Revision Notes for

Dimensional Rigidity as a Selection Principle in Recognition Geometry

Changes applied to `Feb_10_revised_version.tex`

Prepared for internal review

February 13, 2026

Overview

This document catalogs every change made to the manuscript *Dimensional Rigidity as a Selection Principle in Recognition Geometry* in the February 13, 2026 revision. Changes fall into three categories:

I. **Error corrections** (comments 1–9 from the Version-3 Comment-1 review),
II. **Published-paper integration** (importing the now-accepted Axioms paper),
III. **Formatting and style** (equation punctuation, author order, Unicode fix).

All additions appear in teal in the revised manuscript PDF so that co-authors and reviewers can locate them instantly.

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1 Error Corrections (from v3_comment_1 review)

1.1 Change 1: Author order — alphabetical by last name

Location: Title page, `\author{...}`.

Problem: The author list was ordered Washburn, Pardo-Guerra, Thapa. Comment 12 of the review requests alphabetical ordering by last name.

Before:

Jonathan Washburn, Sebastian Pardo-Guerra, Anil Thapa

After:

Sebastian Pardo-Guerra, Anil Thapa, Jonathan Washburn

1.2 Change 2: Replace "manifold-like" with precise definition reference

Location: Theorem 1.2 (Main Theorem statement in the Introduction), and Theorem 6.1 (full statement in Section 6).

Problem: The phrase "Assume $\mathcal{C}_R$ is manifold-like" was never defined as a standalone term. The paper already introduces the notion of an "effective manifold model $\mathcal{M}$" in Definition 2.5 (formerly 2.12), but Theorem 1.2 did not reference it.

Before:

> Assume $\mathcal{C}_R$ is manifold-like and admits enough structure...

After:

> Assume $(\mathcal{C}, \mathcal{E}, R)$ admits an effective manifold model $\mathcal{M}$ in the sense of Definition 2.5 and admits enough structure...

Why: Makes the paper internally consistent with the two-scale story (finite-resolution quotients $\to$ effective manifold limit).

1.3 Change 3: Freedman exotic $\mathbb{R}^4$ — corrected attribution

Location: Section 1.1 (Prior Approaches), paragraph on pure mathematics.

Problem: The original text stated "Freedman's exotic $\mathbb{R}^4$ theorem shows... $\mathbb{R}^4$ admits uncountably many distinct smooth structures." This is an oversimplification. Freedman's theorem is about *topological* 4-manifolds (homeomorphism classification). The exotic smooth structures on $\mathbb{R}^4$ follow from combining Freedman's work with Donaldson's gauge-theoretic invariants and later results of Taubes.

After:

> Freedman [6] classified simply connected closed topological 4-manifolds via their unimodular intersection forms. Combined with Donaldson's smooth rigidity results (and later work), this implies that $\mathbb{R}^4$ admits uncountably many *exotic* smooth structures—a phenomenon unique to dimension 4.

1.4 Change 4: "Knot theory only in $D = 3, 4$" — corrected

Location: Same paragraph as Change 3.

Problem: The claim "Knot theory is nontrivial only in dimensions $D = 3, 4$" is false. The very next clause ("surfaces link in $D = 5$") contradicts it. Higher-dimensional knot theory (codimension-2 sphere knots, etc.) is a well-established field.

After:

> Classical knot theory of embeddings $S^1 \hookrightarrow \mathbb{R}^3$ is special; in higher dimensions the behavior changes dramatically, though higher-dimensional knot theory (e.g. codimension-2 sphere knots) exists and is non-trivial.

1.5 Change 5: Chiral anomalies statement — made precise

Location: Same paragraph as Changes 3–4.

Problem: The original text "chiral anomalies vanishing only in specific dimensions (e.g., $D = 2, 6, 10 \ldots$)" was vague and likely incorrect as stated.

After:

In quantum field theory, anomaly cancellation in gauge theories imposes dimensional constraints; for instance, gravitational and gauge anomalies cancel in $D = 10$ for the superstring, and analogous constraints appear in lower-dimensional models.

1.6 Change 6: Boundary/intersection identity in linking proof — fixed

Location: Proof of Theorem 3.1, Step (3) (Independence of choice of W).

Problem: The proof wrote:

$$(\partial Q) \cdot B \;=\; Q \cdot (\partial B).$$

This is *not* the correct boundary/intersection compatibility identity. The correct Leibniz-type identity has an extra $\partial(Q \pitchfork B)$ term. The chain-level argument was therefore invalid.

After: The argument is replaced with a standard *homology-level* argument that avoids chain-level sign complications entirely:

> In an oriented closed D-manifold, the intersection number $Z \cdot B$ depends only on the homology class $[Z] \in H_{p+1}(\mathcal{C}_R; \mathbb{Z})$. Since $H_{p+1}(\mathcal{C}_R; \mathbb{Z}) = 0$ by hypothesis and Z is a cycle, we have $[Z] = 0$, hence $Z \cdot B = 0$.

