

# Neutrino Masses from the Deep $\varphi$ -Ladder: Fractional Rungs, Mass Splittings, and the $\varphi^7$ Ratio

Paper III of V: The Neutrino Sector

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## Abstract

Papers I and II established the Recognition Science (RS) mechanism of mass and derived predictions for all nine charged fermions from integer positions on the golden-ratio ( $\varphi$ ) ladder. This paper extends the framework to neutrinos, which occupy the *deep* (low-mass) end of the ladder.

The charged sectors use integer rungs; the neutrino sector requires *fractional* (quarter-step) rungs, reflecting the vastly finer mass resolution needed at the sub-eV scale. We assign a specific rung triple  $(r_1, r_2, r_3) = (-239/4, -231/4, -217/4)$  and derive:

- absolute masses  $m_1 \approx 0.00354$  eV,  $m_2 \approx 0.00926$  eV,  $m_3 \approx 0.0499$  eV,
- a mass sum  $\Sigma m_\nu \approx 0.063$  eV (below current cosmological bounds),
- normal ordering ( $m_1 < m_2 < m_3$ ) as a structural consequence (not a fit choice),
- the **key structural prediction**: an exact squared-mass ratio  $(m_3^2/m_2^2) = \varphi^7 \approx 29.03$ , independent of the eV calibration seam,
- mass-squared splittings  $\Delta m_{21}^2 \approx 7.33 \times 10^{-5}$  eV<sup>2</sup> and  $\Delta m_{31}^2 \approx 2.48 \times 10^{-3}$  eV<sup>2</sup>, consistent with NuFIT summary windows.

The ratio of splittings  $R_\Delta = \Delta m_{31}^2/\Delta m_{21}^2 = (\varphi^{11} - 1)/(\varphi^4 - 1) \approx 33.82$  is seam-free (the calibration parameter cancels) and constitutes the most robust falsifiable prediction of the deep-ladder hypothesis. We also discuss the no-go result for integer rungs (0, 11, 19) with  $Z_\nu = 0$  at the anchor, explaining why the neutrino sector requires a qualitatively different approach from the charged sectors, and we provide a comprehensive set of falsifiers.

## Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
1.1	The neutrino puzzle . . . . .	3
1.2	The no-go for integer rungs and its resolution . . . . .	3
1.3	Organization of this paper . . . . .	3
<b>2</b>	<b>The Deep <math>\varphi</math>-Ladder: Fractional Rungs</b>	<b>3</b>
2.1	Ladder coordinate . . . . .	3
2.2	Quarter-step convention . . . . .	4
2.3	Rung assignment . . . . .	4
<b>3</b>	<b>Neutrino Mass Predictions</b>	<b>4</b>
3.1	The eV reporting seam . . . . .	4
3.2	Mass law for neutrinos . . . . .	4
3.3	Predicted absolute masses . . . . .	5

<b>4</b>	<b>Mass-Squared Splittings and the <math>\varphi^7</math> Ratio</b>	<b>5</b>
4.1	Splitting definitions . . . . .	5
4.2	Predicted splittings . . . . .	5
4.3	The exact $\varphi^7$ squared-mass ratio . . . . .	5
4.4	Seam-free splitting ratio . . . . .	5
<b>5</b>	<b>Normal Ordering as a Structural Consequence</b>	<b>6</b>
5.1	Monotonicity of the ladder map . . . . .	6
5.2	Rung ordering implies mass ordering . . . . .	6
<b>6</b>	<b>Cosmological Consistency</b>	<b>6</b>
6.1	The mass sum constraint . . . . .	6
6.2	Near-future sensitivity . . . . .	6
6.3	Kinematic endpoint . . . . .	6
<b>7</b>	<b>Dirac vs. Majorana Nature</b>	<b>6</b>
<b>8</b>	<b>The Integer-Rung No-Go and Why Fractional Rungs Are Needed</b>	<b>7</b>
8.1	The formal rung triple (0, 11, 19) . . . . .	7
8.2	Why the charged sector works and the neutrino sector doesn't . . . . .	7
8.3	Fractional rungs as the minimal modification . . . . .	7
<b>9</b>	<b>Falsifiers</b>	<b>7</b>
9.1	Seam-free falsifiers (depend only on $\varphi$ and rung differences) . . . . .	7
9.2	Scale falsifiers (test the eV reporting seam) . . . . .	7
<b>10</b>	<b>Conclusions</b>	<b>8</b>
10.1	What is structural . . . . .	8
10.2	What is hypothesized . . . . .	8
10.3	What the validation indicates . . . . .	8
10.4	The core falsifiers . . . . .	8

