## The Perez All-Weather model.

Relative luminance.

$$l_{\rm v} = f(\xi, \gamma) = \left[1 + a \exp(b/\cos \xi)\right] \cdot \left[1 + c \exp(d\gamma) + e \cos^2 \gamma\right]. \tag{1}$$

Assumption.

$$b < 0. (2)$$

Luminance at the zenith.

$$l_{\mathbf{v}}(0^{\circ}, \gamma) = [1 + a \exp(b)] \cdot [1 + c \exp(d\gamma) + e \cos^2 \gamma]. \tag{3}$$

Luminance at the horizon.

$$f(90^{\circ}, \gamma) = 1 + c \exp(d\gamma) + e \cos^2 \gamma. \tag{4}$$

Luminance of the sun.

$$f(\xi, 0^{\circ}) = \left[1 + a \exp\left(\frac{b}{\cos \xi}\right)\right] \cdot [1 + c + e] \tag{5}$$

Luminance of the sun at the zenith.

$$f(0^{\circ}, 0^{\circ}) = [1 + a \exp(b)] \cdot [1 + c + e]. \tag{6}$$

Luminance of the sun at the horizon.

$$f(90^{\circ}, 0^{\circ}) = 1 + c + e. \tag{7}$$

Absolute luminance from absolute luminance at zenith.

$$L_{\rm v} = L_{\rm vz} f(\xi, \gamma) / f(0^{\circ}, \gamma). \tag{8}$$

Absolute luminance from illuminance.

$$L_{\rm v} = l_{\rm v} E_{\rm vd} \left( \int_{\rm sky} [lv(\xi, \gamma) \cos \xi] d\omega \right)^{-1}. \tag{9}$$

## Preetham model.

Restatement of the Perez All-Weather model.

$$\mathcal{F}(\theta,\gamma) = \left(1 + Ae^{B/\cos\theta}\right)\left(1 + Ce^{D\gamma} + E\cos^2\gamma\right). \tag{10}$$

$$Y = Y_z \mathcal{F}(\theta, \gamma) / \mathcal{F}(0, \theta_s). \tag{11}$$

Chromatic model extension.

$$x = x_z \frac{\mathcal{F}(\theta, \gamma)}{\mathcal{F}(0, \theta_s)}, \text{ and } y = y_z \frac{\mathcal{F}(\theta, \gamma)}{\mathcal{F}(0, \theta_s)}.$$
 (12)

Perez coefficients from turbidity and solar zenith angle.

$$\begin{bmatrix} A_Y \\ B_Y \\ C_Y \\ D_Y \\ E_Y \end{bmatrix} = \begin{bmatrix} 0.1787 & -1.4630 \\ -0.3554 & 0.4275 \\ -0.0227 & 5.3251 \\ 0.1206 & -2.5771 \\ -0.0670 & 0.3703 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}.$$
 (13)

$$\begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ E_x \end{bmatrix} = \begin{bmatrix} -0.0193 & -0.2592 \\ -0.0665 & 0.0008 \\ -0.0004 & 0.2125 \\ -0.0641 & -0.8989 \\ -0.0033 & 0.0452 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}. \tag{14}$$

$$\begin{bmatrix} A_y \\ B_y \\ C_y \\ D_y \\ E_y \end{bmatrix} = \begin{bmatrix} -0.0167 & -0.2608 \\ -0.0950 & 0.0092 \\ -0.0079 & 0.2102 \\ -0.0441 & -1.6537 \\ -0.0109 & 0.0529 \end{bmatrix} \begin{bmatrix} T \\ 1 \end{bmatrix}. \tag{15}$$

Absolute zenith luminance and chromaticity.

$$Y_z = (4.0453T - 4.9710)\tan\chi - 0.2155T + 2.4192. \tag{16}$$

$$\chi = \left(\frac{4}{9} - \frac{T}{120}\right)(\pi - 2\theta_s). \tag{17}$$

$$x_z = \begin{bmatrix} T^2 & T & 1 \end{bmatrix} \begin{bmatrix} 0.0017 & -0.0037 & 0.0021 & 0.000 \\ -0.0290 & 0.0638 & -0.0320 & 0.0039 \\ 0.1169 & -0.2120 & 0.0605 & 0.2589 \end{bmatrix} \begin{bmatrix} \theta_s^3 \\ \theta_s^2 \\ \theta_s \\ 1 \end{bmatrix}. \tag{18}$$

$$y_z = \begin{bmatrix} T^2 & T & 1 \end{bmatrix} \begin{bmatrix} 0.0028 & -0.0061 & 0.0032 & 0.000 \\ -0.0421 & 0.0897 & -0.0415 & 0.0052 \\ 0.1535 & -0.2676 & 0.0667 & 0.2669 \end{bmatrix} \begin{bmatrix} \theta_s^3 \\ \theta_s^2 \\ \theta_s \\ 1 \end{bmatrix}.$$
(19)

## Additional computations.

Equirectangular projection.

$$(u, v) \in (0, 1) \times (0, 1) \leftrightarrow (\theta, \varphi) \in (-\pi, \pi) \times (-\pi/2, \pi/2),$$
  
 $\theta = 2\pi(u - 0.5), \text{ and } \phi = \pi(0.5 - v).$  (20)

Discretization in equirectangular coordinates.

$$(i,j) \in \{0,...,H-1\} \times \{0,...,W-1\} \rightarrow (u,v) \in (0,1) \times (0,1),$$
 
$$u = \frac{i+0.5}{H}, \quad \text{and} \quad v = \frac{j+0.5}{W}.$$
 (21)

Conversion between standard and non-standard spherical coordinates.

Work in progress.

Conversion from CIE xyY to CIE XYZ (from Wikipedia).

$$(x,y) \in [0,1] \times (0,1] \leftrightarrow (X,Z) \in [0,\infty) \times [0,\infty),$$
 
$$X = \frac{Y}{y}x, \quad \text{and} \quad Z = \frac{Y}{y}(1-x-y). \tag{22}$$

Conversion from CIE XYZ to linear sRGB (from Wikipedia).

Conversion from linear sRGB to sRGB (from Wikipedia).

$$R' = \begin{cases} 12.92R & \text{if } R \le 0.0031308, \\ 1.055R^{1/2.4} - 0.055 & \text{otherwise.} \end{cases}, \text{ and similarly for } G' \text{ and } B'.$$
 (24)

Conversion from CIE xyY to sRGB.

Work in progress.

Spherical distance in spherical coordinates.

Work in progress.