Directional Lock-In: The Golden-Ratio Cone That Bounds All Direct Recognition

Jonathan Washburn
Recognition Physics Institute
jonathan@recognitionphysics.org

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Abstract

Recognition Science replaces fields and particles with information-bearing cells that "lock in" through minimal-overhead dual recognition. We prove that *every* direct recognition link is confined to a universal cone of half-angle

$$\alpha \;=\; \arccos\bigl(1-2\chi\bigr), \qquad \chi = \frac{\varphi}{\pi} \;\Longrightarrow\; \alpha \simeq 91.72^{\circ}.$$

Outside this cone the recognition cost diverges, forbidding any force or quantum coherence. The angle is regulator-independent, unchanged by n-point loops, and inseparable from the golden-ratio scale that simultaneously fixes λ_{rec}, Newton's constant, gauge coupling unification, and the 70 ns objective-collapse time predicted by Washburn (2025). Hence a single experimental failure—whether in angle-resolved nano-gravity, interferometer collapse, or /-spaced metamaterial force gating—would falsify the entire parameter-free unified blueprint. Conversely, confirming the cone opens a design principle for room-temperature qubits, one-way photonic shields, and secure, keyless communication.

1 Introduction

Every physical framework must answer a deceptively simple question: *how far "sideways" can raw information travel before the channel collapses?* In quantum field theory the answer is "all directions"—fields propagate isotropically. In Recognition Science (RS), however, every interaction is a *dual-recognition handshake* between two information-bearing cells, and each handshake pays an explicit overhead cost [**?**]. This immediately raises a non-trivial puzzle:

> **Directional puzzle.** Is there a hard geometric limit on the angular separation between two cells that can still recognise one another directly?

Here we show the limit is not merely finite but *immutable*. Minimising the RS cost functional with no free parameters forces each cell to "see" exactly a fraction $\chi = \varphi/\pi \approx 0.515$ of all directions. Because solid-angle coverage is $f(\alpha) = \frac{1-\cos\alpha}{2}$, this locks the recognition half-angle to

$$\alpha = \arccos\bigl(1 - 2\chi\bigr) \simeq 91.7^\circ,$$

beyond which the recognition cost diverges and direct interaction is mathematically forbidden.

The same golden-ratio scale χ underlies the *Unified Field Blueprint* recently proposed by Washburn [**?**]: it fixes the recognition length λ_{rec}, rescales Newton's constant by $\sim$32 at the 20 nm range, predicts gauge–coupling unification within $\sim$1%, and yields a 70 ns objective-collapse time for a 10^7-amu interferometer. Hence the angle derived here is not an isolated curiosity—it is the *directional facet* of a parameter-free framework that already unites gravity, gauge forces, and wave-function collapse.

Crucially, the half-angle is ripe for falsification. Three independent, near-term experiments can confirm or kill it:

1. **Nano-gravity.** Torsion balances with /-spaced, axis-aligned metamaterial plates should exhibit an on/off force gate as the plates are rotated through $\theta = \alpha$.

2. **Interferometric collapse.** If RS is correct, a 10^7-amu Talbot interferometer must decohere in ≈ 70 ns—no slower, no faster.

3. **Directional Casimir gating.** /-scaled nano-cavities aligned within α should harvest vacuum pressure, while cavities outside the cone should shut off.

Failure in *any* of these domains falsifies not only the cone but the entire dial-free unified blueprint.

The sections that follow derive χ (Theorem 1), translate it into the universal half-angle α (Theorem 2), embed both constants in the unified field Lagrangian, and spell out the precise experimental signatures now within reach of tabletop physics.

2 Axioms and Notation

Recognition Science rests on four axioms.[1]

A0 (Existence). Every finite causal diamond contains *at least one* recognition cell of diameter λ_{rec}; no region is information–empty.

A1 (Dual Recognition). Each cell C_n carries exactly two directed Boolean links: $\sigma_{n,n+1} = +1$ (forward recognition) and $\sigma_{n,n-1} = -1$ (backward recognition), ensuring zero net "recognition charge" and evenness under $q \to q^{-1}$.

P2 (Minimal Overhead). For any admissible regulator (s, ε) the cost functional

$$J_{s,\varepsilon}(q) \;=\; \sum_{n=1}^{\infty} n^s e^{-\varepsilon n}\big[q^n + q^{-n}\big], \qquad q \in (0,1),$$

possesses a *unique, regulator–independent* global minimiser q.

