Internal Memo: The Missing Move
Turning "Ledger Latency" into the ILG Power-Law Kernel
(RRF $\leftrightarrow$ ILG Bridge: fractional response operator, pinned constants, falsifiers)

Jonathan Washburn   +   Internal synthesis (Cursor / GPT-assisted)

December 18, 2025

Abstract

This document records the creative and technical bridge between two strands of the Recognition Science program: (i) the mature *Information-Limited Gravity* (ILG) phenomenology, expressed as a source-side kernel $w(k,a) = 1 + C\,(a/(k\tau_0))^\alpha$; and (ii) the emerging *Reality Recognition Framework* (RRF), which identifies mass with ledger density and gravity with ledger/strain curvature.

The missing move, historically, was that "finite refresh / latency" was a compelling story but not a mechanism that forces the observed (and Lean-encoded) power-law multiplier $k^{-\alpha}$. Here we give a minimal derivation: a ledger-latency model is formalized as a *power-law memory* operator in time (a fractional integral). In Fourier space this produces $(\omega\tau_0)^{-\alpha}$. Using causality/refresh arguments to relate temporal frequency ω to comoving wave-number k via $\omega \sim ck/a$, the model yields $w(k,a) = 1 + C\,(a/(k\tau_0))^\alpha$. Equivalently, the same construction is a fractional *spatial* response operator with symbol $|k|^{-\alpha}$, i.e. $(-\Delta)^{-\alpha/2}$.

We then pin α and C to the RS constants already defined in the primary Lean repository (`github.com/jonwashburn/reality`): $\alpha = (1 - 1/\varphi)/2$ and $C = \varphi^{-3/2}$ (RS-canonical), with an alternate 8-tick-aligned amplitude option used in some modules. Finally, we list falsifiers implied by the mechanism (not just by the kernel), and we propose the next Lean steps to formalize the fractional-operator bridge explicitly.

Contents

1 Background: how we got here (internal narrative)

1.1 From protein folding to universal strain

This project did not begin as a gravity project. It began as a protein-folding optimizer and a search for a tractable, mechanistic surrogate for AlphaFold-like performance. In the process we repeatedly encountered the same formal pattern:

- A system has a notion of *validity* (consistency, closure, equilibrium).

- Deviations from validity are measured by a scalar functional J ("strain", "cost").

- Optimization drives $J \to 0$ subject to constraints.

This manifested as:

- **Proteins:** local and global energetic/steric penalties; folding as J minimization.

- **Sound/sonification:** detuning/dissonance as a measurable J; correlation with fold quality.

- **Logic/traces:** invariants / non-closure as a scalar measure of inconsistency.

That cross-domain identity motivated a more general ontology: instead of treating physics, cognition, and computation as separate substances, treat them as distinct *vantages* on the same underlying closure/recognition process.

1.2 The Reality Recognition Framework (RRF)

RRF is the name for the systematized ontology:

- **Recognition / existence:** to exist is to be recognized by the self-consistent whole.

- **Ledger:** recognition events are bookkeeping; closure constraints enforce conservation-like laws.

- **Strain J:** the universal penalty for non-closure; balanced states are $J = 0$.

- **Octaves:** the same pattern appears at multiple scales (domains), related by scaling hypotheses.

In Lean, RRF was implemented as a module tree under: `IndisputableMonolith/RRF/` plus an umbrella `IndisputableMonolith/RRF.lean`.

1.3 Information-Limited Gravity (ILG)

Independently, the gravity work matured into a precise phenomenology:

- Modify gravity on the *source side*, leaving the operator form classical:

$$k^2 \Phi(\mathbf{k}, a) = 4\pi G a^2 \rho_s(a) \, w(k, a) \, \delta_s(\mathbf{k}, a).$$

- Use a parameter-fixed kernel:

$$w(k, a) = 1 + \varphi^{-3/2} \left(\frac{a}{k\tau_0} \right)^\alpha, \qquad \alpha = \frac{1}{2} \left(1 - \varphi^{-1} \right).$$

- Derive many linear-observable predictions as functions of the variable $X = k\tau_0/a$ ("X-only").

This is expressed across the internal papers: `Dark-Energy.tex`, `Pressure-Gravity.tex`, `Quantum-Gravity-New.tex`, `Information-Limited-Gravity-Paper1-Sept26.tex`.

1.4   The tension: story vs. mechanism

The ILG story is intuitively framed as *finite refresh / information limit / bandwidth triage*: slow, extended systems cannot be updated with the same immediacy as compact, fast systems. The kernel was motivated as a rigorous instantiation of this idea.

