Neutrino Masses and the Deep φ-Ladder

Fractional rungs, mass-squared splittings, and falsifiable structural ratios

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Abstract

Neutrino masses are tiny, but their mass splittings and ordering exhibit rigid structure. This paper extends the single-anchor φ-ladder framework to the deep (low-mass) end by placing neutrinos on *fractional* rungs of the ladder. [HYPOTHESIS]The model yields a normal hierarchy with absolute masses at the level $m_1 \approx 0.00354$ eV, $m_2 \approx 0.00926$ eV, $m_3 \approx 0.0499$ eV, and hence $\sum m_\nu \approx 0.0627$ eV. [VALIDATION]

The key structural prediction is an exact φ-power relation among squared masses implied by the deep rung spacing: $(m_3^{\text{pred}})^2/(m_2^{\text{pred}})^2 = \varphi^7$. [HYPOTHESIS]The individual splittings Δm_{21}^2 and Δm_{31}^2 fall within standard NuFIT windows at current precision (validation). [VALIDATION]The predicted mass sum is consistent with present cosmological bounds without introducing per-species fitting knobs. [VALIDATION]

All eV-reported values are stated under an explicit, single-scalar calibration seam (a declared reporting convention), and the framework forbids particle-by-particle tuning. [CERT]We conclude with falsifiers: if future oscillation analyses or absolute-mass probes rule out the φ^7 ratio, the normal ordering implied by the deep ladder, or the predicted mass scale, the framework is refuted. [VALIDATION]

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1 Introduction

Section summary. Neutrino oscillations reveal precise structure in *differences* of squared masses, but do not determine absolute masses. This paper proposes that neutrinos live on the deep end of the same φ-ladder used for charged sectors, with fractional rungs. We state a conservative claim contract: no per-state fitting, explicit unit seams for eV reporting, and falsifiers that future experiments can settle.

1.1 What oscillations measure (and what they do not)

In the Standard Model extended to include neutrino masses, oscillation experiments primarily measure:

- two independent mass-squared splittings (Δm^2_{21} and Δm^2_{31}), and
- mixing parameters (the PMNS angles and phases).

These observables establish that neutrinos are massive and that the three mass eigenstates are non-degenerate. However, oscillations do *not* fix the absolute mass scale: adding a common offset to all three masses leaves Δm^2 values unchanged. [PROVED]

Independent constraints on the absolute scale come from:

- kinematic measurements (e.g. beta-decay endpoints, summarized by PDG [1]), and
- cosmological bounds on the sum $\sum m_\nu$ (survey-dependent; we use only the qualitative requirement that $\sum m_\nu$ remain below current limits). [VALIDATION]

1.2 Framework overview: the deep φ-ladder

Papers 1–2 develop a discrete-geometry program in which hierarchies are encoded by a φ-ladder coordinate. This paper extends that ladder into the deep regime where masses are far below the charged sectors. [HYPOTHESIS]

The central modeling premise is that the neutrino rungs are *fractional* (quarter-step) positions on the φ-ladder. [HYPOTHESIS]Once those rungs are fixed, the model implies:

- a normal ordering of the three neutrino masses, [HYPOTHESIS]
- concrete values for Δm^2_{21} and Δm^2_{31}, [HYPOTHESIS]
- and a rigid structural relation between them (an exact φ-power ratio). [PROVED]

1.3 Claim contract: no per-species fitting and explicit unit seams

This paper follows the same contract as the earlier papers in the series:

- **No per-eigenstate fitting.** The model does not permit choosing separate coefficients for ν_1, ν_2, ν_3. All structure is shared (ladder base φ, rung rule, and global constants). [PROVED]
- **Explicit eV reporting seam.** Converting dimensionless ladder outputs to eV is a declared reporting convention (a single-scalar calibration seam), and is not claimed to be derived from the neutrino sector alone. [CERT]
- **Validation labeling.** Agreement with NuFIT windows for Δm^2 is labeled as validation and never used as an input. [VALIDATION]

1.4 What this paper delivers (and what it does not)

Deliverables:

- a precise definition of the deep ladder and the fractional rung convention used for neutrinos, [HYPOTHESIS]
- predicted absolute masses m_1, m_2, m_3 under an explicit reporting seam, [HYPOTHESIS]
- predicted splittings Δm^2_{21} and Δm^2_{31} and an exact ratio constraint, [PROVED]
- cosmological and kinematic consistency checks stated only as validation, [VALIDATION]
- falsifiers that future oscillation and absolute-mass measurements can confirm or refute. [VALIDATION]

Non-deliverables: we do not introduce a new global fit, we do not tune phases to match any particular dataset, and we do not claim that the structural ratio is insensitive to future revisions of experimental systematics. [HYPOTHESIS]

1.5 Notation and classical correspondences

Table 1 summarizes the key terms used in this paper and their correspondences to standard physics concepts.

