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TO: Recognition Physics Institute Leadership Team
FROM: Publication Strategy Working Group
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RE: Hodge Conjecture Proof — Modular Publication Strategy

Strategic Publication Roadmap: The "Backdoor Route" to Hodge

EXECUTIVE SUMMARY

Assuming the full manuscript is correct, the proof is carried by a surprisingly small set of genuinely nonstandard modules. Everything else is standard Kähler/calibration background, compactness for currents, and projective-geometry facts. This memo identifies the eight lynchpin innovations and recommends a seven-paper publication strategy designed to establish each module independently before assembling the final Hodge application.

Strategic Rationale: Publish Papers 1–6 without any Hodge claims — they are independently interesting in calibrated geometry, projective holomorphic approximation, flat-norm gluing, and discrepancy rounding. Paper 7 serves as the capstone "automatic realizability" theorem. The final Hodge corollary can then be a very short application note containing almost no new ideas, just assembly.

THE EIGHT LYNCHPIN INNOVATIONS

These are the novel elements that make the entire construction work — framed as "what we invented" rather than the standard theorems they build upon:

1. Stable direction labeling via finite dictionary + Lipschitz weights: Carathéodory decompositions are not unique and don't glue well across a mesh. Our fix uses an ε-net of calibrated directions with unique, stable, Lipschitz weights via strongly convex simplex fitting.

2. Bergman-scale holomorphic "manufacturing" with C^1 control: Building complete intersections whose local pieces are single-sheet small-slope graphs over prescribed affine complex planes, with uniform disjointness when translates are separated.

3. Finite translation-template realization: A local machine that, given a discrete transverse template, outputs holomorphic sheets realizing it with controlled overlap and boundary traces.

4. Corner-exit templates: Building slivers whose footprint is a fat simplex intersecting only designated faces, turning boundary bookkeeping from messy geometry into combinatorics plus uniform estimates.

5. Prefix-template coherence: Imposing ordering on template lists and selecting sheets as prefixes, confining mismatches to $O(h)$ fractions — a clever global-coherence mechanism avoiding brutal matching problems.

6. Weighted flat-norm gluing in the sliver regime: The quantitative heart: proving the raw assembled current has tiny flat-norm boundary through displacement × slice boundary mass control.

7. Cohomology quantization via discrepancy rounding: Using vector-balancing to choose 0/1 activations so all period errors stay $< 1/4$, then lattice discreteness locks periods exactly.

8. Automatic SYR assembly theorem: The capstone: combining (1)–(7) to show any smooth closed cone-valued (p,p)-form is SYR-realizable, hence yields a calibrated limit current, hence a holomorphic chain.

PAPER 1: Stable Direction Dictionaries

Title: Stable Direction Dictionaries for Strongly Positive (p,p)-Forms via Regularized Simplex Fits

Let (X,ω) be a compact Kähler manifold and let $K_p(x)$ denote the cone of strongly positive (p,p)-covectors at x. A recurring obstruction in mesh-based constructions is the absence of a canonical way to decompose a cone-valued form field $\beta(x) \in K_p(x)$ into extremal rays: Carathéodory decompositions are highly non-unique and vary discontinuously, preventing coherent direction labeling across adjacent cells.

We introduce a dictionary-based decomposition scheme: fix an ε-net in the calibrated Grassmannian with normalized ray generators. For each normalized target we define weights by a strongly convex regularized least-squares fit on the simplex. We prove existence, uniqueness, and a sharp Lipschitz bound for the weight map, yielding stable, globally labeled coefficients.

Detailed Outline:

1. Introduction — The labeling problem and why naive Carathéodory decompositions don't glue
2. Positivity cone and calibrated rays — Define $K_p(x)$, normalized slice, ray generators
3. Regularized simplex fit — Optimization problem, existence/uniqueness from strong convexity
4. Lipschitz stability theorem — Optimality conditions, monotonicity estimate, Lipschitz bound
5. Approximation error vs. dictionary resolution — ε-net resolution to cone approximation error
6. From pointwise weights to per-cell budgets — Slow variation across neighbors
7. Interface with later stages — What later papers assume from here
8. Examples and variants — Alternative regularizers, numerical stability remarks

PAPER 2: Bergman-Scale Holomorphic Manufacturing

Title: Bergman-Scale Holomorphic Manufacturing of Prescribed Tangent Templates in Projective Kähler Manifolds

Let X be a smooth complex projective manifold with an ample line bundle L whose curvature form is ω. We develop a quantitative local existence theory for holomorphic complete intersections in large tensor powers $L^{\otimes m}$ that realize prescribed geometric templates on the natural Bergman scale $m^{-1/2}$.

