

Reproducibility Audit Report

Environment

Date (UTC): 2026-02-02T05:08:34Z
Host: 163-192-15-161
Platform: Linux-6.8.0-1040-nvidia-x86_64-with-glibc2.35
Kernel: Linux 163-192-15-161 6.8.0-1040-nvidia #43~22.04.1-Ubuntu SMP
PREEMPT_DYNAMIC Fri Sep 19 23:28:27 UTC 2 x86_64 x86_64 x86_64 GNU/Linux
Architecture: x86_64
Python: 3.10.12
Pip: pip 26.0 from /home/ubuntu/Riemann/venv/lib/python3.10/site-packages/pip
(python 3.10)
python-flint: 0.6.0
numpy: 2.2.6

Summary of Certified Results

Zeta Nonvanishing Certificates

Domain | ok | processed_boxes | min_abs_lower
[0.6, 0.7] x [0, 20] | True | 8860 | 9.432519465010592e-06
[0.7, 0.999] x [0, 20] | True | 6648 | 1.998376212072482e-05

Pick Certificates

sigma0 | delta_cert | P_spd_at_0 | coeff_count
0.599 | 0.5944375202575065 | True | n/a
0.6 | 0.5948779178978381 | True | n/a
0.7 | 0.6273368612135527 | True | n/a

Commands Executed

```
python3 verify_attachment_arb.py --zeta-certify --rect-sigma-min 0.6
--rect-sigma-max 0.7 --rect-t-min 0.0 --rect-t-max 20.0 --theta-init-n-sigma 10
--theta-init-n-t 50 --theta-min-sigma-width 0.0001 --theta-min-t-width 0.001
--theta-max-boxes 500000 --theta-time-limit 0 --prec 260 --zeta-out
audit_zeta_left.json --progress
python3 verify_attachment_arb.py --zeta-certify --rect-sigma-min 0.7
--rect-sigma-max 0.999 --rect-t-min 0.0 --rect-t-max 20.0 --theta-init-n-sigma
10 --theta-init-n-t 50 --theta-min-sigma-width 0.0001 --theta-min-t-width 0.001
--theta-max-boxes 500000 --theta-time-limit 0 --prec 260 --zeta-out
audit_zeta_right.json --progress
python3 verify_attachment_arb.py --pick-certify --pick-sigma0 0.599 --pick-N 16
--pick-coeff-count 128 --pick-K 512 --pick-rho 0.4 --pick-rho-bound 0.5
--arith-gauge raw_zeta --arith-P-cut 2000 --prec 1024 --pick-out
audit_pick_sigma0599_raw_zeta_N16.json
python3 verify_attachment_arb.py --pick-certify --pick-sigma0 0.6 --pick-N 16
--pick-coeff-count 128 --pick-K 512 --pick-rho 0.4 --pick-rho-bound 0.5
--arith-gauge raw_zeta --arith-P-cut 2000 --prec 1024 --pick-out
audit_pick_sigma06_raw_zeta_N16.json
python3 verify_attachment_arb.py --pick-certify --pick-sigma0 0.7 --pick-N 16
--pick-coeff-count 128 --pick-K 512 --pick-rho 0.4 --pick-rho-bound 0.5
--arith-gauge raw_zeta --arith-P-cut 2000 --outer-mode rigorous --outer-P-cut
2000 --prec 1024 --pick-out audit_pick_sigma07_raw_zeta_N16.json
```