Term	Classical Equivalent	Status	Notes
Deep φ-ladder	log-scale mass coordinate	Bridge	Extends Paper 1 ladder
Fractional rungs	quarter-step positions	Novel	No classical analog
κ_{eV}	eV calibration seam	Cert	Declared convention
Δm^2_{ij}	mass-squared splitting	Twin	Standard oscillation observable
φ^7 ratio	seam-free structural ratio	Bridge	Scale cancels
Normal hierarchy	$m_1 < m_2 < m_3$ ordering	Twin	Standard NuFIT convention

Table 1: Dictionary of framework terms and their classical correspondences. *Twin*: mathematically identical to the classical object. *Bridge*: corresponds via an explicit mapping. *Novel*: no direct classical analog. *Cert*: declared calibration convention.

2 The Deep Ladder: Fractional Rungs

Section summary. We define the ladder coordinate used for neutrinos and introduce the "deep" regime where rungs are taken to be fractional. The ladder mathematics is elementary ([PROVED]); the physical claim is the fractional-rung assignment and its specific step size ([HYPOTHESIS]). This section fixes notation so that later sections can state masses and splittings without hidden fitting knobs.

2.1 Ladder coordinate and rungs

As in Papers 1–2, we encode multiplicative hierarchy by a base-φ scale coordinate. For a positive quantity x, define its ladder coordinate by

$$r(x) := \log_{\varphi}(x). \text{ [PROVED]} \tag{1}$$

Equivalently, specifying a rung r specifies a pure ladder factor φ^r. [PROVED]

In the charged sectors we treated rungs as integers. For neutrinos we extend the rung set to rationals:

$$r \in \tfrac{1}{4}\mathbb{Z}. \text{ [HYPOTHESIS]} \tag{2}$$

Equation (2) is a convention for the deep ladder: it asserts that the relevant rung lattice is a quarter-step lattice. No numerical value is being fit here; the claim is that neutrinos exhibit a finer rung resolution than the charged sectors. [HYPOTHESIS]

2.2 Why quarter steps (motivation, not a fit)

The quarter-step convention is motivated by two qualitative constraints:

- **Resolution.** Neutrino splittings are extremely small compared to charged sectors, suggesting that the deep ladder must resolve much smaller exponent increments than integer rungs provide. [HYPOTHESIS]
- **Compatibility with the octave clock.** The series uses an eight-tick closure as a canonical cycle; quarter rungs provide a simple compatible refinement that is still discrete and auditable. [HYPOTHESIS]

These motivations are not proofs; the quarter-step lattice is judged by falsifiers (Sec. 8). [VALIDATION]

2.3 Rung differences and squared-mass ratios

A key reason to use a ladder coordinate is that ratios become differences. If two masses $m_a, m_b > 0$ differ by rung offset $\Delta r := r(m_a) - r(m_b)$, then

$$\frac{m_a}{m_b} = \varphi^{\Delta r}. \text{ [PROVED]} \tag{3}$$

For squared masses this becomes

$$\frac{m_a^2}{m_b^2} = \varphi^{2\Delta r}. \text{ [PROVED]} \tag{4}$$

Later, the neutrino rung assignments will imply a rigid φ-power ratio for the atmospheric-to-solar splitting scale. [PROVED]

2.4 Rung assignment (to be used in later sections)

We denote the three neutrino rungs by $r_1 < r_2 < r_3$ (normal ordering). [HYPOTHESIS]In later sections we will use the specific deep-ladder assignment

$$(r_1, r_2, r_3) := \left(-\frac{239}{4}, -\frac{231}{4}, -\frac{217}{4}\right). \text{ [HYPOTHESIS]} \tag{5}$$

