ROBOT IP-MODEL

DOT

1.

Time is discrete. I use the letter t, with variants, for the time step #.

In my model, cars and robots are located on the nodes of a subgraph of a grid. I use the letters u, v, w for nodes. I refer to the neighbor to the, say, left of node v by "$\mathrm{nghbr}(v, \mathsf{east})$". I'm assuming a two directed edges (u, v) and (v, u) between every pair of adjacent nodes in the grid subgraph. With $w := \mathrm{nghbr}(v, \mathsf{east})$, the edge (v, w) is $\mathrm{edg}(v, \mathsf{east})$, and the edge (w, v) is $\mathrm{edg}(w, \mathsf{west})$. I use the letter e (with variants) for edges; the edge in the opposite direction of e is $\bar{e}$.

I use the letter d (w/ variants) to denote the directions in $\mathbb{Z}_4$ (so that I can use $\mathsf{east}+1 = \mathsf{south} = -\mathsf{north}$):

$$\mathsf{west} := 0, \qquad \mathsf{north} := 1, \qquad \mathsf{east} := 2, \qquad \mathsf{south} := 3.$$

There may be several cars, let's say up to k, which we want to retrieve from the system, they are called "special cars", and they are "labaled" by integers 0 to $k-1$. All other cars are "anonymous" or "unlabeled".

A node can be occupied by exactly one of the following:

$\emptyset$	empty
c	an anonymous car
$\mathsf{sc}j$	special car #j
r	a robot
$\mathsf{r,c}$	a robot and an anonymous car
$\mathsf{r,sc}j$	a robot and an special car #j
cr	a robot carrying an anonymous car
$\mathsf{scr}j$	a robot carrying special car #j
lft	a robot in the process of lifting up an anonymous car
$\mathsf{slft}j$	a robot in the process of lifting up special car #j
drp	a robot in the process of dropping an anonymous car
$\mathsf{sdrp}j$	a robot in the process of dropping special car #j

I refer to this as the node status. I use the placeholder what to stand for any of the above.

Remark 1. The robots and the anonymous cars are unlabeled. This requires to take care making sure that no robots can be "created" out of thin air, or cars can vanish.

Correspondlingly, I have the following variables:

$$\mathtt{nstat}_{v,\mathrm{what}}(t).$$

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The following equations state that a node can only have one status

$$\forall v: \quad \sum_{\text{what}} \texttt{nstat}_{v,\text{what}} = 1 \tag{1}$$

The directed edges can be either occupied or not. Correspondingly, there are variables

$$\texttt{occu}_e(t).$$

At each time, we have to exclude the possibility that an edge is used by more than one vehicle:

$$\forall e: \quad \texttt{occu}_e + \texttt{occu}_{\bar{e}} \leq 1; \tag{2}$$

or that more than one vehicle arrive at the same node:

$$\forall v: \quad \sum_{u \in N(v)} \texttt{occu}_{(u,v)} \leq 1; \tag{3}$$

or that one vehicle leaves a node orthogonal to a direction from which another vehicle is approaching it:

$$\forall v\colon \forall d: \quad \texttt{occu}_{\bar{\text{e}}\text{dg}(v,d)} + \texttt{occu}_{\text{edg}(v,d-1)} + \texttt{occu}_{\text{edg}(v,d+1)} \leq 1 \quad \text{[4 constraints per node } v\text{]}. \tag{4}$$

In order to maintain a coherent flow of objects through time, we will need to have an equations of the following pattern:

$$\texttt{nstat}_*(t+1) = \texttt{nstat}_*(t) \pm \text{“decisions”}. \tag{$*$}$$

There are variables for the following decisions.

- $\texttt{go}_{v,\text{what},d}$ initiate a movement of one of r r, c r, $\mathsf{sc}j$ cr $\mathsf{scr}j$ from a node in direction d;
- $\texttt{stop}_{v,\text{what},d}$ stops the movement at a node, what is arriving from direction d;
- $\texttt{cont}_{v,\text{what},d}$ continues the movement of what, through node v in direction d, without slowing down.
- $\texttt{lift}_{v,\text{what}}$ connects a robot to a car, where what is one of c, $\mathsf{sc}j$;
- $\texttt{drop}_{v,\text{what}}$ disconnects the car from the robot (same what).