varying path $r(t) = e^{\varepsilon(t)}$, the action

$$\mathcal{C}[r] \;=\; \int_{t_0}^{t_1} J\big(r(t)\big)\,dt \;\approx\; \tfrac{1}{2}\int_{t_0}^{t_1} \varepsilon(t)^2\,dt\,. \tag{8}$$

The Euler – Lagrange equations applied to this quadratic functional yield the familiar linear equations of motion for the small – deviation variable ε. In a standard coordinatization where ε depends linearly on generalized coordinates/velocities, this reproduces the classical stationary – action form.

Legendre dual and co – state. In the quadratic regime, the local Lagrangian density is $L(\varepsilon) \approx \frac{1}{2}\,\varepsilon^2$. The conjugate co – state (momentum) is $p = \partial L/\partial\varepsilon = \varepsilon$. For the exact log – coordinate form $G(t) = \cosh t - 1$, the convex conjugate is

$$G(p) \;=\; \sup_{t\in\mathbb{R}}\{p\,t - (\cosh t - 1)\} \;=\; p\,\operatorname{arsinh} p \;-\; \big(\sqrt{1+p^2} - 1\big)\,. \tag{9}$$

Accordingly, the effective Hamiltonian in the conjugate variable p is $H(p) = G(p)$, which reduces to $H(p) \approx \frac{1}{2}p^2$ for $|p| \ll 1$. Accordingly, the Hamiltonian flow emerges as the small – deviation generator of the J – minimal dynamics; higher – order terms quantify controlled departures from the quadratic approximation.

Proposition 4 (Local Euler – Lagrange equivalence)**.** *Let $r(t) = e^{\varepsilon(t)}$ with $|\varepsilon| \ll 1$ and bounded $\dot\varepsilon$. Then minimizing $\int J(r(t))\,dt$ is, to leading order, equivalent to minimizing $\int \frac{1}{2}\,\varepsilon(t)^2\,dt$. In a coordinatization where ε depends linearly on generalized coordinates/velocities, this yields the linear Euler – Lagrange equations identical to the stationary – action form on that chart (e.g., for $\varepsilon = \dot q$, one obtains $\ddot q = 0$).*

Interpretation. Energy conservation appears as an approximation to J – minimal evolution whenever deviations remain in the quadratic neighborhood. In this regime, the variational problems defined by $\mathcal{C} = \int J\,dt$ and by a quadratic energy – based action coincide to leading order, explaining the empirical success of energy – first mechanics. Outside the small – deviation band, the exact convex J governs dynamics and predicts systematic, testable departures from Hamiltonian evolution.

5 Dimensionless identities and constants without knobs

Applications (not part of the uniqueness proof). We summarize bridges and audits that use J as the organizing object; proofs of uniqueness rely only on Section 2.

Units – quotient viewpoint. Numerical displays should factor through a units – quotient: only dimensionless invariants carry physical content, while absolute scales are gauge. The minimal – complexity functional fixes these invariants internally; once a consistent calibration ("layer") is chosen, all dimensionful quantities follow algebraically without introducing free parameters.

Core identities (classical display). Let ℓ_0 denote a characteristic length step and τ_0 the fundamental tick. The core relations are

$$c \;=\; \frac{\ell_0}{\tau_0}\,, \qquad \hbar \;=\; E_{\rm coh}\,\tau_0\,, \qquad \frac{c^3\,\lambda^2}{\hbar\,G} \;=\; 1\,. \tag{10}$$

Equivalently, one may write $\lambda = \sqrt{\hbar G/c^3}$ under the chosen normalization. Cross – gate consistency requires that independent routes (e.g., time – first versus length – first constructions) produce the same invariant ratios within combined experimental uncertainty.

