Overdispersion

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Overdispersion

- Poisson model:

$$
\begin{aligned}
y_i &\sim Poisson(\lambda_i) \\
E[y_i] &= \lambda_i \\
V(y_i) &= \lambda_i
\end{aligned}
$$

- GLM estimation notation

$$
\begin{aligned}
\mu_i &= E(y_i) \\
V(\mu_i) &= \phi V(y_i) = \phi \mu_i (\text{for Poisson})
\end{aligned}
$$

- When $\phi > 1$, there is overdispersion in the model

Likelihood Equations

$$
\sum_{i=1}^{N} \frac{(y_i - \mu_i) x_{ij}}{\phi V(y_i)} \frac{\partial \mu_i}{\partial \eta_i} = 0, \text{for } j = 1, ..., p
$$

- These depend on the distribution of y through μ, $V(y)$
- Having $1/\phi$ in these equations doesn't change the MLE for the β's.
- However, with overdispersion the covariance changes:

$$
Cov(\hat{\beta}) = (X^T W X)^{-1} = \phi Cov(\hat{\beta})
$$

- W is a matrix with diagonal elements w_i such that:

$$
w_i = \frac{\left(\frac{\partial \mu_i}{\partial \eta_i}\right)^2}{V(y_i)} = \frac{\left(\frac{\partial \mu_i}{\partial \eta_i}\right)^2}{\phi \mu_i)} (\text{for Poisson})
$$