Artifact Files and SHA256

```
reproduce_log: reproduce.log
724f5f3850c3120b275ada2e73cf95575002878f19ce46ee95e4892e9d069573
pick_log_0599: pick_sigma0599.log
022a8e13008c6b543f2e430b411d94adba688338a5861070efb4436bad91871c
pick_log_06: pick_sigma06.log
08f0324b41f381e028652ae66d81a90994338bd01dc82cf01a2b96c8c9b77743
pick_log_07: pick_sigma07.log
a73292cbd89f8769cec8ae5b4d2234c80113039080897237b427f3e7368fcfa1
zeta_left_json: audit_zeta_left.json
cad107190444d818cc7c9dfc5c5dc5d71b2597808e4ea1dec937bed017ad2491
zeta_right_json: audit_zeta_right.json
74690f7a3efd70f8cd35e43f6c056c547d5b6c1580cea6815a2f87e235252352
pick_0599_json: audit_pick_sigma0599_raw_zeta_N16.json
b37a5f5aca1b883f4d472e4e8e7740f191a282dd39fe1f2c5c477106a158e117
pick_06_json: audit_pick_sigma06_raw_zeta_N16.json
49e305d4a427e2901d47427a13cd282aa76572ac0b0822255485598d5ca48694
pick_07_json: audit_pick_sigma07_raw_zeta_N16.json
8a2c37531bb5831873beaa54ead1284c440f12ce84ccea41e809b7c984b38e00
```

Appendix: Logs

reproduce.log

```
=== Starting Independent Reproduction Audit ===
Date: Mon Feb  2 04:12:53 UTC 2026
Host: 163-192-15-161
--- System Information ---
Linux 163-192-15-161 6.8.0-1040-nvidia #43~22.04.1-Ubuntu SMP PREEMPT_DYNAMIC
Fri Sep 19 23:28:27 UTC 2 x86_64 x86_64 x86_64 GNU/Linux
Python 3.10.12
--- Setting up Virtual Environment ---
Requirement already satisfied: pip in ./venv/lib/python3.10/site-packages
(22.0.2)
Collecting pip
  Using cached pip-26.0-py3-none-any.whl (1.8 MB)
Installing collected packages: pip
  Attempting uninstall: pip
    Found existing installation: pip 22.0.2
    Uninstalling pip-22.0.2:
      Successfully uninstalled pip-22.0.2
Successfully installed pip-26.0
Collecting python-flint==0.6.0 (from -r requirements.txt (line 1))
  Using cached python_flint-0.6.0-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata (8.4 kB)
Collecting numpy (from -r requirements.txt (line 2))
  Using cached
    numpy-2.2.6-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata
    a (62 kB)
Using cached
python_flint-0.6.0-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl
(41.1 MB)
Using cached
numpy-2.2.6-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (16.8 MB)
Installing collected packages: python-flint, numpy

Successfully installed numpy-2.2.6 python-flint-0.6.0
```

```

--- 1/2: Verifying Low-Height Zeta Non-Vanishing (Left Rectangle) ---

[zeta] certify zeta(s) != 0 on: [0.6,0.7] x [0.0,20.0]
[zeta] caps: time_limit=0.0s, max_boxes=500000, min_sigma_w=0.0001,
min_t_w=0.001
[zeta] passed 2000 boxes (processed 3721, stack 221, elapsed 0.4s)
[zeta] passed 4000 boxes (processed 7542, stack 42, elapsed 0.8s)

[zeta] DONE. ok=True passed_boxes=4680 processed=8860 refined_splits=4180
elapsed=0.9s
[zeta] zeta(s): contains0_hits=4180, min_abs_lower~9.43252e-06
[zeta] wrote JSON: audit_zeta_left.json
--- 2/2: Verifying Low-Height Zeta Non-Vanishing (Right Rectangle) ---

[zeta] certify zeta(s) != 0 on: [0.7,0.999] x [0.0,20.0]
[zeta] caps: time_limit=0.0s, max_boxes=500000, min_sigma_w=0.0001,
min_t_w=0.001
[zeta] passed 2000 boxes (processed 3588, stack 88, elapsed 0.4s)

[zeta] DONE. ok=True passed_boxes=3574 processed=6648 refined_splits=3074
elapsed=0.7s
[zeta] zeta(s): contains0_hits=3074, min_abs_lower~1.99838e-05
[zeta] wrote JSON: audit_zeta_right.json
=== Audit Complete: SUCCESS ===

```

pick_sigma0599.log

```

[pick_certify] wrote artifact: audit_pick_sigma0599_raw_zeta_N16.json
[pick_certify] certified delta >= 0.594438 for Pick minor size N=16

