

## Calibration Remark (Limit Form vs. Second Derivative)

**Remark 1** (Calibration: limit form vs. second derivative at 0). Define  $G(t) := F(e^t)$ . Normalization  $F(1) = 0$  implies  $G(0) = 0$ . If  $F$  is reciprocal ( $F(x) = F(x^{-1})$  for  $x > 0$ ), then  $G$  is even, hence  $G'(0) = 0$  (assuming differentiability at 0). If moreover  $G$  is  $C^2$  at 0, then the Taylor expansion gives

$$G(t) = G(0) + G'(0)t + \frac{G''(0)}{2}t^2 + o(t^2) = \frac{G''(0)}{2}t^2 + o(t^2),$$

so

$$\lim_{t \rightarrow 0} \frac{2F(e^t)}{t^2} = \lim_{t \rightarrow 0} \frac{2G(t)}{t^2} = G''(0).$$

Therefore the calibration condition  $\lim_{t \rightarrow 0} \frac{2F(e^t)}{t^2} = 1$  is equivalent (under the above regularity) to  $G''(0) = 1$ , i.e.  $(F \circ \exp)''(0) = 1$ .