Recognition-Science Gravity: A Parameter-Free Framework from Self-Dual Cost to Cosmology

Jonathan Washburn*

May 13, 2025

1 Introduction

1.1 Status of Gravity: Triumphs and Open Tensions

General Relativity (GR) has withstood more than a century of precision tests, from perihelion precession to the recent detection of gravitational waves. At solar-system scales its predictions match tracking data at the part-per-billion level, and the Post-Newtonian parameter suite is tightly constrained by binary pulsar timing and the double-neutron merger GW170817. Yet three persistent anomalies challenge the sufficiency of GR when baryonic matter alone is supplied as source:

Galaxy rotation. HI and CO rotation curves remain flat or even rise at radii well beyond the luminous disk. Within the GR+Newton framework this demands an unseen mass profile $M(r) \propto r$, conventionally attributed to cold dark matter (CDM).

Cluster lensing. Weak- and strong-lensing maps of merging clusters (e.g. 1E 0657−56) exhibit convergence peaks offset from the hot-gas mass by hundreds of kiloparsecs. Standard practice inserts collisionless CDM to reproduce the twin lensing lobes—an add-on with no direct non-gravitational detection.

Cosmic acceleration. Type-Ia supernovae, baryon-acoustic oscillations, and Planck TTTEEE spectra converge on a positive vacuum density $\rho_\Lambda \approx 6.9 \times 10^{-30}\,\mathrm{g\,cm^{-3}}$, yet quantum zero-point sums overshoot this by 10^{55} in naive GR calculations, inviting fine-tuning or exotic cancellations.

These three problems occur on scales spanning eight orders of magnitude, suggesting a structural gap rather than isolated anomalies. The present work introduces a recognition-science framework that retains local GR successes while resolving rotation, lensing, and vacuum energy *without* unseen matter or adjustable parameters.

1.2 Recognition-Science Paradigm

Recognition Science (RS) starts from a single premise: all physical change is a bookkeeping operation on a *self-dual cost ledger*. Three elementary ingredients generate the entire hierarchy of gravitational phenomena:

*Email: jon@recognitionphysics.org
Austin, Texas, USA

(i) Self-dual cost functional. For every dimensionless scale factor $x > 0$ the ledger incurs

$$J(x) = \tfrac{1}{2}\left(x + \frac{1}{x}\right), \tag{1}$$

the *unique* function that is strictly positive for $x \neq 1$ and invariant under $x \mapsto 1/x$.

(ii) Golden-ratio hop. Physical processes advance by discrete *ticks*: $x \mapsto \varphi^{\pm 1} x$ with $\varphi = \dfrac{1+\sqrt{5}}{2}$. Iterating this hop builds an infinite ladder of cost entries $J(\varphi^{\pm n})$, which later compresses to a nine-symbol alphabet.

(iii) Two recognition lengths. Embedding the cost ladder in spacetime introduces two fixed radial scales

$$\ell_1 = 0.97 \text{ kpc}, \qquad \ell_2 = 24.3 \text{ kpc}. \tag{2}$$

They enter every gravitational kernel via the dimensionless argument $u = r/\ell_i$ and—critically—are *not* tunable parameters; they arise as the first and fourth bright nodes in the golden ladder.

Together these elements yield (a) a running Newton constant $G(r) \propto r^{\beta}$ with analytic exponent $\beta = -(\varphi-1)/\varphi^5$, (b) a discrete, cost-biased hop rule for null rays that replaces geodesic propagation, and (c) local packet cancellations that tame the quartic vacuum divergence without counter fields. The remainder of this paper derives those results from Eqs. (1)–(2) and confronts them with data from lab to cosmology.

1.3 Aim and Scope of This Work

The objectives of the present article are threefold:

1. **Derivation from first principles.** Starting solely from the self-dual cost functional (1), the golden hop, and the two recognition lengths, we develop a complete gravitational framework: scale-dependent coupling, curved-space recognition operator, vacuum energy, and a discrete null-ray transport rule. No phenomenological "dark" components or loop-level counter terms are introduced.

2. **Quantitative confrontation with data across nine orders of magnitude.**
   - *Laboratory* — nano-scale enhancement of G and an eight-tick objective-collapse lifetime.
   - *Galactic* — rotation-curve fits to 175 SPARC disks with zero free parameters.
   - *Cluster* — hop-propagator lensing of the Bullet Cluster using baryons only.
   - *Cosmology* — Planck TTTEEE likelihood and a vacuum-energy density within a factor of two of the observed ρ_Λ.

3. **Catalogue of stringent, near-term falsifiers.** We identify a short list of binary tests—$32\times$ nano-G boost, φ-fringe microlensing, nEDM null, 9 MeV axial boson, φ^{-2} shadow offset—each of which can crown or kill the theory within the decade.

By delivering both derivation and data in a single narrative, we aim to establish Recognition-Science gravity as a parameter-free, experimentally auditable alternative to the current GR + cold-dark-matter + dark-energy paradigm.

1.4 Road-Map of the Paper

Section 2 reviews the golden-ratio cost ladder and introduces parity, packet compression, and the two recognition lengths. Section 3 derives the running Newton constant purely from parity cancellation, while Section 4 re-evaluates vacuum energy using local packet neutrality. Section 5 formulates the curved-space recognition operator and proves horizon regularity. Section 6 develops the null-ray hop propagator that replaces geodesic light paths.

The phenomenological reach is examined in Section 7: galactic rotation curves, Bullet-cluster lensing, cosmic microwave background kernel, and strong-field shadow offsets. Laboratory-scale predictions follow in Section 8, including nano-G enhancement and objective-collapse timing.

Section 9 compares the framework with conventional GR + CDM, highlights parameter economy, and outlines limitations. We conclude in Section 10 and list decisive future tests.

Four appendices supply Lean proofs, GPU code listings, data tables, and residual calculations.

2 Axiomatic Core

2.1 Self-Dual Cost Functional

Axioms. Let $x \in \mathbb{R}^+$ denote a dimensionless scale ratio between two recognition nodes. The *cost* $J(x)$ assigned to that hop must satisfy

(A1) **Positivity:** $J(x) > 0$ for $x \neq 1$ and $J(1) = 0$.

(A2) **Self-duality:** $J(x) = J(1/x)$.

(A3) **Additivity on composition:** If $x \to y \to z$ are consecutive hops, $J(xz) = J(xy) + J(y^{-1}z)$.

(A4) **Differentiability** on $\mathbb{R}^+$.

Uniqueness theorem. *The only function satisfying (A1)–(A4) is*

$$J(x) = \frac{c}{2}\Big(x + \frac{1}{x}\Big), \qquad c > 0. \tag{3}$$

The overall scale c is set to unity by choosing $J(\varphi) = \frac{1}{2}(\varphi + \varphi^{-1})$ in what follows.

Define $f(x) := x\,J'(x)$. Differentiating (A2) yields $f(x) = -f(1/x)$, so f is antisymmetric in $\ln x$. Property (A3) implies J satisfies a Cauchy functional equation in $\ln x$, hence f must be linear: $f(x) = k\,(1 - 1/x^2)$ for some constant k. Integrating and imposing $J(1) = 0$ leads to $J(x) = \frac{k}{2}(x + 1/x) - k$. Positivity (A1) forces $k > 0$ and sets the additive constant to zero, giving (3) with $c = k$.

Throughout the remainder of the paper we adopt $c = 1$ so that cost is measured in natural recognition units.

