The hydraulic jump in radially spreading flow: A new model and new experimental data

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A new model for the hydraulic jump in radially spreading flow is presented. The equation of motion for a liquid annulus spreading out under the influence of hydrostatic pressure gradient and frictional drag is developed. The resulting nonlinear differential equation for the liquid depth, $h(r)$, is solved by computer simulation. The jump is assumed to begin when the laminar flow is engulfed by the underlying boundary layer liquid, as suggested recently in the literature. This complicated mixing process is crudely modeled by a drag term which slows the flow and initiates a positive feedback mechanism culminating at a new critical depth, beyond which the depth increases asymptotically to a final value. The model predicts a new relationship between the laminar flow depth just before the jump and the final depth. An experimental apparatus was built to make detailed measurements of the depth $h(r)$, both in the region before the jump and beyond the jump. The theoretical predictions were compared to the experimental data, and gave surprisingly good agreement by suitable adjustment of the two parameters k and C of the model. The parameter k determines the growth rate of the boundary layer thickness, and C determines the drag force. The results suggest that the usual textbook assumption of zero momentum loss across the jump is not appropriate for this type of hydraulic jump. The case of a hydraulic jump in the absence of gravity is considered also and a much different behavior is predicted, which could be tested by experiment in a microgravity environment.

I. INTRODUCTION

The hydraulic jump in radially spreading flow is commonly observed in the kitchen sink. As the vertical liquid column from the tap spreads out radially across the sink bottom a jump occurs in which the liquid depth increases, and the velocity decreases. This fascinating phenomenon has been discussed by many authors in the past (see Refs. 1–4 for example) but no completely satisfactory explanation has been given, and experimental data are sparse. Recently Godwin[5] pointed out the important role played by the viscous boundary layer in determining the critical radius, r_1, at which the jump initiates. As the liquid spreads out from $r \sim 0$ the boundary layer thickens while at the same time the laminar flow depth decreases. Godwin proposed that the jump initiates when the depth, h_1, equals the boundary layer thickness, Δ, defined as the thickness at which the flow velocity has reached $1/e$ of the bulk value. Using this model Godwin obtained quite good agreement with experiment for the viscosity dependence of the jump radius, over a viscosity range of ν=0.01 to 10 cm^2/s. We also realized the importance of friction due to the boundary by observing the effect of the boundary roughness using various grades of wet sandpaper on the sink bottom. Coarser grades yielded smaller values of the jump radius, just as higher viscosity did in Godwin's experiments.

In previous studies of such hydraulic jumps it was usual to relate the liquid depth, h_1, just before the jump to the depth, h_2, just after the jump by equating the momentum flux before the jump to the pressure head after the jump. Most textbooks on fluid dynamics use this approach, but they generally do not point out that it neglects the drag force exerted on the fluid by the fixed boundary. Godwin[5] used this approach as well, but the resulting predictions for the depth h_2 were not particularly good. It was also assumed that the jump is discontinuous and that the depth remains constant after the jump, neither of which agrees with experiment where the jump width is finite and the depth increases asymptotically after the jump. As far as we are aware, no previous studies have addressed the problem of how the liquid is slowed in the jump region, or how the depth varies with radius.

In the present study the problem of predicting the depth profile $h(r)$ is directly addressed. A Lagrange-type equation of motion, which follows a fluid element, is developed for a liquid annulus spreading out under the influence of hydrostatic-pressure-gradient and frictional-drag forces. The resulting nonlinear differential equation for the depth $h(r)$ is solved numerically by computer. It is assumed that frictional forces are negligible in the laminar flow region prior to the jump. Following Godwin[5] the critical depth h_1, and corresponding radius r_1, is reached when the boundary layer thickness, Δ, is of the same order of magnitude as the depth of the laminar flow, whereupon the boundary layer engulfs the flow and frictional forces are assumed to become dominant. This initiates a positive feedback mechanism in which the velocity starts decreasing rapidly with a corresponding rapid increase in depth. The pressure gradient associated with the positive $\partial h/\partial r$ adds to the deceleration. This compounding culminates in a second critical point, characterized by a critical depth h_c, at which $\partial h/\partial r$ is very large. Beyond this point the flow enters a new regime in which friction is assumed to be negligible again, and the depth increases asymptotically with radius as observed in real jumps. The model predicts a new relationship between the depth of the laminar flow, h_1, just before the jump and the final depth h_f. This relationship differs considerably from that previously assumed to hold at such hydraulic jumps, and is in much better agreement with experiment. Momentum loss to the boundary (sink bottom) during the friction dominated region is responsible for this new relationship, but, surprisingly, the relationship does not depend strongly on the form of the

