

Calibration Remark (Limit Form vs. Second Derivative)

Remark 1 (Calibration: limit form vs. second derivative at 0). Define $G(t) := F(e^t)$. Normalization $F(1) = 0$ implies $G(0) = 0$. If F is reciprocal ($F(x) = F(x^{-1})$ for $x > 0$), then G is even, hence $G'(0) = 0$ (assuming differentiability at 0). If moreover G is C^2 at 0, then the Taylor expansion gives

$$G(t) = G(0) + G'(0)t + \frac{G''(0)}{2}t^2 + o(t^2) = \frac{G''(0)}{2}t^2 + o(t^2),$$

so

$$\lim_{t \rightarrow 0} \frac{2F(e^t)}{t^2} = \lim_{t \rightarrow 0} \frac{2G(t)}{t^2} = G''(0).$$

Therefore the calibration condition $\lim_{t \rightarrow 0} \frac{2F(e^t)}{t^2} = 1$ is equivalent (under the above regularity) to $G''(0) = 1$, i.e. $(F \circ \exp)''(0) = 1$.