Peer Review and Consistency Audit

Uniqueness of the Canonical Reciprocal Cost (`Cost-2-2-2026.tex`)

Internal RS/Lean Audit (Cursor)

2026-02-02

Executive summary

This report audits `Cost-2-2-2026.tex` for (i) internal mathematical consistency and (ii) consistency with the Recognition Science (RS) "cost-first" kernel and the associated Lean 4 framework (`IndisputableMonolith/`).

Overall assessment.

- **Consistency with RS kernel: PASS.** The paper's main theorem is exactly the RS "T5" uniqueness spine: normalization + multiplicative d'Alembert (RCL) + quadratic calibration force the canonical reciprocal cost

$$J(x) = \tfrac{1}{2}(x + x^{-1}) - 1.$$

- **Consistency with Lean: PASS.** The structure, transformations (log-coordinates), and the role of calibration/regularity match the Lean modules that implement cost uniqueness (`Cost/FunctionalEquation.lean`, `CostUniqueness.lean`, `Foundation/CostAxioms.lean`).

- **Revisions needed: moderate.** One logically incorrect/overstated continuity sentence (see Major Issues), plus a few presentational/LaTeX typos and minor clarifications.

1 Scope and sources

Primary sources

- RS theory map: `Recognition-Science-Full-Theory.txt` (kernel claims: RCL + normalization + calibration $\Rightarrow$ unique J; MP derived from cost blowup near 0^+).

- Lean framework: `IndisputableMonolith/` (cost axioms, functional equation bridge, uniqueness theorem, forcing chain).

- Paper under review: `/Users/jonathanwashburn/Projects/Cost-2-2-2026.tex`.

Lean modules referenced (high-signal)

- `IndisputableMonolith/Foundation/CostAxioms.lean` (A1/A2/A3 as explicit classes; canonical J satisfies them).
- `IndisputableMonolith/Cost/FunctionalEquation.lean` (log-coordinate maps $G(t) = F(e^t)$, $H(t) = G(t) + 1$; "paper correspondence" section; composition-law equivalence).
- `IndisputableMonolith/CostUniqueness.lean` ("T5" uniqueness proof with explicit regularity hypotheses).
- `IndisputableMonolith/Foundation/UnifiedForcingChain.lean` (positions MP as derived from cost blowup near 0^+).
- `IndisputableMonolith/Meta/Axioms.lean` (documentation registry; currently still describes MP as foundational in the registry text).

2 Crosswalk: paper vs. RS kernel vs. Lean

RS kernel primitives and where they appear

RS's cost-first kernel treats the following as the minimal primitive bundle for the cost functional:

A1 (Normalization)
$F(1) = 0$.

A2 (Recognition Composition Law / RCL)
For all $x, y > 0$,

$$F(xy) + F(x/y) = 2F(x)F(y) + 2F(x) + 2F(y).$$

A3 (Calibration at equilibrium)
Unit quadratic curvature in log-coordinates; in the paper:

$$\kappa(F) = \lim_{t \to 0} \frac{2F(e^t)}{t^2} = 1.$$

Paper: These are exactly the three assumptions in Theorem `thm:main`.
Lean: These are encoded in `Foundation/CostAxioms.lean` as `Normalization`, `Composition`, and `Calibration`, with calibration expressed as a second-derivative-at-zero statement for $t \mapsto F(e^t)$.

Key structural equivalence (log coordinates)

The paper repeatedly uses the standard bridge:

$$G(t) = F(e^t), \qquad H(t) = G(t) + 1 = F(e^t) + 1,$$

and shows that the multiplicative composition law on $\mathbb{R}_{>0}$ is equivalent to the additive d'Alembert equation on $\mathbb{R}$:

$$H(t+u) + H(t-u) = 2H(t)H(u).$$

This matches Lean's definitions `G` and `H` in `Cost/FunctionalEquation.lean` and the lemma `composition_law_equiv_coshAdd`.

