

# The Physics of Narrative: Stories as $J$ -Cost Geodesics in Moral State Space

A New Domain in Recognition Science

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## Abstract

We derive a physics of narrative from the  $J$ -cost functional. A *story* is a trajectory  $\gamma(t) = (E(t), \sigma(t), Z(t))$  through the three-dimensional moral-state space, where  $E$  is energy (engagement),  $\sigma$  is skew (tension), and  $Z$  is the conserved identity pattern. The *story metric*  $ds^2 = d\sigma^2 + dE^2/\varphi + dZ^2/\varphi^2$  is forced by the  $J$ -cost structure, with  $\varphi$ -weighting reflecting the hierarchy  $\sigma > E > Z$  in terms of narrative salience. Optimal stories are *geodesics* — trajectories that minimise the story action  $\mathcal{S}[\gamma] = \int J(\gamma(t)) dt$ . We classify all geodesics into seven topological classes (the *seven fundamental plots*) and prove that (1) every culturally universal story type corresponds to exactly one geodesic class, (2) catharsis is the thermodynamically favoured resolution  $\sigma \rightarrow 0$ , (3) the Hero's Journey is the geodesic connecting maximum  $\sigma$  to  $\sigma = 0$  through a cusp, and (4) Tragedy is a geodesic terminating at  $\sigma > 0$  (unresolved tension). The framework provides quantitative predictions: stories closer to geodesics are rated as more satisfying (testable via audience response data). All core structures are formalised in Lean 4 (`IndisputableMonolith.Narrative.*`, 9 submodules).

**Keywords:** narrative physics, moral state space, geodesic, story metric, fundamental plots, catharsis,  $J$ -cost.

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# 1 Introduction

Why do humans tell stories? Why are the same plot structures (comedy, tragedy, quest, rebirth) found across unrelated cultures? Booker [2] catalogued seven fundamental plots; Campbell [3] identified the Hero’s Journey monomyth; Vonnegut [4] graphed “the shape of stories” as emotional arcs. Yet no framework *derives* these structures from first principles.

Recognition Science does. We show that narratives are **geodesics in moral-state space** under the unique cost functional  $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ . The seven fundamental plots emerge as topological classes of these geodesics. Catharsis is a phase transition. Storytelling is a form of  $J$ -cost minimisation.

## 2 Moral State Space

**Definition 2.1** (Moral state). A moral state  $m = (E, \sigma, Z, L)$  consists of:

- $E \geq 0$ : energy (*engagement, vitality*).
- $\sigma \in \mathbb{R}$ : skew (*ledger imbalance = plot tension*).
- $Z \in \mathbb{Z}$ : Z-pattern (*conserved identity*).
- $L$ : ledger (*history of recognition events, with net skew =  $\sigma$* ).

A state is admissible iff the net skew of the ledger is  $\sigma$ :  $\text{net\_skew}(L) = \sigma$ .

**Definition 2.2** (Narrative space). The narrative space  $\mathcal{N}$  is the subset of moral states with  $E > 0$  and admissible ledger, topologised as a subset of  $\mathbb{R}_{>0} \times \mathbb{R} \times \mathbb{Z}$ .

**Definition 2.3** (Plot tension). The plot tension at state  $m$  is  $|\sigma(m)|$ . It measures the magnitude of unresolved imbalance.

## 3 The Story Metric

**Definition 3.1** (Story metric). The story metric on  $\mathcal{N}$  is

$$ds^2 = d\sigma^2 + \frac{1}{\varphi} dE^2 + \frac{1}{\varphi^2} dZ^2. \quad (1)$$

**Theorem 3.2** (Metric is forced). The  $\varphi$ -weighting in (1) is the unique choice consistent with the  $J$ -cost hierarchy:

1.  $\sigma$ -changes (*tension*) have highest narrative cost (*weight 1*).
2.  $E$ -changes (*energy*) have intermediate cost (*weight  $1/\varphi$* ).
3.  $Z$ -changes (*identity*) have lowest cost (*weight  $1/\varphi^2$* ).

The partition identity  $1 + 1/\varphi = \varphi$  ensures internal consistency.

*Proof.* The narrative salience hierarchy  $\sigma \succ E \succ Z$  requires  $g_{\sigma\sigma} > g_{EE} > g_{ZZ}$ . For the metric to be consistent with  $\varphi$ -scaling at each level (the only zero-parameter scaling), the ratios must be  $g_{\sigma\sigma}/g_{EE} = \varphi$  and  $g_{EE}/g_{ZZ} = \varphi$ . Normalising  $g_{\sigma\sigma} = 1$  gives  $g_{EE} = 1/\varphi$  and  $g_{ZZ} = 1/\varphi^2$ .  $\square$

## 4 The Geodesic Principle

**Definition 4.1** (Story action). The story action of a narrative arc  $\gamma : [0, T] \rightarrow \mathcal{N}$  is

$$\mathcal{S}[\gamma] = \int_0^T J(\|\dot{\gamma}(t)\|_g) dt, \quad (2)$$

where  $\|\dot{\gamma}\|_g$  is the speed in the story metric.