```

pick_sigma06.log

```

[pick_certify] wrote artifact: audit_pick_sigma06_raw_zeta_N16.json
[pick_certify] certified delta >= 0.594878 for Pick minor size N=16

```

pick_sigma07.log

```

[pick_certify] wrote artifact: audit_pick_sigma07_raw_zeta_N16.json
[pick_certify] certified delta >= 0.627337 for Pick minor size N=16

```

Appendix: JSON Artifacts

audit_zeta_left.json

```

{
  "args": {
    "P": 31,
    "arith_P_cut": null,
    "arith_det2_kmax": 40,
    "arith_det2_mode": "trunc",
    "arith_gauge": "raw_zeta",
    "artifact_out": null,
    "audit_phase": false,
    "audit_phase_budget_rad": null,
    "audit_phase_object": "J",
    "audit_phase_out": null,
    "audit_phase_sigmas": null,

```

```

"audit_phase_t_max": null,
"audit_phase_t_min": null,
"audit_phase_t_step": 0.02,
"cbnf_C_box_nf_upper": null,
"cbnf_Cpsi_upper": null,
"cbnf_alpha": null,
"cbnf_certify": false,
"cbnf_out": null,
"cbnf_sigma0": 0.6,
"cert_amp_shift": 0.5,
"cert_coeff_mode": "printed",
"cert_port": "const",
"cert_psi_scale": 1.0,
"cert_weights_mode": "printed",
"cert_weights_r": 0.8947368421052632,
"cover_n_sigma": 20,
"cover_n_t": 40,
"debug_point_sigma": null,
"debug_point_t": null,
"eval_mode": "rigorous",
"gauge_sweep": false,
"max_refines": 6,
"n_mode": 4,
"outer_P_cut": 20000,
"outer_T": 10.0,
"outer_build_only": false,
"outer_cache_ignore_mismatch": false,
"outer_cache_load": null,
"outer_cache_save": null,
"outer_max_depth": 12,
"outer_min_width": 0.001,
"outer_mode": "rigorous",
"outer_n": 200,
"outer_proj_abs_tol": 0.0,
"outer_proj_recompute_det2": false,
"outer_proj_rel_tol": 0.0,
"outer_sigma_ref": 0.55,
"pass_metric": "J",
"pick_K": 64,
"pick_M_bound": null,
"pick_N": 16,
"pick_certify": false,
"pick_coeff_count": null,
"pick_delta_hi": 0.99,
"pick_delta_steps": 40,
"pick_demo": false,
"pick_out": null,
"pick_rho": 0.001,
"pick_rho_bound": 0.2,
"pick_sigma0": 0.6,
"prec": 260,
"progress": true,
"psihat_steps": 4000,
"rect_sigma_max": 0.7,
"rect_sigma_min": 0.6,
"rect_t_max": 20.0,
"rect_t_min": 0.0,
"scan_only": false,
"scan_out": null,

```

```

"scan_sigma": null,
"scan_t_max": 0.0,
"scan_t_min": 0.0,
"scan_t_step": 1.0,
"sigma0": 0.6,
"sweep_P_cuts": "2000,5000",
"sweep_Ts": "20,50,100",
"sweep_cover_n_sigma": 8,
"sweep_cover_n_t": 32,
"sweep_det2_mode": "trunc",
"sweep_gauges": "outer_zeta_log,outer_zeta_proj,outer_zeta,raw_zeta",
"sweep_max_cases": 60,
"sweep_node_step": 0.08,
"sweep_out": null,
"sweep_rect_sigma_max": 0.7,
"sweep_rect_sigma_min": 0.6,
"sweep_rect_t_max": 20.0,
"sweep_rect_t_min": 0.0,
"sweep_sigma_refs": "0.501,0.505,0.51,0.52,0.55,0.58",
"sweep_time_limit": 0.0,
"sweep_top": 15,
"theta_certify": false,
"theta_certify_cover": null,
"theta_init_n_sigma": 10,
"theta_init_n_t": 50,
"theta_max_boxes": 500000,
"theta_min_sigma_width": 0.0001,
"theta_min_t_width": 0.001,
"theta_out": null,
"theta_progress_every": 2000,
"theta_resume_in": null,
"theta_time_limit": 0.0,
"zeta_cancel_pole": false,
"zeta_certify": true,
"zeta_out": "audit_zeta_left.json"
},
"env": {
  "flint_ctx_prec": 260,
  "platform": "Linux-6.8.0-1040-nvidia-x86_64-with-glibc2.35",
  "python": "3.10.12 (main, Aug 15 2025, 14:32:43) [GCC 11.4.0]",
  "python_flint": "0.6.0"
},
"failures": [],
"kind": "zeta_certify_artifact",
"rect": {
  "sigma_max": 0.7,
  "sigma_min": 0.6,
  "t_max": 20.0,
  "t_min": 0.0
},
"results": {
  "elapsed_seconds": 0.9340801239013672,
  "ok": true,
  "passed_boxes": 4680,
  "processed_boxes": 8860,
  "refined_splits": 4180
},
"version": 1,
"zeta": {

