Multiple start destination pairs

$$\mathcal{L}(\vec{f_e}^{1}, \ldots, \vec{f_e}^{w}, \lambda_n^1, \ldots, \lambda_n^w) = \sum_{e\in E} \frac{1}{2}\left(\sum_w f_e^w\right)^2 \cdot \alpha_e + \beta_e \cdot \sum_w f_e^w$$
$$\sum_{n\in V}\sum_w \lambda_n^w \left(p_n^w - \sum_{e\in E} E_{ne} \cdot f_e^w\right)$$

$$= \sum_{n,m}\sum_w \left(\sum_{w_2} \frac{1}{2} f_{n,m}^{w} \cdot f_{n,m}^{w_2} \cdot \alpha_{n,m}\right) + \beta_{n,m} \cdot f_e^w$$
$$+ \lambda_n^w \cdot \left(p_n^w - f_{n,m}^w + f_{m,n}^w\right)$$

$$\frac{\partial \mathcal{L}}{\partial f_{n',m'}^{w'}} = \alpha_{n',m'}\left(f_{n',m'}^{w'} + \sum_{w_2 \neq w'} f_{n',m'}^{w_2}\right) + \beta_{n',m'} - \lambda_{n'}^{w'} + \lambda_{m'}^{w'} \overset{!}{=} 0$$

$$f_{n',m'}^{w'} = -\sum_{w_2 \neq w'} f_{n',m'}^{w_2} + \frac{1}{\alpha_{n',m'}} \cdot \left(\lambda_{n'}^{w'} - \lambda_{m'}^{w'} - \beta_{n',m'}\right)$$

$$f_{n',m'}^{w'} = \max\left(0, -\sum_{w_2 \neq w'} f_{n',m'}^{w_2} + \frac{1}{\alpha_{n',m'}} \cdot \left(\lambda_{n'}^{w'} - \lambda_{m'}^{w'} - \beta_{n',m'}\right)\right)$$

Artifical to garanti $f_{n',m'}^{w'} \geq 0$

$$f[:] - f.\text{sum}(0)[\text{None},:] + \frac{1}{\alpha[\text{None},:]} \cdot \left(\lambda[:, \text{edges}.T[0]] - \lambda[:, \text{edges}.T[1]] - \beta[\text{None},:]\right)$$

$$\alpha_{n,m} \sum_{w_2} f_{n,m}^{w_2} - \lambda_n^{w} + \lambda_m^{w} = -\beta_{n,m}$$

$$p_n^{w} = \sum_m f_{n,m}^{w} - f_{m,n}^{w}$$

$$\alpha_{n,m} \cdot \left(\sum_{w_2} f_{n,m}^{w_2} \right) - \lambda_n^{w} + \lambda_m^{w} = -\beta_{n,m} \quad \text{if } f_{n,m}^{w} > 0$$

$$p_n^{w} = \sum_m \max(0, f_{n,m}^{w}) - \max(f_{m,n})$$

case $W=2$

$$\left(\begin{array}{c|c} D_1 & O_1 \\ \hline O_2 & D_2 \end{array}\right) \begin{pmatrix} \vec{f}^1 \\ \vec{f}^2 \\ \lambda^1 \\ \lambda^2 \end{pmatrix} = \begin{pmatrix} -\vec{\beta}^1 \\ -\vec{\beta}^2 \\ p_n^1 \\ p_n^2 \end{pmatrix}$$

$$D_1 = \left(\begin{array}{ccc|ccc} \alpha_1^1 & & 0 & \alpha_1^2 & & 0 \\ & \ddots & & & \ddots & \\ 0 & & \alpha_E^1 & 0 & & \alpha_E^2 \\ \hline \alpha_1^1 & & 0 & \alpha_1^2 & & 0 \\ & \ddots & & & \ddots & \\ 0 & & \alpha_E^1 & 0 & & \alpha_E^2 \end{array}\right) \qquad O_1 = \left(\begin{array}{c|c} \begin{matrix} 0 \cdots -1\ 0 \cdots 1 \cdots 0 \cdots \\ \cdots 1\ 0 \cdots -1\ 0 \cdots \\ \cdots 0 \cdots 0 \cdots \end{matrix} & 0 \\ \hline 0 & \begin{matrix} 0 \cdots -1\ 0 \cdots 1 \cdots 0 \\ 1\ 0 \cdots 1\ 0 \cdots -1\ 0 \\ \cdots 0 \cdots 0\ 0 \end{matrix} \end{array}\right)$$

$$O_2 = \left(\begin{array}{c|c} \begin{matrix} 0 \cdots 1\ 0 \cdots -1\ 0 \\ 0 \cdots -1 \cdots 0 \cdots \\ 0 \cdots 0 \end{matrix} & 0 \\ \hline 0 & \begin{matrix} 0 \cdots 1 \cdots 1 \cdots 0 \\ 1\ 0 \cdots 1\ 0 \cdots -1 \\ \cdots \end{matrix} \end{array}\right) \qquad D_2 = \begin{pmatrix} 0 \end{pmatrix}$$

- delete columns where $f_{n,m}^{w} = 0$, because here also derivative is zero.

O_2

$$p_n^{w} = \sum_m \max(0, f_{n,m}^{w}) - \max(0, f_{m,n}^{w})$$

$$O_2 = \left(\begin{array}{c|c} w=1 & 0 \\ \hline 0 & w=2 \end{array}\right) \begin{pmatrix} \vec{f}^{1} \\ \vec{f}^{2} \end{pmatrix}$$

1 where edge.T[0] = i and my_mask[edge.i]

-1 where edge.T[1] = i and my_mask[edge.i]

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edge[my_mask].T[0] $\leftarrow$ i

np.arange[edge.sum()][my_mask] $\leftarrow$ j add variable $f_{n,m} = \sum_{w} f^{w}_{n,m}$? (would not.

1 if n,m is part of w