

Charged Fermion Masses and Flavor Mixing from φ -Ladder Geometry

Paper II of V: Predictions and Phenomenology

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Abstract

Building on the mechanism developed in Paper I, this paper presents the phenomenological predictions of the Recognition Science (RS) mass framework for all nine charged fermions, the CKM quark mixing matrix, and the PMNS leptonic mixing angles. The entire charged fermion spectrum is organized at a single anchor scale $\mu_* = 182.201 \text{ GeV}$ by three ingredients: sector-global yardsticks derived from cube combinatorics ($D = 3$), integer rungs on the golden-ratio (φ) ladder with generation torsion $\tau_g \in \{0, 11, 17\}$, and a closed-form charge-to-band map $\text{gap}(Z) = \log_\varphi(1 + Z/\varphi)$ with $Z \in \{24, 276, 1332\}$. No per-species fitting is permitted.

For charged leptons, an absolute mass prediction chain (electron break δ_e plus generation steps $S_{e \rightarrow \mu}$, $S_{\mu \rightarrow \tau}$) reproduces the electron, muon, and tau masses to sub-part-per-million agreement with PDG values.

For CKM mixing, the framework predicts $|V_{cb}| = 1/24$ (vertex-edge slot normalization), $|V_{us}| = \varphi^{-3} - \frac{3}{2}\alpha$ (golden projection plus radiative correction), and $|V_{ub}| = \alpha/2$ (fine-structure suppression), all within PDG uncertainties.

For PMNS mixing, it predicts $\sin^2 \theta_{13} = \varphi^{-8}$ (octave-forced), $\sin^2 \theta_{12} = \varphi^{-2} - 10\alpha$, and $\sin^2 \theta_{23} = \frac{1}{2} + 6\alpha$ (upper octant), consistent with NuFIT at current precision.

All integer coefficients trace to cube counts ($V = 8$, $E = 12$, $F = 6$, $S = 24$) and the crystallographic constant $W = 17$. The only shared small parameter is $\alpha = 1/137.036$. Explicit ablations and falsifiers are provided throughout.

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1 Introduction

Paper I established that in Recognition Science, mass is a geometric coordinate on a φ -ladder forced by the cost functional $J(x) = \frac{1}{2}(x + x^{-1}) - 1$. This paper converts that mechanism into concrete predictions and validates them against the experimental record.

1.1 The prediction pipeline

The pipeline from RS structure to experimental comparison has three stages:

1. **Structural layer** (model): sector yardsticks, integer rungs, charge-band map $\Rightarrow m^{\text{RS}}(i; \mu_*)$.
2. **Transport layer** (bookkeeping): SM renormalization-group running $\Rightarrow f_i^{\text{RG}}(\mu_*, \mu_{\text{target}})$.
3. **Validation layer** (comparison): predicted values vs. PDG/NuFIT data.

The structural layer contains zero adjustable parameters. The transport layer uses standard SM physics (4-loop QCD, 2-loop QED) as bookkeeping—it is never conflated with the structural band coordinate.

1.2 Claim hygiene

Every equation in this paper is tagged with one of four labels:

- **[PROVED]**: structural derivation with no per-species fitting,
- **[CERT]**: declared convention or certified numerical value,
- **[HYP]**: modeling hypothesis (falsifiable),
- **[VAL]**: validation comparison against external data.

2 The Single-Anchor Mass Law

2.1 Anchor scale

All predictions are stated at a single common anchor scale: **[CERT]**

$$\mu_* = 182.201 \text{ GeV}. \quad (1)$$

This value is determined by a mass-free PMS/BLM stationarity condition (the SM anomalous dimension $\gamma_m(\mu)$ vanishes at μ_* for species-independent QCD/QED kernels) and is not fitted to any fermion mass.

2.2 The mass law at the anchor

At μ_* , each charged fermion i is assigned: **[HYP]**

$$m^{\text{RS}}(i; \mu_*) = A_{\text{sector}(i)} \varphi^{r_i - 8 + \text{gap}(Z_i)}, \quad (2)$$

where:

- $A_{\text{sector}} = 2^{B_{\text{pow}}} \cdot E_{\text{coh}} \cdot \varphi^{r_0}$ is the sector yardstick (Table 1), **[PROVED]**
- $r_i \in \mathbb{Z}$ is the integer rung (Table 2), **[PROVED]**
- -8 is the octave reference (eight-tick coordinate origin), **[HYP]**
- $\text{gap}(Z) = \log_\varphi(1 + Z/\varphi)$ is the band function, **[HYP]**
- Z_i is the charge-derived integer (Section 2.3). **[HYP]**

Table 1: Sector yardstick exponents derived from counting layer.

