Entry and Multiple Equilibria

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Outline

Incomplete Econometric Models

- Multiple equilibria raise significant challenges for empirical analysis
  - Non-unique mapping from exogenous to endogenous variables
    - May imply non- or partial identification of model parameters
  - Even if model is identified, counterfactual predictions non-unique

- Approaches we've seen so far:
  1. Restrict model structure / parameters until multiplicity disappears
  2. Transform the problem: look at outcomes that are unique across multiple equilibria (Bresnahan & Reiss '91, Berry '92)
  3. Specify an Equilibrium Selection Mechanism (ESM) (Berry '92)

- Today, we'll consider two other possibilities
  1. Identify parameters using equilibria that never appear in regions of multiplicity
  2. Directly engage with multiplicity; use as basis for estimation
     - $\longrightarrow$ Moment inequality models

Outline

1. Tamer, ReStud 2003
    - Framework
    - Identification
2. Ciliberto & Tamer, ECMA 2009

Model Incompleteness

- Tamer's paper offers two key insights into incomplete models
  1. Incompleteness and identification are logically distinct
     - Incompleteness: for certain values of exogenous variables, econometric model predicts multiple outcomes
     - Parameters of incomplete models may be identified
     - "More" multiplicity need not imply a larger identified set
  2. Information from regions of multiplicity can yield a more efficient estimator
     - Proposes efficient semiparametric maximum likelihood (SML) estimator
     - Appropriate estimation method depends on nature of multiplicity and computational challenges
     - e.g. Moment inequality estimators needed in other cases
- Points developed through a 2x2 discrete game

Outline

Bivariate Discrete Game

- Econometric model of a 2x2 discrete response game

$$y_1^* = x_1\beta_1 + y_2\Delta_1 + u_1$$
$$y_2^* = x_2\beta_2 + y_1\Delta_2 + u_2$$

where $y_j = 1\{y_j^* \geq 0\}$ for $j = 1, 2$

  - $(x_1, x_2) \in \mathbb{R}^d$ - observed exogenous variables
  - $(u_1, u_2) \in \mathbb{R}^2$ - unobserved exogenous variables distributed F_u
  - $(y_1, y_2) \in \{0, 1\}^2$ - entry decisions
  - $(\beta_1, \beta_2, \Delta_1, \Delta_2)$ - model parameters
  - Note $\Delta_1 \neq \Delta_2$ departs from Berry '92, B&R '91

- Suppose $\Delta_1, \Delta_2 < 0$. $(0, 1)$ and $(1, 0)$ are both equilibria in region

$$-x_1\beta_1 < u_1 \leq -x_1\beta_1 - \Delta_1$$
$$-x_2\beta_2 < u_2 \leq -x_2\beta_2 - \Delta_2$$

Restrictions with Multiplicity

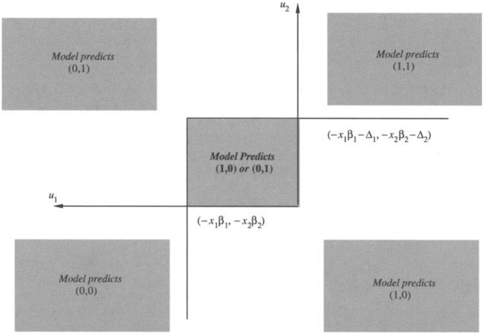

FIGURE 1
Incomplete model with multiple equilibria

$$
\begin{aligned}
P_3(x,\beta) =& Pr(u_1 < -x_1\beta_1 - \Delta_1; u_2 > -x_2\beta_2 - \Delta_2) \\
&+ Pr(u_1 < -x_1\beta_1; -x_2\beta_2 < u_2 < -x_2\beta_2 - \Delta_2) \\
P_4(x,\beta) =& Pr(u_1 < -x_1\beta_1 - \Delta_1; u_2 > -x_2\beta_2)
\end{aligned}
$$

Then we have the following restriction: $P_3(x,\beta) \leq Pr[(0,1) \mid x] \leq P_4(x,\beta)$

Example: Coordination Game (Jovanovic '89)

- Two players make binary decisions y_1, y_2. Payoffs:

$$\Pi_1(y_1, y_2, u_1) = y_1 \times (\theta_1 y_2 - u_1)$$
$$\Pi_2(y_1, y_2, u_2) = y_2 \times (\theta_2 y_1 - u_2)$$

  Player types $u_i \sim U(0,1)$

- Assumes unprofitable to be only entrant
  - Technology adoption and complementary products (e.g. electric vehicles and charging stations)

- Only Nash equilibria are
  - $(1,1)$ if $u_1 < \theta_1,\ u_2 < \theta_2$
  - $(0,0)$ for all (u_1, u_2)