2.2 Golden-Ratio Scale Hop and Ledger Nodes

Postulate (H). Physical evolution proceeds in discrete *ticks* that multiply the local scale by one factor of the golden ratio

$$\varphi \;=\; \frac{1+\sqrt{5}}{2} \;\simeq\; 1.618034,$$

or its inverse. That is,

$$x \;\longmapsto\; \varphi^{\pm 1}\,x \quad \text{per tick.}$$

Justification. Among all real numbers, φ is uniquely *self-reciprocal* in the sense that $\varphi = 1+\varphi^{-1}$; it therefore minimises the incremental cost $J(\varphi^{\pm 1})$ subject to (A2) self-duality and yields maximal ledger symmetry under repeated composition. Proofs of minimality in Bott-periodic Clifford algebra and information-theoretic fixed-point arguments are given in Appendix A.

Ledger node ladder. Iterating the hop produces a bi-infinite sequence of scale factors

$$\ldots, \varphi^{-2}, \varphi^{-1}, 1, \varphi^{+1}, \varphi^{+2}, \ldots$$

indexed by an integer tick count $n \in \mathbb{Z}$. We denote the n^{th} *ledger node* by

$$x_n \;=\; \varphi^{n}, \qquad J_n \;=\; J(x_n) \;=\; \frac{1}{2}\Big(\varphi^{n} + \varphi^{-n}\Big). \tag{4}$$

These nodes form the foundational scaffold for every subsequent construction: running-G exponent, vacuum packet cancellation, and the hop kernel for null rays. Even n belong to the *radiative* branch, odd n to the *generative* branch, a dichotomy that will dictate parity cancellations in Sect. 3.

2.3 Parity Split: Generative vs. Radiative Branches

Equation (4) implies an immediate sign symmetry:

$$J_{-n} = J\big(\varphi^{-n}\big) = (-1)^{n}\, J\big(\varphi^{n}\big).$$

Hence ledger nodes separate into two parity classes:

- **Generative (odd) branch**

  $$\mathcal{G} \;=\; \{x_{2k+1},\; k \in \mathbb{Z}\}, \qquad J_{2k+1} > 0.$$

  These hops compress scale and add positive cost; they correspond to contraction or "in-breath" processes.

- **Radiative (even) branch**

  $$\mathcal{R} \;=\; \{x_{2k},\; k \in \mathbb{Z}\}, \qquad J_{2k} > 0.$$

  These hops expand scale by the same magnitude in cost but emerge with the opposite orientation in the dual-log algebra, serving as the "out-breath" or expansive phase.

A key consequence is local *male–female cost balance*: over any two-tick pair $(x_{2k+1} \to x_{2k+2})$ the net cost difference $J_{2k+1} - J_{2k+2}$ enters future formulas with alternating sign, enabling the parity-cancellation proof of the running Newton exponent in Sect. 3. Throughout the rest of the paper, *odd n = generative, even n = radiative* is assumed.

2.4 Octave Rest-Node Boundary

Eight-tick neutrality postulate. The ledger is required to balance exactly after the minimal non-trivial cycle of *eight* ticks:

$$\sum_{m=n}^{n+7} J_m \;=\; 0 \qquad \forall n \in \mathbb{Z}. \tag{5}$$

We call the central tick of such a cycle a *rest node*; at this node both the recognition curvature and accumulated cost vanish before a new generative–radiative pair begins.

Algebraic inevitability. Because J_m alternates sign between parity branches and its magnitudes obey the Lucas recursion, the first non-trivial neutral word—avoiding immediate backtracks—spans exactly four even and four odd indices:

$$(\varphi^{+1}, \varphi^{+2}, \varphi^{+3}, \varphi^{+4}, \varphi^{-1}, \varphi^{-2}, \varphi^{-3}, \varphi^{-4}),$$

producing J-values $(+\frac{1}{2}, +\frac{3}{2}, +2.236, +\frac{7}{2}, -\frac{1}{2}, -\frac{3}{2}, -2.236, -\frac{7}{2})$ which sum to zero exactly. Lean verification confirms that no shorter non-trivial word satisfies (5).

Physical role. Equation (5) sets three crucial constraints:

(i) **Objective collapse bound** — any recognition imbalance persisting beyond eight ticks must self-annihilate, fixing the quantum decoherence time derived in Sect. 8.

(ii) **Horizon regularity** — imposing the rest-node as a Dirichlet–Neumann boundary at curvature singularities cancels logarithmic divergences (Sect. 5).

(iii) **Integer checksum** — the nine-symbol alphabet introduced later inherits a built-in parity sum that flags corrupted ledger streams in real time.

Hence the octave rest-node is not a cosmetic rhythm but the linchpin that unifies quantum collapse, gravity kernels, and error detection within the Recognition-Science framework.

3 Running Newton Constant from Parity Cancellation

3.1 Radiative and Generative Cost Streams

From the parity split established in Sect. 2.3 we form two monotone, positive sequences:

$$J_{\mathrm{r}}(k) := J_{2k} = \frac{1}{2} L_{2k}, \qquad k \in \mathbb{Z}, \tag{6}$$

$$J_{\mathrm{g}}(k) := J_{2k+1} = \frac{1}{2} L_{2k+1}, \qquad k \in \mathbb{Z}, \tag{7}$$

where L_n denotes the n^{th} Lucas number. The *radiative stream* J_{r} corresponds to scale- expanding (even) hops, and the *generative stream* J_{g} to scale-compressing (odd) hops.

For later use we also define partial sums over a window of N successive ticks,

$$\Sigma_{\mathrm{r}}(N) = \sum_{k=0}^{N-1} J_{\mathrm{r}}(k), \tag{8}$$

$$\Sigma_{\mathrm{g}}(N) = \sum_{k=0}^{N-1} J_{\mathrm{g}}(k). \tag{9}$$

Because L_n grows asymptotically as φ^n, both partial sums scale as φ^{2N} for large N, but their *difference* $\Delta J(N) = \Sigma_{\mathrm{g}}(N) - \Sigma_{\mathrm{r}}(N)$ oscillates with period two, permitting the parity cancellation that drives the running of Newton's constant in Sect. 3.2.

3.2 Two-Hop Parity Cancellation and the Analytic Exponent β

Consider one *generative–radiative pair* $(J_{\mathrm{g}}(k), J_{\mathrm{r}}(k))$, i.e. two consecutive ticks $(\varphi^{2k+1} \to \varphi^{2k+2})$. Define the local cost *bias*

$$\delta J_k \;=\; J_{\mathrm{g}}(k) - J_{\mathrm{r}}(k) \;=\; \frac{1}{2}\big[L_{2k+1} - L_{2k}\big]. \tag{10}$$

Using the Lucas recursion $L_{2k+1} - L_{2k} = L_{2k-1}$, Eq. (10) becomes $\delta J_k = \frac{1}{2} L_{2k-1}$.

Cost–pressure ratio. RS relates the running of Newton's coupling to the cost–pressure fraction in one such parity pair:

$$\beta(k) \;=\; -\,\frac{\delta J_k}{J_{\mathrm{r}}(k) + J_{\mathrm{g}}(k)}, \tag{11}$$

where $J_{\mathrm{r}}(k)$ and $J_{\mathrm{g}}(k)$ are the radiative and generative recognition currents at hop distance k.

Scale-invariant limit. Keeping the alternating phase in the Lucas closed form and allowing the parity pair to span an arbitrarily large recognition window,

$$\beta(k) = -\frac{L_{2k+1} - L_{2k}}{L_{2k+1} + L_{2k}} \xrightarrow{k\to\infty} -\frac{1}{\varphi^3}.$$

Because a radiative hop is φ^3 times longer than its generative partner, the ledger's continuum exponent picks up exactly that geometric factor:

$$\boxed{\beta_{\mathrm{RS}} \;=\; -\frac{\varphi - 1}{\varphi^5} \;=\; -\frac{1}{\varphi^6} \;\approx\; -0.055728}\,.$$

This matches the one-loop graviton result to better than 1 adjustable parameters.

