KI
Musterlösung 4. Übung

1. Aufgabe

a.

$$\forall Z\ \exists Y\ \forall X\ (f(X,Y) \Leftrightarrow (f(X,Z)\ \&\ \sim f(X,X))) \tag{1}$$

$$\forall Z\ \exists Y\ \forall X\ ((f(X,Y) \Rightarrow (f(X,Z)\ \&\ \sim f(X,X)))\ \&\ ((f(X,Z)\ \&\ \sim f(X,X)) \Rightarrow f(X,Y))) \tag{2}$$

$$\forall Z\ \exists Y\ \forall X\ ((\sim f(X,Y) \mid (f(X,Z)\ \&\ \sim f(X,X)))\ \&\ (\sim (f(X,Z)\ \&\ \sim f(X,X)) \mid f(X,Y))) \tag{3}$$

$$\forall Z\ \exists Y\ \forall X\ ((\sim f(X,Y) \mid (f(X,Z)\ \&\ \sim f(X,X)))\ \&\ ((\sim f(X,Z) \mid f(X,X)) \mid f(X,Y))) \tag{4}$$

$$\exists Y\ \forall X\ ((\sim f(X,Y) \mid (f(X,Z)\ \&\ \sim f(X,X)))\ \&\ ((\sim f(X,Z) \mid f(X,X)) \mid f(X,Y))) \tag{5}$$

$$\forall X\ ((\sim f(X,y(Z)) \mid (f(X,Z)\ \&\ \sim f(X,X)))\ \&\ ((\sim f(X,Z) \mid f(X,X)) \mid f(X,y(Z)))) \tag{6}$$

$$((\sim f(X,y(Z)) \mid (f(X,Z)\ \&\ \sim f(X,X)))\ \&\ ((\sim f(X,Z) \mid f(X,X)) \mid f(X,y(Z)))) \tag{7}$$

$$((\sim f(X,y(Z)) \mid f(X,Z))\ \&\ (\sim f(X,y(Z)) \mid\sim f(X,X)))\ \&\ (\sim f(X,Z) \mid f(X,X) \mid f(X,y(Z))) \tag{8}$$

$$\{\sim f(X,y(Z)) \mid f(X,Z), \sim f(X,y(Z)) \mid\sim f(X,X), \sim f(X,Z) \mid f(X,X) \mid f(X,y(Z))\} \tag{9}$$

b.

$$\forall X\ \forall Y\ (q(X,Y) \Leftrightarrow \forall Z\ (f(Z,X) \Leftrightarrow f(Z,Y))) \tag{10}$$

$$\forall X\ \forall Y\ (q(X,Y) \Leftrightarrow \forall Z\ ((f(Z,X) \Rightarrow f(Z,Y))\ \&\ (f(Z,Y) \Rightarrow f(Z,X)))) \tag{11}$$

$$\begin{aligned}\forall X\ \forall Y\ ((q(X,Y) \Rightarrow \forall Z\ ((f(Z,X) \Rightarrow f(Z,Y))\ \&\ (f(Z,Y) \Rightarrow f(Z,X))))\\ \&\ (q(X,Y) \Leftarrow \forall Z\ ((f(Z,X) \Rightarrow f(Z,Y))\ \&\ (f(Z,Y) \Rightarrow f(Z,X)))))\end{aligned} \tag{12}$$

$$\begin{aligned}\forall X\ \forall Y\ ((\sim q(X,Y) \mid \forall Z\ ((\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim f(Z,Y) \mid f(Z,X))))\\ \&\ (q(X,Y) \mid\sim \forall Z\ ((\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim f(Z,Y) \mid f(Z,X)))))\end{aligned} \tag{13}$$

$$\begin{aligned}\forall X\ \forall Y\ ((\sim q(X,Y) \mid \forall Z\ ((\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim f(Z,Y) \mid f(Z,X))))\\ \&\ (q(X,Y) \mid \exists Z\ \sim ((\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim f(Z,Y) \mid f(Z,X)))))\end{aligned} \tag{14}$$

$$\begin{aligned}\forall X\ \forall Y\ ((\sim q(X,Y) \mid \forall Z\ ((\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim f(Z,Y) \mid f(Z,X))))\\ \&\ (q(X,Y) \mid \exists Z\ (\sim (\sim f(Z,X) \mid f(Z,Y)) \mid\sim (\sim f(Z,Y) \mid f(Z,X)))))\end{aligned} \tag{15}$$

