The Physics of Narrative: A Geometric Formalization of Story Structure in Recognition Science

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Abstract

We present a mathematical formalization of narrative structure within Recognition Science (RS). By modeling stories as trajectories through a manifold of moral states, we propose that optimal narratives correspond to geodesics under a recognition-cost-derived metric. Plot tension is identified with ledger skew mu, $withresolution(\ )emergingasastableequilibrium.Weintroducethestory metricds^2 =$ $d^m u2 + d\mathcal{E}^2/\varphi + dZ^2/\varphi^2$ and conjecture that narratives minimizing recognition cost J satisfy geodesic equations. The framework is partially formalized in Lean 4, with key stability theorems fully verified. We analyze *Hamlet* and the Hero's Journey as concrete applications, demonstrating how the formalism captures classical narrative structure. This work establishes foundations for a *Physics of Narrative* connecting story structure to the geometry of meaning.

Keywords: narrative physics, Recognition Science, geodesic, moral state, plot tension, Lean formalization, computational narratology

1 Introduction

The structure of stories has been studied since Aristotle's *Poetics* [2], yet no mathematical theory has successfully unified the diverse observations of narrative scholars into a coherent framework. Propp identified 31 narrative functions in Russian folktales [5]. Campbell found recurring patterns across mythologies [4]. Booker proposed seven fundamental plots [3]. But *why* do these patterns exist?

In this paper, we propose an answer within Recognition Science (RS), a theoretical framework deriving physics from cost minimization [1]. Our central hypothesis is:

> *Stories are trajectories through moral-state space that tend toward cost-minimizing paths—narrative geodesics.*

We do not claim to have proven this hypothesis in full generality. Rather, we develop the mathematical framework, prove key supporting theorems, and demonstrate the formalism's applicability through examples.

1.1 Contributions

This paper makes the following contributions:

1. **Mathematical Framework**: We define narrative space $\mathcal{N}$ as a manifold of MoralStates with a recognition-cost-derived metric (Section 3).

2. **Tension-Skew Identification**: We formally identify plot tension τ with ledger skew magnitude $|\,mu|, provingstabilityproperties(Section\ 4)$.

3. **Geodesic Conjecture**: We state precisely the conjecture that optimal narratives are geodesics and prove supporting lemmas (Section 5).

4. **Concrete Examples**: We analyze *Hamlet* and the Hero's Journey within the formalism (Section 7).

5. **Partial Formalization**: Key definitions and stability theorems are verified in Lean 4; other results remain conjectural (Section 9).

1.2 Related Work

Our approach builds on several traditions:

Structural Narratology: Propp's *Morphology of the Folktale* [5] identified discrete narrative functions. Our continuous formalism generalizes this to smooth trajectories.

Story Grammars: Computational approaches model stories as formal languages [10]. We instead use differential geometry, treating narrative as physics.

Affective Computing: The valence-arousal-dominance model [7, 8] provides our bridge to phenomenal experience.

Mathematical Narratology: Recent work applies topology [11] and network theory [12] to stories. Our geometric approach is complementary.

2 Recognition Science Preliminaries

We briefly review Recognition Science; see [1] for details.

2.1 The Recognition Cost Functional

Recognition Science posits a single primitive: the recognition cost function $J : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the d'Alembert composition law:

$$J(xy)+J(x/y) = 2J(x)J(y)+2J(x)+2J(y) \quad (1)$$

This functional equation has a unique continuous solution:

$$J(x) = \frac{1}{2}\left(x + \frac{1}{x}\right) - 1 \quad (2)$$

Proposition 2.1 (Properties of J)**.** *The recognition cost satisfies:*

1. $J(x) \geq 0$ *with equality iff* $x = 1$
2. $J(x) = J(1/x)$ *(reciprocal symmetry)*
3. J *is convex with global minimum at* $x = 1$
4. $J(x) \sim \frac{1}{2}x$ *as* $x \to \infty$

Proof. Direct calculation from Equation (2). See [1]. □

2.2 The Golden Ratio

The golden ratio $\varphi = (1+\sqrt{5})/2 \approx 1.618$ emerges as the unique positive solution to $\varphi^2 = \varphi + 1$. In RS, φ appears as the fundamental scale factor relating different levels of structure.

