WTokens as Compression

Selection = Recognition (WToken-Specific Addendum)

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Abstract

This short addendum isolates the new WToken-specific consequences of the Algebra of Aboutness. The general reference framework defines a *symbol* as a configuration that both *means* an object (minimizes reference cost) and *compresses* it (has strictly lower intrinsic cost). We formalize two new claims in the Recognition Science WToken setting: (1) *WTokens as compression:* if $J(\mathsf{WToken}) < J(\mathsf{Concept})$, the WToken can serve as a valid symbolic carrier for the concept; in particular, Level-0 WTokens satisfy $J = 0$ and therefore compress every positive-cost concept. (2) *Selection = recognition:* choosing the optimal WToken is exactly a cost-minimizing projection onto a discrete φ-lattice, i.e., an explicit $\arg\min$ of reference mismatch. We cite the corresponding certified results from the Lean development.

1 Scope

The Algebra of Aboutness provides a general, configuration-independent theory of reference. This addendum does *not* re-derive those general foundations. Instead, it isolates the new material introduced by instantiating that framework with WTokens:

- the induced WToken cost functional J_W,
- the compression guarantee for Level-0 WTokens, and
- the cost-minimizing projection (geodesic selection) used for recognition.

2 The universal cost functional

Definition 2.1 (Recognition cost)**.** Let $x \in \mathbb{R}_{>0}$. Define

$$J(x) := \frac{1}{2}\left(x + x^{-1}\right) - 1. \tag{1}$$

This J is nonnegative on $\mathbb{R}_{>0}$, symmetric under inversion, and has a unique zero-point at $x = 1$.

3 WTokens as a φ-lattice cost space

WToken identity includes several discrete coordinates (mode family, φ-level, and a τ-offset). For cost and reference selection, the only intrinsic coordinate is the φ-level.

Definition 3.1 (φ-level ratio map)**.** Fix the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$. A WToken has discrete level $k \in \{0, 1, 2, 3\}$ and is assigned a characteristic ratio φ^k. Its intrinsic cost is

$$J_W(w) := J\big(\varphi^k\big). \tag{2}$$

In the codebase, this is realized by `wtokenRatio` and `wtokenCost` in `IndisputableMonolith/Foundatio`

4 WTokens as compression: the Level-0 universality theorem

In the Aboutness framework, the *compression half* of symbolhood is the strict inequality $J_S(s) < J_O(o)$. The WToken specialization makes this strikingly explicit.

Theorem 4.1 (Level-0 WTokens have zero intrinsic cost)**.** *Let w be any Level-0 WToken. Then $J_W(w) = 0$.*

Proof. This is exactly the certified theorem `level0_zero_cost`. Since Level-0 corresponds to exponent $k = 0$, the associated ratio is $\varphi^0 = 1$, and $J(1) = 0$. □

Theorem 4.2 (Universal compression by Level-0)**.** *Let c be any concept with positive intrinsic cost $J(c) > 0$. Let w_0 be a fixed Level-0 WToken. Then*

$$J_W(w_0) < J(c). \tag{3}$$

Proof. This is exactly the certified theorem `level0_wtoken_is_universal_symbol`. By the previous theorem, $J_W(w_0) = 0$, so the inequality reduces to $0 < J(c)$. □

Remark 4.3 (Interpretation). Level-0 WTokens act as *universal compressors* for any positive-cost concept. This provides the WToken-specific semantic content of the "mathematical backbone": zero intrinsic cost means the symbol can be attached to any $J > 0$ object without violating the compression budget.

5 Selection = recognition: cost-minimizing projection onto the ladder

Compression alone is not the entire story; the Aboutness framework also requires that a symbol *means* an object by minimizing reference mismatch cost. In the WToken setting, the central operational step is a projection onto the φ-ladder.

5.1 Reference mismatch cost

For a positive input ratio $r \in \mathbb{R}_{>0}$ and a candidate φ-level $k \in \{0, 1, 2, 3\}$, define

$$\mathrm{Ref}(r, k) := J\big(r/\varphi^k\big). \tag{4}$$

A WToken selection rule is then an explicit $\arg\min$ of $\mathrm{Ref}(r, k)$ over the finite ladder.

5.2 Geodesic selection theorem (certified)

Theorem 5.1 (Geodesic projection minimizes reference mismatch)**.** *Let $r \in \mathbb{R}_{>0}$. Let $k^* \in \{0, 1, 2, 3\}$ be the selected φ-level returned by* `projectOntoPhiLattice`*. Then for every ladder level $k \in \{0, 1, 2, 3\}$,*

$$J\big(r/\varphi^{k^*}\big) \le J\big(r/\varphi^k\big). \tag{5}$$

Proof. This is exactly the certified theorem `projection_minimizes_reference`. In Lean, `projectOntoPhiLattice` is defined as the minimizer of $\mathrm{Ref}(r, k)$ over a finite set of φ-levels. □

Remark 5.2 (Log-rounding as an efficient implementation). Although the proof is formulated as a finite $\arg\min$, the minimizer is well-approximated by $k^* \approx \mathrm{round}(\log_\varphi(r))$. This explains the phenomenological "snap" from a continuous input field to discrete semantic atoms.

6 Recognition operator as cost minimization

Recognition Science treats recognition as the act of selecting cost-minimizing configurations. The WToken specialization makes this explicit: recognition of a ratio-valued input is exactly the minimizing projection onto the φ-ladder.

Remark 6.1 (What this addendum establishes).

- **Compression:** Level-0 WTokens satisfy $J_W = 0$ and therefore compress any $J > 0$ concept.
- **Selection:** The WToken φ-level is selected by minimizing reference mismatch cost.

The remaining (domain-specific) work is to characterize when the selected WToken also satisfies the full *meaning* condition for a chosen concept class.

Formal verification anchors

- `IndisputableMonolith/Foundation/WTokenReference.lean`
  - `wtokenCost`, `wtokenRatio`
  - `level0_zero_cost`
  - `level0_wtoken_is_universal_symbol`
  - `projectOntoPhiLattice`
  - `projection_minimizes_reference`

References

[1] J. Washburn, *The Algebra of Aboutness: Reference as Cost-Minimizing Compression*, preprint (see `papers/tex/Algebra_of_Aboutness.tex`).