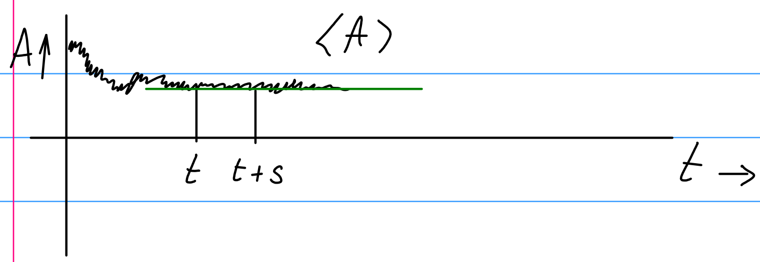


$$K(s) = \langle A(t)\, A(t+s) \rangle = \lim_{T \to \infty} \int_0^T A(t)\, A(t+s)\, dt$$

$\langle A(t) \rangle = 0$ Otherwise $A(t) \to A(t) - \langle A \rangle$

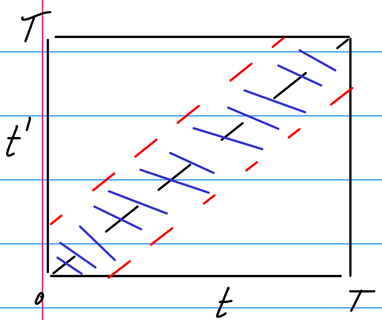
$$K(s) = K(-s)$$

$$\tilde{A}(\omega) = \frac{1}{\sqrt{T}} \int_0^T A(t)\, e^{i\omega t}\, dt$$

$S(\omega) = \tilde{A}(\omega)\, \tilde{A}^*(\omega)$ power spectrum

$$S(\omega) = \tilde{A}(\omega)\,\tilde{A}^*(\omega) \qquad \text{'power spectrum'.}$$

$$S(\omega) = \frac{1}{T} \int_0^T \underbrace{A(t)\, e^{i\omega t}\, dt \int_0^T A(t')}_{K(t'-t)}\, e^{-i\omega t'}\, dt' =$$



Significant contributions come from the blue shaded area.

New variables: $\tau = \frac{t+t'}{2}$, $s = t' - t$ $\Big\}$ $t = \tau - s/2$, $t' = \tau + s/2$

Then: $$S(\omega) = \frac{1}{T} \int_0^T d\tau \int_{-\infty}^{\infty} ds\, e^{i\omega(\tau - s/2)}\, e^{-i\omega(\tau + s/2)}\, \underbrace{A(\tau - s/2) A(\tau + s/2)}_{K(s)\ (\text{with } \int d\tau)}$$

Hence: $$S(\omega) = \frac{1}{T} \int_{-\infty}^{\infty} ds\, K(s)\, e^{-i\omega s} = K(\omega).$$

Wiener - Kintchine theorem.

Let us apply this to a Brownian particle starting off with $v = 0$

$(m\dot{v} = -\gamma v + R)$ R may now be correlated!

$$\rightarrow m v(t) = e^{-\gamma t/m} \int_0^{t'} e^{\gamma t'/m} R(t')\, dt'$$

$\langle \frac{m}{2} v^2(t) \rangle$ = kinetic energy

$$= \frac{1}{2m} \left\langle e^{-2\gamma t/m} \int_0^t e^{\gamma t_1/m} R(t_1)\, dt_1 \int_0^t e^{\gamma t_2/m} R(t_2)\, dt_2 \right\rangle$$

$$\left(\langle R(t_1) R(t_2) \rangle = K_F(t_1 - t_2) \right)$$

$$= \frac{1}{2m} e^{-2\gamma t/m} \int_0^t d\tau \int_0^t e^{2\gamma\tau/m} K(s)\, ds\; e^{i\underset{0}{\underset{\|}{\omega}} s}$$

$$= \frac{1}{4\gamma}\left(1 - e^{-2\gamma t/m}\right) \tilde{K}_F(0)$$

Therefore: $\frac{k_B T}{2} = \frac{1}{4\gamma} \tilde{K}_F(0) \rightarrow \tilde{K}_F(0) = 2\gamma k_B T$.

For a Langevin particle (R(t) uncorrelated).

$$\langle v(0) v(t) \rangle = \frac{k_B T}{m} e^{-\gamma |t|/m}$$

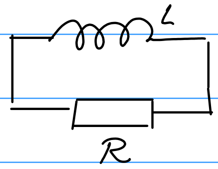
Then: $K_v(\omega) = \frac{k_B T}{m} \int_{-\infty}^{\infty} e^{-\gamma |t|/m + i\omega t} dt$

$$= \frac{2 k_B T}{m} \mathrm{Re} \int_0^{\infty} e^{-\gamma t/m + i\omega t} dt = \frac{2 k_B T}{m} \mathrm{Re} \frac{1}{\gamma/m - i\omega}$$

$$= \frac{2 k_B T}{\gamma} \mathrm{Re} \frac{1}{1 - i\omega m/\gamma} = \frac{2 k_B T}{\gamma} \frac{1}{1 + \omega^2 m^2/\gamma^2}.$$

So, $\langle |v(\omega)|^2 \rangle = S(\omega) = K_v(\omega) = \frac{2 k_B T}{\gamma} \frac{1}{1 + \omega^2 m^2/\gamma^2}$

For an electric current:



$I = \dot{q}$ where q is the transported charge.

$V_L = L\dot{I}$; $V_R = IR$

$L\dot{I} = -RI + \eta$, so $L \leftrightarrow m$ and $R \leftrightarrow \gamma$.

$m\dot{v} = -\gamma v + R$

Energy $= \frac{1}{2}LI^2 +$ other terms. $\rightarrow$ $\frac{1}{2}\langle LI^2\rangle = \frac{k_B T}{2}$

$$\langle \tilde{I}(\omega)\tilde{I}^*(\omega)\rangle = \frac{2k_BT}{R}\,\frac{1}{1+\omega^2L^2/R^2}$$. 'white noise'.

$$S_V = R^2\langle \tilde{I}(\omega)\tilde{I}^*(\omega)\rangle = 2k_BTR \qquad \omega \ll R/L$$

In a frequency band $\Delta\omega$:

$\sqrt{\langle V^2\rangle} = \sqrt{2k_BTR\Delta\omega}$ Take $\Delta\omega \simeq 10^4\,Hz$ (audio)

$R = 10^6\,\Omega$, $k_BT = 30\,meV$ (room temp)

$$\sqrt{\langle V^2\rangle} = \sqrt{2\cdot 30\cdot 10^{-3}\cdot 10^{6}\cdot 10^{4}\,1.6\cdot 10^{-19}} \sim 10^{-6}\,V.$$