$$\sum_w f_e^w = \left(\sum_n \lambda_n^w E_n\right) \frac{1}{\alpha_e} - \frac{\beta_e}{\alpha_e}$$

Let effective $\tilde{\alpha}_e = \alpha_e \cdot \frac{\sum_w f_e^w}{f_e^w}$

$$\Leftrightarrow \alpha_e = \tilde{\alpha}_e \cdot \frac{f_e^w}{\sum_w f_e^w}$$

$$\Rightarrow \sum_w f_e^w = \left(\sum_n \lambda_n^w E_n\right) \frac{1}{\tilde{\alpha}_e} \cdot \frac{\sum_w f_e^w}{f_e^w} - \frac{\beta_e}{\tilde{\alpha}_e} \cdot \frac{\sum_w f_e^w}{f_e^w}$$

$$\Leftrightarrow 1 = \left(\sum_n \lambda_n^w E_n\right) \frac{1}{\tilde{\alpha}_e} \cdot \frac{1}{f_e^w} - \frac{\beta_e}{\tilde{\alpha}_e} \cdot \frac{1}{f_e^w}$$

$$\Leftrightarrow f_e^w = \left(\sum_n \lambda_n^w E_n\right) \frac{1}{\tilde{\alpha}_e} - \frac{\beta_e}{\tilde{\alpha}_e}$$