  We can only learn from markets with entrants. Identification depends on a priori restrictions on parameter values (θ_1, θ_2):

  - $(\theta_1, \theta_2) \geq 1$: no unique outcome, no ID - $0 \leq Pr[(1,1)] \leq 1$
  - $\theta_1 = \theta_2 = \theta < 1$: partial ID - $0 \leq Pr[(1,1)] \leq \theta^2$

Outline

Identification when $\Delta_1 \times \Delta_2 > 0$

- Back to general 2x2 game:

$$
\begin{aligned}
y_1^* =& x_1\beta_1 + y_2\Delta_1 + u_1 \\
y_2^* =& x_2\beta_2 + y_1\Delta_2 + u_2
\end{aligned}
$$

with $y_j = 1\{y_j^* > 0\}$

- For identification of (β, Δ), assume:
    - F_u known and independent of x (can be relaxed)
    - Random sample of markets
    - Key exclusion restriction: $\exists$ continuous regressor x_{1k} excluded from 2's payoff

- Argument ignores regions of multiplicity: β identified by $Pr[(0,0) \mid x]$, Δ by $Pr[(1,1) \mid x]$
    - "Identification at infinity" argument
    - β_2 identified by $\lim_{x_{1k} \to \infty} Pr[(0,0) \mid x]$
    - β_1 identified by rank condition
    - Apply similar argument to $Pr[(1,1) \mid x]$ for Δ

Outcome Regions for $\Delta_1 \times \Delta_2 < 0$

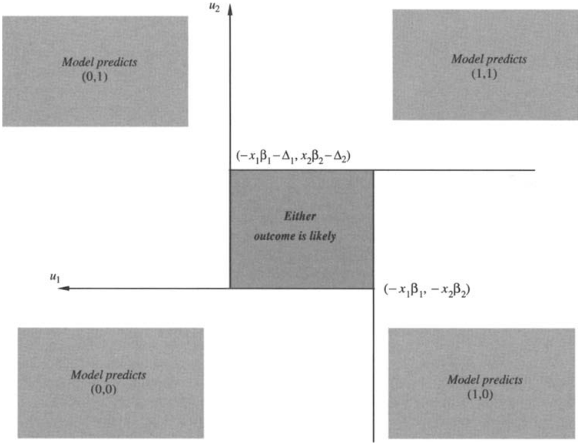

FIGURE 2
Incomplete bivariate model for the case where $\Delta_1 > 0$, $\Delta_2 < 0$

- No PSNE in region of multiplicity $\Longrightarrow$ all four outcomes possible
  - *All* equilibra intersect region of multiplicity

Identification when $\Delta_1 \times \Delta_2 < 0$

- Build an identification argument based on upper and lower bounds on probability of each outcome
  - For the $(0,0)$ outcome, define

$$
\begin{aligned}
P_2(x,\beta) =& Pr[u_1 \leq -x_1\beta_1, u_2 \leq -x_2\beta_2] \\
P_{\text{square}}(x,\beta) =& Pr[-x_1\beta_1 - \Delta_1 \leq u_1 \leq -x_1\beta_1; -x_2\beta_2 \leq u_2 \leq -x_2\beta_2 - \Delta_2]
\end{aligned}
$$

  - Then $P_2(x,\beta) \leq Pr[(0,0) \mid x] \leq P_2(x,\beta) + P_{\text{square}}(x,\beta)$
  - Similarly for other outcomes

- Identification requires a full-support $x_{1k} \neq x_{2k}$
  - Idea: for $b \neq \beta$, can find region X' where $\forall x' \in X'$, $P_2(x',\beta) > P_2(x',b) + P_{\text{square}}(x',b)$
  - For any $b \neq \beta$, at some x's the observed outcome probabilities will violate the predicted bounds

- Suggests a moment inequality estimator
  - No outcome has an exact probability given the model
  - Develops semiparametric maximum likelihood (SML) estimator for other cases

Outline

Overview

- Revisits entry in U.S. airline industry
- Goal: allow for richer competitive effects δ_j^i. May depend on:
    - Characteristics of potential entrants vs incumbents
        - Large carriers (American) vs low-cost carriers (Southwest)
        - Effects of entry may be heterogeneous and asymmetric
    - Pre-existing market structure
        - Airport presence may deter entry of competitors
- Heterogeneous competitive effects breaks uniqueness properties
    - Unique equilibrium in # firms not guaranteed
    - Possibly no (empirically relevant) regions without multiplicity
- Approach: Inequality restrictions based on revealed preference
    - Entrants make positive profits; non-entrants would not
    - Partial identification of model parameters
    - Estimation and inference using moment inequalities

