Introduction to Symbolic AI Tasks WS 2018/19

Kevin Schneider (389667)
Heinrich Lukas Weil (389347)

Task 1 - The Resolution Method

a)

- $C_1 \equiv (X \lor F \lor \neg M \lor R)$
- $C_2 \equiv (\neg F \lor X \lor B)$
- $C_3 \equiv (\neg R \lor \neg F \lor X \lor \neg B)$
- $C_4 \equiv (X \lor F \lor M)$
- $C_5 \equiv (X \lor \neg M \lor \neg R \lor F)$
- $C_6 \equiv (\neg F \lor X \lor R \lor \neg A \lor \neg B)$
- $C_7 \equiv \neg X$

Knowledge $W \equiv C_1 \land C_2 \land C_3 \land C_4 \land C_5 \land C_6 \land C_7$

Hypothesis $H \equiv \neg(\neg F \lor X \lor \neg B \lor R \lor A)$

Proof that $W \models H$ holds using the resolution method.

Solution:

Too proof that $W \models H$, show that $W \land \neg H$ is not satisfiable.

$\neg H \equiv (\neg F \lor X \lor \neg B \lor R \lor A)$

C_7 is an atom which can resolve all X in C_1 - C_6:

- $C_8 \equiv C_1 \land C_7 \rightarrow (F \lor \neg M \lor R)$
- $C_9 \equiv C_2 \land C_7 \rightarrow (\neg F \lor B)$
- $C_{10} \equiv C_3 \land C_7 \rightarrow (\neg R \lor \neg F \lor \neg B)$
- $C_{11} \equiv C_4 \land C_7 \rightarrow (F \lor M)$
- $C_{12} \equiv C_5 \land C_7 \rightarrow (\neg M \lor \neg R \lor F)$
- $C_{13} \equiv C_6 \land C_7 \rightarrow (\neg F \lor R \lor \neg A \lor \neg B)$
- $C_{14} \equiv \neg H \land C_7 \rightarrow (\neg F \lor \neg B \lor R \lor A)$

Further Resolutions:

- $C_{15} \equiv C_8 \land C_{11} \rightarrow (F \lor R)$

- $C_{16} \equiv C_{11} \wedge C_{12} \rightarrow (\neg R \vee F)$
- $C_{17} \equiv C_{15} \wedge C_{16} \rightarrow F$
- $C_{18} \equiv C_9 \wedge C_{17} \rightarrow \mathbf{B}$
- $C_{19} \equiv C_{13} \wedge C_{17}$ -> $(R \vee \neg A \vee \neg B)$
- $C_{20} \equiv C_{14} \wedge C_{14} \rightarrow (\neg B \vee R \vee A)$
- $C_{21} \equiv C_{20} \wedge C_{19} \rightarrow (\neg B \vee R)$
- $C_{22} \equiv C_{17} \wedge C_{10} \rightarrow (\neg B \vee \neg R)$
- $C_{23} \equiv C_{22} \wedge C_{21} \rightarrow \mathbf{\neg B}$
- $C_{24} \equiv C_{23} \wedge C_{18} \rightarrow \mathbf{()}$

Therefore, W ∧ ¬H is not satisfiable and W |= H holds.

b)

Determine whether the following formulas are satisfiable or not. If yes, provide a model (i.e., an interpretation that satisfies the formula); if no, prove that by applying the resolution method.

(S ∨ W) ∧ (S ∨ ¬W) ∧ (¬S ∨ W) ∧ (¬S ∨ ¬W)

Solution: The formula is not satisfiable:

- (S ∨ W) ∧ (S ∨ ¬W) -> S
- (¬S ∨ ¬W) ∧ (¬S ∨ W) -> ¬S
- ¬S ∨ S -> ()

(B ∨ ¬A) ∧ (¬B ∨ ¬X ∨ F) ∧ (¬Z ∨ X) ∧ (Z ∨ X ∨ P) ∧ (¬B ∨ ¬X ∨ ¬F) ∧ (Z ∨ X ∨ ¬P)

Solution: The formula is satisfiable. ¬B ¬A X ¬F ¬Z ¬P is a possible solution(table is a bit too large for this pdf sorry, you can copy the code in the pre tag to check it):

B	A	X	F	Z	P	(((((B ∨	-A)	∧	((-B ∨	-X)	∨	F)) ∧	(-Z ∨	X)) ∧	((Z ∨	X) ∨	P)) ∧	((-B ∨ -X) ∨ -F))	∧	((Z ∨	X) ∨	-P)
0	0	1	0	0	0	1	1	1	1 1	0	1	1	1	1	1	1	1	1 1 1 0 1	**1**	1	1	1

```
B A X F Z P | (((((B V -A) ∧ ((-B V -X) V F)) ∧ (-Z V X)) ∧ ((Z V X) V
P)) ∧ ((-B V -X) V -F)) ∧ ((Z V X) V -P)
────────────────────────────────────────────────────────────────────────
─────────────────────────────────────

0 0 1 0 0 0 |       1 1   1   1  1 0   1     1  1  1     1     1    1
1   1  1 0   1 1    1     1    1 1
```

Task 2 - Unification

Unify the following pairs of clauses. First, select suitable candidates that may be unifiable from each set, then apply the method described in the lecture. Finally, apply the resulting substitution to the whole clause.

Hint: Imagine you are trying to apply the resolution rule. Only one literal needs to match.

{p(ƒ (X, Y), Z), q(a, X)} and {¬q(W, b), r(a, c)}

Solution:

- p(ƒ (X, Y), Z) cant be unified with ¬q(W, b) or r(a, c) because the predicate symbols dont match.
- q(a, X) and r(a, c) cant be unified because the predicate symbols dont match.
- S{a/W,b/X} is a unifier for ¬q(W, b) and q(a, X):

  S(q(a, X)) = q(W, X)

  S(¬q(W, b)) = ¬q(W, X)

  S(q(a, X)) = ¬S(¬q(W, b)).

- the substituted clauses are then {p(ƒ (X, Y), Z), q(W, X)} and {¬q(W, X), r(a, c)}, which have the resolvant:

  {p(ƒ (X, Y), Z),r(a, c)}

{q(ƒ (ƒ (X, Y), X)), ¬p(Z)} and {q(ƒ (ƒ (g(c), Z), g(Z)))}

- p(Z) cant be unified with q(ƒ (ƒ (X, Y), X)) because the predicate symbols dont match.
- q(ƒ (ƒ (X, Y), X)) and q(ƒ (ƒ (g(c), Z), g(Z))) are candidates for unification, but substitution of two different constants fails:

{q(ƒ (ƒ (X, Y), X)) =? q(ƒ (ƒ (g(c), Z), g(Z)))}	**{}**
{(ƒ (ƒ (X, Y), X) =? (ƒ (ƒ (g(c), Z), g(Z))}	{}
{ƒ (X, Y) =? ƒ (g(c), Z) , X =? g(Z)}	{g(c)/X)}
{Y =? Z}	⊥

Task 3 -

see "FamiliyTree_KevinSchneider_LukasWeil.pl"

Task 4 -

see "FamiliyTree_KevinSchneider_LukasWeil.pl"

Task 5 -

see "Resolver_KevinSchneider_LukasWeil.py"