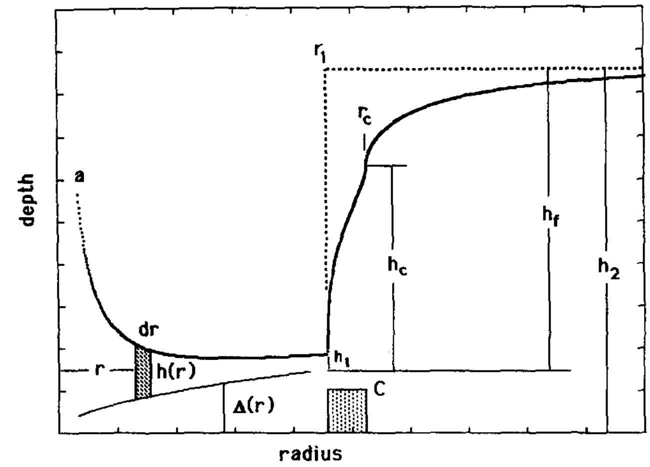

Fig. 1. The heavy line depicts the depth profile in a typical hydraulic jump occurring at the jump radius r_1 and depth h_1. The vertical stream impinging on the plate has radius a. Previous models assumed that the jump is a sharp step with a constant depth h_2 beyond the jump, as shown by the dashed line. The shaded region on the left depicts the cross section of a liquid annulus of radius r, thickness dr and height h, used in the present model. The height h and thickness dr both change as the annulus spreads radially. The boundary layer of thickness $\Delta(r)$ is shown, as well as the depths h_c, h_f, and h_2 used in the present model. The shaded region at the bottom depicts the effective drag coefficient used in the model. Note that the vertical scale is greatly exaggerated.

frictional drag. The results suggest that the usual assumption of zero momentum loss at hydraulic jumps needs to be reexamined.

We are not aware of the existence in the literature of any detailed experimental data for such hydraulic jumps. Thus, in order to test the predictions of the theory an apparatus was designed to measure the depth, $h(r)$, for r values both in the region before the jump as well as beyond the jump. Considerable care was necessary to eliminate the effect of the vertical sink boundary at large r since this causes erroneously large depths in the region beyond the jump, and small values for the jump radius.

The case of a hydraulic jump in the absence of gravity was considered also and a much different behavior is predicted, which could be tested by experiment in a microgravity environment.

II. THE MODEL

Consider a liquid annulus of radius r, width dr, depth $h(r)$, and velocity $v(r)$, spreading radially outwards on a flat plate from an initial vertical stream of radius a, as shown in Fig. 1. Incompressible flow is assumed. The flow rate Q is determined by the stream velocity v_s, i.e.,

$$Q=\pi a^2 v_s. \tag{1}$$

Conservation of mass requires that Q is constant, so that

$$2\pi rhv=Q \tag{2}$$

must be satisfied for all r well beyond the impact point of the stream, i.e., $r \gg a$. The equation of motion for the annulus follows from Newton's second law with the forces arising from drag, and from the pressure gradient associated with the surface slope, i.e.,

$$2\pi r\rho h\, dr\, \frac{dv}{dt}=-C\nu\left(\frac{v}{h}\right)^n dr-2\pi rh\left(\rho g\,\frac{dh}{dr}\right)dr, \tag{3}$$

where time t is the independent variable, ρ is the density, g is the acceleration of gravity, ν is viscosity, and C is an effective drag coefficient. The exact form for the drag term is not known, so the power law form in the shear, $(v/h)^n$, was chosen as being physically reasonable. The value of n was not very important for the outcome of the model, as similar results were obtained for $n=1$, 2, 3 or 4. For the results presented here $n=2$ was used, as it is felt to be the most physical since drag forces in liquids are often proportional to v^2. The width dr of the fluid annulus changes as it moves outward and both its depth and velocity change also. In order to maintain a constant volume of the annulus, while simultaneously satisfying Eq. (2), one has $dr=v\ dt$. The latter relation can be combined with Eqs. (2) and (3) to give a differential equation for velocity, v, with t as the independent variable. The computer simulation would then proceed with independent time steps dt. Alternatively, the equations can be combined to eliminate v and t, so as to obtain a differential equation for $h(r)$ with r as the independent variable, Eq. (4). The simulation then proceeds with independent steps dr. We have done the simulations using both methods and found identical results. The dr method is used here since the predicted $h(r)$ can then be directly compared to the measured depths:

