Combinatorics

Problem 1

(a) In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are next to each other?

(b) In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are not next to each other?

(c) In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are next to each other but a and c are not?

Problem 2

(a) How many well-formed formulas can be constructed from one $\vee$; one $\wedge$; two parenthesis pairs (,); and the three literals p, $\neg p$, and q?

(b) Under the equivalence relation defined by **logical equivalence**, how many equivalence classes do the formulas in part (a) form?

Problem 3
Let A be a set with m elements and B be a set with n elements.

(a) How many functions from A to B are there?

(b) How many injective functions?

(c)* How many surjective functions?

(d) How many binary relations are there on A?

(e) How many reflexive binary relations are there on A?

(f) How many symmetric binary relations are there on A?

Problem 4† (20T2)
You are taking an exam that has 6 easy questions and 4 difficult questions. Assuming all questions are distinguishable, how many ways are there of ordering the questions so that:

(a) All the easy questions come first. (4 marks)

† indicates a previous exam question
* indicates a difficult/advanced question.

(b) Each pair of difficult questions is separated by at least 2 easy questions.

(c) Each pair of difficult questions is separated by at least 1 easy question.

(d) Each pair of difficult questions is separated by at most 1 easy question.

(e) Each pair of difficult questions is separated by exactly 1 easy question.

Problem 5
We want to tile a $2 \times n$ rectangle with 2×1 tiles so that the rectangle is completely covered and no tiles are overlapping. For example, here are two different ways to tile a 2×3 rectangle:

How many different ways (ignoring symmetry) are there of tiling a $2 \times n$ rectangle with 2×1 tiles in this way?

Problem 6
A tennis doubles match consists of two teams of two players per team. Ordering between teams, and within teams is not considered.

(a) How many different tennis doubles matches can be made with 4 players?

(b) How many different tennis doubles matches can be made with 5 players?

(c) How many different tennis doubles matches can be made from n players?