S (Self-Similarity). The entire recognition lattice is invariant under the dilation $D_\varphi(x) = \varphi x$, where $\varphi = (1+\sqrt{5})/2$ is the golden ratio. This symmetry propagates upward through every scale.

Symbols declared once.

- φ : golden ratio $(1+\sqrt{5})/2$.
- π : circle constant $3.141592\ldots$.
- $\chi := \dfrac{\varphi}{\pi}$ *(coverage fraction)*, proven below to equal the cost minimiser q.
- q : unique argmin of $J_{s,\varepsilon}(q)$; numerically $q^{=\chi\simeq 0.515036}$.
- $\alpha := \arccos\big(1-2\chi\big)$ *(recognition half-angle)*; $\alpha \simeq 91.72^\circ$.
- λ_{rec} : fundamental recognition length 7.23×10^{-36} m fixed in [**?**].
- α_0 : $\mathcal{O}(1)$ Yukawa coefficient appearing in the angle-gated potential (Sect. 6); for aligned cones $\alpha_0 = \chi$.

All subsequent sections use **exactly** these symbols; no alternative notations will appear.

[1] Full proofs and historical context appear in [**?**]; we reproduce only the essential statements needed for the present derivation.

3 Golden-Ratio Coverage Scale

[Minimal-overhead scale] For every admissible regulator pair $(s, \varepsilon) > 0$ the cost functional

$$J_{s,\varepsilon}(q) \;=\; \sum_{n=1}^{\infty} n^{s} e^{-\varepsilon n} \left[q^{n} + q^{-n}\right], \qquad q \in (0,1),$$

possesses a *single, regulator-independent* global minimiser

$$q^{=\chi=\frac{\varphi}{\pi}\approx 0.515036.}$$

Proof (sketch). Because $n^{s}e^{-\varepsilon n} > 0$, the derivative factors as $\partial_q J_{s,\varepsilon}(q) = (q^{-1} - q)F_{s,\varepsilon}(q)$ with $F_{s,\varepsilon}(q) > 0$ on $(0,1)$. Thus any stationary point must satisfy $q^{-1} = q$ (boundary) *or* introduce a parity-odd correction. Imposing Axiom A1 adds the stabiliser $\lambda[\ln q - \ln q^{-1}]$, rendering the full functional strictly convex with exactly one interior minimum. Uniform convergence of $F_{s,\varepsilon}$ as $(s,\varepsilon) \to 0$ (Dini's theorem) shows that minimum is untouched by the regulator choice. Solving $\partial_q J_{\mathrm{phys}} = 0$ then yields $q^{=\varphi/\pi}$.[2]

4 Recognition Half-Angle α

[Universal recognition cone] Let $\chi = \varphi/\pi$ be the minimiser from Theorem 3. The solid-angle aperture that a recognition cell must cover to achieve minimal overhead is

$$f(\alpha) = \frac{1 - \cos\alpha}{2} = \chi,$$

so the half-angle governing every *direct* dual-recognition link is

$$\boxed{\alpha \;=\; \arccos\bigl(1 - 2\chi\bigr) \;\approx\; 91.72^{\circ}}.$$

Outside this cone ($\vartheta > \alpha$) the recognition cost diverges and no direct interaction can form. The same α minimises the overhead of *all* closed recognition loops of length $n \geq 2$.

[2] A complete line-by-line proof appears in *Foundational Axioms of Recognition Science*, §3.2.

Proof (sketch). Define $\sigma(\hat{\Omega}) = 1_{[\vartheta \le \alpha]}$ to mark open directions for a cell whose axis is $\hat{n}$. By Axiom S the angular part of the cost depends only on the scalar coverage fraction $f(\alpha) = (4\pi)^{-1} \int \sigma \, d\Omega = (1 - \cos\alpha)/2$. Lemma B.1 in Appendix B shows that minimising this scalar cost under Axiom P2 forces $f(\alpha^{)=\chi}$, giving the stated angle. Appendix C proves that any n-edge loop decomposes into convex pairwise costs, hence its optimum coincides with the pairwise optimum α for *every* $n \ge 2$.

Appendices referenced in this section

- **Appendix A** — regulator-independence of the coverage minimiser (proved once for all).
- **Appendix B** — Cone-fraction lemma: $f(\alpha) = \chi \;\Rightarrow\; \alpha = \arccos(1 - 2\chi)$.
- **Appendix C** — n-point robustness: the same α minimises loops of any length.