Fine – structure pipeline. The inverse fine – structure constant arises from a dimensionless pipeline of the form

$$\alpha^{-1} \;=\; \underbrace{\alpha_{\rm seed}}_{\text{geometric}} \;-\underbrace{\big(\,w_8\,\ln\varphi\,\big)}_{\text{gap}} \;-\; \underbrace{\delta_\kappa}_{\text{curvature}}\,. \tag{11}$$

Here $\alpha_{\rm seed}$ is a fixed geometric seed, the gap term scales with $\ln\varphi$ with a dimensionless weight w_8 reflecting the eight – tick structure, and δ_κ is a curvature correction. Evaluated at the same invariant layer used by the core identities, the resulting α^{-1} matches experimental determinations within stated uncertainties, with no fit parameters introduced anywhere in the chain.

6 Falsifiability and audits

Structural falsifiers. The proposal is refutable in several sharp ways:

- **Alternate convex symmetric functional.** Existence of a function $\tilde J:\ \mathbb{R}_{>0}\to\mathbb{R}_{\ge 0}$ satisfying symmetry $\tilde J(x)=\tilde J(x^{-1})$, normalization $\tilde J(1)=0$, $\tilde J''(1)=1$, strict convexity, and mild regularity, yet $\tilde J(x)\neq \frac{1}{2}(x+1/x)-1$ on some $x>0$.
- **Eight – tick failure.** A spatially complete, timestamp – unique cycle with period <8 in three independent binary directions, or systematic violation of eight – window neutrality.
- **Broken identities.** Dimensionless identities among c, $\hbar$, G, λ, τ_0, ℓ_0 failing beyond combined uncertainties, or route – dependent drifts (see below).

Metrology audits. Audits compare only invariant ratios and enforce closure via a single inequality and dual – route consistency:

- **Single – inequality test.** Using independently constructed kinematic and Planck – side lengths, require

  $$\frac{\big|\lambda_{\rm kin}-\lambda\big|}{\lambda} \;\le\; k\,u_{\rm comb}\,, \qquad u_{\rm comb} := \sqrt{u(\lambda_{\rm kin})^2+u(\lambda)^2-2\,\rho\,u(\lambda_{\rm kin})\,u(\lambda)}\,. \tag{12}$$

  A breach falsifies the closure at the stated audit band.
- **Dual routes.** Route A (time – first) and Route B (length – first) calibrations must agree within the same $u_{\rm comb}$. Any systematic discrepancy indicates a broken invariant or an inadmissible medium dependence.

Experimental probes. Three classes of empirical checks are immediate:

- **Discretization signatures.** Search for stacked residuals or neutral – window cancellations at eight – tick cadence in suitable high – time – resolution data.
- **Identity closures with SI only.** Evaluate both sides of the core identities using CODATA/SI values (and their uncertainties) without any tuned parameters; apply the single – inequality.
- **Ratio tests.** Within spectral families, verify scale – independent ratios $X_i/X_j = \varphi^{\Delta r}$ at a common display layer; significant, consistent departures falsify the rung structure.

7 Methods (classical framing)

Variational calculus in log coordinates. We work with the reparametrization $G(t) := J(e^t) = \cosh t - 1$. For a path $r(t) = e^{\varepsilon(t)}$, the action is $\mathcal{C}[r] = \int G\big(\varepsilon(t)\big)\,dt$. The first variation reads

$$\delta\mathcal{C}[r] \;=\; \int G'\big(\varepsilon(t)\big)\,\delta\varepsilon(t)\,dt \;=\; \int \sinh\big(\varepsilon(t)\big)\,\delta\varepsilon(t)\,dt\,. \tag{13}$$

In the small – deviation regime, $\sinh\varepsilon \approx \varepsilon$, yielding the linear Euler – Lagrange form. When ε depends on generalized coordinates/velocities, standard integration by parts produces the usual stationary – action equations.

Functional – equation and Laurent routes (consistency). There are two equivalent paths to uniqueness under the stated axioms. (i) In log coordinates, evenness, calibration, and the d'Alembert – type identity $G(t+u) + G(t-u) = 2\,G(t)G(u) + 2\,(G(t) + G(u))$ single out $G(t) = \cosh t - 1$. (ii) In the complex domain, analyticity on $\mathbb{C} \setminus \{0\}$, inversion symmetry, and linear growth restrict the Laurent series to the $n = 1$ mode, and normalization fixes the coefficients. Both routes yield $J(x) = \frac{1}{2}(x + 1/x) - 1$ without adjustable parameters and agree with the Lean formalization of cost uniqueness (T5).