# 1 Introduction

## 1.1 The neutrino puzzle

Neutrinos present unique challenges. Oscillation experiments measure mass-squared *differences*  $\Delta m_{21}^2$  and  $\Delta m_{31}^2$  with remarkable precision, but do not determine the absolute mass scale. The mass ordering (normal vs. inverted) remains an open question. The absolute scale is constrained by cosmological bounds on  $\Sigma m_\nu$  and kinematic measurements ( $\beta$ -decay endpoints), but neither has produced a definitive value.

Within the Recognition Science framework, the charged fermion masses are organized by integer rungs on the  $\varphi$ -ladder at the anchor scale  $\mu_\star$ . Neutrinos, however, are qualitatively different for two reasons:

1. **Vanishing charge band:** Neutrinos have  $Q = 0$ , so the integerized charge  $\tilde{Q} = 6Q = 0$  and the band label  $Z_\nu = 0$ . The gap function gives  $\text{gap}(0) = \log_\varphi(1 + 0/\varphi) = 0$ —there is no band correction. The structural ingredient that splits the charged families is absent.
2. **Deep ladder:** Neutrino masses ( $\sim 10^{-2}$  eV) are  $\sim 10^{10}$  times smaller than the electron mass. On the  $\varphi$ -ladder this corresponds to rungs in the deep negative region (around  $r \sim -55$  to  $-60$ ), far from the charged sector rungs ( $r \sim 2$  to  $21$ ).

## 1.2 The no-go for integer rungs and its resolution

A natural first attempt applies the same integer rung convention to neutrinos with the charged-sector generation torsion  $\{0, 11, 17\}$ , giving the formal rung triple  $(r_1, r_2, r_3) = (0, 11, 19)$ . However, as documented in the companion analysis, this triple fails the acceptance test: neither normal nor inverted ordering produces splittings consistent with NuFIT data under  $Z_\nu = 0$  with a single neutrino yardstick.

This no-go is instructive rather than fatal. It identifies the precise structural point where the neutrino sector diverges from the charged sectors: the *rung resolution*. The resolution is to allow **fractional rungs**—specifically, quarter-step positions  $r \in \frac{1}{4}\mathbb{Z}$ —on the deep ladder.

## 1.3 Organization of this paper

Section 2 defines the deep ladder and the fractional rung convention. Section 3 derives the neutrino mass predictions under a specific rung triple. Section 4 computes mass-squared splittings and derives the seam-free  $\varphi^7$  ratio. Section 5 proves that normal ordering is a structural consequence. Section 6 checks cosmological consistency. Section 7 discusses the relationship to Dirac vs. Majorana nature. Section 8 lists falsifiers. Section 9 concludes.

# 2 The Deep $\varphi$ -Ladder: Fractional Rungs

## 2.1 Ladder coordinate

As in the charged sectors, the base- $\varphi$  logarithm defines the ladder coordinate: [PROVED]

$$r(x) := \log_\varphi(x) = \frac{\ln x}{\ln \varphi}. \quad (1)$$

For two masses  $m_a, m_b > 0$  separated by rung offset  $\Delta r$ : [PROVED]

$$\frac{m_a}{m_b} = \varphi^{\Delta r}, \quad \frac{m_a^2}{m_b^2} = \varphi^{2\Delta r}. \quad (2)$$

## 2.2 Quarter-step convention

For the neutrino sector, we extend the rung lattice: [HYP]

$$r \in \frac{1}{4}\mathbb{Z}. \quad (3)$$

This extension is motivated by:

- **Resolution:** neutrino splittings are extremely small compared to charged sectors, requiring finer exponent increments than integer steps provide,
- **Octave compatibility:** quarter steps are the simplest refinement compatible with the eight-tick period ( $8 \times \frac{1}{4} = 2$ , an integer).

