Geometric mass residue versus Standard-Model RG transport: separation, no-go, and Lean-certified gap properties

Jonathan Washburn*

Abstract

An earlier "single-anchor" draft (retained in this file for provenance) implicitly conflated two distinct quantities: a closed-form geometric residue

$$\mathscr{F}(Z) = \frac{\ln(1 + Z/\varphi)}{\ln \varphi}$$

and the Standard-Model running-mass transport residue

$$f_i^{\mathrm{RG}}(\mu_\star, \mu) := \frac{1}{\ln \varphi} \int_{\ln \mu_\star}^{\ln \mu} \gamma_m^{\mathrm{SM}}(\mu') \, d\ln \mu',$$

where γ_m^{SM} is the usual QCD/QED/EW mass anomalous dimension in $\overline{\mathrm{MS}}$. This manuscript records the corrected framework: $\mathscr{F}(Z)$ is taken as the Recognition-side (structural) residue f^{Rec}, while f_i^{RG} is a small perturbative transport factor used only to compare structural predictions to PDG masses in a fixed scheme. We formalize in Lean: strict concavity / diminishing increments of $\mathscr{F}$, certified interval bounds for $\mathscr{F}(24)$ and $\mathscr{F}(276)$ (matching the existing $\mathscr{F}(1332)$ style), and a no-go separation showing that small RG residues cannot equal $\mathscr{F}(1332)$.

*e-mail: jon@recognitionphysics.org

One-line result (scan-friendly). Define the geometric (Recognition) residue

$$f^{\mathrm{Rec}}(Z) \equiv \mathscr{F}(Z) \equiv \mathrm{gap}(Z) := \frac{\ln(1 + Z/\varphi)}{\ln \varphi}$$

with the fixed integer map

$$Z = \begin{cases} 4 + (6Q)^2 + (6Q)^4, & \text{quarks (color fundamental)}, \\ (6Q)^2 + (6Q)^4, & \text{charged leptons}, \\ 0, & \text{Dirac neutrinos}. \end{cases}$$

Separately define the Standard-Model transport residue

$$f_i^{\mathrm{RG}}(\mu_\star, \mu) := \frac{1}{\ln \varphi} \int_{\ln \mu_\star}^{\ln \mu} \gamma_m^{\mathrm{SM}}(\mu') \, d\ln \mu'.$$

Framework separation (no cheating): f_i^{RG} is not identified with f^{Rec}; Lean certifies concavity/bounds for f^{Rec} and proves a no-go inequality separating small f^{RG} from $\mathrm{gap}(1332)$.

Status (Lean formalization). The geometric residue $f^{\mathrm{Rec}}(Z)$ and its key analytic/numerical properties are now machine-verified in Lean (concavity / diminishing increments, and certified interval bounds at $Z \in \{24, 276, 1332\}$). A no-go separation is also proved showing that a small perturbative transport residue cannot equal $\mathrm{gap}(1332)$.

Operational non-circularity rule (framework-separated). PDG masses are external validation targets. Structural quantities ($f^{\text{Rec}}(Z)$ and the sector yardsticks used to form $m_{\text{pred}}(\mu_\star)$) are computed without PDG masses; SM RG (QCD/QED/EW) is used only as a common transport map so comparisons are made like-for-like in a fixed scheme/scale.

$$m_i^{\text{PDG}\to\mu_\star} = m_i^{\text{PDG}}(\mu_{\text{ref}})\exp\left(\int_{\ln\mu_{\text{ref}}}^{\ln\mu_\star}\gamma_i(\mu)\,d\ln\mu\right),$$

Note: a previous draft used this transport to assert an "anchor identity" $f^{\text{RG}}\approx\mathscr{F}(Z)$. That identification is withdrawn; the corrected framework separates f^{RG} from f^{Rec}.

1 Framework update (2025-12): separating f^{Rec} from f^{RG}

Two residues, different roles. We distinguish:

$$f^{\text{Rec}}(Z) := \mathscr{F}(Z) = \frac{\ln(1+Z/\varphi)}{\ln\varphi} \quad \text{(geometric/structural; zero-parameter),}$$

(1)

$$f_i^{\text{RG}}(\mu_\star,\mu) := \frac{1}{\ln\varphi}\int_{\ln\mu_\star}^{\ln\mu}\gamma_m^{\text{SM}}(\mu')\,d\ln\mu' \quad \text{(SM transport; scheme-/scale-wise).}$$

(2)

The earlier identification $f_i^{\text{RG}}(\mu_\star,m_i)\overset{?}{\approx}f^{\text{Rec}}(Z_i)$ is *not* adopted. In the corrected framework, f^{Rec} is the Recognition-side band coordinate; f^{RG} is used only to transport between schemes/scales when comparing to PDG masses.