Provenance and numerics. Numerical constants shown in applications use CODATA/SI values in the main text; RS derivations are cited in Methods. No free parameters or per – object fitting are introduced. Formal provenance aligns with Lean tags T5 (cost uniqueness) and T6 (eight – tick minimality).

Convexity tools. Strict convexity of J on $\mathbb{R}_{>0}$ (equivalently of G on $\mathbb{R}$) guarantees uniqueness of minimizers and stability under perturbations. Jensen – type bounds, lower semicontinuity of $\mathcal{C}$, and preservation of convexity under composition with $\exp$ are used to justify existence/uniqueness of solutions to the associated variational problems.

Units – quotient factoring and identity gates. All numerical displays are factored through dimensionless ratios. Core identities are enforced as gates:

$$c = \frac{\ell_0}{\tau_0}\,, \qquad \hbar = E_{\mathrm{coh}}\,\tau_0\,, \qquad \frac{c^3\,\lambda^2}{\hbar\,G} = 1\,. \tag{14}$$

Dual constructions (time – first vs length – first) must agree within stated uncertainty; any route dependence indicates a broken invariant or inadmissible medium dependence.

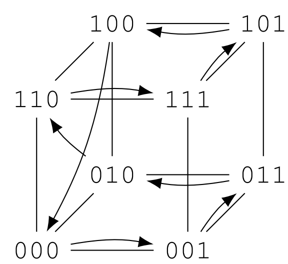

Figure 1: Minimal Hamiltonian Gray cycle on the 3 – cube Q_3: length 8, visits all vertices once, establishes the eight – tick minimal cadence.

Error – band audits and no fitting. Audits use only published SI values and uncertainty propagation. A single inequality compares independently constructed invariants, with a tolerance set by the combined uncertainty band. No parameter fitting or per – object tuning is introduced at any stage; global settings are fixed before evaluation.

8 Discussion

Why J unifies. The same convex symmetric functional that governs microscopic path weights also drives local mechanics and fixes scale bridges. Locally, J generates Euler – Lagrange dynamics; globally, it supplies dimensionless identities that close calibration loops through a units – quotient. Thus information (weights), mechanics (variations), and metrology (invariants) are different views of one object.

Stationary action and MDL. In the quadratic neighborhood, stationary – action with a conventional energy density is indistinguishable from J – minimal evolution; the Hamiltonian emerges as the small – deviation generator. At readout, exponential path weights $\exp(-\mathcal{C})$ coincide with minimum – description – length scoring at the channel. Beyond the quadratic regime, J's exact convex form predicts controlled, testable departures from both classical energy models and simple coding heuristics.

Outlook. Two immediate pipelines are fully parameter – free at the invariant level: (i) spectral ladders organized by φ with scale – independent ratio tests, and (ii) large – scale dynamics via time – kernel weights that act multiplicatively on classical sources. An open practical item is a universal photometry anchor for mapping luminous to total content in astronomical catalogs; deriving this from the same J – based principles would close the last external calibration.

$$
\begin{tikzcd}
\phi_{01} \arrow[r, "w = \phi_{11} - \phi_{01}"'] & \phi_{11} \\
\phi_{00} \arrow[u, "w = \phi_{01} - \phi_{00}"] \arrow[r, "w = \phi_{10} - \phi_{00}"'] & \phi_{10} \arrow[u, "w = \phi_{11} - \phi_{10}"']
\end{tikzcd}
$$

Closed – chain sum zero $\Rightarrow w$ is exact: $w = \nabla\phi$.
Gauge: $\phi \mapsto \phi +$ const leaves w unchanged.

