Charged Fermion Masses from Octave Closure and φ-Ladder Geometry

A single-anchor, no-per-flavor-tuning mass framework with explicit transport hygiene

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Abstract

The Standard Model treats the charged-fermion masses as inputs (Yukawa couplings) rather than outputs. This paper presents a structural mass framework in which charged fermion masses are organized by two ingredients: (i) an eight-tick closure that supplies a canonical "octave" reference for ladder coordinates, and (ii) a φ-ladder scale coordinate that encodes multiplicative hierarchy by fixed rung steps. At a single common anchor scale $\mu_\star$, each charged fermion is assigned an integer rung within its sector and a charge-derived band label; the mass at the anchor is then determined by sector-global yardsticks together with these discrete coordinates. No per-species fitting is permitted: the only inputs at the model layer are sector-wide integers and the charge-to-band map shared across families.

To compare to experimental conventions at other scales or schemes, Standard-Model renormalization-group running is used *only* as bookkeeping transport under a declared policy ([CERT]), and is never conflated with the structural band coordinate ([PROVED]). We report reproducible tables for the charged-lepton sector (electron, muon, tau), including accuracy against PDG values ([VALIDATION]), and provide explicit ablations and falsifiers that would refute the framework if observed.

Contents

1 Introduction

1.1 What the Standard Model does and does not explain about masses

In the Standard Model (SM), charged-fermion masses arise from Yukawa couplings. Those couplings are empirical inputs: the SM is extraordinarily successful once the numbers are specified, but it does not explain why the electron, muon, and tau have the numerical values they do, nor why the charged-fermion spectrum spans many orders of magnitude. This motivates the search for *structure*: a small set of organizing principles that constrain the spectrum without introducing per-particle tuning knobs.

1.2 Framework overview: discrete closure and ladder coordinates

The framework developed here treats stable particles as stable *recognition boundaries* whose persistence is tied to discrete closure. Two ingredients organize the mass hierarchy:

- **Octave (eight-tick closure).** A minimal closure in three dimensions yields an 8-tick cycle. In this paper, that cycle supplies a canonical reference for ladder coordinates: the appearance of a universal "-8" offset is an octave reference (a coordinate origin), not a fitted correction.

- **φ-ladder (multiplicative scale coordinate).** We represent scale hierarchies by a ladder coordinate in which integer rung shifts correspond to fixed multiplicative factors. The choice of φ is structural within the framework: it is the unique self-similar scaling factor compatible with the closure and cost constraints that define the model.

At a single common anchor scale $\mu_{\star}$, each charged fermion is described by: (i) a sector-wide yardstick (shared across all members of a sector), (ii) an integer rung, and (iii) a charge-derived band label shared across equal-charge families.

1.3 Claim contract and non-circularity (referee-facing)

This paper enforces a strict claim contract.

- **No per-species fitting.** The model layer permits only sector-global integers and shared maps (e.g. a common charge-to-band rule). There are no particle-by-particle offsets or "hand-tuned" exponents.

- **Transport hygiene.** Any comparison to Particle Data Group (PDG) conventions requires an explicit choice of scheme and scale. Standard-Model renormalization-group running is used *only* as bookkeeping transport under a declared policy and is never identified with the structural band coordinate.

- **No circular testing.** No measured mass may appear on the right-hand side of its own prediction. Where external conventions enter (e.g. PDG values), they enter only on the validation side.

Throughout, we distinguish structural derivations from declared transport conventions and from validation comparisons against external data.

1.4 What this paper delivers (and what it does not)

The main deliverables are:

- a single-anchor mass framework for charged fermions built from sector-global yardsticks and discrete coordinates,
- an explicit derivation of the charged-lepton masses (electron, muon, tau) as a reproducible absolute prediction pipeline,
- ablations and falsifiers designed to make the framework refutable rather than merely descriptive.

We will cite PDG values for comparison [1]; those comparisons are labeled as validation rather than derivation.

This paper does *not* claim that a structural band coordinate is invariant under SM running away from the anchor, and it does *not* claim that bookkeeping transport exponents are the same object as the structural band map.

1.5 Notation and classical correspondences

Table 1 summarizes the key terms used in this paper and their correspondences to standard physics concepts. This dictionary is intended to help readers familiar with SM phenomenology navigate the structural framework without requiring prior exposure to the underlying discrete-geometry program.

Term	Classical Equivalent	Status	Notes
φ-ladder	log-scale mass coordinate	Bridge	Self-similar scaling; $\varphi^2 = \varphi + 1$
Octave (8-tick)	—	Novel	Minimal 3-bit closure; no classical analog
Rung (r)	discrete scale index	Bridge	Integer step on φ-ladder
gap(Z)	family band correction	Bridge	Closed-form from charge; see Eq. (14)
f^{RG}	SM RG running	Twin	Standard QED/QCD transport
Sector yardstick	SM sector mass scale	Bridge	Discrete cube-derived input
$\mu_\star$	anchor scale (GeV)	Bridge	Common reference for all species

Table 1: Dictionary of framework terms and their classical correspondences. *Twin*: mathematically identical to the classical object. *Bridge*: corresponds via an explicit mapping or limit. *Novel*: no direct classical analog exists.

2 The Octave: Why Eight Ticks Are Forced

Section summary. This framework models the minimal stable closure in three dimensions as an *eight-step* cycle (an "octave"). Two ingredients matter: (i) a counting statement (a 3-bit context space has eight states), and (ii) a dynamics statement (atomic updates move by one-bit steps), which together motivate a Gray-adjacent 8-cycle as the canonical closure schedule. These are conditional modeling statements: if the underlying closure is not three-bit or not atomic, the Octave premise would need revision.

2.1 Minimal closure: why the period is eight in a three-bit context space

We begin with a clean separation between *what is assumed* and *what follows*.

