The Lighthouse Efficiency Parameter $\eta_L = 0.2936$

Cubic Metric Coupling in φ-Spiral Phased Arrays

Derived from the Recognition Composition Law

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Abstract

We derive the *Lighthouse efficiency parameter* η_L, a dimensionless constant that quantifies how effectively a φ-spiral electromagnetic coil array converts field energy into directional metric perturbation via the cubic non-linearity of the Recognition Science cost functional $\mathrm{J}(x) = \frac{1}{2}(x + x^{-1}) - 1$. For the canonical 8-coil configuration with pitch parameter $\kappa = 1$ and bipolar neutral schedule, we obtain the exact value

$$\eta_L = \frac{\left|\sum_{i=0}^{7} \varphi^{-3i/8} s_i\right|}{\sum_{i=0}^{7} \varphi^{-2i/8}} = 0.293\,615\,975\,3\ldots$$

where $\varphi = (1+\sqrt{5})/2$ is the golden ratio and $s_i \in \{+1, -1\}$ is the bipolar drive kernel. We prove that $\eta_L = 0$ for any *uniform* coil array (the cubic terms cancel by symmetry), establishing that the φ-spiral geometry is essential for metric coupling. We compute η_L as a function of spiral tightness κ, number of coils n, and schedule choice, and show that η_L is maximized by the canonical bipolar schedule among all binary neutral schedules. All geometric sums admit closed forms as rational functions of φ, making η_L a computable algebraic invariant of the Lighthouse architecture. The result is formalized in the Lean 4 proof assistant (module `Foundation.MetricPerturbation`).

Contents

1 Introduction

The Lighthouse project seeks to create a directional metric perturbation using a solid-state electromagnetic phased array whose geometry is derived from Recognition Science (RS) [1]. The central claim is that the φ-spiral arrangement of coils, driven with an 8-tick neutral schedule, accesses a non-standard coupling between electromagnetic fields and spacetime curvature through the *cubic non-linearity* of the RS cost functional.

In standard physics, the gravitational effect of electromagnetic energy is governed by the stress-energy tensor $T_{\mu\nu}$, which is *quadratic* in the field strength. This coupling is suppressed by $G/c^4 \sim 10^{-44}$ in SI units, making it unmeasurably small for laboratory fields. The RS framework, however, contains a *cubic* correction to the cost functional that, under specific geometric and temporal conditions, produces a *directional* metric perturbation whose sign depends on the commutation direction.

The efficiency of this cubic coupling depends entirely on the coil geometry and drive schedule. This paper derives the quantity η_{L} that governs this efficiency.

1.1 Summary of Results

(i) The cubic metric coupling vanishes identically for uniform coil arrays (§4.3).

(ii) For φ-spiral arrays, the coupling is non-zero and equals $\eta_{\mathrm{L}} = 0.2936\ldots$ for the canonical v0 configuration (§4).

(iii) η_{L} admits a closed-form expression as a ratio of geometric sums in φ (§5).

(iv) The canonical bipolar schedule maximizes η_{L} among binary neutral schedules (§6).

(v) η_{L} increases monotonically with spiral tightness κ, approaching 1 as $\kappa \to \infty$ (§7).

2 The Cost Functional and Its Cubic Non-Linearity

2.1 The Recognition Cost Functional

The unique cost functional satisfying the Recognition Composition Law (RCL) with normalization $\mathrm{J}(1) = 0$ and calibration $\mathrm{J}''(1) = 1$ is:

$$\mathrm{J}(x) = \frac{1}{2}\left(x + \frac{1}{x}\right) - 1 = \frac{(x-1)^2}{2x}, \qquad x > 0. \tag{1}$$

This is proved in the Lean module `Cost.JcostCore` with zero `sorry` statements.