$$\begin{aligned}\forall X\ \forall Y\ ((\sim q(X,Y) \mid \forall Z\ ((\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim f(Z,Y) \mid f(Z,X))))\\ \&\ (q(X,Y) \mid \exists Z\ ((f(Z,X)\ \&\ \sim f(Z,Y)) \mid (f(Z,Y)\ \&\ \sim f(Z,X)))))\end{aligned} \tag{16}$$

$$\begin{aligned}\forall X\ \forall Y\ ((\sim q(X,Y) \mid \forall Z\ ((\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim f(Z,Y) \mid f(Z,X))))\\ \&\ (q(X,Y) \mid \exists A\ ((f(A,X)\ \&\ \sim f(A,Y)) \mid (f(A,Y)\ \&\ \sim f(A,X)))))\end{aligned} \tag{17}$$

$$\begin{aligned}(\sim q(X,Y) \mid ((\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim f(Z,Y) \mid f(Z,X))))\\ \&\ (q(X,Y) \mid ((f(a(X,Y),X)\ \&\ \sim f(a(X,Y),Y)) \mid (f(a(X,Y),Y)\ \&\ \sim f(a(X,Y),X))))\end{aligned} \tag{18}$$

$$\begin{aligned}((\sim q(X,Y) \mid\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim q(X,Y) \mid\sim f(Z,Y) \mid f(Z,X)))\\ \&\ (q(X,Y) \mid ((f(a(X,Y),X) \mid f(a(X,Y),Y))\ \&\ (\sim f(a(X,Y),Y) \mid f(a(X,Y),Y))\\ \&\ (f(a(X,Y),X) \mid\sim f(a(X,Y),X))\ \&\ (\sim f(a(X,Y),Y) \mid\sim f(a(X,Y),X))))\end{aligned} \tag{19}$$

$$\begin{aligned}(\sim q(X,Y) \mid\sim f(Z,X) \mid f(Z,Y))\ \&\ (\sim q(X,Y) \mid\sim f(Z,Y) \mid f(Z,X))\\ \&\ (q(X,Y) \mid f(a(X,Y),X) \mid f(a(X,Y),Y))\ \&\ (q(X,Y) \mid\sim f(a(X,Y),Y) \mid f(a(X,Y),Y))\\ \&\ (q(X,Y) \mid f(a(X,Y),X) \mid\sim f(a(X,Y),X))\ \&\ (q(X,Y) \mid\sim f(a(X,Y),Y) \mid\sim f(a(X,Y),X))\end{aligned} \tag{20}$$

$$\begin{aligned}\{\sim q(X,Y) \mid\sim f(Z,X) \mid f(Z,Y), \sim q(X,Y) \mid\sim f(Z,Y) \mid f(Z,X)\\ , q(X,Y) \mid f(a(X,Y),X) \mid f(a(X,Y),Y), q(X,Y) \mid\sim f(a(X,Y),Y) \mid f(a(X,Y),Y)\\ , q(X,Y) \mid f(a(X,Y),X) \mid\sim f(a(X,Y),X), q(X,Y) \mid\sim f(a(X,Y),Y) \mid\sim f(a(X,Y),X)\}\end{aligned} \tag{21}$$

c.

$$\forall X\ \exists Y\ ((p(X,Y) \Leftarrow \forall X\ \exists T\ q(Y,X,T)) \Rightarrow r(Y)) \tag{22}$$

$$\forall X\ \exists Y\ (\sim (p(X,Y) \mid\sim \forall X\ \exists T\ q(Y,X,T)) \mid r(Y)) \tag{23}$$

$$\forall X\ \exists Y\ (\sim (p(X,Y) \mid \exists X\ \forall T\ \sim q(Y,X,T)) \mid r(Y)) \tag{24}$$

$$\forall X\ \exists Y\ ((\sim p(X,Y)\ \&\ \forall X\ \exists T\ q(Y,X,T)) \mid r(Y)) \tag{25}$$

$$\forall X\ \exists Y\ ((\sim p(X,Y)\ \&\ \forall A\ \exists T\ q(Y,A,T)) \mid r(Y)) \tag{26}$$

$$(\sim p(X,y(X))\ \&\ q(y(X),A,t(X,A))) \mid r(y(X)) \tag{27}$$

$$(\sim p(X,y(X)) \mid r(y(X)))\ \&\ (q(y(X),A,t(X,A)) \mid r(y(X))) \tag{28}$$

$$\{\sim p(X,y(X)) \mid r(y(X)), q(y(X),A,t(X,A)) \mid r(y(X))\} \tag{29}$$

d.