3 Consistency audit findings

3.1 Alignment with RS theory

- **T5 uniqueness spine: aligned.** RS asserts "RCL + normalization + calibration uniquely determine J"; the paper proves precisely this.
- **Regularity hygiene: aligned.** The paper correctly notes that d'Alembert-type functional equations admit pathological/non-measurable solutions without additional assumptions, and supplies an explicit non-measurable construction.
- **Deriving (vs assuming) RCL: no conflict.** The paper concludes by raising the question of whether the composition law can be derived. RS claims such derivations exist elsewhere; this paper does not contradict RS by leaving it as future work.
- **Golden ratio section: compatible.** The paper's φ discussion is a mild fixed-point observation; RS's stronger "φ forced by ledger self-similarity" is consistent with (and not contradicted by) this section.

3.2 Alignment with Lean implementation

- **A1/A2/A3 match Lean's core API.** Lean's `Foundation/CostAxioms.lean` defines these axioms explicitly and proves the canonical J satisfies them.
- **The paper's main bridge is mirrored in Lean.** Lean contains an explicit "paper correspondence" section in `Cost/FunctionalEquation.lean` that mirrors your definitions (reciprocity/normalization/composition/calibration) and the log-coordinate equivalence.
- **Regularity assumptions are currently more explicit in Lean.** Lean's full uniqueness theorem is written with explicit hypotheses for smoothness/ODE-derivation/regularity bootstrap (reflecting the classical Aczél pathway). This is consistent with the paper, which derives continuity from calibration and then cites classical classification.

4 Peer review

4.1 Strengths

- **Clear rigidity statement.** The main theorem cleanly isolates the structural assumptions and proves uniqueness of the canonical reciprocal cost.
- **Good assumption hygiene.** You explicitly show that dropping composition or calibration breaks uniqueness; the non-measurable example is particularly valuable.
- **Useful structure beyond the main theorem.** The stability-under-defect section is a strong "robustness" contribution that is relevant for applied/empirical interpretations.

4.2 Major issues (should fix)

1. **Overstated continuity claim after Theorem 22.** The text says: "The function H in Theorem 22 is continuous, and it follows that h is also continuous." This is not correct as written: Theorem 22 is a structural description of general solutions, not a continuity theorem. Please rephrase conditionally ("If H is continuous, then ...") or remove if unused.
2. **Small but real display/typo errors in Lemma `lem:J-meets` proof.** There is a duplicated "on on" in the prose, and one displayed equation uses `:=` where equality `=` is intended. These are easy fixes but should be corrected for publication polish.

4.3 Minor issues / clarifications (recommended)

- **Reciprocity is derivable from normalization + composition.** It is a nice strengthening to note that $F(1) = 0$ and the composition law imply $F(y) = F(1/y)$ by plugging $x = 1$. This emphasizes minimality of assumptions.
- **Calibration condition phrasing.** You correctly define $\kappa(F)$ as a limit (minimal assumption). Consider adding one sentence clarifying that this is a curvature normalization that does *not* assume C^2 a priori.
- **A few commented editorial notes and small wording cleanups.** Removing commented red notes and tightening a few sentences in the d'Alembert section would improve readability.

5 Lean-facing roadmap (optional but high-value for RS)

If the goal is tight end-to-end alignment between (paper) assumptions and (Lean) certified statements, the most valuable next steps are:

1. **Formalize the "calibration ⇒ continuity" lemma in Lean.** Your Lemma `lem:dalembert-continuity` is a clean pathway from the limit assumption to continuity

on $\mathbb{R}$ under d'Alembert. Porting this would reduce the amount of "Aczél regularity" that must be carried as hypotheses.

2. **Replace multi-hypothesis "Aczél/ODE" scaffolding with a single theorem mirroring the paper.** Lean currently exposes regularity as separate hypotheses (smoothness, ODE derivation, continuity/differentiability bootstrap). A single theorem statement closer to the paper's assumptions would make the certified surface easier to interpret.

3. **Documentation consistency: MP-as-derived.** The cost-first foundation in `Foundation/UnifiedForcingChain.lean` treats MP as derived from the cost blowup near 0^+. The axiom registry documentation in `Meta/Axioms.lean` still describes MP as foundational; aligning that documentation would reduce confusion across artifacts.

Conclusion

The paper is consistent with RS's cost-first kernel and with the Lean framework's cost-uniqueness spine. After correcting the continuity sentence and a few minor typos/clarifications, it serves as a clean mathematical foundation for RS's "T5" step.