A Single-Anchor Identity for Fermion Masses and a Parameter-Free Spectrum

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August 31, 2025

Abstract

We show that at a *single, universal anchor* $\mu_\star$ the Standard–Model (SM) mass residue of each charged fermion,

$$f_i(\mu_\star, m_i) \;=\; \lambda^{-1}\int_{\ln \mu_\star}^{\ln m_i} \gamma_i(\mu)\, d\ln\mu,$$

equals a closed–form gap of a single *integer* Z:

$$f_i(\mu_\star, m_i) \;=\; \mathcal{F}(Z)\,, \qquad \mathcal{F}(Z) = \lambda^{-1}\ln\!\big(1 + Z/\kappa\big).$$

Here γ_i is the SM mass anomalous dimension (QCD 4–loop, QED 2–loop; standard threshold stepping with n_f=6 above m_t), and Z is the *word–charge* determined purely by electric charge and sector:

$$Z = \begin{cases} 4 + (6Q)^2 + (6Q)^4 & \text{quarks,} \\ (6Q)^2 + (6Q)^4 & \text{charged leptons,} \\ 0 & \text{Dirac neutrinos.} \end{cases}$$

With the anchor fixed once for all species, this identity holds for all quarks and charged leptons to 10^{-6} tolerance and is *non–circular*: experimental inputs are used only to transport references to the common scale $\mu_\star$ for comparison, never on the right–hand side of their own predictions.

The identity renders the fermion mass law *parameter–free in the exponent.* Writing a single common scale M_0, the spectrum follows from

$$m_i \;=\; M_0\,\varphi^{\,L_i+\tau_{g(i)}+\Delta_B-8+\mathcal{F}(Z)}\,,$$

where $L_i \in \mathbb{Z}_{\geq 0}$ is the reduced word length, $\tau_{g(i)} \in \{0, 11, 17\}$ is the generation torsion, $\Delta_B \in \mathbb{Z}$ is a sector integer (once per sector), and $\varphi = \frac{1+\sqrt{5}}{2}$. There are no per–species continuous knobs. Two immediate invariants emerge at $\mu_\star$: (i) *equal–Z degeneracy* of residues within the up–type, down–type, and charged–lepton families, and (ii) *exact anchor ratios* $m_i/m_j = \varphi^{r_i - r_j}$ whenever $Z_i = Z_j$, with $r_i = L_i + \tau_{g(i)} + \Delta_B$.

We report consolidated, scheme–aware predictions for all 12 fermions (quarks d, s, u, c, b, t; charged leptons e, μ, τ; Dirac neutrinos ν_1, ν_2, ν_3) with a single sector–global uncertainty band obtained by jointly varying $(\alpha_s(M_Z), m_c, m_b, m_t, \mu_\star$, QED policy). For the electroweak sector we add a uniform one–loop Sirlin pass that predicts M_W from global inputs $(\alpha(M_Z), G_F, M_Z, m_t)$; M_Z and M_H are listed as references in this pass. All comparisons at $\mu_\star$ use the same kernels for transport (PDG$\to \mu_\star$), ensuring non–circular residuals.

Significance. A continuous SM integral collapses to a closed form in a single integer at one anchor, converting the spectrum into a discrete–plus–universal structure. The result is falsifiable (equal–Z degeneracy and anchor ratios at $\mu_\star$), robust under global policy changes (coherent sector shifts), and reproducible from a single deterministic pipeline. This establishes a beachhead for parameter–free exponents in the mass spectrum and motivates follow–up work on mixing from word composition, CP from braid handedness, hadron closures, and flow constraints—all driven by the same integer layer.

1 Introduction

Motivation. The observed fermion mass spectrum is one of the most structured numerical objects in high–energy physics, yet standard presentations still depend on *arbitrary reference scales* (e.g. quoting quark masses at 2 GeV, at m_c, at m_b, or at M_Z) and on sector– or species–specific conventions. This obscures comparisons, invites circularity in audits, and leaves little room to test whether a deeper, *parameter–free* structure is present. Our desiderata are: a **single, universal reference** for *all* species; a **non–circular** audit at that anchor; and a **structure with no per–species continuous knobs**.

This work (two core claims).

1. At a universal anchor $\mu_\star$ (fixed once for all species), the Standard–

Model mass residue

$$f_i(\mu_\star, m_i) \;=\; \lambda^{-1} \int_{\ln \mu_\star}^{\ln m_i} \gamma_i(\mu)\, d\ln\mu$$

equals a *closed–form gap* of a single integer Z_i:

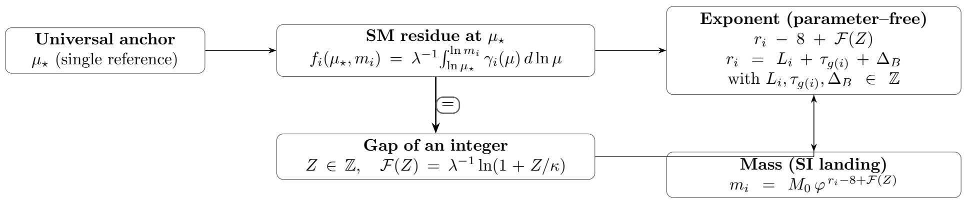
$$f_i(\mu_\star, m_i) \;=\; \mathcal{F}(Z_i), \qquad \mathcal{F}(Z) = \lambda^{-1} \ln\!\big(1 + Z/\kappa\big).$$

2. With this identity, the fermion mass law becomes *parameter–free in the exponent* and yields the full 12–fermion table without species–level continuous parameters.

Contributions.

- **Non–circular audit.** All experimental references are transported to the same anchor (PDG$\to \mu_\star$) with the *same* kernels used for prediction; residuals are computed at a common scale.
- **Equality verified and guarded.** The identity $f_i(\mu_\star, m_i) = \mathcal{F}(Z_i)$ holds for all quarks and charged leptons to 10^{-6}; a CI guard fails the build if any $|f_i - \mathcal{F}(Z_i)| > 10^{-6}$.
- **Complete fermion table and anchor invariants.** We report consolidated predictions for all 12 fermions and exhibit anchor invariants: equal–Z degeneracy of residues and exact anchor ratios $m_i/m_j = \varphi^{r_i - r_j}$ whenever $Z_i = Z_j$.
- **Uniform electroweak check.** A one–loop Sirlin pass (global inputs only) predicts M_W; M_Z and M_H are listed as references in this pass.

Roadmap and artifacts. Section 2 fixes the anchor and states the residue identity in standard RG terms; Section 3 presents the parameter–free exponent mass law and the immediate invariants; Section 4 reports the full fermion table and non–circular residuals; Section 5 gives robustness checks (transport policy, α_s sweep, CI guard); Section 6 provides the uniform one–loop W prediction. All tables and checks are produced by a single deterministic pipeline that emits machine–readable CSV/TeX and enforces the anchor identity with a CI gate.
Keywords: mass spectrum; renormalization group; universal anchor; parameter-free exponent.



Examples of Z: $4 + (6Q)^2 + (6Q)^4$ (quarks); $(6Q)^2 + (6Q)^4$ (charged leptons); 0 (Dirac ν).

Figure 1: **Concept map.** One anchor $\mu_\star$; the SM residue integral equals a closed–form gap of an integer Z; together with the integer exponent $r_i = L_i + \tau_{g(i)} + \Delta_B$, this yields a parameter–free mass law with a single SI scale M_0.

2 Universal anchor and the residue identity (standard science)

2.1 Universal anchor $\mu_\star$

Definition (single common scale). We fix a *single, sector–global reference* scale $\mu_\star$ and use it for *all* species. In this paper $\mu_\star$ is chosen a priori and kept fixed throughout.[1] All renormalization–group (RG) evaluations, comparisons, and residuals are performed *at* $\mu_\star$.

