Entropy of the ideal gas (P 3.8; LN 1.4)

$S(\mathcal{E}) = k_B \ln \Omega(\mathcal{E})$; $\Omega =$ nr of states accessible at $\mathcal{E}$

Volume $V = L \times L \times L$

$\psi = e^{i \underline{k} \cdot \underline{r}}$ Periodic boundary conditions.

$\underline{k} = \frac{2\pi}{L}(n_x, n_y, n_z)$

Grid in 3D. $\{\underline{p}_i\}$: grid in 3N dim. $\{\underline{p}_i\} = \underline{P}$ (3N)

Grid const: $\frac{2\pi\hbar}{L} = hk.$ $(\underline{p} = \hbar \underline{k})$

States have energy $\mathcal{E} = \sum_i \frac{\underline{p}_i^2}{2m} = \frac{\underline{P}^2}{2m}$

States with " $\leq \mathcal{E}$: volume sphere with radius P $V(P, 3N)$

$$\rightarrow \quad \Omega(\mathcal{E}) = \frac{dV(P,3N)}{dP} dP \underbrace{\left(\frac{L}{h}\right)^{3N}}_{1/\text{vol of grid cell}}$$

$$V(P, 3N) = \frac{\pi^{3N/2}}{(3N/2 + 1)!} P^{3N}$$

$$\text{So} \quad \Omega(\mathcal{E}) = \frac{\pi^{3N/2}}{(3N/2+1)!} 3N (2m\mathcal{E})^{\frac{3N-1}{2}} \frac{V^N}{h^{3N}}$$

Use $\ln N! = N(\ln N - 1)$

So $S = k_B\left\{N\ln\left(\frac{V}{h^3}(2\pi m\mathcal{E})^{3/2}\right) + \ln 3N - \left(\frac{3N+1}{2}\right)\left(\ln\left(\frac{3N+1}{2}\right)-1\right)\right\} + k_B\ln\cancel{\Delta p}$

$$\simeq k_B N\left[\ln\left(\frac{V}{h^3}\left(\frac{4\pi m\mathcal{E}}{3N}\right)^{3/2}\right) + \frac{3}{2}\right] + \cancel{O(\ln N)} +$$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial \mathcal{E}}\right)_{N,V} = \frac{3}{2}\frac{Nk_B}{\mathcal{E}}, \text{ or: } \underline{\mathcal{E} = \frac{3}{2}Nk_BT}$$

$$C_V = \frac{3}{2}Nk_B$$

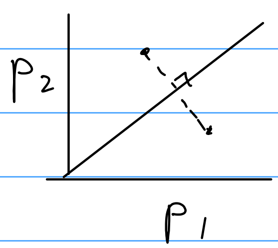
Furthermore: $\frac{P}{T} = \left.\frac{\partial S}{\partial V}\right)_{\mathcal{E},N} = \frac{Nk_B}{V} \rightarrow \underline{PV = Nk_BT}$

$$\neq$$

$$S = k_B N\left[\ln\left(\frac{V}{h^3}\left(\frac{4\pi m\mathcal{E}}{3N}\right)^{3/2}\right) + \frac{3}{2}\right] =$$

$$k_B N\left[\ln V/h^3 + \frac{3}{2}\ln\frac{4\pi m}{3} + \frac{3}{2}\ln\frac{\mathcal{E}}{N} + \frac{3}{2}\right]$$

$$S(2\mathcal{E}, 2V, 2N) = 2S(\mathcal{E},V,N) + 2k_BN\ln 2$$



Extra factor $\frac{1}{N!} \rightarrow$

$$S_{QM} = S_{cl} - k_B N(\ln N - 1)$$

$$= k_B N\left[\ln\frac{V}{N} + \frac{3}{2}\ln\frac{4\pi m}{3h^2} + \frac{3}{2}\ln\frac{\mathcal{E}}{N} + 5/2\right]$$

This is extensive.