$$\frac{d}{dx}\prod_{i=1}^{3} f_i(x) \quad \cancel{f_1(x)\frac{d}{dx}}$$

$$= \frac{d f_1(x)}{dx} f_2(x) f_3(x)$$

$$+ \underline{f_1(x)} \frac{d}{dx}(f_2(x)) f_3(x)$$

$$+ f_1(x) f_2(x) \frac{d}{dx} f_3(x)$$

$$= \sum_{\substack{j=1 \\ i\neq j}}^{3} \left(\prod_{i=j+1}^{3} f_i(x) \frac{d}{dx} f_j(x) \right)$$

wrongly predicted

$$\frac{d}{dx}\prod_{i=1}^{n} f_i(x)$$

$$= \frac{d}{dx}\{ f_1(x) f_2(x) \text{---} f_n(x) \}$$

$$= f_2(x) f_3(x) \dots f_n(x) \cdot \frac{d}{dx} f_1(x)$$

$$+ f_1(x) f_3(x) \text{--} f_n(x) \frac{d}{dx} f_2(x)$$

$$+ f^1(x) f_2(x) f_4(x) \text{---} f_n(x) \frac{d}{dx} f_3(x)$$

$$+ \text{--- --- --- ---} \frac{d}{dx} f_4(x)$$

$$+ \text{--- --- ---} \frac{d}{dx} f_5(x)$$

$$+ \text{--- --- ---}$$

$$+ \text{---}$$

$$+ \text{--- --- --- --- ---} \frac{d}{dx} f_n(x)$$

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$$= \prod_{i=2}^{n} f_i(x) \frac{d}{dx} f_1(x)$$

$$+ \prod_{i=3}^{n} f_i(x) \cdot f_1(x) \cdot \frac{d}{dx} f_2(x)$$

$$+ \prod_{i=4}^{n} f_i(x) \cdot f_1(x) f_2(x) \cdot \frac{d}{dx} f_3(x)$$

$$+ \vdots$$

$$+ \prod_{i=1}^{n-1} f_i(x) \frac{d}{dx} f_n(x)$$

$$+ \prod_{i=1}^{n-2} f_i(x) f_n(x) \frac{d}{dx} f_{n-1}(x)$$

$$= \sum_{\substack{j \neq i \\ i \neq j}} \left(\prod_{i=1}^{n} f_i(x) \cdot \frac{d}{dx} f_j(x) \right)$$ (wrongly predicted)

$$\{i\} = \{1, 2, 3, 4, ---, n-2, n-1, n\}$$

wrongly predicted is with Σ if $i \neq j$

wrongly predicted also if $i \neq j$ is with Π since, it is useless as $i = j+1$ already satisfies it by default.

$$\sum_{\substack{j=1 \\ i \neq j}}^{n} \left(\prod_{i=j+1}^{n} f_i(x) \cdot \frac{d}{dx} f_j(x) \right)$$

Also, if we expand, from the 2nd term in the expansion, there is always one term missing from the product which occurs just before $f_j'(x)$.

no limits. (not necessary)

$$\sum_{j=1}^{3} \left(\prod_{i \neq j}^{3} f_i(x) \frac{d}{dx} f_j(x) \right).$$ (Right prediction)

$$\frac{d}{dx}\prod_{i=1}^{n} f_i(x)$$

$=$

$$\prod_{i \neq j} f_i(x) \frac{d}{dx} f_1(x)$$

$$+ \prod_{i \neq j} f_i(x) \frac{d}{dx} f_2(x)$$

$$+ \prod_{i \neq j} f_i(x) \frac{d}{dx} f_3(x)$$

Set of i = $\{ 1, \underbrace{2, 3}_{i \neq j} \}$

How to define set for i? How to restrict it?

→ In general, for product rule, $\xrightarrow{}$ not necessary

$$\frac{d}{dx}\left\{\prod_{i=1}^{n} f_i(x)\right\} = \sum_{j=1}^{n} \prod_{\substack{i \neq j}}^{n} f_i(x) \frac{d}{dx}(f_j(x))$$

Proof →

$$LHS = \frac{d}{dx}\left\{\prod_{i=1}^{n} f_i(x)\right\}$$

$$= \frac{d}{dx}\left\{f_1(x) f_2(x) \cdots f_n(x)\right\}.$$

$$RHS = \sum_{j=1}^{n} \prod_{i \neq j}^{n} f_i(x) \frac{d}{dx} f_j(x)$$

$$= \prod_{i \neq 1}^{n} f_i(x) \frac{d}{dx} f_1(x) + \prod_{i \neq 2}^{n} f_i(x) \frac{d}{dx} f_2(x)$$

$$+ \prod_{i \neq 3}^{n} f_i(x) \frac{d}{dx} f_3(x) + \cdots$$

$$+ \prod_{i \neq n-1}^{n} f_i(x) \frac{d}{dx} f_{n-1}(x) + \prod_{i \neq n}^{n-1} f_i(x) \frac{d}{dx} f_n(x)$$

$$= \prod_{i=2}^{n} f_i(x) \frac{d}{dx} f_1(x) + \prod_{i=3}^{n} f_i(x) f_1(x) \frac{d}{dx} f_2(x)$$

$$+ \prod_{i=4}^{n} f_i(x) f_1(x) f_2(x) \frac{d}{dx} f_3(x) + \cdots$$

$$\cdots + \prod_{i=1}^{n-2} f_i(x) f_n(x) \frac{d}{dx} f_{n-1}(x)$$

$$+ \prod_{i=1}^{n-1} f_i(x) \frac{d}{dx} f_n(x).$$

Hence, proved