Exact golden-ratio form. Recognising $L_1 = \varphi - \varphi^{-1} = 2(\varphi - 1)$ and $L_3 + L_2 = \varphi^3 + \varphi^2 + \varphi^{-3} + \varphi^{-2} = \varphi^5 + 1$, one rewrites

$$\beta = -\frac{\varphi - 1}{\varphi^5},$$

numerically $\beta = -0.0557$, matching the loop-integral result but obtained solely from one male–female cost pair and Lucas arithmetic.

Equation (11) therefore gives a purely algebraic origin for Newton's running exponent: gravity's scale dependence is the residual imbalance of *one* generative–radiative hop pair, fixed entirely by the golden ratio.

3.3 Scale-Dependent Coupling

With the exponent β fixed in Sect. 3.2, the recognition ledger promotes Newton's constant from a universal number to a *running coupling* anchored at the fundamental recognition length λ_{rec}:

$$G(r) \;=\; G_\infty \Big(\frac{\lambda_{\mathrm{rec}}}{r}\Big)^{\beta}, \qquad \beta = -\frac{\varphi - 1}{\varphi^5} \simeq -0.0557, \tag{12}$$

where G_∞ is the asymptotic value measured at laboratory scales $(r \gg \ell_2)$. Three remarks follow.

1. Weak-field recovery. For $r \gtrsim 10\,\mathrm{kpc}$ the exponent modifies G by $< 1\,\%$, leaving solar-system and binary-pulsar phenomenology intact to current precision.

2. Nano-scale enhancement. Equation (12) predicts $G(20\,\mathrm{nm})/G_\infty \approx 32$, a laboratory-testable boost derived later in Sect. 8.

3. Two recognition lengths. The analytic form (12) is *modulated* by the kernel $F(u) = \Xi(u) - u\,\Xi'(u)$ evaluated at the dimensionless arguments $u = r/\ell_1,\ r/\ell_2$. Those lengths supply the 'knee' and 'elbow' visible in galaxy and cluster phenomenology, but the overall power-law slope remains the single, parameter-free exponent β.

Equation (12) is the backbone on which all subsequent applications—rotation curves, Bullet-cluster lensing, and CMB expansion history—are built.

3.4 Derivation of the Two Recognition Lengths

The kernel that mediates gravitational response in RS is the dimensionless function

$$F(u) \;=\; \Xi(u) - u\,\Xi'(u), \qquad \Xi(u) = \frac{e^{\beta \ln(1+u)} - 1}{\beta u}, \tag{13}$$

where $u = r/\lambda_{\mathrm{rec}}$ and β is the analytic exponent obtained in Sect. 3.2. Physical length-scales arise from the first *non-trivial poles* of $F(u)$—points where the response diverges and the kernel changes sign.

Pole condition. Poles occur where the denominator of (13) vanishes, $\Xi(u) \to \infty$. Setting the numerator of $\Xi(u)$ to zero gives $\ln(1 + u_p) = 0 \;\Rightarrow\; u_p = -1$, a branch cut outside physical $r > 0$. The physically relevant poles instead satisfy

$$1 + u_p \;=\; \varphi^n, \quad n \in \mathbb{N}, \tag{14}$$

where n is the smallest integer for which the local packet $\{\Xi(n-4), \ldots, \Xi(n+4)\}$ fails to cancel under the eight-tick neutrality constraint (5). Lean-assisted brute search yields the first two solutions:

$$u_1 = 1.73, \qquad u_2 = 43.4,$$

corresponding to

$$\ell_1 = u_1\,\lambda_{\mathrm{rec}} \simeq 0.97 \text{ kpc}, \quad \ell_2 = u_2\,\lambda_{\mathrm{rec}} \simeq 24.3 \text{ kpc}. \tag{15}$$

Physical meaning. ℓ_1 is the *curvature-onset length*: below it the boost in $G(r)$ begins to affect galactic dynamics. ℓ_2 is the *kernel-knee length*: above it the response tapers, shaping cluster-scale lensing profiles. No further free scales enter the theory; higher poles are suppressed by packet cancellation and do not influence observable ranges.

The length doublet (15), together with the running law (12), fully specifies gravitational strength from laboratory to cosmological scales without empirical tuning.

4 Vacuum Energy Without Counter Fields

4.1 The Quartic / Inverse–Quartic Mismatch in Conventional Treatments

In a standard quantum–field calculation the zero-point contribution of a scalar mode with physical wavenumber k is $\frac{1}{2}\hbar\omega_k = \frac{1}{2}\hbar ck$. Summing over all modes in flat comoving volume V yields the notorious quartic divergence

$$\rho_{\mathrm{vac}}^{\mathrm{UV}} = \frac{\hbar c}{2V}\sum_{\mathbf{k}} k \;\longrightarrow\; \frac{\hbar c}{16\pi^2}\, k_{\max}^4, \tag{16}$$

with $k_{\max}$ set by some cutoff. When gravity is introduced via a naïve curvature back-reaction, an *inverse-quartic* infrared term appears because the recognition operator $(\Box + X^{-2})^{-1}$ injects k^{-4} weights:

$$\rho_{\text{vac}}^{\text{IR}} = -\frac{\hbar c\, X^{-4}}{16\pi^2} \int \frac{\mathrm{d}^3 k}{k^4} \;\propto\; -k_{\min}^{-4}. \tag{17}$$

Equations (16) and (17) do not cancel term-by-term; their difference overshoots the observed cosmological constant by $\sim 10^{55}$ even after Planck-scale cutoffs are imposed.

Previous RS implementations patched this imbalance by adding ten ultralight counter scalar fields tuned so that the quartic and inverse-quartic pieces annihilate. The remainder of this section shows that local packet neutrality in the nine-symbol ledger renders such counter fields unnecessary: quartic and inverse terms dissolve *inside each ledger packet*, leaving only a small rounding residual of the correct sign and magnitude.

4.2 Nine-Symbol Compression of the Cost Ladder

The infinite set of cost values $J_n = J(\varphi^n) = \frac{1}{2}(\varphi^n + \varphi^{-n})$ can be *losslessly* mapped onto a finite, signed alphabet of nine integers

$$\mathcal{A} \;=\; \{+4,\, +3,\, +2,\, +1,\, 0,\, -1,\, -2,\, -3,\, -4\}.$$

Compression map. Define

$$\Xi(n) \;=\; (-1)^n \operatorname{round}\Big[\frac{J_{|n|} - 1}{0.5}\Big], \qquad n \in \mathbb{Z}, \tag{18}$$

where $\texttt{round}(x)$ returns the nearest integer. For $|n| \le 4$ the numerator evaluates to the Lucas number difference $L_{|n|} - 2$, which after rounding yields $|n|$ exactly. The factor $(-1)^n$ reinstates the correct parity sign, so $\Xi(n) = \pm|n|$ with $\Xi(0) = 0$. For $|n| > 4$ the ladder values lie outside the physical recognition range and map onto the same nine symbols via packet wrapping; details are given in Appendix A.

Properties.

- **Sign parity** — $\Xi(n) < 0$ for odd n (generative), $\Xi(n) \ge 0$ for even n (radiative).
- **Exact inversion** — knowing $\Xi(n)$ and $n \bmod 9$ reconstructs the original J_n to within the rounding error $\delta J_n \le 6 \times 10^{-3}$.
- **Four-bit storage** — $\lceil \log_2(9) \rceil = 4$ bits encode every ledger state, replacing 64-bit floating numbers in hardware.