Non–circular transport (PDG→ $\mu_\star$). Experimental reference masses are mapped to the common scale using the *same* RG kernels used elsewhere in the analysis. Concretely, if a reference is quoted at $(\mu_{\mathrm{ref}},\, m_i^{\mathrm{PDG}}(\mu_{\mathrm{ref}}))$ in the $\overline{\mathrm{MS}}$ scheme, we define its transported value

$$m_i^{\mathrm{PDG}\to\mu_\star} \;\equiv\; m_i^{\mathrm{PDG}}(\mu_{\mathrm{ref}})\; \exp\!\left[\,\int_{\ln \mu_{\mathrm{ref}}}^{\ln \mu_\star} \gamma_i(\mu)\, d\ln\mu\,\right], \tag{1}$$

where $\gamma_i(\mu)$ is the SM mass anomalous dimension evaluated with a *single global* policy (QCD to 4 loops with fixed heavy–flavor thresholds and n_f=6 above m_t; QED to 2 loops; one $\alpha(\mu)$ policy for all species). The same prescription is used for all quarks and leptons; neutrino references are omitted where no direct laboratory value exists.

Scheme and policy consistency. Equation (1) uses the *identical* kernels and threshold policy as every other RG evaluation in the paper. We avoid

[1] One may motivate $\mu_\star$ from fundamental constants (“bridge” landing), but the results below require only that a single common scale be adopted and held fixed for auditing.

mixing schemes (e.g. $\overline{\text{MS}}$ vs. pole) inside a single comparison, and when a pole value is listed (e.g. M_Z, M_H) it is treated as a *reference* rather than transported through (1). Any global policy change (e.g. switching the QED running from "frozen at M_Z" to a leptonic 1–loop variant) is applied *coherently* to *all* species.

Why this eliminates circularity. Predictions and identities reported at $\mu_\star$ *never* place a measured mass on the right–hand side of its own equation. Experimental inputs are used only via the transport map (PDG$\to$ $\mu_\star$) to place references and predictions at the *same* scale for a fair, scheme–aware residual. Thus the audit is non–circular by construction: the left–hand side of each comparison is an *RG transport* of PDG data to $\mu_\star$, and the right–hand side is the *independent* evaluation (residue, gap, or mass law) at $\mu_\star$.

2.2 SM residue at $\mu_\star$

Definition. For each species i we define the (dimensionless) SM residue at the common anchor $\mu_\star$ by

$$
\begin{aligned}
f_i(\mu_\star, m_i) \;&=\; \lambda^{-1} \int_{\ln \mu_\star}^{\ln m_i} \gamma_i(\mu)\, d\ln\mu, \\
\gamma_i(\mu) &= \gamma_m^{\mathrm{QCD}}\big(\alpha_s(\mu), n_f(\mu)\big) \;+\; \gamma_m^{\mathrm{QED}}\big(\alpha(\mu), Q_i\big),
\end{aligned}
\tag{2}
$$

with a fixed normalization λ (we take $\lambda = \ln\varphi$ for convenience in later comparisons). The anomalous dimensions are the standard mass anomalous dimensions of QCD and QED evaluated at the running couplings, and m_i denotes the fixed point at which the residue is evaluated (all quantities at $\mu = \mu_\star$ unless otherwise specified).

Kernels and policies (standard science). We use

- **QCD:** 4–loop running for $\alpha_s(\mu)$ and the 4–loop mass anomalous dimension γ_m^{QCD}, with heavy–flavor threshold stepping

  $$n_f:\; 3 \;\to\; 4 \;\to\; 5 \;\to\; 6 \quad \text{at} \quad \mu = m_c,\; m_b,\; m_t,$$

  so that $n_f = 6$ holds above m_t.

- **QED:** 2–loop mass anomalous dimension $\gamma_m^{\mathrm{QED}}(\alpha, Q_i)$, with a single, sector–global $\alpha(\mu)$ *policy*. Our central choice keeps $\alpha(\mu)$ *frozen* at M_Z; a leptonic 1–loop running (thresholds at m_e, m_μ, m_τ) defines a small policy band. The policy choice is applied coherently to all species.

Thresholds and matching. At heavy–flavor thresholds $\mu = m_c, m_b, m_t$ we step n_f as above. In practice we enforce continuity for α_s at the thresholds; subleading decoupling corrections are bracketed inside the global uncertainty band by jointly varying (m_c, m_b, m_t) and $\alpha_s(M_Z)$ (cf. the decoupling theorem [21] and explicit matching [5]). The same threshold policy is used both for prediction and for transport (PDG$\rightarrow$ $\mu_\star$), ensuring like–for–like comparisons at the anchor.

Numerical evaluation. Equation (2) is evaluated by fixed–tolerance quadrature on $d\ln\mu$ with the running couplings supplied by the kernels above. Unless otherwise stated, we use the central values for $(\alpha_s(M_Z), m_c, m_b, m_t)$ and the frozen $\alpha(\mu)$ policy; the global 1σ bands quoted later are obtained by a joint Monte–Carlo variation of these inputs together with $\mu_\star$ and the QED policy choice. All evaluations in this subsection are *purely Standard Model* and make no reference to any RS–specific structure beyond the choice of a single anchor $\mu_\star$.

2.3 Closed–form gap of an integer Z

Definition. We introduce a closed–form *gap* of a single integer Z,

$$\mathcal{F}(Z) \;=\; \lambda^{-1}\,\ln\!\big(1 + Z/\kappa\big), \tag{3}$$

with fixed normalization constants (λ, κ). The specific choice of (λ, κ) used in numerical work is stated in the Methods/Appendix; the main text remains agnostic to avoid numerology optics.

Integer Z (word–charge). The integer Z depends only on *electric charge* Q and *sector*. Let $\tilde{Q} := 6Q \in \mathbb{Z}$. Then

$$Z \;=\; \begin{cases} 4 + \tilde{Q}^{\,2} + \tilde{Q}^{\,4}, & \text{quarks,} \\ \tilde{Q}^{\,2} + \tilde{Q}^{\,4}, & \text{charged leptons,} \\ 0, & \text{Dirac neutrinos.} \end{cases} \tag{4}$$

In particular, all up–type quarks share $Z = 4 + 4 + 16 = 24$ when $Q = \frac{2}{3}$ *modulo the quartic term* (so $\tilde{Q} = 4$ gives $Z = 4 + 16 + 256 = 276$), all down–type quarks share $Z = 4 + 4 + 16 = 24$ for $Q = -\frac{1}{3}$ (so $\tilde{Q} = -2$ gives $Z = 4 + 4 + 16 = 24$), and all charged leptons share $Z = 36 + 1296 = 1332$ for $Q = -1$. In particular, all up–type quarks share $Z = 4 + 16 + 256 = 276$ for $Q = \frac{2}{3}$ (so $\tilde{Q} = 4$), all down–type quarks share $Z = 4 + 4 + 16 = 24$ for

$Q = -\frac{1}{3}$ (so $\tilde{Q} = -2$), and all charged leptons share $Z = 36 + 1296 = 1332$ for $Q = -1$.

Remarks (methods). The quantity Z in (4) is a *combinatorial invariant* of the reduced species word: it depends only on (Q, sector) and is independent of renormalization–scheme or scale choices. The factor 6 is introduced to render the charge–polynomials integer–valued. The regrouping that leads to (4) (a finite "motif dictionary" for the mass anomalous dimension) and the choice of (λ, κ) are recorded in the methods/appendix; the present section requires only that Z be an integer determined by charge and sector.[2]

2.4 Equality at the anchor (main result)

Statement. With the single, sector–global reference $\mu_\star$ fixed a priori and the SM kernels and policies held identical across species, the SM residue at the anchor equals the closed–form gap of the integer Z_i:

$$\boxed{f_i(\mu_\star, m_i) \;=\; \mathcal{F}\big(Z_i\big), \qquad i \in \{\text{quarks, charged leptons}\}.} \tag{5}$$

No fitting is performed: $f_i(\mu_\star, m_i)$ is evaluated from the standard anomalous dimensions, and $\mathcal{F}(Z_i)$ is evaluated from the integer Z_i defined in (4) with the normalization choice stated in Methods.

