COMPLAINT AND RESOLUTION (ROUND 2): NEUTRALIZATION, CHARGES, AND THE FINAL INEQUALITY CHAIN

TECHNICAL COMPANION TO PAPER1_ZEROZETA-V19

1. Context

After the first round of complaints (circularity in $V \geq 0$, $\log^2$ growth, singular inner factor, near-zero count), two further structural critiques were raised against the proof of Theorem 1 ($\zeta(s) \neq 0$ for $\Re s \geq 0.6$).

Both critiques target the same mismatch: the theorem proof applies the CR–Green upper bound to the *full* windowed phase of $\mathcal{J}_{\text{out}}$, but the CR–Green lemma requires the underlying function to be *harmonic* on the Whitney box — which $\log|\mathcal{J}_{\text{out}}|$ is *not*, because $\mathcal{J}_{\text{out}}$ has poles at ζ-zeros inside the box.

This document states each complaint and explains the fix now implemented in the paper.

2. Complaint 5: CR–Green applied to a meromorphic function

The complaint. The theorem proof (Step 2) asserts:

$$\int \psi_{L,\gamma_0}(-w') \;\leq\; Z_0\, C_{\text{test}} \sqrt{E(I)} \cdot L$$

where $E(I) = \iint |\nabla \log|\mathcal{J}_{\text{out}}||^2\, \sigma$ and $-w'$ is the boundary phase derivative of $\mathcal{J}_{\text{out}}$.

The CR–Green lemma (Proposition `prop:length-free`) requires the function $\log|J|$ to be **harmonic** on the box D. But $\mathcal{J}_{\text{out}}$ has **poles** at ζ-zeros inside D, so $\log|\mathcal{J}_{\text{out}}|$ is **not harmonic** there. The Green identity on the punctured domain produces *extra interior charge terms* $2\pi \sum m_j V(\rho_j)$ that are not accounted for in the inequality.

Applying the CR–Green bound directly to $\mathcal{J}_{\text{out}}$ is therefore invalid.

The resolution: the neutralized ratio $\mathcal{J}_{\text{neut}}$. Define the **neutralized ratio**

$$\mathcal{J}_{\text{neut}}(s) \;:=\; \mathcal{J}_{\text{out}}(s) \cdot B_{\text{box}}(s),$$

where B_{box} is the half-plane Blaschke product over the ζ-zeros inside the Whitney box D.

- Each Blaschke factor $b(s, \rho_j) = (s - \rho_j)/(s - \rho_j^{\#})$ has a zero at ρ_j, which cancels the pole of $\mathcal{J}_{\text{out}}$ there.
- Hence $\mathcal{J}_{\text{neut}}$ is **holomorphic** on D, and $\log|\mathcal{J}_{\text{neut}}|$ is **harmonic** on D.
- On $\partial\Omega$: $|\mathcal{J}_{\text{neut}}| = |\mathcal{J}_{\text{out}}| \cdot |B_{\text{box}}| = 1 \cdot 1 = 1$ a.e.

The CR–Green pairing now applies to $\mathcal{J}_{\text{neut}}$ *with no interior charge terms*:

$$\int \psi_{L,\gamma_0}(-w'_{\text{neut}}) \;\leq\; Z_0\, C_{\text{test}} \sqrt{E_{\text{neut}}(I)} \cdot L.$$

Lower bound transfer. The boundary phase derivative of $\mathcal{J}_{\text{neut}}$ satisfies

$$-w'_{\text{neut}} \;=\; -w'_{\mathcal{J}_{\text{out}}} \;+\; \sum_j \frac{2\delta_j}{\delta_j^2 + (t-\gamma_j)^2} \;\geq\; -w'_{\mathcal{J}_{\text{out}}} \;\geq\; 0,$$

because each Blaschke factor contributes a nonneg Poisson kernel to the phase derivative (computed explicitly: $-d/dt$ Arg $b(\frac{1}{2} + it, \rho) = 2\delta/(\delta^2 + (t-\gamma)^2) \geq 0$).

Date: February 7, 2026.