Outline

Heterogeneous Competitive Effects

- A firm chooses to serve a market iff

$$\pi_{im} = S'_m\alpha_i + Z'_{im}\beta_i + W'_{im}\gamma_i + \sum_{j\neq i}(\delta^i_j + Z'_{jm}\phi^i_j)y_{jm} + \epsilon_{im} \geq 0$$

  - θ - finite parameter of interest
  - y_{jm} - entry decision of firm j in market m
  - $X_m = (S_m, Z_m, W_m)$ - observable market characteristics
  - S_m - market characteristics common to all firms in market m
  - $Z_m = (Z_{1m}, ..., Z_{km})$ - firm characteristics affecting all π_{im}
  - $W_m = (W_{1m}, ..., W_{km})$ - firm characteristics excluded from competitors' payoffs $\pi_{i,-m}$
  - ϵ_{im} - iid profit shock

- Equilibrium not even unique in number of firms
  - True even if $\delta^i_j = \delta_j$, $\phi^i_j = 0\ \forall i, j$
  - Previous studies set $\delta^i_j = \delta$, and often $(\alpha_i, \beta_i) = (\alpha, \beta)$

Model Implications

- $(\delta_j^i + Z_{jm}'\phi_j^i)y_{jm}$ allows for rich heterogeneity in competitive effects
  - Firm-pair-specific competitive effects δ_j^i
  - Depends on market and competitor characteristics (e.g. airport presence) through ϕ_j^i
  - Interpretations: product differentiation, cost externalities, competitive conduct
  - What restrictions does the CT payoff structure maintain?
- Model maintains a reduced-form structure for profits
  - Expected payoffs in a "long-run equilibrium"
  - Data do exist on prices and quantities
    - To incorporate, would need instruments, an explicit model of supply and demand

General Approach to Estimation

- y - observed endogenous variables; X - observed exogenous variables
- ϵ - unobserved exogenous variables, F_ϵ known up to finite dimension
- Random sample of observations $(\mathbf{y}_m, \mathbf{X}_m)$ for markets $m = 1, ..., n$
- For each y', X, and θ, define two regions:
    - $R_1(\theta, X)$ - y' is the unique outcome
    - $R_2(\theta, X)$ - y' is one of multiple possible outcomes
- Model restricts probability of observing y' given X:

$$\begin{aligned} Pr(y' \mid X) &= \int_{R_1(\theta,X)} Pr(y' \mid \epsilon, X) dF + \int_{R_2(\theta,X)} Pr(y' \mid \epsilon, X) dF \\ &= \int_{R_1(\theta,X)} dF + \int_{R_2(\theta,X)} Pr(y' \mid \epsilon, X) dF \end{aligned}$$

- Exact probability depends on equilibrium selection rule
    - Could estimate using semiparametric likelihood (Tamer '03)
    - Difficult to implement in practice

Objective Function

- Regardless of equilibrium selection rule,

$$\int_{R_1(\theta,X)} dF \leq Pr(y' \mid X) \leq \int_{R_1(\theta,X)} dF + \int_{R_2(\theta,X)} dF$$

- The upper and lower bounds are summarized

$$\mathbf{H}_1(\theta,\mathbf{X}) \equiv \begin{bmatrix} H_1^1(\boldsymbol{\theta},X) \\ \vdots \\ H_1^{2^K}(\boldsymbol{\theta},X) \end{bmatrix} \leq \begin{bmatrix} Pr(\mathbf{y}_1 \mid X) \\ \vdots \\ Pr(\mathbf{y}_{2^K} \mid X) \end{bmatrix} \leq \begin{bmatrix} H_2^1(\boldsymbol{\theta},X) \\ \vdots \\ H_2^{2^K}(\boldsymbol{\theta},X) \end{bmatrix} \equiv \mathbf{H}_2(\theta,\mathbf{X})$$

- Moment inequality estimator penalizes violations of these bounds

$$Q(\boldsymbol{\theta}) = \int [\|(P(\mathbf{X}) - H_1(\boldsymbol{\theta},\mathbf{X}))_-\| + \|(P(\mathbf{X}) - H_2(\boldsymbol{\theta},\mathbf{X}))_+\|] \, dF_x$$

Implementation

- Estimation and inference based on empirical analogue

$$Q_n(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n}\left(\|(P_n(x_i) - \hat{H}_1(\boldsymbol{\theta}, x_i))_-\| + \|(P_n(x_i) - \hat{H}_2(\boldsymbol{\theta}, x_i))_+\|\right)$$