$$\frac{dh}{dr}=\frac{C\nu/2\pi\rho rh^2-h/r}{1-4\pi^2 gr^2h^3/Q^2}. \tag{4}$$

A fourth-order Runge–Kutta[6] algorithm written in True-Basic was used to solve Eq. (4) for $h(r)$, using sufficiently small steps dr to achieve stable solutions. At each step the corresponding velocity $v(r)$ was calculated from Eq. (2). A convenient starting point was chosen with $r>a$, $v=v_s$, and h consistent with Q from Eqs. (1) and (2). The drag coefficient C was assumed to be zero in the laminar flow region prior to the jump. In this region the numerator of Eq. (4) is negative, and the denominator is positive since the second term in the denominator is much less than unity for the values involved. The depth $h(r)$ is thus a decreasing function, while the velocity $v(r)$ increases only slightly, $v(r)\approx v_s \approx v_1$ being a good approximation in this region. The laminar flow is assumed to ride over the comparatively stagnant boundary layer, so that the total depth is given approximately by $h(r)+\Delta(r)$, as shown in Fig. 1.

Godwin[5] determined the critical radius, r_1, as the point where the laminar flow depth, $h(r)$, equals the boundary layer thickness, $\Delta(r)$. The same concept was used here to determine r_1, but better agreement with experiment was found when $\Delta(r)$ was assumed to be several times larger than the laminar flow depth. A subroutine was included to calculate the increasing boundary layer thickness $\Delta(r)$ using the expression given by Godwin,[5] i.e.,

$$\Delta=k(\nu t)^{1/2}, \tag{5}$$

where k is a dimensionless constant of order unity,[5] and the time $t\approx r/v$. The value of $h(r)$ was compared to Δ at every step and when $h(r)\leq\Delta/m$ was reached ($m\approx 3$ to 5) the critical radius r_1 was thereby determined. Beyond r_1 the laminar flow liquid is assumed to mix with the boundary layer liquid, which has the effect of slowing the flow. To account for this in the model the drag coefficient C was then increased, see Fig. 1, to a value sufficient to initiate and maintain the positive feedback mechanism leading to the second critical point, h_c, in $h(r)$. In this way the slowing of the flow by interaction with the boundary layer, which is a very complicated process, was crudely approximated in the model by an effective drag term. The drag coefficient C must be large enough so that the numerator of Eq. (4) is dominated by the first term, causing a rapid increase in $h(r)$ and a corresponding decrease in $v(r)$. The exact value of C is not too important since larger values simply make the transition region narrower but do not greatly change the value of h_c [see Eq. (6) below], so C was chosen to give the best agreement with experiment. If C was chosen too small then the critical depth h_c was not reached at all and the depth would decrease again with no jump occurring (Fig. 7). Note that k and C are the only adjustable parameters in the model. Furthermore, k is effective only in determining the jump radius r_1, while C is effective only in the active transition region just beyond r_1, Fig. 1.

As $h(r)$ increases, the first term in the numerator of Eq. (4) decreases and the second term increases. More importantly, the second term in the denominator increases towards unity. As the denominator of Eq. (4) passes through zero the slope $\partial h/\partial r$ becomes vertical causing a sharp rise in h. This defines the *second critical point*, h_c in the transition. Beyond this point the flow is assumed to become laminar again ($C\approx 0$) and $h(r)$ increases asymptotically to a final value h_f, as shown in Fig. 1. Thus the predicted behavior is much different from previous models, which assumed a single discontinuous jump followed by a constant depth beyond the jump, as shown by the dashed line in Fig. 1.