Equation (5) is the core discrete input for the neutrino sector in this paper. It is not tuned per mass eigenstate; it is a single rung triple whose consequences are then checked against external oscillation summaries. [HYPOTHESIS]

Classical correspondence. The logarithmic ladder coordinate $r(x) = \log_\varphi(x)$ is a standard change of variables; what is novel is the fractional-rung lattice $r \in \frac{1}{4}\mathbb{Z}$. There is no direct classical analog to discrete quarter-step rungs: in continuum field theory, masses vary continuously. The closest conceptual relative is a discrete quantum number (like spin projection or isospin component) that restricts allowed states to a lattice. The compatibility of quarter steps with the eight-tick octave ($8 \times \frac{1}{4} = 2$) is an internal consistency check, analogous to requiring that lattice refinements divide evenly into fundamental periods (T6 bridge). [HYPOTHESIS]

3 Neutrino Mass Predictions

Section summary. Given a discrete rung triple (r_1, r_2, r_3) and a single global calibration seam for eV reporting, the deep ladder yields three absolute neutrino masses. The rung triple is the only neutrino-sector discrete input ([HYPOTHESIS]); the eV scale is fixed once for the overall framework ([CERT]); the numerical values quoted here are the consequence of those declarations (and are later compared to external constraints as validation).

3.1 From rungs to eV masses (explicit reporting seam)

Section 2 fixes the neutrino rung triple $(r_1, r_2, r_3) \in (\frac{1}{4}\mathbb{Z})^3$ (Eq. (5)). [HYPOTHESIS]To report absolute masses in eV, we require a declared calibration seam that converts one ladder "coherence quantum" to SI energy. We represent that seam by a single scalar τ_0 (seconds per ladder tick), and define the corresponding eV scale

$$\kappa_{\mathrm{eV}} := \frac{\hbar}{\tau_0}\Big/ (1\,\mathrm{eV})\,. \text{ [CERT]} \tag{6}$$

This seam is global: it is fixed once for the framework and is not adjusted per neutrino eigenstate. [CERT]

With κ_{eV} fixed, the deep-ladder mass hypothesis is:

$$m_i^{\mathrm{pred}} := \kappa_{\mathrm{eV}}\, \varphi^{r_i}, \qquad i \in \{1,2,3\}. \text{ [HYPOTHESIS]} \tag{7}$$

3.2 Predicted absolute masses (numerical evaluation under the seam)

Evaluating Eq. (7) for the rung triple (5) under the declared seam yields the absolute mass window:

$$0.00352 < m_1^{\mathrm{pred}} < 0.00355 \text{ eV, [CERT]} \tag{8}$$

$$0.00924 < m_2^{\mathrm{pred}} < 0.00928 \text{ eV, [CERT]} \tag{9}$$

$$0.04987 < m_3^{\mathrm{pred}} < 0.04993 \text{ eV. [CERT]} \tag{10}$$

The implied mass sum is therefore

$$0.06263 < \sum_{i=1}^{3} m_i^{\mathrm{pred}} < 0.06276 \text{ eV. [CERT]} \tag{11}$$

Compatibility with cosmological and kinematic constraints is assessed later as validation, not used to set τ_0 or the rungs. [VALIDATION]

Classical correspondence. The mass prediction $m_i^{\text{pred}} = \kappa_{\text{eV}}\,\varphi^{r_i}$ is an instance of the single-anchor mass law used throughout the series: a global scale factor times a pure φ-power determined by a discrete rung. This corresponds to the phenomenological "quantum ladder" structure observed in many hierarchical mass spectra, where ratios between adjacent states follow a geometric progression. The calibration seam κ_{eV} plays the role of an overall unit conversion (Bridge to SI), analogous to fixing $\hbar$ or c when reporting energies in eV rather than inverse seconds. No per-eigenstate fitting is introduced; the entire mass hierarchy is encoded in the rung triple. [HYPOTHESIS]

4 Mass-Squared Splittings

Section summary. Oscillation experiments measure mass-squared splittings rather than absolute masses. Given the deep-ladder masses from Sec. 3, we define the two independent splittings and evaluate them under the declared eV reporting seam. The resulting values are then compared to NuFIT summary windows as validation.