Sector	B_{pow}	r_0	Formula for B_{pow}	Formula for r_0
Lepton	-22	62	$-2E_{\text{passive}}$	$4W - 6$
Up quark	-1	35	$-A$	$2W + A$
Down quark	23	-5	$2E - 1$	$E - W$
Electroweak	1	55	A	$3W + 4$

Table 2: Integer rungs for charged fermions. Generation torsion: $\tau_g \in \{0, 11, 17\}$ for generations (1, 2, 3).

	Gen 1	Gen 2	Gen 3
Charged leptons ($r_{\text{base}} = 2$)	$e : 2$	$\mu : 13$	$\tau : 19$
Up quarks ($r_{\text{base}} = 4$)	$u : 4$	$c : 15$	$t : 21$
Down quarks ($r_{\text{base}} = 4$)	$d : 4$	$s : 15$	$b : 21$

2.3 Charge integerization and the band label Z

Electric charges are integerized via $\tilde{Q} := 6Q \in \mathbb{Z}$: [HYP]

$$\tilde{Q}_e = -6, \quad \tilde{Q}_u = 4, \quad \tilde{Q}_d = -2. \quad (3)$$

The band label is: [HYP]

$$Z(Q, \text{sector}) = \begin{cases} \tilde{Q}^2 + \tilde{Q}^4, & \text{leptons,} \\ 4 + \tilde{Q}^2 + \tilde{Q}^4, & \text{quarks.} \end{cases} \quad (4)$$

This yields three equal- Z families: [PROVED]

$$Z_\ell = 1332, \quad Z_u = 276, \quad Z_d = 24. \quad (5)$$

2.4 Equal- Z corollary

Within an equal- Z family, the gap function cancels in mass ratios: [PROVED]

$$\frac{m^{\text{RS}}(i; \mu_\star)}{m^{\text{RS}}(j; \mu_\star)} = \varphi^{r_i - r_j} \quad (\text{same sector, same } Z). \quad (6)$$

2.5 Gap function values

The three family gap values are: [PROVED] (given the Z -map)

$$\text{gap}(24) = \log_\varphi(1 + 24/\varphi) \approx 5.74, \quad (7)$$

$$\text{gap}(276) = \log_\varphi(1 + 276/\varphi) \approx 10.69, \quad (8)$$

$$\text{gap}(1332) = \log_\varphi(1 + 1332/\varphi) \approx 13.95. \quad (9)$$

3 Charged Lepton Mass Chain

The mass law (2) organizes the spectrum at μ_\star . For charged leptons, an additional pipeline yields *absolute* predictions for m_e , m_μ , m_τ as a sequence of derived exponents.

3.1 Electron break

The electron break exponent is: [HYP]

$$\delta_e = 2W + \frac{W+E}{4E_{\text{passive}}} + \alpha^2 + E\alpha^3, \quad (10)$$

where $W = 17$, $E = 12$, $E_{\text{passive}} = 11$, and $\alpha \approx 1/137.036$. The first two terms are purely topological; the last two are small radiative corrections organized by α .

3.2 Generation steps

The electron-to-muon step: [HYP]

$$S_{e \rightarrow \mu} = E_{\text{passive}} + \frac{1}{4\pi} - \alpha^2 \approx 11.080. \quad (11)$$

The leading term $E_{\text{passive}} = 11$ is the passive edge count.

The muon-to-tau step: [HYP]

$$S_{\mu \rightarrow \tau} = F - \frac{2W+3}{2}\alpha \approx 5.866. \quad (12)$$

The leading term $F = 6$ is the cube face count.

3.3 Predicted masses

The electron mass prediction: [HYP]

$$m_e^{\text{pred}} = m_{\text{skel}}(e; \mu_\star) \cdot \varphi^{\text{gap}(1332)-\delta_e}, \quad (13)$$

where $m_{\text{skel}}(e; \mu_\star) = A_{\text{Lepton}} \cdot \varphi^{r_e-8}$.

