Tau-Step "Exclusivity" Does Not Establish a Fermion Mass Law

Here I evaluate the claim that the revised muon-to-tau step coefficient,

$$C_\tau = W + \frac{D}{2}, \tag{1}$$

derived in `tau_step_exclusivity.tex`, resolves the criticism that the lepton-generation formulas are hand-specific and admit many alternatives.

I show that while the algebra is correct, the exclusivity claim fails at a structural level. The failure is not numerical but logical: the construction does not uniquely determine the functional form of the tau step from prior axioms and therefore cannot support a mass *law* claim.

Most of the sections below are already done in the previous note, here I again briefly give it to be self contained.

1 What must be proven to claim a mass law

As mentioned in the earlier note: A fermion mass law is a mapping

$$\mathcal{L} : \{\text{fermion species}\} \to \mathbb{R}_{>0}$$

that is:

1. *Uniquely derived* from stated axioms or mechanisms,
2. *Identifiable*: no alternative inequivalent constructions exist that reproduce the same validated data,
3. *Predictive*: functional forms are fixed *before* comparison to experimental masses.

Numerical agreement alone is insufficient. What is required is uniqueness of the derivation.

2 Summary of the tau-step revision

We have

$$S_{\mu\to\tau} = F - \frac{2W+3}{2}\alpha, \tag{2}$$

with $W = 17$, giving a coefficient $C_\tau = 18.5$.

The exclusivity note rewrites

$$\frac{2W+3}{2} = W + \frac{D}{2}, \tag{3}$$

interpreting the integer 3 as the spatial dimension $D = 3$.

This removes an *aesthetic* arbitrariness but does not establish necessity.

3 Rewriting a number is not a derivation

Re-expressing a numerically fitted constant in terms of other constants does not constitute a derivation unless the expression is uniquely forced by prior axioms.

Proof: The exclusivity note explicitly states that the value $C_\tau = 18.5$ was *numerically determined* first. The expression $W + D/2$ is chosen afterward to reproduce this value.

Formally, let C be a real number determined from data. Any identity of the form

$$C = f(c_1, c_2, \dots) \tag{4}$$

is a *representation*, not a derivation, unless the theory proves:

$$\text{(axioms)} \;\Rightarrow\; C = f(c_1, c_2, \dots) \tag{5}$$

and proves that no other f' is admissible.

The note does not provide such a proof.

Remark 1. Lean verification of $17 + 3/2 = 18.5$ certifies arithmetic, not physical necessity.

4 Explicit non-uniqueness using the same constants

Even fixing the counting-layer constants

$$W = 17, \quad D = 3, \quad F = 6, \quad E_{\text{total}} = 12, \quad E_{\text{passive}} = 11, \tag{6}$$

there are many inequivalent formulas that evaluate to 18.5.

Exact degeneracy at $D = 3$: The coefficient $C_\tau = 18.5$ admits multiple distinct expressions built solely from the same counting-layer constants.

Proof: Using cube identities $F = 2D$, $E_{\text{total}} = 2F$, and $E_{\text{total}} - E_{\text{passive}} = 1$:

$$\begin{aligned} C_\tau &= W + \frac{D}{2}, && (7)\\ &= W + \frac{F}{4}, && (8)\\ &= W + \frac{E_{\text{total}}}{8}, && (9)\\ &= \frac{2W + D}{2}, && (10)\\ &= \frac{4E_{\text{total}} - E_{\text{passive}}}{2}. && (11) \end{aligned}$$

Each expression equals 18.5 at $D = 3$, yet they are algebraically distinct.

Remark 2. No principle in the exclusivity note rules out the alternatives above. Selecting $D/2$ is therefore a choice, not a consequence.

5 Additivity assumptions merely restrict the hypothesis class

The exclusivity note enforces a condition of the form

$$\Delta(D_1 + D_2) = \Delta(D_1) + \Delta(D_2), \tag{12}$$

from which linearity $\Delta(D) = kD$ follows.

Imposing additivity does not derive the tau-step coefficient; it restricts the space of allowed functions after the fact.

Proof: On $\mathbb{N}$, additivity implies $\Delta(D) = D\Delta(1)$. Fixing $\Delta(3) = 3/2$ yields $\Delta(D) = D/2$.

However:

1. Additivity is not derived from the mass framework axioms,
2. Many non-additive functions agree at $D = 3$,
3. RS already fixes $D = 3$, so cross-D behavior is untestable.

Thus additivity functions as an *exclusion rule*, not a derivation.

6 Collapse of cross-dimensional arguments

The exclusivity argument implicitly appeals to behavior across dimensions D. But RS independently claims that only $D = 3$ is physically realized.

If only $D = 3$ is physically meaningful, then functional uniqueness in D cannot be inferred.

Proof: Let $f(D)$ be any function such that $f(3) = 3/2$. Define

$$g(D) := \frac{D}{2} + (D-3)^2 h(D), \tag{13}$$

for arbitrary $h(D)$. Then $g(3) = 3/2$ exactly, but $g \neq f$ as a function.

Since RS forbids testing at $D \neq 3$, all such functions are observationally indistinguishable.

Remark 3. Thus dimensional rigidity removes, rather than enforces, functional uniqueness.

7 General non-identifiability of the lepton chain

Theorem 1 (Non-identifiability of the lepton mass pipeline)**.** *The lepton mass chain cannot uniquely determine its step formulas from lepton masses alone.*

Proof. The chain has the structure

$$\frac{m_\mu}{m_e} = \varphi^{S_{e\to\mu}}, \qquad \frac{m_\tau}{m_\mu} = \varphi^{S_{\mu\to\tau}}.$$

Hence

$$S_{e\to\mu} = \log_\varphi(m_\mu/m_e), \quad S_{\mu\to\tau} = \log_\varphi(m_\tau/m_\mu).$$

These two real numbers always exist for any positive masses. Therefore:

- Introducing $S_{e\to\mu}$ and $S_{\mu\to\tau}$ introduces two degrees of freedom,
- Writing them in terms of α, W, D, etc. does not remove those degrees unless the functional forms are uniquely derived,
- No such uniqueness proof is provided.

□

8 Conclusion

The revised tau-step formula is:

- Algebraically correct,
- Aestheticly cleaner,
- Formally verifiable in Lean.

However, it does *not*:

- Eliminate functional non-uniqueness,
- Derive the coefficient from prior axioms,
- Prevent alternative constructions using the same constants,
- Establish identifiability required for a mass law.