However, there remained a conceptual gap:

> Why does latency produce a *power-law* gain with symbol $k^{-\alpha}$, rather than an exponential cutoff, a rational filter, a log, etc.?

RRF also introduced a new explanatory vocabulary (ledger closure, strain, recognition density), but did not automatically force the ILG kernel either.

This memo records the missing bridge: **turn latency into a fractional operator**.

2   Goal: derive the ILG kernel from ledger latency (not just motivate it)

We want a derivation that:

- is **minimal**: few assumptions, explicit and auditable;

- forces **power-law** behavior in a controlled regime;

- naturally yields the ILG form $w(k, a) = 1 + C(a/(k\tau_0))^\alpha$;

- makes **falsifiers** easy to state (and different from arbitrary kernel fits);

- is consistent with the already-pinned RS constants in Lean: $\alpha = (1 - 1/\varphi)/2$ and $C = \varphi^{-3/2}$.

3   The minimal ledger-latency model (assumptions)

Assumption 1 (Ledger-consistent sourced density). *The gravitational potential responds to a recognition-consistent (ledger-closed) source ρ_{rec} rather than the instantaneous source ρ. Equivalently, in the pressure language: $\nabla^2 \Phi \propto \rho_{\mathrm{rec}}$ (or its contrast).*

Assumption 2 (Latency as backlog memory (scale-free)). *Ledger closure is not instantaneous; unclosed transactions persist as a backlog. Backlog is* scale-free *in the low-frequency regime and therefore appears as a power-law memory kernel in time. Concretely, for some $\alpha \in (0, 1)$ and amplitude $C \geq 0$,*

$$\rho_{\mathrm{rec}}(t, \mathbf{x}) = \rho(t, \mathbf{x}) + C\,\tau_0^{-\alpha}\,(I_t^\alpha \rho)(t, \mathbf{x}), \tag{1}$$

where I_t^α is the Riemann–Liouville fractional integral in time:

$$(I_t^\alpha f)(t) := \frac{1}{\Gamma(\alpha)} \int_0^\infty s^{\alpha-1} f(t - s)\,\mathrm{d}s. \tag{2}$$

Assumption 3 (Causal identification of time-scale with spatial scale). *For a comoving spatial Fourier mode k at scale factor a, the fastest effective refresh frequency scales like the causal rate:*

$$\omega(k, a) \sim \frac{c\,k}{a}. \tag{3}$$

In units with $c = 1$, this is $\omega \sim k/a$. This is the bridge that carries time-latency into scale-dependent spatial response.

Remark 1 (Why these assumptions are minimal). *Assumption 2 is the core: it is exactly the claim "latency/backlog has power-law memory". This is the only step that fixes a power-law rather than an arbitrary filter family. Assumption 3 is a causality/refresh statement: the ledger is local and finite-speed, so a mode's refresh frequency must scale with its physical wavelength.*

3.1 Why power-law memory is the unique scale-free latency kernel

The word "scale-free" can be made precise. The point is not that we *like* power laws; it is that *if the ledger latency has no characteristic scale in the slow regime*, then the only stable functional form for the memory kernel is a homogeneous (power) kernel.

Definition 1 (Backlog kernel as a convolution). *Write the backlog contribution to the recognition-consistent source as a time-convolution:*

$$(Bf)(t) := \int_0^\infty K(s)\, f(t - s)\, \mathrm{d}s,$$

for some kernel $K(s) \geq 0$ on $s > 0$.

Assumption 4 (Homogeneity / scale invariance of backlog). *In the slow regime, the backlog kernel has a scaling symmetry: for some exponent $\alpha \in (0, 1)$ and all $\lambda > 0$,*

$$K(\lambda s) = \lambda^{\alpha - 1} K(s). \tag{4}$$

Proposition 1 (Scale invariance forces a power kernel). *If K is locally integrable on $(0, \infty)$ and satisfies (4), then there exists a constant $c \geq 0$ such that*

$$K(s) = c\, s^{\alpha - 1}.$$

Proof sketch. Equation (4) is exactly the definition of a homogeneous function of degree $(\alpha - 1)$ on $(0, \infty)$. On the positive reals, such functions are of the form $K(s) = cs^{\alpha - 1}$. $\qquad\square$

Remark 2 (Connection to fractional calculus). *Choosing $K(s) \propto s^{\alpha - 1}$ produces the Riemann–Liouville fractional integral (up to normalization), which is precisely why fractional operators are the canonical language for scale-free latency.*

3.2 Alternative physical interpretation: heavy-tailed waiting times

Another way to justify the same mathematics is via renewal processes / queueing: if ledger "commit" events have waiting-time distribution with a Pareto tail $\mathbb{P}(\text{wait} > s) \sim s^{-\alpha}$, then the effective response to a driving signal is known to exhibit fractional (power-law) memory, and the low-frequency gain behaves like $|\omega|^{-\alpha}$. This memo does not require the full stochastic derivation; the point is that *power-law tails* and *fractional operators* are equivalent descriptions of "latency without a characteristic scale."

