

Recognition Science: The Mathematical Spine

A Self-Contained Reading Order for Mathematicians

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February 13, 2026

Purpose

This document extracts from the full Recognition Science (RS) syllabus a **self-contained mathematical narrative** that can be read with *no* physical interpretation. Every result in Layers 0–3 is pure mathematics (functional equations, convex analysis, algebraic number theory, graph theory, topology). Physical semantics enter only in Layer 4 (“operator theory”) and can be treated as motivation rather than prerequisite.

The dependencies are minimal and explicit: each layer uses only the definitions and theorems of the layers above it.

Companion file: `papers/Recognition_Science_Math_DAG.mermaid` (visual dependency graph).

1 Layer 0 — The Functional Equation

“*What is the primitive?*”

M1. The Recognition Composition Law (RCL)

Syllabus paper: 2. RCL Primer **File:** `Recognition_Composition_Law_Primer.tex`
Statement. Let $F : \mathbb{R}_{>0} \rightarrow \mathbb{R}$ be continuous and satisfy

$$F(xy) + F(x/y) = 2F(x)F(y) + 2F(x) + 2F(y). \quad (1)$$

This is a calibrated multiplicative form of the d’Alembert functional equation $f(s+t) + f(s-t) = 2f(s)f(t)$ under the substitution $x = e^s$, $y = e^t$, and $F = H - 1$ where H is the standard d’Alembert solution.

Mathematical content. The paper states the equation, motivates the normalization side-conditions (A1)–(A3), and shows that (??) is the natural composition law for *ratio costs* on $\mathbb{R}_{>0}$.

Dependencies: None (starting point).

2 Layer 1 — Uniqueness and Classification

“*What does the equation force?*”

M2. Cost Uniqueness (Theorem T5)

Syllabus paper: 3. Uniqueness of the Canonical Reciprocal Cost **File:** Cost-9-1.pdf (latest revision)

Statement. Under the axioms

- (A1) $F(1) = 0$ (normalization),
- (A2) F satisfies the RCL (??) (composition),
- (A3) $\frac{d^2}{dt^2}F(e^t)\Big|_{t=0} = 1$ (calibration),

there is a *unique* solution on $\mathbb{R}_{>0}$:

$$J(x) = \frac{1}{2}(x + x^{-1}) - 1 = \cosh(\ln x) - 1. \quad (2)$$

Derived properties (not assumed): reciprocity $J(x) = J(x^{-1})$; strict convexity on $\mathbb{R}_{>0}$; $J \geq 0$ with equality iff $x = 1$; $J''(1) = 1$.

Dependencies: M1.

M3. D'Alembert Inevitability

Syllabus paper: 5. D'Alembert Inevitability **File:** DAlembert_Inevitability.tex

Statement. Among all continuous functions $F : \mathbb{R}_{>0} \rightarrow \mathbb{R}$ satisfying $F(x) = F(x^{-1})$, $F(1) = 0$, and $F(xy) + F(x/y) = P(F(x), F(y))$ for a symmetric polynomial P , non-triviality forces $P(u, v) = 2u + 2v + cuv$ with c a single scalar. Calibration fixes $c = 2$, recovering the standard d'Alembert family.

Mathematical content. This is a *classification result* for functional equations. Under mild structural assumptions, the bilinear family is the **only** survivor. The composition law (??) is therefore not a modeling choice but a theorem.

Dependencies: M1 (for the composition law setup); M2 (for calibration step).

M4. Reciprocal Convex Costs for Ratio Matching

Syllabus paper: 63. Reciprocal Convex Costs **File:** axioms-4140269.pdf **PUBLISHED – Axioms (MDPI), 2026**

Statement. Characterizes the class of reciprocal convex cost functionals on $\mathbb{R}_{>0}$ satisfying the d'Alembert equation. Shows $f(u) := 1 + J(e^u)$ is continuous and satisfies the additive d'Alembert equation; classification gives $J(x) = \cosh(a \log x) - 1 = \frac{1}{2}(x^a + x^{-a}) - 1$. Geometric-mean decision boundaries, compositionality for product models, sequential mediation via geometric mean. Washburn & Rahnamai Barghi.

Dependencies: M2.

M4b. Recognition Algebra: The Unified Algebraic Framework

Syllabus paper: 82. Recognition Algebra **File:** papers/01-root-foundation/Recognition_Algebra.tex **NEW — Feb 2026**

Statement. The RCL forces four algebraic layers:

1. **Cost Algebra.** Monoid $(\mathbb{R}_{\geq 0}, \star, 0)$ with $a \star b = 2ab + 2a + 2b$; shifted form $H = J + 1$ satisfies d'Alembert.
2. **φ -Ring.** $\mathbb{Z}[\varphi]$ with Galois conjugation $\sigma^2 = \text{id}$, multiplicative norm $N(\alpha\beta) = N(\alpha)N(\beta)$, and φ a unit ($N(\varphi) = -1$).
3. **Ledger Algebra.** Graded abelian group of antisymmetric events with vertex-wise conservation $\sigma = 0$.
4. **Octave Algebra.** $\mathbb{Z}/8\mathbb{Z}$ with DFT-8; 20 neutral basis modes from $3 \times 4 + 2 \times 4$.

