

# Mathematical Modeling Project

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## 1 the belle sux model

$$\begin{aligned}\dot{v} &= v(r - ax) = f_1(v, x), \\ \dot{x} &= -bx + cv = f_2(v, x)\end{aligned}\tag{1}$$

With fixed points

$$\begin{aligned}\dot{v}(0, 0) &= \dot{x}(0, 0) = 0 \\ \dot{v}\left(\frac{ba}{rc}, \frac{a}{r}\right) &= \dot{x}\left(\frac{ba}{rc}, \frac{a}{r}\right) = 0\end{aligned}\tag{2}$$

Where  $v$  is the viral strain and  $x$  is the specific immune response to the strain.  $r$  is the rate at which the virus reproduces.  $a$  is the rate at which the immune cells destroy the virus.  $b$  is the rate at which the immune cells die off.  $c$  is rate at which the immune cells reproduce. This is dependent on the number of viruses present,  $v$ .

### 1.1 Getting the fixed points

First we take the Jacobian

$$J = \begin{pmatrix} r - ax & -av \\ c & -b \end{pmatrix}\tag{3}$$

Then we find the characteristic equation. Let  $\alpha = \frac{ba}{rc}$  and let  $\beta = \frac{a}{r}$

$$\begin{aligned}J|_{(0,0)} - \lambda I &= \lambda^2 + \lambda(b - r) - br = 0 \\ \implies \lambda_{1,2} &= r, -b \\ J|_{(\alpha,\beta)} - \lambda I &= \end{aligned}\tag{4}$$