2.2 Taylor Expansion Near Equilibrium

Setting $x = 1 + \varepsilon$ with $|\varepsilon| < 1$:

$$\mathrm{J}(1+\varepsilon) = \frac{\varepsilon^2}{2(1+\varepsilon)} = \frac{1}{2}\varepsilon^2 - \frac{1}{2}\varepsilon^3 + \frac{1}{2}\varepsilon^4 - \cdots \tag{2}$$

The leading term $\frac{1}{2}\varepsilon^2$ gives standard (quadratic) physics: energy, Maxwell equations, linearized Einstein equations. The *first non-linear correction* is the cubic term $-\frac{1}{2}\varepsilon^3$.

2.3 Physical Interpretation of the Cubic Term

The quadratic term is *symmetric*: $\varepsilon^2 = (-\varepsilon)^2$. It cannot produce a directional effect. The cubic term is *antisymmetric*: $\varepsilon^3 = -(-\varepsilon)^3$. It distinguishes the sign of ε and therefore produces a directional effect.

Remark 2.1. *This antisymmetry is the mathematical origin of the "sign flip" prediction: reversing the commutation direction (which flips the sign pattern of ε) reverses the cubic contribution to the metric perturbation.*

2.4 Metric Perturbation from Cost

In RS, the effective metric coefficient at a bond with recognition multiplier x is:

$$g_{\text{eff}}(x) = 1 - 2\,\mathrm{J}(x). \tag{3}$$

The metric deviation from flat space is:

$$h(x) \;=\; g_{\text{eff}}(x) - 1 \;=\; -2\,\mathrm{J}(x) \;=\; -(x-1)^2/x. \tag{4}$$

Expanding for $x = 1 + \varepsilon$:

$$h = -\varepsilon^2 + \varepsilon^3 - \varepsilon^4 + \cdots \tag{5}$$

The first term ($-\varepsilon^2$, always negative) is standard gravity (attractive). The second term ($+\varepsilon^3$, sign-dependent) is the *metric coupling* that the Lighthouse exploits.

3 The Lighthouse Coil Array

3.1 φ-Spiral Geometry

An n-coil Lighthouse array with pitch parameter $\kappa \in \mathbb{Z}_{>0}$ places coils at:

$$\theta_i = \frac{2\pi i}{n}, \qquad r_i = r_0 \cdot \varphi^{\kappa i/n}, \qquad i = 0, 1, \ldots, n-1. \tag{6}$$

The coil at position i is assigned to phase group $\psi_i = i \bmod 8$.

3.2 Amplitude Profile

The electromagnetic field amplitude at the observation point due to coil i decays with distance. For a fixed observation point (e.g., the center of the array), the amplitude from coil i scales inversely with radius:

$$A_i = A_0 \cdot \left(\frac{r_0}{r_i}\right) = A_0 \cdot \varphi^{-\kappa i/n}, \tag{7}$$

where A_0 is the amplitude from the innermost coil. Without loss of generality, set $A_0 = 1$.

3.3 The 8-Tick Neutral Schedule

The drive schedule assigns a sign $s_i \in \{+1, -1\}$ to each coil at each tick, subject to the *neutrality constraint*:

$$\sum_{i=0}^{n-1} s_i = 0. \tag{8}$$

The *canonical bipolar schedule* for $n = 8$ is:

$$\mathbf{s} = (+1, +1, +1, +1, -1, -1, -1, -1). \tag{9}$$

4 Definition and Computation of η_{L}

4.1 Definition

Definition 4.1 (Lighthouse Efficiency Parameter)**.** *For an n-coil array with amplitude profile (A_i) and drive signs (s_i), the* Lighthouse efficiency parameter *is:*

$$\boxed{\eta_{\mathrm{L}} = \frac{\left|\sum_{i=0}^{n-1} A_i^3\, s_i\right|}{\sum_{i=0}^{n-1} A_i^2}.} \tag{10}$$

The numerator measures the residual cubic coupling (which produces the directional metric effect), and the denominator measures the total quadratic energy (which is always present). Their ratio η_{L} is the fraction of EM energy that participates in metric coupling.

4.2 Computation for the v0 Configuration

For $n = 8$, $\kappa = 1$, canonical bipolar schedule, with $A_i = \varphi^{-i/8}$:

Table 1: Coil-by-coil contributions to η_{L} for the v0 Lighthouse.

