The Geometry of Inquiry: Questions as Cost Gaps, Forced Answers, and the Meta-Closure of Recognition Science

A Theorem in Recognition Science

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Abstract

We formalize a *Geometry of Inquiry* in which questions are not free-floating semantic objects but structures determined by the J-cost landscape. A question Q maps a context (configuration space with cost) to a set of candidate answers, each carrying a cost. We classify questions into three types: *well-formed* (some answer has finite cost), *dissolved* (all answers have infinite cost — the fate of Gödel-type self-referential paradoxes), and *forced* (exactly one answer has zero cost). We prove that each theorem T0–T8 in the Recognition Science forcing chain is a forced question with a unique zero-cost answer. We then establish a *meta-closure theorem*: Recognition Science is the unique zero-cost theory in the space of physical frameworks, and the question "Why RS?" is itself forced. This achieves a non-paradoxical form of self-reference: RS explains why RS is the explanation, with the self-referential loop being cost-decreasing (stable) rather than cost-increasing (paradoxical). All definitions and core theorems are formalized in Lean 4 (`IndisputableMonolith.Foundation.Inquiry`, `QuestionTaxonomy`, `InquiryForcingConnection`, `MetaClosure`).

Keywords: cost geometry, inquiry, forced questions, meta-closure, Gödel dissolution, self-reference, Recognition Science.

Contents

1 Introduction

What is a question? In classical logic, a question is an interrogative sentence; in formal semantics, it is a partition of logical space [5]. Neither framework explains *why some questions have unique answers* while others are paradoxical.

Recognition Science offers a precise answer: a question is a **cost gap** in the J-landscape. The gap's geometry determines whether the question is well-formed, dissolved, or forced. The act of inquiry is gradient descent on the cost manifold — moving from high cost to zero cost.

This paper formalizes that insight and proves two main results:

1. **The forcing chain is a sequence of forced questions.** Each theorem T0–T8 corresponds to a question whose unique zero-cost answer is the RS result (logic, discreteness, ledger, J, φ, 8-tick, D=3).

2. **Meta-closure.** In the space of physical theories, RS has zero cost (zero free parameters, unique J, unique φ). All alternatives have positive cost. The question "Which theory is correct?" is forced, and the answer is RS.

The self-referential character of statement (2) — RS explaining why RS — is resolved by showing that the self-reference loop is cost-*decreasing*: each iteration reduces residual cost toward zero, unlike Gödelian loops that increase complexity without bound.

Foundational dependencies.
1. J-uniqueness (T5) [1].
2. Gödel dissolution [2]: self-referential queries have infinite cost.
3. Reference theory [3]: reference as cost-minimising compression.
4. Law of Existence [4]: x exists iff $J(x) = 0$ iff $x = 1$.

2 Questions as Cost Structures

Definition 2.1 (Context)**.** *A* context $\mathcal{X} = (X, J_X)$ *is a non-empty set* X *(the configuration space) equipped with a non-negative cost function* $J_X : X \to \mathbb{R}_{\geq 0}$*.*

Definition 2.2 (Question)**.** *A* question $Q = (\mathcal{X}, \mathcal{A}, C, \iota)$ *consists of:*
- *A context* $\mathcal{X}$ *(where the question is asked).*
- *An answer space* $\mathcal{A} = (A, J_A)$ *(where answers live).*
- *A non-empty set of candidates* $C \subseteq A$*.*
- *An embedding* $\iota : C \hookrightarrow A$*.*

Definition 2.3 (Answer cost)**.** *The* cost *of an answer* $a \in C$ *is* $J_A(\iota(a))$*.*

3 Classification of Questions

Definition 3.1 (Well-formed)**.** Q *is* well-formed *iff* $\exists\, a \in C : J_A(a) < \infty$*.*

Definition 3.2 (Dissolved)**.** Q *is* dissolved *iff* $\forall\, a \in C : J_A(a) = \infty$ *(or exceeds any fixed threshold).*

Definition 3.3 (Forced)**.** Q *is* forced *iff* $\exists!\, a \in C : J_A(a) = 0$*.*

Definition 3.4 (Determinate)**.** Q *is* determinate *iff* $\exists\, a \in C : J_A(a) = 0$*.*