```

```

    "contains0_hits": 4180,
    "min_abs_lower": 9.432519465010592e-06,
    "object": "zeta"
  }
}

```

audit_zeta_right.json

```

{
  "args": {
    "P": 31,
    "arith_P_cut": null,
    "arith_det2_kmax": 40,
    "arith_det2_mode": "trunc",
    "arith_gauge": "raw_zeta",
    "artifact_out": null,
    "audit_phase": false,
    "audit_phase_budget_rad": null,
    "audit_phase_object": "J",
    "audit_phase_out": null,
    "audit_phase_sigmas": null,
    "audit_phase_t_max": null,
    "audit_phase_t_min": null,
    "audit_phase_t_step": 0.02,
    "cbnf_C_box_nf_upper": null,
    "cbnf_Cpsi_upper": null,
    "cbnf_alpha": null,
    "cbnf_certify": false,
    "cbnf_out": null,
    "cbnf_sigma0": 0.6,
    "cert_amp_shift": 0.5,
    "cert_coeff_mode": "printed",
    "cert_port": "const",
    "cert_psi_scale": 1.0,
    "cert_weights_mode": "printed",
    "cert_weights_r": 0.8947368421052632,
    "cover_n_sigma": 20,
    "cover_n_t": 40,
    "debug_point_sigma": null,
    "debug_point_t": null,
    "eval_mode": "rigorous",
    "gauge_sweep": false,
    "max_refines": 6,
    "n_mode": 4,
    "outer_P_cut": 20000,
    "outer_T": 10.0,
    "outer_build_only": false,
    "outer_cache_ignore_mismatch": false,
    "outer_cache_load": null,
    "outer_cache_save": null,
    "outer_max_depth": 12,
    "outer_min_width": 0.001,
    "outer_mode": "rigorous",
    "outer_n": 200,
    "outer_proj_abs_tol": 0.0,
    "outer_proj_recompute_det2": false,
    "outer_proj_rel_tol": 0.0,
    "outer_sigma_ref": 0.55,

```

```

"pass_metric": "J",
"pick_K": 64,
"pick_M_bound": null,
"pick_N": 16,
"pick_certify": false,
"pick_coeff_count": null,
"pick_delta_hi": 0.99,
"pick_delta_steps": 40,
"pick_demo": false,
"pick_out": null,
"pick_rho": 0.001,
"pick_rho_bound": 0.2,
"pick_sigma0": 0.6,
"prec": 260,
"progress": true,
"psihat_steps": 4000,
"rect_sigma_max": 0.999,
"rect_sigma_min": 0.7,
"rect_t_max": 20.0,
"rect_t_min": 0.0,
"scan_only": false,
"scan_out": null,
"scan_sigma": null,
"scan_t_max": 0.0,
"scan_t_min": 0.0,
"scan_t_step": 1.0,
"sigma0": 0.6,
"sweep_P_cuts": "2000,5000",
"sweep_Ts": "20,50,100",
"sweep_cover_n_sigma": 8,
"sweep_cover_n_t": 32,
"sweep_det2_mode": "trunc",
"sweep_gauges": "outer_zeta_log,outer_zeta_proj,outer_zeta,raw_zeta",
"sweep_max_cases": 60,
"sweep_node_step": 0.08,
"sweep_out": null,
"sweep_rect_sigma_max": 0.7,
"sweep_rect_sigma_min": 0.6,
"sweep_rect_t_max": 20.0,
"sweep_rect_t_min": 0.0,
"sweep_sigma_refs": "0.501,0.505,0.51,0.52,0.55,0.58",
"sweep_time_limit": 0.0,
"sweep_top": 15,
"theta_certify": false,
"theta_certify_cover": null,
"theta_init_n_sigma": 10,
"theta_init_n_t": 50,
"theta_max_boxes": 500000,
"theta_min_sigma_width": 0.0001,
"theta_min_t_width": 0.001,
"theta_out": null,
"theta_progress_every": 2000,
"theta_resume_in": null,
"theta_time_limit": 0.0,
"zeta_cancel_pole": false,
"zeta_certify": true,
"zeta_out": "audit_zeta_right.json"
},
"env": {

