The Perez All-Weather model.

Relative luminance.

$$l_{\mathrm{v}} = f(\xi, \gamma) = [1 + a \exp(b / \cos \xi)] \cdot [1 + c \exp(d\gamma) + e \cos^2 \gamma]. \tag{1}$$

Assumption.

$$b < 0. \tag{2}$$

Luminance at the zenith.

$$l_{\mathrm{v}}(0°, \gamma) = [1 + a \exp(b)] \cdot [1 + c \exp(d\gamma) + e \cos^2 \gamma]. \tag{3}$$

Luminance at the horizon.

$$f(90°, \gamma) = 1 + c \exp(d\gamma) + e \cos^2 \gamma. \tag{4}$$

Luminance of the sun.

$$f(\xi, 0°) = \left[1 + a \exp\left(\frac{b}{\cos} \xi\right)\right] \cdot [1 + c + e] \tag{5}$$

Luminance of the sun at the zenith.

$$f(0°, 0°) = [1 + a \exp(b)] \cdot [1 + c + e]. \tag{6}$$

Luminance of the sun at the horizon.

$$f(90°, 0°) = 1 + c + e. \tag{7}$$

Absolute luminance from absolute luminance at zenith.

$$L_{\mathrm{v}} = L_{\mathrm{vz}} f(\xi, \gamma) / f(0°, \gamma). \tag{8}$$

Absolute luminance from illuminance.

$$L_{\mathrm{v}} = l_{\mathrm{v}} E_{\mathrm{vd}} \left(\int_{\mathrm{sky}} [\mathrm{lv}(\xi, \gamma) \cos \xi] \mathrm{d}\omega \right)^{-1}. \tag{9}$$

Preetham model.

Restatement of the Perez All-Weather model.

$$\mathcal{F}(\theta,\gamma) = \left(1 + Ae^{B/\cos\theta}\right)\left(1 + Ce^{D\gamma} + E\cos^2\gamma\right). \tag{10}$$

$$Y = Y_z \mathcal{F}(\theta,\gamma)/\mathcal{F}(0,\theta_s). \tag{11}$$

Chromatic model extension.

$$x = x_z \frac{\mathcal{F}(\theta,\gamma)}{\mathcal{F}(0,\theta_s)}, \quad \text{and} \quad y = y_z \frac{\mathcal{F}(\theta,\gamma)}{\mathcal{F}(0,\theta_s)}. \tag{12}$$

Perez coeffiecients from turbidity and solar zenith angle.

$$\begin{bmatrix} A_Y \\ B_Y \\ C_Y \\ D_Y \\ E_Y \end{bmatrix} = \begin{bmatrix} 0.1787 & -1.4630 \\ -0.3554 & 0.4275 \\ -0.0227 & 5.3251 \\ 0.1206 & -2.5771 \\ -0.0670 & 0.3703 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}. \tag{13}$$

$$\begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ E_x \end{bmatrix} = \begin{bmatrix} -0.0193 & -0.2592 \\ -0.0665 & 0.0008 \\ -0.0004 & 0.2125 \\ -0.0641 & -0.8989 \\ -0.0033 & 0.0452 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}. \tag{14}$$

$$\begin{bmatrix} A_y \\ B_y \\ C_y \\ D_y \\ E_y \end{bmatrix} = \begin{bmatrix} -0.0167 & -0.2608 \\ -0.0950 & 0.0092 \\ -0.0079 & 0.2102 \\ -0.0441 & -1.6537 \\ -0.0109 & 0.0529 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}. \tag{15}$$

Absolute zenith luminance and chromaticity.

$$Y_z = (4.0453T - 4.9710)\tan\chi - 0.2155T + 2.4192. \tag{16}$$

$$\chi = \left(\frac{4}{9} - \frac{T}{120}\right)(\pi - 2\theta_s). \tag{17}$$

$$x_z = \begin{bmatrix} T^2 & T & 1 \end{bmatrix} \begin{bmatrix} 0.0017 & -0.0037 & 0.0021 & 0.000 \\ -0.0290 & 0.0638 & -0.0320 & 0.0039 \\ 0.1169 & -0.2120 & 0.0605 & 0.2589 \end{bmatrix} \begin{bmatrix} \theta_s^3 \\ \theta_s^2 \\ \theta_s \\ 1 \end{bmatrix}. \tag{18}$$

$$y_z = \begin{bmatrix} T^2 & T & 1 \end{bmatrix} \begin{bmatrix} 0.0028 & -0.0061 & 0.0032 & 0.000 \\ -0.0421 & 0.0897 & -0.0415 & 0.0052 \\ 0.1535 & -0.2676 & 0.0667 & 0.2669 \end{bmatrix} \begin{bmatrix} \theta_s^3 \\ \theta_s^2 \\ \theta_s \\ 1 \end{bmatrix}. \tag{19}$$

Reinhard tone mapping.