## 2.3 Rung assignment

The specific deep-ladder rung triple is: [HYP]

$$(r_1, r_2, r_3) = \left( -\frac{239}{4}, -\frac{231}{4}, -\frac{217}{4} \right). \quad (4)$$

The rung differences are:

$$r_2 - r_1 = \frac{-231 - (-239)}{4} = 2, \quad (5)$$

$$r_3 - r_2 = \frac{-217 - (-231)}{4} = \frac{7}{2}, \quad (6)$$

$$r_3 - r_1 = \frac{-217 - (-239)}{4} = \frac{11}{2}. \quad (7)$$

The appearance of  $11/2$  for  $r_3 - r_1$  and  $7/2$  for  $r_3 - r_2$  reflects the deep-ladder signature: the “11” of the charged sector generation torsion appears halved, while the “7” carries echoes of the  $\varphi^7$  structural identity that governs the atmospheric-to-solar hierarchy.

## 3 Neutrino Mass Predictions

### 3.1 The eV reporting seam

Absolute neutrino masses in eV require a global calibration seam: [CERT]

$$\kappa_{\text{eV}} := \frac{\hbar}{\tau_0 \cdot (1 \text{ eV})} \approx 1.086 \times 10^{10} \text{ eV}, \quad (8)$$

where  $\tau_0$  is the fundamental tick (from the eight-tick closure). This seam is fixed once for the entire framework and is *not* adjusted per neutrino.

Equivalently, pinning the seam algebraically: [CERT]

$$\kappa_{\text{eV}} = 2^{-22} \varphi^{51} \times 10^6 \text{ eV}. \quad (9)$$

### 3.2 Mass law for neutrinos

The deep-ladder mass hypothesis is: [HYP]

$$m_i^{\text{pred}} = \kappa_{\text{eV}} \cdot \varphi^{r_i}, \quad i \in \{1, 2, 3\}. \quad (10)$$

Note the absence of a gap function ( $Z_\nu = 0 \Rightarrow \text{gap}(0) = 0$ ).

### 3.3 Predicted absolute masses

Evaluating (10) with the rung triple (4): [CERT]

$$m_1^{\text{pred}} \approx 0.00354 \text{ eV}, \quad (11)$$

$$m_2^{\text{pred}} \approx 0.00926 \text{ eV}, \quad (12)$$

$$m_3^{\text{pred}} \approx 0.0499 \text{ eV}. \quad (13)$$

The mass sum: [CERT]

$$\Sigma m_{\nu}^{\text{pred}} \approx 0.063 \text{ eV}. \quad (14)$$

## 4 Mass-Squared Splittings and the $\varphi^7$ Ratio

### 4.1 Splitting definitions

Standard definitions: [PROVED]

$$\Delta m_{21}^2 := m_2^2 - m_1^2, \quad \Delta m_{31}^2 := m_3^2 - m_1^2. \quad (15)$$

### 4.2 Predicted splittings

From the mass law (10): [PROVED]

$$\Delta m_{ij}^2 = \kappa_{\text{eV}}^2 (\varphi^{2r_i} - \varphi^{2r_j}). \quad (16)$$

Numerically: [CERT]

$$\Delta m_{21}^2 \approx 7.33 \times 10^{-5} \text{ eV}^2, \quad (17)$$

$$\Delta m_{31}^2 \approx 2.48 \times 10^{-3} \text{ eV}^2. \quad (18)$$

Both fall within NuFIT summary windows for normal ordering. [VAL]

### 4.3 The exact $\varphi^7$ squared-mass ratio

The seam  $\kappa_{\text{eV}}$  cancels in the squared-mass ratio: [PROVED]

$$\frac{(m_3^{\text{pred}})^2}{(m_2^{\text{pred}})^2} = \varphi^{2(r_3-r_2)} = \varphi^{2 \times 7/2} = \varphi^7. \quad (19)$$

This is the single most important structural prediction of the neutrino sector: [HYP]

$$\frac{m_3^2}{m_2^2} = \varphi^7 \approx 29.03.$$

(20)

It is *seam-free* (independent of  $\kappa_{\text{eV}}$ ) and testable with oscillation data alone once absolute mass information becomes available.

### 4.4 Seam-free splitting ratio

The ratio of mass-squared splittings is also seam-free: [PROVED]

$$R_{\Delta} := \frac{\Delta m_{31}^2}{\Delta m_{21}^2} = \frac{\varphi^{2(r_3-r_1)} - 1}{\varphi^{2(r_2-r_1)} - 1} = \frac{\varphi^{11} - 1}{\varphi^4 - 1} \approx 33.82. \quad (21)$$

This depends only on  $\varphi$  and the rung differences, not on any calibration convention.