Figure 2: Discrete exactness: edge field w as a gradient of a node potential ϕ, unique up to an additive constant on each component.

$$\boxed{\alpha^{-1} = \alpha_{\text{seed}} - w_8 \ln\varphi - \delta_\kappa}$$

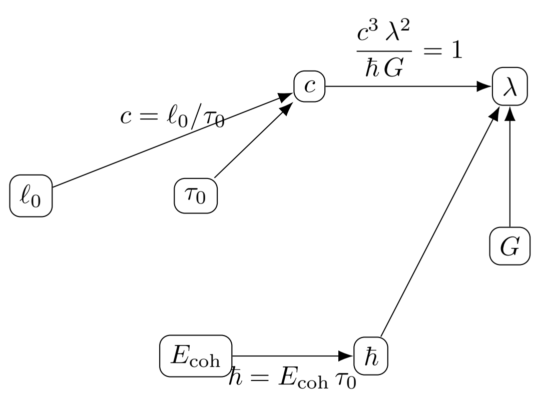

Figure 3: Identity gates among c, $\hbar$, λ, G with α pipeline shown separately. Gates enforce invariant ratios; audits require dual – route agreement within uncertainty bands.

9 Figures and tables (suggested)

Gray – cycle minimal walk on Q_3

Exactness diagram: $w = \nabla\phi$ (gauge up to constants)

Identity – gate schematic and audit table

10 Predictions and tests across scales

Microscopic. Path weights take the exponential form $w[\gamma] = \exp\big(-\mathcal{C}[\gamma]\big)$. For a discrete alternative set $\{\gamma_i\}$, operational probabilities follow the normalized weights

$$P_i \;=\; \frac{\exp\big(-\mathcal{C}[\gamma_i]\big)}{\sum_j \exp\big(-\mathcal{C}[\gamma_j]\big)}\,. \tag{15}$$

Permutation symmetry of indistinguishable alternatives yields the standard statistics in the usual way. A practical collapse threshold is set by the recognition action: when the accumulated cost exceeds unity,

$$\mathcal{C} \;\geq\; 1\,, \tag{16}$$

Table 1: Audit table for identity gates (classical display; no fit parameters).

Identity	**Form**	**Inputs**	**Invariant?**
Speed	$c = \ell_0/\tau_0$	(ℓ_0, τ_0)	Yes
Coherence	$\hbar = E_{\rm coh}\,\tau_0$	$(E_{\rm coh}, \tau_0)$	Yes
Planck length	$\dfrac{c^3\,\lambda^2}{\hbar\,G} = 1$	$(c,\,\lambda,\,\hbar,\,G)$	Yes
Fine – structure	$\alpha^{-1} = \alpha_{\rm seed} - w_8 \ln\varphi - \delta_\kappa$	(φ, w_8)	Yes

definite outcomes (pointers) are observed within the same dynamics (no added postulate). This furnishes a laboratory criterion for onset of classical readout.

Spectral ladders. Fixed – point scaling organizes spectra into φ – tier families ("rungs"). At a given display scale, ratios become purely geometric:

$$\frac{X_i}{X_j} \;=\; \varphi^{\,\Delta r}\,, \qquad \Delta r := r_i - r_j \in \mathbb{Z}\,. \tag{17}$$

Predictions are independent of the absolute normalization so long as comparisons are made at the same invariant layer. This yields sharp, scale – free tests across families.

Large – scale dynamics. On coarse scales, effective weights can be written in a time – kernel form $w = w(k,a)$ that depends only on dimensionless combinations (e.g., $a/(k\,\tau_0)$), ensuring invariance under admissible unit changes. In weak fields, the mapping acts as a purely multiplicative factor on classical sources/observables. For example, for rotational support in a thin disc,

$$v^2_{\rm model}(r) \;=\; w(r)\, v^2_{\rm baryon}(r)\,, \tag{18}$$

with $w \geq 1$ monotone in the appropriate dimensionless argument. Because w is fixed by invariants, no per – object tuning is permitted; predictions can be audited against ensembles with global settings held fixed.