Log-average luminance (corrected).

$$\overline{L}_w = \exp\left(\frac{1}{N} \sum_{x,y} \log\Big(\delta + L_{w(x,y)}\Big) \right). \tag{20}$$

Scaled luminance.

$$L(x,y) = \frac{\alpha}{\overline{L}_w} L_{w(x,y)}. \tag{21}$$

Global burning-free tone mapping.

$$L_d(x,y) = \frac{L(x,y)}{1 + L(x,y)}. \tag{22}$$

Global tone mapping with a white point.

$$L_d(x,y) = \frac{L(x,y)\Big(1 + \frac{L(x,y)}{L_{\text{white}}^2}\Big)}{1 + L(x,y)}. \tag{23}$$

Additional equations.

Discretization in equirectangular coordinates.

$$
\begin{gathered}
(i, j) \in \{0, ..., W-1\} \times \{0, ..., \lfloor H/2 \rfloor - 1\} \to (u, v) \in [0, 1) \times (0, 1), \\
u = \frac{i + 0.5}{W}, \quad \text{and} \quad v = \frac{2j + 1}{H}.
\end{gathered} \tag{24}
$$

Equirectangular projection (From Wikipedia).

$$
\begin{gathered}
(u, v) \in [0, 1) \times (0, 1) \leftrightarrow (\psi, \phi) \in [-\pi, \pi) \times (0, \pi/2), \\
\psi = 2\pi u, \quad \text{and} \quad \phi = \frac{\pi}{2}(1 - v).
\end{gathered} \tag{25}
$$

Conversion between standard and non-standard spherical coordinates.

$$
\begin{gathered}
(\psi, \phi, \psi_s, \phi_s) \in [-\pi, \pi) \times (0, \pi/2) \times [-\pi, \pi) \times (0, \pi/2) \\
\to \\
(\theta, \theta_s, \gamma) \in (0, \pi/2) \times (0, \pi/2) \times [0, \pi).
\end{gathered} \tag{26}
$$

Zenith angles.

$$
\theta = \frac{\pi}{2} - \phi, \quad \text{and} \quad \theta_s = \frac{\pi}{2} - \phi_s. \tag{27}
$$

Spherical distance in spherical coordinates (from Wikipedia).

$$
\gamma = \arccos(\sin\phi \sin\phi_s + \cos\phi \cos\phi_s \cos(\Delta\psi)). \tag{28}
$$

Haversine formula (from Wikipedia).

$$
\gamma = \operatorname{archav}(\operatorname{hav}(\Delta\phi) + (1 - \operatorname{hav}(\phi + \phi_s)) \operatorname{hav}(\Delta\psi)). \tag{29}
$$

Vincenty formula (from Wikipedia).

$$
\begin{gathered}
\gamma = \operatorname{atan2}\Bigg(\sqrt{(\cos\phi_s \sin(\Delta\psi))^2 + (\cos\phi \sin\phi_s - \sin\phi \cos\phi_s \cos(\Delta\psi))^2}, \\
\sin\phi \sin\phi_s + \cos\phi \cos\phi_s \cos(\Delta\psi)\Bigg).
\end{gathered} \tag{30}
$$

Conversion from CIE xyY to CIE XYZ (from Wikipedia).

$$
\begin{gathered}
(x, y) \in [0, 1] \times (0, 1] \leftrightarrow (X, Z) \in [0, \infty) \times [0, \infty), \\
X = \frac{Y}{y} x, \quad \text{and} \quad Z = \frac{Y}{y}(1 - x - y).
\end{gathered} \tag{31}
$$

Conversion from CIE XYZ to linear sRGB (from Wikipedia).

$$
\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} +3.2406255 & -1.5372073 & -0.4986286 \\ -0.9689307 & +1.8757561 & +0.0415175 \\ +0.0557101 & -0.2040211 & +1.0569959 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{32}
$$

Conversion from linear sRGB to sRGB (from Wikipedia).

$$
R' = \begin{cases} 12.92R & \text{if } R \le 0.0031308, \\ 1.055R^{1/2.4} - 0.055 & \text{otherwise.} \end{cases}, \quad \text{and similarly for } G' \text{ and } B'. \tag{33}
$$