COMP9020

Foundations of Computer Science

Acknowledgement of Country

I would like to acknowledge and pay my respect to the Bedegal people who are the Traditional Custodians of the land on which UNSW is built, and of Elders past and present.

COMP9020 21T2 Staff

Lecturer:	Paul Hunter
Email:	paul.hunter@unsw.edu.au
Lectures:	Wednesdays 12-2pm and Fridays 12-2pm
Consults:	Tuesdays 8-9pm and Thursdays 1-2pm
Research:	Theoretical CS: Algorithms, Formal verification

Online arrangements

Lectures:

- Zoom: https://unsw.zoom.us/j/96612524105 (passcode: comp9020)
- Recordings available on echo360 (through Moodle)

Consultations:

- Zoom: https://unsw.zoom.us/j/87192636642 (passcode: 1+1=2)
- Group-based, student-driven
- Wiki for questions

Other points of contact:

- Course forums
- Email

What you can expect from me

What is this course about?

What is Computer Science?

> *"Computer science no more about computers than astronomy is about telescopes"*
>
> – E. Dijkstra

Course Aims

Computer Science is about exploring the ability, and limitation, of computers to solve problems. It covers:

- **What** are computers capable of solving?
- **How** can we get computers to solve problems?
- **Why** do these approaches work?

This course aims to increase your level of mathematical maturity to assist with the fundamental problem of **finding, formulating, and proving** properties of programs.

Course Aims

The actual content is taken from a list of subjects that constitute the basis of the tool box of every serious practitioner of computing:

• number theory	week 1
• sets and languages	week 2
• relations and functions	week 3
• recursion; algorithmic analysis	week 4
• induction	week 5

• logic	week 7
• graph theory	week 8
• combinatorics	week 9
• probability and expectation	week 10

Course Material

All course information is placed on the course website

www.cse.unsw.edu.au/∼cs9020/

Content includes:

- Lecture slides and recordings
- Quizzes and Assignments
- Course Forums
- Practice questions
- Challenge questions
- Study groups

Course Material

Textbooks:

- KA Ross and CR Wright: Discrete Mathematics
- E Lehman, FT Leighton, A Meyer: Mathematics for Computer Science

Alternatives:

- K Rosen: Discrete Mathematics and its Applications

Assessment Summary

50% exam, 40% assignments, 10% quizzes:

- 9 quizzes, worth up to 1.67 marks each
- 3 assignments, worth up to 13.33 marks each
- final exam (2 hours) worth up to 50 marks

Quizzes are available for 48 hours before the Wednesday lecture.
Assignments due on Mondays of weeks 5, 8 and 11.

You must achieve 40% on the final exam to pass

Your final score will be taken from your 6 best quiz results, 3 assignments and final exam.

Late policy and Special Consideration

All assessments are submitted through the course website

Lateness policy

- Assignments: 10% off raw mark per 12 hours or part thereof
- Quizzes: Late submissions not accepted
- Exam: Late submissions not accepted

If you cannot meet a deadline through illness or misadventure you need to apply for Special Consideration.

More information

View the course outline at:

https://webcms3.cse.unsw.edu.au/COMP9020/21T2/outline

Particularly the sections on **Student conduct** and **Plagiarism**.

What I will expect from you

Assessments

To achieve good marks in this course you need to demonstrate:

- Your understanding of the material
- Your ability to work with the material

NB

How you get an answer is as, if not more important than what the answer is.