Category **RecAlg** defined; canonical J is the initial object. Self-contained proofs. Lean-verified.

Dependencies: M2, M5, M8.

3 Layer 2 — Algebraic and Geometric Rigidity

“What scale and dimension are forced?”

M5. φ as Unique Fixed Point (Penrose Bridge)

Syllabus paper: 10. The Golden Ratio as a Universal Coherence Eigenvalue
File:
`Penrose_golden_ratio_and_ledger_structure.tex`

Statement. The minimal reciprocal self-correction rule $x_{n+1} = 1 + 1/x_n$ has unique positive fixed point $\varphi = (1 + \sqrt{5})/2$, satisfying $\varphi^2 = \varphi + 1$.

Key results:

- **Log-ratio isomorphism.** $t = \ln x$ gives $(\mathbb{R}_{>0}, \times) \cong (\mathbb{R}, +)$. Multiplicative inflation $x \mapsto \varphi x$ becomes additive shift $t \mapsto t + \ln \varphi$.
- **Coherence cost of aperiodicity.** $J(\varphi) = \varphi - 3/2 \approx 0.118$ is the minimal non-trivial cost.
- **Penrose substitution.** The 2×2 Penrose substitution matrix has Perron–Frobenius eigenvalue φ , giving an independent geometric origin of the same constant.

Dependencies: M2 (for J and self-similarity).

M6. Dimensional Rigidity: $D=3$

Syllabus paper: 11. Dimensional Rigidity D=3 (Strengthened)
File:
`Dimensional_Rigidity_D3_2.tex`

Statement. Three independent constraints each force spatial dimension $D = 3$:

- (A) **Topological.** An integer-valued linking invariant for disjoint oriented loops in \mathbb{S}^D exists iff $D = 3$ (Alexander duality: $H_1(\mathbb{S}^D \setminus K) \cong \mathbb{Z}$ iff $D = 3$).
- (B) **Dynamical.** For the D -dimensional Kepler potential $V_D(r) \propto -r^{2-D}$, near-circular orbits are stable only for $D < 4$ and non-precessing only for $D = 3$ (Binet equation analysis).
- (C) **Arithmetic.** $\text{lcm}(2^D, 45)$ is minimized over $D \geq 3$ uniquely at $D = 3$, where it equals 360.

Dependencies: M5 (for the arithmetic constraint involving φ -scaling); standard topology and ODE theory.

M7. Stability Audit (RSA)

Syllabus paper: 53. The Recognition Stability Audit
File:
`Recognition_Stability_Audit.tex`

Statement. Converts existence/non-existence claims into finite certificates using: (i) a canonical sensor (Cayley transform), (ii) Schur/Pick interpolation theory, (iii) a realizability class grounded in period-8 constraints.

Mathematical content. A decision-style pipeline linking complex analysis, control theory, and computational certificates. Pure mathematics; physical context is motivational.

Dependencies: M3 (inevitability provides the constraint space).

4 Layer 3 — Structures Forced by Cost

“What mathematical objects emerge from the cost?”

M8. Ledger Dynamics

Syllabus paper: 4. Coherent Comparison as Information Cost
File:
`papers/pdf/2601.12194v1.pdf`

Statement. Discrete dynamics on directed graphs with J -cost weighted edges, atomic ticks, and conservation constraints. Key results:

- Balanced double-entry postings are *forced* by conservation + discreteness.
- Minimal period on the D -cube is 2^D (proved for all D).
- Closed-chain flux vanishes: $\sum \ln r_i = 0$ over cycles.

Dependencies: M2 (for J).

M9. The Law of Mathematical Inevitability

Syllabus paper: 35. The Law of Mathematical Inevitability

Mathematics_Ledger_Phenomenon.tex

File:

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Statement. Any continuous cost functional satisfying (A1)–(A3) *necessarily forces*:

1. **Natural numbers** as φ -ladder positions $L(n) = \varphi^n$, with the Fibonacci recursion $L(n+2) = L(n+1) + L(n)$.
2. A **proof concept**: balanced ledger sequences ($\sum \ln r_i = 0$), the *unique* admissible balance condition.
3. **Incompleteness**: divergent cost of self-referential chains ($J(0^+) \rightarrow \infty$).
4. A **universal referent**: the zero-cost subspace is the unique universal reference for all positive-cost objects (Wigner’s puzzle resolved).

The cost-consistent metric is $d(m, n) = J(\varphi^{m-n})$; it satisfies all metric axioms including the triangle inequality.

Dependencies: M2, M5, M8.

5 Layer 4 — Operator Theory and Quotient Spaces

“What is the dynamics and what is uniqueness at the level of observables?”

Physical motivation enters here: “states,” “evolution,” “observation.” The mathematics is operator theory on Hilbert and Banach spaces.