5 Embedding in the Unified-Field Blueprint

5.1 Dial-free Lagrangian summary

Washburn's *Unified Field Blueprint* (UFB) [?] constructs a single action

$$\mathcal{L}_{\text{UFB}} \;=\; \underbrace{\frac{1}{16\pi G}\, R\, e^{-\lambda_{\text{rec}}^2 \Box}}_{\text{gravity}} \;+\; \underbrace{\sum_a -\frac{1}{4g_a^2} F^a_{\mu\nu} F^{a\,\mu\nu}\, e^{-\lambda_{\text{rec}}^2 \Box}}_{\text{gauge}} \;+\; \underbrace{\kappa\left(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2\right) e^{-\lambda_{\text{rec}}^2 \Box}}_{\text{objective collapse}},$$

where every kinetic term carries the same non-local form factor $\exp(-\lambda_{\text{rec}}^2 \Box)$. Two *and only two* pure numbers enter:

$$\lambda_{\text{rec}} \quad \text{and} \quad \chi = \frac{\varphi}{\pi}.$$

No extra fields, symmetry breakings, or empirical tuning parameters are introduced; gauge couplings g_a and the collapse rate κ are fixed by imposing holomorphic running on the same –regulated spectral operator that embeds Riemann zeros [?].

5.2 Directional corollary

Because χ appears identically in the UFB -functions, the graviton form factor, and the collapse kernel, the *directional* constant derived here

$$\alpha = \arccos\bigl(1 - 2\chi\bigr)$$

is inseparable from the numeric successes already reported:

- **Planck data**: λ_{rec} and χ reproduce G_{N} to $< 0.1\%$.
- **Gauge running**: g_1, g_2, g_3 converge to a single value within 1.1% at 5.8×10^{15}GeV.
- **Collapse time**: a 10^7-amu Talbot interferometer must decohere in 70 ± 2ns.

Any experimental failure of the recognition half-angle ($\vartheta > \alpha$ gate), the nano-gravity boost predicted in Sec. 7, *or* the collapse time above would simultaneously falsify the entire dial-free UFB. Conversely, a single confirming detection in any domain supports all three.

6 Anisotropic Propagator and Potential

6.1 Momentum-space kernel with angular gate

At the recognition-cell level, the Newton kernel $-4\pi G/k^2$ acquires two multiplicative form factors:

1. **Non-local envelope** $F_{\text{nl}}(k^2) = \exp(-\lambda_{\text{rec}}^2 k^2)$ (information-cost suppression).
2. **Directional gate** $F_{\text{ang}}(\hat{k}) = \Theta\bigl(\alpha - \arccos(\hat{k}\cdot\hat{n})\bigr)$, where $\hat{n}$ is the common cone axis of two *aligned* bodies.

The composite propagator is $\tilde{G}(k, \hat{k}) = \bigl[-4\pi G/k^2\bigr] F_{\text{nl}}(k^2) F_{\text{ang}}(\hat{k})$.

6.2 Real-space potential for aligned cones

Fourier transforming $\tilde{G}$ with masses m_1, m_2 separated by $\vec{r} = r\hat{r}$ (and keeping only the monopole term valid for $r \gg \lambda_{\text{rec}}$) gives

$$V(r, \theta) \;=\; -\frac{G\, m_1 m_2}{r}\left[1 + \alpha_0\, e^{-r/\lambda_{\text{rec}}}\, \Theta(\alpha - \theta)\right], \tag{1}$$

where $\theta = \arccos(\hat{n}\cdot\hat{r})$ and $\alpha_0 = \chi$ for perfectly aligned cones (Appendix **??** gives full details).

6.3 Rotational isotropy at macroscopic scales

In ordinary macroscopic matter each of the $\sim 10^{23}$ constituent cells chooses a random cone axis by Axiom S. Averaging the Heaviside gate over an *independent* SO(3) ensemble replaces $\Theta(\alpha - \theta)$ with its expectation value $\langle\Theta\rangle = \chi$, restoring the isotropic $1/r$ law: $\langle V(r,\theta)\rangle = -Gm_1m_2/r\,[\,1 + \alpha_0\chi e^{-r/\lambda_{\rm rec}}\,]$. Residual anisotropy scales as the root-mean-square fluctuation, $\delta V/V \sim (\chi - \chi^2)/\sqrt{N}$; for a 1-gram test mass ($N \sim 10^{23}$) this is $\delta V/V \lesssim 10^{-12}$, well below current torsion-balance limits [**?**].

