Recognition Geometry (RG) — Lean Formalization Summary

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Purpose

This note is a collaborator-facing map of the Lean 4 development for Recognition Geometry (RG). It lists the modules under `IndisputableMonolith/RecogGeom/` and summarizes what each module defines and what it proves.

Status tags

- **PROVED**: stated and proved in Lean.
- **DEF**: definition/structure only (no substantive theorem claim).
- **MODEL**: a structural bridge or placeholder modeling choice (useful for orientation, not the final RS instantiation).
- **TODO**: explicitly marked incomplete or awaiting a stronger theorem statement.

High-level coverage (today)

- **Core RG spine (PROVED)**: configuration space, event space, locality structure, recognizers, indistinguishability, recognition quotient, and the canonical factorization map on the quotient.
- **Refinement under composition (PROVED)**: combining recognizers refines the quotient; composite indistinguishability is conjunction.

- **Finite local resolution (PROVED)**: formal definition + the "no injection on infinite neighborhood with finite events" obstruction.
- **Symmetry / gauge (PROVED)**: event-preserving maps induce quotient actions; automorphisms + gauge-equivalence relation; gauge implies indistinguishable.
- **Comparative recognition (PROVED)**: comparative recognizers induce order-type relations; definition of recognition pseudometric.
- **Charts/dimension layer (MIXED)**: core chart/atlas definitions are in place, but several "geometry" claims are currently phrased as hypotheses (see the module notes below).
- **RS bridge (MODEL)**: a structural template showing how ledger states / 8-tick finite resolution / J-cost would instantiate RG.

Module-by-module map

`RecogGeom/Core.lean` **(PROVED)**

- **DEF**: `ConfigSpace`, `EventSpace`, `DecEventSpace`, `RecognitionTriple`.
- **PROVED**: `config_exists`, `event_nontrivial`.

`RecogGeom/Locality.lean` **(PROVED)**

- **DEF**: `LocalConfigSpace` with neighborhood assignment `N` and axioms (`mem_of_mem_N`, `N_nonempty`, `intersection_closed`, `refinement`).
- **PROVED**: `has_neighborhood`, `self_mem_neighborhood`, `common_refinement`, `sub_neighborhood`.
- **DEF**: neighborhood filter base construction (`neighborhoodFilterBase`).

`RecogGeom/Recognizer.lean` **(PROVED)**

- **DEF**: `Recognizer` (nontrivial function `R : C → E`), `LocalRecognizer`.
- **PROVED**: nontriviality lemmas and basic fiber facts such as `Recognizer.fiber`, `Recognizer.fibers_partition`.

`RecogGeom/Indistinguishable.lean` **(PROVED)**

- **DEF**: `Indistinguishable r` c_1 c_2 `:= (r.R` c_1 `= r.R` c_2`)` and the setoid it induces.
- **PROVED**: `indistinguishable_equivalence`; resolution-cell lemmas such as `resolutionCell_eq_fiber` and `resolutionCells_partition`.

`RecogGeom/Quotient.lean` **(PROVED)**

- **DEF**: `RecognitionQuotient r := Quotient(indistinguishableSetoid r)` (i.e. the quotient of C by indistinguishability).
- **PROVED**: `quotientMk_eq_iff`; `quotientEventMap` and `quotientEventMap_injective`.
- **PROVED**: `quotient_equiv_image :` `C_R` $\simeq$ `range(R)`.
- **PROVED**: `liftToQuotient` + `liftToQuotient_spec` (quotient universal mapping property for functions constant on resolution cells).
- **DEF**: induced quotient neighborhoods (`quotientNeighborhoods`) as a construction-level locality lift.

`RecogGeom/Composition.lean` **(PROVED)**

- **DEF**: `CompositeRecognizer` with notation $\mathtt{r}_1 \otimes \mathtt{r}_2$.
- **PROVED**: `composite_indistinguishable_iff` and `composite_resolutionCell`.
- **PROVED**: quotient projection maps `quotientMapLeft`, `quotientMapRight` and their surjectivity.
- **PROVED**: `refinement_theorem` (composite quotient refines both components).

`RecogGeom/FiniteResolution.lean` **(PROVED)**

- **DEF**: `HasFiniteLocalResolution` and `HasFiniteResolution`.
- **PROVED**: `no_injection_on_infinite_finite` (finite events on an infinite neighborhood prevents injectivity).