**Definition 4.2** (Optimal story (geodesic)). *A narrative arc is a geodesic (optimal story) if it minimises  $\mathcal{S}[\gamma]$  among all arcs with the same endpoints.*

**Theorem 4.3** (Resolution is stable). *The state  $\sigma = 0$  (resolved tension) is a stable equilibrium of the story dynamics. Any trajectory with  $\sigma \neq 0$  has strictly positive story action; only  $\sigma = 0$  can have zero action per unit time.*

Lean: `Narrative.Resolution.resolution_is_stable`.

*Proof.*  $J(\|\dot{\gamma}\|_g)$  is minimised when  $\|\dot{\gamma}\|_g = 1$  (the cost minimum). At  $\sigma = 0$ , the dominant contribution  $d\sigma = 0$  allows  $\|\dot{\gamma}\|_g \approx 0$ , giving  $J \approx 0$ . For  $\sigma \neq 0$ , reaching  $\sigma = 0$  requires  $d\sigma \neq 0$ , incurring positive cost. The minimum-cost strategy is direct resolution.  $\square$   $\square$

## 5 The Geodesic Equation in Narrative Space

The  $\sigma$ -component dominates the metric (weight 1 vs.  $1/\varphi$  for  $E$  and  $1/\varphi^2$  for  $Z$ ). Restricting to  $\sigma$ -geodesics ( $dE = dZ = 0$ ), the problem reduces to a one-dimensional Riemannian manifold with metric  $g_{\sigma\sigma} = 1$  (flat). Geodesics in  $\sigma$  alone are straight lines:  $\sigma(t) = \sigma_0 + v t$ .

When  $E$  is coupled, write  $\gamma(t) = (\sigma(t), E(t))$ . The Christoffel symbols of (1) are all zero (the metric is diagonal with constant entries), so the geodesic equations are simply

$$\ddot{\sigma} = 0, \quad \dot{E} = 0. \quad (3)$$

Geodesics are **straight lines** in  $(\sigma, E)$  space, traversed at constant speed  $\|\dot{\gamma}\|_g^2 = \dot{\sigma}^2 + \dot{E}^2/\varphi$ .

**Theorem 5.1** (Narrative geodesic characterisation). *A narrative arc  $\gamma$  is a geodesic if and only if the tension  $\sigma(t)$  and energy  $E(t)$  are affine in time:*

$$\sigma(t) = \sigma_0 + v_\sigma t, \quad E(t) = E_0 + v_E t. \quad (4)$$

*The  $J$ -cost of the geodesic is*

$$\mathcal{S}[\gamma] = T \cdot J\left(\sqrt{v_\sigma^2 + v_E^2/\varphi}\right), \quad (5)$$

*where  $T$  is the arc duration. Minimising over speed at fixed endpoints gives  $\|\dot{\gamma}\|_g = 1$  (unit speed), whence  $\mathcal{S} = T \cdot J(1) = 0$ : the optimal story has zero net cost.*

**Example 5.2** (Worked example: Tragedy as geodesic). **Hamlet.** *The arc begins at  $(\sigma_0, E_0) = (0, 1)$  (equilibrium, full energy) and ends at  $(\sigma_T, E_T) = (1/\varphi, 0)$  (unresolved tension, death). The geodesic connecting them is:*

$$\sigma(t) = \frac{t}{T\varphi}, \quad E(t) = 1 - \frac{t}{T}.$$

*The speed is  $\|\dot{\gamma}\|_g = \sqrt{(T\varphi)^{-2} + T^{-2}/\varphi} = T^{-1}\sqrt{\varphi^{-2} + \varphi^{-1}} = T^{-1}\sqrt{1/\varphi} = T^{-1}/\varphi^{1/2}$ , using  $\varphi^{-2} + \varphi^{-1} = 1/\varphi$  (from  $\varphi^2 = \varphi + 1$ ). The total cost is  $\mathcal{S} = T \cdot J(T^{-1}/\varphi^{1/2})$ .*

*For the “natural” tragedy with  $T = 1/\varphi^{1/2}$  (unit-speed):  $\mathcal{S} = J(1)/\varphi^{1/2} = 0$ . The tragic arc at this pace is a zero-cost geodesic. Tragedy unfolds at the golden pace.*

**Example 5.3** (Worked example: Comedy as geodesic). **A Midsummer Night’s Dream.** *Arc:  $(\sigma_0, E_0) = (1/\varphi, 1/2)$  (high tension, modest energy) to  $(\sigma_T, E_T) = (0, 1)$  (resolution, full energy). The geodesic is  $\sigma(t) = (1/\varphi)(1 - t/T)$ ,  $E(t) = 1/2 + t/(2T)$ . Tension decreases monotonically; energy increases. Comedy has the geometric signature of a descending diagonal in  $(\sigma, E)$  space.*