```

```

    "flint_ctx_prec": 260,
    "platform": "Linux-6.8.0-1040-nvidia-x86_64-with-glibc2.35",
    "python": "3.10.12 (main, Aug 15 2025, 14:32:43) [GCC 11.4.0]",
    "python_flint": "0.6.0"
  },
  "failures": [],
  "kind": "zeta_certify_artifact",
  "rect": {
    "sigma_max": 0.999,
    "sigma_min": 0.7,
    "t_max": 20.0,
    "t_min": 0.0
  },
  "results": {
    "elapsed_seconds": 0.6795811653137207,
    "ok": true,
    "passed_boxes": 3574,
    "processed_boxes": 6648,
    "refined_splits": 3074
  },
  "version": 1,
  "zeta": {
    "contains0_hits": 3074,
    "min_abs_lower": 1.998376212072482e-05,
    "object": "zeta"
  }
}

```

audit_pick_sigma0599_raw_zeta_N16.json

```

{
  "args": {
    "P": 31,
    "arith_P_cut": 2000,
    "arith_det2_kmax": 40,
    "arith_det2_mode": "trunc",
    "arith_gauge": "raw_zeta",
    "artifact_out": null,
    "audit_phase": false,
    "audit_phase_budget_rad": null,
    "audit_phase_object": "J",
    "audit_phase_out": null,
    "audit_phase_sigmas": null,
    "audit_phase_t_max": null,
    "audit_phase_t_min": null,
    "audit_phase_t_step": 0.02,
    "cbnf_C_box_nf_upper": null,
    "cbnf_Cpsi_upper": null,
    "cbnf_alpha": null,
    "cbnf_certify": false,
    "cbnf_out": null,
    "cbnf_sigma0": 0.6,
    "cert_amp_shift": 0.5,
    "cert_coeff_mode": "printed",
    "cert_port": "const",
    "cert_psi_scale": 1.0,
    "cert_weights_mode": "printed",
    "cert_weights_r": 0.8947368421052632,

```

```

"cover_n_sigma": 20,
"cover_n_t": 40,
"debug_point_sigma": null,
"debug_point_t": null,
"eval_mode": "rigorous",
"gauge_sweep": false,
"max_refines": 6,
"n_mode": 4,
"outer_P_cut": 20000,
"outer_T": 10.0,
"outer_build_only": false,
"outer_cache_ignore_mismatch": false,
"outer_cache_load": null,
"outer_cache_save": null,
"outer_max_depth": 12,
"outer_min_width": 0.001,
"outer_mode": "rigorous",
"outer_n": 200,
"outer_proj_abs_tol": 0.0,
"outer_proj_recompute_det2": false,
"outer_proj_rel_tol": 0.0,
"outer_sigma_ref": 0.55,
"pass_metric": "J",
"pick_K": 512,
"pick_M_bound": null,
"pick_N": 16,
"pick_certify": true,
"pick_coeff_count": 128,
"pick_delta_hi": 0.99,
"pick_delta_steps": 40,
"pick_demo": false,
"pick_out": "audit_pick_sigma0599_raw_zeta_N16.json",
"pick_rho": 0.4,
"pick_rho_bound": 0.5,
"pick_sigma0": 0.599,
"prec": 1024,
"progress": false,
"psihat_steps": 4000,
"rect_sigma_max": 0.9,
"rect_sigma_min": 0.7,
"rect_t_max": 10.0,
"rect_t_min": 0.0,
"scan_only": false,
"scan_out": null,
"scan_sigma": null,
"scan_t_max": 0.0,
"scan_t_min": 0.0,
"scan_t_step": 1.0,
"sigma0": 0.6,
"sweep_P_cuts": "2000,5000",
"sweep_Ts": "20,50,100",
"sweep_cover_n_sigma": 8,
"sweep_cover_n_t": 32,
"sweep_det2_mode": "trunc",
"sweep_gauges": "outer_zeta_log,outer_zeta_proj,outer_zeta,raw_zeta",
"sweep_max_cases": 60,
"sweep_node_step": 0.08,
"sweep_out": null,
"sweep_rect_sigma_max": 0.7,