## 5 Normal Ordering as a Structural Consequence

### 5.1 Monotonicity of the ladder map

Since  $\varphi > 1$ , the map  $r \mapsto \kappa_{\text{eV}} \cdot \varphi^r$  is strictly increasing in  $r$  for any  $\kappa_{\text{eV}} > 0$ . [PROVED]

### 5.2 Rung ordering implies mass ordering

The rung triple satisfies  $r_1 < r_2 < r_3$ : [HYP]

$$-\frac{239}{4} < -\frac{231}{4} < -\frac{217}{4}. \quad (22)$$

By monotonicity: [PROVED]

$$m_1^{\text{pred}} < m_2^{\text{pred}} < m_3^{\text{pred}}. \quad (23)$$

**Normal ordering is not a choice in RS; it is forced by the discrete rung assignment.** If future experiments decisively establish inverted ordering, the rung triple (4) is refuted.

## 6 Cosmological Consistency

### 6.1 The mass sum constraint

Current cosmological analyses within  $\Lambda\text{CDM}$ -like frameworks constrain: [VAL]

$$\Sigma m_\nu \lesssim 0.12 \text{ eV} \quad (\text{representative bound}). \quad (24)$$

The predicted sum  $\Sigma m_\nu^{\text{pred}} \approx 0.063 \text{ eV}$  is comfortably within this bound. [VAL]

### 6.2 Near-future sensitivity

Next-generation surveys (e.g., DESI, Euclid, CMB-S4) may tighten the bound toward  $\Sigma m_\nu \lesssim 0.06 \text{ eV}$ . If the bound crosses below 0.063 eV, the deep-ladder mass scale is directly pressured. [VAL]

### 6.3 Kinematic endpoint

The KATRIN experiment constrains  $m_\beta < 0.45 \text{ eV}$  (90% CL) from tritium  $\beta$ -decay. The predicted effective mass  $m_\beta^{\text{pred}} \approx \sqrt{\sum |U_{ei}|^2 m_i^2} \sim 0.01 \text{ eV}$  is far below current sensitivity but within reach of proposed future experiments. [VAL]

## 7 Dirac vs. Majorana Nature

The deep-ladder framework treats neutrinos as Dirac fermions with  $Z_\nu = 0$  at the anchor. Under this assignment:

- Lepton number is conserved,
- The effective Majorana mass  $m_{\beta\beta}$  for neutrinoless double-beta decay is zero,
- The mass hierarchy is governed purely by the rung triple and the single neutrino yardstick.

The Majorana alternative would require  $Z_\nu \neq 0$  (a nonzero anchor residue) or additional discrete structure (writhe parity of the neutral braid triple). The current framework does not exclude the Majorana possibility in principle, but the simplest realization ( $Z_\nu = 0$ , Dirac) is the one tested here.

**Falsifier:** detection of neutrinoless double-beta decay at a rate inconsistent with zero  $m_{\beta\beta}$  would require modification of the  $Z_\nu = 0$  assignment.

## 8 The Integer-Rung No-Go and Why Fractional Rungs Are Needed

### 8.1 The formal rung triple $(0, 11, 19)$

If one applies the charged-sector generation torsion  $\{0, 11, 17\}$  directly to neutrinos (with the same baseline rung conventions), the formal triple is  $(r_1, r_2, r_3) = (0, 11, 19)$ . The splitting ratio would be: [PROVED]

$$R_{\Delta}^{\text{integer}} = \frac{\varphi^{38} - 1}{\varphi^{22} - 1} \approx 1.85 \times 10^3, \quad (25)$$

which is more than 50 times larger than the observed  $R_{\Delta} \approx 33$ . Both normal and inverted orderings fail the acceptance test under this triple.

### 8.2 Why the charged sector works and the neutrino sector doesn't

In the charged sectors, the band function  $\text{gap}(Z)$  provides a large exponent shift ( $\sim 6\text{--}14$ ) that separates the three families. For neutrinos,  $\text{gap}(0) = 0$ , so there is no band correction. The entire hierarchy must come from the rung differences alone, and integer torsion values  $\{0, 11, 17\}$  produce splittings that are too widely separated.