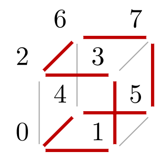

Figure 1: A 2D projection of the 3-cube with the Gray-8 cycle highlighted: $[0, 1, 3, 2, 6, 7, 5, 4]$. Each step flips exactly one bit, representing an 8-tick atomic closure schedule.

Assumption (context encoding). We assume the relevant "closure state" of a stable boundary is representable as three independent binary degrees of freedom (a 3-bit context). [HYPOTHESIS]This is the minimal discrete state space compatible with a three-dimensional cell and with a nontrivial notion of adjacency.

Counting consequence. A 3-bit context has eight distinct states:

$$2^3 = 8. \text{ [PROVED]}$$

Therefore any periodic schedule that fully covers the state space requires at least eight ticks. [PROVED]In this paper we take an 8-tick cover as the canonical closure clock used to define ladder coordinates. [HYPOTHESIS]

2.2 Gray adjacency: why one-bit steps are the natural "atomic" evolution

Assumption (atomic updates). We assume the closure evolves by a single elementary update per tick ("one posting per tick"), so the observable cannot jump arbitrarily between distant states. [HYPOTHESIS]

Adjacency consequence. Under a one-update-per-tick rule, each tick flips exactly one bit of the 3-bit context, i.e. the next state differs by one-bit adjacency (Hamming distance 1). [PROVED]In combinatorics, an ordering of all cube vertices with one-bit adjacency at each step is a *Gray code*.

Why this matters for masses (motivation only). If closure evolution is constrained to one-bit steps, then stable boundaries are naturally indexed by discrete step counts. This is the conceptual reason a rung-based ladder coordinate is appropriate in the mass framework: the hierarchy is encoded by integer step structure rather than by continuous per-particle tuning. [HYPOTHESIS]

2.3 A concrete Gray-8 cycle (the canonical octave schedule)

There are many Gray codes on the cube; the mass framework needs only the existence of an 8-step, one-bit-adjacent cycle. For concreteness we display the standard Gray-8 cycle

$$[0, 1, 3, 2, 6, 7, 5, 4],$$

which traverses all eight cube states by one-bit steps and returns to the start after eight ticks. [PROVED]

Connection to the "-8" reference. Later, when we define the mass law at the anchor, the appearance of a universal "-8" in the ladder exponent will be interpreted as a coordinate origin tied to this eight-tick closure clock, not as a fitted, particle-by-particle correction. [HYPOTHESIS]

Classical correspondence. The eight-tick minimal period (2^D with $D = 3$) has no direct classical analog: in continuum field theory, microscopic periodicity is hidden beneath coarse-grained dynamics. The closest conceptual relative is the minimal cell traversal in discrete dynamical systems, where a state space of 2^D vertices requires at least 2^D steps for a Hamiltonian path. In signal processing, the Nyquist–Shannon sampling theorem provides a related bound: faithful reconstruction of a band-limited signal requires at least two samples per period. Here the 8-tick cycle plays an analogous role as the minimal closure period for a 3-bit context. [HYPOTHESIS]

3 The φ-Ladder: Self-Similarity and Scale Coordinates

3.1 Why a multiplicative ladder is the natural coordinate for mass hierarchies

Particle masses span many orders of magnitude. In such a setting, differences are less informative than *ratios*: a model that treats "one step" as a fixed multiplicative change is more stable than a model that treats "one step" as a fixed additive change. This motivates using a logarithmic coordinate for scale.

3.2 A self-similarity constraint that singles out φ

To choose a canonical base for the ladder, we impose a simple self-similarity requirement: the growth factor should be compatible with a two-step recurrence in which the next scale decomposes into the sum of the previous two scales. [HYPOTHESIS]At the level of a unitless scaling factor x, this is captured by

$$x^2 = x + 1. \text{ [HYPOTHESIS]} \tag{1}$$

Equation (1) has a unique positive solution:

$$\varphi := \frac{1+\sqrt{5}}{2}. \text{ [PROVED]} \tag{2}$$

We use this φ as the base of the ladder coordinate. [HYPOTHESIS]The mathematical content is unambiguous (the solution exists and is unique); the modeling content is the claim that the relevant closure/self-similarity constraint for stable recognition boundaries is correctly represented by (1).

3.3 Scale coordinates in base φ

Define the base-φ logarithm as

$$\log_\varphi(x) := \frac{\ln x}{\ln \varphi}. \text{ [PROVED]} \tag{3}$$

This coordinate has the standard ratio property:

$$\log_\varphi(m_1) - \log_\varphi(m_2) = \log_\varphi\left(\frac{m_1}{m_2}\right). \text{ [PROVED]} \tag{4}$$

3.4 Rungs, step size, and what "integer" means in this paper

We will describe masses by integer *rungs* on the φ-ladder at a single anchor scale $\mu_\star$. Concretely, "rung differences" are modeled as integer differences in the scale coordinate. [HYPOTHESIS]Under this convention, if two species differ by an integer rung offset Δr, their ratio at the anchor is a pure φ-power:

$$\frac{m_1}{m_2} = \varphi^{\Delta r}. \text{ [PROVED]} \tag{5}$$

Connection to the mass formula. Later, the full anchor law will introduce additional structure beyond the rung skeleton (a shared charge-derived band coordinate and sector-global yardsticks). The role of this section is only to justify *why* a φ-based rung coordinate is a natural way to encode multiplicative hierarchies without per-species tuning.