- Non-parametric frequency estimator for $P_n(x)$
  - Inference method requires discretizing X into $S_x = \{x_1, ..., x_s\}$
- $\hat{H}_1, \hat{H}_2$ estimated via simulation (no analytic characterization)
  1. Fixed set of $r = 1, ..., R$ simulation draws of ϵ_m^r in each market
  2. Calculate profit vectors for each possible outcome $y^1, ..., y^{2^K}$:
  $$\boldsymbol{\pi}(\mathbf{y}^j, \mathbf{X}, \theta, \epsilon^r) = [\pi_1(\mathbf{y}^j_{-1}, \mathbf{X}, \theta, \epsilon_1^r), ..., \pi_K(\mathbf{y}^j_{-K}, \mathbf{X}, \theta, \epsilon_K^r)]$$
  3. Find all equilibria of the game. For $r = 1, ..., R$
     - If $\mathbf{y}^j$ is an equilibrium, $\hat{H}_2^j = \hat{H}_2^j + \frac{1}{R}$
     - If $\mathbf{y}^j$ is the unique equilibrium, $\hat{H}_1^j = \hat{H}_1^j + \frac{1}{R}$
- Inference conducted using recently developed methods in Chernozhukov, Hong, Tamer (ECMA 2007)
  - Interlude to cover key concepts and methods

Outline

Problem Formulation

- Consider a population criterion function $Q(\theta) \geq 0$
  - e.g. based on moment inequalities $E[m_i(\theta)] \leq 0$:

$$Q(\theta) = \|E[m_i(\theta)]_+ W^{1/2}(\theta)\|^2$$

- $\hat{\theta} \in \Theta \subset \mathbb{R}^d$ satisfies the model's restrictions only if $Q(\hat{\theta}) = 0$

$$\Theta_I = \{\theta \in \Theta : Q(\theta) = 0\} = \arg\min_{\theta \in \Theta} Q(\theta)$$

- Θ_I is the **identified set**. Moment inequality case:

$$\Theta_I = \{\theta : E[m_i(\theta)] \leq 0\}$$

- We want to provide an estimator $\hat{\Theta}_I$ of Θ_I which is
  - Consistent
  - Yields valid confidence sets for Θ_I (or $\theta \in \Theta_I$)
  - Is computationally tractable

Estimator

- Chernozhukov, Hong, and Tamer (ECMA 2007) show how to consistently estimate Θ_I and construct valid confidence regions
- Estimator chooses parameter values that make objective function small

$$\hat{\Theta}_I = C_n(\hat{c}) \equiv \{\theta \in \Theta \mid a_n Q_n(\theta) \leq \hat{c}\}$$

  The level, $\hat{c}$, may be data dependent. The normalizing sequence a_n is n or $\frac{n}{\log n}$

- Challenge is choosing appropriate $\hat{c}$
  - Use data to determine tolerance
  - CHT propose a generic subsampling method

Desired Properties

- **Consistency:** what does this mean for set estimation?
  - Need a notion of closeness for sets $\hat{\Theta}_I = C_n(\hat{c})$ and Θ_I
  - Hausdorf distance measures maximum discrepancy between two sets. If $d(a, B) \equiv \inf_{b \in B} ||b - a||$, define

$$d_H(A, B) \equiv \max \left\{ \sup_{a \in A} d(a, B), \sup_{b \in B} d(A, b) \right\}$$

  - Consistency means

$$d_H(C_n(\hat{c}), \Theta_I) \xrightarrow{p} 0$$

- **Confidence Region:** the estimate $C_n(\hat{c})$ should contain the identified set Θ_I with probability at least $\alpha \in (0, 1)$:

$$\lim_{n \to \infty} P(\Theta_I \subseteq C_n(\hat{c})) \geq \alpha$$

Inferential Statistic

- For results on consistency and coverage, a key statistic is

$$\mathcal{C}_n \equiv \sup_{\theta \in \Theta_I} a_n Q_n(\theta)$$

This is called the *inferential statistic*

  - Maximum value of objective function on identified set

- We don't observe Θ_I, so $\mathcal{C}_n$ is never observed directly
- However, constructing valid confidence regions depends crucially on estimating the distribution of $\mathcal{C}_n$, because

$$\Theta_I \subseteq C_n(c) \Longleftrightarrow \sup_{\theta \in \Theta_I} a_n Q_n(\theta) \leq c \Longleftrightarrow \mathcal{C}_n \leq c$$

  - Consistency: select $\hat{c} \geq \mathcal{C}_n$ with probability approaching one
  - Coverage: estimate α-quantile of the $\mathcal{C}_n$ distribution