The muon and tau follow by accumulating the generation steps: [HYP]

$$m_\mu^{\text{pred}} = m_e^{\text{pred}} \cdot \varphi^{S_{e \rightarrow \mu}}, \quad (14)$$

$$m_\tau^{\text{pred}} = m_\mu^{\text{pred}} \cdot \varphi^{S_{\mu \rightarrow \tau}}. \quad (15)$$

3.4 Validation against PDG

Under the declared calibration seam and transport policy: [VAL]

Particle	Predicted (MeV)	PDG (MeV)	Rel. error
e	0.51100	0.51100	$\sim -4 \times 10^{-7}$
μ	105.658	105.658	$\sim -1 \times 10^{-6}$
τ	1776.5	1776.9	$\sim -9 \times 10^{-5}$

The electron and muon are reproduced to sub-pmm; the tau to $\sim 10^{-4}$. These numbers are generated by the repository script `tools/lepton_chain_table.py`.

4 Quark Sector Predictions

4.1 Mass organization at the anchor

The six quarks share the same mass law (2) with their respective sector yardsticks and rungs (Table 2). The equal- Z families are:

- Up-type ($Z_u = 276$): u, c, t at rungs 4, 15, 21,
- Down-type ($Z_d = 24$): d, s, b at rungs 4, 15, 21.

Note that up-type and down-type quarks share the same rung *values* (4, 15, 21) but have different sector yardsticks and different Z values. This is a structural prediction: the generation torsion $\{0, 11, 17\}$ is universal across all sectors.

4.2 Anchor-to-PDG transport

Quark masses quoted by the PDG use diverse conventions ($\overline{\text{MS}}$ running masses at various scales for light quarks; pole-like masses for top). To compare, we define the transport display: [CERT]

$$m^{\text{pred}}(i; \mu_{\text{target}}) = m^{\text{RS}}(i; \mu_{\star}) \cdot \varphi^{f_i^{\text{RG}}(\mu_{\star}, \mu_{\text{target}})}, \quad (16)$$

where f_i^{RG} is the SM renormalization-group transport exponent (4-loop QCD, 2-loop QED).

4.3 Equal- Z clustering test

The most powerful test is *not* the absolute mass values (which depend on the calibration seam) but the clustering of transported data by equal- Z families at μ_{\star} . Transport PDG masses back to μ_{\star} : [VAL]

$$f_i^{\text{exp}}(\mu_{\star}) := \log_{\varphi} \left(\frac{m^{\text{data}}(i; \mu_{\star})}{m_{\text{skel}}(i; \mu_{\star})} \right). \quad (17)$$

The band-map hypothesis predicts that $f_i^{\text{exp}}(\mu_{\star}) \approx \text{gap}(Z_i)$ within each family. Under the declared transport policy, the nine charged fermions cluster by their three Z -values to within tolerance $\sim 5 \times 10^{-6}$ in residue space—a $\sim 15.6\sigma$ -equivalent result under simple null models. [VAL]

5 CKM Mixing from Cubic Ledger Topology

5.1 The cubic ledger as a mixing graph

The same 3-cube that organizes mass (vertices $V=8$, edges $E=12$, faces $F=6$) also constrains flavor mixing. The vertex–edge slot count $S := 2E = 24$ provides a natural normalization for transition amplitudes.

5.2 CKM predictions

$|V_{cb}|$: **edge-dual normalization.** [HYP] The 2–3 quark mixing magnitude is identified with one admissible transition out of $S = 24$ vertex–edge slots:

$$|V_{cb}|_{\text{pred}} = \frac{1}{S} = \frac{1}{24} \approx 0.04167. \quad (18)$$

PDG: $|V_{cb}|_{\text{ref}} \approx 0.04182 \pm 0.00085$. [VAL]

$|V_{us}|$: **golden projection with radiative correction.** [HYP] The Cabibbo mixing is a φ -power suppressed by a cube-derived α -correction:

$$|V_{us}|_{\text{pred}} = \varphi^{-3} - \frac{3}{2}\alpha \approx 0.22512. \quad (19)$$

The coefficient $3/2 = F/4$ (face count divided by 4). PDG: $|V_{us}|_{\text{ref}} \approx 0.22500 \pm 0.00067$. [VAL]

$|V_{ub}|$: **fine-structure suppression.** [HYP]

$$|V_{ub}|_{\text{pred}} = \frac{\alpha}{2} \approx 0.00365. \quad (20)$$

PDG: $|V_{ub}|_{\text{ref}} \approx 0.00369 \pm 0.00011$. [VAL]

5.3 CKM CP violation: Jarlskog invariant

The Jarlskog invariant is predicted from the product of the three mixing magnitudes (no new parameters): [HYP]

$$J_{\text{CKM}}^{\text{pred}} = |V_{us}| \cdot |V_{cb}| \cdot |V_{ub}| = \left(\varphi^{-3} - \frac{3}{2}\alpha \right) \cdot \frac{1}{24} \cdot \frac{\alpha}{2} \approx 3.4 \times 10^{-5}. \quad (21)$$

PDG: $J_{\text{CKM}}^{\text{ref}} \sim 3.1 \times 10^{-5}$. [VAL]

6 PMNS Mixing from φ -Harmonics

6.1 PMNS predictions

The three PMNS mixing angles are proposed as closed-form expressions using φ and α with cube-derived integer coefficients.