Theorem 3.5 (Trichotomy)**.** *Every question is either dissolved, forced, determinate-but-not-forced, or well-formed-but-indeterminate. These four classes are exhaustive and mutually exclusive.*

Proof. Case analysis on the infimum of J_A over C:

- $\inf = \infty$: dissolved.
- $\inf = 0$, achieved by exactly one a: forced.
- $\inf = 0$, achieved by multiple a: determinate, not forced.
- $0 < \inf < \infty$: well-formed, indeterminate. □

Theorem 3.6 (Forced implies unique answer)**.** *If Q is forced, the zero-cost answer a^* is unique.*
Lean: `forced_answer_unique`.

Proof. By the definition of forced ($\exists!$). □ □

Theorem 3.7 (Dissolved questions are not real questions)**.** *A dissolved question has no practically accessible answer. In the RS ontology, dissolved questions correspond to self-referential or paradoxical constructions whose cost diverges.*
Lean: `dissolved_question_no_answer`.

Example 3.8 (The Liar Paradox as dissolved question)**.** *Consider the question "Is the sentence 'This sentence is false' true or false?" Model this as Q with $C = \{\text{True}, \text{False}\}$ and answer cost:*

- $J_A(\text{True}) = \infty$*: if the sentence is true, it is false, creating a self-referential loop with divergent cost ($J(0^+) \to \infty$).*
- $J_A(\text{False}) = \infty$*: if the sentence is false, it is true, same divergence.*

Both answers have infinite cost, so Q is dissolved*. The Liar is not a paradox but a cost singularity — a question that the J-landscape renders inaccessible. This is the RS mechanism behind Gödel dissolution [2].*

Example 3.9 ("Why does anything exist?" as forced question)**.** $C = \{$*"Something exists"*, *"Nothing exists"*$\}$.

- $J_A($*"Something"*$) = 0$*: at $x = 1$ (the unique existent), $J(1) = 0$.*
- $J_A($*"Nothing"*$) = \infty$*: $J(0^+) \to \infty$.*

This is forced*: exactly one answer has zero cost. Existence is not contingent but inevitable.*

4 The Forcing Chain as Forced Questions

Each theorem T0–T8 can be recast as a forced question.

Definition 4.1 (T0 Question: "What is logic?")**.** $C = \{\mathit{True}, \mathit{False}\}$*;* $J_A(\mathit{True}) = 0$, $J_A(\mathit{False}) > 0$*. This is forced at True (consistency has zero cost).*
Lean: `T0Question`, `t0_forced`.

Definition 4.2 (T5 Question: "What is the cost function?")**.** $C = \{F : \mathbb{R}_{>0} \to \mathbb{R}_{\geq 0} \mid F(1) = 0,\ F \text{ symmetric, smooth}\}$*;* $J_A(F) = 0$ *iff* $F = J$*. Forced at $F = J$ by T5 uniqueness.*
Lean: `T5Question`, `t5_forced_at_one`.

Definition 4.3 (T6 Question: "What is the scaling ratio?")**.** $C = \{r \in \mathbb{R}_{>0} \mid r^2 = r + 1\}$*;* $J_A(r) = 0$ *iff* $r = \varphi$*. Forced at* $\varphi = (1+\sqrt{5})/2$*.*
Lean: `T6Question`, `t6_forced_at_phi`.

Definition 4.4 (T7 Question: "What is the period?")**.** $C = \{2^k : k \in \mathbb{N}\}$*;* $J_A(n) = 0$ *iff* $n = 8$*. Forced at* $n = 8 = 2^3$*.*
Lean: `T7Question`, `t7_forced_at_eight`.

Definition 4.5 (T8 Question: "What is the dimension?")**.** $C = \mathbb{N}$*;* $J_A(D) = 0$ *iff* $2^D = 8$*, i.e.* $D = 3$*. Forced at* $D = 3$*.*
Lean: `T8Question`, `t8_forced_at_three`.

Theorem 4.6 (Complete forcing)**.** *All of T0, T5, T6, T7, T8 are forced questions. Their answers form the RS foundation: logic, J, φ, 8-tick, D=3.*
Lean: `forcing_chain_as_inquiry`.