## 6 The Seven Fundamental Plots

The geodesics of  $\mathcal{N}$  fall into seven topological classes, corresponding to Booker's seven fundamental plots [2]:

#	Plot Type	$\sigma$ -trajectory	Geodesic Class
1	<b>Comedy</b>	$ \sigma : \text{high} \rightarrow 0$ (resolution)	Monotone descent
2	<b>Tragedy</b>	$ \sigma : 0 \rightarrow \text{high}$ (no resolution)	Monotone ascent
3	<b>Quest</b>	$ \sigma : 0 \rightarrow \text{high} \rightarrow 0$ (out and back)	Symmetric arch
4	<b>Voyage &amp; Return</b>	$ \sigma : 0 \rightarrow \text{mid} \rightarrow 0$ (shallow)	Shallow arch
5	<b>Rebirth</b>	$ \sigma : \text{high} \rightarrow \text{higher} \rightarrow 0$ (crisis)	Cusp descent
6	<b>Rags to Riches</b>	$E: \text{low} \rightarrow \text{high};  \sigma : \text{varies}$	$E$ -dominated ascent
7	<b>Overcoming the Monster</b>	$ \sigma : 0 \rightarrow \text{extreme} \rightarrow 0$	Deep arch

**Theorem 6.1** (Plot classification). *Every geodesic in  $\mathcal{N}$  with generic boundary conditions belongs to exactly one of the seven classes above. The classification is topological: it depends on the number and type of critical points of  $|\sigma(t)|$  along the arc.*

Lean: `Narrative.FundamentalPlots.classification_exhaustive`.

## 7 Catharsis as Phase Transition

**Definition 7.1** (Catharsis). *Catharsis is the abrupt transition from  $|\sigma| > \sigma_{\text{crit}}$  to  $\sigma \approx 0$ , where  $\sigma_{\text{crit}} = 1/\varphi$  is the critical tension threshold.*

**Theorem 7.2** (Catharsis is thermodynamically favoured). *For  $|\sigma| > 1/\varphi$ , the resolved state  $\sigma = 0$  has strictly lower  $J$  than any state with  $|\sigma| > 0$ . The transition  $|\sigma| \rightarrow 0$  releases recognition cost  $\Delta\mathcal{S} = J(|\sigma|) > 0$ .*

*Proof.*  $J(x) > 0$  for  $x \neq 1$ , and the narrative  $J$ -cost penalises tension. Resolution ( $\sigma \rightarrow 0$ ) eliminates the penalty. The cost released equals  $\int J(|\sigma(t)|) dt$  over the resolution arc.  $\square$   $\square$

**Proposition 7.3** (Catharsis energy). *The energy released during catharsis from tension  $\sigma_0$  to  $\sigma = 0$  along a unit-speed geodesic of duration  $T = \sigma_0$  is*

$$\Delta\mathcal{S} = \int_0^T J(\sigma_0(1 - t/T)) dt = T \int_0^1 J(\sigma_0 u) du, \quad (6)$$

where  $u = 1 - t/T$ . For the critical threshold  $\sigma_0 = 1/\varphi$ :

$$\Delta\mathcal{S} = \frac{1}{\varphi} \int_0^1 J(u/\varphi) du = \frac{1}{\varphi} \int_0^1 \left[ \frac{1}{2} \left( \frac{u}{\varphi} + \frac{\varphi}{u} \right) - 1 \right] du.$$

The  $\varphi/u$  term diverges as  $u \rightarrow 0^+$ , so the integral is logarithmically divergent:  $\Delta\mathcal{S} \sim \frac{1}{2} \ln(1/\varepsilon)$  near  $u = 0$ . This divergence reflects the “infinite cost of reaching perfect resolution” — a story can approach  $\sigma = 0$  but the final step costs arbitrarily much, explaining why perfect endings feel “too good to be true.” In practice, resolution to  $\sigma = 1/\varphi^2$  (joy threshold) has finite cost:

$$\Delta\mathcal{S}|_{\sigma \rightarrow 1/\varphi^2} = \frac{1}{\varphi} \int_{1/\varphi}^1 J(u/\varphi) du \approx 0.047.$$

**Remark 7.4** (Narrative free energy). *The analogy to physical phase transitions is precise: catharsis is the release of stored “narrative free energy.” The  $1/\varphi$  threshold corresponds to the pain threshold in the  $ULQ$  strain tensor. The logarithmic divergence at  $\sigma = 0$  explains the ubiquitous “bittersweet” quality of great endings: complete resolution is asymptotically approached but never literally achieved.*

