Bose Einstein Condensation

$$\epsilon_{\underline{k}} = \frac{\hbar^2 k^2}{2m} \qquad \text{Massive, free Bosons.}$$

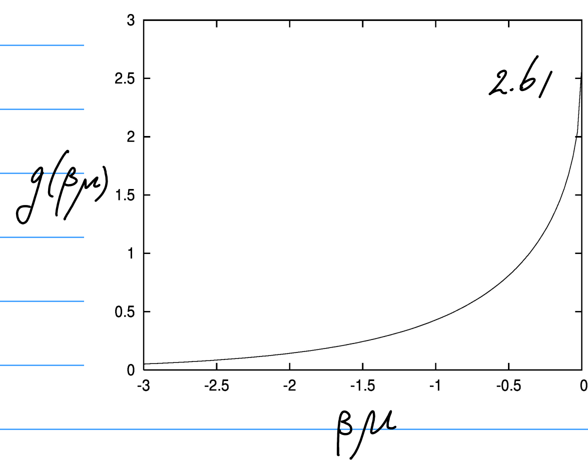
$$N = \sum_{\underline{k}} \frac{1}{e^{\beta(\epsilon_{\underline{k}} - \mu)} - 1} = \frac{V}{(2\pi)^3} \int_0^\infty 4\pi k^2 dk \; \frac{1}{e^{\beta(\hbar^2 k^2/2m - \mu)} - 1}$$

$$\left(x^2 = \beta \frac{\hbar^2 k^2}{2m} \right)$$

$$\frac{N}{V} \lambda^3 = \frac{4}{\sqrt{\pi}} \int_0^\infty \frac{x^2}{e^{x^2 - \beta\mu} - 1} dx \equiv g(\beta\mu).$$

$$\mu < \epsilon_{\underline{k}} \Rightarrow \beta\mu < 0$$

$$n_{max} \simeq \frac{2.61}{\lambda^3}$$

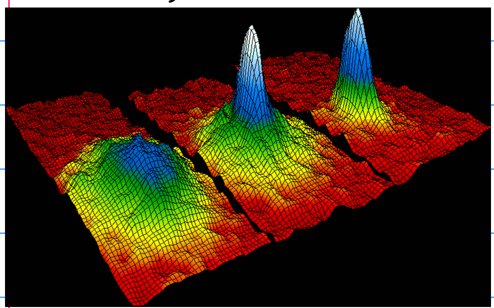


$$n_g = \frac{1}{e^{\beta(\epsilon_g - \mu)} - 1} \quad \text{very large}$$

$$\frac{N}{V} = n = \frac{2.61}{\lambda^3} + \frac{1}{e^{\beta(\epsilon_g - \mu)} - 1}$$

Bose Einstein condensation

Cornell, Wiemann, 1995



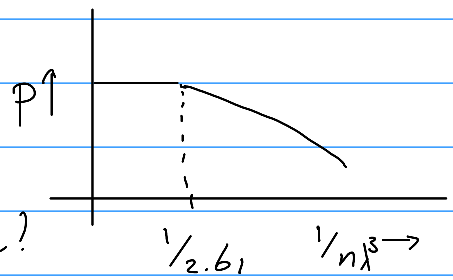
Rubidium

Ketterle: Sodium.

$$\frac{P}{k_B T} = \frac{1}{V} \ln Z_{gr} = -\frac{4}{\lambda^3\sqrt{\pi}} \int_0^\infty \ln\left(1 - e^{-x^2+\beta\mu}\right) x^2\,dx = f(\beta\mu)$$

$$\frac{N}{V} = n = \lambda^3 g(\beta\mu) \quad \mu < 0$$

BE : $\mu = 0$



11.21.48 over?