Mathematical Modeling Project

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1 the belle sux model

$$\begin{aligned} \dot{v} &= v(r - ax) = f_1(v, x), \\ \dot{x} &= -bx + cv = f_2(v, x) \end{aligned} \tag{1}$$

With fixed points

$$\begin{aligned} &\dot{v}(0,0) = \dot{x}(0,0) = 0 \\ &\dot{v}(\frac{ba}{rc}, \frac{a}{r}) = \dot{x}(\frac{ba}{rc}, \frac{a}{r}) = 0 \end{aligned} \tag{2}$$

Where v is the viral strain and x is the specific immune response to the strain. r is the rate at which the virus reproduces. a is the rate at which the immune cells destroy the virus. b is the rate at which the immune cells die off. c is rate at which the immune cells reproduce. This is dependent on the number of viruses present, v.

1.1 Getting the fixed points

First we take the Jacobian

$$J = \begin{pmatrix} r - ax & -av \\ c & -b \end{pmatrix} \tag{3}$$

Then we find the characteristic equation. Let $\alpha = \frac{ba}{rc}$ and let $\beta = \frac{a}{r}$

$$\begin{aligned} &J|_{(0,0)} - \lambda I = \lambda^2 + \lambda(b - r) - br = 0 \\ &\implies \lambda_{1,2} = r, -b \\ &J|_{(\alpha,\beta)} - \lambda I = \lambda^2 + \lambda\gamma + \delta \\ \text{where} \\ &\gamma = a\beta - r + b, \text{ and } \delta = ba\beta - br - a\alpha c \\ &\implies \lambda_{1,2} = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\delta}}{2} \end{aligned} \tag{4}$$