Coil i	$A_i = \varphi^{-i/8}$	s_i	A_i^2	$A_i^3 \cdot s_i$
0	1.00000000	+1	1.00000000	+1.00000000
1	0.94162189	+1	0.88665178	+0.83489072
2	0.88665178	+1	0.78615138	+0.69704252
3	0.83489072	+1	0.69704252	+0.58195433
4	0.78615138	−1	0.61803399	−0.48586827
5	0.74025734	−1	0.54798094	−0.40564691
6	0.69704252	−1	0.48586827	−0.33867084
7	0.65635049	−1	0.43079597	−0.28275315
Σ			5.45252484	+1.60094840

$$\boxed{\eta_{\mathrm{L}} = \frac{1.60094840}{5.45252484} = 0.293\,615\,975\,3\ldots} \tag{11}$$

4.3 Vanishing for Uniform Arrays

Proposition 4.2 (Uniform arrays have zero metric coupling)**.** *If $A_i = A$ for all i (uniform array), then $\eta_{\mathrm{L}} = 0$ for any neutral schedule.*

Proof. With $A_i = A$, the cubic sum becomes:

$$\sum_{i=0}^{n-1} A^3\, s_i = A^3 \sum_{i=0}^{n-1} s_i = A^3 \cdot 0 = 0$$

by the neutrality constraint (8). Hence $\eta_{\mathrm{L}} = 0/(\text{positive}) = 0$. □

Remark 4.3. *This is the fundamental result: the φ-spiral geometry is* necessary *for metric coupling. A uniform array, regardless of schedule, produces zero cubic residual. The broken amplitude symmetry of the φ-spiral is what makes the Lighthouse work.*

5 Closed-Form Expression

Both sums in (10) are finite geometric series in powers of φ.

5.1 Quadratic Sum

$$P_n(\kappa) \;=\; \sum_{i=0}^{n-1} \varphi^{-2\kappa i/n} \;=\; \frac{1-\varphi^{-2\kappa}}{1-\varphi^{-2\kappa/n}}. \tag{12}$$

For $n = 8$, $\kappa = 1$:

$$P_8(1) = \frac{1-\varphi^{-2}}{1-\varphi^{-1/4}} = 5.452\,524\,84\ldots$$

5.2 Cubic Sum

For the canonical bipolar schedule with the first $n/2$ coils at $+1$ and the rest at -1:

$$C_n(\kappa) \;=\; \sum_{i=0}^{n/2-1} \varphi^{-3\kappa i/n} - \sum_{i=n/2}^{n-1} \varphi^{-3\kappa i/n}. \tag{13}$$

Setting $q = \varphi^{-3\kappa/n}$, this becomes:

$$\begin{aligned} C_n(\kappa) &= \sum_{i=0}^{n/2-1} q^i - \sum_{i=n/2}^{n-1} q^i \\ &= \frac{1-q^{n/2}}{1-q} - q^{n/2} \cdot \frac{1-q^{n/2}}{1-q} \\ &= \frac{(1-q^{n/2})^2}{1-q}. \end{aligned} \tag{14}$$

For $n = 8$, $\kappa = 1$, $q = \varphi^{-3/8}$:

$$C_8(1) = \frac{(1-\varphi^{-3/2})^2}{1-\varphi^{-3/8}} = 1.600\,948\,40\ldots$$

5.3 Closed Form for η_{L}

Theorem 5.1 (Closed form of η_{L})**.** *For the canonical n-coil Lighthouse with pitch κ and bipolar schedule:*

$$\boxed{\eta_{\mathrm{L}}(n,\kappa) = \frac{(1-\varphi^{-3\kappa/2})^2}{(1-\varphi^{-3\kappa/n})} \cdot \frac{(1-\varphi^{-2\kappa/n})}{(1-\varphi^{-2\kappa})}.} \tag{15}$$

Proof. Direct substitution of (12) and (14) into Definition 4.1. □

Remark 5.2. *The expression (15) is a rational function of fractional powers of φ. Since φ is algebraic ($\varphi^2 = \varphi + 1$), η_{L} belongs to an algebraic extension of $\mathbb{Q}$. It is a* computable algebraic invariant *of the Lighthouse architecture, determined entirely by the golden ratio and the coil/schedule parameters.*

6 Dependence on Schedule

We compare η_L for several neutral schedules, all using the same φ-spiral geometry ($n = 8$, $\kappa = 1$).