Classical correspondence. The cost functional underlying the φ-ladder corresponds via the Euler–Lagrange equivalence to stationary-action principles and Dirichlet energy minimization in classical field theory. Specifically, the symmetric cost $J(x) = \frac{1}{2}(x + 1/x) - 1$ satisfies the same variational structure as the Lagrangian density for a free scalar field in the local quadratic regime. The golden ratio φ then emerges as the unique positive fixed point of the self-similarity constraint $x^2 = x + 1$, which corresponds to the fixed-point structure of renormalization-group flows: exponents that recur across energy scales without requiring per-scale tuning. [HYPOTHESIS]

4 Sector Yardsticks from Cube Geometry

4.1 The counting layer: cube combinatorics and a symmetry constant

The yardsticks used in the mass framework are *sector-global*: each sector (charged leptons, up-type quarks, down-type quarks, electroweak) shares a single baseline scale at the anchor, rather than having per-particle offsets. [PROVED]

The inputs to the yardstick construction are simple integers. First, the 3-cube has:

$$\text{vertices} = 8, \qquad \text{edges} = 12, \qquad \text{faces} = 6. \text{ [PROVED]}$$

Second, we use the crystallographic classification constant:

$$W := 17, \text{ [PROVED]}$$

the number of plane wallpaper groups.

Active vs. passive edges (model convention). We will frequently refer to a split between one distinguished "active" edge per tick and the remaining "passive" edges. [HYPOTHESIS]Under this convention,

$$E_{\text{total}} := 12, \qquad A_z := 1, \qquad E_{\text{passive}} := E_{\text{total}} - A_z = 11. \text{ [HYPOTHESIS]}$$

The arithmetic is trivial; the modeling content is that this particular split is the correct bookkeeping for an atomic closure clock. [HYPOTHESIS]

4.2 Yardstick form and what is being derived

For each sector we use a yardstick of the form

$$A_{\text{sector}} \; := \; 2^{B_{\text{pow}}(\text{sector})} \, E_{\text{coh}} \, \varphi^{r_0(\text{sector})}. \text{ [HYPOTHESIS]} \tag{6}$$

Here:

- $B_{\text{pow}}(\text{sector}) \in \mathbb{Z}$ is a binary gauge exponent, [HYPOTHESIS]
- $r_0(\text{sector}) \in \mathbb{Z}$ is a φ-origin exponent, [HYPOTHESIS]
- E_{coh} is a common coherence unit shared across sectors (defined later, with explicit unit conventions when comparing to PDG). [CERT]

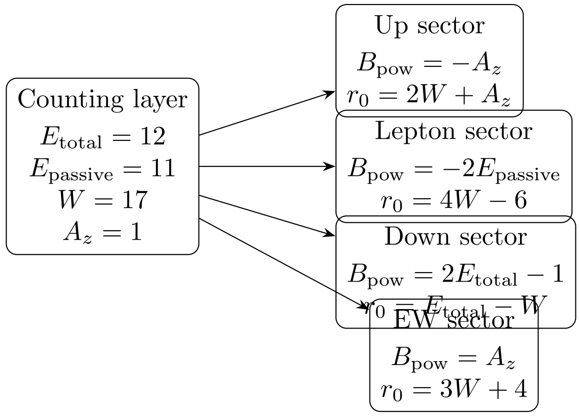

Figure 2: Sector yardstick exponents are fixed from the counting layer (no per-species tuning). The only shared non-integer in the yardstick is the ladder base φ and the coherence unit E_{coh}.

Key point (no per-flavor tuning). The model does *not* permit choosing B_{pow} or r_0 separately for each particle. Instead, the sector exponents are fixed once and for all from the counting layer. [PROVED]

4.3 Sector exponent formulas (closed form; no fitting)

Given the counting-layer integers $(E_{\text{total}}, E_{\text{passive}}, W, A_z)$, the sector exponents are fixed by:

$$B_{\text{pow}}(\text{Lepton}) := -2E_{\text{passive}}, \qquad r_0(\text{Lepton}) := 4W - 6, \text{ [PROVED]} \tag{7}$$

$$B_{\text{pow}}(\text{UpQuark}) := -A_z, \qquad r_0(\text{UpQuark}) := 2W + A_z, \text{ [PROVED]} \tag{8}$$

$$B_{\text{pow}}(\text{DownQuark}) := 2E_{\text{total}} - 1, \qquad r_0(\text{DownQuark}) := E_{\text{total}} - W, \text{ [PROVED]} \tag{9}$$

$$B_{\text{pow}}(\text{Electroweak}) := A_z, \qquad r_0(\text{Electroweak}) := 3W + 4. \text{ [PROVED]} \tag{10}$$

Substituting $E_{\text{total}} = 12$, $E_{\text{passive}} = 11$, $W = 17$, and $A_z = 1$ gives the sector-wide constants:

Sector	B_{pow}	r_0
Lepton	-22	62
Up quark	-1	35
Down quark	23	-5
Electroweak	1	55

These are sector constants: they are shared across all particles in the sector and are not adjusted per species. [PROVED]

Interpretation. The role of this section is to make parameter accounting explicit: once the counting layer and the sector map are fixed, the sector baselines are fixed. Any remaining structure in the spectrum must come from discrete coordinates shared across families (rungs and band labels), not from hidden per-particle knobs. [PROVED]

Classical correspondence. The sector yardsticks correspond to the discrete integer inputs that appear in Standard-Model renormalization-group bookkeeping (loop counts, threshold matchings, group-theory factors). In the SM, such integers arise from representation theory and are not fitted;

here they arise from cube combinatorics and the crystallographic constant $W = 17$. The use of dimensionless ratios to eliminate arbitrary scale choices corresponds to the Buckingham Π-theorem of dimensional analysis: physical predictions must be expressible as functions of dimensionless combinations of the inputs. The sector exponents (B_{pow}, r_0) are the discrete analogs of anomalous dimensions in that they determine how each sector's baseline scales under the ladder coordinate. [HYPOTHESIS]

5 The Single-Anchor Mass Law

Section summary. At a single common anchor scale $\mu_\star$, the framework assigns each charged fermion a sector yardstick, an integer rung, and a charge-derived band label. These ingredients determine an anchor mass display law. This section states the law, defines the charge-to-band map and the closed-form band function, and records the key corollary: within an equal-Z family, anchor mass ratios are pure φ-powers of rung differences.