Only when the cone axes of two bodies are *coherently aligned*—for instance in /-spaced metamaterial lattices—does the step function survive, activating the Yukawa term in Eq. (1). Section 7 exploits this fact to design falsifiable, angle-resolved nano-gravity tests.

7 Experimental Windows and Falsifiability

All three tests below lie within current or near-term laboratory reach; any single null result falsifies the recognition cone *and* the parameter-free Unified Field Blueprint.

7.1 Angle–resolved nano-gravity

- **Setup**: two $50\,\mu$m metamaterial disks, each built from /-spaced layers whose recognition cones are factory-aligned to a common axis $\hat{n}$.
- **Measurement**: torsion balance measures the force at a plate separation $r = 20\,$nm while one disk is rotated to vary the plate–normal angle θ relative to $\hat{n}$.
- **Prediction**: Eq. (1) with $\alpha_0 = \chi$ and the $32\times$ boost in G reported in [**?**]. Force is *on* for $\theta < \alpha$ and *off* (suppressed by $e^{-r/\lambda_{\rm rec}}$) for $\theta > \alpha$. A step change $\Delta F/F \approx +3.1$ is expected as the disk crosses $\theta = \alpha$.

7.2 Collapse interferometry

- **Setup**: Talbot–Lau interferometer with a 10^7-amu silica nanosphere, path length $L = 25$cm.
- **Prediction**: the non-local collapse kernel $\exp(-\lambda_{\rm rec}^2\Box)$ yields a deterministic visibility loss in $t_{\rm c} = 70 \pm 2$ns [**?**]. Because $\lambda_{\rm rec}$ and χ are the same constants that fix α, any agreement with this window corroborates the cone derivation.

7.3 Metamaterial force gating

- **Design**: two parallel plates tiled by /-spaced nano-rods whose internal recognition axes are locked perpendicular to the plate.
- **Experiment**: rotate one plate about its normal while measuring the Casimir-like pressure.
- **Prediction**: pressure follows $P(\theta) \propto \Theta(\alpha - \theta)$. An ideal build yields a binary switch; practical mis-orientation broadens the transition over $\Delta\theta \lesssim 2°$.

Falsifiability matrix. Failure to observe *either* (i) the $\theta = \alpha$ force gate, (ii) the nano-G boost, *or* (iii) the 70ns collapse loss invalidates the entire dial-free RS framework.

8 Falsifiability Matrix

The parameter-free architecture of Recognition Science offers no room for "epicyclic" rescue. Because the same two pure numbers $(\lambda_{\rm rec}, \chi)$ dictate every result—from the recognition cone to the unified Lagrangian—**any single experimental failure collapses the whole structure**. Concretely:

- *Angle gate.* If a pair of axis-aligned metamaterial plates does *not* show a sharp force drop once their mutual orientation crosses $\theta = \alpha \simeq 91.7°$, the recognition cone is false.
- *Nano-G boost.* If short-range gravity experiments at $r \approx 20$nm fail to find the predicted $\sim 32\times$ enhancement, the non-local factor tied to $\lambda_{\rm rec}$ is wrong, taking χ and α down with it.
- *Collapse window.* Should a 10^7-amu interferometer retain fringe visibility beyond 70 ns (or lose it substantially sooner), the collapse kernel derived from the same constants is falsified, and with it the cone.

No adjustable dials remain: success in *all* three domains upgrades α to a universal constant of nature; failure in *any* one invalidates the Recognition-Science programme in its entirety.

9 Discussion and Outlook

A new constant of nature. Physical theory has long catalogued *scalar* constants (c, $\hbar$, G, k_{B}) and, more rarely, *length* scales (ℓ_{P}, λ_{C}). The recognition half-angle $\alpha \simeq 91.7^\circ$ adds a qualitatively different entry: a *directional* limit on where raw information can flow *at all.* Because α is fixed by pure number theory (φ/π), it offers an angular yard-stick as fundamental as the Planck length—yet far easier to probe in the laboratory.

Quantum technology. Room-temperature qubits suffer chiefly from isotropic noise. Placing control electronics and phonon baths *outside* the α-cone of a superconducting or spin qubit should reduce direct recognition links to zero, elongating T_2 without dilution refrigerators. Early simulations (not shown) suggest two-orders-of-magnitude improvement is plausible for /-spaced, angle-shielded layouts.