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Mathematical communication

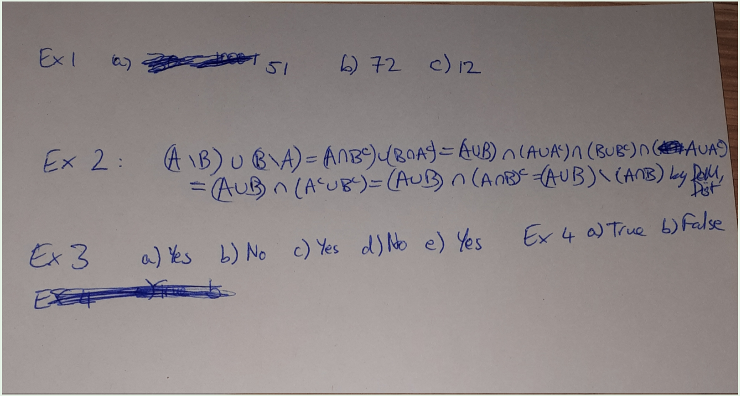
Guidelines for good mathematical writing

Mathematical writing should be:

- Clear
- Logical
- Sensical

Examples

Example (Bad)



Examples

Example (Good)

Ex. 2

$$\begin{aligned}
(A \setminus B) \cup (B \setminus A) &= (A \cap B^c) \cup (B \cap A^c) && \text{(Def.)} \\
&= ((A \cap B^c) \cup B) \cap ((A \cap B^c) \cup A^c) && \text{(Dist.)} \\
&= (A \cup B) \cap (B^c \cup B) \\
&\qquad \cap (A \cup A^c) \cap (B^c \cup A^c) && \text{(Dist.)} \\
&= (A \cup B) \cap (A^c \cup B^c) && \text{(Ident.)} \\
&= (A \cup B) \cap (A \cap B)^c && \text{(DeM.)} \\
&= (A \cup B) \setminus (A \cap B) && \text{(Def.)}
\end{aligned}$$

Examples

Example (Good)

Ex. 4a

We will show that if R_1 and R_2 are symmetric, then $R_1 \cap R_2$ is symmetric.

Suppose $(a, b) \in R_1 \cap R_2$.

Then $(a, b) \in R_1$ and $(a, b) \in R_2$.

Because R_1 is symmetric, $(b, a) \in R_1$; and because R_2 is symmetric, $(b, a) \in R_2$.

Therefore $(b, a) \in R_1 \cap R_2$.

Therefore $R_1 \cap R_2$ is symmetric.

Proofs

A large component of your work in this course is giving **proofs** of **propositions.**

A **proposition** is a statement that is either true or false.

Example

Propositions:

- $3 + 5 = 8$
- All integers are either even or odd
- There exist a, b, c such that $1/a + 1/b + 1/c = 4$

Not propositions:

- $3 + 5$
- x is even or x is odd
- $1/a + 1/b + 1/c = 4$

Proposition structure

Common proposition structures include:

If A then B	$(A \Rightarrow B)$
A if and only if B	$(A \Leftrightarrow B)$
For all x, A	$(\forall x.A)$
There exists x such that A	$(\exists x.A)$

$\forall$ and $\exists$ are known as **quantifiers**.

Proofs

A large component of your work in this course is giving **proofs** of **propositions.**

A proof of a proposition is an argument to convince the reader/marker that the proposition is true.

A **proof** of a proposition is a finite sequence of logical steps, starting from base assumptions (**axioms** and **hypotheses**), leading to the proposition in question.

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$3 \times 2 = (2+1) \times 2$$

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$\begin{aligned} 3 \times 2 &= (2+1) \times 2 \\ &= (2 \times 2) + (1 \times 2) \end{aligned}$$

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$\begin{aligned} 3 \times 2 &= (2+1) \times 2 \\ &= (2 \times 2) + (1 \times 2) \\ &= (1 \times 2) + (2 \times 2) \end{aligned}$$

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$\begin{aligned} 3 \times 2 &= (2+1) \times 2 \\ &= (2 \times 2) + (1 \times 2) \\ &= (1 \times 2) + (2 \times 2) \\ &= 2 + (2 \times 2) \end{aligned}$$