General Consistency Result

- **CHT (2007), Theorem 3.1:** Suppose mild regularity conditions on Θ and Q, as well as the following properties:
  - Q_n is uniformly no smaller than Q on Θ: $\sup_{\theta \in \Theta}(Q(\theta) - Q_n(\theta))_+ = O_p(1/b_n)$ for a sequence of constants $b_n \to \infty$
  - $Q_n \to 0$ uniformly on Θ_I: $\sup_{\theta \in \Theta_I} Q_n(\theta) = O_p(1/a_n)$ for a sequence of constants $a_n \to \infty$
    - Requirement of convergence to 0 unrestrictive
  - $P(\hat{c} \geq \mathcal{C}_n) \to 1$, but $\hat{c}/a_n \xrightarrow{p} 0$

  Then $P(\Theta_I \subseteq \hat{\Theta}_I) \to 1$ and $d_H(\hat{\Theta}_I, \Theta_I) = o_p(1)$
- CHT verify these conditions for moment inequality models, with $b_n = \sqrt{n}$, $a_n = n$
  - Recommend $\hat{c} \propto \log n$ to satisfy $\hat{c} \geq \mathcal{C}_n$ wp $\to 1$

Confidence Regions for Θ_I

- Choose appropriate level of inequality violation tolerance, $\hat{c}$, so that

$$P(\Theta_I \subseteq \hat{\Theta}_I) = P\left(\mathcal{C}_n = \sup_{\theta \in \Theta_I} nQ_n(\theta) < \hat{c}\right) \geq \alpha$$

- The confidence region is

$$C_n(\hat{c}) = \{\theta \in \Theta : nQ_n(\theta) < \hat{c}\}$$

- But how to find the right $\hat{c}$?
  - Idea: if model correctly specified, sampling error creates inequality violations
  - Use data to determine how large the violations are likely to be
- Additional regularity conditions needed for valid coverage
  - Related to distribution of inferential statistic $\mathcal{C}_n$
  - Usually satisfied in applications

Subsampling Approach

1. **Initial estimate** $C_n(c_0)$ of Θ_I, e.g. $C_n(q_n)$ where $q_n \equiv \inf_{\theta\in\Theta} a_n Q_n(\theta)$
2. **Subsample** the statistic $n(Q_n(\theta_0) - \min_t Q_n(t))$ $\forall\theta_0 \in C_n(c_0)$, obtain α-quantiles $c_1(\theta_0)$:
   - Draw iid subsamples $j = 1, \ldots, B_n$ of size b_n
   - For each $\theta_0 \in C_n(c_0)$, calculate subsample analog
     $$T_j(\theta_0) = b_n[Q_{b_n,j}(\theta_0) - \min_t Q_{b_n,j}(t)]$$
     for $j = 1, \ldots, B_n$
   - Take α quantile of $\{T_j(\theta_0)\}$, $c_1(\theta_0)$
3. **Construct confidence region** using $c_1 = \sup_{\theta_0\in C_n(c_0)} c_1(\theta_0)$:
   $$\hat{\Theta}_I = C_n(c_1) = \{\theta \in \Theta \mid n(Q_n(\theta) - \min_t Q_n(t)) \leq c_1\}$$
4. **Repeat** (optional) using $C_n(c_1)$ to obtain c_2, etc

Comments

- Size and number of subsamples must be chosen appropriately
  - $\lim_{n\to\infty} b_n = \infty$, $\lim_{n\to\infty} B_n = \infty$, $\lim_{n\to\infty} \frac{b_n}{n} = 0$
  - Ciliberto and Tamer choose $b_n = n/4$
- Could continue iterating procedure to get $c_2, c_3, \ldots$
  - Further iteration does not affect validity, but can yield higher-order refinements (Romano and Shaikh, 2009)
- Simulation error is an important issue in practice
  - Simulation draws $S \to \infty$ quickly to preserve consistency
  - Re-simulate in each subsample to account for additional error
- Can also perform inference for a specific $\theta \in \Theta_I$.
  - Ciliberto & Tamer take this approach. Estimate distribution of $nQ_n(\theta)$ under null of $\theta_0 = \theta$, obtain α-quantile $c_n(\theta)$. Then

$$\hat{\Theta}_I = \left\{ \theta \in \Theta \mid n(Q_n(\theta) - \min_t Q_n(t)) \leq \min\{c_2, c_n(\theta)\} \right\}$$

Outline

The Wright Amendment

- Counterfactuals focus on 2006 repeal of the Wright Amendment
- Passed by Congress in 1979 to stimulate growth of Dallas/Fort Worth
  - Restricted airline service out of Dallas Love, its competitor
    - Non-stop flights to states neighboring Texas
    - Prohibited through service or ticketing outside of those states
    - Commuter planes with fewer than 56 seats
- Southwest claimed this was anti-competitive
  - SW relied exclusively on Boeing 737 plane to simplify operations
  - All Boeing 737s have capacity over 100
  - Counter-argument: Southwest could fly out of Dallas/Fort Worth
- Repeal took full effect in 2014
  - Goal: predict full repeal's effects before they are observed