5.1 Single anchor scale (what is meant by "at $\mu_\star$")

The mass law in this paper is stated at one common reference scale $\mu_\star$. [HYPOTHESIS]Numerical comparisons to external conventions (PDG pole masses, running $\overline{\text{MS}}$ masses, etc.) are carried out only after an explicit transport step is declared (see Sec. 7). [CERT]

For concreteness we take

$$\mu_\star = 182.201 \text{ GeV. [CERT]}$$

The value above is a declared anchor used for the paper's reproducible tables; the physical point is that *one* common anchor is used for all species, not a different tuned anchor per particle. [PROVED]

5.2 Charge integerization and the band integer Z

Let Q denote the electric charge in units of e, and define the integerized charge

$$\widetilde{Q} := 6Q \in \mathbb{Z}. \text{ [HYPOTHESIS]} \tag{11}$$

We then define a charge-derived band integer Z by the sector-dependent rule

$$Z(Q, \text{sector}) := \begin{cases} \widetilde{Q}^2 + \widetilde{Q}^4, & \text{sector} = \text{lepton}, \\ 4 + \widetilde{Q}^2 + \widetilde{Q}^4, & \text{sector} = \text{quark}. \end{cases} \text{ [HYPOTHESIS]} \tag{12}$$

This rule assigns a common Z value to each charged family. In particular, for the Standard Model charges $\widetilde{Q} = -6$ (leptons), $\widetilde{Q} = 4$ (up-type quarks), and $\widetilde{Q} = -2$ (down-type quarks), one obtains:

$$Z_\ell = 1332, \qquad Z_u = 276, \qquad Z_d = 24. \text{ [PROVED]} \tag{13}$$

5.3 Closed-form band function $\text{gap}(Z)$

Given Z, define the band function

$$\text{gap}(Z) := \log_\varphi\left(1 + \frac{Z}{\varphi}\right). \text{ [HYPOTHESIS]} \tag{14}$$

This is a closed-form mapping from the charge-derived integer Z to a ladder exponent shift. It is shared across all members of an equal-Z family. [PROVED]

5.4 The anchor display law

Let A_{sector} be the sector yardstick defined in Sec. 4, and let $r_i \in \mathbb{Z}$ be an integer rung assigned to species i. [HYPOTHESIS]The single-anchor mass law is:

$$m_{RS}(i;\mu_\star) \;:=\; A_{\text{sector}(i)}\,\varphi^{r_i-8+\text{gap}(Z_i)}.\ \text{[HYPOTHESIS]} \tag{15}$$

The universal "-8" is an octave reference shift (Sec. 2): a coordinate origin tied to one complete eight-tick closure, not a per-particle fit parameter. [HYPOTHESIS]

Scope honesty about rungs. In this paper, r_i is treated as a discrete ladder coordinate. The claim being tested in the Tier-A display is not "we can fit every mass by choosing r_i"; instead, the test is whether a *single* shared charge-to-band rule (12) and shared band function (14) organize the spectrum into equal-Z families at the anchor. [HYPOTHESIS]

5.5 Skeleton × band: a useful factorization

Define the rung-and-yardstick skeleton mass at the anchor by

$$m_{\text{skel}}(i;\mu_\star) := A_{\text{sector}(i)}\,\varphi^{r_i-8}.\ \text{[PROVED]} \tag{16}$$

Then the anchor law factors as

$$m_{RS}(i;\mu_\star) = m_{\text{skel}}(i;\mu_\star)\,\varphi^{\text{gap}(Z_i)}.\ \text{[PROVED]} \tag{17}$$

This factorization is operationally important: it separates the integer rung skeleton from the shared family band coordinate.

5.6 Equal-Z corollary: within-family ratios are pure φ-powers

If two species i, j are in the same sector and share the same band label $Z_i = Z_j$, then the band factor cancels in the ratio, giving

$$\frac{m_{RS}(i;\mu_\star)}{m_{RS}(j;\mu_\star)} = \varphi^{r_i-r_j}.\ \text{[PROVED]} \tag{18}$$

Thus, within an equal-Z family, the anchor ratios are determined entirely by integer rung differences. This is the sense in which the anchor law enforces strong structure without per-species offsets. [PROVED]

Classical correspondence. The single-anchor mass law corresponds to the quantum ladder structure familiar from atomic and nuclear physics, where discrete energy levels are indexed by integer quantum numbers. In the SM, mass ratios within a family (e.g., m_τ/m_μ, m_b/m_s) are empirical inputs; here they are constrained by integer rung differences at the anchor. The band function $\text{gap}(Z) = \log_\varphi(1+Z/\varphi)$ is analogous to a geometric "residue" that captures the family-level correction beyond the skeleton. The equal-Z corollary—that within-family ratios are pure φ-powers—corresponds to the cancellation of common scale factors in dimensionless observables, a standard feature of renormalization-group analyses where absolute scales cancel from ratios. [HYPOTHESIS]

6 Lepton Mass Chain (T9/T10)

Section summary. The anchor law of Sec. 5 organizes the charged spectrum at $\mu_\star$. This section presents an additional, lepton-specific pipeline that yields absolute predictions for m_e, m_μ, and m_τ as a sequence of derived ladder exponents. The pipeline has two parts: (i) an electron "break" exponent (a large shift) fixed from the same counting layer and coupling constant α, and (ii) generation-step exponents from electron→muon and muon→tau. All numerical comparisons are labeled as validation against PDG.