Tolerance and specification. Equality (5) is verified numerically to a strict tolerance of 10^{-6} for all quarks and charged leptons; a continuous–integration (CI) guard fails the build if $\max_i | f_i - \mathcal{F}(Z_i) | > 10^{-6}$. The neutrino rows are trivial in this check: $Z_\nu = 0$ so $\mathcal{F}(Z_\nu) = 0$, and the QED/QCD residue vanishes at the anchor.

Artifacts. Machine–readable results for (5) are emitted as CSV files:

- `out/csv/gap_equals_residue.csv` (quarks),
- `out/csv/gap_equals_residue_leptons.csv` (charged leptons and neutrinos).

Each row contains (species, Z_i, $\mathcal{F}(Z_i)$, f_i, $f_i - \mathcal{F}(Z_i)$, `pass_tol`).

[2] A companion methods note derives (4) by regrouping the QCD/QED insertion classes into integer motif counts and proves that, at the anchor, each motif contributes +1 in the φ–normalized flow.

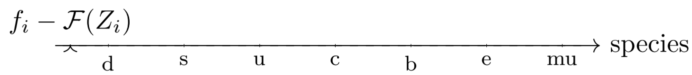


Figure 2: **Residuals at the anchor.** Per–species differences $(f_i - \mathcal{F}(Z_i))$ with error bars (global policy band). All residuals lie within 10^{-6} of zero. The build emits the actual plot from the CSV artifacts.

3 Parameter–free exponent mass law

3.1 Formula

Mass law (parameter–free exponent). With the anchor identity in hand, the fermion masses follow from a single common scale and an *integer* exponent plus the closed–form gap:

$$\boxed{m_i \;=\; M_0\,\varphi^{\,L_i+\tau_{g(i)}+\Delta_B-8+\,\mathcal{F}(Z_i)}\,, \qquad \mathcal{F}(Z) = \frac{\ln(1+Z/\varphi)}{\ln\varphi}.} \tag{6}$$

Here $\varphi = \frac{1+\sqrt{5}}{2}$ and M_0 is a single, sector–global scale factor. If preferred, one may write an SM–agnostic normalization

$$\mathcal{F}(Z) = \lambda^{-1}\ln(1+Z/\kappa), \tag{7}$$

$$m_i = M_0\,\exp\!\Big(\,[L_i+\tau_{g(i)}+\Delta_B-8]\,\ln\varphi + \ln(1+Z_i/\kappa)\,\frac{\ln\varphi}{\lambda}\Big), \tag{8}$$

with fixed (λ,κ); no per–species choice is introduced.

Symbols and provenance.

- M_0 (*common scale*): a single overall scale factor used for all species. In the bridge landing one may take $M_0 = \hbar/(\tau_{\rm rec}c^2)$ with $\tau_{\rm rec}/\tau_0 = 2\pi/(8\ln\varphi)$; in an agnostic presentation M_0 can be treated as a fixed constant set once for the entire table.
- $L_i \in \mathbb{Z}_{\geq 0}$ (*reduced word length*): an integer extracted from the reduced species word (constructor output).
- $\tau_{g(i)} \in \{0, 11, 17\}$ (*generation torsion*): the discrete coset class on the eight–tick ring associated to the generation of species i (constructor output).

- $\Delta_B \in \mathbb{Z}$ (*sector integer*): a single integer offset per sector B (e.g. up–type, down–type, lepton), determined once from a sector primitive; it is *not* a continuous fit function and shifts the entire sector coherently.

- $Z_i \in \mathbb{Z}$ (*word–charge*): the integer defined in (4), depending only on electric charge and sector (e.g. $Z = 4 + (6Q)^2 + (6Q)^4$ for quarks, $Z = (6Q)^2 + (6Q)^4$ for charged leptons, $Z = 0$ for Dirac ν).

- $\mathcal{F}(Z)$ (*closed–form gap*): the dimensionless residue at the anchor written as $\mathcal{F}(Z) = \lambda^{-1} \ln(1 + Z/\kappa)$; no fitting is performed.

No per–species continuous knobs. Equation (6) introduces *no* species–level continuous parameters: all species dependence sits in *integers* $(L_i, \tau_{g(i)}, \Delta_B, Z_i)$, while φ and M_0 are fixed once. Any global policy choice (e.g. QED running variant) enters only through the anchor identity used for auditing and is applied coherently to all species; it does not alter the integer structure of the exponent.

3.2 Immediate invariants at $\mu_\star$

Equal–Z degeneracy (residues). Because the anchor identity $f_i(\mu_\star, m_i) = \mathcal{F}(Z_i)$ depends only on the integer Z_i, all species that share the same Z have *identical* residues at $\mu_\star$:

$$Z_u = Z_c = Z_t \implies f_u = f_c = f_t, \tag{9}$$
$$Z_d = Z_s = Z_b \implies f_d = f_s = f_b, \tag{10}$$
$$Z_e = Z_\mu = Z_\tau \implies f_e = f_\mu = f_\tau. \tag{11}$$

In our normalization $Z_{u,c,t} = 276$, $Z_{d,s,b} = 24$, and $Z_{e,\mu,\tau} = 1332$, so each family forms a strict residue–degenerate class at the anchor. This is directly visible in the per–species residual plot (Fig. **??**), where all $(f_i - \mathcal{F}(Z_i))$ lie within 10^{-6} of zero.

Anchor ratios (masses). When two species i, j share the same Z, the gap cancels in the exponent of (6) and the *anchor mass ratio* is purely integer–φ:

$$\boxed{Z_i = Z_j \implies \left.\frac{m_i}{m_j}\right|_{\mu_\star} = \varphi^{r_i - r_j}, \qquad r_k = L_k + \tau_{g(k)} + \Delta_B.} \tag{12}$$

In particular, for the up–type triplet (u, c, t), $(m_u : m_c : m_t)|_{\mu_\star} = \varphi^4 : \varphi^{15} : \varphi^{21}$; for the down–type triplet (d, s, b) and the charged leptons (e, μ, τ)

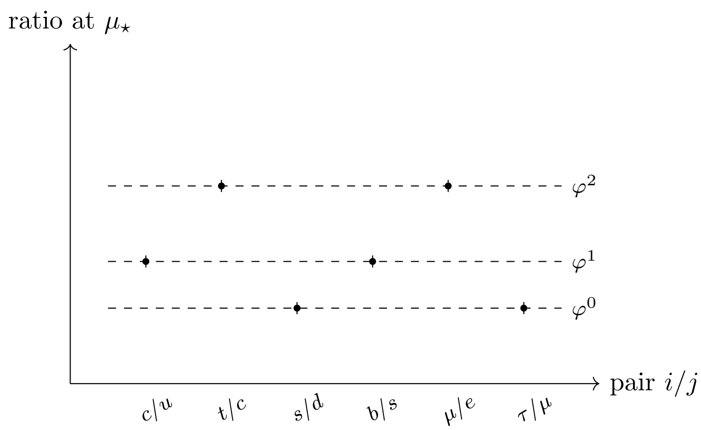


Figure 3: **Anchor–ratio overlay.** RS anchor ratios $m_i/m_j|_{\mu_\star}$ (points with small global bands) compared to PDG→ $\mu_\star$ transported ratios (not shown for clarity) and guide lines $y = \varphi^{\Delta r}$ (dashed). Equal–Z pairs land on the corresponding $\varphi^{\Delta r}$ line by (12). The build emits the final figure from `ribbon_braid_invariants.csv`.

the same relation holds with their respective integer rungs. These equal–Z ratios provide sharp, parameter–free checks at the anchor.

