

Mathematical Modeling Project

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1 The Simple Population Dynamcis Model

$$\begin{aligned}\dot{v} &= v(r - ax) = f_1(v, x), \\ \dot{x} &= -bx + cv = f_2(v, x)\end{aligned}\tag{1}$$

With fixed points

$$\begin{aligned}\dot{v}(0, 0) &= \dot{x}(0, 0) = 0 \\ \dot{v}\left(\frac{ba}{rc}, \frac{a}{r}\right) &= \dot{x}\left(\frac{ba}{rc}, \frac{a}{r}\right) = 0\end{aligned}\tag{2}$$

Where v is the viral strain and x is the specific immune response to the strain. r is the rate at which the virus reproduces. a is the rate at which the immune cells destroy the virus. b is the rate at which the immune cells die off. c is rate at which the immune cells reproduce. This is dependent on the number of viruses present, v .

1.1 Getting the fixed points

First we take the Jacobian

$$J = \begin{pmatrix} r - ax & -av \\ c & -b \end{pmatrix}\tag{3}$$

Then we find the characteristic equation. Let $\alpha = \frac{ba}{rc}$ and let $\beta = \frac{a}{r}$

$$\begin{aligned}J|_{(0,0)} - \lambda I &= \lambda^2 + \lambda(b - r) - br = 0 \\ \implies \lambda_{1,2} &= r, -b \\ J|_{(\alpha,\beta)} - \lambda I &= \lambda^2 + \lambda\gamma + \delta\end{aligned}\tag{4}$$

where

$$\begin{aligned}\gamma &= a\beta - r + b, \text{ and } \delta = ba\beta - br - a\alpha c \\ \implies \lambda_{1,2} &= \frac{-\gamma \pm \sqrt{\gamma^2 - 4\delta}}{2}\end{aligned}$$

Figure 1: Population size of the viral load and the immune response for a single virus strain with $r = 2.4$, $a = 2$, $b = 0.1$, $c = 1$.

