Reductio5

May 13, 2021

Axiom 1.

$$(x = y \text{ and } y = z) \implies x = z \tag{1}$$

Axiom 2.

$$1 = a \tag{2}$$

Axiom 3.

$$\neg(1 = 2) \tag{3}$$

Theorem 1.

$$\neg a = 2 \tag{4}$$

Proof. We will assume the contrary:

$$a = 2 \tag{5}$$

Next, we use the transitivity of equality.

$$\forall y : (\forall z : (1 = y \text{ and } y = z \implies 1 = z)) \tag{6}$$

$$\forall z : (1 = a \text{ and } a = z \implies 1 = z) \tag{7}$$

$$1 = a \text{ and } a = 2 \implies 1 = 2 \tag{8}$$

Using the axiom a = 1 as well as our initial assumtion, this reduces to:

$$1 = 2 \tag{9}$$

Obvoiously, a statement and its negation cannot both hold true.

$$1 = 2 \text{ and } \neg 1 = 2 \implies \text{False} \tag{10}$$

However, we just deduced the former equality, and we presupposed the latter. This leads to a contradiction.

$$\text{False} \tag{11}$$

This means our assumption must have been wrong.

$$\neg a = 2 \tag{12}$$

$\square$