In this model the second critical point, h_c, is determined by the zero crossing of the denominator of Eq. (4), which can be rewritten as

$$h_c=\sqrt[3]{\frac{Q^2}{4\pi^2 g r_c^2}}, \tag{6}$$

where r_c is the radius corresponding to the critical depth, h_c. [Note that by combining Eqs. (6) and (2) h_c can also be expressed in the simpler form $h_c=v_c^2/g$, where v_c is the corresponding critical velocity.] The final depth, h_f, is larger than h_c by the asymptotic final increase shown in Fig. 1. An approximate analytical expression for h_f can be obtained by integrating Eq. (4) approximately for the region beyond h_c. In this region $C=0$ is assumed, and if the denominator is assumed to be dominated by the second term then

$$\frac{dh}{dr}\approx\frac{Q^2}{4\pi^2 g r^3 h^2}, \tag{7}$$

which can be integrated directly to give

$$h_f^3-h_c^3\approx\frac{3Q^2}{8\pi^2 g r_c^2}. \tag{8}$$

Using Eq. (6) for h_c one then obtains for h_f

$$h_f\approx\sqrt[3]{\frac{5Q^2}{8\pi^2 g r_c^2}}\approx 1.36 h_c. \tag{9}$$

The exact value of h_f is slightly larger than Eq. (9) predicts and its numerical value for any particular case is given more accurately by the simulation. The total final depth is $h_2=h_f+\Delta(r_1)$, as shown in Fig. 1.

An important aspect of the model is that h_f, or h_c, does not depend strongly on the drag coefficient C. This can be seen analytically from Eq. (6) where the only dependence on C is via the radius r_c, and the results in Sec. IV show this to be rather weak. Thus, although a minimum value of C is needed to produce a reasonable jump, it may exceed the minimum by a factor of 10 or more without having much effect on the final height of the jump (Fig. 3).

If the transition region is rather sharp then r_c in Eq. (9) is approximately the jump radius r_1, which can be expressed in terms of the depth h_1 just before the jump by combining Eqs. (1) and (2), that is

$$r_1\approx a^2/2h_1. \tag{10}$$

Combining Eqs. (1), (9), and (10), one then obtains the following relation between h_f and h_1,

$$h_f\approx\sqrt[3]{\frac{5v_1^2h_1^2}{2g}}. \tag{11}$$

Equation (11) differs fundamentally from the relation

$$h_f=\sqrt{\frac{2h_1v_1^2}{g}} \tag{12}$$

commonly used to describe such hydraulic jumps. Equation (12) is obtained by assuming that the momentum flux before the jump is canceled by the pressure head beyond the jump, i.e., momentum loss by drag forces on the boundary is neglected.[4,5] In Eq. (11), which follows naturally from the present model, the momentum flux is reduced partly by the pressure head and partly by the drag force with the boundary in the jump region. For the parameter values typical of hydraulic jumps in a sink Eq. (11) gives much smaller values of h_2 than Eq. (12) gives, in much better agreement with experiment as discussed below.

III. EXPERIMENT

Considerable care was required in setting up the experiment since the model assumed a flat plate extending to large r in all directions, whereas a typical sink has a sloping bottom surrounded by vertical walls, and a drain hole. The actual sink bottom was not used since the vertical sink walls can cause erroneously large liquid depths beyond the jump, and also erroneously small values of the jump radius, r_1. (These effects can be observed by starting the flow in an initially empty sink and watching the jump radius decrease as the water depth increases. Changing the size of the drain hole can also cause depth and radius changes.) To eliminate these effects a flat circular brass plate (10 cm radius by 1.2 cm thick) was placed on the sink bottom and carefully leveled. The apparatus designed to measure the liquid depth is shown in Fig. 2. A stainless-steel wire probe (diam=1 mm) with a sharpened point was attached to a micrometer-controlled stage moving in a vertical track, which itself was attached to the stage of a horizontal traveling microscope. The traveling microscope was supported by a wooden platform resting on the top of the sink, and its horizontal track was carefully leveled.

The water surface was detected by measuring the electrical resistance between the wire probe and the metal plate. The resistance dropped abruptly to about 10 KΩ on first contact with the water surface and dropped again to ≈ 0 Ω when the probe touched the plate. The initial contact with the surface was done very carefully and the effect of the probe on the flow was negligible. The difference in the two micrometer readings gave the depth with a precision of ±0.005 cm. The

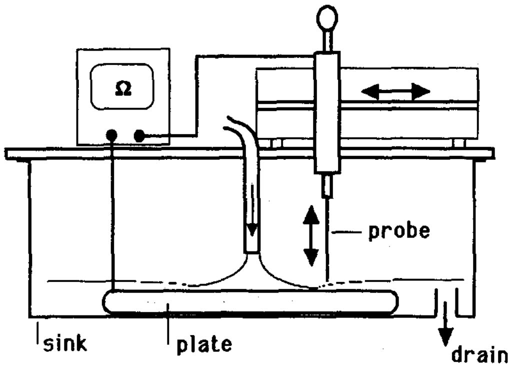

Fig. 2. Sketch of the experimental apparatus used to measure the liquid depths.

radial position (±0.02 cm) of the probe was determined by reading the scale of the traveling microscope.