4 Derivation: fractional time-memory $\Rightarrow (\omega\tau_0)^{-\alpha}$

We work in temporal Fourier domain (fixing spatial $\mathbf{x}$ or a spatial mode k). Let $\widehat{f}(\omega)$ denote the Fourier transform in time with convention

$$\widehat{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)\; \mathrm{d}t.$$

Proposition 2 (Fractional integral multiplier). *For sufficiently regular f (tempered distributions suffice for the multiplier statement), the Riemann–Liouville fractional integral satisfies*

$$\widehat{I_t^\alpha f}(\omega) = (i\omega)^{-\alpha}\, \widehat{f}(\omega), \tag{5}$$

with the standard branch choice for $(i\omega)^{-\alpha}$.

Therefore Assumption 2 implies:

$$\widehat{\rho}_{\mathrm{rec}}(\omega, \mathbf{x}) = \left[1 + C\,(\tau_0\, i\omega)^{-\alpha}\right] \widehat{\rho}(\omega, \mathbf{x}). \tag{6}$$

In magnitude (ignoring the phase of $i^{-\alpha}$), this is a gain of the form:

$$|\widehat{\rho}_{\mathrm{rec}}| \approx \left[1 + C\,(\tau_0\,|\omega|)^{-\alpha}\right] |\widehat{\rho}|. \tag{7}$$

Remark 3 (Interpretation). *The backlog behaves like "old source contributions do not decay exponentially; they decay as a power". At low frequency (slow dynamics, small $|\omega|$) the enhancement grows as $|\omega|^{-\alpha}$. At high frequency (fast dynamics) it vanishes and $\rho_{\mathrm{rec}} \to \rho$.*

5 Causality bridge: $(\omega\tau_0)^{-\alpha} \Rightarrow (a/(k\tau_0))^\alpha$

Now consider a comoving Fourier mode k of the source field in an expanding background. Causal refresh identifies the effective update frequency for that mode as $\omega \sim ck/a$ (Assumption 3). Substituting into (7) yields:

$$|\widehat{\rho}_{\mathrm{rec}}(k, a)| \approx \left[1 + C\left(\frac{a}{k\,c\,\tau_0}\right)^\alpha\right] |\widehat{\rho}(k, a)|. \tag{8}$$

Absorb c into τ_0 (or set $c = 1$ in natural units) to obtain:

$$w(k, a) = 1 + C\left(\frac{a}{k\tau_0}\right)^\alpha. \tag{9}$$

Claim 1 (This is the missing move). *Equation (9) is the ILG kernel, derived from a ledger-latency model whose essential structure is a fractional-memory backlog plus a causal scale–time identification. The power law $k^{-\alpha}$ is not an* ansatz; *it is the Fourier symbol of a fractional operator.*

6 Equivalent spatial form: a fractional response operator with symbol $|k|^{-\alpha}$

Because (9) depends on $k^{-\alpha}$, it can be expressed as an operator acting on spatial fields at fixed epoch.

Definition 2 (Fractional spatial integral / Riesz potential)**.** *Define the fractional spatial operator I_α by its Fourier symbol:*

$$\widehat{(I_\alpha f)}(k) := |k|^{-\alpha}\,\widehat{f}(k).$$

Formally, $I_\alpha = (-\Delta)^{-\alpha/2}$ on mean-zero fields (to avoid the $k = 0$ singularity).

Then the same latency mechanism can be written (schematically) as:

$$\rho_{\text{rec}}(\mathbf{x}, a) = \rho(\mathbf{x}, a) + C\left(\frac{a}{\tau_0}\right)^\alpha (-\Delta)^{-\alpha/2}\rho(\mathbf{x}, a), \tag{10}$$

whose Fourier transform is exactly (9).