5 Theory Space and Meta-Closure

Definition 5.1 (Theory space)**.** *The* theory space $\mathcal{T}$ *is the set of all candidate physical frameworks. Each theory* $T \in \mathcal{T}$ *has a* theory cost

$$J_{\mathcal{T}}(T) = \underbrace{N_{params}(T)}_{parameter\ count} + \underbrace{J_{mismatch}(T)}_{prediction\ error} + \underbrace{J_{complexity}(T)}_{description\ length} . \tag{1}$$

Theorem 5.2 (RS has zero cost)**.** $J_{\mathcal{T}}(RS) = 0$.

$N_{params} = 0$ *(proven: zero adjustable parameters).* $J_{mismatch} = 0$ *(at the model layer: exact derivations).* $J_{complexity} = 0$ *(minimal: one functional equation).*

Lean: `rs_zero_cost`.

Theorem 5.3 (Alternatives have positive cost)**.** *For any theory* $T \neq RS$ *with at least one free parameter:* $J_{\mathcal{T}}(T) > 0$.

Lean: `alternatives_positive_cost`.

Theorem 5.4 (Meta-closure)**.** *The question "Which physical framework is correct?" is* forced *in* $\mathcal{T}$*. The unique zero-cost answer is RS.*

Lean: `meta_closure`.

Proof. Combine Theorems 5.2 and 5.3: RS is the unique T with $J_{\mathcal{T}}(T) = 0$, so the question is forced by Definition 3.3. □ □

6 Self-Reference Without Paradox

Definition 6.1 (Self-referential question)**.** *A question is* self-referential *if its answer space includes the theory in which it is formulated.*

Theorem 6.2 (Stable self-reference)**.** *The meta-closure question ("Why RS?") is self-referential but* stable*: the self-reference loop is cost-decreasing.*

Proof. Let T_n denote the n-th iteration of "apply RS to explain RS."

- $T_0 =$ RS has cost 0 in $\mathcal{T}$.
- $T_1 =$ RS applied to T_0: since RS is self-consistent, $T_1 = T_0$ and cost remains 0.

The sequence $\{T_n\}$ is constant at cost 0. No divergence, no paradox. Contrast with Gödelian self-reference, where cost increases without bound.

Lean: `self_ref_stable`. □

Remark 6.3. *This is the precise sense in which RS dissolves Gödel's objection. Self-referential* arithmetic *sentences can have unbounded cost* ($J(0^+) \to \infty$)*. Self-referential* cost-minimisation *is stable (cost stays at zero). These are different mathematical objects in different categories.*

7 The Eight Fundamental Inquiry Modes

A systematic analysis of question structure yields eight fundamental inquiry modes, corresponding to the eight independent directions in cost space:

#	Mode	Canonical Form	Cost Signature
1	Existence	"Does X exist?"	$J(X) = 0$ vs > 0
2	Identity	"What is X?"	argmin J
3	Relation	"How does X relate to Y?"	$J(X/Y)$
4	Cause	"Why X?"	Gradient flow toward X
5	Possibility	"Can X occur?"	$J(X) < \infty$
6	Necessity	"Must X occur?"	$J(\neg X) = \infty$
7	Composition	"What are X's parts?"	Subadditivity of J
8	Purpose	"What is X for?"	Direction of $-\nabla J$

Theorem 7.1 (Completeness of inquiry modes)**.** *Every well-formed question can be decomposed into a combination of the eight fundamental modes.*

Proof. We show that the eight modes span the possible relationships between a question context $\mathcal{X}$ and its answer space $\mathcal{A}$ under the J landscape. A question probes one of:

1. **Existence**: Is $J_A(a) = 0$ for some a? (Probes the zero set.)
2. **Identity**: What is $\arg\min J_A$? (Probes the minimiser.)
3. **Relation**: What is $J(a_1/a_2)$ for given pairs? (Probes the cost between two candidates.)
4. **Cause**: What is $-\nabla J_A(a)$? (Probes the gradient — the direction of cost decrease.)
5. **Possibility**: Is $J_A(a) < \infty$? (Probes finiteness.)
6. **Necessity**: Is $J_A(\neg a) = \infty$? (Probes inevitability of a.)
7. **Composition**: Is $J_A(a_1 + a_2) \leq J_A(a_1) + J_A(a_2)$? (Probes sub/superadditivity.)
8. **Purpose**: In which direction does $-\nabla J$ point from a? (Probes teleology — what a is "for" in the cost landscape.)

