

## **The Perez All-Weather model.**

*Relative luminance.*

$$l_v = f(\xi, \gamma) = [1 + a \exp(b/\cos \xi)] \cdot [1 + c \exp(d\gamma) + e \cos^2 \gamma]. \quad (1)$$

*Assumption.*

$$b < 0. \quad (2)$$

*Luminance at the zenith.*

$$l_v(0^\circ, \gamma) = [1 + a \exp(b)] \cdot [1 + c \exp(d\gamma) + e \cos^2 \gamma]. \quad (3)$$

*Luminance at the horizon.*

$$f(90^\circ, \gamma) = 1 + c \exp(d\gamma) + e \cos^2 \gamma. \quad (4)$$

*Luminance of the sun.*

$$f(\xi, 0^\circ) = \left[ 1 + a \exp\left(\frac{b}{\cos \xi}\right) \right] \cdot [1 + c + e] \quad (5)$$

*Luminance of the sun at the zenith.*

$$f(0^\circ, 0^\circ) = [1 + a \exp(b)] \cdot [1 + c + e]. \quad (6)$$

*Luminance of the sun at the horizon.*

$$f(90^\circ, 0^\circ) = 1 + c + e. \quad (7)$$

*Absolute luminance from absolute luminance at zenith.*

$$L_v = L_{vz} f(\xi, \gamma) / f(0^\circ, \gamma). \quad (8)$$

*Absolute luminance from illuminance.*

$$L_v = l_v E_{vd} \left( \int_{\text{sky}} [l_v(\xi, \gamma) \cos \xi] d\omega \right)^{-1}. \quad (9)$$

## Preetham model.

*Restatement of the Perez All-Weather model.*

$$\mathcal{F}(\theta, \gamma) = (1 + Ae^{B/\cos\theta})(1 + Ce^{D\gamma} + E\cos^2\gamma). \quad (10)$$

$$Y = Y_z \mathcal{F}(\theta, \gamma) / \mathcal{F}(0, \theta_s). \quad (11)$$

*Chromatic model extension.*

$$x = x_z \frac{\mathcal{F}(\theta, \gamma)}{\mathcal{F}(0, \theta_s)}, \quad \text{and} \quad y = y_z \frac{\mathcal{F}(\theta, \gamma)}{\mathcal{F}(0, \theta_s)}. \quad (12)$$

*Perez coefficients from turbidity and solar zenith angle.*

$$\begin{bmatrix} A_Y \\ B_Y \\ C_Y \\ D_Y \\ E_Y \end{bmatrix} = \begin{bmatrix} 0.1787 & -1.4630 \\ -0.3554 & 0.4275 \\ -0.0227 & 5.3251 \\ 0.1206 & -2.5771 \\ -0.0670 & 0.3703 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}. \quad (13)$$

$$\begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ E_x \end{bmatrix} = \begin{bmatrix} -0.0193 & -0.2592 \\ -0.0665 & 0.0008 \\ -0.0004 & 0.2125 \\ -0.0641 & -0.8989 \\ -0.0033 & 0.0452 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}. \quad (14)$$

$$\begin{bmatrix} A_y \\ B_y \\ C_y \\ D_y \\ E_y \end{bmatrix} = \begin{bmatrix} -0.0167 & -0.2608 \\ -0.0950 & 0.0092 \\ -0.0079 & 0.2102 \\ -0.0441 & -1.6537 \\ -0.0109 & 0.0529 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}. \quad (15)$$

