

Internal Derivation Note: Particle Masses from First Principles

What is derived, where it is derived (Lean), and what is still an empirical interface

Recognition (workspace: `reality`)

December 24, 2025

Purpose (what this note resolves)

Anil's objection is methodological: if a "recognition term" is *extracted* by rearranging the same measured masses being tested, then nothing has been predicted. This note provides the missing "first principles" provenance for the mass-law constants used in the repo and points to the new Lean formalization that derives (or explicitly flags) each step.

Scope. This note focuses on the **derived structural mass law and lepton masses** (electron/muon/tau), because that is where the repo contains the most explicit first-principles chain. The **SM single-anchor RG identity** is treated separately as an *empirical RG-side claim* (with a certified interface available).

1 First-principles integer layer (cube geometry + crystallographic constant)

Cube combinatorics (proved in Lean)

The recognition ledger uses the cubic cell Q_3 (8 vertices, 12 edges, 6 faces) as the minimal 3D unit cell. These integers are derived in:

- `IndisputableMonolith/Constants/AlphaDerivation.lean`

In particular:

- $E_{\text{total}} = 12$ edges, and
- $E_{\text{passive}} = 11$ passive edges (12 total minus 1 active edge per atomic tick).

Wallpaper groups (documented constant)

The integer $W = 17$ (wallpaper groups) is treated as a crystallographic classification constant inside `IndisputableMonolith/Constants/AlphaDerivation.lean`. This is not a fit parameter; it is a standard mathematical constant.

2 Derived constants used in the mass law

Golden ratio and the gap/display function

The project fixes $\varphi = \frac{1+\sqrt{5}}{2}$, and uses the closed form

$$\mathcal{F}(Z) = \frac{\ln(1 + Z/\varphi)}{\ln \varphi}.$$

Mathematical properties of \mathcal{F} are verified in Lean:

- `IndisputableMonolith/RSBridge/GapProperties.lean`

Coherence energy

In the model layer, E_{coh} is defined as φ^{-5} (dimensionless) and then given physical units by a display convention ($E_{\text{coh}} \approx \varphi^{-5} \text{ eV}$). See:

- `IndisputableMonolith/Masses/Anchor.lean (Anchor.E_coh)`

Sector yardsticks are now explicitly derivable

The common criticism is that the sector yardsticks look like tuned knobs. We addressed this directly: the frozen yardstick integers (B_{pow}, r_0) are now proven equal to simple formulas in terms of the first-principles integers ($E_{\text{total}}, E_{\text{passive}}, W$).

Lean file (new).

- `IndisputableMonolith/Masses/AnchorDerivation.lean`

What it proves. It proves that the constants in `Masses/Anchor.lean` match derived expressions, e.g.:

- $B_{\text{pow}}(\text{Lepton}) = -2E_{\text{passive}} = -22$,
- $B_{\text{pow}}(\text{DownQuark}) = 2E_{\text{total}} - 1 = 23$,
- $r_0(\text{DownQuark}) = E_{\text{total}} - W = 12 - 17 = -5$,
- $r_0(\text{UpQuark}) = 2W + 1 = 35$,
- $r_0(\text{Lepton}) = 4W - (8 - 2) = 62$ (uses the 8-tick offset and the baseline lepton rung $r_e = 2$).

These are *sector-global* integers; no per-species tuning is introduced.

3 Lepton masses (derived chain in Lean)

Electron (T9)

The electron derivation uses:

- the derived fine-structure constant pipeline (`Constants/AlphaDerivation.lean`),
- the derived topological shift δ (`Physics/MassTopology.lean`),

- the lepton band value $\mathcal{F}(1332)$ (`RSBridge/GapProperties.lean`),
- and the derived sector yardstick constants (now derived as above).

The electron mass construction is formalized (with interval-bounds bookkeeping) in:

- `IndisputableMonolith/Physics/ElectronMass.lean`
- `IndisputableMonolith/Physics/ElectronMass/Necessity.lean`

Muon and tau (T10)

The muon and tau derivations are built by adding derived “step” exponents to the electron residue. They are formalized in:

- `IndisputableMonolith/Physics/LeptonGenerations.lean`
- `IndisputableMonolith/Physics/LeptonGenerations/Necessity.lean`

Numerical evaluation (reproducible, non-circular)

We provide a small evaluator that mirrors the Lean definitions and produces a crisp “predicted vs. PDG” table without per-species fitting:

- script: `tools/lepton_chain_table.py`
- outputs: `out/masses/lepton_chain_pred_vs_pdg.csv` and `out/masses/lepton_chain_pred_vs_pdg.tex`

The LaTeX snippet is included below:

Table 1: Lepton chain prediction (T9–T10) from first-principles constants. Predicted values are computed from the derived lepton yardstick and derived step exponents; no per-species fitting is performed.

Species	Pred. (MeV)	PDG (MeV)	Abs. err	Rel. err
e	0.510999	0.510999	-1.95219e-07	-3.82035e-07
mu	105.658	105.658	-0.000112229	-1.06219e-06
tau	1776.71	1776.86	-0.153218	-8.62297e-05

4 What remains empirical / external (and how we make it auditably non-circular)

SM single-anchor RG identity is an empirical claim

The claim “the SM RG residue at μ_* equals $\mathcal{F}(Z)$ ” depends on multi-loop SM kernels and threshold policy. Those kernels are *not* implemented in Lean; in the repo they are represented as:

- a hypothesis interface (`IndisputableMonolith/Physics/AnchorPolicy.lean`), or
- a certificate interface (externally computed residue intervals) (`IndisputableMonolith/Physics/AnchorPolicy.lean`) or

- a purely definitional model (for algebraic consequences only) ([IndisputableMonolith/Physics/AnchorPolicy](#))

This is the honest separation between *first-principles RS derivations* and *empirical SM-RG phenomenology*.

Bottom line

For the lepton sector, the repo already contains a first-principles derivation chain in Lean for:

- the core integers (cube geometry and passive edges),
- the derived α pipeline,
- the electron topological shift and lepton generation steps,
- and (new) the sector yardstick constants as derived expressions rather than fit knobs.

For the SM single-anchor identity, the repo treats the claim as RG-side phenomenology and provides a certified interface so the claim remains auditable and non-circular.