RecogGeom/Connectivity.lean (PROVED)

- **DEF**: IsRecognitionConnected, IsLocallyRegular, SatisfiesRG5.
- **PROVED**: basic connectivity lemmas and locally_regular_cell_connected.

RecogGeom/Symmetry.lean (PROVED)

- **DEF**: RecognitionPreservingMap (event-preserving map), RecognitionAutomorphism (bijective).
- **PROVED**: symmetry preserves indistinguishability (symmetry_preserves_indistinguishable) and induces quotient action (symmetryQuotientMap).
- **DEF+PROVED**: GaugeEquivalent and gauge_implies_indistinguishable; gauge equivalence is an equivalence relation.

RecogGeom/Comparative.lean (PROVED, with TODO notes)

- **DEF**: ComparativeRecognizer, InducesPreorder, InducesPartialOrder.
- **PROVED**: construction of induced preorder/partial order; supporting lemmas like preorder_refl, metric_from_comparisons.
- **DEF**: RecognitionDistance (pseudometric structure) and RecognitionDistance.IsMetric.
- **TODO (in-module docs)**: the file contains a documentation note about bridging RS J-cost to RecognitionDistance; this bridge is realized structurally in RSBridge.lean (see below).

RecogGeom/Charts.lean (MIXED: DEF + hypothesis-based theorems)

- **DEF**: RecognitionChart, ChartCompatible, RecognitionAtlas.
- **PROVED**: chart respects indistinguishability (chart_respects_equiv); atlases cover the quotient (atlas_covers_quotient).
- **MODEL/TODO**: several "geometry" claims are phrased as explicit hypotheses, e.g. recognition_dimension_unique_hypothesis and finite_resolution_no_chart_hypothesis. Theorems that use these are conditional.
- **TODO**: IsSmoothRecognitionGeometry is currently a placeholder definition.

`RecogGeom/Dimension.lean` **(PROVED, with interpretive docs)**

- **DEF**: separating recognizers (`IsSeparating`), pair separation (`PairSeparates`), independence (`IndependentRecognizers`).
- **PROVED**: `separating_quotient_bijective`, `separating_singleton_cells`, `pairSeparates_iff`, `independent_strict_refines`.
- **NOTE**: "spacetime is 4D" content is currently documentation/TODO, not a proved theorem.

`RecogGeom/Foundations.lean` **(PROVED, with scope notes)**

- **PROVED**: pillar theorems packaging earlier results; `fundamental_theorem` ($[c_1]=[c_2] \leftrightarrow R(c_1)=R(c_2)$).
- **PROVED**: `universal_property` in the operational sense used in the paper (surjective projection, injective event map, factorization).
- **NOTE**: full category-theoretic uniqueness statements are explicitly marked as future work.

`RecogGeom/RSBridge.lean` **(MODEL / structural bridge)**

- **MODEL**: structural interfaces for RS ledger states (`RSConfigSpace`), locality from an `RHat`-operator (`RSLocalityFromRHat`), and measurements (`RSMeasurement`).
- **MODEL+PROVED**: `EightTickFiniteResolution` and `eight_tick_implies_RG4` (RS finite-resolution hypothesis $\Rightarrow$ RG finite resolution).
- **PROVED**: `physical_space_is_quotient` (specialization of `quotient_equiv_image`).
- **PROVED**: `toRecognitionDistance` (J-cost axioms packaged as a `RecognitionDistance`).

`RecogGeom/Examples.lean` **(PROVED)**

- **PROVED**: small concrete recognizer examples (finite cyclic, sign/magnitude on $\mathbf{Z}$) and a composition-refinement example.

`RecogGeom/Integration.lean` **(DEF + documentation)**

- **DEF**: an integrated `RecognitionGeometry` bundle type.
- **NOTE**: provides a human-readable, in-Lean summary of modules and theorem names.

What is *not* yet claimed as proved (important)

- A full "recognition manifold theorem" (conditions under which the quotient is a smooth manifold) is not presented as a proved Lean theorem in the current library.
- Uniqueness of recognition dimension is currently stated via an explicit hypothesis in `Charts.lean`.
- Any fully concrete RS instantiation of locality via an implemented ledger/`RHat` model is still a modeling bridge rather than an end-to-end physics formalization.