4.1 Definitions

We use the standard definitions (normal ordering conventions are discussed later):

$$\Delta m_{21}^2 \;:=\; m_2^2 - m_1^2, \qquad \Delta m_{31}^2 \;:=\; m_3^2 - m_1^2. \text{ [PROVED]} \tag{12}$$

If $m_1 < m_2 < m_3$ (normal ordering), then both splittings are positive. [PROVED]

4.2 Predicted splittings from the deep ladder

Using the mass law of Sec. 3, Eq. (7), the predicted splittings are

$$\Delta m_{ij}^{2\,\text{pred}} \;=\; \left(m_i^{\text{pred}}\right)^2 - \left(m_j^{\text{pred}}\right)^2 \;=\; \kappa_{\text{eV}}^2\left(\varphi^{2r_i} - \varphi^{2r_j}\right). \text{ [PROVED]} \tag{13}$$

Thus, while the absolute eV-scale splittings depend on the global seam parameter κ_{eV}, the *ratio* of splittings depends only on rung differences:

$$\frac{\Delta m_{31}^{2\,\text{pred}}}{\Delta m_{21}^{2\,\text{pred}}} \;=\; \frac{\varphi^{2r_3} - \varphi^{2r_1}}{\varphi^{2r_2} - \varphi^{2r_1}}. \text{ [PROVED]} \tag{14}$$

The next section derives the exact φ-power relation $(m_3^{\text{pred}})^2/(m_2^{\text{pred}})^2 = \varphi^7$ implied by the deep rung spacing, and records the resulting closed-form (seam-free) prediction for the splitting ratio as a fixed function of φ. [HYPOTHESIS]

4.3 Numerical evaluation and validation

Evaluating the splittings using the mass bounds from Sec. 3 (Eq. (10)) yields the representative values

$$\Delta m_{21}^{2\,\text{pred}} \approx 7.33 \times 10^{-5}\ \text{eV}^2, \text{ [CERT]} \tag{15}$$
$$\Delta m_{31}^{2\,\text{pred}} \approx 2.48 \times 10^{-3}\ \text{eV}^2. \text{ [CERT]} \tag{16}$$

As a validation check, we compare to NuFIT 5.x summary windows for normal ordering [2]. At the level of precision used in this paper, the predictions satisfy:

$$7.21 \times 10^{-5} < \Delta m_{21}^{2\,\text{pred}} < 7.62 \times 10^{-5}\ \text{eV}^2,\ \text{[VALIDATION]} \tag{17}$$

$$2.455 \times 10^{-3} < \Delta m_{31}^{2\,\text{pred}} < 2.567 \times 10^{-3}\ \text{eV}^2.\ \text{[VALIDATION]} \tag{18}$$

These comparisons are strictly validation: the NuFIT windows are not used to set the rungs or the calibration seam. [VALIDATION]

Classical correspondence. The mass-squared splittings Δm_{21}^2 and Δm_{31}^2 are mathematically identical to the standard oscillation observables used in NuFIT and PDG summaries (Twin status). The key structural addition is Eq. (14): the *ratio* of splittings is seam-free because the global scale κ_{eV}^2 cancels. This mirrors how mass-ratio predictions in QFT are often more robust than absolute-scale predictions, since renormalization-scale dependence cancels in ratios. The framework predicts concrete numerical values for both splittings (Bridge to oscillation phenomenology), but the falsifiable core is the seam-free ratio. [HYPOTHESIS]

5 The φ^7 Ratio

Section summary. The deep rung spacing implies an *exact* φ-power relation among the neutrino squared masses. This exact ratio is independent of the eV calibration seam. We also record the induced (seam-free) closed form for the ratio of the measured mass-squared splittings.

5.1 An exact squared-mass ratio from rung differences

From the mass law $m_i^{\text{pred}} = \kappa_{\text{eV}}\varphi^{r_i}$ (Eq. (7)), the seam cancels in squared-mass ratios:

$$\frac{\left(m_3^{\text{pred}}\right)^2}{\left(m_2^{\text{pred}}\right)^2} = \frac{\kappa_{\text{eV}}^2\varphi^{2r_3}}{\kappa_{\text{eV}}^2\varphi^{2r_2}} = \varphi^{2(r_3-r_2)}.\ \text{[PROVED]} \tag{19}$$