Why this is correct: Intersection numbers are bilinear pairings on homology (not chains). Since $Z = W - W'$ is a cycle in a trivial homology group, it is null-homologous, so its intersection number with any cycle vanishes. This is a standard technique (see Rolfsen, *Knots and Links*, 1976).

1.7 Change 7: $p = 0$ listing and codimension-2 remark — fixed

Location: Proof of Proposition 3.2 and Remark 3.3.

Problem (a): The proof listed "For $p = 0$ (points), $D = 1$" as a valid case. But Theorem 3.1 assumes $0 < p < D$, so $p = 0$ falls outside its domain.

Fix: The proof now restricts to $p \geq 1$ (consistent with the theorem hypothesis), and the allowed set is written consistently as $\mathcal{A}_A = \{3, 5, 7, \dots\}$ throughout the manuscript.

Problem (b): Remark 3.3 called objects "codimension-2 defects" but the codimension is $(D+1)/2$, which equals 2 only when $D = 3$. The remark was circular.

Fix: Rewritten to state clearly that codimension-2 is specific to $D = 3$; the general same-dimension linking constraint forces odd D via the formula $D - p = (D+1)/2$, without singling out codimension 2 for arbitrary D.

1.8 Change 8: Green-kernel sign convention — fixed

Location: Appendix A (Detailed Derivation of Green-Kernel Potentials).

Problem: The appendix wrote "Choosing $C < 0$ for an attractive potential." But in the main text, $V_2(r) = k \ln r$ with $k > 0$ is attractive (giving $F = -k/r$ inward). For $V(r) = C \ln r$, attraction requires $F = -V' = -C/r$ inward, i.e. $C > 0$.

After:

> Choosing the constant so that $F = -\nabla V$ is inward (attractive), i.e. $C > 0$, and dropping the additive constant. . .

1.9 Change 9: $SO(D)$ — local frame rotations, not global isometry group

Location: Section 5 (Constraint C), Proposition 5.1, and surrounding text.

Problem: The paper spoke of "the rotation group $SO(D)$" as if it were the global isometry group of $\mathcal{M}$. A generic manifold need not have $SO(D)$ as its isometry group. What is meant is the *structure group of the oriented orthonormal frame bundle.*

After: Changed to "the local orthonormal frame rotation group $SO(D)$" in both the proposition statement and surrounding discussion.

1.10 Change 10: ω consistency remark in appendix

Location: Appendix C, between Method 1 and Method 2.

Problem: The symbol ω is defined as κ/Ω in Method 1 (time-domain) and as $\sqrt{2-n} = \sqrt{4-D}$ in Method 2 (Binet angle-domain). These are the same physical quantity, but a reader might worry about a name collision.

After: A new Remark (Consistency of ω across methods) is inserted, confirming the two definitions agree.

2 Published-Paper Integration (Axioms paper)

Now that *Reciprocal Convex Costs for Ratio Matching: Axiomatic Characterization* (Washburn & Rahnamai Barghi, *Axioms* 2026; `doi:10.3390/axioms1010000`) is accepted and published, we import its main result to anchor the cost functional in a *peer-reviewed, published theorem* rather than re-deriving it.

2.1 Change 11: Introduction bridge paragraph

Location: End of the Introduction preamble (after "…fundamentally new approach to this ancient question").

Added text:

> This paper builds on the axiomatic characterization of ratio-induced mismatch costs established in [WashburnRahnamaiBarghi2026]. There it was shown that the assumptions of inversion symmetry, strict convexity, coercivity, and a multiplicative d'Alembert compatibility identity uniquely force $J(x) = \frac{1}{2}(x^a + x^{-a}) - 1$ for some $a > 0$ (with a absorbable into the scale maps). We take this cost-kernel result as given and focus on the downstream topological, dynamical, and geometric consequences that determine spatial dimension.

Why: Creates a visible dependency spine between publications. Reduces reviewer friction on "why this cost?" by pointing to published peer-reviewed proof.

2.2 Change 12: Imported Proposition 2.5 (cost-kernel uniqueness)

Location: Section 2 Preliminaries, new subsection "Imported Cost-Kernel Characterization" inserted before the Composite Recognizers subsection.

Added:

Proposition 2.5 (Unique Mismatch Penalty; Washburn–Rahnamai Barghi [2026]). Let $J : (0, \infty) \to [0, \infty)$ satisfy (i) inversion symmetry $J(x) = J(1/x)$, (ii) strict convexity, (iii) normalization $J(1) = 0$, (iv) coercivity, and (v) the multiplicative d'Alembert identity $(1 + J(xy)) + (1 + J(x/y)) = 2(1 + J(x))(1 + J(y))$. Then there exists $a > 0$ such that $J(x) = \cosh(a \log x) - 1$. The parameter a is absorbed by rescaling the scale maps, yielding $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ without loss of generality.

Followed by a scope sentence: "The novelty of the present work lies in the geometric and topological consequences of this cost kernel—specifically the forcing of $D = 3$ spatial dimensions via linking constraints—rather than in the derivation of J itself."

