AE640A: Autonomous Navigation (Spring'22)
Assignment #5: State Estimation and Filtering

Due Date: 25/04/2022 Maximum Marks: 150

Instructions

- This assignment is based on topics from **Module 3 & 4** and is designed to get you familiar with the basic mathematical foundations and gain hands-on experience
- You may choose to typeset your answers in LaTeX or Markdown etc. We encourage this and acknowledge the efforts put in by you. Hence, if the document is submitted with proper source code is not plagiarised and is properly typeset, a **5% bonus** will be added to whatever score you obtain in that assignment.
- **Set-2 is a programming assignment.** You may choose any language of your choice, but the code should be clearly written, properly commented and submitted with a README file to execute the code.
- For handwritten answers, use a scanning application on your phone to scan the handwritten answers and convert them to a pdf. Use ilovepdf to merge various pdf files. As the name suggests, we love pdf's, so please submit a single pdf file as **rollnumber.pdf.**
- The use of any unfair means such as plagiarism by any student would be severely punished. For details regarding the mode of submission and penalties imposed for late submissions, compilation errors, etc., refer to this document.
- Contact the course staff ae640a@gmail.com for any further queries.

Set 1 - Kalman Filter Theory (70 points)

1. Show that Kalman Filter is a minimum mean-square error (MMES) estimator. You can use error covariance to show this. [25]
2. Consider the below polar to rectangular coordinate transformation: [45]

$$x = r\cos(\theta)\,,$$

$$y = r\sin(\theta)$$

where, r is the range and θ is the angle or bearing and can be measured using sensors. The x and y coordinates are the function of range and angle. In a generic form, the above equations can be written as:

$$f = h(r, \theta)$$

I. **The range and bearings can be further written as**

$$r = \bar{r} + r_e$$

$$\theta = \bar{\theta} + \theta_e$$

where, the true means of the range ($\bar{r}$) and bearing ($\bar{\theta}$) are 1 m and π/2 degrees respectively, and r_e and θ_e are the zero-mean deviations from their means. **Derive the expression for mean and covariance for the 'f'.**

Matlab exercise: Generate 1000 random range and bearing points which are uniformly distributed between ±0.01 m and ±10 degrees. Plot the mean and covariance ellipse using the above-derived expression.

II. **Derive the expression for mean and covariance for the nonlinear function 'f' using the concepts of first-order linearization (EKF).**

Matlab exercise: Generate 1000 random range and bearing points which are uniformly distributed between ±0.01 m and ±10 degrees. Plot the mean and covariance ellipse using the expressions obtained from first-order linearization.

III. **Derive the unscented sigma points and weights for the nonlinear function 'f'.**

Matlab exercise: Generate 1000 random range and bearing points which are uniformly distributed between ±0.01 m and ±10 degrees. Plot the mean and covariance ellipse of 'f' using unscented transformation.

IV. **Compare the mean and covariance for true, first-order approximation, and unscented transformations.**

Set 2 - Kalman Filter Practical (80 points)

1. Find the time stamped raw data from IMU (Roll Pitch and Yaw) in the file named raw.txt. The data in the file is raw, and hence contains noise. There are other fields as well, you are required to extract the relevant fields for yourself. Filter the noise off the data using Kalman Filter, and plot the filtered data over time. You will have to tune the filter using the basic principles taught in class.

   For your reference, the plots visualising the filtered data will be released in due time.Use them as reference to compare the performance of your filter.