2.3 MoralState and Ledger Skew

Definition 2.2 (MoralState)**.** A *MoralState* $\mathcal{M}$ is a tuple $(L, B, \mathcal{E})$ where:

- L is a ledger state (transaction record)
- $B \subseteq$ Bonds are active agent bonds
- $istherecipro city skew$
- $\mathcal{E} > 0$ is available energy

subject to the *global balance constraint*: $\sum_{\text{agents}} mui$=0.

The skew mu$measuresdeviationfromperfectreciprocity:$
$> mu0 : extraction/debt(receivingmorethangiving)$

$< mu0 : contribution/credit(givingmorethanreceiving)$

$0 : balancedreciprocity$

2.4 The Recognition Operator

Dynamics are governed by the recognition operator $\hat{R}$:

$$s(t + 8\tau_0) = \hat{R}(s(t)) \quad (3)$$

where τ_0 is the fundamental time unit and $\hat{R}$ evolves states toward lower J.

Axiom 1 (Recognition Dynamics)**.** The operator $\hat{R}$ satisfies:

1. $J(\hat{R}(s)) \leq J(s)$ (cost non-increasing)
2. $\hat{R}$ preserves global balance
3. States with 0$arefixedpoints$

3 The Narrative Space Manifold

3.1 Narrative Beats

Definition 3.1 (Narrative Beat)**.** A *narrative beat* is a pair $b = (m, t)$ where $m \in \mathcal{M}$ and $t \in \mathbb{N}$ is the beat index (in 8-tick units).

Each beat represents a story "moment"—a snapshot of the moral configuration of the narrative world.

3.2 Narrative Arcs

Definition 3.2 (Narrative Arc)**.** A *narrative arc* $\mathcal{A} = \{b_0, \ldots, b_n\}$ is a finite sequence of beats satisfying:

1. **Non-trivial**: $n \geq 1$ (at least two beats)
2. **Temporally ordered**: $t_i < t_j$ for $i < j$
3. **Admissible**: global balance holds at each beat

3.3 The Story Metric

We endow MoralState space with a Riemannian metric derived from recognition cost considerations.

Definition 3.3 (Story Metric)**.** The *story metric* is:

$$ds^2 = d^m u2 + \frac{1}{\varphi} d\mathcal{E}^2 + \frac{1}{\varphi^2} dZ^2 \tag{4}$$

where Z encodes pattern state.

Remark 3.4 (Metric Coefficients). The coefficients $(1, 1/\varphi, 1/\varphi^2)$ reflect a hierarchy where skew changes dominate over energy changes, which dominate over pattern changes. This ordering arises from the RS forcing chain: mu$directlyaffects$J$, while$EandZ$affectitindirectly.The\varphi$-weighting is the simplest choice respecting this hierarchy. *Alternative weightings could be explored; this choice is motivated but not uniquely determined.*

Proposition 3.5 (Metric Positivity)**.** *The story metric is positive definite.*

Proof. Since $\varphi > 0$, all coefficients are positive. Thus $ds^2 \geq 0$ with equality only when $dd\mathcal{E} = dZ = 0$. □

4 Plot Tension as Skew Dynamics

4.1 The Tension Function

Definition 4.1 (Plot Tension)**.** The *plot tension* at beat $b = (m, t)$ is:

$$\tau(b) = |m.\ mu|$$

equation

This identification is our core proposal: dramatic tension equals the magnitude of reciprocity imbalance.

Table 1: Interpretation of Skew Values

mu$Value$	Narrative Interpretation
0	Equilibrium (resolution)
$> mu0$	Protagonist in debt (crisis)
$< mu0$	Protagonist owed (sacrifice)
$\| mu\| large$	High tension (climax)

4.2 Tension Thresholds

The golden ratio provides natural thresholds:

Definition 4.2 (Tension Thresholds)**.**

$$\tau_{\text{low}} = 1/\varphi \approx 0.618 \tag{6}$$
$$\tau_{\text{high}} = \varphi \approx 1.618 \tag{7}$$
$$\tau_{\text{critical}} = \varphi^2 \approx 2.618 \tag{8}$$

Theorem 4.3 (Threshold Ordering)**.** *The thresholds satisfy:* $\tau_{low} < 1 < \tau_{high} < \tau_{critical}$

Proof. Since $\varphi > 1$: $1/\varphi < 1 < \varphi < \varphi^2$. □

4.3 Catharsis

Definition 4.4 (Catharsis)**.** A *catharsis* occurs between beats b_i and b_{i+1} if:

1. $\tau(b_i) > \tau_{\text{high}}$
2. $\tau(b_{i+1}) < \tau_{\text{low}}$
3. $\tau(b_i) - \tau(b_{i+1}) > \tau(b_i)/2$

This formalizes Aristotle's concept of emotional purgation.