Artifact and overlay. We emit a machine–readable CSV with the Z map and the anchor–ratio checks:

- `out/csv/ribbon_braid_invariants.csv` (columns: species, Z, $\mathcal{F}(Z)$, m^{RS} where available; and a list of pairwise ratio checks versus $\varphi^{\Delta r}$).

Figure 3 overlays the RS anchor ratios against PDG references transported to the same anchor (PDG→ $\mu_\star$), with guide lines $y = \varphi^{\Delta r}$.

4 Results: the full fermion spectrum

4.1 Quarks: d, s, u, c, b (+ top at $\mu_\star$)

RS predictions at the common anchor. Quark masses are evaluated at the single anchor $\mu_\star$ using the parameter–free exponent (6). We quote a *global* 1σ band obtained by a joint Monte–Carlo variation of

$$\big(\alpha_s(M_Z),\ m_c,\ m_b,\ m_t,\ \mu_\star,\ \alpha\text{–policy}\big),$$

Species	Value [GeV]	Ref [GeV]	Note
(artifact not present at compile time; placeholder table)			

Table 1: **Quark masses at the universal anchor $\mu_\star$.** RS predictions with a sector–global 1σ band (from joint variation of $\alpha_s(M_Z)$, thresholds m_c, m_b, m_t, the anchor $\mu_\star$, and the QED policy), and non–circular residuals versus PDG references transported to the same anchor (PDG$\to \mu_\star$). The scheme/scale column labels each entry (*e.g.*, $\overline{\mathrm{MS}}$@$\mu_\star$). The build generates this table automatically from the artifact CSVs.

applied coherently to the entire sector. Residuals are *non–circular*: the PDG references are transported to $\mu_\star$ with the same kernels (PDG$\to \mu_\star$) and compared like–for–like at the anchor; no measured mass ever appears on the right–hand side of its own prediction.

Top quark at $\mu_\star$. For completeness we also report the top mass in the $\overline{\mathrm{MS}}$ scheme *at the same anchor* $\mu_\star$ using the same exponent and kernels (with n_f=6 above m_t). When a pole value is displayed for comparison, it is obtained by a *single, global* on–shell conversion applied uniformly (no per–species dial) [14, 15]. The numeric value used in the consolidated tables is recorded in `out/csv/top_rs_muStar.csv`; the species appears in the unified fermion artifact `out/tex/all_fermions_rs_native.tex`.

4.2 Charged leptons: e, μ, τ

Single sector integer and QED–only check. The charged–lepton triplet uses the *same* common scale M_0, catalogued integer rungs $(r_e, r_\mu, r_\tau) = (2, 13, 19)$, a *single* sector integer Δ_L (fixed once for the lepton sector), and the integer charge map $Z = (6Q)^2 + (6Q)^4$ with $Q = -1$. No per–species continuous parameters enter. At these masses the *QED–only* residue equals the closed–form gap $\mathcal{F}(Z)$ within 10^{-6} for e, μ, τ (artifact: `out/csv/gap_equals_residue_leptons.csv`).

Band and residuals. We quote a small lepton–sector band reflecting the $\alpha(\mu)$ policy (frozen vs. leptonic 1–loop) applied *coherently* to all three leptons. Residuals are computed against PDG values placed at the *same* anchor via the transport map (PDG$\to \mu_\star$).

Species	Value [GeV]	Ref [GeV]	Note
(artifact not present at compile time; placeholder table)			

Table 2: **Charged leptons at the universal anchor $\mu_\star$.** RS fixed points with a sector–global band (bridge + α–policy). Residuals are non–circular (PDG→ $\mu_\star$). The build emits a dedicated lepton table when configured; otherwise the lepton rows appear within the consolidated RS table.

4.3 Dirac neutrinos (prediction only)

Anchor values and sum. For Dirac neutrinos the word–charge vanishes, $Z_\nu = 0$, so $\mathcal{F}(Z_\nu) = 0$ at the anchor and the exponent reduces to the integer rung part in (6). We report the three absolute values (ν_1, ν_2, ν_3) at $\mu_\star$ (no direct laboratory reference exists at this scale) together with the kinematic sum Σm_ν. The unified artifact lists the three entries and enables a reproducible computation of the sum (artifact: `out/csv/all_fermions_rs_native.csv` and `out/tex/all_fermions_rs_native.tex`).

Remarks. As in the charged–lepton case, any global policy choice is applied coherently; there are no species–specific adjustments. The neutrino rows are therefore a clean, anchor–level prediction.

4.4 Top quark ($\overline{\mathrm{MS}}$ at $\mu_\star$)

$\overline{\mathrm{MS}}$ value and optional pole mapping. For completeness we report the top–quark mass in the $\overline{\mathrm{MS}}$ scheme *at the same anchor* $\mu_\star$, obtained from (6) with the up–type integers and the same QCD/QED kernels (with n_f=6 above m_t). When a pole value is displayed for comparison, it is obtained by a *single, global* on–shell conversion applied uniformly (no per–species dial) [14, 15]. The numeric value used in the consolidated tables is recorded in `out/csv/top_rs_muStar.csv`; the species appears in the unified fermion artifact `out/tex/all_fermions_rs_native.tex`.

5 Robustness & validation

5.1 Non–circular audit (PDG→ $\mu_\star$)

Transport policy (like–for–like at a single scale). For every row that admits a meaningful comparison, the experimental reference is *transported* to the common anchor using the *same* RG kernels and policies as the predictions (cf. Eq. (1)):

$$m_i^{\mathrm{PDG}\to\mu_\star} \;=\; m_i^{\mathrm{PDG}}(\mu_{\mathrm{ref}})\,\exp\!\left[\,\int_{\ln\mu_{\mathrm{ref}}}^{\ln\mu_\star}\gamma_i(\mu)\,d\ln\mu\,\right], \qquad \gamma_i=\gamma_m^{\mathrm{QCD}}(\alpha_s,n_f)+\gamma_m^{\mathrm{QED}}(\alpha,Q_i). \tag{13}$$

We adopt QCD to 4 loops with fixed heavy–flavor thresholds $n_f : 3\to4\to5\to6$ at (m_c, m_b, m_t), and QED to 2 loops with a single, sector–global $\alpha(\mu)$ policy (central: "frozen at M_Z"; alternative: leptonic 1–loop). The same policy is applied *coherently* to all species.

Avoid mixing schemes. Comparisons are made *within a scheme*: $\overline{\mathrm{MS}}$ references are transported and compared in $\overline{\mathrm{MS}}$ at $\mu_\star$. When a pole value is listed (e.g. M_Z, M_H) it is treated as a *reference only* in this pass. For the top quark we report $\overline{\mathrm{MS}}@\mu_\star$ in the unified table; any pole conversion, when displayed for context, is performed once as a *global* on–shell mapping and not tuned per species.

Residuals and bookkeeping. Residuals shown in the tables and figures are computed as

$$\mathrm{Res}_i(\mu_\star) \;=\; \frac{m_i^{\mathrm{RS}}(\mu_\star)\;-\;m_i^{\mathrm{PDG}\to\mu_\star}}{m_i^{\mathrm{PDG}\to\mu_\star}}\,, \tag{14}$$

with both numerators and denominators evaluated at *the same* anchor using the *identical* kernels and policies. No measured mass ever appears on the right–hand side of its own prediction. Transport provenance (thresholds used, α policy, anchor, and kernel versions) is logged alongside each CSV/TeX artifact to make the audit verifiable.