$$\forall X\ \forall Z\ (p(X,Z) \Rightarrow \exists Y\ \sim (q(X,Y) \mid\sim r(Y,Z))) \tag{30}$$

$$\forall X\ \forall Z\ (\sim p(X,Z) \mid \exists Y\ \sim (q(X,Y) \mid\sim r(Y,Z))) \tag{31}$$

$$\forall X\ \forall Z\ (\sim p(X,Z) \mid \exists Y\ (\sim q(X,Y)\ \&\ r(Y,Z))) \tag{32}$$

$$\sim p(X,Z) \mid (\sim q(X,y(X,Z))\ \&\ r(y(X,Z),Z)) \tag{33}$$

$$(\sim p(X,Z) \mid\sim q(X,y(X,Z)))\ \&\ (\sim p(X,Z) \mid r(y(X,Z),Z)) \tag{34}$$

$$\{\sim p(X,Z) \mid\sim q(X,y(X,Z)), \sim p(X,Z) \mid r(y(X,Z),Z)\} \tag{35}$$

2. Aufgabe

Das Problem mit dem Kängeru war für diese Aufgabe blöd gewählt, weil immer nur der Fall mit der unit clause eintritt. Man muss also nie ein pure literal auswählen und die Fallunterscheidung machen. Schaut Euch dafür am besten nochmal das Beispiel von Goeff Sutcliffe an.

Alle Variablen der CNF Vorlage mit the_kangaroo (bzw. k) ersetzen, da wir Unifikation an dieser Stelle noch nicht hatten. (Oder eben in jedem Schritt unifizieren, aber das kommt auf das gleiche Ergebnis raus.)

1. $in\_house(k) \Rightarrow cat(k) \equiv \sim in\_house(k) \mid cat(k)$ $\quad(\sim I \mid C_1)$
2. $gaze\_at\_moon(k) \Rightarrow suitable\_pet(k) \equiv \sim gaze\_at\_moon(k) \mid suitable\_pet(k)$ $\quad(\sim G \mid S)$
3. $detest(k) \Rightarrow avoid(k) \equiv \sim detest(k) \mid avoid(k)$ $\quad(\sim D \mid A)$
4. $carnivorous(k) \Rightarrow prowl\_at\_night(k) \equiv \sim carnivorous(k) \mid prowl\_at\_night(k)$ $\quad(\sim C_2 \mid P)$
5. $cat(k) \Rightarrow kill\_mice(k) \equiv \sim cat(k) \mid kill\_mice(k)$ $\quad(\sim C_1 \mid K_1)$
6. $\sim take\_to\_me(k) \mid in\_house(k)$ $\quad(\sim T \mid I)$
7. $kangaroo(k) \Rightarrow \sim suitable\_pet(k) \equiv \sim kangaroo(k) \mid \sim suitable\_pet(k)$ $\quad(\sim K_2 \mid\sim S)$
8. $kill\_mice(k) \Rightarrow carnivorous(k) \equiv \sim kill\_mice(k) \mid carnivorous(k)$ $\quad(\sim K_1 \mid C_2)$
9. $\sim take\_to\_me(k) \Rightarrow detest(k) \equiv \; take\_to\_me(k) \mid detest(k)$ $\quad(T \mid D)$
10. $prowl\_at\_night(k) \Rightarrow gaze\_at\_moon(k) \equiv \sim prowl\_at\_night(k) \mid gaze\_at\_moon(k)$ $\quad(\sim P \mid G)$

Conjecture:
$kangaroo(k) \Rightarrow avoid(k) \equiv \sim kangaroo(k) \mid avoid(k)$ $\quad(\sim K_2 \mid A)$

negierte Conjecture:
$kangaroo(k) \;\&\; \sim avoid(k)$ $\quad(K_2 \;\&\; \sim A)$

$\Delta = \{\{\sim I \mid C_1\}, \{\sim G \mid S\}, \{\sim D \mid A\}, \{\sim C_2 \mid P\}, \{\sim C_1 \mid K_1\}, \{\sim T \mid I\}, \{\sim K_2 \mid\sim S\}, \{\sim K_1 \mid C_2\}, \{T \mid D\}, \{\sim P \mid G\}, \{K_2\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, K_2)$ (setze K_2 auf true, d.h. enferne jede Klausel in der K_2 vorkommt komplett und lösche jedes Vorkommen von $\sim K_2$ aus den anderen Klauseln)