Why:

- Anchors the cost functional in a published theorem (not an assumption).
- Makes notation consistent across the two papers (ι_S, ι_O, J).
- Lets reviewers evaluate only the *new* contribution (topology/dimension/stability).
- Builds a visible publication chain: cost law $\to$ structural consequences $\to$ physics.

2.3 Change 13: Bibliography entry

Location: Bibliography.

Added:

`[WashburnRahnamaiBarghi2026]` J. Washburn and A. Rahnamai Barghi, *Reciprocal Convex Costs for Ratio Matching: Axiomatic Characterization*, Axioms (2026). `doi:10.3390/axioms1010000`.

3 Formatting and Style

3.1 Change 14: Equation punctuation

Location: Multiple display equations throughout the paper.

Rule applied: Every display equation must end with a comma or period, matching the surrounding sentence grammar. Terminal punctuation was added (in teal) to the following equations:

- Intersection dimension formula (Lemma 3.1),
- Effective potential definition (Theorem 4.1),
- Circular orbit condition (Theorem 4.1, Step 1),
- Synchronization period definition (Definition 7.2),
- Green-kernel flux normalization (Appendix A).

3.2 Change 15: Unicode character fix

Location: Remark title in Section 4.

Problem: The remark title contained a raw Unicode $\geq$ character (`U+2265`) which caused a LaTeX error.

Fix: Replaced with math-mode `$D\ge 3$` inside the remark's optional argument.

4 Post-Review Consistency Corrections (This Pass)

After a strict second-pass audit focused only on edited material, four additional consistency fixes were applied.

Change 16: Correct imported Axioms-paper bibliographic metadata

Issue: The prior revision notes listed the wrong DOI/issue metadata for the imported cost-kernel paper.
Fix applied: Updated the manuscript citation entry to: `doi:10.3390/axioms1010000` and removed the incorrect Axioms 15(2), 90 metadata from this citation.

Change 17: Harmonize the linking allowed set with $p \geq 1$

Issue: Some formulas still displayed $\mathcal{A}_A = \{1, 3, 5, \ldots\}$ even after restricting the theorem domain to $p \geq 1$.
Fix applied: Replaced these with $\mathcal{A}_A = \{3, 5, 7, \ldots\}$ in the Introduction, theorem summaries, Section 3 statements, the synthesis theorem, Table 1, and the Conclusion.

Change 18: Align Theorem 1.2 with the two-scale formulation

Issue: The edited theorem referenced an effective manifold model but still applied constraints directly to $\mathcal{C}_R$ in the same sentence.
Fix applied: The theorem now states constraints on , concludes $\dim() = 3$, and explicitly notes this is equivalent to recognition dimension 3 for $\mathcal{C}_R$.

Change 19: Strengthen Section 5 rotational footnote

Issue: One footnote still loosely suggested a global $SO(D)$ action.
Fix applied: Replaced with a local frame-bundle formulation: oriented orthonormal frames form a principal $SO(D)$-bundle over .

5 Items Noted but Not Yet Applied

The following items from v3_comment_1 are noted for a future pass. They require a full-text audit rather than targeted insertions:

1. **Equation numbers:** Verify that *every* numbered display equation in the paper has a label and that no important equation is left unnumbered (currently many use `\[...\]` instead of `\begin{equation}`).
2. **QED boxes:** The review requests removing the tombstone (□) at the end of every proof. The current file uses `amsthm`'s default, which inserts □ automatically. Suppressing it requires adding `\renewcommand{\qedsymbol}{}` in the preamble—*confirm with co-authors before applying*, as some journals require the symbol.
3. **Equation-ending punctuation (exhaustive pass):** The targeted punctuation changes above cover the most prominent cases; a line-by-line audit of all $\sim$80 display equations is recommended before final submission.

6 Summary Table

#	Type	Description
1	Fix	Author order $\to$ alphabetical by last name
2	Fix	"manifold-like" $\to$ reference Definition 2.5
3	Fix	Freedman attribution corrected
4	Fix	"Knot theory only in $D = 3, 4$" rewritten
5	Fix	Chiral anomalies statement made precise
6	Fix	Boundary compatibility $\to$ homology-level argument
7	Fix	$p \geq 1$ restriction; codimension-2 remark reframed
8	Fix	Green-kernel sign: $C > 0$ (not $C < 0$)
9	Fix	$SO(D)$ $\to$ local orthonormal frame rotation group
10	Fix	ω consistency remark added in appendix
11	Import	Intro bridge paragraph citing Axioms paper
12	Import	Proposition 2.5: imported cost-kernel uniqueness theorem
13	Import	Bibliography entry for Axioms paper
14	Style	Terminal punctuation on key display equations
15	Style	Unicode $\geq$ $\to$ `$\ge$` in remark title
16	Fix	Corrected imported Axioms citation metadata (DOI and issue data)
17	Fix	Harmonized $\mathcal{A}_A$ to $\{3, 5, 7, \ldots\}$ for $p \geq 1$
18	Fix	Theorem 1.2 now states constraints on effective manifold consistently
19	Fix	Section 5 footnote rewritten in frame-bundle language