Surface-tension blocking effects at the outer edge of the plate were found to also cause erroneously large liquid depths on the plate. The surface tension acts to produce an *effective wall* at the plate edge. For example, with the flow stopped the static liquid depth was 2.8 mm everywhere on the plate. This problem was greatly reduced by adjusting the static liquid level in the sink to a fraction of a millimeter above the level of the plate, thereby greatly reducing the surface tension. In this way the static level over the plate could be reduced to 0.4 mm, nearly an order of magnitude reduction. The liquid level in the sink was controlled by the height of a tube inserted into the sink drain, Fig. 2. The surface tension effect was further reduced by rounding-off ($R \approx 7$ mm) the outer edge of the plate using a lathe, thereby reducing the static layer thickness over the plate to 0.3 mm. Proper experimental results could not be obtained without taking these precautions, since otherwise the depths were greatly influenced by the boundary at the outer edge of the plate, rather than by the intrinsic properties of the jump. (It is possible that the outer boundary condition could be incorporated into the theory, but this was not attempted.)

The flow rate Q was determined by measuring the time to fill a 200 ml beaker, and the stream radius a was measured with a vernier caliper at a height of ≈ 1 cm above the plate. The nozzle was a brass tube (8 mm i.d.) situated with its lower end about 8 cm above the plate, and connected to a constant level container by gum-rubber tubing. The flow rate Q was controlled by a clamp on the tubing. The over-flow from the constant-level container was directed into the sink drain. The maximum value of Q with this setup was $Q \approx 50$ cm^3/s, but the steadiest conditions were obtained for $Q < 20$ cm^3/s. Also, for $Q \approx 50$ cm^3/s the jump radius was too close to the plate edge for getting reliable depths in the region beyond the jump.

IV. MODEL PREDICTIONS COMPARED TO EXPERIMENT

The model predictions for $h(r)$ are compared to experiment in Figs. 3–5 for values of the flow rate $Q = 5.7$, 16, 40 cm^3/s, respectively. The corresponding values of the measured stream radius a and the chosen parameters k and C of the model are given in the figure captions. The parameter k was chosen to obtain the best agreement with the measured

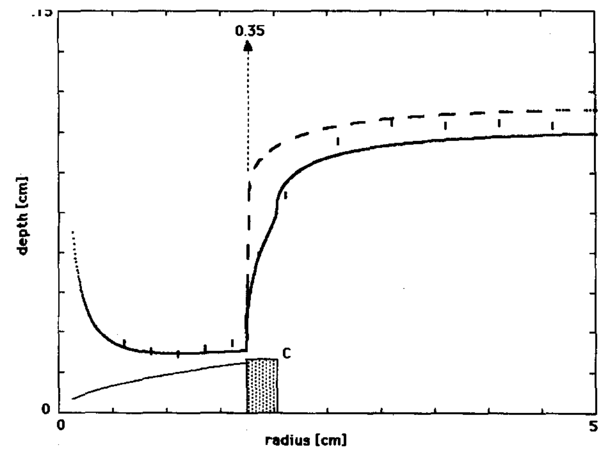

Fig. 3. Comparison of theory and experiment for $Q = 5.7$ cm^3/s and $a = 0.13$ cm. The model parameters used were $k = 1.8$ and $C = 0.2$ gm s/cm^2. The depth profile predicted by the present model is shown by the heavy line, while the experimental data are shown by the bars, and the vertical dashed line shows the prediction of earlier theories. The heavy dashed line shows the effect of increasing the drag constant to $C = 2.0$ gm s/cm^2. The boundary layer is shown by the fine line.

jump radius, Godwin.[5] The drag coefficient C was set equal to zero, except in the region between the critical radius r_1 and the radius corresponding to the critical depth h_c, as shown by the shaded region in the figures. The value of C was chosen to best fit the data, but increasing it beyond this value by a factor of 10 or more did not cause much change in the height predictions. An example of this can be seen in Fig. 3 where the heavy dashed line shows the effect of increasing C by a factor of 10. The jump width has become very narrow but the final depth has changed by less than 10%. A viscosity of $\nu = 0.01$ cm^2/s was used in all cases.