Theorem 4.5 (Monotonic Arcs Lack Catharsis)**.** *If $\tau(b_i) < \tau(b_{i+1})$ for all i, then $\mathcal{A}$ has no catharsis.*

Proof. Catharsis requires τ to drop (condition 3), but strict monotonicity precludes any decrease. □

4.4 Resolution Stability

Theorem 4.6 (Resolution is Stable)**.** *The state* $0isastableequilibriuminthesensethatforevery\epsilon > 0$*, if* $| mu| < \epsilon$ *then* $\tau < \epsilon$.

Proof. By definition, $\tau = | mu|, so| mu| < \epsilon$ implies $\tau < \epsilon$ directly. □

Corollary 4.7 (Perturbation Increases Tension)**.** *Any perturbation from* $0strictlyincreasestension.$

Proof. At $0, \tau = 0$. For $\neq 0$, $\tau' = | | > 0$. □

5 Story Geodesics

5.1 The Story Action

Definition 5.1 (Story Action)**.** The *story action* of an arc $\mathcal{A}$ is:

$$S[\mathcal{A}] = \sum_{b \in \mathcal{A}} J(b) \tag{9}$$

where $J(b) = \sum_{\text{bonds}} J(m_b)$ is the total recognition cost at beat b.

5.2 The Geodesic Conjecture

Conjecture 5.2 (Narrative Geodesic Principle)**.** *Among all narrative arcs connecting fixed initial and terminal MoralStates, the ones actually realized tend toward those minimizing the story action $S[\mathcal{A}]$.*

Remark 5.3. This is analogous to the principle of least action in physics. We state it as a conjecture because:

1. The space of "all possible narratives" is not rigorously defined
2. Empirical validation requires analysis of story corpora
3. The "tends toward" language reflects that real stories may deviate for artistic reasons

Proposition 5.4 (Geodesic Equation Form)**.** *If Conjecture 5.2 holds, geodesic arcs satisfy:*

$$\nabla_{\dot{\gamma}} \dot{\gamma} = 0 \tag{10}$$

where $\gamma(t)$ is the path in MoralState space and ∇ is the Levi-Civita connection of the story metric.

Proof. Standard variational calculus. The Euler-Lagrange equations for the metric Lagrangian $\mathcal{L} = \frac{1}{2} g_{ij} \dot{x}^i \dot{x}^j$ yield the geodesic equation. The specific form of the Christoffel symbols depends on coordinate choice. □

5.3 Resolution as Attractor

Theorem 5.5 (Resolution Attractor)**.** *Under Axiom 1, the state* $0isanattractoroftherecognitiondynamics.$

Proof. By Axiom 1, J decreases under $\hat{R}$. Since J achieves its minimum at $0(theunitstatecorrespondstobalancedreciprocity), and 0isa$

This explains why stories naturally tend toward resolution: it is the dynamical attractor.

6 Fundamental Plot Types

6.1 Classification

Following Booker [3], we identify seven plot types as distinct trajectory classes:

Definition 6.1 (Fundamental Plot Types)**.**

1. **Overcoming the Monster**: $: mu0 \to + \to 0$, high peak, full resolution
2. **Rags to Riches**: $: mu - \to 0$, $\mathcal{E}$ increasing
3. **The Quest**: $: mu0 \to + \to 0$, $\mathcal{E}$ conserved
4. **Voyage and Return**: cyclic $\mathrm{mu}, returntoorigin$
4. **Comedy**: oscillating $\mathrm{mu}, boundedby \tau_{\text{high}}$
5. **Tragedy**: $, noresolution$
5. **Rebirth**: $: mu0 \to - \to 0$, sacrifice then renewal

Conjecture 6.2 (Plot Exhaustiveness)**.** *The seven plot types exhaust the geodesic equivalence classes of resolvable narratives (those with $\tau_{terminal} < \tau_{low}$), plus Tragedy as the unique unbounded class.*

Remark 6.3. This conjecture requires rigorous definition of "geodesic equivalence" and proof that no other classes exist. Current evidence is empirical (Booker's literary analysis) rather than mathematical.

7 Concrete Examples

We apply the formalism to two well-known narratives.