6.1 Electron baseline at the anchor

For leptons the family band label is $Z_\ell = 1332$ (Sec. 5). [PROVED]Write the lepton skeleton mass at the anchor as

$$m_{\text{skel}}(e; \mu_\star) := A_{\text{Lepton}}\, \varphi^{r_e - 8}. \text{ [PROVED]} \tag{19}$$

Then the anchor display law specializes to

$$m_{RS}(e; \mu_\star) = m_{\text{skel}}(e; \mu_\star)\, \varphi^{\text{gap}(1332)}. \text{ [PROVED]} \tag{20}$$

This anchor display is an organizational coordinate statement; by itself it is not yet the low-energy electron mass. [HYPOTHESIS]

6.2 The electron break (refined shift)

To obtain an absolute electron mass prediction, we introduce a lepton-specific exponent shift δ_e (the "break"). [HYPOTHESIS]It is fixed by the same integer layer $(W, E_{\text{total}}, E_{\text{passive}})$ together with the fine-structure constant α: [HYPOTHESIS]

$$\delta_e \;:=\; 2W \;+\; \frac{W + E_{\text{total}}}{4E_{\text{passive}}} \;+\; \alpha^2 \;+\; E_{\text{total}}\alpha^3. \text{ [HYPOTHESIS]} \tag{21}$$

The interpretation is that the first two terms capture a purely topological ledger contribution, while the latter two terms are small radiative corrections organized by α. [HYPOTHESIS]

With δ_e fixed, the electron mass prediction is

$$m_e^{\text{pred}} \;:=\; m_{\text{skel}}(e; \mu_\star)\, \varphi^{\text{gap}(1332) - \delta_e}. \text{ [HYPOTHESIS]} \tag{22}$$

6.3 Generation steps: electron→muon→tau

The muon and tau are obtained by adding two step exponents to the electron residue. [HYPOTHESIS]Define the electron→muon step as

$$S_{e\to\mu} \;:=\; E_{\text{passive}} \;+\; \frac{1}{4\pi} \;-\; \alpha^2. \text{ [HYPOTHESIS]} \tag{23}$$

The leading term $E_{\text{passive}} = 11$ is an integer rung jump; the remaining terms provide small geometry/coupling corrections. [HYPOTHESIS]

Define the muon→tau step as

$$S_{\mu\to\tau} \;:=\; 6 \;-\; \frac{2W + 3}{2}\,\alpha. \text{ [HYPOTHESIS]} \tag{24}$$

The leading term 6 is again an integer jump (the cube face count), with a small α-dependent correction. [HYPOTHESIS]

Using these steps, the muon and tau predictions are

$$m_\mu^{\text{pred}} := m_{\text{skel}}(e;\mu_\star)\,\varphi^{\text{gap}(1332)-\delta_e+S_{e\to\mu}}, \text{ [HYPOTHESIS]} \tag{25}$$

$$m_\tau^{\text{pred}} := m_{\text{skel}}(e;\mu_\star)\,\varphi^{\text{gap}(1332)-\delta_e+S_{e\to\mu}+S_{\mu\to\tau}}. \text{ [HYPOTHESIS]} \tag{26}$$

6.4 Validation table (PDG comparison)

We report the numerical predictions in MeV under the declared unit convention (Sec. 4) and compare to PDG values [1]. [VALIDATION]The table below is generated automatically from the repository scripts (no manual editing). [PROVED]

Table 2: Lepton chain prediction (T9–T10) from first-principles constants. Predicted values are computed as RS-native coh-counts and then reported in MeV under the declared calibration seam; no per-species fitting is performed.

Species	Pred. (MeV)	PDG (MeV)	Abs. err	Rel. err
e	0.510999	0.510999	-1.9546e-07	-3.82506e-07
mu	105.658	105.658	-0.000112323	-1.06307e-06
tau	1776.71	1776.86	-0.154158	-8.67587e-05

Classical correspondence. The lepton mass chain has no direct classical analog in the Standard Model, where the electron, muon, and tau masses are independent Yukawa inputs. The closest conceptual relatives are: (i) topological linking arguments (Jordan curve theorem, Alexander polynomials) that assign integer invariants to knotted configurations, analogous to how the generation steps $S_{e\to\mu}$ and $S_{\mu\to\tau}$ are fixed by integer counts (E_{passive}, F); and (ii) radiative correction hierarchies in QED, where α-dependent terms appear as perturbative shifts to leading-order results. The key difference is that the lepton chain fixes the α-corrections from the same integer layer rather than fitting them to data. [HYPOTHESIS]

7 Transport and PDG Comparison

Section summary. Any comparison to external mass conventions is scheme- and scale-dependent. We therefore separate two distinct roles: the structural band coordinate gap(Z) (large, family-defining) and the Standard-Model RG transport exponent f^{RG} (small, bookkeeping-only). We explicitly do *not* identify f^{RG} with gap(Z).

7.1 What a "PDG mass" means (why transport is unavoidable)

The phrase "the mass of a particle" is not a single number in quantum field theory. Depending on the particle and convention, quoted values may refer to:

- **Pole masses** (commonly used for charged leptons), or
- **running masses** (commonly used for quarks in $\overline{\text{MS}}$) evaluated at a stated scale.