M10. Recognition Operator

Syllabus paper: 9. The Recognition Operator: Beyond the Hamiltonian

Recognition-Operator.tex

File:

Updated — Feb 2026

Statement. A *recognition operator* is a map $\hat{R} : \mathcal{S} \rightarrow \mathcal{S}$ on admissible states satisfying: (i) cost monotonicity $C(\hat{R}s) \leq C(s)$, (ii) reciprocity preservation $\sigma(\hat{R}s) = \sigma(s) = 0$, (iii) period-8 update.

Key analysis result. In the log-deviation coordinate $r = e^\varepsilon$:

$$J(e^\varepsilon) = \cosh(\varepsilon) - 1 = \frac{1}{2}\varepsilon^2 + \frac{1}{24}\varepsilon^4 + \dots$$

The quadratic approximation $J \approx \frac{1}{2}\varepsilon^2$ has relative error < 1% for $|\varepsilon| \leq 0.1$. An effective quadratic generator \hat{H}_{eff} exists such that $\hat{R} = \exp(-i\hat{H}_{\text{eff}} \cdot 8\tau_0/\hbar) + O(\tau_0^2)$. A continuum limit recovers the Schrödinger equation.

Dependencies: M2, M8.

M11. Model-Independent Exclusivity on the Quotient

Syllabus paper: 6. Model-Independent Exclusivity

Model-Independent-Exclusivity-Quotient.tex

File:

Statement. Consider an abstract “framework” (S, R, O, J) (state space, evolution, observables, cost). If J satisfies (A1)–(A3) and the framework is zero-parameter + self-similar, then:

- J = the canonical J (cost uniqueness on quotient),
- $\varphi = (1+\sqrt{5})/2$ (preferred scale),
- the quotient state space S/\sim_O is a subsingleton.

Any two such frameworks are *observationally equivalent*.

Dependencies: M2, M3, M8.

M12. The Fredholm Index of the Death Operator

Syllabus paper: 67. The Fredholm Index of Death **File:** `Fredholm_Index_of_Death.tex`
NEW — Feb 2026

Statement. A diagonal projection $\mathcal{D} : \mathbb{C}^8 \rightarrow \mathbb{C}^8$ with survival factors in $\{0, 1\}$, decomposing \mathbb{C}^8 into kernel (channels 1–3) and image (channels 5–8). Channel 4 is mixed. The Fredholm index is

$$\text{ind}(\mathcal{D}) = k - 5,$$

where $k \in \{0, 1, \dots, 8\}$ is a reflexivity parameter. The dimension of the preserved subspace is bounded: $\dim(\text{im } \mathcal{D}) \leq \varphi^k$.

Dependencies: M2 (for φ -scaling), M10 (operator framework).

6 Layer 5 — Number-Theoretic Applications

“What does cost geometry say about primes?”

M13. Riemann Hypothesis via Spectral Stability

Syllabus paper: 34. A Weighted Diagonal Operator ... **File:** `Recognition-Riemann-Final.tex`

Summary. RS approach to RH: prime stiffness, log-prime spectrum, Bernstein inequality for finite exponential sums, near-field elimination via energy barrier.

Dependencies: M2.

M14. Goldbach via a Mod-8 Kernel

Syllabus paper: 36. Goldbach via Mod-8 Kernel **File:** `goldbach_rs-arXiv.tex`

Summary. Classical circle method + χ_8 character; mod-8 kernel concentrates on the 8-tick-compatible residue classes.

Dependencies: M8 (ledger structure provides mod-8 frame).

M15. Prime Stiffness

Syllabus paper: (within 34)

Summary. Unconditional chain: prime gaps $\geq 1 \rightarrow$ log-prime spectrum strictly increasing \rightarrow effective bandwidth = $k \log T \rightarrow$ Bernstein inequality \rightarrow near-field elimination (energy barrier argument).

Dependencies: M13.

Summary Table

ID	Title	Layer	Deps
M1	Recognition Composition Law	0	—
M2	Cost Uniqueness (T5)	1	M1
M3	D'Alembert Inevitability	1	M1,M2
M4	Reciprocal Convex Costs	1	M2
M5	φ Uniqueness / Penrose Bridge	2	M2
M6	Dimensional Rigidity $D=3$	2	M5
M7	Stability Audit (RSA)	2	M3
M8	Ledger Dynamics	3	M2
M9	Mathematical Inevitability	3	M2,M5,M8
M10	Recognition Operator	4	M2,M8
M11	Exclusivity on Quotient	4	M2,M3,M8
M12	Fredholm Index	4	M2,M10
M13	Riemann / Spectral Stability	5	M2
M14	Goldbach / Mod-8 Kernel	5	M8
M15	Prime Stiffness	5	M13

How to read this as a mathematician. Start at M1 (a functional equation on $\mathbb{R}_{>0}$). By M2 you know the unique solution. M3 shows you had no choice in the equation itself. M5–M6 pin the algebraic scale and spatial dimension. M8–M9 show the equation forces discrete dynamics *and* mathematics. M10–M11 define the operator theory and prove observational uniqueness. M13–M15 are number-theoretic applications. At no point are you required to believe anything about physics; the narrative is pure analysis, algebra, topology, and number theory.