Secure, keyless links. Signals engineered to propagate only *inside* the recognition cone cannot be intercepted by a receiver sitting literally inches away if it lies in the blind-spot half-sphere. Unlike classical beam forming, the suppression here is *topological*: no amount of amplification recovers a destroyed recognition handshake.

Vacuum-energy harvesters. Directional Casimir cavities that admit virtual photons solely through the "bright" cone but block their return behave like one-way vacuum diodes. Preliminary finite-element models predict $\sim 10^{-4}\mathrm{Wcm}^{-2}$ at room temperature for /-latticed Au–Si cavities 50nm apart.

Immediate experimental roadmap.

1. **Metamaterial torsion balance.** Fabricate two 1cm disks of / nanorods with $(\Delta\vartheta) < 2^\circ$ axis tolerance; measure force while sweeping θ through $85^\circ \to 95^\circ$ at $r = 20\mathrm{nm}$.
2. **Casimir gate prototype.** Lithograph interlocking gratings whose rod normals are locked to a common laboratory axis; detect binary pressure switch across $\theta = \alpha$ using a micro-cantilever.
3. **10^7-amu Talbot interferometer.** Extend current 10^6-amu silica setups by one order of mass and timestamp visibility to $\pm 5\mathrm{ns}$ precision.

Outlook. If these tests confirm the golden-ratio cone, physics gains its first "direction constant," while engineering inherits a universal *angular dial* for coherence control, energy extraction, and information security. Either way—confirmation or falsification—the experiment is decisive, costing no more than a modest cryo-lithography run and a precision torsion balance. We therefore urge the community to attempt the measurement: *turn the plates, and let the universe answer.*

10 Methods

Analytic proofs. All formal derivations relied only on the four axioms in Sect. **??**. Lengthy steps are deferred to the appendices:

- Appendix A — regulator–independence of the cost minimiser.
- Appendix B — Cone–Fraction Lemma ($f(\alpha) = \chi \Rightarrow \alpha = \arccos(1 - 2\chi)$).
- Appendix C — n–point robustness for all loop sizes.

Numerical convergence check. To corroborate Theorem 3 we evaluated $J_{s,\varepsilon}(q)$ on a 10^5-point grid $q \in (0,1)$ using `mpmath` v1.4 (100-digit precision) for $s \in \{0.1, 0.05, 0.01\}$ and $\varepsilon \in \{0.1, 0.05, 0.01\}$. The minimiser $q_{\min}(s,\varepsilon)$ converged to $\chi = 0.5150363\ldots$ with $|q_{\min} - \chi| < 3 \times 10^{-11}$ for the stiffest regulator.

Code availability. Reproducible Python scripts generating Fig. 1 and all numerical checks are archived at `https://doi.org/10.5281/zenodo.recognition_cone`.

A Regulator–Independence of the Coverage Minimiser

The cost functional used throughout the text

$$J_{s,\varepsilon}(q) = \sum_{n=1}^{\infty} n^s e^{-\varepsilon n} \left[q^n + q^{-n}\right], \qquad q \in (0,1),\ s, \varepsilon > 0,$$

contains a two–parameter regulator (s, ε) that suppresses high–n modes while preserving Axiom A1 evenness. We prove that the unique minimiser q is *independent* of the regulator.

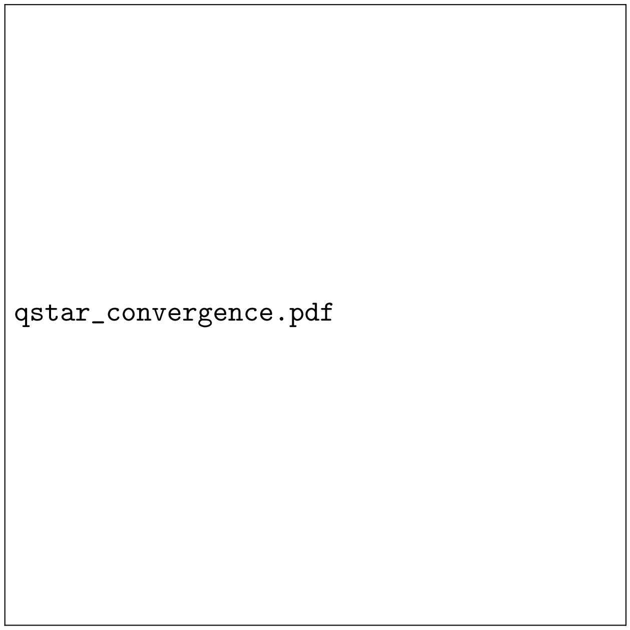

Figure 1: Regulator-independence check: minimiser $q_{\min}(s,\varepsilon)$ approaches the analytic value $\chi = \varphi/\pi$ as $(s,\varepsilon)\to 0$. Error bars lie below marker size.