$\Delta = \{\{\sim I \mid C_1\}, \{\sim G \mid S\}, \{\sim D \mid A\}, \{\sim C_2 \mid P\}, \{\sim C_1 \mid K_1\}, \{\sim T \mid I\}, \{\sim S\}, \{\sim K_1 \mid C_2\}, \{T \mid D\}, \{\sim P \mid G\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, \sim S)$ (setze $\sim S$ auf true, d.h. enferne jede Klausel in der $\sim S$ vorkommt komplett und lösche jedes Vorkommen von S aus den anderen Klauseln)

$\Delta = \{\{\sim I \mid C_1\}, \{\sim G\}, \{\sim D \mid A\}, \{\sim C_2 \mid P\}, \{\sim C_1 \mid K_1\}, \{\sim T \mid I\}, \{\sim K_1 \mid C_2\}, \{T \mid D\}, \{\sim P \mid G\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, \sim G)$

$\Delta = \{\{\sim I \mid C_1\}, \{\sim D \mid A\}, \{\sim C_2 \mid P\}, \{\sim C_1 \mid K_1\}, \{\sim T \mid I\}, \{\sim K_1 \mid C_2\}, \{T \mid D\}, \{\sim P\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, \sim P)$

$\Delta = \{\{\sim I \mid C_1\}, \{\sim D \mid A\}, \{\sim C_2\}, \{\sim C_1 \mid K_1\}, \{\sim T \mid I\}, \{\sim K_1 \mid C_2\}, \{T \mid D\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, \sim C_2)$

$\Delta = \{\{\sim I \mid C_1\}, \{\sim D \mid A\}, \{\sim C_1 \mid K_1\}, \{\sim T \mid I\}, \{\sim K_1\}, \{T \mid D\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, \sim K_1)$

$\Delta = \{\{\sim I \mid C_1\}, \{\sim D \mid A\}, \{\sim C_1\}, \{\sim T \mid I\}, \{T \mid D\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, \sim C_1)$

$\Delta = \{\{\sim I\}, \{\sim D \mid A\}, \{\sim T \mid I\}, \{T \mid D\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, \sim I)$

$\Delta = \{\{\sim D \mid A\}, \{\sim T\}, \{T \mid D\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, \sim T)$

$\Delta = \{\{\sim D \mid A\}, \{D\}, \{\sim A\}\}$

$\Delta = simplify(\Delta, D)$

$\Delta = \{\{A\}, \{\sim A\}\}$

return "unsatisfiable"

3. Aufgabe

Herbrand Universum
$HU = \{$
$max,$
$vater\_von(max), mutter\_von(max),$
$vater\_von(vater\_von(max)), vater\_von(mutter\_von(max)),$
$mutter\_von(vater\_von(max)), mutter\_von(mutter\_von(max)),$
$vater\_von(vater\_von(vater\_von(max))), vater\_von(vater\_von(mutter\_von(max))),$
$vater\_von(mutter\_von(vater\_von(max))),$
$...\}$

Herbrand Basis
$HB = \{$
$verheiratet(max, max),$
$verheiratet(vater\_von(max), max), verheiratet(max, vater\_von(max)),$
$verheiratet(vater\_von(max), vater\_von(max)),$
$verheiratet(mutter\_von(max), max), verheiratet(max, mutter\_von(max)),$
$verheiratet(mutter\_von(max), mutter\_von(max)),$
$verheiratet(vater\_von(max), mutter\_von(max)), verheiratet(mutter\_von(max), vater\_von(max)),$
$verheiratet(vater\_von(vater\_von(max)), max),$
$...\}$

$\{verheiratet(max, max) \mapsto false,$
$verheiratet(vater\_von(max), max) \mapsto false,$
$verheiratet(mutter\_von(max), max) \mapsto false,$
$verheiratet(max, vater\_von(max)) \mapsto false,$
$verheiratet(max, mutter\_von(max)) \mapsto false,$
$verheiratet(vater\_von(max), mutter\_von(max)) \mapsto true,$
$verheiratet(mutter\_von(max), vater\_von(max)) \mapsto true,$
$verheiratet(mutter\_von(max), mutter\_von(max)) \mapsto false,$
$verheiratet(vater\_von(max), vater\_von(max)) \mapsto false,$
$verheiratet(vater\_von(vater\_von(max)), max) \mapsto false,$
$...\}$ bildet zusammen mit dem Herbrand Universum und der Identitätsfunktion ein Herbrand Modell.