Table 2: η_L for different neutral schedules on the 8-coil φ-spiral.

Schedule	$\mathbf{s}$	η_L
Canonical bipolar	$(+, +, +, +, -, -, -, -)$	**0.2936**
Asymmetric	$(+, +, +, -, -, -, -, +)$	0.1839
Gradient $(\pm 4, \pm 3, \ldots)$	weighted	0.1202
Alternating	$(+, -, +, -, +, -, +, -)$	0.0764
Uniform (any schedule)	any neutral	0.0000

Proposition 6.1 (Canonical bipolar maximizes η_L among binary schedules)**.** *Among all binary ($s_i \in \{+1, -1\}$) neutral schedules on $n = 8$ coils with monotone-decreasing amplitudes $A_0 > A_1 > \cdots > A_7 > 0$, the canonical bipolar schedule* $(+1, +1, +1, +1, -1, -1, -1, -1)$ *maximizes* $\left|\sum A_i^3 s_i\right|$.

Proof sketch. Since A_i^3 is monotone decreasing, the sum $\sum A_i^3 s_i$ is maximized by assigning $s_i = +1$ to the largest A_i^3 terms and $s_i = -1$ to the smallest, subject to equal counts (neutrality). This is exactly the canonical bipolar assignment. □

7 Dependence on Spiral Tightness κ

Table 3: η_L as a function of spiral tightness κ ($n = 8$, canonical bipolar).

κ	η_L	Interpretation
0	0	Uniform (degenerate spiral)
0.5	0.1626	Gentle spiral
1	**0.2936**	**v0 baseline**
2	0.4823	Moderate spiral
3	0.6015	Tight spiral
5	0.7258	Very tight
8	0.8044	Extreme
10	0.8364	Near-maximum
$\to \infty$	$\to 1$	Limiting value

Proposition 7.1 (η_L is monotone in κ)**.** *For fixed n and canonical bipolar schedule, $\eta_L(\kappa)$ is strictly increasing in κ for $\kappa > 0$, with $\eta_L(0) = 0$ and $\lim_{\kappa\to\infty} \eta_L = 1$.*

Proof sketch. As $\kappa \to \infty$, the outer coils have amplitude $\varphi^{-\kappa i/n} \to 0$ for $i > 0$, so only coil 0 contributes: $\eta_L \to |A_0^3 \cdot (+1)|/A_0^2 = A_0 = 1$. For $\kappa = 0$, all amplitudes are equal and Proposition 4.2 applies. Monotonicity follows from the increasing asymmetry of the amplitude profile. □

Remark 7.2. *Tighter spirals give higher η_L but concentrate the field at the innermost coil. There is a practical trade-off between η_L and the spatial extent of the field pattern. The v0 choice $\kappa = 1$ balances efficiency against field coverage.*

8 The Complete Lighthouse Coupling Equation

Combining the results from §2 and §4, the total metric perturbation produced by the Lighthouse is:

$$\boxed{\frac{\delta g}{g} = \alpha_{\mathrm{em}}^2 \, P_{\mathrm{em}} \left(1 \, - \, \eta_{\mathrm{L}} \cdot \alpha_{\mathrm{em}} \cdot A_{\mathrm{peak}}\right) \, \cdot \, Q} \tag{16}$$

where:

- $\alpha_{\mathrm{em}} = \sqrt{\alpha_{\mathrm{fine}}} \approx 0.0854$ is the EM coupling in natural units,
- $P_{\mathrm{em}} = \sum A_i^2$ is the total EM power (dimensionless),
- $\eta_{\mathrm{L}} = 0.2936$ is the Lighthouse efficiency (this paper),
- A_{peak} is the peak field amplitude (in Planck units),
- Q is the resonance quality factor of the "metric cavity" (to be measured).

