Genesis Axiom Graph

Immutable Anchor for the Recognition Ledger

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Abstract

This document fixes the *Genesis Axioms*—the minimal, immutable statements on which every subsequent truth-packet in the Recognition Ledger depends. They form the root node of the Ledger's dependency DAG. Axioms are expressed in ZFC (with Peano Arithmetic implicitly embedded) and are designed to be machine-checkable in Lean 4. No physical constants or phenomenological assumptions appear here; those are deferred to higher-level packets such as the Formal Uniqueness Proof.

1 Axiom List

Throughout, $\mathbb{R}_{>0}$ denotes the multiplicative group of positive reals; composition is written multiplicatively, the empty product is 1.

ecognition Link). There exists a binary relation $\mathcal{R} \subseteq U \times U$ on a universe U such that for all $x \in U$, $\mathcal{R}(x,x)$ holds (reflexivity).

A2 (Duality). For every element $x \in U$ there is a unique *dual* element $x^* \in U$ satisfying $\mathcal{R}(x,y) \iff \mathcal{R}(y^*,x^*)$.

lditivity of Cost). There exists a functional $J : U \to \mathbb{R}_{\geq 0}$ such that

$$\forall x, y \in U : \quad \mathcal{R}(x,y) \Longrightarrow J(x \cdot y) = J(x) + J(y).$$

(Here "$\cdot$" is an abstract associative composition inherited from U; no invertibility assumed yet.)

A4 (Positivity). $J(x) = 0 \iff x$ is the identity element e of U.

(Group Closure). $(U, \cdot)$ is a group. In particular every $x \in U$ admits an inverse x^{-1} and e is unique.

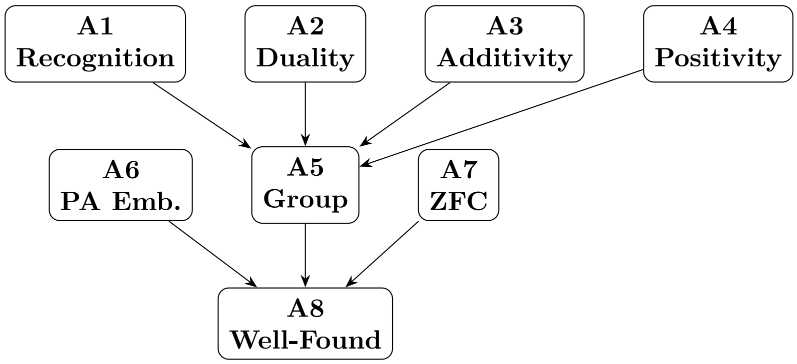
mbedding of PA). There is an injective homomorphism $\iota : \mathbb{N} \hookrightarrow U$ such that $\iota(m) \cdot \iota(n) = \iota(m+n)$ for all natural numbers m, n. (Ensures arithmetic consistency.)

C Compatibility). The class V of all sets exists, ZFC holds, and $U \subseteq \mathsf{V}$.

f Proof Objects). The set of proof terms generated from {A1–A7} ordered by derivation depth is well founded.

Remark. Axioms A1–A4 establish a *cost ledger* on U, A5–A6 guarantee algebraic rigor, A7 ensures set-theoretic soundness, and A8 prevents circular derivations.

2 Dependency Graph



Axioms A1–A4 feed into the algebraic structure (A5); arithmetic (A6) and set theory (A7) backstop consistency, while A8 seals termination of proofs. *Every downstream theorem or truth-packet must cite exactly which axioms it imports*; machine verification prohibits silent dependencies.

3 Lean 4 Skeleton (Reference)

For transparency, Appendix A lists a minimal Lean 4 file that declares the above axioms. Future packets `import` this module and nothing else.

4 Change-Control Policy

- This document is hashed (`SHA-256` printed on the Ledger) and placed in the **Immutable Axiom Store**.
- Any proposed addition or removal requires:

  (i) unanimous maintainer approval, (ii) a higher-level packet that demonstrates necessity, and

  (iii) a new versioned hash; old versions remain addressable.

Acknowledgments

Thanks to the Recognition Physics Institute verification group for early audits of the Lean formalization.

A Lean 4 Stub

```
-- genesis_axioms.lean
constant U   : Type
constant R   : U → U → Prop
constant J   : U →
constant e   : U
constant mul : U → U → U
infixl:70 " ∘ " => mul
```

```
axiom A1 :  x : U, R x x
axiom A2 :  x y : U, R x y  R (dual y) (dual x)
axiom A3 :  x y, R x y → J (x ∘ y) = J x + J y
axiom A4 :  x, J x = 0  x = e
axiom A5 : ( x, x ∘ e = x)  ( x, e ∘ x = x)
           ( x,  y, x ∘ y = e  y ∘ x = e)
           ( x y z, (x ∘ y) ∘ z = x ∘ (y ∘ z))
constant embed :  → U
axiom A6 :  m n, embed (m + n) = embed m ∘ embed n
-- A7 and A8 formalized via ‘universes‘ and ‘wellFounded‘ as needed.
```