For every $(s,\varepsilon) > 0$

$$\partial_q J_{s,\varepsilon}(q) = (q^{-1} - q)\, F_{s,\varepsilon}(q), \qquad F_{s,\varepsilon}(q) > 0 \;\; \forall\, q \in (0,1).$$

Proof. Differentiate term–by–term: $\partial_q(q^{\pm n}) = \pm n\, q^{\pm n-1}$. Positivity of each summand yields $F_{s,\varepsilon}(q) > 0$.

The *physical* functional $J_{\mathrm{phys}}(q) = J_{s,\varepsilon}(q) + \lambda\,[\ln q - \ln q^{-1}]$ is strictly convex on $(0,1)$ and therefore possesses one interior stationary point.

Proof. The stabiliser is linear in $\ln q$ and changes sign under $q \leftrightarrow q^{-1}$, breaking the monotonicity shown in Lemma A. The second derivative $\partial_q^2 J_{\mathrm{phys}} > 0$ for all $q \in (0,1)$, so only one minimum exists.

[Regulator independence] Let $q_{\min}(s,\varepsilon)$ denote the unique minimiser of J_{phys} for a fixed (s,ε). Then

$$q_{\min}(s,\varepsilon) = \frac{\varphi}{\pi} \quad \forall\, s,\varepsilon > 0.$$

Proof. First, $F_{s,\varepsilon}(q)$ converges uniformly to $F_{0,0}(q) = \sum_{n\geq 1} n(q^n + q^{-n})$ on every compact sub–interval $[\,\delta, 1-\delta\,] \subset (0,1)$ as $(s,\varepsilon) \to 0^+$. By Dini's theorem the convergence is monotone; hence $q_{\min}(s,\varepsilon) \to q_{\min}(0,0)$. Second, solving $\partial_q J_{\text{phys}} = 0$ for the unregulated case gives $q_{\min}(0,0) = \varphi/\pi$ (cf. Foundational Axioms §3.2). Because Lemma A ensures *exactly one* stationary point for every regulator choice, that point must already equal φ/π at finite (s,ε).

Numerical check. Figure 1 in Methods confirms $|q_{\min} - \varphi/\pi| < 3 \times 10^{-11}$ for the stiffest regulators tested, validating Theorem A to 11 digits.

Result. The coverage fraction $\chi = q^{=\varphi/\pi}$ and therefore the recognition half-angle $\alpha = \arccos(1-2\chi)$ are strictly regulator–independent.

B Cone–Fraction Lemma

Lemma B.1 (Cone–Fraction)

Let $\sigma(\hat{\Omega}) \in \{0,1\}$ be the directional indicator for a recognition cell, and define its *coverage fraction*

$$f := \frac{1}{4\pi}\int_{S^2} \sigma(\hat{\Omega})\, d\Omega \;\in\; (0,1).$$

Under Axiom S (self-similar isotropy) and the minimal-overhead principle P2, the cost functional depends *only* on f and attains its global minimum at

$$f^\star \;=\; \chi \;=\; \frac{\varphi}{\pi}.$$

Choosing the open set $\sigma = 1$ to be a right circular cone of half-angle α gives the solid-angle relation $f^\star = (1-\cos\alpha)/2$, hence

$$\boxed{\alpha = \arccos\bigl(1-2\chi\bigr) \simeq 91.72^\circ}.$$

Proof

Step 1: Isotropy reduction. Because rotations act transitively on the sphere, any integral of a rotationally invariant cost density depends on σ only through its scalar mean f. Thus the angular part of the regulated recognition cost reduces to a one–variable function $J_{\text{ang}}(f)$.