Remark 4 (Why this matters)**.** *This exhibits the ILG kernel as a* nonlocal *response: recognition/ledger closure is not pointwise; it couples long wavelengths more strongly. This makes the "dark-sector" behavior a direct consequence of nonlocal closure rather than an extra matter component.*

7 Pinning the constants: α and C

7.1 What is already pinned in Lean (reality repo)

In the primary Lean repository (`github.com/jonwashburn/reality`), the ILG kernel parameters are encoded directly:

- **Kernel form:** $w(k, a) = 1 + C(a/(k\tau_0))^\alpha$.

- **Exponent:** $\alpha = (1 - 1/\varphi)/2$ (named `alphaLock`).

- **Amplitude (RS canonical):** $C = \varphi^{-3/2}$ (in `rsKernelParams`).

- **Alternate amplitude (8-tick aligned):** $C = 49/162$ (in `eightTickKernelParams`).

The exact code location:

```
-- reality/IndisputableMonolith/ILG/Kernel.lean (excerpt)
structure KernelParams where
  alpha : Real
  C : Real
  tau0 : Real
  tau0_pos : 0 < tau0
  alpha_nonneg : 0 <= alpha
  C_nonneg : 0 <= C
```

```
noncomputable def rsKernelParams (tau0 : Real) (h : 0 < tau0) :
   KernelParams := {
  alpha := alphaLock,                    -- alphaLock = (1 - 1/phi)/2
  C := phi ^ (-(3 : Real) / 2),          -- RS-canonical amplitude
  tau0 := tau0,
  tau0_pos := h,
  alpha_nonneg := le_of_lt alphaLock_pos,
  C_nonneg := le_of_lt (Real.rpow_pos_of_pos phi_pos _)
}
```

7.2 Interpretation: why $C = \varphi^{-3/2}$

Internally, the $\varphi^{-3/2}$ factor has an intended geometric meaning:

- There are **three orthogonal recognition channels** in $D = 3$ (x,y,z).

- Each channel has a **primal/dual** traversal across an 8-tick schedule.

- Ledger link penalty fixes the *product* of primal/dual weights per channel to φ^{-1}.

- Minimality / equal split gives each traversal weight $\varphi^{-1/2}$ per channel.

- Multiplying three channels yields $\varphi^{-3/2}$.

This is exactly the "3-channel factorization" used in the ILG papers' appendices.

7.3 Interpretation: why $\alpha = (1 - 1/\varphi)/2$

The exponent α is the reciprocity/self-similarity exponent. In this memo, *the mechanism that produces a power law* is fractional memory. But fractional memory alone does not pin the exponent numerically. Pinning α requires an additional selection rule.

The RS program pins α by **self-similarity at the golden fixed point**: the scaling transform by φ is the unique stable recursion under the cost/coverage constraints, and the exponent becomes a function of φ.

In Lean, this is encoded as:

$$\alpha = \frac{1}{2}\left(1 - \frac{1}{\varphi}\right)$$

and treated as a derived constant from φ (see parameter provenance certificate).

8 Falsifiers implied by the mechanism (not just by kernel fitting)

The fractional-latency mechanism yields falsifiers stronger than "fit a kernel":

Falsifier 1 (Slope of enhancement term). *In the regime where $w-1$ is dominated by the enhancement term and the $\max(\varepsilon, \cdot)$ cutoff is inactive,*

$$\frac{\partial \ln(w - 1)}{\partial \ln k} = -\alpha \quad \text{(constant slope)}.$$

Large, systematic deviation from constant slope falsifies pure fractional-memory latency.

Falsifier 2 (X-only reciprocity identity). *If $w = w(X)$ with $X = k\tau_0/a$, then for any such observable $Q(X)$:*

$$\partial_{\ln a} \ln Q(a, k) = -\partial_{\ln k} \ln Q(a, k).$$

This is the "single-plot" falsifier family emphasized in `Dark-Energy.tex`. *Violation falsifies the hypothesis that the enhancement is purely a function of X (hence purely causal-latency based).*

Falsifier 3 (No free prefactor). *If C is pinned to $\varphi^{-3/2}$ by the 3-channel ledger geometry, then the amplitude is not adjustable. Any empirical requirement for a different amplitude in the same regime falsifies the channel-factorization story (or the identification of the relevant channels/dimension).*

Remark 5 (Why these falsifiers are "mechanism-level"). *They do not just test whether a kernel can be tuned. They test whether a* fractional-memory + causality *origin is correct. Other kernel families can fit some data but will generally not satisfy all three falsifiers simultaneously.*

9 How this connects to the four gravity manuscripts (internal alignment)

9.1 `Dark-Energy.tex`

Already uses $w(k, a) = 1 + \varphi^{-3/2}(a/(k\tau_0))^\alpha$ and emphasizes the $X = k\tau_0/a$ universal-variable structure, including the reciprocity identity and single-plot falsifiers. This memo supplies a deeper *mechanism* for why $k^{-\alpha}$ appears: fractional latency.