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$
\begin{aligned}
3 \times 2 &= (2+1) \times 2 \\
&= (2 \times 2) + (1 \times 2) \\
&= (1 \times 2) + (2 \times 2) \\
&= 2 + (2 \times 2) \\
&= (2 \times 1) + (2 \times 2)
\end{aligned}
$$

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$
\begin{aligned}
3 \times 2 &= (2+1) \times 2 \\
&= (2 \times 2) + (1 \times 2) \\
&= (1 \times 2) + (2 \times 2) \\
&= 2 + (2 \times 2) \\
&= (2 \times 1) + (2 \times 2) \\
&= 2 \times (1 + 2)
\end{aligned}
$$

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$\begin{aligned}
3 \times 2 &= (2+1) \times 2 \\
&= (2 \times 2) + (1 \times 2) \\
&= (1 \times 2) + (2 \times 2) \\
&= 2 + (2 \times 2) \\
&= (2 \times 1) + (2 \times 2) \\
&= 2 \times (1+2) \\
&= 2 \times 3.
\end{aligned}$$

Proofs: How much detail?

- Depends on the context (question, expectation, audience, etc)
- Each **step** should be justified (excluding basic algebra and arithmetic)

Proofs: pitfalls

Starting from the proposition and deriving true **is not valid**.

Example

Prove: $0 = 1$

$$
\begin{array}{lrcll}
 & 0 & = & 1 & \\
\text{So (mult. by 2)} & 0 & = & 2 & \\
\text{So (subtract 1)} & -1 & = & 1 & \\
\text{So} & (-1)^2 & = & (1)^2 & \\
\text{So} & 1 & = & 1 & \text{which is true.}
\end{array}
$$

Does this mean that $0 = 1$?

Proofs: pitfalls

Make sure each step is logically valid

Example

$$-20 = -20$$

Proofs: pitfalls

Make sure each step is logically valid

Example

$$-20 = -20$$

So $$25 - 45 = 16 - 36$$

Proofs: pitfalls

Make sure each step is logically valid

Example

$$
\begin{array}{lrcl}
 & -20 & = & -20 \\
\text{So} & 25 - 45 & = & 16 - 36 \\
\text{So} & 5^2 - 2 \cdot 5 \cdot \frac{9}{2} & = & 4^2 - 2 \cdot 4 \cdot \frac{9}{2}
\end{array}
$$

Proofs: pitfalls

Make sure each step is logically valid

Example

$$
\begin{array}{lrcl}
 & -20 & = & -20 \\
\text{So} & 25-45 & = & 16-36 \\
\text{So} & 5^2-2\cdot 5\cdot \frac{9}{2} & = & 4^2-2\cdot 4\cdot \frac{9}{2} \\
\text{So} & 5^2-2\cdot 5\cdot \frac{9}{2}+\left(\frac{9}{2}\right)^2 & = & 4^2-2\cdot 4\cdot \frac{9}{2}+\left(\frac{9}{2}\right)^2
\end{array}
$$

Proofs: pitfalls

Make sure each step is logically valid

Example

$$
\begin{array}{lrcl}
 & -20 & = & -20 \\
\text{So} & 25-45 & = & 16-36 \\
\text{So} & 5^2 - 2\cdot 5\cdot \frac{9}{2} & = & 4^2 - 2\cdot 4\cdot \frac{9}{2} \\
\text{So} & 5^2 - 2\cdot 5\cdot \frac{9}{2} + \left(\frac{9}{2}\right)^2 & = & 4^2 - 2\cdot 4\cdot \frac{9}{2} + \left(\frac{9}{2}\right)^2 \\
\text{So} & (5-\frac{9}{2})^2 & = & (4-\frac{9}{2})^2
\end{array}
$$

