Planck Distribution - The Chemical Potential of Photons

Quantum Harmonic oscillator

$$Z = \sum_{n=0}^{\infty} e^{-\beta\hbar\omega(n+1/2)}$$

Average n: $$\langle n\rangle = \frac{\sum_{n=1}^{\infty} n\, e^{-\beta\hbar\omega(n+\cancel{1/2})}}{\sum_{n=0}^{\infty} e^{-\beta\hbar\omega(n+\cancel{1/2})}} =$$

$$= \frac{\sum_{n=1}^{\infty} n\cdot\xi^n}{\sum_{n=0}^{\infty}\xi^n} \;\tilde{Z} \quad ; \quad \xi = e^{-\beta\hbar\omega}$$

$$= \frac{\partial \ln \tilde{Z}}{\partial \xi}\cdot\xi \quad ; \quad \tilde{Z} = \frac{1}{1-\xi}$$

$$\rightarrow \langle n\rangle = \frac{+\xi}{1-\xi} = \frac{1}{e^{\beta\hbar\omega}-1}$$

Bose-Einstein distribution with $\mu = 0$.

Electromagnetic field:

$$H = \sum_{\text{modes}} H^{QM}_{HO}(\omega_{\text{mode}})$$

mode: polarization & wavevector $\underline{k}$; $\omega_{\underline{k}} = c|\underline{k}|$.

$$\Rightarrow \langle n_{\underline{k},\epsilon}\rangle = \frac{1}{e^{\beta\hbar\omega_{\underline{k}}}-1}$$

Another argument for $\mu = 0$ Interaction with electrons

$$e^- + \gamma \rightleftarrows e^- \rightarrow \mu_e + \mu_\gamma = \mu_e \qquad \left(\sum_\nu n_\nu \mu_\nu = 0\right)$$

$$\rightarrow \mu_\gamma = 0$$

Electrons - positrons

$e^- + e^+ \rightleftarrows \gamma$ Then $\mu_- + \mu_+ = \mu_\gamma$

$N_- - N_+ = \text{const}: \quad \mu_- = \mu_+$

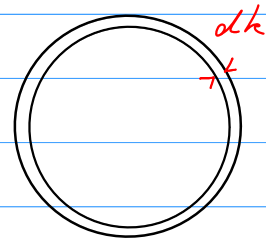
Additional arguments: $\mu_- = \mu_+ = 0.$

Modes: $\underline{k} = \frac{2\pi}{L}(n_x, n_y, n_z).$

Dispersion relation: $\omega_{\underline{k}} = c|\underline{k}|$

Number of modes between ω and $\omega + d\omega$; per vol

$$\underset{\substack{\uparrow\\ \text{polarizations}}}{2} \cdot \frac{V}{V} \cdot \frac{4\pi k^2 dk}{(2\pi)^3} = \frac{k^2 dk}{\pi^2}$$



Energy between ω and $\omega + d\omega$: $\qquad \omega = ck$

$$\frac{k^2 dk}{\pi^2} \cdot \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1} = \underline{\frac{\hbar}{\pi^2 c^3} \frac{\omega^3 d\omega}{e^{\beta\hbar\omega} - 1}}$$