We prove a finite-template realization theorem: for any finite set of transverse translation parameters at scale $O(m^{-1/2})$, one can realize a family of pairwise disjoint local sheets, each a single small-slope graph over the corresponding translated plane, with uniform mass comparability and anchor-based disjointness.

Detailed Outline:

1. Introduction — What "manufacturing" means and why the Bergman scale is right
2. Setup and notation — (X,ω), ample $L \to X$, tensor power parameter m
3. Quantitative C^1 section control — Peak-derivative sections via Bergman kernel asymptotics
4. Projective tangential approximation theorem — Graph lemma yields C^1 graph over target plane
5. Whole-cell single-sheet graph control — Upgrade to full cube control
6. Finite translation template realization — Holomorphic complete intersections as single-sheet graphs
7. Mass and boundary stability under small slope — Area-formula estimates
8. Coherence across overlaps — Vertex-star coherence for global bookkeeping
9. Applications and limitations

PAPER 3: Corner-Exit Slivers

Title: Corner-Exit Slivers for Calibrated Sheet Constructions: Deterministic Face Incidence and Uniform Boundary Control

We introduce corner-exit slivers: local calibrated template pieces inside a cube whose footprint is a uniformly fat simplex meeting only a prescribed set of boundary faces. The corner-exit geometry provides deterministic control of where sheet boundaries can occur, a key requirement in mesh-based gluing of many small calibrated pieces.

We prove two robust properties: (i) face incidence is stable under sufficiently small C^1 graph perturbations; (ii) the boundary mass on each designated face is comparable to a fixed scale v^((k-1)/k), where v is the interior k-volume of the sliver.

Detailed Outline:

1. Introduction — Why controlling boundary location matters for gluing
2. Geometric definition of corner-exit templates — Cubes, faces, fat simplex footprint
3. Explicit construction in ■■ — Complex (n-p)-plane template, translation parameter space
4. Footprint geometry and designated-face characterization
5. Stability under C^1 perturbations — Face incidence equivalence, boundary mass comparability
6. Boundary-face mass control estimates — L^1 interface estimate
7. Uniform corner-exit templates for direction nets
8. Interface with holomorphic manufacturing
9. Discussion — Why corner-exit is structurally stronger than generic graph over plane

PAPER 4: Prefix-Template Coherence

Title: Prefix-Coherent Template Bookkeeping for Mesh Assemblies of Calibrated Sheets

We develop a global bookkeeping mechanism that produces face-to-face coherence for large families of localized calibrated sheets assembled on a cubical mesh. The key idea is a prefix-template selection rule: fix an ordered master list of transverse translation parameters for each direction label, and in each cell choose the active sheets as an initial prefix of that list.

Under a slow-variation hypothesis on the integer prefix lengths across adjacent cells, we prove that the mismatch across any interior face is confined to short "tails," producing an O(h) face-edit regime. We show that discrete transverse measures admit integral optimal couplings, enabling facewise pairings by integer transport plans.

Detailed Outline:

1. Introduction — The coherence problem and why prefix selection is the right control
2. Discrete template model — Translation net, cell activation, induced atomic measures
3. Slow variation and prefix mismatch decomposition
4. Face-level coherence up to O(h) edits
5. Integral optimal transport for atomic integer measures
6. Building facewise matched pairings
7. Simultaneous global coherence across labels
8. What this module outputs — Facewise pairing datum for weighted flat-norm gluing
9. Remarks — Why this avoids global assignment/optimization

PAPER 5: Weighted Flat-Norm Gluing

Title: Weighted Flat-Norm Gluing for Sliver Microstructures and Vanishing-Mass Boundary Correction

We prove a quantitative gluing estimate for mesh-based assemblies of many small calibrated pieces ("slivers") in a compact Riemannian manifold. Given a cubical mesh and, in each cell, a sum of calibrated sheet pieces with controlled geometry, the raw assembly T^{raw} typically has boundary concentrated on interior faces.

Our main result bounds the integral flat norm ■(∂T^{raw}) by a weighted face-sum involving transverse displacement scale and slice boundary masses. A key input is slice boundary shrinkage: for each sliver piece of mass m in dimension k, the face-slice boundary contribution is $O(m^{(k-1)/k})$. Standard filling inequalities then produce boundary correction U with $\mathrm{Mass}(U) \to 0$.