The agreement between theory and experiment is generally quite good, both regarding the shape of $h(r)$ and the final depths, and certainly better than expected considering the simplicity of the model. The agreement is particularly good for the smallest value of Q, Fig. 3, where the flow was steadiest and the measured jump was sharpest. The agreement is least for the largest value of Q, Fig. 5, where the measured jump is more gradual. The agreement in the latter

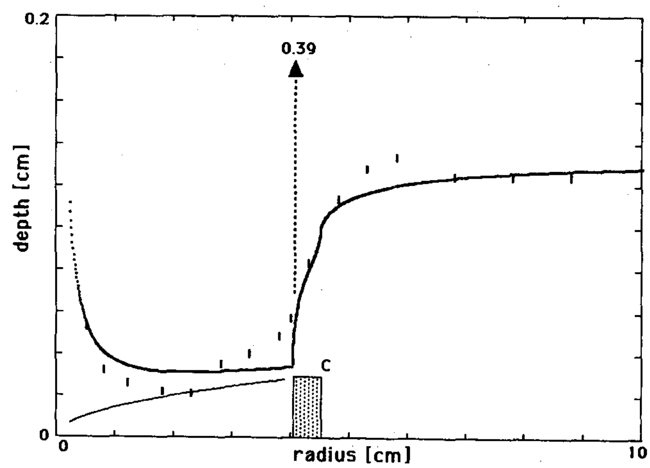

Fig. 4. Comparison of theory and experiment for $Q = 16$ cm^3/s and $a = 0.22$ cm. The model parameters used were $k = 1.5$ and $C = 0.4$ gm s/cm^2.

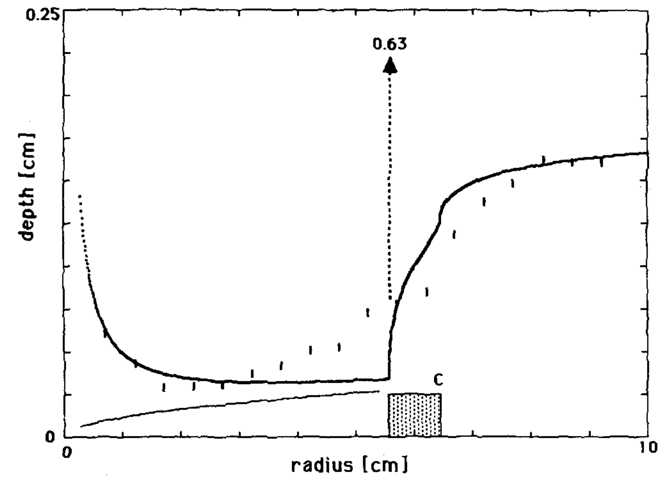

Fig. 5. Comparison of theory and experiment for $Q=40$ cm^3/s and $a=0.28$ cm. The model parameters used were $k=1.5$ and $C=1.0$ gm s/cm^2.

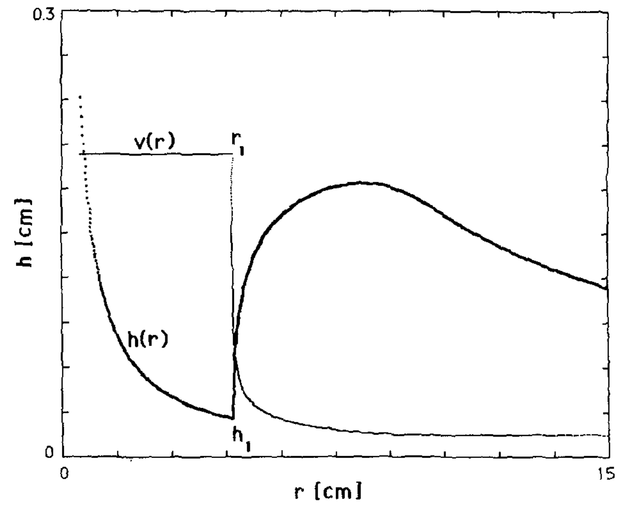

Fig. 7. Predicted hydraulic jump in a microgravity environment.

