AE640A: Autonomous Navigation (Spring'22)
Assignment #3: State Estimation and Dynamics

Due Date: TBD Maximum Marks: 100

Instructions

- This assignment is based on topics from **Module 3** and is designed to get you familiar with the basic mathematical foundations required for the module and gain hands-on experience
- You may choose to typeset your answers in LaTeX or Markdown etc. We encourage this and acknowledge the efforts put in by you. Hence, if the document is submitted with proper source code is not plagiarised and is properly typeset, a **5% bonus** will be added to whatever score you obtain in that assignment.
- For handwritten answers, use a scanning application on your phone to scan the handwritten answers and convert them to a pdf. Use ilovepdf to merge various pdf files. As the name suggests, we love pdf's, so please submit a single pdf file as **rollnumber.pdf.**
- Refer to the previous lectures from the course for an understanding of these topics. Contact the course staff ae640a@gmail.com for any further queries.

Set 1 - Dynamics (20 points)

1. Assume that a frame is rotated by Euler angels, derive the kinematic relationship in terms of the rotation matrix. [Hint: You can consider a planer case and relate rotation matrix from one frame to another frame.] [10]
2. Prove that an exponential map of a skew-symmetric matrix is a rotation matrix. [10]

Set 2 - Kalman Filter Theory (40 points)

1. Show that Kalman Filter is a minimum mean-square error (MMES) estimator. You can use error covariance to show this. [15]

2. Consider the following linear Gaussian system: [25]

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t + w_t, \\ y_t &= Hx_t + \eta_t, \end{aligned}$$

where $x_t \in R^{n}{}_{x}$ is the state vector, $u_t \in R^{n}{}_{u}$ is the input vector, $w_t \in R^{n}{}_{w}$ is a disturbance vector acting on the system, $y_t \in R^{n}{}_{y}$ is the output vector, and $\eta_t \in R^{n}{}_{\eta}$is a noise vector acting on the output.

The initial state is assumed to be Gaussian $x_0 \sim N(\hat{x}_0, \Sigma x_0)$. The disturbance w_t and η_t have the known probability distribution $w_t \sim N(0, \Sigma w_t)$ and $\eta_t \sim N(0, \Sigma \eta_t)$. Show that the Kalman filter is optimal using the covariance matrix

$$P_{t|t} = \mathbf{E}[(x_t - x_{t|t})(x_t - x_{t|t})^T]$$

Set 3 - Least Square Estimation (40 points)

1. Consider the localization problem given in Fig.1. The unit vectors from the vehicle to beacon and respective ranges are observed as follows:

Beacon no	Unit Vector	Range	Landmark location
1	$[0.50 \quad 0.87]^T$	12.2	$[16.0 \quad 25.4]^T$
2	$[-0.77 \quad 0.64]^T$	6.9	$[4.6 \quad 19.5]^T$
3	$[-0.50 \quad -0.87]^T$	14.2	$[3.0 \quad 2.9]^T$
4	$[0.34 \quad -0.94]^T$	8.9	$[13.0 \quad 6.5]^T$

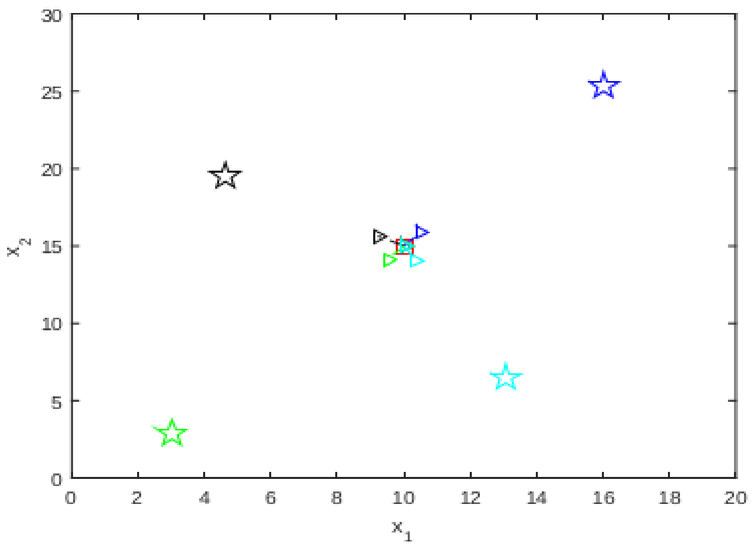

Fig 1: Localization problem with four beacons

Determine the position of the vehicle using the least-squares approach. **[20]**

2. Assume that in the above problem the unit vectors are not available. The locations of beacons are now available, formulate the nonlinear least-square problem and determine the solution. Assume that each sensor has the following accuracy: Sensor 1 -95%, Sensor 2 -98%, Sensor 3 -90%, Sensor 4 -92%), how would use this information in your solutions? [20]