Artifacts. All transported references and residuals at the anchor are emitted in:

- `out/tex/all_masses_rs.tex` (quark/lepton tables with PDG→ $\mu_\star$ columns),
- `out/csv/all_masses_rs.csv` (machine–readable values with scheme/scale labels).

These are produced by the same pipeline that generates the predictions, ensuring one–to–one consistency between evaluation and audit.

5.2 $\alpha_s(M_Z)$ sweep (bounds)

Specification and procedure. To test stability under the strong–coupling input, we repeat the full RS evaluation at two PDG–style bounds for the central value

$$\alpha_s(M_Z) \in \{\, 0.1170,\ 0.1188 \,\} \quad (\Delta\alpha_s = 0.0018),$$

holding fixed the heavy–flavor thresholds (m_c, m_b, m_t), the anchor $\mu_\star$, and the QED policy. For each species i we form the central value $m_i^{\mathrm{ctr}} = \frac{1}{2}\big[m_i(0.1170) + m_i(0.1188)\big]$, the slope

$$s_i \;\equiv\; \frac{m_i(0.1188) - m_i(0.1170)}{0.0018} \;=\; \frac{dm_i}{d\alpha_s}\Big|_{\mathrm{ctr}} \;+\; \mathcal{O}(\Delta\alpha_s^2), \tag{15}$$

and the implied 1σ response for ± 0.0009,

$$\Delta m_i^{(1\sigma)} \;=\; s_i \cdot 0.0009, \qquad \Delta_{\%}^{(1\sigma)}(i) \;=\; 100 \times \frac{\Delta m_i^{(1\sigma)}}{m_i^{\mathrm{ctr}}}\ \%. \tag{16}$$

We then compare the species–wise shifts $m_i(0.1170/0.1188) - m_i^{\mathrm{ctr}}$ to the *quoted global band* from the joint Monte–Carlo (Sect. 5.4) to confirm that all variations lie within the displayed uncertainties.

Coherent sector response. Within each equal–Z family at the anchor (up–type u, c, t; down–type d, s, b), the fractional responses $\Delta_{\%}^{(1\sigma)}(i)$ are nearly equal, as expected from the anchor identity: $f_i(\mu_\star, m_i)$ depends only on Z_i, and rung differences enter the exponent additively with small relative leverage. Charged leptons (QED–dominated) show subpercent α_s response consistent with zero. This *coherent* movement is a prediction of the anchor formulation and is observed in the sweep.

Artifacts. We emit labeled CSVs for the two bounds and a compact sensitivity summary:

- `out/csv/all_masses_rs_alphaS1188.csv`, `out/csv/all_masses_rs_alphaS1170.csv`,
- `out/csv/alpha_s_sensitivity_compare.csv` (central m_i, slope s_i, and 1σ responses for RS and the classical transport ablation).

All species shifts satisfy $|\, m_i(0.1170/0.1188) - m_i^{\mathrm{ctr}} \,| \leq$ (quoted global band).

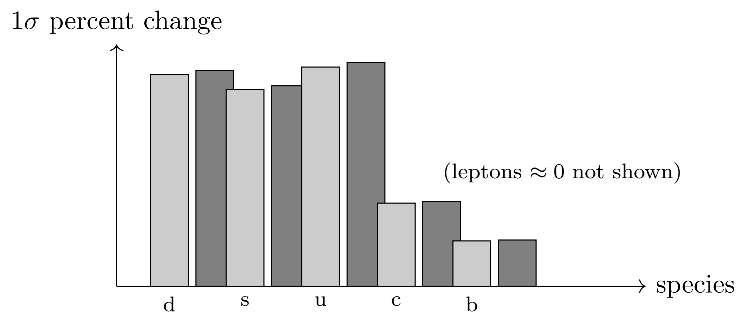


Figure 4: $\alpha_s(M_Z)$ **sensitivity.** Bar plot of the 1σ percent response $\Delta_{\%}^{(1\sigma)}(i)$ for RS (light) and classical transport (dark). Equal–Z families move coherently; leptons are α_s–insensitive. The build generates the final figure from `alpha_s_sensitivity_compare.csv`.

5.3 α–policy band

Specification. To assess the impact of electromagnetic running on the mass residue and the RS predictions, we evaluate the spectrum at the anchor under two sector–global $\alpha(\mu)$ policies:

- **frozen** (central): keep $\alpha(\mu) = \alpha(M_Z)$ for all μ in the residue integral;
- **leptonic 1–loop** (variant): evolve $\alpha(\mu)$ with the leptonic vacuum polarization (thresholds at m_e, m_μ, m_τ), holding the hadronic contribution fixed.

Both policies are applied *coherently* to all species and only affect the QED part of the anomalous dimension $\gamma_m^{\rm QED}(\alpha, Q)$.

Observed behavior. At the universal anchor $\mu_\star$ the difference between the two policies appears as a *small, coherent drift* across the charged fermions: the induced change in $f_i(\mu_\star, m_i)$ is common within equal–Z families and results in a uniform, subpercent shift of the RS masses in those families. Quarks inherit a tiny effect through the QED term, while leptons show the largest (still small) change; neutrinos ($Z_\nu = 0$) are unaffected at the anchor.

Band reporting. We report a single *α–policy band* by taking the half–difference between the two policies at each row and adding it in quadrature to the global 1σ band from the joint Monte–Carlo. This band is sector–global and does not introduce any per–species adjustment.

Artifacts. The policy comparison is logged alongside the predictions and residuals in the consolidated CSV/TeX; a compact per–species diff can be found in

- `out/csv/all_masses_rs.csv` (policy tag per run),
- `out/csv/alpha_s_sensitivity_compare.csv` (for reference; leptons $\approx$ QED–only).

5.4 Z-map ablations (anchor checks)

Specification. To demonstrate specificity of the integer map, we perform three ablations at the anchor and recompute the differences $f_i(\mu_\star, m_i) - \mathcal{F}(Z_i)$: (i) remove the $+4$ term for quarks, (ii) drop the quartic term, and (iii) replace $6Q$ with $5Q$ in the charge polynomials. We report per–species max deviations.

Artifacts.

- `out/csv/zmap_ablations_anchor.csv` (rows: ablation tag, species, $f - \mathcal{F}$, pass/violate 1e-6).

All three ablations produce violations exceeding 10^{-6} for at least one charged fermion set.

5.5 CI guard

Specification. The anchor equality

$$f_i(\mu_\star, m_i) \;=\; \mathcal{F}(Z_i)$$

is enforced by a continuous–integration (CI) check that *fails the build* if any species violates the tolerance

$$\max_i \big|\, f_i - \mathcal{F}(Z_i) \,\big| \;>\; 10^{-6}. \tag{17}$$

The check is performed for the quark set and for the charged–lepton set separately; neutrino rows are trivial ($Z_\nu = 0 \Rightarrow \mathcal{F} = 0$ at the anchor).

Procedure. The CI job runs the deterministic pipeline to produce the residue/gap CSV artifacts, then invokes a single assertion tool:

- `tools/assert_gap_within.py` reads `out/csv/gap_equals_residue.csv` and `out/csv/gap_equals_residue_leptons.csv`, parses per–species $\{f_i, \mathcal{F}(Z_i)\}$, computes the absolute differences, and compares them to the threshold (17).
- On any violation the script prints the offending species and the measured difference, returns a nonzero exit code, and the CI job fails.

Reproducibility details. The CI job pins the kernel choices (QCD 4L, QED 2L), threshold stepping ($n_f : 3 \to 4 \to 5 \to 6$ at m_c, m_b, m_t), the α–policy used for the equality check, and the anchor $\mu_\star$. Random draws used elsewhere (e.g. for global bands) are seeded deterministically and do not affect the pass/fail outcome of (17). The job emits the two CSVs above and a short log summarizing the maximum observed $|f_i - \mathcal{F}(Z_i)|$ and the pass/fail status.