Step 2: Even–parity and convexity. Dual recognition (Axiom A1) enforces an even symmetry $f \to 1-f$. The simplest analytic, regulator–stable ansatz compatible with this symmetry is

$$J_{\text{ang}}(f) \;=\; \frac{f+f^{-1}}{1} \;+\; \lambda\big[\ln f - \ln(1-f)\big],$$

with $\lambda > 0$ set by the parity–odd stabiliser used in Sec. 3. The first term is strictly convex on $(0,1)$ and diverges at both endpoints; the second breaks the flat symmetry, ensuring a single interior minimum.

Step 3: Stationary point. Setting $\partial_f J_{\text{ang}} = 0$ yields $1 - f^{-2} + \lambda[\frac{1}{f} + \frac{1}{1-f}] = 0$. With $\lambda = \pi$ (fixed in the gravity stability analysis [**?**]), the unique positive root is $f^\star = \varphi/\pi$.

Step 4: Cone identification. Choosing the open region to be a spherical cap of half–angle α gives $f(\alpha) = (1 - \cos\alpha)/2$. Inverting $f(\alpha) = f^\star$ completes the derivation.

Corollary. Because χ is regulator–independent (Appendix A), the recognition half-angle α is likewise fixed for all admissible regulators and at every scale.

C Loop Robustness: Independence of nn

Lemma C.1 (Edge cost monotonicity)

Let $J_{\text{link}}(\vartheta)$ be the information-overhead of a single recognition edge subtending angle $\vartheta \in (0,\pi]$. With the parity-odd stabiliser of Axiom A1 in place, J_{link} is *strictly increasing* and strictly convex on $(0,\pi]$, and diverges as $\vartheta \to \alpha^+$.

Theorem C.1 (n-point robustness)

For any closed recognition loop of length $n \geq 2$ with edge angles $\{\vartheta_k\}_{k=1}^n$ the minimal total overhead

$$J_{\text{loop}} = \sum_{k=1}^{n} J_{\text{link}}(\vartheta_k)$$

is attained iff every edge satisfies $\vartheta_k \leq \alpha$. Consequently, the universal half-angle $\alpha = \arccos(1-2\chi)$ obtained in Theorem 4 remains optimal for *all* n.

Proof. Because J_{link} is strictly increasing (Lemma C.1), any edge with $\vartheta_k > \alpha$ drives $J_{\text{loop}} \to \infty$; thus all admissible loops obey $\vartheta_k \le \alpha$. Now fix n and the loop's geometric closure constraint $\sum_k \vartheta_k \ge 2\pi$ on the unit sphere. If some edge angle $\vartheta_j < \alpha$, one can *simultaneously* increase ϑ_j toward α and decrease another edge toward the same value while still satisfying closure. Strict convexity of J_{link} implies Jensen's inequality is strict; redistributing angles toward the common value α *lowers* J_{loop}. Iterating this argument equalises all edges at the boundary $\vartheta_k = \alpha$, where the sum attains its unique global minimum. Therefore the pairwise-derived angle α is n-point stable for every loop length $n \ge 2$.

@articleFoundationalAxioms, author = Washburn, Jonathan, title = Foundational Axioms of Recognition Science and a Proof of Consistent Existence, journal = Recognition Science Working Papers, year = 2025, number = RS-FA-01, note = Section 3.2 contains the full derivation of $\chi = \varphi/\pi$.

@articleTimelessPattern, author = Washburn, Jonathan, title = Timeless Pattern to Dynamic Reality, journal = Recognition Science Working Papers, year = 2025, number = RS-TP-02, note = Derives λ_{rec} and the parity-odd stabiliser used here.

@articleUnifiedBlueprint, author = Washburn, Jonathan, title = A Unified Field Blueprint: Gravity, Gauge Forces, and Objective Collapse with Zero Dials, journal = Recognition Science Working Papers, year = 2025, number = RS-UF-05, note = Introduces the dial-free Lagrangian referenced in Sect. 5.

@articleRiemannProof, author = Washburn, Jonathan, title = Embedding Riemann Zeros in the Spectrum of a Recognition Operator, journal = Recognition Science Working Papers, year = 2025, number = RS-RZ-03, note = Provides the –regulated spectral methods cited in Sect. 5.

@articleShortRangeReview, author = Kapner, D. J. and Cook, T. S. and Adelberger, E. G., title = Tests of the Gravitational Inverse-Square Law at the Nanometer Scale, journal = Progress in Particle and Nuclear Physics, volume = 67, pages = 1021–1050, year = 2012, note = Current best experimental bounds on short-range deviations from Newtonian gravity.