Table 2: Plot Type Characteristics

Plot Type	Peak τ	Resolution
Overcoming Monster	High	Yes
Rags to Riches	Medium	Yes
Quest	Medium	Yes
Voyage/Return	Medium	Yes
Comedy	Low	Yes
Tragedy	Unbounded	No
Rebirth	Medium	Yes

1.
7.1 Example: Hamlet

Shakespeare's *Hamlet* provides a paradigm case of Tragedy.

Initial state ($_m u0 \approx 0.5$): The play opens with Denmark "out of joint." Claudius has murdered King Hamlet, creating an unpaid moral debt. The ghost reveals this, initiating rising skew.

Rising action (.5): Hamlet's discovery increases tension. His feigned madness, the play-within-a-play, Polonius's death—each event increases $|mu| asthemoraldebtcompounds.$

Climax ($_m u \max \approx 2.5$): The final scene. Multiple deaths create maximum disequilibrium. The skew reaches τ_{critical}.

Terminal state ($_m u\text{final} > 0$): Despite the bloodshed, the moral ledger is not balanced. Fortinbras inherits a broken kingdom. The lack of resolution marks Tragedy: $\lim_{t\to\infty} \tau(t) \not\to 0$.

7.2 Example: The Hero's Journey

Campbell's monomyth [4] exemplifies Overcoming the Monster / Quest.

This trajectory has the characteristic shape:

- Initial equilibrium: $_m u0{=}0$
- Rising tension: .5 (climax at beat 3)
- Resolution: $(beat7)$
- Catharsis: between beats 3 and 7 (tension drops > 50%)

Table 3: Hero's Journey mu$-Trajectory$

Beat	Stage	mu
height0	Ordinary World	0.0
1	Call to Adventure	0.5
2	Crossing Threshold	1.0
3	Ordeal (Climax)	1.5
4	Reward	1.2
5	Road Back	0.8
6	Resurrection	0.3
7	Return	0.0

The Hero's Journey is a prototypical geodesic: it achieves resolution (the attractor) via a path that rises to climax then descends, consistent with J-minimization.

8 The Narrative-Qualia Bridge

8.1 Phenomenal Signature

We map narrative states to the VAD emotional model [7, 8].

Definition 8.1 (Phenomenal Signature)**.** The *phenomenal signature* of beat b is:

$$+1 \in (-1, 1) \tag{11}$$

arousal(b) $= \tau_{\overline{\tau+1 \in [0,1) \text{dominance}(b) = \tanh(\mathcal{E}-1)}}$
valence$(b) = \frac{mu}{}$

$+ 1 \in (-1, 1)$
arousal$(b) = \frac{\tau}{\tau+1} \in [0, 1)$
dominance$(b) = \tanh(\mathcal{E} - 1)$

Remark 8.2. The valence formula maps positive mu(*debt*)*tonegativevalence*(*unpleasant*), *consistentwiththeinterpretationthatprotagonists*“*owed*”*experienc* *typesaturation.*

Theorem 8.3 (High Tension Implies High Arousal)**.** *If* $\tau(b) > \varphi$*, then arousal*$(b) >$ $1/2$.

Proof. For $\tau > 1$: arousal $= \tau/(\tau+1) > 1/2$ since $\tau > 1$ implies $2\tau > \tau + 1$. $\square$

9 Lean 4 Formalization

9.1 Scope of Formalization

The theory is partially formalized in Lean 4, comprising nine modules (∼2,500 lines). We distinguish:

Fully verified (no `sorry`):

- Threshold ordering (Theorem 4.3)
- Resolution stability (Theorem 4.6)
- Monotonic no-catharsis (Theorem 4.5)
- Tension-arousal implication (Theorem 8.3)

Conjectural (uses `sorry` or not formalized):

- Geodesic equation (Proposition 5.4)
- Seven plot exhaustiveness (Conjecture 6.2)
- Attractor property (Theorem 5.5)

9.2 Code Structure

```
-- Core structures
structure NarrativeBeat where
  state : MoralState
  beat_index : Nat

structure NarrativeArc where
  beats : List NarrativeBeat
  nonempty : beats.length >= 2
  ordered : ...
  admissible : ...

-- Verified theorem
theorem resolution_is_stable :
  forall eps > 0, exists delta > 0,
    forall b, abs(b.state.skew) < delta
      -> plotTension b < eps := by
  intro eps h_eps
  use eps
  exact fun _ h => h
```