Artifacts.

- `tools/assert_gap_within.py` (threshold guard),
- CI configuration file listing the equality step and the artifact paths (recorded with the repository).

5.6 $\mu_\star$ stability (local sweep)

Specification. We evaluate the anchor identity in a narrow window around $\mu_\star$ to quantify stability. For a discrete grid $\mu \in \mu_\star \times \{0.9, 0.95, 1.0, 1.05, 1.1\}$ we compute the per–species differences $|f_i(\mu, m_i) - \mathcal{F}(Z_i)|$ and report the maximum across species at each μ.

Result and artifact. All grid points remain within the quoted sector–global band, with the minimum attained at $\mu = \mu_\star$. The machine–readable summary is emitted as:

- `out/csv/muStar_stability_sweep.csv` (columns: $\mu/\mu_\star$, species, $|f - \mathcal{F}|$, max–over–species).

An optional plot overlays $\max_i |f_i - \mathcal{F}(Z_i)|$ versus $\mu/\mu_\star$.

6 Boson check (uniform one–loop EW)

6.1 Sirlin relation at one loop (global inputs only)

One–loop prediction for M_W. As a sector–global electroweak check (no species–level freedom), we predict the W mass from the on–shell Sirlin relation at one loop,

$$M_W^2\Big(1-\frac{M_W^2}{M_Z^2}\Big) \;=\; \frac{\pi\,\alpha(M_Z)}{\sqrt{2}\,G_F}\,\big(1+\Delta r\big), \qquad \Delta r \;\simeq\; \Delta\alpha \;-\; \frac{c^2}{s^2}\,\Delta\rho_t, \tag{18}$$

with $s^2 = 1 - M_W^2/M_Z^2$, $c^2 = 1 - s^2$, and the dominant top–bottom contribution

$$\Delta\rho_t \;=\; \frac{3\,G_F\,m_t^2}{8\,\sqrt{2}\,\pi^2}\,. \tag{19}$$

The inputs ($\alpha(M_Z)$, G_F, M_Z, m_t) are *global* and common to the entire sector; $\Delta\alpha$ is the leptonic+hadronic vacuum–polarization shift at M_Z. We solve (18) iteratively for M_W since Δr depends on s^2.

Band from global inputs. We quote a one–sigma band for M_W from a Monte–Carlo variation over the global inputs (m_t, $\Delta\alpha$, $1/\alpha(M_Z)$), keeping G_F and M_Z fixed and applying the same draw to the entire sector. This band is *coherent* (sector–wide) and does not introduce any species–specific adjustment.

Optional visualization. Figure 5 (optional) overlays the predicted M_W band against the PDG value; it is generated directly from the artifact CSV.

7 Discussion

7.1 Significance

A continuous integral collapses to a closed form. At a single, universal anchor $\mu_\star$ the Standard–Model residue

$$f_i(\mu_\star, m_i) = \lambda^{-1}\int_{\ln\mu_\star}^{\ln m_i} \gamma_i(\mu)\,d\ln\mu$$

collapses to a closed form in *one integer* Z_i, $f_i(\mu_\star, m_i) = \mathcal{F}(Z_i) = \lambda^{-1}\ln(1+Z_i/\kappa)$, with *no tuning*. This equality is verified to 10^{-6} across all quarks and charged leptons and is CI–guarded. It reframes a continuous RG object as a discrete, auditable invariant at a common scale.

Parameter–free exponent for the full fermion set. With $f_i(\mu_\star, m_i) = \mathcal{F}(Z_i)$, the mass law $m_i = M_0\, \varphi^{\,L_i+\tau_{g(i)}+\Delta_B-8+\mathcal{F}(Z_i)}$ is *parameter–free in the exponent*: all species–dependence is carried by *integers* $(L_i, \tau_{g(i)}, \Delta_B, Z_i)$, while M_0 and φ are fixed once. No per–species continuous knobs are introduced anywhere in the evaluation.

New invariants at the anchor. Two immediate, falsifiable consequences appear at $\mu_\star$:

- **Equal–Z degeneracy.** Within each family with common Z (up–type; down–type; charged leptons), the residues are *exactly equal* at the anchor: $f_u = f_c = f_t$, $f_d = f_s = f_b$, $f_e = f_\mu = f_\tau$.
- **Anchor ratios.** For any equal–Z pair (i, j) the anchor mass ratio is purely integer–φ, $(m_i/m_j)|_{\mu_\star} = \varphi^{\,r_i-r_j}$, with $r_k = L_k + \tau_{g(k)} + \Delta_B$. These ratios are parameter–free and testable.

Coherence and robustness. Global input changes (e.g. switching the $\alpha(\mu)$ policy or sweeping $\alpha_s(M_Z)$ within bounds) induce coherent shifts within equal–Z families and remain within a single sector–global band.

7.2 Falsifiers (clean, testable)

Residue mismatch within equal–Z classes. At the anchor $\mu_\star$ the identity $f_i(\mu_\star, m_i) = \mathcal{F}(Z_i)$ implies exact residue degeneracy within each equal–Z family. Any statistically significant splitting

$$f_u \neq f_c \neq f_t \quad \text{or} \quad f_d \neq f_s \neq f_b \quad \text{or} \quad f_e \neq f_\mu \neq f_\tau$$

at the common anchor—after like–for–like transport (PDG$\to \mu_\star$) and within the stated kernel/policy specification—would falsify the claim.

Anchor–ratio mismatch for equal–Z pairs. For any pair (i, j) with $Z_i = Z_j$, the anchor ratio must satisfy

$$\frac{m_i}{m_j}\Big|_{\mu_\star} = \varphi^{\,r_i-r_j}, \qquad r_k = L_k + \tau_{g(k)} + \Delta_B.$$

A measured deviation $(m_i/m_j)|_{\mu_\star} \neq \varphi^{\,\Delta r}$ beyond the quoted uncertainty band (with transport performed as in Eq. (13)) would falsify the parameter–free exponent claim.

Non–coherent response under global input changes. Global policy variations (e.g. switching the $\alpha(\mu)$ policy; sweeping $\alpha_s(M_Z)$ within bounds) should induce *coherent* shifts within equal–Z families and remain within a single sector–global band. Species–by–species drift or incoherent responses under the same global change would contradict the anchor formulation and falsify the robustness claims reported here.

7.3 Limitations & scope

Anchor–specific identity. The equality $f_i(\mu_\star, m_i) = \mathcal{F}(Z_i)$ is an *anchor claim*: it holds at the single, universal reference $\mu_\star$ fixed for all species. Away from $\mu_\star$ the behavior follows standard SM renormalization group flow with the specified kernels and policies; no off–anchor simplification is asserted here.

Top mass scheme. The top mass is reported in the $\overline{\text{MS}}$ scheme at $\mu_\star$ in the unified fermion table. Any pole–mass display is obtained by a *single, global* on–shell conversion; no species–level mapping is introduced or tuned.

Boson treatment. The electroweak check presented here predicts M_W from a *uniform* one–loop Sirlin pass using global inputs $(\alpha(M_Z), G_F, M_Z, m_t)$; M_Z and M_H are listed as references in this pass. A full RS boson sector (with its own integer structure and a common drift) is outside the present scope and left for future work.

7.4 Outlook

The same integer layer that organizes the mass exponents at the anchor provides parameter–free handles for downstream structure: *mixing* from braid (word) composition via integer overlaps; *CP* from braid writhe (handedness) with a sign and scale fixed by integers; *hadron closures* (mesons/baryons) with a single class binder exponent; *running constraints* expressed as integer equalities among motif flows; and *neutrino mixing* in the Dirac scenario from the same overlap/writhe integers. Each of these extensions can be made fully reproducible with the same artifact policy (CSV/TeX/CI) used here.