Proofs: pitfalls

Make sure each step is logically valid

Example

$$\begin{array}{lrcl}
 & -20 & = & -20 \\
\text{So} & 25-45 & = & 16-36 \\
\text{So} & 5^2 - 2\cdot 5\cdot \frac{9}{2} & = & 4^2 - 2\cdot 4\cdot \frac{9}{2} \\
\text{So} & 5^2 - 2\cdot 5\cdot \frac{9}{2} + \left(\frac{9}{2}\right)^2 & = & 4^2 - 2\cdot 4\cdot \frac{9}{2} + \left(\frac{9}{2}\right)^2 \\
\text{So} & (5-\frac{9}{2})^2 & = & (4-\frac{9}{2})^2 \\
\text{So} & 5-\frac{9}{2} & = & 4-\frac{9}{2}
\end{array}$$

Does this mean that $5 = 4$?

Proofs: pitfalls

Make sure each step is logically valid

Example

Suppose $a = b$. Then,

$$
\begin{array}{lrcl}
 & a^2 & = & ab \\
\text{So} & a^2 - b^2 & = & ab - b^2 \\
\text{So} & (a-b)(a+b) & = & (a-b)b \\
\text{So} & a+b & = & b \\
\text{So} & a & = & 0
\end{array}
$$

This is true no matter what value a is given at the start, so does that mean everything is equal to 0?

Proofs: pitfalls

For propositions of the form $\forall x.A$ where x can have infinitely many values:

- You cannot enumerate infinitely many cases in a proof.
- Only considering a finite number of cases is not sufficient.

Example

$$\text{For all } n,\ n^2 + n + 41 \text{ is prime}$$

True for $n = 0, 1, 2, \ldots, 39$. Not true for $n = 40$.

Proofs: pitfalls

The order of quantifiers matters when it comes to propositions:

Example

- For every number x, there is a number y such that y is larger than x
- There is a number y such that for every number x, y is larger than x

Proof strategies: direct proof

Proposition form	You need to do this
$A \Rightarrow B$	Assume A and prove B
$A \Leftrightarrow B$	Prove "If A then B" and "If B then A"
$\forall x.A$	Show A holds for every possible value of x
$\exists x.A$	Find a value of x that makes A true

Proof strategies: contradiction

To prove A is true, assume A is false and derive a contradiction.
That is, start from the negation of the proposition and derive false.

Example

Prove: $\sqrt{2}$ is irrational
Proof: Assume $\sqrt{2}$ is rational ...

Negating propositions

Proposition form	Its negation
A and B	
A or B	
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x.A$	
$\exists x.A$	

Negating propositions

Proposition form	Its negation
A and B	not A or not B
A or B	
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x.A$	
$\exists x.A$	

Negating propositions

Proposition form	Its negation
A and B	not A or not B
A or B	not A and not B
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x.A$	
$\exists x.A$	

Negating propositions

Proposition form	Its negation
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Negating propositions

Proposition form	Its negation
A and B	not A or not B
A or B	not A and not B
$A \Rightarrow B$	A and not B
$A \Leftrightarrow B$	A and not B, or B and not A
$\forall x.A$	
$\exists x.A$	

Negating propositions

Proposition form	Its negation
A and B	not A or not B
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$\forall x.A$	$\exists x.$ not A
$\exists x.A$	

Negating propositions

Proposition form	Its negation
A and B	not A or not B
A or B	not A and not B
$A \Rightarrow B$	A and not B
$A \Leftrightarrow B$	A and not B, or B and not A
$\forall x.A$	$\exists x.$ not A
$\exists x.A$	$\forall x.$ not A

Proof strategies: contrapositive

To prove a proposition of the form "If A then B" you can prove "If not B then not A"

Example

Prove: If $m + n \geq 73$ then $m \geq 37$ or $n \geq 37$.

Proof strategies: dealing with $\forall$

How can we check infinitely many cases?

- Choose an **arbitrary** element: an object with no assumptions about it (may have to check several cases)
- Induction (see week 8)

Example

Prove: For every integer n, n^2 will have remainder 0 or 1 when divided by 4.

Note: "Arbitrary" is not the same as "random".