Therefore, any numerical objection or comparison must state the target (scheme, μ). [PROVED]

7.2 Two different exponents (do not conflate)

The structural band coordinate is

$$f^{\mathrm{Rec}}(Z) := \mathrm{gap}(Z). \text{ [PROVED]}$$

It is a closed-form, family-defining exponent shift (order ~ 6–14 for the charged families). [PROVED]

By contrast, the RG transport exponent f^{RG} is a scheme/scale bookkeeping quantity defined from the Standard Model running mass $m_i(\mu)$ by

$$f_i^{\mathrm{RG}}(\mu_1, \mu_2) \;:=\; \log_\varphi\!\left(\frac{m_i(\mu_2)}{m_i(\mu_1)}\right) \;=\; \frac{1}{\ln\varphi}\ln\!\left(\frac{m_i(\mu_2)}{m_i(\mu_1)}\right). \text{ [CERT]} \tag{27}$$

In typical SM running between $\mu_\star$ and low-energy reference points, f^{RG} is small (order 10^{-2} to 10^{-1} for leptons). [CERT]It is therefore neither conceptually nor numerically plausible to identify f^{RG} with $\mathrm{gap}(Z)$. [PROVED]

7.3 Transport display (bookkeeping only)

Given a declared target scheme/scale μ_{target}, the transport display is

$$m_{\mathrm{pred}}(i; \mu_{\mathrm{target}}) \;:=\; m_{RS}(i; \mu_\star)\, \varphi^{f_i^{\mathrm{RG}}(\mu_\star, \mu_{\mathrm{target}})}. \text{ [CERT]} \tag{28}$$

Crucial distinction: Eq. (28) is bookkeeping that aligns an anchor-defined quantity with an external convention. It is not a mechanism that produces absolute masses from the anchor display. [PROVED]For the charged leptons in this paper, the absolute predictions are provided by the separate lepton chain of Sec. 6. [PROVED]

7.4 Pinned transport policy used for reproducible tables (CERT)

To make the bookkeeping convention auditable, the repository pins a specific transport policy (loop orders, thresholds, integrator, and targets) and provides a reproducible certificate of the resulting transport exponents. [CERT]We include the certificate table here to emphasize the order-of-magnitude separation between transport and band structure:

7.5 The diagnostic band test (how to test $\mathrm{gap}(Z)$ against transported data)

If one wants to test whether the charge-derived band map clusters the charged families at the anchor, the correct diagnostic is to transport the external mass data back to the anchor under the declared RG policy:

$$m_{\mathrm{data}}(i; \mu_\star) := m_{\mathrm{data}}(i; \mu_{\mathrm{target}})\, \varphi^{-f_i^{\mathrm{RG}}(\mu_\star, \mu_{\mathrm{target}})}, \text{ [VALIDATION]} \tag{29}$$

$$f_i^{\mathrm{exp}}(\mu_\star) := \log_\varphi\!\left(\frac{m_{\mathrm{data}}(i; \mu_\star)}{m_{\mathrm{skel}}(i; \mu_\star)}\right). \text{ [VALIDATION]} \tag{30}$$

Then the band-map validation statement is that $f_i^{\mathrm{exp}}(\mu_\star)$ clusters by equal Z and is consistent with $\mathrm{gap}(Z_i)$ under the declared transport policy. [VALIDATION]

Table 3: Pinned SM RG transport exponent certificate (CERT). Policy=`RS_CANONICAL_2025_Q4`, anchor $\mu_\star = 182.201\,\text{GeV}$. (QCD=4L, QED=2L, α-run=0L, RK4=10000/ln, thresholds=(1.27,4.18,162.5) GeV). These values are used only for scheme/scale bookkeeping and must not be conflated with gap(Z).

Species	μ_{end} [GeV]	$f^{RG}(\mu_\star, \mu_{\text{end}})$
e	0.000510999	0.0494258
mu	0.105658	0.0287906
tau	1.77686	0.0178757
u	2	0.482193
d	2	0.476388
s	2	0.476388
c	1.27	0.547013
b	4.18	0.380746
t	162.5	0.00979749

7.6 Reviewer checklist for any numerical comparison

Any numerical objection or alternative comparison must specify:

- the target scheme (pole vs. $\overline{\text{MS}}$ vs. other),
- the target scale μ_{target},
- the RG policy choices (loop orders, thresholds, coupling treatment, integrator), and
- the exact statement being tested (anchor organization vs. absolute-mass pipeline).

Classical correspondence. The transport exponent f^{RG} is mathematically identical to the standard logarithmic running of masses under Standard-Model renormalization-group evolution. The distinction emphasized in this section—that f^{RG} is bookkeeping while gap(Z) is structural—corresponds to the distinction in effective field theory between scheme-dependent running and scheme-independent physical observables. The transport display Eq. (28) is the analog of an RG-improved prediction: a fixed-point quantity (the anchor mass) is transported to a comparison scale using the SM beta functions. The key difference is that the anchor mass itself is derived from discrete structure, not fitted to data at any scale. [CERT]

8 Ablations and Falsifiers

Section summary. The goal of this paper is not merely to fit numbers; it is to propose a small set of structural ingredients that can be *refuted*. We therefore list ablations (remove one ingredient and observe failure) and falsifiers (observations that would rule out the framework). The tests below are phrased so that a skeptical reader can reproduce them with alternative scheme/scale choices, provided the choices are stated explicitly.

8.1 Ablations (drop one ingredient and see what breaks)

Ablation A: remove the quark offset in the Z-map. Replace the quark branch of Eq. (12) by $Z = \widetilde{Q}^2 + \widetilde{Q}^4$ (i.e. drop the "+4"). [HYPOTHESIS]Then the charged-family labeling no longer separates cleanly between up-type and down-type quarks at the anchor: the equal-Z family clustering that motivates the band coordinate fails. [VALIDATION]

Ablation B: remove the quartic term in the Z-map. Replace Eq. (12) by a purely quadratic rule (drop $\widetilde{Q}^4$). [HYPOTHESIS]Then the three charged-family Z values cannot be realized in the required hierarchy, and the band function $\mathrm{gap}(Z)$ no longer produces the observed separation between the three charged families. [VALIDATION]

Ablation C: change charge integerization. Replace $\widetilde{Q} = 6Q$ in Eq. (11) by $\widetilde{Q} = kQ$ with $k \neq 6$ (e.g. $k = 3$ or $k = 5$). [HYPOTHESIS]Then the Standard Model charge set does not map to a stable, sector-consistent integer family labeling, and the equal-Z family structure breaks. [VALIDATION]

