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Modeling Fatality Distances

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A probability density function (pdf) of fatalities found around turbines is needed to calculate an area correction factor. The pdf is a function of the distance of the carcass from the turbine, assuming isotropy.

If all of the carcasses are assumed to be detected with equal probability the carcass distances can be modeled in a typical fashion and standard maximum likelihood estimates can be obtained. In practice the probability of detection is not equal for all carcasses and in particular the probability of detection is a function of the distance from the turbine mast.

This document presents some of the ideas to account for differing probabilities of detection and some of the questions and difficulties associated with this process. The first section describes the weighted distribution, as introduced by Dan Dalthorp, with the two versions depending on the choice of weights and the second section introduces the weighted likelihood. This paper does not solve the problem but is a foundation for further discussion to answer the remaining questions.

1 Weighted Distribution

Let $f(x|\theta)$ be some known pdf (e.q. Gamma, Weibull, etc.) with θ being the associated vector of parameters. In the context of carcass fatalities at wind turbines x represents distances from the turbine.

The weighted distribution (WD) is defined as

$$f_j^*(x|\theta) = \frac{w_j(x)f(x|\theta)}{\int_{I_f} w_j(y)f(y|\theta)dy} \tag{1}$$

where I_f is the support associated with $f(x|\theta)$ and $w_j(x)$ is the weighting function.

Q1: How should $w_j(x)$ be defined?

We believe that $w_j(x)$ should be

$$w_1(x) \propto \text{ probability of detection at distance x} \tag{2}$$

or

$$w_2(x) \propto \frac{1}{\text{probability of detection at distance x}} \tag{3}$$

It should be noted that $w_j(x)$ is a function of distance and if two carcasses are the same distance from the turbine the weight is the same for both carcasses.

In either case of the choice of $w_j(x)$, an estimate of θ is obtained from maximizing the log likelihood of $f_j^*(x|\theta)$,

$$l_j^*(\theta|\underline{\mathrm{x}}) = \sum_{i=1}^{n} log(w_j(x_i)) + \sum_{i=1}^{n} log(f(x_i|\theta)) - nlog(\int_{I_f} w_j(y)f(y|\theta)dy) \tag{4}$$

where x_i is the distance from the turbine for the i^{th} $(i = 1, 2, \cdots, n)$ carcass. The result from maximizing $l_j^*(\theta|\underline{\mathrm{x}})$ (Eq. 4) produces $\hat{\theta}_j^*$.

Once the choice of $w_j(x)$ has been made and $\hat{\theta}_j^*$ obtained, the question becomes how to apply $\hat{\theta}_j^*$ to calculate the area correction. The estimate $\hat{\theta}_j^*$ can be applied to f, f_j^* or

$$f_j^\dagger(x|\theta) = \frac{f(x|\theta)}{\int_{I_f} w_j(y)f(y|\theta)dy} \tag{5}$$

Q2: $\hat{\theta}_j^*$ can be used to calculate the densities of f_j^*, f or $f_j^\dagger$. Which of the three are

meaninful in describing either the observed dat or the true underlying distribution fo carcasses using the weighting function $w_1(x)$? Or using $w_2(x)$?

2 Weighted Likelihood

The weighted likelihood (WL) applies the weights to the likelihood and not the distribution itself. Assuming some distribution $f(x|\theta)$ that models the distances of carcasses from a turbine, which has corresponding likelihood $L(\theta|x_i)$. The weighted likelihood is

$$WL(\theta|\underline{x}) = \prod_{i=1}^{n} L(\theta|x_i)^{w_i} \tag{6}$$

where w_i is the weight associated with the i^{th} carcass. Please note, w_i is not a function but simply a weight value. We believe the choice for the weights (w_i) should be proportional to the inverse of the probability of detection. This changes the likelihood to a pseudo increase in sample size. For example if the probability of detection is one half at distance x_i, then $w_i = 2$, which is analogous to finding two carcasses at distance x_i.

Q3: Does the WL allow weights to be unique to each carcass?

Q4: Is the assumption of the WL, that the observed data does not follow $f(x|\theta)$ but the estimate $\hat{\theta}$ from Equation 6 does correspond to $f(x|\theta)$?

3 Next Step?

We see three possible choices: the WD with the weights being the inverse probability of detection (Eq. 3), the WD with the weights being the non-inverse of probability of detection (Eq. 2), and the WL with the weights being the inverse of probability of detection (Eq. 6). The main question is what should be done next?