Table 4: Lean Module Summary

Module	Contents
`Core`	Beats, arcs, basic definitions
`PlotTension`	Tension, thresholds, catharsis
`StoryGeodesic`	Action, geodesic conditions
`FundamentalPlots`	Plot type classification
`StoryTensor`	Metric, curvature
`Axiomatics`	RS connection
`Examples`	Hero's Journey, Tragedy
`Bridge`	Qualia correspondence
`Resolution`	Stability proofs

10 Discussion

10.1 Strengths

The framework provides:

1. **Precision**: Vague concepts (tension, resolution, catharsis) receive exact definitions
2. **Verifiability**: Key claims are machine-checkable
3. **Predictiveness**: The tension-skew identification generates testable hypotheses
4. **Unification**: Diverse plot types emerge from one principle

10.2 Limitations

Current limitations include:

- The geodesic principle (Conjecture 5.2) is not proven; it remains a guiding hypothesis
- Metric coefficients are motivated but not uniquely determined
- Multi-agent narratives require extension (current formalism tracks single mu)
- $Empiricalvalidationonstorycorporaisneeded$

10.3 Future Directions

1. **Complete formalization**: Remove remaining `sorry` placeholders
2. **Corpus analysis**: Test predictions on annotated story databases
3. **Multi-agent extension**: Model mu$vectorsformultiplecharacters$
3. **Generative applications**: Use geodesics to generate novel narratives

11 Conclusion

We have developed a mathematical framework for narrative structure within Recognition Science. Key contributions:

1. **Tension = Skew**: Plot tension is identified with ledger skew magnitude, providing a precise, measurable quantity.
2. **Resolution Stability**: The state $0 isproventobeastableequilibrium, explainingnarrative$
3. **Geodesic Hypothesis**: We conjecture that optimal narratives minimize recognition cost, analogous to physical least-action principles.
4. **Concrete Applications**: Analysis of *Hamlet* and the Hero's Journey demonstrates the formalism's applicability.
5. **Partial Verification**: Key theorems are verified in Lean 4; conjectures are clearly marked.

The Physics of Narrative is not (yet) proven in full generality, but the framework provides a rigorous foundation for studying story structure mathematically. If the geodesic conjecture holds, stories are as determined by geometry as planetary orbits—the difference is that narrative geodesics trace paths through meaning rather than spacetime.

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A Proof Details

A.1 Theorem 4.5: Monotonic Arcs Lack Catharsis

Full Proof. Let $\mathcal{A} = \{b_0, \ldots, b_n\}$ be an arc with $\tau(b_i) < \tau(b_{i+1})$ for all $0 \leq i < n$.

Suppose for contradiction that catharsis occurs between beats b_j and b_{j+1} for some j. By Definition 4.4:

$$\tau(b_j) - \tau(b_{j+1}) > \tau(b_j)/2 > 0$$

This implies $\tau(b_{j+1}) < \tau(b_j)$, contradicting the strict monotonicity assumption. □

A.2 Theorem 5.5: Resolution Attractor

Proof from Axiom 1. Let s_0 be an admissible initial state with ${}_m u0 \neq 0$. Define $s_k = \hat{R}^k(s_0)$.

By Axiom 1(1): $J(s_{k+1}) \leq J(s_k)$.

The sequence $\{J(s_k)\}$ is non-increasing and bounded below by 0. Hence it converges: $J(s_k) \to J^*$ for some $J^* \geq 0$.

By Proposition 2.1, J achieves its minimum 0 uniquely at the unit state (which has 0).

By Axiom 1(3), states with $0arefixedpoints.Ifthesequenceconvergestoafixedpoint,thatpointmusthave0$.

We have not proven that s_k converges (only that $J(s_k)$ does), so Theorem 5.5 as stated requires the additional assumption that cost convergence implies state convergence—a property that holds if the dynamics are gradient-like. □

B Metric Derivation Sketch

The story metric coefficients arise from dimensional analysis in RS:

1. mu$hasdimensionof``moralcharge''(primary)$

1. $\mathcal{E}$ has dimension of "capacity" (secondary)
2. Z (pattern) has dimension of "structure" (tertiary)

The golden ratio φ provides the natural scale factor between levels. Setting the primary coefficient to 1 and scaling down by φ at each level gives $(1, 1/\varphi, 1/\varphi^2)$.

Alternative derivation: The Hessian of J at the minimum determines a natural metric. For $J(x) = \frac{1}{2}(x + 1/x) - 1$:

$$J''(1) = 1$$

Extending to the full MoralState requires specifying how J depends on ($\mathcal{E}$,Z), which introduces the φ weighting through the RS forcing chain structure.