The first factor ($\alpha_{\mathrm{em}}^2 P_{\mathrm{em}}$) is the standard quadratic EM-gravity coupling ($\sim 10^{-44}$ for lab fields). The second factor ($1 - \eta_{\mathrm{L}} \cdot \alpha_{\mathrm{em}} \cdot A_{\mathrm{peak}}$) contains the Lighthouse correction. The third factor (Q) accounts for resonant accumulation.

8.1 The Sign-Flip Prediction

Reversing the commutation direction replaces $\mathbf{s} \to -\mathbf{s}$, which flips $A_{\mathrm{peak}} \to -A_{\mathrm{peak}}$ in the cubic term. The quadratic term is unchanged. Therefore:

$$\left.\frac{\delta g}{g}\right|_{\mathrm{forward}} - \left.\frac{\delta g}{g}\right|_{\mathrm{reverse}} = 2\, \alpha_{\mathrm{em}}^3 \, P_{\mathrm{em}} \, \eta_{\mathrm{L}} \, A_{\mathrm{peak}} \, Q. \tag{17}$$

This differential signal is the primary experimental observable.

8.2 The Null Prediction

For a scrambled (random) phase assignment, the expected value of the cubic sum is zero by symmetry: $\mathbb{E}[\sum A_i^3 s_i] = 0$ when s_i are i.i.d. uniform on $\{+1, -1\}$. Thus scrambled-phase runs should show no sign-flip signal, serving as a null control.

9 Discussion

9.1 What $\eta_{\mathrm{L}} = 0.2936$ Means

The number 0.2936 tells us that the φ-spiral geometry converts approximately 29% of the electromagnetic energy into cubic metric coupling per unit of $\alpha_{\mathrm{em}} \cdot A_{\mathrm{peak}}$. This is a substantial geometric advantage over uniform arrays (which achieve 0%).

However, η_{L} alone does not determine whether the effect is measurable. The perturbative estimate gives $\delta g/g \sim 10^{-105}$ for lab-scale fields without resonance enhancement. The entire experimental program rests on the *resonance hypothesis*: that the φ-spiral geometry, combined with 8-tick scheduling synchronized to the fundamental ledger update rate, creates a high-Q "metric cavity" that amplifies the perturbation to measurable levels.

9.2 What η_{L} Does *Not* Depend On

The value 0.2936 is independent of:

- The coil current (it cancels in the ratio),
- The physical scale of the array (r_0 cancels),
- The drive frequency (it enters only through Q),

- The SI calibration seam (it is purely RS-native).

It depends only on φ, the number of coils ($n = 8$), the pitch ($\kappa = 1$), and the schedule.

9.3 Optimization Pathways

Table 3 suggests that tighter spirals ($\kappa > 1$) could substantially increase η_L. A $\kappa = 3$ design achieves $\eta_L = 0.60$, doubling the coupling. This motivates future iterations beyond v0.

9.4 Formal Verification

The structural properties of η_L — including the sign-flip theorem, the null prediction, and the vanishing for uniform arrays — are formalized in Lean 4 in the modules:

- `Foundation.EMRecognitionCost` (EM cost, coil arrays, neutral schedules)
- `Foundation.MetricPerturbation` (coupling equation, sign-flip proof)

Both modules compile with zero `sorry` statements.

10 Conclusion

The Lighthouse efficiency parameter $\eta_L = 0.2936$ is a computable, algebraic invariant of the 8-coil φ-spiral architecture that:

(1) Is *derived from first principles* (the golden ratio φ and the cost functional J),

(2) *Vanishes for uniform arrays* (proving the φ-spiral is essential),

(3) Is *maximized by the canonical bipolar schedule* among binary neutral schedules,

(4) Enters the coupling equation as the coefficient of the directional (cubic) correction,

(5) Is *formally verified* in Lean 4 with zero unresolved proof obligations.

The critical remaining unknown is the resonance quality factor Q, which determines whether the cubic coupling accumulates to measurable levels. This is the target of the v0 experimental program.

References

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