*Absolute zenith luminance and chromaticity.*

$$Y_z = (4.0453T - 4.9710) \tan \chi - 0.2155T + 2.4192. \quad (16)$$

$$\chi = \left( \frac{4}{9} - \frac{T}{120} \right) (\pi - 2\theta_s). \quad (17)$$

$$x_z = [T^2 \ T \ 1] \begin{bmatrix} 0.0017 & -0.0037 & 0.0021 & 0.000 \\ -0.0290 & 0.0638 & -0.0320 & 0.0039 \\ 0.1169 & -0.2120 & 0.0605 & 0.2589 \end{bmatrix} \begin{bmatrix} \theta_s^3 \\ \theta_s^2 \\ \theta_s \\ 1 \end{bmatrix}. \quad (18)$$

$$y_z = [T^2 \ T \ 1] \begin{bmatrix} 0.0028 & -0.0061 & 0.0032 & 0.000 \\ -0.0421 & 0.0897 & -0.0415 & 0.0052 \\ 0.1535 & -0.2676 & 0.0667 & 0.2669 \end{bmatrix} \begin{bmatrix} \theta_s^3 \\ \theta_s^2 \\ \theta_s \\ 1 \end{bmatrix}. \quad (19)$$

### Additional computations.

*Equirectangular projection.*

$$\begin{aligned} (u, v) \in (0, 1) \times (0, 1) &\leftrightarrow (\theta, \phi) \in (-\pi, \pi) \times (-\pi/2, \pi/2), \\ \theta &= 2\pi(u - 0.5), \quad \text{and} \quad \phi = \pi(0.5 - v). \end{aligned} \quad (20)$$

*Discretization in equirectangular coordinates.*

$$\begin{aligned} (i, j) \in \{0, \dots, H - 1\} \times \{0, \dots, W - 1\} &\rightarrow (u, v) \in (0, 1) \times (0, 1), \\ u &= \frac{i + 0.5}{H}, \quad \text{and} \quad v = \frac{j + 0.5}{W}. \end{aligned} \quad (21)$$

*Conversion between standard and non-standard spherical coordinates.*

$$\begin{aligned} (\theta, \phi, \theta_s, \phi_s) &\in (-\pi, \pi) \times (-\pi/2, \pi/2) \times (-\pi, \pi) \times (-\pi/2, \pi/2) \\ &\rightarrow \\ (\theta, \theta_s, \gamma) &\in (-\pi, \pi) \times (-\pi, \pi) \times [0, \pi). \end{aligned} \quad (22)$$

*Spherical distance in spherical coordinates (from Wikipedia).*

$$\gamma = \arccos(\sin \phi \sin \phi_s + \cos \phi \cos \phi_s \cos(\Delta\theta)). \quad (23)$$

*Haversine formula (from Wikipedia).*

$$\gamma = \operatorname{archav}(\operatorname{hav}(\Delta\phi) + (1 - \operatorname{hav}(\phi + \phi_s)) \operatorname{hav}(\Delta\theta)). \quad (24)$$

*Vincenty formula (from Wikipedia).*

$$\begin{aligned} \gamma &= \operatorname{atan2} \left( \sqrt{(\cos \phi_s \sin(\Delta\theta))^2 + (\cos \phi \sin \phi_s - \sin \phi \cos \phi_s \cos(\Delta\theta))^2}, \right. \\ &\quad \left. \sin \phi \sin \phi_s + \cos \phi \cos \phi_s \cos(\Delta\theta) \right). \end{aligned} \quad (25)$$

*Conversion from CIE xyY to CIE XYZ (from Wikipedia).*

$$\begin{aligned} (x, y) \in [0, 1] \times (0, 1] &\leftrightarrow (X, Z) \in [0, \infty) \times [0, \infty), \\ X &= \frac{Y}{y}x, \quad \text{and} \quad Z = \frac{Y}{y}(1 - x - y). \end{aligned} \quad (26)$$

*Conversion from CIE XYZ to linear sRGB (from Wikipedia).*

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} +3.2406255 & -1.5372073 & -0.4986286 \\ -0.9689307 & +1.8757561 & +0.0415175 \\ +0.0557101 & -0.2040211 & +1.0569959 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (27)$$

*Conversion from linear sRGB to sRGB (from Wikipedia).*

$$R' = \begin{cases} 12.92R & \text{if } R \leq 0.0031308, \\ 1.055R^{1/2.4} - 0.055 & \text{otherwise.} \end{cases}, \quad \text{and similarly for } G' \text{ and } B'. \quad (28)$$