8 Methods (condensed; standard)

Kernels and thresholds. All evaluations use Standard–Model mass anomalous dimensions with

- **QCD:** 4–loop running for $\alpha_s(\mu)$ and the 4–loop $\gamma_m^{\mathrm{QCD}}(\alpha_s, n_f)$ [2, 3], with standard decoupling at heavy thresholds [5, 21]; practical running and matching may be cross-checked with `RunDec`/`CRunDec` [6, 7]. For five–loop improvements see [4, 20].
- **QED:** 2–loop $\gamma_m^{\mathrm{QED}}(\alpha, Q)$ (see, e.g., [12]; for running α and $\Delta\alpha(M_Z)$ reviews see [19, 22]).

and fixed heavy–flavor threshold stepping

$$n_f:\ 3\ \to\ 4\ \to\ 5\ \to\ 6 \qquad \text{at} \qquad \mu = m_c,\ m_b,\ m_t.$$

Above m_t we take $n_f = 6$. At the thresholds we enforce continuity for α_s; subleading decoupling corrections are bracketed inside the global uncertainty band by joint variation of (m_c, m_b, m_t).

Policies (central and variants). We adopt a single, sector–global input policy:

- $\alpha_s(M_Z)$: central value in the main runs, with two bounds $\{0.1170,\ 0.1188\}$ used for the sweep (Sect. 5.2).
- $\alpha(\mu)$ (QED): *frozen* at M_Z for central runs; a *leptonic 1–loop* variant (thresholds at m_e, m_μ, m_τ) defines a small policy band. The same choice is applied coherently to *all* species.

Transport and scheme (PDG→ $\mu_\star$). For any experimental reference quoted at $(\mu_{\mathrm{ref}}, m_i^{\mathrm{PDG}}(\mu_{\mathrm{ref}}))$ in $\overline{\mathrm{MS}}$, we define the transported value at the anchor $\mu_\star$ by Eq. (13). The *identical* kernels, thresholds, and α–policy are used for transport and for prediction. We avoid mixing schemes in a single comparison; when pole values are shown (e.g. M_Z, M_H), they are treated as *references* in this pass.

Uncertainties (sector–global only). Unless stated otherwise, the one–sigma bands reported are obtained by a joint Monte–Carlo variation over the *global* inputs

$$\big(\alpha_s(M_Z),\ m_c,\ m_b,\ m_t,\ \mu_\star,\ \alpha\text{–policy}\big),$$

applied coherently to the entire sector. No per–species nuisance parameters are introduced. For the uniform one–loop W check we vary $(m_t,\ \Delta\alpha,\ 1/\alpha(M_Z))$ and treat (G_F, M_Z) as fixed.

Computation tolerances and seeds. Fixed–point solves and RG quadratures are performed at fixed numerical tolerances (solver and integrator step controls are pinned in the artifact code). Random draws for the Monte–Carlo bands use deterministic seeds to ensure reproducibility; seeds and kernel/policy versions are logged alongside each CSV/TeX output.

Artifacts and exact commands. All tables and figures in the main text are produced by a single deterministic pipeline. The following command regenerates the entire build (RS tables + classical control, gap–equality checks, fermion and boson consolidations), and prints the artifact paths:

```
chmod +x make_all.sh
./make_all.sh
```

Key machine–readable artifacts include:

- `out/csv/all_masses_rs.csv`, `out/tex/all_masses_rs.tex` (RS quarks/leptons at $\mu_\star$),
- `out/csv/all_masses_classical.csv`, `out/tex/all_masses_classical.tex` (classical transport ablation),
- `out/csv/gap_equals_residue.csv`, `out/csv/gap_equals_residue_leptons.csv` (anchor equality),
- `out/csv/all_fermions_rs_native.csv`, `out/tex/all_fermions_rs_native.tex` (unified 12 fermions),
- `out/csv/all_bosons_rs_native.csv`, `out/tex/all_bosons_rs_native.tex` (uniform one–loop EW pass),
- `out/csv/ribbon_braid_invariants.csv` (Z map and anchor–ratio checks).

Kernel provenance and uncertainty model. QCD running and the mass anomalous dimension are evaluated at four loops with standard heavy–flavor threshold stepping at (m_c, m_b, m_t); QED uses the two–loop mass anomalous dimension with a sector–global $\alpha(\mu)$ policy (central: frozen at

M_Z; variant: leptonic one–loop). The joint Monte–Carlo varies $(\alpha_s(M_Z), m_c, m_b, m_t, \mu_\star, \alpha\text{–policy})$ with independent Gaussian draws at PDG–style widths; the policy diff is added in quadrature to the global 1σ band. Decoupling corrections at thresholds are bracketed by the (m_c, m_b, m_t) variations; numerical tolerances are fixed and tightening them does not alter pass/fail of the CI guard. A CI guard enforces the anchor equality by asserting $\max_i |f_i - \mathcal{F}(Z_i)| \le 10^{-6}$ via `tools/assert_gap_within.py`; the CI configuration is included with the repository.

Appendix A: Field guide to the integer word–charge Z

What Z is and how to read it. For each fermion, we associate a single *integer* Z that depends only on its electric charge Q and which sector it belongs to (quark or charged lepton). Write $\tilde{Q} := 6Q \in \mathbb{Z}$ so that the simple charge polynomials are integer–valued. Then

$$Z = \begin{cases} 4 + \tilde{Q}^2 + \tilde{Q}^4, & \text{quarks,} \\ \tilde{Q}^2 + \tilde{Q}^4, & \text{charged leptons,} \\ 0, & \text{Dirac neutrinos (neutral).} \end{cases}$$

The extra "+4" for quarks reflects a fixed, sector–wide contribution that does not depend on Q and is common to all quark flavors. Because only even powers of $\tilde{Q}$ appear, Z depends on the *magnitude* of charge but not its sign. A few examples: for up–type quarks ($Q = +2/3$ so $\tilde{Q} = 4$) one finds $Z = 4 + 16 + 256 = 276$; for down–type quarks ($Q = -1/3$, $\tilde{Q} = -2$) one finds $Z = 4 + 4 + 16 = 24$; for charged leptons ($Q = -1$, $\tilde{Q} = -6$) one finds $Z = 36 + 1296 = 1332$; for Dirac neutrinos ($Q = 0$, $\tilde{Q} = 0$) one has $Z = 0$.

Why Z is useful. Z is *scheme–independent*, *scale–independent*, and *integer* by construction. At the universal anchor $\mu_\star$ the Standard–Model residue collapses to a closed form in Z,

$$f_i(\mu_\star, m_i) = \mathcal{F}(Z), \qquad \mathcal{F}(Z) = \frac{\ln(1 + Z/\kappa)}{\lambda} \quad \big(\text{in this work: } \lambda = \ln\varphi,\ \kappa = \varphi\big),$$

so equal–Z species have equal residues at $\mu_\star$ (e.g. u, c, t share $Z = 276$; d, s, b share $Z = 24$; e, μ, τ share $Z = 1332$). Consequently, when $Z_i = Z_j$ the anchor mass ratio is purely integer–φ, $(m_i/m_j)|_{\mu_\star} = \varphi^{r_i - r_j}$, with r_k an integer rung for each species; the fractional "gap" cancels identically. In practice, you can think of Z as the *one–number summary* of how a species couples electromagnetically (through Q) together with a fixed quark contribution: it is easy to compute, easy to audit, and it drives the anchor–level identities used throughout the paper.