Under the specific deep rung assignment of Eq. (5), the rung gap is

$$r_3 - r_2 = \frac{7}{2}.\ \text{[HYPOTHESIS]} \tag{20}$$

Substituting (20) into (19) yields the advertised exact ratio:

$$\frac{\left(m_3^{\text{pred}}\right)^2}{\left(m_2^{\text{pred}}\right)^2} = \varphi^7.\ \text{[HYPOTHESIS]} \tag{21}$$

Equivalently, $m_3^{\text{pred}}/m_2^{\text{pred}} = \varphi^{7/2}$. [HYPOTHESIS]

5.2 Induced prediction for the splitting ratio (seam-free)

While the squared-mass ratio is a pure φ-power, the *splitting* ratio depends on m_1 as well. Using Eq. (14) together with the rung differences from Eq. (5): $r_2 - r_1 = 2$ and $r_3 - r_1 = 11/2$, one obtains the closed form

$$\frac{\Delta m_{31}^{2\,\text{pred}}}{\Delta m_{21}^{2\,\text{pred}}} = \frac{\varphi^{11}-1}{\varphi^4-1} \approx 33.823.\ \text{[HYPOTHESIS]} \tag{22}$$

This ratio is *seam-free*: it depends only on the discrete rung differences and on φ, not on κ_{eV}. [PROVED]Its agreement with experimental summaries is assessed as validation (Sec. 4 and Sec. 8). [VALIDATION]

Classical correspondence. The exact relation $(m_3^{\mathrm{pred}})^2/(m_2^{\mathrm{pred}})^2 = \varphi^7$ is the neutrino-sector analog of the mass-family-ratio predictions in Paper 1: when two species share the same ladder base, their mass ratio is a pure φ-power determined by the rung difference. This corresponds to the general phenomenological observation that hierarchical mass spectra often exhibit geometric-progression structure (Bridge to Yukawa texture models). The crucial feature is that the seam cancels: the ratio is a dimensionless, convention-free prediction testable by oscillation experiments alone. This is the falsifiable core of the deep-ladder hypothesis. [HYPOTHESIS]

6 Normal Hierarchy from Geometry

Section summary. The ordering of neutrino masses (normal vs inverted) is an experimental question, but in the deep-ladder framework the ordering is not an independent fit knob: it is fixed by the rung ordering together with monotonicity of the φ-ladder map. We record this implication explicitly and state what would falsify it.

6.1 Monotonicity of the ladder map

The ladder base satisfies $\varphi > 1$. [PROVED]For any fixed $\kappa_{\mathrm{eV}} > 0$, the mapping

$$r \mapsto m(r) := \kappa_{\mathrm{eV}}\, \varphi^r \text{ [PROVED]} \tag{23}$$

is strictly increasing in r. [PROVED]Therefore rung ordering implies mass ordering. [PROVED]

6.2 Normal ordering implied by the deep rungs

Section 2 fixes the neutrino rungs (r_1, r_2, r_3) with

$$r_1 < r_2 < r_3. \text{ [HYPOTHESIS]} \tag{24}$$

Combining Eq. (24) with the monotonicity of Eq. (23) yields

$$m_1^{\mathrm{pred}} < m_2^{\mathrm{pred}} < m_3^{\mathrm{pred}}. \text{ [PROVED]} \tag{25}$$

Thus, within this framework, "normal ordering" is not a choice made to match an external fit; it is the direct consequence of the discrete rung assignment. [HYPOTHESIS]

6.3 Validation and falsifier

Global oscillation analyses currently favor normal ordering, but the ordering remains an experimental output rather than an input to this paper. [VALIDATION]If future oscillation and matter-effect measurements decisively establish inverted ordering, then the deep rung hypothesis (and in particular the rung triple of Eq. (5)) is refuted. [VALIDATION]

Classical correspondence. The statement "rung ordering implies mass ordering" has no direct classical analog: in continuum field theory, mass eigenvalues can be permuted freely by relabeling. The closest conceptual relative is the constraint that quantum numbers (e.g. principal quantum number in atomic physics) order energy levels. Here the rung triple is a discrete topological input (T9), and the monotonicity of φ^r enforces a rigid mass ordering without additional fitting. This is a *Novel* structural claim: the ordering is not a choice but a consequence of the deep-ladder assignment. [HYPOTHESIS]

7 Cosmological Constraints

Section summary. Cosmological data constrain neutrino masses primarily through the sum $\sum m_\nu$. In the deep-ladder framework, $\sum m_\nu$ is predicted once the rung triple and the global eV reporting seam are fixed. Cosmological bounds are model-dependent and are used only for validation, never as an input.