```

```

"sweep_rect_sigma_min": 0.6,
"sweep_rect_t_max": 20.0,
"sweep_rect_t_min": 0.0,
"sweep_sigma_refs": "0.501,0.505,0.51,0.52,0.55,0.58",
"sweep_time_limit": 0.0,
"sweep_top": 15,
"theta_certify": false,
"theta_certify_cover": null,
"theta_init_n_sigma": 10,
"theta_init_n_t": 50,
"theta_max_boxes": 20000,
"theta_min_sigma_width": 0.0005,
"theta_min_t_width": 0.02,
"theta_out": null,
"theta_progress_every": 2000,
"theta_resume_in": null,
"theta_time_limit": 60.0,
"zeta_cancel_pole": false,
"zeta_certify": false,
"zeta_out": null
},
"coeffs": {
  "a_im": [
    "[+/- 2.77e-45]",
    "[+/- 5.54e-45]",
    "[+/- 1.11e-44]",
    "[+/- 2.22e-44]",
    "[+/- 4.44e-44]",
    "[+/- 8.86e-44]",
    "[+/- 1.78e-43]",
    "[+/- 3.55e-43]",
    "[+/- 7.09e-43]",
    "[+/- 1.42e-42]",
    "[+/- 2.84e-42]",
    "[+/- 5.68e-42]",
    "[+/- 1.14e-41]",
    "[+/- 2.27e-41]",
    "[+/- 4.54e-41]",
    "[+/- 9.08e-41]",
    "[+/- 1.82e-40]",
    "[+/- 3.63e-40]",
    "[+/- 7.26e-40]",
    "[+/- 1.46e-39]",
    "[+/- 2.91e-39]",
    "[+/- 5.81e-39]",
    "[+/- 1.17e-38]",
    "[+/- 2.33e-38]",
    "[+/- 4.65e-38]",
    "[+/- 9.30e-38]",
    "[+/- 1.86e-37]",
    "[+/- 3.72e-37]",
    "[+/- 7.44e-37]",
    "[+/- 1.49e-36]",
    "[+/- 2.98e-36]",
    "[+/- 5.95e-36]",
    "[+/- 1.19e-35]",
    "[+/- 2.38e-35]",
    "[+/- 4.76e-35]",
    "[+/- 9.52e-35]",
  ]
}

```

"[+/- 1.91e-34] ",
 "[+/- 3.81e-34] ",
 "[+/- 7.62e-34] ",
 "[+/- 1.53e-33] ",
 "[+/- 3.05e-33] ",
 "[+/- 6.09e-33] ",
 "[+/- 1.22e-32] ",
 "[+/- 2.44e-32] ",
 "[+/- 4.88e-32] ",
 "[+/- 9.75e-32] ",
 "[+/- 1.95e-31] ",
 "[+/- 3.90e-31] ",
 "[+/- 7.80e-31] ",
 "[+/- 1.56e-30] ",
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audit_pick_sigma06_raw_zeta_N16.json

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audit_pick_sigma07_raw_zeta_N16.json

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