No-go (no cheating). Lean proves that a small transport residue cannot equal the large geometric band value (e.g. for $Z = 1332$): see 'Indisputable-Monolith/Physics/MassResidueNoGo.lean'.

Certified properties of $\mathscr{F}$. Lean proves strict concavity / diminishing increments for the real extension of $\mathscr{F}$, and certifies interval bounds for $\mathscr{F}(24)$ and $\mathscr{F}(276)$ (matching the style for $\mathscr{F}(1332)$): see 'IndisputableMonolith/RSBridge/GapProperties.lean'.

Measurability and prediction. The experimental observable remains the PDG mass m_i. The theory predicts a mass (or band coordinate) from RS structure and SM RG is used only to transport for like-for-like comparison. The geometric residue $f^{\text{Rec}}(Z)$ is not "measured from PDG"; it is a closed-form prediction from the integer invariant Z.

$\gamma_i = \gamma_i^{\text{QCD}} + \gamma_i^{\text{QED}}$

Legacy note. The remainder of this file contains the legacy single-anchor SM-identity draft and CI machinery retained for provenance; it is excluded from compilation while the public-facing RS mass manuscript is rewritten under the separated-residue framework.

References

[1] R. L. Workman *et al.* (Particle Data Group), "Review of Particle Physics," Prog. Theor. Exp. Phys. **2022**, 083C01 (2023 update), doi:10.1093/ptep/ptac097.

[2] J. A. M. Vermaseren, S. A. Larin, and T. van Ritbergen, "The 4-loop quark mass anomalous dimension and the invariant quark mass," Phys. Lett. B **405**, 327 (1997).

[3] T. van Ritbergen, J. A. M. Vermaseren, and S. A. Larin, "The 4-loop QCD beta-function in the MS scheme," Phys. Lett. B **400**, 379 (1997), doi:10.1016/S0370-2693(97)00370-5.

[4] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, "Decoupling relations to $\mathscr{O}(\alpha_s^3)$

and their connection to low-energy theorems," Nucl. Phys. B **510**, 61 (1998), doi:10.1016/S0550-3213(97)00649-4.

[5] F. Herren and M. Steinhauser, "Version 3 of RunDec and CRunDec: Running and decoupling of the strong coupling and quark masses," Comput. Phys. Commun. **224**, 333 (2018), doi:10.1016/j.cpc.2017.11.014.

[6] M. E. Machacek and M. T. Vaughn, "Two-Loop Renormalization Group Equations in a General Quantum Field Theory. II. Yukawa Couplings," Nucl. Phys. B **236**, 221 (1984), doi:10.1016/0550-3213(84)90533-9. (See also I: Nucl. Phys. B **222**, 83 (1983), doi:10.1016/0550-3213(83)90610-7.)

[7] M.-x. Luo, H.-w. Wang, and Y. Xiao, "Two-loop renormalization group equations in the Standard Model," Phys. Rev. D **67**, 065019 (2003), doi:10.1103/PhysRevD.67.065019.

[8] P. M. Stevenson, "Optimized Perturbation Theory," Phys. Rev. D **23**, 2916 (1981), doi:10.1103/PhysRevD.23.2916.

[9] S. J. Brodsky, G. P. Lepage, and P. B. Mackenzie, "On the Elimination of Scale Ambiguities in Perturbative Quantum Chromodynamics," Phys. Rev. D **28**, 228 (1983), doi:10.1103/PhysRevD.28.228.

[10] J. Washburn, "Empirical Regularity in Standard Model Fermion Masses: Code and Data," Git repository mirrored in the artifact manifest (repository name and commit hash recorded in `out/log/manifest.txt`).

Kernels and policies

Unless stated otherwise, we use: (i) QCD four-loop running and quark mass anomalous dimensions with n_f stepping $3 \to 4 \to 5 \to 6$ at fixed $\overline{\text{MS}}$ thresholds (μ_c, μ_b, μ_t) held constant for all species and all scans, with standard one-step decoupling and matching at each threshold; (ii) QED two-loop running for charged leptons and quarks with a single, global choice of $\alpha(\mu)$ policy; (iii) a single global input for $\alpha_s(M_Z)$. Sensitivity bands are produced by coherent, sector-global variations of these inputs (no per-species tuning or offsets).

Statements and Declarations

Ethics approval, Consent, and Human/Animal research. Not applicable.

Use of AI tools. No generative AI was used to produce scientific content; standard editing tools were used for grammar and typesetting only.