7.1 What cosmology constrains

In standard cosmological analyses, the leading sensitivity to neutrino masses is through the total mass sum

$$\Sigma_\nu \ := \ m_1 + m_2 + m_3. \text{ [PROVED]} \tag{26}$$

The exact numerical bound on Σ_ν depends on the assumed cosmological model (e.g. ΛCDM vs extensions) and the datasets included. For this reason, we treat cosmological constraints strictly as validation checks rather than as part of the model layer. [VALIDATION]

7.2 Deep-ladder prediction for the mass sum

Section 3 derived the predicted mass-sum window under the declared eV seam:

$$0.06263 < \Sigma_\nu^{\mathrm{pred}} < 0.06276 \text{ eV. [CERT]} \tag{27}$$

This value is not obtained by fitting cosmological data; it is implied by the rung triple and the single global reporting seam. [CERT]

7.3 Validation against current cosmological bounds

The Particle Data Group summarizes cosmological limits on Σ_ν and emphasizes their model dependence [1]. [VALIDATION]Using representative current bounds (typically at the $\Sigma_\nu \lesssim 0.12\,\mathrm{eV}$ level in ΛCDM-like analyses), the predicted range (27) is comfortably allowed. [VALIDATION]Future tightening of cosmological bounds toward $\Sigma_\nu < 0.06\,\mathrm{eV}$ would directly pressure or refute the deep-ladder mass scale. [VALIDATION]

8 Falsifiers

This section lists experimental outcomes that would refute the deep-ladder hypothesis class proposed in this paper. We distinguish *seam-free* falsifiers (independent of the eV calibration seam) from *scale* falsifiers (which test the declared eV reporting seam). [PROVED]

8.1 Seam-free falsifiers (depend only on rung differences and φ)

F1: splitting-ratio mismatch. Define the experimentally inferred splitting ratio

$$R_\Delta \; := \; \frac{\Delta m^2_{31}}{\Delta m^2_{21}}. \text{ [PROVED]} \tag{28}$$

Under the rung triple of Eq. (5), the model predicts the seam-free value

$$R_\Delta^{\text{pred}} \; = \; \frac{\varphi^{11}-1}{\varphi^4-1} \; \approx \; 33.823. \text{ [HYPOTHESIS]} \tag{29}$$

This hypothesis is falsified if the best-fit R_Δ inferred from oscillation data (for the stated ordering and dataset release) becomes inconsistent with R_Δ^{pred} beyond the quoted experimental uncertainty. [VALIDATION]

F2: ordering mismatch. The deep rungs are ordered $r_1 < r_2 < r_3$ (Eq. (24)), which implies normal mass ordering $m_1 < m_2 < m_3$ (Eq. (25)). [HYPOTHESIS]If future oscillation and matter-effect measurements decisively establish inverted ordering, the rung triple hypothesis is refuted. [VALIDATION]

F3: squared-mass ratio mismatch (requires absolute-mass information). The rung gap $r_3 - r_2 = 7/2$ implies the exact squared-mass ratio $(m_3^{\text{pred}})^2/(m_2^{\text{pred}})^2 = \varphi^7$ (Eq. (21)). [HYPOTHESIS]If future absolute-mass information (together with ordering identification) determines m_3^2/m_2^2 in a way that excludes φ^7, this rung-gap hypothesis is refuted. [VALIDATION]

8.2 Scale falsifiers (test the declared eV reporting seam)

F4: exclusion by oscillation windows for Δm^2. The deep ladder predicts specific eV-scale splittings (Sec. 4) once the global seam is fixed. [CERT]If updated NuFIT (or successor) summary windows for the stated ordering exclude $\Delta m^2_{21}{}^{\text{pred}}$ or $\Delta m^2_{31}{}^{\text{pred}}$ at high significance, then either the rung triple or the declared eV seam is refuted. [VALIDATION]

F5: cosmological exclusion of Σ_ν. The predicted mass sum is $\Sigma_\nu^{\text{pred}} \approx 0.0627\,\text{eV}$ (Eq. (27)). [CERT]If cosmological analyses (under clearly stated model assumptions) establish an upper bound $\Sigma_\nu < 0.0626\,\text{eV}$, then the deep-ladder mass scale is ruled out. [VALIDATION]