case can be improved slightly by assuming a more complicated form for the drag coefficient C, as shown in Fig. 6 where C increases gradually as r increases from r_1 to r_c. In Fig. 6 it was also assumed that the boundary layer does not begin to develop at $r=0$ but rather at $r=3a$. This assumption is physically reasonable and gives a better fit to the data in the region $r<r_1$. Note that, despite the different form of the drag term, the predicted final depth in Fig. 6 has not changed significantly from that of Fig. 5. This is a further example of our earlier statements that the predicted final depth depends only weakly on the form of the drag coefficient C, provided only that C exceeds some minimum value.

The predicted second critical point at h_c is not very pronounced and would be difficult to check by experiment, due to small fluctuations of the flow pattern which were particularly noticeable in the jump region.

The predictions of earlier theories,[4,5] Eq. (12), are shown by the vertical dashed lines in the figures, and the predicted final depths indicated by the numbers over the arrow are very much larger and in poor agreement with experiment.

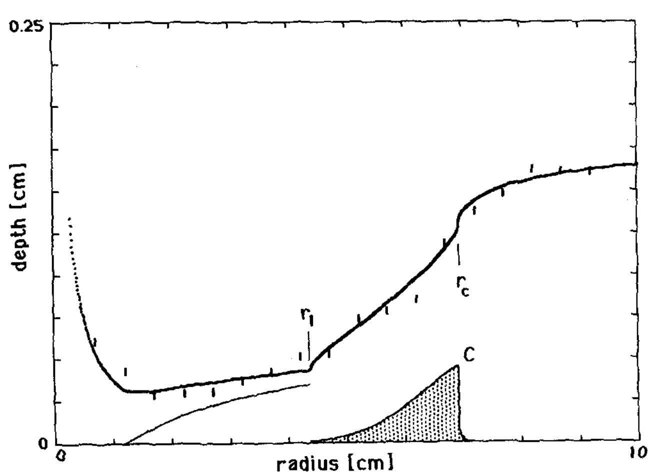

Fig. 6. Comparison of theory and experiment for $Q=40$ cm^3/s and $a=0.28$ cm. The model parameter $k=2.5$, and C was assumed to increase gradually to a maximum of 1.0 gm s/cm^2 as shown by the shaded region. Note that the boundary layer was assumed to begin at $r=3a$ rather than at $r=0$. The experimental data are the same as in Fig. 5.

A. Effect of microgravity

The present model predicts a strikingly different behavior for this type of hydraulic jump in a microgravity environment. Equation (6) predicts that h_c will increase as g decreases, and this would probably happen for moderate decreases of g. However, for very small values of g it is unlikely that h would rise to the critical value of h_c needed to make the transition to the asymptotically increasing flow regime. Instead the decreasing effect of drag, due to the larger h values, may allow the depth to start decreasing again before h_c is reached, as shown in Fig. 7. As h continues to decrease, a new boundary layer may develop and the process may be repeated at a larger r. This prediction could be tested by experiments in an orbiting space craft, or in a land-based microgravity facility.

V. DISCUSSION

One might reasonably ask whether the present model can be applied in the limit of the earlier models, i.e., zero momentum loss to the boundary. Strictly speaking, the answer is no since the present model produces no jump at all when the drag term is zero. This is physically reasonable, as it is simply saying that the liquid will not begin to slow down without the effect of the drag term. If the drag is sufficiently large the jump will occur via the positive feedback mechanism described in Sec. II, otherwise it will not occur. The previous models,[4,5] Eq. (12), simply predicted what the final depth would be if the flow suddenly slowed without momentum loss to the boundary. They did not provide a mechanism for initiating or continuing the slowdown. The fact that the final depths predicted by Eq. (12) are about three times larger than the measured values suggests that momentum loss to the boundary is an important factor.