Data

- Augments Origin and Destination Survey (from Berry '92) with flight- and ticket-level data
  - Crucially, can distinguish airports within a city
  - Model assumes independence across airports even within a city
    - May not be innocuous for counterfactual of interest
  - Sample restrictions similar (but not identical) to Berry's
    - Keep markets into/out of Dallas Love of distance $>$ 500 miles
- Important modeling decision: aggregate smaller airlines
  - Focus on American, Delta, United, and Southwest
  - Medium airlines (MA) and low-cost carriers (LCC) combined
  - Aids computation – # equilibria exponential in # firms
    - Valid if combined firms behave in strategically similar ways
- Quarterly panel from Q1 '96 through Q4 '07
  - Estimation sample based on Q2 '01

Market Size and Entry

TABLE II

DISTRIBUTION OF THE NUMBER OF CARRIERS BY MARKET SIZE[a]

Number of Firms	Large	Medium	Small	Total
0	7.07	7.31	7.73	7.29
1	41.51	22.86	20.91	30.63
2	29.03	24.30	22.14	25.93
3	12.23	19.67	16.34	15.72
4	8.07	15.14	14.59	11.93
5	1.66	9.58	16.17	7.48
6	0.42	1.13	2.11	1.02
Number	1202	971	569	2742

- Many markets, including large ones, highly concentrated
- Larger markets do not have more firms (driven by MA, LCC?)

Exclusion Restrictions, Parametric Assumptions

- Identification relies on market-specific variables affecting one firms' profits but not others. Two types of variables considered:
  1. *Airport presence*: fraction of markets served out of an airport
  2. *Costs*: proxied by distance from an airline's nearest hub
     - Exclusion restriction valid if hub distance affects fixed costs, but not variable costs (why? is this reasonable?)

- Exclusion restrictions depend on richness of model specification
  1. *Fixed Competitive Effects* ($\phi_j^i = 0\ \forall i, j$): Both airport presence and costs excluded from competitors' profits
  2. *Variable Competitive Effects*: only costs excluded; airport presence may affect competitors' profits
     - Incumbent with large airport presence may deter entry

- Distribution of unobservables parameterized as jointly normal:

$$\epsilon_{im} = u_{im} + u_m + u_m^o + u_m^d$$

  Allows correlation across firms by origin, destination, and market

Outline

Fixed Competitive Effects $\phi_j^i = 0$

TABLE III

EMPIRICAL RESULTS[a]

	Berry (1992)	Heterogeneous Interaction	Heterogeneous Control	Firm-to-Firm Interaction
Competitive fixed effect	[−14.151, −10.581]			
AA		[−10.914, −8.822]	[−9.510, −8.460]	
DL		[−10.037, −8.631]	[−9.138, −8.279]	
UA		[−10.101, −4.938]	[−9.951, −5.285]	
MA		[−11.489, −9.414]	[−9.539, −8.713]	
LCC		[−19.623, −14.578]	[−19.385, −13.833]	
WN		[−12.912, −10.969]	[−10.751, −9.29]	
LAR on LAR				
LAR: AA, DL, UA, MA				[−9.086, −8.389]
LAR on LCC				[−20.929, −14.321]
LAR on WN				[−10.294, −9.025]
LCC on LAR				[−22.842, −9.547]
WN on LAR				[−9.093, −7.887]
LCC on WN				[−13.738, −7.848]
WN on LCC				[−15.950, −11.608]
Airport presence	[3.052, 5.087]	[11.262, 14.296]	[10.925, 12.541]	[9.215, 10.436]
Cost	[−0.714, 0.024]	[−1.197, −0.333]	[−1.036, −0.373]	[−1.060, −0.508]
Wright	[−20.526, −8.612]	[−14.738, −12.556]	[−12.211, −10.503]	[−12.092, −10.602]
Dallas	[−6.890, −1.087]	[−1.186, 0.421]	[−1.014, 0.324]	[−0.975, 0.224]
Market size	[0.972, 2.247]	[0.532, 1.245]	[0.372, 0.960]	[0.044, 0.310]
WN			[0.358, 0.958]	
LCC			[0.215, 1.509]	

(*Continues*)