F6: direct absolute-mass detection above the predicted scale. Any direct kinematic or laboratory measurement that robustly implies a neutrino mass scale well above the predicted window of Eq. (10) (under the same declared reporting seam) refutes the deep-ladder mass assignment. [VALIDATION]

9 Conclusions

This paper presented a deep-ladder neutrino mass framework in which neutrino masses occupy fractional rungs of the φ-ladder. The goal was not to "fit" oscillation numbers, but to propose a small, auditable set of discrete inputs (a rung triple and a rung lattice) whose consequences are sharply falsifiable. [HYPOTHESIS]

9.1 What is structural vs what is a seam vs what is hypothesized

Structural (mathematics). Once a ladder base $\varphi > 1$ is fixed, ladder coordinates turn ratios into differences, and seam parameters cancel from ratios (Secs. 2,4–5). [PROVED]In particular, the ratio of mass-squared splittings depends only on rung differences and φ, not on the eV reporting seam. [PROVED]

Declared seam (units). Absolute masses in eV require an explicit reporting seam (Sec. 3), represented by a single global scalar τ_0 (or equivalently κ_{eV}). This seam is fixed framework-wide and is not a neutrino-sector tuning knob. [CERT]

Model hypotheses (physics). The physical content of this paper is the deep rung lattice choice $r \in \frac{1}{4}\mathbb{Z}$ and the specific rung triple (r_1, r_2, r_3) used for neutrinos (Sec. 2), together with the deep-ladder mass law $m_i^{\mathrm{pred}} = \kappa_{\mathrm{eV}}\varphi^{r_i}$ (Sec. 3). [HYPOTHESIS]These hypotheses are not asserted as inevitable consequences of the Standard Model; they are proposed structural laws to be tested. [HYPOTHESIS]

9.2 What the validation indicates

Under the declared seam and rung triple, the predicted Δm^2_{21} and Δm^2_{31} lie within standard NuFIT summary windows at current precision (Sec. 4). [VALIDATION]The predicted mass sum $\Sigma_\nu^{\mathrm{pred}} \approx 0.0627\,\mathrm{eV}$ is consistent with representative cosmological bounds (Sec. 7), with the caveat that such bounds are model-dependent. [VALIDATION]

9.3 The core falsifiers

The most robust tests are seam-free:

- the splitting ratio $R_\Delta = \Delta m^2_{31}/\Delta m^2_{21}$ (Eq. (29)), [VALIDATION]
- the ordering prediction (normal ordering implied by rung ordering; Eq. (25)), [VALIDATION]
- and, once absolute-mass information becomes available, the squared-mass ratio implied by the deep rung gap (Eq. (21)). [VALIDATION]

Additional falsifiers test the absolute scale, including future cosmological tightening and direct absolute-mass probes (Sec. 8). [VALIDATION]

9.4 Next steps

The remaining work for completing Paper 3 is to document the interval/rounding bounds used for the φ-power evaluations in Appendix A, so that each quoted numerical window can be audited from a single place. [PROVED]

A Interval Bounds (for auditing)

This appendix records the explicit numerical interval bounds used to produce the certified windows in the main text. All bounds are stated as outward-rounded intervals (to avoid over-claiming precision) and are treated as certified numerical facts. [CERT]

A.1 Tight bounds for φ and quarter steps

We use the following tight bracket for the golden ratio:

$$1.61803395 < \varphi < 1.6180340. \text{ [CERT]} \tag{30}$$

From this one may conservatively bound any fixed integer power φ^n by monotonicity. [PROVED]

For quarter-step reasoning, we also record a bound on $\varphi^{1/4}$:

$$1.12783847 < \varphi^{1/4} < 1.12783849. \text{ [CERT]} \tag{31}$$

A.2 Certified bounds for the deep-ladder φ-powers

The neutrino rungs in Eq. (5) require the following negative quarter-rung powers:

$$4.594 \times 10^{-12} < \varphi^{-217/4} < 4.598 \times 10^{-12}, \text{ [CERT]} \tag{32}$$

$$8.515 \times 10^{-13} < \varphi^{-231/4} < 8.538 \times 10^{-13}. \text{ [CERT]} \tag{33}$$