These exhaust the first- and second-order properties of J_A: zero set (1,6), minimiser (2), gradient (4,8), pairwise cost (3), finiteness (5), and composition structure (7). Any well-formed question about the landscape reduces to a combination of these probes.

Lean: `QuestionTaxonomy.modes_complete`. □

Remark 7.2 (Why eight?)**.** *The count "eight" is not arbitrary. The cost landscape J on $\mathbb{R}_{>0}$ is a one-dimensional function with: a zero (mode 1), a minimum (mode 2), a first derivative (modes 4, 8), pairwise structure (mode 3), global finiteness properties (modes 5, 6), and composition rules (mode 7). These are the* complete *set of qualitatively distinct probes of a smooth convex function — parallelling the 8 DFT modes of the eight-tick cycle. The coincidence of "8 inquiry modes" with "8-tick period" is structural, not numerical.*

8 Implications

1. **The regress problem is dissolved.** "Why does X hold?" terminates when $J(X) = 0$. The chain of "why" questions converges to the zero-cost ground state, not to an infinite regress.

2. **Inquiry IS physics.** Questions are cost gaps; inquiry is gradient descent; answers are cost minima. There is no distinction between "the universe finding its ground state" and "an agent finding an answer."

3. **RS is self-justifying.** The meta-closure theorem shows RS is the unique zero-cost framework, and the question "Why RS?" is forced. This is not circular — it is a fixed point.

9 Comparison with Existing Approaches

Feature	Standard	RS (this paper)
Groenendijk–Stokhof [5]	Partition semantics	Cost-gap semantics
Hintikka [6]	Game-theoretic questions	Forced questions ($J = 0$)
Floridi [7]	Levels of abstraction	Cost hierarchy (J values)
Gödel	Incompleteness (syntactic)	Dissolved questions (cost $= \infty$)
Tarski	Undefinability of truth	Truth $=$ J-minimality (defined)

Remark 9.1. *The RS framework does not contradict Gödel or Tarski. Gödel sentences are* dissolved *(infinite cost), not "false" or "unprovable in the strong sense." Tarski's result applies to* truth predicates within formal languages*; the RS notion of truth ($J = 0$) is a* physical *criterion, not a linguistic one. The two are compatible because they operate in different categories.*

10 Discussion

Claims and non-claims

We formalise questions as cost structures and prove that the RS forcing chain T0–T8 consists of forced questions. We do *not* claim that all philosophical questions can be formulated within this framework, or that the meta-closure theorem constitutes a "proof of RS" in the circular sense. The meta-closure is a *fixed point*: RS is self-consistent, not self-proving.

Open problems

(Q1) Can the eight inquiry modes be axiomatised independently (i.e. proved to be a basis for a "logic of questions")?

(Q2) Is there a category-theoretic formulation (e.g. questions as morphisms in a topos)?

(Q3) Can dissolved questions be "regularised" by modifying the cost landscape (analogous to renormalisation)?

(Q4) Does the theory-space cost $J_{\mathcal{T}}$ have a measurable proxy (e.g. description length + prediction error)?

11 Falsification Criteria

Falsification Criterion 11.1 (Non-forced physical constant)**.** *If a fundamental constant (e.g. φ, α^{-1}) can be shown to have multiple zero-cost explanations in $\mathcal{T}$ (the question is determinate but not forced), meta-closure fails.*

Falsification Criterion 11.2 (Zero-cost alternative)**.** *If a zero-parameter framework $T \neq RS$ is exhibited with $J_{\mathcal{T}}(T) = 0$, the uniqueness claim is falsified.*

12 Lean Formalization

Module	**Content**
`Foundation.Inquiry`	Context, Question, answerCost, classification
`Foundation.QuestionTaxonomy`	8 modes, completeness
`Foundation.InquiryForcingConnection`	T0–T8 as forced questions
`Foundation.MetaClosure`	Theory space, RS zero cost, meta-closure

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