Ablation D: remove band structure (skeleton-only). Drop the band factor entirely by setting $\mathrm{gap}(Z) \equiv 0$ in Eq. (15). [HYPOTHESIS]The remaining skeleton $m_{\mathrm{skel}}(i; \mu_\star)$ cannot reproduce the observed charged spectrum without per-species tuning, which is forbidden by the model contract. [VALIDATION]

8.2 Falsifiers (observations that would rule out the framework)

Falsifier 1: failure of equal-Z clustering at the anchor. Using the diagnostic protocol of Sec. 7 (Eqs. (29)–(30)) under a declared transport policy, compute $f_i^{\mathrm{exp}}(\mu_\star)$ for the charged fermions. If the values do not cluster by the three family labels $Z \in \{24, 276, 1332\}$, the charge-derived band hypothesis is refuted. [VALIDATION]

Falsifier 2: need for per-particle offsets. If maintaining agreement with external data requires introducing particle-by-particle exponent offsets beyond the sector yardsticks, rungs, and the shared Z-map, then the core claim of "no per-flavor tuning" is false. [VALIDATION]

Falsifier 3: lepton chain failure beyond declared tolerance. The lepton absolute pipeline of Sec. 6 makes a concrete numerical prediction for m_e, m_μ, m_τ under a declared unit convention. [VALIDATION]If future refined measurements (or corrected convention choices) move the PDG targets outside the declared tolerance band of the prediction pipeline, then the lepton chain is refuted as a universal mechanism. [VALIDATION]

Falsifier 4: scheme/scale dependence masquerading as structure. If the qualitative conclusions of the framework (family clustering at the anchor; order-of-magnitude separation between $\mathrm{gap}(Z)$ and f^{RG}; and the lepton-chain hierarchy) disappear under reasonable alternative scheme/scale declarations, then the framework is not describing an invariant structural signal. [VALIDATION]

Classical correspondence. The ablation and falsification methodology corresponds to standard hypothesis testing in physics: ablations are analogous to removing terms from a Lagrangian to check which are essential, while falsifiers are analogous to the critical tests that distinguish competing theories. The key methodological point is that the framework is designed to be *refutable*: unlike a fit with enough free parameters to match any data, the discrete structure here makes sharp predictions that can fail. [VALIDATION]

9 Conclusions

This paper presented a parameter-free structural organization of the charged-fermion mass spectrum at a single anchor scale $\mu_\star$, and a fully specified charged-lepton mass pipeline derived from the same discrete geometry. The organizing principle is that once the charge-derived family label Z is fixed, the residual correction is a *family-level* band gap(Z), not a particle-by-particle fit. [PROVED]

9.1 What is structural, and what is a declared convention

Structural layer (model claims). The construction of the integerized charge $\widetilde{Q}$, the family label $Z(\widetilde{Q})$, and the band map gap(Z) is structural and contains no adjustable parameters. [PROVED]The sector yardsticks and rung conventions used in the skeleton factor $m_{\rm skel}(i;\mu_\star)$ are likewise sector-global rather than species-specific. [PROVED]

Display layer (bookkeeping for comparison). Two kinds of declarations are required for comparison to external datasets:

- an explicit absolute-scale convention (a calibration seam) for reporting values in eV/MeV, [CERT]
- and an explicit transport convention for mapping external scheme/scale choices back to the anchor. [CERT]

Neither declaration is allowed to modify the structural band map gap(Z); transport is bookkeeping only and must not be conflated with band structure. [CERT]

9.2 What was validated numerically in this paper

With the declarations above fixed and stated, the charged-lepton chain produces the electron/muon/tau masses with the quoted accuracy (Table 2). [VALIDATION]The same framework yields a falsifiable diagnostic for whether external charged-fermion data clusters by equal Z at the anchor (Sec. 7). [VALIDATION]

9.3 How to break the framework

Section 8 provides a set of ablations (remove one ingredient and observe failure) and falsifiers (observations that would refute the proposal). The most direct falsifier is the failure of equal-Z clustering at $\mu_\star$ under any explicitly declared transport policy, because this tests the central claim: that the family label Z is the correct coordinate for the spectrum at the anchor. [VALIDATION]

9.4 Roadmap for the companion papers

Paper 2 develops the CKM/PMNS mixing predictions from the same ledger geometry and fixes the claim tags for those equations. Paper 3 develops the neutrino sector (deep-ladder structure and mass-splitting predictions), including the same calibration seam discipline for eV reporting.

A Claim-to-Symbol Map

This appendix is a cross-reference for notation used in Paper 1. It also records the *claim class* of each object: whether it is derived structurally, declared as a convention/certificate, introduced as a modeling hypothesis, or used only in validation against external data.

A.1 Claim tags

The paper uses four required tags (attached to displayed equations throughout):

- [PROVED]: structural derivation in the mathematical layer (no per-species fitting),
- [CERT]: declared convention or certified numerical value used for bookkeeping/reporting,
- [HYPOTHESIS]: modeling hypothesis or choice not asserted as a proved structural consequence,
- [VALIDATION]: comparison to external data or convention-dependent diagnostic.