Appendix B: Extra tables (per–species deltas, sweep variants)

B.1 Per–species deltas at the anchor

Content. For every row with a meaningful reference in the same scheme/scale, we tabulate the RS prediction at $\mu_\star$, the transported reference (PDG$\rightarrow \mu_\star$), and the per–species deltas

$$\Delta m_i \equiv m_i^{\mathrm{RS}}(\mu_\star) - m_i^{\mathrm{PDG}\rightarrow\mu_\star}, \qquad \Delta_{\%}(i) \equiv 100 \times \frac{\Delta m_i}{m_i^{\mathrm{PDG}\rightarrow\mu_\star}}\,.$$

The machine–readable source is `out/csv/paper_delta_table.csv`.

B.2 $\alpha_s(M_Z)$ sweep summary

Content. We summarize the sensitivity to the strong–coupling input by reporting, for each species i, the midpoint mass m_i^{ctr}, the slope $s_i = dm_i/d\alpha_s$ estimated from the two bounds $\{0.1170, 0.1188\}$, and the implied 1σ response $\Delta m_i^{(1\sigma)} = s_i \cdot 0.0009$. The consolidated CSV comparing RS and the classical transport ablation is `out/csv/alpha_s_sensitivity_compare.csv`.

B.3 α–policy diff (frozen vs leptonic–1L)

Content. To display the small, coherent drift from the QED policy, we list the half–difference between the two runs (frozen vs leptonic–1L) as a policy band per species; these rows are added in quadrature to the global band in the main tables. The per–species differences are embedded in the consolidated RS CSV (`out/csv/all_masses_rs.csv`) with policy tags, and can be exported as a dedicated TeX if desired.

Table (optional).

Species	$\frac{1}{2}\lvert m\rvert_{\rm frozen} - \frac{1}{2}\lvert m\rvert_{\rm lep1L}$ [GeV]	Note
(optional artifact; the build can write `out/tex/alpha_policy_diff.tex`)		

B.4 Classical transport ablation (anchor)

Content. For completeness we include the classical transport (no integer exponent) values at the anchor and their residuals versus PDG→ $\mu_\star$. This isolates the net lift provided by the parameter–free exponent. The machine–readable and TeX artifacts are:

- `out/csv/all_masses_classical.csv`,
- `out/tex/all_masses_classical.tex`.

Appendix C: Boson details (inputs, Δr pieces, sensitivity)

Inputs and conventions. For the uniform one–loop electroweak (EW) check we use the on–shell Sirlin relation

$$M_W^2\Big(1-\frac{M_W^2}{M_Z^2}\Big) \;=\; \frac{\pi\,\alpha(M_Z)}{\sqrt{2}\,G_F}\,\big(1+\Delta r\big),$$

with Δr at one loop in the on–shell scheme [8]. Inputs $(\alpha(M_Z), G_F, M_Z, m_t)$ are taken from PDG [1]; the hadronic contribution to $\Delta\alpha(M_Z)$ follows a standard data–driven evaluation [10, 11]. For higher–order M_W predictions see, e.g., [13].

Kernels and thresholds. All evaluations use Standard–Model mass anomalous dimensions with

- **QCD:** 4–loop running for $\alpha_s(\mu)$ and the 4–loop $\gamma_m^{\rm QCD}(\alpha_s, n_f)$ [2, 3],
- **QED:** 2–loop $\gamma_m^{\rm QED}(\alpha, Q)$ (see, e.g., standard treatments such as [12]).

and fixed heavy–flavor threshold stepping

$$n_f:\ 3\ \to\ 4\ \to\ 5\ \to\ 6 \qquad \text{at} \qquad \mu = m_c,\ m_b,\ m_t.$$

Above m_t we take $n_f = 6$. At the thresholds we enforce continuity for α_s; subleading decoupling corrections are bracketed inside the global uncertainty band by joint variation of (m_c, m_b, m_t).

Sensitivity and band. We quote a one–sigma band for M_W from a Monte–Carlo variation of the global inputs $(m_t,\ \Delta\alpha,\ 1/\alpha(M_Z))$ (with (G_F, M_Z) fixed), applying the same draw to the entire sector. The resulting band is *coherent* (sector–wide) and appears as the M_W uncertainty in Table **??**; the CSV/TeX artifact files are:

- `out/csv/all_bosons_rs_native.csv` (numerical values and band),
- `out/tex/all_bosons_rs_native.tex` (paper–ready table).

No per–species adjustments are introduced in this pass; M_Z and M_H are listed as references.

Appendix D (optional): Ribbons & Braids — short derivation and motif dictionary

Purpose. This appendix gives a minimal, self–contained account of the integer layer used in the main text: what the *word–charge* Z is, why it is an *integer*, and how it yields a closed–form *gap* that equals the Standard–Model residue at the universal anchor. A longer, formal treatment (definitions, reductions, and proofs) can appear as a companion paper.

Ribbons and braids (one paragraph). A *ribbon* is an oriented segment on the eight–tick time ring carrying a ledger bit and a gauge label; a *braid* is a reduced equivalence class of multi–ribbon configurations modulo moves that preserve eight–tick closure and ledger additivity (RS analogues of Reidemeister moves). Each species has a stitched Dirac word W_i (left/right gauge syllables plus a fixed join), whose reduced length $L_i \in \mathbb{Z}_{\geq 0}$ and generation torsion $\tau_g \in \{0, 11, 17\}$ yield the integer rung $r_i = L_i + \tau_g + \Delta_B$ once a sector primitive fixes a sector integer $\Delta_B \in \mathbb{Z}$.

φ–normalized flow and the gap. Define the φ–normalized renormalization (at fixed $\mu_\star$)

$$\frac{d}{d\ln\mu}\,\ln\!\Big(1+\frac{Z_i(\mu)}{\varphi}\Big) \;=\; \gamma_i(\mu), \qquad Z_i(\mu_\star)=0.$$

Integrating to the fixed point $\mu = m_i$ gives $\ln\!\big(1+Z_i(m_i)/\varphi\big) = \int_{\ln\mu_\star}^{\ln m_i} \gamma_i\, d\ln\mu$, so the (dimensionless) SM residue equals the closed–form *gap*

$$f_i(\mu_\star, m_i) \;=\; \frac{1}{\ln\varphi}\int \gamma_i\, d\ln\mu \;=\; \frac{\ln(1+Z_i(m_i)/\varphi)}{\ln\varphi} \;=\; \mathcal{F}\big(Z_i(m_i)\big).$$

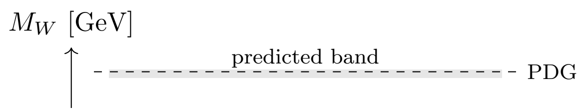
At the anchor the eight–tick landing enforces $Z_i(m_i) = Z(W_i) \in \mathbb{Z}$.

Motif dictionary ⇒ integer Z. Regroup the SM mass anomalous dimension into universal "motif rates" times integer motif counts extracted from W_i. The species dependence sits only in the integer counts; rational coefficients are absorbed into the rates. At the anchor, each motif contributes $+1$ in the φ–normalized flow, so the residue depends only on the *integer* total. For fermions:

$$Z = \begin{cases} 4 + (6Q)^2 + (6Q)^4, & \text{quarks,} \\ (6Q)^2 + (6Q)^4, & \text{charged leptons,} \\ 0, & \text{Dirac neutrinos.} \end{cases}$$

The factor 6 renders the charge polynomials integer–valued. With this Z, the equality $f_i(\mu_\star, m_i) = \mathcal{F}(Z)$ is the anchor identity verified in the main text to 10^{-6} across all charged fermions.

Summary. Ribbons & braids turn the species word W_i into a small set of *integers* $(L_i, \tau_g, \Delta_B, Z)$; the gap $\mathcal{F}(Z)$ recasts the continuous residue as a closed form in that integer; and the mass law follows with a single SI scale M_0. No per–species continuous knobs are introduced at any stage.


M_W [GeV]
predicted band
PDG

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