Fixed Competitive Effects $\phi_j^i = 0$

TABLE III—*Continued*

	Berry (1992)	Heterogeneous Interaction	Heterogeneous Control	Firm-to-Firm Interaction
Market distance	[4.356, 7.046]	[0.106, 1.002]	[0.062, 0.627]	[−0.057, 0.486]
WN			[−2.441, −1.121]	
LCC			[−0.714, 1.858]	
Close airport	[4.022, 9.831]	[−0.769, 2.070]	[−0.289, 1.363]	[−1.399,−0.196]
WN			[1.751, 3.897]	
LCC			[0.392, 5.351]	
U.S. center distance	[1.452, 3.330]	[−0.932, −0.062]	[−0.275, 0.356]	[−0.606, 0.242]
WN			[−0.357, 0.860]	
LCC			[−1.022, 0.673]	
Per capita income	[0.568, 2.623]	[−0.080, 1.010]	[0.286, 0.829]	[0.272, 1.073]
Income growth rate	[0.370, 1.003]	[0.078, 0.360]	[0.086, 0.331]	[0.094, 0.342]
Constant	[−13.840, −7.796]	[−1.362, 2.431]	[−1.067, −0.191]	[0.381, 2.712]
MA			[−0.016, 0.852]	
LCC			[−2.967, −0.352]	
WN			[−0.448, 1.073]	
Function value	1756.2	1644.1	1627	1658.3
Multiple in identity	0.837	0.951	0.943	0.969
Multiple in number	0	0.523	0.532	0.536
Correctly predicted	0.328	0.326	0.325	0.308

[a] These set estimates contain the set of parameters that cannot be rejected at the 95% confidencet level. See Chernozhukov, Hong, and Tamer (2007) and the Supplemental Material for more details on constructing these confidence regions.

Variable Competitive Effects

VARIABLE COMPETITIVE EFFECTS

	Independent Unobs	Variance–Covariance	Only Costs
Fixed effect			
AA	[−9.433, −8.485]	[−8.817, −8.212]	[−11.351, −9.686]
DL	[−10.216, −9.255]	[−9.056, −8.643]	[−12.472, −11.085]
UA	[−6.349, −3.723]	[−4.580, −3.813]	[−10.671, −8.386]
MA	[−9.998, −8.770]	[−7.476, −6.922]	[−11.906, −10.423]
LCC	[−28.911, −20.255]	[−14.952, −14.232]	[−11.466, −8.917]
WN	[−9.351, −7.876]	[−6.570, −5.970]	[−12.484, −10.614]
Variable effect			
AA	[−5.792, −4.545]	[−4.675, −3.854]	
DL	[−3.812, −2.757]	[−3.628, −3.030]	
UA	[−10.726, −5.645]	[−8.219, −7.932]	
MA	[−6.861, −4.898]	[−7.639, −6.557]	
LCC	[−9.214, 13.344]		
WN	[−10.319, −8.256]	[−11.345, −10.566]	
Airport presence	[14.578, 16.145]	[10.665, 11.260]	
Cost	[−1.249, −0.501]	[−0.387, −0.119]	
AA			[−0.791, 0.024]
DL			[−1.236, 0.069]
UA			[−1.396, −0.117]
MA			[−1.712, 0.072]
LCC			[−17.786, 1.045]
WN			[−0.802, 0.169]
Wright	[−17.800, −16.346]	[−16.781, −15.357]	[−14.284, −10.479]
Dallas	[0.368, 1.323]	[0.839, 1.132]	[−5.517, −2.095]

Reporting Counterfactuals

- With partially identified / incomplete models, counterfactual statements require careful treatment
  - Model only provides lower and upper bounds on probability of each outcome
- E.g. suppose we want to predict effects of exogenous variables on the probability that American enters a market
  - Many market structures in which American enters: monopoly, duopoly with each competitor, etc.
  - Summing over upper bounds across market structures cannot be correct because they are mutually exclusive events
  - CT approach:
    1. In each market, calculate upper bound for each of the 64 market structures at actual and perturbed observables
    2. For each structure, average upper bounds across markets
    3. Report largest positive and negative changes in average upper bounds across structures

Effects on Entry Probability

MARGINAL EFFECTS[a]

	AA	DL	UA	MA	LCC	WN	No Firms
Market size							
Positive	*0.1188*	*0.1136*	*0.0571*	*0.1188*	*0.0849*	*0.1118*	−0.0033
Negative	−0.0494	−0.0720	−0.0001	−0.0442	*−0.1483*	−0.0300	−0.0033
Market distance							
Positive	0.0177	0.0165	0.0106	0.0177	0.0099	0.0000	0.0006
Negative	*−0.0354*	*−0.0377*	*−0.0110*	*−0.0360*	*−0.0128*	*−0.0377*	0.0006
Close airport							
Positive	*0.1178*	*0.1122*	*0.0312*	*0.1048*	0.0662	*0.1178*	−0.0033
Negative	−0.0375	−0.0518	−0.0004	−0.0318	*-0.0911*	−0.0175	−0.0033
Change income							
Positive	*0.0283*	*0.0265*	*0.0149*	*0.0283*	0.0171	*0.0277*	−0.0007
Negative	−0.0140	−0.0193	−0.0001	−0.0120	*−0.0339*	−0.0086	−0.0007
Per capita income							
Positive	*0.0576*	*0.0546*	*0.0291*	*0.0576*	0.0364	*0.0573*	−0.0015
Negative	−0.0270	−0.0377	−0.0002	−0.0237	*−0.0699*	−0.0160	−0.0015