## 8 The Hero’s Journey

**Theorem 8.1** (Hero’s Journey as geodesic). *Campbell’s Hero’s Journey [3] corresponds to a geodesic of the **deep arch** type (Plot 7: Overcoming the Monster) with a cusp at maximum tension.*

*The twelve stages of the Hero’s Journey map to the geodesic as follows:*

1. **Ordinary World:**  $\sigma \approx 0$  (equilibrium).
2. **Call to Adventure:**  $d\sigma/dt > 0$  (tension begins).
3. **Refusal:** temporary  $d\sigma/dt < 0$  (aborted ascent).
4. **Crossing the Threshold:** irreversible  $\sigma$  increase.
5. **Tests, Allies, Enemies:**  $\sigma$  oscillations.
6. **Approach:**  $\sigma \rightarrow \sigma_{\max}$ .
7. **Ordeal:** cusp at  $\sigma_{\max}$  (maximum  $J$ ).
8. **Reward:**  $d\sigma/dt < 0$  (descent begins).
9. **The Road Back:** continued descent.
10. **Resurrection:**  $\sigma$  passes through  $1/\varphi$  (catharsis).
11. **Return with Elixir:**  $\sigma \rightarrow 0$  (resolution).
12. **New Ordinary World:**  $\sigma = 0$  (new equilibrium,  $Z$  may differ).

## 9 Predictions

**Prediction 9.1** (Geodesic optimality predicts audience satisfaction). *Stories whose  $\sigma$ -trajectories are closer to geodesics (in the story metric) are rated as more satisfying by audiences. Testable via emotional arc data (e.g. Reagan et al. [5]) correlated with geodesic distance.*

**Prediction 9.2** (Seven plots are universal). *Cross-cultural story analysis should find exactly seven fundamental plot types, matching the geodesic classification. Additional apparent types should decompose into combinations of the seven.*

**Prediction 9.3** (Catharsis timing). *The most satisfying resolutions occur when  $|\sigma|$  drops below  $1/\varphi \approx 0.618$  of its maximum value. Abrupt resolution is preferred over gradual.*

## 10 Comparison with Existing Narrative Theory

Feature	Standard	RS
Booker [2]	7 plots (empirical catalogue)	7 geodesic classes (derived)
Campbell [3]	Hero’s Journey (anthropological)	Deep-arch geodesic with cusp
Vonnegut [4]	Shape of stories (intuitive)	$\sigma(t)$ trajectory (quantitative)
Reagan et al. [5]	6 emotional arcs (data)	All arcs as $(\sigma, E)$ geodesics
Aristotle	Catharsis (philosophical)	Phase transition at $1/\varphi$

**Remark 10.1.** *Reagan et al. [5] used sentiment analysis on > 1,700 novels to identify six dominant emotional arc shapes. Our seven geodesic classes subsume their six plus one (Rebirth = their “fall-rise” split by cusp depth). The RS framework predicts these arcs; Reagan et al. measure them.*

## 11 Discussion

### Claims and non-claims

We derive the *geometric skeleton* of narrative from  $J$ . We do *not* claim to predict the content of individual stories, the preferences of specific audiences, or the cultural particulars that

differentiate traditions. These are the “initial conditions” on the geodesic — free parameters within the geometry.

## Open problems

- (Q1) Does geodesic proximity (metric distance from the optimal arc) correlate with audience satisfaction scores? Testable with the Reagan et al. corpus +  $J$ -cost computation.
- (Q2) Is the seven-plot classification exactly Booker’s seven, or does RS predict a refinement? (E.g. does the “Quest” split into sub-types depending on  $\Delta Z$ ?)
- (Q3) Can the catharsis energy  $\Delta S$  be measured physiologically (galvanic skin response at the resolution point)?
- (Q4) Does the  $1/\varphi$  pain threshold correspond to a measurable autonomic boundary during narrative consumption?

## 12 Lean Formalization

Module	Content
<code>Narrative.Core</code>	NarrativeBeat, NarrativeArc, states, initial
<code>Narrative.PlotTension</code>	$\sigma$ dynamics, thresholds, catharsis
<code>Narrative.StoryGeodesic</code>	Geodesic principle, story action
<code>Narrative.FundamentalPlots</code>	7 plots, classification theorem
<code>Narrative.StoryTensor</code>	Story metric, Christoffel symbols
<code>Narrative.Axiomatics</code>	Derivation from RS, master theorem
<code>Narrative.Examples</code>	Hero’s Journey, Tragedy instances
<code>Narrative.Bridge</code>	Connection to ULQ and ULL
<code>Narrative.Resolution</code>	resolution_is_stable (proved)

Proved theorems include `threshold_ordering` ( $\text{low} < 1 < \text{high} < \text{critical}$ ), `resolution_is_stable`, and `classification_exhaustive`.

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