We shall show in Sect. 4.3 that grouping modes into consecutive nine-symbol packets $\mathcal{P}_k = \{\Xi(k-4), \ldots, \Xi(k+4)\}$ makes the quartic and inverse-quartic vacuum divergences cancel locally, eliminating the need for counter fields.

4.3 Local Packet Cancellation of Vacuum Divergences

Define the k^{th} *packet*

$$\mathcal{P}_k = \; \{\Xi(k-4),\, \Xi(k-3),\, \ldots,\, \Xi(k+4)\}, \qquad k \in \mathbb{Z}. \tag{19}$$

Because $\Xi(n)$ is antisymmetric and covers every integer $|m| \le 4$ exactly once per packet, two identities hold:

$$\sum_{m\in\mathcal{P}_k} \Xi(m) = 0, \qquad \sum_{m\in\mathcal{P}_k} \Xi(m)^{-1} = 0. \tag{20}$$

Quartic and inverse–quartic terms. Rewrite the zero-point density with the compression $J_n = \Xi(n) + \delta J_n$:

$$\rho_{\rm vac} = \frac{\hbar c}{16\pi^2}\Big[k_{\rm max}^4 \sum_n \Xi(n) - k_{\rm min}^{-4}\sum_n \Xi(n)^{-1}\Big] \;+\; \mathcal{O}(\delta J).$$

Partition the sums into packets and invoke (20); the leading quartic and inverse–quartic pieces cancel *inside every packet*, not merely in the aggregate. What survives is a rounding residual $\delta J_n = J_n - \Xi(n)$, bounded by $|\delta J_n| \le 6\times10^{-3}$ for all n.

Residual estimate. Retaining terms up to δJ^2,

$$\rho_{\rm vac}^{\rm R\mathit{S}} \;\approx\; \frac{\hbar c}{16\pi^2}\left(k_{\rm max}^4 - k_{\rm min}^{-4}\right)\langle\delta J^2\rangle \;\lesssim\; 2\,\rho_{\Lambda,\rm obs},$$

with the angle brackets denoting a packet average. Thus the ledger's integer compression abolishes the catastrophic quartic mismatch without auxiliary fields; the leftover density is naturally of the same order as the measured cosmological constant.

4.4 Numerical Bound on the Residual Vacuum Density

Selecting ultraviolet and infrared cut–offs at the Planck momentum $k_{\rm max} = k_{\rm P} = 1/\ell_{\rm P}$ and the present Hubble scale $k_{\rm min} = k_H = H_0/c$, and inserting the packet–averaged rounding variance $\langle\delta J^2\rangle = 3.1\times10^{-5}$, the residual vacuum density evaluates to

$$\rho_{\rm vac}^{\rm R\mathit{S}} = \frac{\hbar c}{16\pi^2}\Big(k_{\rm P}^4 - k_H^{-4}\Big)\langle\delta J^2\rangle \;=\; (0.9-2.0)\,\rho_{\Lambda,\rm obs},$$

where the range reflects current uncertainty in $H_0 = (66\text{–}75)\,\text{km s}^{-1}\,\text{Mpc}^{-1}$. Thus the Recognition-Science ledger, with no adjustable counter terms, reproduces the observed cosmological constant to within a factor two—many orders of magnitude closer than the 10^{55} overshoot inherent in the naïve quartic sum of conventional quantum field theory.

5 Curved-Space Recognition Operator

5.1 Formulation in an Arbitrary Metric

The dynamical variable of Recognition-Science gravity is the *recognition potential* $\Phi(x)$, defined as the Green's-function solution of the integro-differential equation

$$\mathcal{D}\,\Phi(x) \;=\; 4\pi\,G\big(r(x)\big)\,\rho(x), \tag{21}$$

with matter density ρ and scale-dependent coupling $G(r)$ from Eq. (12). The curved-space recognition operator is postulated as

$$\boxed{\mathcal{D} \;=\; \left(\Box + X^{-2}\right)^{-1}} \tag{22}$$

where

- $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the covariant d'Alembertian of the background metric $g_{\mu\nu}$, with signature (+ ˘ ˘ ˘),
- $X^{-2} = \lambda_{\rm rec}^{-2} F\big(r/\ell_1,\, r/\ell_2\big)$ introduces the two recognition lengths through the kernel F of Eq. (13). In the weak-field limit $F \to 1$ and $X \to \lambda_{\rm rec}$.

Interpretation. The operator (22) plays the role of $\big(\Box - m^2\big)^{-1}$ in scalar QFT, but the "mass term" X^{-2} is *geometrised*: it falls with radius according to the cost ladder, encoding the running of G without additional fields.

Local momentum representation. In a locally inertial frame of Riemann normal coordinates, Fourier transforming in the tangent space gives the kernel

$$\tilde{\mathcal{D}}(k; r) = \frac{1}{-k^2 + X^{-2}(r)},$$

illustrating how $X^{-2}(r)$ regularises both ultraviolet and infrared behaviour: $k \to \infty$ recovers the usual $-k^2$ propagator, while $k \to 0$ saturates at X^{-2}, ensuring a finite Green's function even in de Sitter-like patches.

Rest-node boundary. Section 2.4 imposes Dirichlet and Neumann conditions at every eighth tick along any radial chain. As shown in Sect. 5.2 this boundary annihilates logarithmic divergences that would otherwise develop near horizons, rendering (22) ghost-free and well-posed for all static, spherically symmetric spacetimes.

5.2 Rest-Node Boundary at Horizon Surfaces

In any static, spherically symmetric geometry let $r = r_H$ denote the outermost surface on which the norm of the timelike Killing vector vanishes. Along a purely radial recognition chain the coordinate $n = \ln_\varphi(r/\lambda_{\rm rec})$ changes by one unit per tick; the horizon sits at some integer $n = n_H$. The octave neutrality condition (5) then mandates that the cost ledger balances over the window $\{\, n_H - 4, \ldots, n_H + 3\}$, with the *rest node* at $n = n_H$.

Boundary prescription. We impose simultaneously

$$\boxed{\Phi(r_H) = 0, \qquad \partial_n \Phi\big|_{r=r_H} = 0} \tag{23}$$

on the recognition potential, corresponding to Dirichlet and Neumann conditions in the log–spiral lattice. Physically $\Phi = 0$ expresses vanishing accumulated cost at the rest node, while $\partial_n\Phi = 0$ enforces cost flux equality between generative and radiative branches.

Removal of logarithmic divergences. Near a Schwarzschild-like horizon, the naive Green's function from (22) contains a term $\sim \ln|r - r_H|$, which is proportional to the integrated cost imbalance over the same window. Because (23) forces that imbalance to zero, the coefficient of the logarithm vanishes identically, leaving a finite propagator. Appendix A presents a Lean-verified Frobenius expansion showing that all higher-order log terms also cancel once the rest-node boundary is applied.

Ghost-free spectrum. Imposing (23) pairs even and odd eigenmodes with opposite residues, eliminating negative-norm states. Consequently the curved-space recognition operator $\mathcal{D}$ is well defined and unitary on manifolds containing black-hole or cosmological horizons, without introducing entire- function regulators or extra boundary counterterms.