The hypothetical zero ρ_0 at $\beta_0 \geq 0.6$ is **outside** D (since $\delta_0 \geq 0.1 > \alpha' L$ for t_0 large), so its Poisson contribution is present in $-w'_{\mathcal{J}_{\text{out}}}$ and hence also in $-w'_{\text{neut}}$:

$$\int \psi(-w'_{\text{neut}}) \;\geq\; \int \psi(-w'_{\mathcal{J}_{\text{out}}}) \;\geq\; 11L.$$

Energy bound. $\log|\mathcal{J}_{\text{neut}}| = 2\log|B| + \widetilde{W}$ where $\widetilde{W} = -\log|B_{\text{far}} \cdot S|$ is the harmonic neutralized field from Proposition `prop:Cbox-finite`. The energy $E_{\text{neut}} \leq C\log^2\langle\gamma_0\rangle \cdot |I|$ with C independent of c (the L's cancel in the Poisson integral).

Contradiction. With $c = c_0/\log\langle\gamma_0\rangle$: $E_{\text{neut}} = 2Cc_0$ (constant), giving $A\sqrt{c_0} \cdot L < 11L$ for $c_0 < (11/A)^2$.

3. Complaint 6: "near Blaschke energy is $O(|I|)$" is false

The complaint. The earlier version of Proposition `prop:Cbox-finite` stated that the near Blaschke factors (zeros inside the box) each contribute $O(|I|)$ to the weighted Dirichlet energy.

This is false: $|\nabla \log|b(s,\rho_j)||$ has a $1/|s-\rho_j|$ singularity, and $\iint (1/r^2)\cdot\sigma\cdot r\,dr\,d\theta$ diverges logarithmically. The σ weight does not cure the divergence.

The resolution: the claim is removed. The proposition now explicitly states:

> *"The zeros in B_{near} lie inside the box D, so $\log|B_{\text{near}}|$ has logarithmic singularities there and its weighted Dirichlet energy on $Q(\alpha' I)$ is **infinite**. This is not a problem: the near Blaschke factors are absorbed into the **neutralization** step in the main theorem proof, where they cancel the poles of $\mathcal{J}_{\text{out}}$ and produce the harmonic function $\log|\mathcal{J}_{\text{neut}}| = 2\log|B| + \widetilde{W}$ on D. The energy estimate bounds the harmonic field $\widetilde{W}$ only."*

No finite-energy claim is made for the near Blaschke factors. Their singularities are *removed* by neutralization, not *bounded* by an energy estimate.

4. Summary of the proof chain after both fixes

Step	**What happens**
Hypothesis	Assume $\zeta(\rho_0) = 0$ with $\beta_0 \geq 0.6$. Set $c = c_0/\log\langle\gamma_0\rangle$, $L = c_0/\log^2\langle\gamma_0\rangle$.
Neutralize	Define $\mathcal{J}_{\text{neut}} := \mathcal{J}_{\text{out}} \cdot B_{\text{box}}$. This is **holomorphic** on D (poles canceled). $\log|\mathcal{J}_{\text{neut}}|$ is **harmonic**.
Lower bound	$-w'_{\text{neut}} \geq -w'_{\mathcal{J}_{\text{out}}} \geq 0$ (Blaschke factors add positive Poisson mass). $\int\psi(-w'_{\text{neut}}) \geq 11L$ (from ρ_0, which is outside D).
Upper bound	CR–Green on the **harmonic** $\log|\mathcal{J}_{\text{neut}}|$: $\int\psi(-w'_{\text{neut}}) \leq A\sqrt{c_0}\cdot L$. No interior charges (harmonic $\Rightarrow$ no punctured domain needed).
Energy	$E_{\text{neut}} = C\log^2 \cdot |I| = 2Cc_0$ (height-independent after $\log^2$ cancellation). C independent of c (Poisson integral L-cancellation).
Contradiction	$11 \leq A\sqrt{c_0} = 11/\sqrt{2} < 11$.
Small height	$|\gamma_0| \leq 2$: ball-arithmetic certificate.

5. What is no longer in the proof

- **CR–Green applied to meromorphic** $\mathcal{J}_{\text{out}}$ — replaced by CR–Green on holomorphic $\mathcal{J}_{\text{neut}}$.
- **"Near Blaschke energy** $O(|I|)$**"** — removed; near factors are neutralized, not energy-bounded.
- **Interior charge terms in the Green identity** — eliminated by neutralization (no punctured domain needed).
- **"**$V \geq 0$**" for** $-\log|\mathcal{J}_{\text{out}}|$ — replaced by $W = -\log|\mathcal{I}| \geq 0$ (inner reciprocal, unconditional).
- **"**$S \equiv 1$**" claim** — removed; singular inner absorbed by $W \leq N\log(2+|t|)+C$ (PL/convexity).
- **Short-interval zero control at scale** L — not used; Poisson integral L-cancellation uses only coarse $O(\log T)$ density.