- E.g. if market size rises by 1 million,
  - One market structure with American entering is 11.88% more likely to be *an* equilibrium
  - Another such structure is 4.94% less likely to be an equilibrium

Competitive Effects

Airport presence	0.0673	0.0498	0.1888	0.0734	0.0599	0.1040
Cost	−0.0102	−0.0068	−0.0117	−0.0120	−0.0054	−0.0125
AA	...	−0.3606	−0.2556	−0.4108	−0.0704	−0.2143
DL	−0.3336	...	−0.2658	−0.3908	−0.0335	−0.2126
UA	−0.2486	−0.2630	...	−0.2696	−0.0675	−0.2015
MA	−0.3877	−0.3941	−0.2717	...	−0.0989	−0.2766
LCC	−0.0998	−0.1579	−0.0721	−0.1415	...	−0.0411
WN	−0.2256	−0.2356	−0.2030	−0.2868	−0.0242	...

- Maximum possible competitive effects: Delta's entry decreases the probability of American entering by as much as 33.36%
- Effects heterogeneous across carriers

Policy Analysis: Repeal of the Wright Amendment

- The Wright Amendment affected 93 markets
  - How many markets would be served afterwards?
  - Which firms would enter?
  - Simulation sets *Wright* dummy variable to 0
- CT report:
  - Change in average probability of each carrier serving a market
    - Max change in avg. upper bounds (as for marginal effects)
  - Change in probability no firms serve a market
- Question of which parameters to report results for
  - Minimum and maximum values across parameters in $\hat{\Theta}_I$
  - Parameter value that minimized the objective function
    - Happened to be unique

Counterfactual Predictions

PREDICTED PROBABILITIES FOR POLICY ANALYSIS: MARKETS OUT OF DALLAS LOVE

Airline	Variance–Covariance	Independent Obs	Only Costs
No firms	[−0.6514, −0.6384, −0.6215]	[−0.7362, −0.6862, −0.6741]	[−0.6281, −0.6162, −0.5713]
AA	[0.4448, 0.4634, 0.4711]	[0.2067, 0.3013, 0.3280]	[0.3129, 0.3782, 0.4095]
DL	[[0.4768, 0.4988, 0.5056]	0.2733, 0.3774, 0.4033]	[0.3843, 0.4315, 0.4499]
UA	[0.1377, 0.1467, 0.1519]	[0.1061, 0.1218, 0.2095]	[0.2537, 0.3315, 0.3753]
MA	[0.4768, 0.4988, 0.5056]	[0.2733, 0.3774, 0.4033]	[0.3656, 0.4143, 0.4342]
LCC	[0.3590, 0.3848, 0.4156]	[0.8369, 0.8453, 0.8700]	[0.2839, 0.3771, 0.3933]
WN	[0.4480, 0.4744, 0.4847]	[0.2482, 0.2697, 0.3367]	[0.3726, 0.4228, 0.4431]

- Huge predicted effects on number of entrants
- As of Oct. 2006, Southwest was planning to serve 43 markets outside of the Wright Amendment area. Success?

Parting Thoughts: Moment Inequality Models

- Moment inequality models are a powerful tool...
  - Allow empirical progress under weaker assumptions
  - Natural "revealed preference" interpretation of model restrictions
  - Today: model incompleteness due to multiple equilibria
  - Next week: weaker assumptions on agents' information sets
- ...but not a panacea
  - Highly computationally demanding
    - Model structure may need to be simplified (e.g. # firms)
  - Model must still be correctly specified
    - Exclusion restrictions, functional form, and parametric assumptions (next week: attempts to relax the latter)
  - Limited ability to perform counterfactuals
- There's also lots more work on computational and econometric issues
  - e.g. Andrews & Soares '09; Galichon & Henry '11
  - Many other applications: auctions, dynamic games

Extensions and Other Work

- We've just scratched the surface of entry models
- In two weeks, we'll discuss extensions before starting dynamic games
    - Post-entry competition and social efficiency (Berry & Waldfogel '99)
    - Endogenous product choice (Mazzeo '02, Seim '06, Jia '08, Fan '13, Wollman '18)
- Next week, other applications of moment inequality models

Next Time

- Pakes, ECMA 2010 (*)
- Pakes, Porter, Ho, Ishii, ECMA 2015 (*)
- Illanes, 2017 (***)