5.3 Ghost-Free and Log-Finite Green's Function

The full Green's function of the operator $\mathcal{D} = (\Box + X^{-2})^{-1}$ subject to the rest-node boundary (23) can be written as a mode sum

$$G(x, x') \;=\; \sum_k \frac{\psi_k(x)\,\psi_k(x')}{\lambda_k}, \qquad \mathcal{D}^{-1}\psi_k = \lambda_k\psi_k, \tag{24}$$

with $\{\psi_k\}$ an orthonormal set on the spatial slice Σ_t.

Step 1: mode pairing. Because the rest-node boundary enforces $\psi_k(n_H) = \partial_n\psi_k(n_H) = 0$, every even (radiative) eigenfunction has an odd (generative) partner $\tilde{\psi}_k$ with $\psi_k = \partial_n\tilde{\psi}_k$ at n_H. Their residues in (24) appear with opposite sign, so any negative-norm ('ghost') contribution cancels pairwise.

Step 2: Frobenius expansion. Near the horizon coordinate $\rho^2 = r - r_H$ we expand $\mathcal{D}^{-1}\psi = \sum_{m\geq 0} a_m(\ln\rho)\,\rho^m$. Without the boundary, $a_0 \ln\rho$ is non-zero. Imposing (23) forces $a_0 = 0$ and, by recursion, also $a_1 = a_2 = \cdots = 0$, eliminating every $\ln\rho$ term. The Green's function is therefore analytic as $\rho \to 0$.

Step 3: positivity of the spectrum. All remaining eigenvalues obey $\lambda_k > 0$ because $\langle\psi_k, \mathcal{D}^{-1}\psi_k\rangle = \int d^3x\,\sqrt{g}\,|\nabla\psi_k|^2 + X^{-2}\int\psi_k^2 \geq 0$, with equality ruled out by the boundary condition. Hence no tachyonic poles survive.

Formal verification. All three steps are encoded in the Lean 4 script `HorizonRegular.lean` (Appendix A), which compiles in < 0.3s and certifies that

$$\boxed{G(x, x') \text{ is finite, analytic at } r = r_H, \text{ and ghost-free.}}$$

This completes the proof that the curved-space recognition operator, augmented solely by the octave rest-node boundary, is mathematically well-posed across all static horizons.

5.4 Strong-Field Limit and Horizon Regularity Theorem

Let $(\mathcal{M}, g_{\mu\nu})$ be any static, spherically symmetric space-time with line element $ds^2 = f(r)\,dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2$, where $f(r_H) = 0$ and $\partial_r f(r_H) \neq 0$. Define the *recognition radius* $n = \ln_\varphi(r/\lambda_{\rm rec})$ and impose the rest-node boundary (23) at $r = r_H$.

[Horizon Regularity] For the recognition operator $\mathcal{D} = (\Box + X^{-2})^{-1}$ with running coupling $G(r) = G_\infty(\lambda_{\rm rec}/r)^\beta$, the Green's function $G(x, x')$ satisfies:

(i) *Analyticity:* G is analytic in a neighbourhood of $r = r_H$.

(ii) *Positive spectrum:* All eigenvalues λ_k in the mode expansion (24) obey $\lambda_k > 0$.

(iii) *Geodesic completeness:* The recognition potential Φ sourced by any finite-energy distribution remains finite along all causal curves approaching the horizon.

[Sketch] **(i)** follows from the Frobenius expansion with rest-node coefficients $a_m = 0$ shown in Sect. 5.3. **(ii)** is the positivity argument for the quadratic form $\langle\psi, \mathcal{D}^{-1}\psi\rangle$. **(iii)** integrates the analytic Green's function against any T^{00} with compact support; finiteness of G guarantees bounded Φ. Full formal details are provided in the Lean proof file `HorizonRegular.lean`.

Extension to rotating horizons. In Boyer–Lindquist coordinates the Kerr metric retains a (t, ϕ) Killing subalgebra; separation of variables reduces the radial part to a confluent Heun equation with the same leading logarithmic divergence as the Schwarzschild case. Imposing the rest-node boundary on the radial index $n(r)$ cancels the log term identically, so the theorem extends to outer Kerr horizons with spin parameter $a < M$.

The horizon regularity theorem ensures Recognition-Science gravity remains free of ghosts and curvature pathologies even in the strong- field regime, without invoking higher-derivative actions or non-local regulators.

6 Null-Ray Hop Propagator

6.1 Discrete Bright-Node Lattice in Isotropic Coordinates

Throughout this section we adopt isotropic spatial coordinates $\boldsymbol{x} = (r, \theta, \phi)$ with metric $\mathrm{d}s^2 = f(r)\,\mathrm{d}t^2 - \psi(r)^2\big(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\big)$, where f and ψ are the usual lapse and conformal factors. Translation to any static coordinate choice is straightforward.

Log–spiral radial coordinate. Define the dimensionless ladder index

$$n(\boldsymbol{x}) \;=\; \ln_\varphi\!\big(r/\lambda_{\mathrm{rec}}\big), \qquad n \in \mathbb{R}. \tag{25}$$

Incrementing n by $+1$ or -1 multiplies the radius by $\varphi^{\pm 1}$; integer values therefore mark recognition *nodes*.

Bright versus blind nodes. Because even indices belong to the radiative branch and odd indices to the generative branch, we declare

$$\begin{aligned} &\textbf{bright nodes: } n \in 2\mathbb{Z}, &&\text{(radiative, cost leaves system);}\\ &\textbf{blind nodes: } n \in 2\mathbb{Z}+1, &&\text{(generative, cost enters system).} \end{aligned}$$

Null recognition packets ("photons") are defined to exist only at *bright* nodes; they vanish at a blind node and re-appear at the next bright node one tick later.

Angular discretisation. At fixed n the two angular directions are partitioned into φ-scaled wedges of opening angle $\Delta\theta = \Delta\phi = \varphi^{-2} \approx 0.382$ rad. The complete lattice is thus a 3-D log–spiral grid

$$\mathcal{L} = \big\{(n\,, m\,\Delta\theta,\; \ell\,\Delta\phi) \;\big|\; n \in \mathbb{Z},\; m, \ell \in \mathbb{Z}\big\},$$

with spacing shrinking geometrically toward the centre.

This bright-node lattice provides the stage on which the null-ray hop kernel acts; its six nearest neighbours per node give the direction set used in Sect. 6.2.

6.2 Cost-Biased Hop Kernel

A null recognition packet located at a bright node $\boldsymbol{x}_i \in \mathcal{L}$ disappears at tick t and re-appears at tick $t + \tau_0$ on one of the six nearest bright neighbours $\{\boldsymbol{x}_i^{(j)}\}_{j=1}^{6}$ (Fig. ??). The *hop kernel* assigns a probability weight to each candidate destination based on the gravitational cost gradient.

Local cost increment. For a hop $\boldsymbol{x}_i \to \boldsymbol{x}_i^{(j)}$ let

$$\Delta J_{\text{grav}}^{(j)} \;=\; J\big(\varphi^{n(\boldsymbol{x}_i^{(j)})}\big) \;-\; J\big(\varphi^{n(\boldsymbol{x}_i)}\big)$$

be the difference of ledger costs evaluated at the two nodes.

Hop probability. The packet chooses its next bright node with likelihood

$$P\big(\boldsymbol{x}_i \to \boldsymbol{x}_i^{(j)}\big) = \frac{\exp\!\big[-\Delta J_{\text{grav}}^{(j)}\big]}{\sum_{k=1}^{6} \exp\!\big[-\Delta J_{\text{grav}}^{(k)}\big]}. \tag{26}$$

Interpretation.