### 8.3 Fractional rungs as the minimal modification

Quarter-step rungs are the smallest extension of the rung lattice that:

1. Provides sufficient resolution for the neutrino mass hierarchy,
2. Maintains compatibility with the eight-tick period ( $8 \times \frac{1}{4} = 2$ ),
3. Requires no new continuous parameters (the positions are still discrete rational numbers).

The specific triple  $(-239/4, -231/4, -217/4)$  is selected by requiring that the predicted splittings fall within NuFIT windows—this is the sense in which the rung triple is “fixed by data” rather than derived purely from structure. The honest status is: *the rung lattice convention is structural (HYP); the specific rung triple within that lattice is constrained by oscillation data (HYP+VAL)*.

## 9 Falsifiers

### 9.1 Seam-free falsifiers (depend only on $\varphi$ and rung differences)

**F1: Splitting-ratio mismatch.** If  $R_{\Delta} = \Delta m_{31}^2 / \Delta m_{21}^2$  departs from the predicted value  $(\varphi^{11} - 1) / (\varphi^4 - 1) \approx 33.82$  beyond experimental uncertainty, the rung triple is refuted. [VAL]

**F2: Ordering mismatch.** If inverted ordering is decisively established, the rung ordering  $r_1 < r_2 < r_3$  is refuted. [VAL]

**F3: Squared-mass ratio mismatch.** If absolute mass information establishes  $m_3^2 / m_2^2 \neq \varphi^7$ , the rung gap hypothesis is refuted. [VAL]

### 9.2 Scale falsifiers (test the eV reporting seam)

**F4: Oscillation windows.** If updated NuFIT windows exclude  $\Delta m_{21}^{2,\text{pred}}$  or  $\Delta m_{31}^{2,\text{pred}}$ , either the rung triple or the seam is refuted. [VAL]

**F5: Cosmological exclusion.** If cosmological bounds establish  $\Sigma m_{\nu} < 0.062$  eV, the predicted mass scale is ruled out. [VAL]

**F6: Direct mass detection.** A kinematic measurement robustly implying a mass scale well above the predicted window refutes the deep-ladder assignment. [VAL]

**F7: Neutrinoless double-beta decay.** Detection of  $0\nu\beta\beta$  at a level inconsistent with the Dirac  $Z_\nu = 0$  assignment would require extending the framework. [VAL]

## 10 Conclusions

This paper has extended the Recognition Science mass framework to the neutrino sector via the deep  $\varphi$ -ladder with fractional (quarter-step) rungs.

### 10.1 What is structural

- The ladder mathematics: ratios are  $\varphi$ -powers of rung differences; the seam cancels from ratios. [PROVED]
- Normal ordering: forced by rung ordering plus  $\varphi > 1$ . [PROVED]
- The  $\varphi^7$  squared-mass ratio: a seam-free structural prediction. [HYP]
- The seam-free splitting ratio  $R_\Delta = (\varphi^{11} - 1)/(\varphi^4 - 1)$ . [HYP]

### 10.2 What is hypothesized

- The quarter-step rung lattice  $r \in \frac{1}{4}\mathbb{Z}$ . [HYP]
- The specific rung triple  $(-239/4, -231/4, -217/4)$ . [HYP]
- The Dirac nature ( $Z_\nu = 0, m_{\beta\beta} = 0$ ). [HYP]

### 10.3 What the validation indicates

Under the declared seam,  $\Delta m_{21}^2$  and  $\Delta m_{31}^2$  both fall within NuFIT windows. The mass sum  $\Sigma m_\nu \approx 0.063$  eV is consistent with current cosmological bounds. The splitting ratio  $R_\Delta \approx 33.82$  is consistent with the experimental value  $R_\Delta^{\text{exp}} \approx 33.4$ . [VAL]

### 10.4 The core falsifiers

The most robust tests are seam-free:

- The splitting ratio  $R_\Delta$  (testable now),
- The mass ordering (testable with current and near-future experiments),
- The  $\varphi^7$  ratio (testable when absolute mass information becomes available).

The neutrino sector represents the frontier of the RS mass program. The charged sectors exhibit remarkable agreement with data; the neutrino sector requires a structural extension (fractional rungs) that is natural within the framework but not yet derived from the same pure counting-layer arguments that fix the charged sector. Closing this gap—deriving the neutrino rung lattice from first principles—remains the primary open problem.

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