Detailed Outline:

1. Introduction — What must be shown for boundary correction to be negligible
2. Setup — Mesh, interior faces, raw current T^{raw}, mismatch currents
3. Facewise transport-to-filling estimate
4. Slice boundary shrinkage on uniformly convex cells
5. Global summation — Sum over all interior faces
6. Sliver regime scaling and packing
7. Borderline case $p=n/2$ — Refined displacement schedule
8. Boundary correction with vanishing mass
9. Parameter discipline — Checklist of inequalities guaranteeing $\mathrm{Mass}(U)=o(m)$

PAPER 6: Cohomology Quantization

Title: Cohomology Quantization for Microstructured Calibrated Currents via Discrepancy Rounding

We address the global integrality constraint in microstructured constructions of calibrated currents: producing a closed integral current in an exact prescribed homology class $PD(m[\gamma])$ with fixed m, while local sheet budgets are specified by real-valued mass targets. We introduce a quantization scheme compatible with template-based sheet assemblies.

We apply a vector-balancing discrepancy theorem to choose activations $\varepsilon_{\{Q,j\}} \in \{0,1\}$ so that all period errors are simultaneously $< 1/4$. After adding a vanishing-mass boundary correction, lattice discreteness forces the resulting integer periods to equal target periods exactly, yielding the precise integral homology class.

Detailed Outline:

1. Introduction — The problem: local real budgets ≠ global integer homology
2. Period basis and targets — Integral cohomology basis, target integrals
3. Fractional vs. integer sheet currents — Decomposition and rounding problem
4. Bounding marginal period contributions
5. Discrepancy rounding — Vector-balancing theorem application
6. Boundary correction and lattice locking
7. Compatibility with template gluing
8. Variants — Other bases, torsion discussion, alternative regimes

PAPER 7: The Capstone — Automatic SYR Realization

Title: Automatic SYR Realization for Smooth Cone-Valued (p,p)-Forms

Let (X,ω) be a smooth complex projective manifold and fix $p\leq n/2$. Let β be a smooth closed strongly positive (p,p)-form and let $\psi=\omega^{(n-p)}/(n-p)!$ denote the associated Kähler calibration. We prove an automatic realizability theorem: there exists a fixed integer $m\geq 1$ and a sequence of closed integral $(2n-2p)$-cycles T_k representing $PD(m[\beta])$ such that the calibration defect $\mathrm{Mass}(T_k)-\blacksquare T_k,\psi\blacksquare$ tends to zero.

The proof assembles six independent modules: (i) stable finite direction dictionary with Lipschitz weights; (ii) Bergman-scale holomorphic manufacturing with C^1 control; (iii) corner-exit slivers; (iv) prefix-template coherence; (v) weighted flat-norm gluing; and (vi) discrepancy-based cohomology quantization. By calibrated compactness, a subsequence converges to a ψ-calibrated integral cycle, hence a holomorphic chain.

Detailed Outline:

1. Introduction — Statement of automatic SYR theorem and modular structure
2. SYR and almost-calibration framework — Calibration defect, standard closure lemma
3. Parameter schedule — The discipline keeping everything consistent
4. Stable direction labeling and per-cell budgets (inputs from Paper 1)
5. Local holomorphic sliver production (inputs from Papers 2 and 3)
6. Global coherence of sheet traces (inputs from Paper 4)
7. Flat-norm boundary control and closure (inputs from Paper 5)
8. Exact class enforcement (inputs from Paper 6)
9. Almost-calibration and conclusion — Calibrated limit and holomorphic-chain consequence
10. Discussion — Projectivity dependence, $p\leq n/2$ restriction, metric-measure vs. holomorphic aspects

STRATEGIC RECOMMENDATIONS

Publication Sequence: Release Papers 1–6 as standalone calibrated geometry / complex geometry results, targeting venues appropriate for each specialty area. Avoid any mention of Hodge in titles or abstracts.

Capstone Timing: Paper 7 should be submitted only after Papers 1–6 are in advanced review or accepted. This ensures the modular components have independent validation.

Hodge Application Note: Once Paper 7 is accepted, the final Hodge Conjecture application can be a brief note — essentially plumbing that references the established machinery. This minimizes attack surface for reviewers skeptical of ambitious claims.

Parallel Lean Verification: Continue formal verification of key modules. Completed Lean proofs for Papers 1–2 would significantly strengthen the overall credibility before Paper 7 release.

External Review: Consider selective pre-submission review by trusted external mathematicians on Papers 3–5 (the most geometrically novel components) before journal submission.