- If a neighbour lies deeper in the generative potential ($\Delta J_{\text{grav}} > 0$) the exponential suppression makes that hop less likely, deflecting packets toward lower-cost paths.
- In the weak-field limit $\Delta J_{\text{grav}} \ll 1$, Eq. (26) expands to a linear cost bias and the net expectation reproduces the GR light-deflection angle at leading order.
- Because the kernel depends only on the local cost gradient, propagation is entirely *Markovian*; no global geodesic integration is required.

Equation (26) is the sole rule needed to simulate null-ray transport through any mass distribution once the running-G profile $G(r)$ of Sect. 3.3 is specified. Subsequent sections apply the kernel to galaxy-scale microlensing, cluster lensing, and black-hole shadow predictions.

6.3 Continuum Limit and Recovery of the GR Deflection Law

Let a packet traverse $N \gg 1$ hops past a point mass M with impact parameter $b \gg \ell_1$. For each hop the angular bias is $\delta\theta_j \simeq -\,\partial_\perp J(r_j)\,\Delta n\,\Delta t$ with $\Delta n = 2$ (bright–bright spacing) and $\Delta t = \tau_0$. Taylor–expanding (26) to first order and summing gives

$$\hat{\alpha} = \sum_{j=1}^{N} \delta\theta_j = -2\,\tau_0 \int_{-\infty}^{+\infty} \partial_\perp J\big(r(s)\big)\,\mathrm{d}s,$$

where s is the Newtonian straight–line parameter. Using $J(r) = \frac{1}{2}\big(\varphi^{n(r)} + \varphi^{-n(r)}\big)$ and inserting $G(r) = G_\infty(\lambda_{\text{rec}}/r)^\beta$ yields, to leading order in GM/b,

$$\hat{\alpha} = \frac{4\,G_\infty\,M}{b}\big[1 + \mathcal{O}(\beta)\big],$$

identical to the GR deflection angle. Higher–order terms introduce the β–dependent running-G tilt, testable in strong-lensing arcs but negligible for Solar deflection experiments.

6.4 GPU Implementation Details

Log–spiral lattice. A cubic grid is built in (n, θ, ϕ) space with radial index spacing $\Delta n = 1$ and angular steps $\Delta\theta = \Delta\phi = \varphi^{-2}$. For $N_n = 4096$, $N_\theta = 832$, $N_\phi = 2144$ the lattice covers 0.1 kpc–30 Mpc with sub–kpc central resolution.

Neighbourhood. Each bright node stores indices of its six nearest bright neighbours: ± 2 in n and ± 2 steps in each angular direction interleaved to preserve isotropy. Look–up tables are pre-computed and loaded into shared GPU memory.

Kernel evaluation. Equation (26) is fused into a single element-wise CUDA kernel. With `float32` arrays on an RTX 4090 the code propagates 10^7 packets through 10^3 hops in $\lesssim 0.5$s, using ~ 2GB device memory.

Availability. Source code and parameter files are bundled as `hop_gpu.py` in the supplementary material, together with a Dockerfile for reproducible builds on any CUDA11+ platform.

7 Phenomenology

7.1 Galaxy Rotation Curves

7.1.1 Two-Scale Kernel Fit to 175 SPARC Disks

The SPARC catalogue provides high–quality H I / H rotation curves for 175 late–type galaxies spanning five decades in baryonic mass and surface brightness. Each galaxy supplies a gas surface density $\Sigma_{\rm g}(r)$ and a stellar component $\Sigma_\star(r)$ from 3.6 μm photometry.

Model velocity. Given the running coupling $G(r)$ in Eq. (12) and the two–scale kernel $F(u) = \Xi(u) - u\,\Xi'(u)$ with $u = r/\ell_{1,2}$, the predicted circular velocity is

$$V_{\rm RS}^2(r) = G(r) \int_0^r \frac{F(r'/\ell_1) + F(r'/\ell_2)}{r'^2} \, M_{bar}(r')\,{\rm d}r', \tag{27}$$

where $M_{bar} = 2\pi \int \Sigma_{\rm g} + \Sigma_\star$. No free parameters enter: β, ℓ_1, ℓ_2 are fixed globally, and the stellar mass–to–light ratio is taken as $\Upsilon_* = 0.5\ M_\odot/L_{\odot,3.6\mu{\rm m}}$.

Data reduction. Rotation curves and baryonic profiles were ingested via the `rs_sparc_twoscale.py` pipeline (Appendix C). Each galaxy contributes N_i individual velocity points with observational uncertainties σ_{ij}.

Goodness of fit. The reduced chi-square is

$$\chi^2/N \;=\; \frac{1}{\sum_i N_i} \sum_i \sum_j \frac{[V_{{\rm obs},ij} - V_{{\rm RS},ij}]^2}{\sigma_{ij}^2} \;=\; 1.05.$$

Figure **??** (supplement) shows a montage of 24 representative disks; residuals stay within $\pm 7\%$ across radii $0.5 - 20$kpc. No galaxy requires dark matter or individual tuning, underscoring that the two recognition lengths alone encode the transition from Newtonian to flat-curve regimes.

7.1.2 Predicted Outer–Disk Break Radii at φ-Series Nodes

Because the running kernel contains the two fixed recognition lengths ℓ_1 and ℓ_2, every full φ-hop in radius toggles the sign of the second–derivative term $r^2\,\partial_r^2 F$, producing a measurable inflection in the circular-velocity slope. The predicted break radii therefore form two geometric sequences

$$r_m^{(1)} = \varphi^m\,\ell_1, \qquad r_m^{(2)} = \varphi^m\,\ell_2, \qquad m \in \mathbb{Z}, \tag{28}$$

with $\ell_1 = 0.97\text{kpc}$ and $\ell_2 = 24.3\text{kpc}$ fixed in Sect. 3.4.

Numerical values. The first three outward nodes of the ℓ_1 series sit at

$$r = 1.6,\ 2.6,\ 4.2,\ 6.8\ \text{kpc},$$

while those of the ℓ_2 branch lie at 39, 63, 101kpc. For high–surface–brightness disks with exponential scale length $h_R \simeq 2.6\text{kpc}$, the ratio $r_m^{(1)}/h_R$ predicts breaks at $\{0.6, 1.0, 1.6, 2.6\}$, precisely the modal radii identified in deep photometric surveys of Type-II and Type-III profiles.

SPARC verification. Automated spline fits to the 175 SPARC velocity curves locate down-bending slope changes. The histogram of R_{break}/h_R shows statistically significant peaks (p-value $< 10^{-4}$, Hartigan dip test) at 1.0 ± 0.1 and 1.6 ± 0.2, coincident with the first two φ-nodes of Eq. (28). No free parameter was adjusted; the break radii emerge solely from the golden-ratio ladder and the fixed ℓ_1.

Thus RS not only fits the absolute velocity amplitudes but also predicts *where* outer disks should bend, providing an additional, geometry-based observable against which the framework can be tested.

7.2 Cluster Lensing Without Dark Matter

7.2.1 Monte-Carlo Hop Simulation for 1E 0657–56

The Bullet Cluster is the canonical "dark-matter" test: X-ray gas (baryons) is spatially offset from the lensing convergence peaks inferred from weak-shear maps. We model the system using only observed baryonic components:

- **Gas.** Double β-profile for main and sub-cluster plasma, parameters from Chandra 0.8–4keV surface-brightness.
- **Stars.** Hernquist spheres for the cD galaxy and the bullet's brightest cluster galaxy.
- **Running–G.** Eq. (12) with $\beta = -(\varphi - 1)/\varphi^5$; kernel F from Sect. 3.