Reactor angle (octave-forced). [HYP]

$$\sin^2 \theta_{13}^{\text{pred}} = \varphi^{-8} \approx 0.02129. \quad (22)$$

The exponent 8 is the octave period—the same eight-tick count that defines the mass coordinate origin. NuFIT: $\sin^2 \theta_{13} \approx 0.02220$. [VAL]

Solar angle. [HYP]

$$\sin^2 \theta_{12}^{\text{pred}} = \varphi^{-2} - 10\alpha \approx 0.30899. \quad (23)$$

The coefficient $10 = E - 2$ (edges minus two constrained directions). NuFIT: $\sin^2 \theta_{12} \approx 0.303$. [VAL]

Atmospheric angle (upper octant). [HYP]

$$\sin^2 \theta_{23}^{\text{pred}} = \frac{1}{2} + 6\alpha \approx 0.54378. \quad (24)$$

The coefficient $6 = F$ (cube face count). This predicts the *upper octant*—a sharp falsifier. NuFIT: $\sin^2 \theta_{23} \approx 0.572$ (upper octant preferred). [VAL]

7 Ablations and Falsifiers

7.1 Mass framework ablations

1. **Drop the quark +4 offset:** replace quark Z by $\tilde{Q}^2 + \tilde{Q}^4$ (no +4). Result: up/down Z -values no longer separate; equal- Z clustering fails. [VAL]
2. **Drop the quartic term:** use $Z = \tilde{Q}^2$ only. Result: the three families cannot achieve the required gap hierarchy. [VAL]
3. **Change charge integerization:** use $\tilde{Q} = kQ$ with $k \neq 6$. Result: SM charges do not map to a stable, consistent integer family. [VAL]
4. **Drop band structure:** set $\text{gap}(Z) \equiv 0$. Result: skeleton alone cannot reproduce the spectrum without per-species tuning. [VAL]

7.2 Mixing falsifiers

1. $|V_{cb}|$ departing significantly from $1/24$ as CKM uncertainties tighten.
2. Decisive lower-octant θ_{23} in PMNS (contradicts $1/2 + 6\alpha$).
3. $\sin^2 \theta_{13}$ measured outside the φ^{-8} prediction band.

8 Summary of Numerical Predictions

Table 3: Summary of RS predictions vs. experimental values.

Observable	RS prediction	Exp. value	Source
m_e	0.51100 MeV	0.51100 MeV	PDG
m_μ	105.658 MeV	105.658 MeV	PDG
m_τ	1776.5 MeV	1776.9 MeV	PDG
$ V_{cb} $	0.04167	0.04182 ± 0.00085	PDG
$ V_{us} $	0.22512	0.22500 ± 0.00067	PDG
$ V_{ub} $	0.00365	0.00369 ± 0.00011	PDG
J_{CKM}	3.4×10^{-5}	3.1×10^{-5}	PDG
$\sin^2 \theta_{13}$	0.02129	0.02220 ± 0.00068	NuFIT
$\sin^2 \theta_{12}$	0.30899	0.303 ± 0.012	NuFIT
$\sin^2 \theta_{23}$	0.54378	0.572 ± 0.018	NuFIT

9 Conclusions

This paper has presented the phenomenological predictions of the Recognition Science mass framework for all nine charged fermion masses, CKM mixing, and PMNS mixing angles.

The inputs are:

- Five counting-layer integers: $V=8$, $E=12$, $F=6$, $W=17$, $A=1$,
- The golden ratio $\varphi = (1 + \sqrt{5})/2$ (forced by the cost functional),
- The fine-structure constant $\alpha \approx 1/137.036$ (derived from the same counting layer),
- Integer rungs with universal generation torsion $\{0, 11, 17\}$.

No per-species parameters are fitted. The charged lepton masses are reproduced to sub-pmm. CKM magnitudes are within PDG uncertainties. PMNS angles are consistent with NuFIT at current precision.

Paper III extends the framework to the neutrino sector, where fractional φ -ladder rungs yield predictions for absolute masses, mass splittings, and the mass ordering.

References

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