9.2 `Pressure-Gravity.tex`

Recasts ILG as classical gravity with an effective source/"pressure" $p = w * \rho_b$. The fractional-operator picture fits naturally: the pressure source is precisely the application of a fractional operator (Fourier multiplier) to the baryonic field.

9.3 `Quantum-Gravity-New.tex`

Builds the discrete recognition calculus, 8-tick minimality, and bridge invariants to GR/QG. This memo connects the QG framing to ILG by adding: "finite update" $\Rightarrow$ fractional memory $\Rightarrow$ power-law nonlocal kernel. This is compatible with discrete-to-continuum limits (fractional operators often appear from heavy-tailed waiting times).

9.4 `Information-Limited-Gravity-Paper1-Sept26.tex`

Frames gravity as latency geometry and motivates triage. The fractional-memory mechanism can be read as the unique scale-free form of triage: rather than a specific scheduler, the ledger imposes power-law persistence of unclosed recognition events.

10 RRF alignment: "mass = ledger density" and "gravity = ledger curvature"

RRF introduces a foundational mapping:

- **Mass** = ledger density

- **Curvature** = constraint pressure / strain

In Lean (RRF module): `IndisputableMonolith/RRF/Foundation/Gravity.lean`.

This memo's mechanism suggests a refinement:

> Mass is the instantaneous ledger transaction density; *effective gravitational sourcing* is the **recognition-consistent ledger density**, i.e. instantaneous density plus backlog. Backlog is the fractional operator.

That is, RRF provides the ontological identification of what is being sourced; ILG provides the phenomenological kernel; the fractional-latency bridge explains why the kernel has a $k^{-\alpha}$ tail.

11 Next Lean steps (optional, but recommended)

If we want this bridge to be more than a memo, the natural formalization target is:

1. Define a Fourier-multiplier operator on a suitable function space (periodic torus is easiest) whose symbol is $|k|^{-\alpha}$ with mean-zero constraint.

2. Prove the identity that applying this operator corresponds to multiplication by $k^{-\alpha}$ in Fourier domain (likely already close to existing Fourier work in the repo).

3. State the ILG kernel as $I + C(a/\tau_0)^\alpha(-\Delta)^{-\alpha/2}$.

4. Prove the X-reciprocity identity at the level of the kernel and derived observables (some of this exists already in the ILG pipeline/papers).

5. Connect to RRF: define a `SpatialLedger` density field and define the recognition-consistent density via the fractional operator; then show monotonicity / positivity constraints.

12 Open questions / guardrails

- **Uniqueness of power-law memory:** This memo assumes power-law memory as the scale-free choice. The deeper question is whether RRF/ledger axioms force this uniquely (e.g., from self-similarity + minimality).

- **IR behavior:** $k^{-\alpha}$ diverges at $k \to 0$; physically the zero mode is fixed by boundary conditions (mean-zero potential in a periodic box, or decay at infinity). This must be treated carefully.

- **Phase of** $(i\omega)^{-\alpha}$**:** For real physical fields we use magnitude or an appropriate symmetric fractional operator (Caputo / Riesz variants). This memo focuses on the amplitude scaling.

- **Relation to** G**:** The kernel is dimensionless; G is still an external scale in the classical presentation. A further creative leap is to derive G itself as the inverse "bandwidth" of the vacuum ledger.

A   Appendix: conventions and quick references

A.1   Fourier conventions

Time Fourier transform: $\widehat{f}(\omega) = \int e^{-i\omega t} f(t)\, \mathrm{d}t$. Spatial Fourier transform conventions vary; the key property used here is: fractional operators correspond to power-law symbols, e.g. $(-\Delta)^{s/2} \leftrightarrow |k|^s$.

A.2   Where the constants live in Lean

- `IndisputableMonolith/ILG/Kernel.lean`: `rsKernelParams` sets $C = \varphi^{-3/2}$ and $\alpha =$ `alphaLock`.

- `IndisputableMonolith/URCGenerators/ParameterProvenanceCert.lean`: records the provenance chain $\mathrm{MP} \to (\varphi, \alpha, C_{\mathrm{lag}}, \alpha_{\mathrm{EM}})$.

- `IndisputableMonolith/RRF/Foundation/Gravity.lean`: records the mapping "mass = ledger density" and "curvature monotone in strain".