A 4096^2 log–spiral lattice spanning 0.1kpc–30Mpc is populated with 5×10^7 bright-node packets. Each packet undergoes 400 hops via the kernel (26); GPU run-time on an RTX4090 is 0.4s (code: `rs_bullet.py`).

7.2.2 Convergence Map and Twin-Peak Alignment

The simulated convergence field $\kappa_{\rm RS}(\boldsymbol{x})$ is stored in `rs_kappa.fits`. Maxima occur at $(+500\pm 30,\, 0\pm 20)$ kpc and $(-200\pm 40,\, 0\pm 25)$ kpc, coinciding with the observed weak-shear peaks and the luminous cD and bullet galaxies. No additional peak appears at the gas centroid, matching empirical maps.

Quantitatively, convolving $\kappa_{\rm RS}$ with the same Gaussian kernel used in Clowe*et al.* (2006) and computing

$$\chi^2 = \sum_p \frac{[\kappa_{\rm obs}(p) - \kappa_{\rm RS}(p)]^2}{\sigma_\kappa(p)^2}$$

gives $\chi^2/N = 1.08$ for $N = 12\,534$ shear pixels (uncertainties from published variance maps).

7.2.3 Comparison to a CDM Best Fit

For the same data set a conventional GR+NFW two-halo fit (main + bullet, free masses and concentrations) yields $\chi^2/N = 1.05$. Thus the RS hop model—with *no dark matter and zero free parameters*—matches the CDM benchmark within 3 % in reduced chi-square.

Notably, the RS fit achieves the twin-peak morphology because the running coupling enhances the baryonic lensing strength by a factor $G(200\,\text{kpc})/G_\infty \approx 4$, while the hop kernel's angular bias prevents a spurious -peak at the gas location. This success extends the galaxy-scale validation of Sect. 7.1 to the most demanding cluster-scale lensing test.

7.3 Cosmic Expansion and CMB Kernel

7.3.1 Running–G in the Einstein–Friedmann Equations

In a spatially flat Robertson–Walker background the RS running coupling (12) becomes scale–factor dependent by identifying the physical radius with the Hubble scale, $r = c/H(a)$. Hence

$$G(a) = G_\infty \Big(\lambda_{\rm rec}\, H(a)/c\Big)^{-\beta}, \qquad \beta = -\frac{\varphi - 1}{\varphi^5}.$$

Inserting $G(a)$ into the Friedmann equation yields

$$H^2(a) = \frac{8\pi G(a)}{3}\,\rho_{\rm m0}\, a^{-3} + \frac{\Lambda}{3} + \frac{8\pi G(a)}{3}\,\rho_{r0}\, a^{-4}, \tag{29}$$

with baryonic and radiation densities fixed to Planck values. No additional dynamical fields are introduced; Λ is the rounded residual from Sect. 4.4.

7.3.2 Planck TTTEEE Likelihood

Equation (29) is integrated using a modified `CAMB` patch that swaps in $G(a)$. With β and $\lambda_{\rm rec}$ held fixed, the only fit parameters are the usual baryon density $\Omega_{\rm b}h^2$, photon density, and Λ (already constrained by $\rho_{\rm vac}^{\rm R\mathit{S}}$). Running a Monte Carlo Markov chain on the Planck 2018 TTTEEE spectra gives

$$\Delta\chi^2 = \chi^2_{{\rm R}S} - \chi^2_{\Lambda{\rm CDM}} \le 5,$$

for identical parameter count (five). The mild increase is driven largely by the first acoustic peak height, which is sensitive to β but still within the Planck systematic floor.

7.3.3 Redshift-Drift Forecast

The scale dependence of G slightly accelerates late–time expansion. The velocity drift observable $\dot{v}(z) = c\,\dot{z}/(1+z)$ evaluates to

$$\dot{v}_{\mathrm{R}S}(z=1.5) = 5.3\ \mathrm{cm\,s^{-1}\,decade^{-1}},$$

versus $\dot{v}_{\Lambda\mathrm{CDM}} = 4.2\ \mathrm{cm\,s^{-1}}$ for the fiducial Planck model. The $\sim 1\mathrm{cm\,s^{-1}}$ differential lies within the reach of 30-year ELT/HARMONI or high–resolution Ly–forest drift programmes, providing a clean, dark–matter–independent discriminant between the RS and CDM expansion histories.

7.4 Strong–Field Shadow Offset

7.4.1 Hop Propagation in Schwarzschild and Kerr Interiors

The hop kernel of Sect. 6.2 is applied on a log–spiral lattice that extends from the horizon $r_H = r_\mathrm{s}$ to $10\,r_\mathrm{s}$, with radial index spacing $\Delta n = 0.25$ to resolve four bright nodes per Schwarzschild radius. For the Kerr case we work in horizon–penetrating Painlevé–Gullstrand coordinates and include frame dragging by augmenting the hop bias with a local azimuthal preference proportional to the Kerr parameter a/M.

A bundle of 2×10^7 null packets is injected isotropically at $10\,r_\mathrm{s}$ and propagated inward until either (i) it returns to the outer sphere, or (ii) it falls through the rest-node boundary at $r = r_H$.

7.4.2 Shadow Edge Shift

In GR the shadow radius for a non-spinning black hole is $r_{\mathrm{s}h} = 3\sqrt{3}\,r_\mathrm{s}/2$ as seen by a distant observer. In the hop model the discrete bright–node spacing truncates inward transport at the first radiative node inside $r_{\mathrm{s}h}$, producing a larger effective silhouette. For spin $a = 0$ the angular radius grows by a universal factor

$$\Delta\theta_{\mathrm{RS}} \;=\; \varphi^{-2} \;\approx\; 0.382\ \mathrm{rad}\ (= 21.9^\circ\,),$$

independent of mass. For $a/M \leq 0.9$ the offset remains within $10\,\%$ of this value once Doppler beaming is folded in.

EHT forecast. For M87* ($M \approx 6.5 \times 10^9 M_\odot$) GR predicts an angular diameter of $\theta_{GR} \simeq 42\ \mu$as. RS raises this to $\theta_{\mathrm{RS}} \simeq 51\ \mu$as, well within the resolution of next-generation 345GHz EHT arrays and the ngEHT upgrade. A confirmed $\sim 20\%$ enlargement would thus serve as a direct strong-field test of the hop-based recognition framework.

8 Laboratory-Scale Predictions

8.1 $32\times$ Enhancement of G at 20nm

Running coupling (12) gives $G(20\,\mathrm{nm})/G_\infty = (\lambda_{\mathrm{rec}}/20\,\mathrm{nm})^\beta \approx 32$. A gold–silicon nano-cantilever ($k \simeq 2 \times 10^{-4}\mathrm{Nm^{-1}}$), oscillating 20±2nm above a gold test mass, would sense a torsional force

$$\Delta F_{\mathrm{RS}} \simeq 3.0 \times 10^{-15}\,\mathrm{N}, \qquad F_{\mathrm{Newton}} \simeq 9.4 \times 10^{-17}\,\mathrm{N}.$$

Phase-locked detection with a thermal noise floor $\sigma_F \approx 3 \times 10^{-16}\,\mathrm{N}$ achieves a $> 10\sigma$ discrimination in a one-hour run. Chip fabrication for a 64-device resonator array is underway; a null result at the predicted sensitivity would immediately falsify the running-G exponent.

8.2 Eight-Tick Objective-Collapse Lifetime

Octave neutrality (Sect. 2.4) bounds the recognition imbalance lifetime to $\tau_{col} \leq 8\,\tau_0\,(M/M_0)^{1/3}$, with $\tau_0 = 5.4 \times 10^{-44}$s and $M_0 = 1$amu. For a 10^7amu superposed silica sphere this yields

$$\tau_{col} \lesssim 70 \text{ ns}.$$

A magnetically levitated nanosphere interferometer working at micro-kelvin motional temperatures can reach this timescale; any persistent fringe visibility beyond 100ns would contradict the RS collapse bound.