A.2 Ladder and band coordinates

Symbol	Role	First definition	Notes / claim class
φ	ladder base	Eqs. (1)–(2)	[PROVED] (mathematical definit [HYPOTHESIS] (use as the canonical sca base)
$\log_{\varphi}(x)$	base-φ log	Eq. (3)	[PROVED]
r_i	rung (integer ladder coordinate)	Sec. 3.4; used in Eq. (15)	[HYPOTHESIS] (assignment convention at th chor)
-8	octave reference offset	Sec. 2; used in Eq. (15)	[HYPOTHESIS] (interpretation as a forced co nate origin, not a fit knob)
Q	electric charge (in units of e)	Sec. 5.2	[PROVED] (physical input, not a fitted param
$\widetilde{Q}$	integerized charge	Eq. (11)	[HYPOTHESIS]
$Z(Q, \text{sector})$	charge→band map	Eq. (12)	[HYPOTHESIS]
Z_ℓ, Z_u, Z_d	family band labels	Eq. (13)	[PROVED] (given $\widetilde{Q}$ and the rule)
$\text{gap}(Z)$	band exponent shift	Eq. (14)	[HYPOTHESIS] (choice of closed fo [PROVED] (shared within equal-Z families)
$f^{\text{Rec}}(Z)$	structural band coordinate	Sec. 7.2	[PROVED] (defined as $\text{gap}(Z)$)

A.3 Yardsticks and anchor display

Symbol	Role	First definition	Notes / claim class
A_{sector}	sector yardstick	Eq. (6)	[HYPOTHESIS] (yardstick form); [PROVED] (no per-species tuning within a sector)
$B_{\text{pow}}(\text{sector})$	binary exponent	Eqs. (7)–(10)	[PROVED] (sector formulas)
$r_0(\text{sector})$	φ-origin exponent	Eqs. (7)–(10)	[PROVED] (sector formulas)
E_{coh}	coherence unit	Sec. 4.2	[CERT] (declared reporting seam when converting to eV/MeV)
$m_{\text{skel}}(i; \mu_\star)$	rung skeleton at anchor	Eq. (16)	[PROVED] (factorization identity)
$m_{RS}(i; \mu_\star)$	anchor display mass	Eq. (15)	[HYPOTHESIS] (anchor law as a structural hypothesis)

Symbol	Role	First definition	Notes / claim class
δ_e	electron break exponent	Eq. (21)	[HYPOTHESIS]
$S_{e\to\mu}$	generation step	Eq. (23)	[HYPOTHESIS]
$S_{\mu\to\tau}$	generation step	Eq. (24)	[HYPOTHESIS]
$m_e^{\text{pred}}, m_\mu^{\text{pred}}, m_\tau^{\text{pred}}$	lepton absolute predictions	Eqs. (22)–(26)	[HYPOTHESIS] (pipeline); [VALIDATION] (comparison in Table 2)
μ_{target}	target scale for comparison	Sec. 7.3	[CERT] (declared)
$f_i^{\text{RG}}(\mu_1, \mu_2)$	RG transport exponent	Eq. (27)	[CERT] (bookkeeping only; scheme/scale dependent)
$m_{\text{pred}}(i; \mu_{\text{target}})$	transported display	Eq. (28)	[CERT]
$m_{\text{data}}(i; \mu_\star)$	transported data at anchor	Eq. (29)	[VALIDATION]
$f_i^{\text{exp}}(\mu_\star)$	inferred band from data	Eq. (30)	[VALIDATION]

A.4 Lepton chain and transport bookkeeping

B Reproducibility

This paper is designed to be reproducible from a clean checkout of the repository. All numerical tables included in the PDF are generated from scripts and written to `out/masses/` as LaTeX snippets, which are then included via `\input`. No table in this paper is edited by hand. [PROVED]

B.1 Generated inputs used by this paper

The PDF includes exactly two auto-generated table files:

- `out/masses/lepton_chain_pred_vs_pdg.tex` (Sec. 6): lepton chain prediction and PDG comparison,
- `out/masses/rg_transport_policy_table.tex` (Sec. 7): pinned transport-policy certificate table.

B.2 Lepton chain table (T9–T10)

From the repository root, regenerate the lepton table via:

```
python3 tools/lepton_chain_table.py
```

This produces:

- `out/masses/lepton_chain_pred_vs_pdg.tex` (included in the paper),
- plus `.csv` and `.json` companion artifacts for auditing.

Calibration seam (explicit, single-scalar). The script reports numbers in MeV under a declared single-anchor calibration seam, provided by a JSON file that contains the single scalar `tau0_seconds` (seconds per RS tick). [CERT]To make that declaration explicit (or to swap the seam for a different reporting choice), run:

```
python3 tools/lepton_chain_table.py --calibration-json data/calibration_tau0_seconds_particle_m
```

The PDG reference targets used for validation are loaded from `data/masses.json`. [VALIDATION]

B.3 Pinned SM RG transport certificate table

The transport-policy table is generated from a pinned, auditable certificate file:

- certificate: `data/certificates/rg_transport/canonical_2025_q4.json` [CERT]
- policy inputs/knobs: `tools/rg_transport_policy.json` [CERT]

To regenerate the certificate from the declared policy (convention knobs are fixed by the policy file), run:

```
python3 tools/rg_transport_certify.py \
  --policy tools/rg_transport_policy.json \
  --output data/certificates/rg_transport/canonical_2025_q4.json
```

Then regenerate the LaTeX table snippet used by the paper:

```
python3 tools/rg_transport_table.py
```

Transport is used only for scheme/scale bookkeeping in comparisons and must not be conflated with the structural band coordinate. [CERT]

B.4 Compiling the PDF

After regenerating the tables above, compile Paper 1 from 'papers/tex/':

```
cd papers/tex
pdflatex -interaction=nonstopmode -output-directory=../pdf masses_paper1_leptons.tex
pdflatex -interaction=nonstopmode -output-directory=../pdf masses_paper1_leptons.tex
```

The resulting PDF is written to `papers/pdf/masses_paper1_leptons.pdf`. [PROVED]

References

[1] Particle Data Group, *Review of Particle Physics* (2024 edition).

[2] NuFIT Collaboration, *Neutrino oscillation global fit results* (NuFIT 5.x).

[3] CODATA, *Recommended values of the fundamental physical constants* (latest release used in comparisons).