We also use a coarse bound for φ^{-2}:

$$0.3818 < \varphi^{-2} < 0.382. \text{ [CERT]} \tag{34}$$

Combining Eqs. (33) and (34) yields a conservative interval for $\varphi^{-239/4} = \varphi^{-231/4}\varphi^{-2}$:

$$3.251 \times 10^{-13} < \varphi^{-239/4} < 3.262 \times 10^{-13}. \text{ [CERT]} \tag{35}$$

A.3 Pinned eV seam used for the reported windows

To evaluate absolute masses in eV, the paper uses the global seam constant κ_{eV} introduced in Eq. (6). [CERT]For the numerical windows reported in Sec. 3, we pin:

$$\kappa_{\text{eV}} \ := \ \left(2^{-22}\varphi^{51}\right) \times 10^6 \text{ eV} \ \approx \ 1.0856997758 \times 10^{10} \text{ eV}. \text{ [CERT]} \tag{36}$$

Equivalently, this corresponds to the single-scalar tick seam $\tau_0 = \hbar/(\kappa_{\text{eV}} \cdot 1\,\text{eV}) \approx 6.0626 \times 10^{-26}\,\text{s}$. [CERT]

A.4 Derived certified windows used in the main text

Combining Eq. (36) with Eqs. (32)–(35) yields the mass windows:

$$0.0035296 < m_1^{\text{pred}} < 0.0035411 \text{ eV}, \text{ [CERT]} \tag{37}$$

$$0.0092447 < m_2^{\text{pred}} < 0.0092698 \text{ eV}, \text{ [CERT]} \tag{38}$$

$$0.0498770 < m_3^{\text{pred}} < 0.0499206 \text{ eV}. \text{ [CERT]} \tag{39}$$

From these, conservative propagation gives splitting windows:

$$7.2926 \times 10^{-5} < \Delta m_{21}^{2\ \text{pred}} < 7.3469 \times 10^{-5} \text{ eV}^2, \text{ [CERT]} \tag{40}$$

$$2.47518 \times 10^{-3} < \Delta m_{31}^{2\ \text{pred}} < 2.47960 \times 10^{-3} \text{ eV}^2. \text{ [CERT]} \tag{41}$$

A.5 Seam-free ratios

Using the rung differences implied by Eq. (5), the splitting ratio has the seam-free closed form of Eq. (22). Bounding φ by Eq. (30) yields

$$33.823291 < \frac{\varphi^{11} - 1}{\varphi^4 - 1} < 33.823298. \text{ [CERT]} \tag{42}$$

Reproducibility note. For a quick numerical cross-check (not a substitute for interval bounds), the repository includes `scripts/neutrino_mass_scale_validate.py`. [PROVED]

B Reproducibility

This appendix summarizes the minimal commands needed to (i) re-evaluate the headline numerical values quoted in the main text and Appendix A, and (ii) rebuild the PDF. All computations are straightforward evaluations of the closed-form expressions already stated in the paper; there is no per-eigenstate fitting workflow. [PROVED]

B.1 Quick numerical cross-check (floating point)

From the repository root, run:

```
python3 scripts/neutrino_mass_scale_validate.py
```

This script prints:

- the implied absolute masses m_1, m_2, m_3 under the declared eV reporting seam,
- the splittings Δm^2_{21}, Δm^2_{31}, and their ratio,
- and basic consistency checks against representative cosmology and direct-mass bounds.

This is a *floating-point* check only; the outward-rounded certified intervals used in the paper are the explicit brackets recorded in Appendix A. [CERT]

B.2 Compiling the PDF

Compile Paper 3 from `papers/tex/`:

```
cd papers/tex
pdflatex -interaction=nonstopmode -output-directory=../pdf masses_paper3_neutrinos.tex
pdflatex -interaction=nonstopmode -output-directory=../pdf masses_paper3_neutrinos.tex
```

The resulting PDF is written to `papers/pdf/masses_paper3_neutrinos.pdf`. [PROVED]

References

[1] Particle Data Group, *Review of Particle Physics* (2024 edition).

[2] NuFIT Collaboration, *Neutrino oscillation global fit results* (NuFIT 5.x).

[3] CODATA, *Recommended values of the fundamental physical constants* (latest release used in comparisons).