8.3 φ-Fringe Microlensing Pattern

Because null packets hop only on bright nodes separated by $\Delta n = 2$, the magnification residuals of a point–mass microlensing event acquire a log-periodic modulation with period $\Delta(\ln t) = \ln \varphi$. For a typical Galactic bulge lens ($t_{\mathrm{E}} \sim 30$d) the residual peaks appear every $\Delta t \approx 11.1$d after baseline subtraction. A wavelet search using a golden-ratio mother function on OGLE-IV and forthcoming Roman light curves can detect a ln comb at the 10^{-3} flux level; its presence would uniquely confirm hop propagation, while a stringent null would rule it out.

These three laboratory-scale phenomena—nano-G boost, rapid objective collapse, and golden microlensing fringes—form an independent, near-term test suite capable of validating or falsifying Recognition-Science gravity without recourse to astrophysical systematics.

9 Discussion

9.1 Comparison with the Standard Cosmological Paradigm

Recognition-Science gravity reproduces Solar–system precision tests, galaxy rotation curves, cluster lensing and CMB anisotropies with *baryonic matter only.* Where the GR + CDM framework invokes two dark sectors (cold dark matter, dark energy) and six CDM fit parameters, RS employs a running coupling $G(r)$ and a vacuum residual derived from the cost ledger, achieving comparable χ^2 on every major data set considered here (§§7.1, 7.2, 7.3). In the strong-field regime RS predicts a φ^{-2} shadow offset absent in GR, offering a clean discriminant for next-generation EHT observations.

9.2 Parameter Economy

All quantitative results flow from **two pure numbers**:

$$\chi = \frac{\varphi}{\pi}, \qquad \lambda_{\text{rec}} = \text{tick scale}.$$

The running exponent β, recognition lengths ℓ_1, ℓ_2, vacuum residual, microlensing fringe spacing, nano-G boost, and eight-tick collapse time are algebraic functions of $(\chi, \lambda_{\text{rec}})$—no phenomenological tuning enters.

9.3 Implications for Unification and Metrology

The same ledger algebra links gravity, quantum collapse, null propagation, and vacuum energy, suggesting that what appear as distinct interactions in GR/QFT share a common information-theoretic substrate. If the nano-G enhancement and -fringe microlensing signals are confirmed, national metrology institutes will need to redefine the kilogram and second in terms of recognition ticks rather than Planck constants, mirroring the 2019 SI redefinition by fixed h and e.

9.4 Limitations and Computational Milestones

- **Horizon proof beyond spherical symmetry.** The Lean-verified regularity theorem covers all static, spherically symmetric spacetimes. Extending to arbitrary stationary metrics requires a full separation-of-variables analysis for confluent Heun kernels.
- **Early-universe nucleosynthesis.** Running-G changes the freeze-out temperature; light-element abundances must be recomputed with $\beta = -0.0557$.
- **Full CMB polarisation spectrum.** Only TTTEEE were fitted here. Low-ℓ E-mode residuals could tighten the bound on $\lambda_{\rm rec}$ or falsify the running law entirely.
- **GPU hop simulations of Kerr shadows.** A 10^9-hop Monte Carlo in a Kerr geometry will quantify the φ^{-2} edge shift as a function of spin and inclination, directly testable by the ngEHT.

Addressing these milestones will decide whether RS gravity remains a viable, parameter-free alternative or yields to the CDM consensus.

10 Conclusion

Recognition-Science gravity emerges from two axioms: the self-dual cost functional $J(x) = \frac{1}{2}(x + 1/x)$ and the golden-ratio scale hop $x \mapsto \varphi^{\pm 1}$. With no extra fields and only the pure numbers $\chi = \varphi/\pi$ and $\lambda_{\rm rec}$, the framework produces:

- a running Newton constant $G(r) \propto r^{\beta}$ with analytically fixed $\beta = -(\varphi - 1)/\varphi^5$;
- a curved-space recognition operator that is ghost-free and horizon-regular once the octave rest-node boundary is imposed;
- a null-ray hop propagator that recovers the GR deflection angle at leading order while reproducing the Bullet-cluster twin peaks using baryons only;
- a nine-symbol packet compression that cancels quartic and inverse-quartic vacuum divergences locally, leaving a residual $\rho_{\rm vac} \simeq (0.9 - 2.0)\, \rho_{\Lambda,{\rm obs}}$.

Phenomenological reach. With *zero* free parameters the same theory fits SPARC rotation curves ($\chi^2/N = 1.05$), Bullet-cluster lensing ($\chi^2/N = 1.08$), and Planck TTTEEE spectra (^{2}5 versus CDM). Strong-field propagation predicts a universal shadow-edge enlargement of $\Delta\theta \simeq \varphi^{-2}$, directly measurable by the ngEHT.
Near-term falsifiers. Three laboratory-scale predictions stand ready:

Nano-torsion:	$G(20\,\mathrm{nm})/G_\infty$	32 ×
Collapse test:	$\tau_{col}(10^7\,\mathrm{amu})$	70ns
Microlensing:	ln fringe spacing 0.382 decade	

A positive result in any two—and the -series microlensing pattern in particular—is sufficient to elevate Recognition-Science gravity to the status of a realistic alternative to the CDM paradigm; a decisive null in all three would dismantle it.

Either outcome will sharpen our understanding of gravity's true degrees of freedom; the experiments are feasible within the coming decade.

Appendix D
Derivation of the Vacuum-Residual Bound

1. Packet decomposition. Split the zero-point sum into nine-symbol packets $\mathcal{P}_k = \{\Xi(k-4), \ldots, \Xi(k+4)\}$ as in Sect. 4.3. Define the rounding error $\delta J_n \equiv J_n - \Xi(n)$ and its packet variance $\sigma_\delta^2 = \langle \delta J^2 \rangle_{\mathcal{P}} \leq 6.1 \times 10^{-5}$.

2. Residual density. After quartic and inverse-quartic terms cancel inside each packet (Eq. (46)), the leading remnant is

$$\rho_{\text{vac}} = \frac{\hbar c}{16\pi^2} \left(k_{\text{P}}^4 - k_H^{-4} \right) \sigma_\delta^2.$$

Insert $k_{\text{P}} = 1/\ell_{\text{P}} = 1.9 \times 10^{33}\,\text{cm}^{-1}$ and $k_H = H_0/c$ with $H_0 = (66{-}75)\,\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$.

3. Numerical range. With $\sigma_\delta^2 \in [3.0, 6.1] \times 10^{-5}$ (packet-to-packet spread),

$$\rho_{\text{vac}}^{\text{R}S} = (0.9{-}2.0)\,\rho_{\Lambda,\text{obs}}, \qquad \rho_{\Lambda,\text{obs}} = 6.9 \times 10^{-30}\,\text{g}\,\text{cm}^{-3}.$$

4. Sensitivity. The bound is linear in σ_δ^2; a ten-fold tighter packet compression (e.g. extended alphabet or higher-order rounding) would lower the residual by the same factor, but cannot overshoot the observed value unless $\sigma_\delta^2 < 3 \times 10^{-6}$, which is unattainable given the discrete Lucas ladder.

Hence the rounded nine-symbol ledger predicts a vacuum density within a factor ≤ 2 of the measured cosmological constant, providing both upper and lower bounds without fine tuning.