It is possible to give a quantitative estimate of the degree of momentum loss to the boundary. If there is no loss to the boundary then one has the equality[4,5]

$$\tfrac{1}{2}g(h_f^2-h_1^2)=(v_1^2h_1-v_f^2h_f), \tag{13}$$

where the left side is proportional to the difference in the *thrust of the pressure head* on the two sides of the jump. The right side is proportional to the change of *momentum flux* from upstream to downstream of the jump. This equation essentially says that the rate of momentum loss is due to the net thrust. [It is assumed in Eq. (13) that the width of the jump is small compared to the jump radius.] Note that Eq. (13) reduces to Eq. (12) when $h_1 \ll h_f$ and $v_f \ll v_1$, as assumed by Godwin.[5] When momentum loss to the boundary does occur then one should therefore expect the inequality

$$\tfrac{1}{2}g(h_f^2-h_1^2)<(v_1^2h_1-v_f^2h_f) \tag{14}$$

and the strength of the inequality would be a measure of the degree of momentum loss. For example, substituting the model results from Fig. 3 ($h_1=0.0046$ cm, $v_1=110$ cm/s, $h_f=0.10$ cm and $v_f=1.9$ cm/s) into Eq. (14) one finds that the left side is about 10% of the right side, indicating that the majority of the upstream momentum flux is lost to boundary drag.

It is also interesting to note that Eqs. (13) and (14) can be derived from the present model, under certain assumptions. If the jump is assumed to be so sharp that $r \approx r_1$ throughout the jump then Eq. (3) can be approximated by

$$\frac{d}{dr}\left(hv^2+\frac{gh^2}{2}\right)=-\frac{C\nu}{2\pi r_1 \rho}\left(\frac{v}{h}\right)^n, \tag{15}$$

where use was made of Eq. (2) in the form $hv=Q/2\pi r_1=$const., and $d/dt=v\ d/dr$ was also used. Integrating Eq. (15) across the jump then gives

$$(g/2)(h_c^2-h_1^2)=(v_1^2h_1-v_c^2h_c)-H, \tag{16}$$

where $-H$ is the integral of the right-hand side of Eq. (15), $H \geqslant 0$. When the boundary drag coefficient $C=0$, Eq. (16) agrees exactly with Eq. (13) which is the standard result of previous models. When boundary drag is not zero Eq. (16) shows that the expected inequality of Eq. (14) is indeed a feature of the model. The present model appears, therefore, to be consistent with the previous models in the limit of a sharp jump. However, the rigorous numerical solution of Eq. (3) shows that a sharp jump occurs only when the drag coefficient C is large, in which case there is large momentum loss to the boundary. The good agreement with the experimental results suggests that this is also the case for real jumps.

VI. CONCLUSION

The hydraulic jump in the kitchen sink is observed by most people on a daily basis, yet an understanding of the physics of why it happens has been lacking. Only recently has the importance of frictional drag with the boundary been recognized. Godwin[5] showed that friction is important for determining the *radius* of the jump. The new model presented here takes frictional drag into account in a crude but physically reasonable way and suggests that friction also has an important effect on the *shape* and *height* of the jump. The results indicate that the usual textbook assumption of zero momentum loss at a hydraulic jump is not appropriate, at least for this case. The physics and mathematics concepts used are at the level of junior/senior undergraduate physics students, and hopefully the paper will help to arouse student interest in the often neglected subject of hydrodynamics.

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[1]Lord Rayleigh, "On the theory of long waves and bores," Proc. R. Soc. London, Ser. A **90**, 324–328 (1914).
[2]J. Walker, "Sink hydraulic jump," in *The Flying Circus of Physics* (Wiley, New York, 1975), p. 93.
[3]R. A. R. Tricker, *Bores, Breakers, Waves and Wakes* (American Elsevier, New York, 1965), Chaps. 5 and 6.
[4]E. J. Watson, "The radial spread of a liquid jet over a horizontal plane," J. Fluid Mech. **20**, 481–499 (1964).
[5]R. P. Godwin, "The hydraulic jump (shocks and viscous flow in the kitchen sink)," Am. J. Phys. **61**, 829–832 (1993).
[6]S. E. Koonin, *Computational Physics* (Addison-Wesley, Reading, MA, 1986), p. 31.

TEACHING: ACCURATE OR INTERESTING?

Poets like Spenser boasted of knowing books which they had never read, and stole their quotations from intermediate authors. But on the whole they knew far more about literature than we do, because they were taught in school that it was interesting and shown how to discover its interest. There is a dichotomy here. Scholarship must be accurate, whether it is interesting or not. But teaching must be interesting, even if it is not one hundred per cent accurate.

Gilbert Highet, *The Art of Teaching* (Vintage Books, New York, 1989; originally published by Alfred A. Knopf, 1950), p. 194.