GLMMs

- estimation
- DF
- interpretation
- example
- overdispersion

Why we use GLMMs (Applied Regression Modelling) (Gelman Hill 11.5) (Ruppert, Wand, Caroll, Sempiparametric Regression)

- interested in estimating group=level regression coefficients that account for individual + group level variation
- Allow us to model individual-level coefficients parsimoniously
- estimating regression coefficients for particular groups. Especially relevant when group size is small
  - GLMMs are best when number of groups > 5
  - best when there is a lot of variation across groups

GLMMS Notation

Recipe for GLMM:

- GLM-like formulation
  - $g(E(Y_{ij}|u_i)) = X_{ij}^T\beta + Z_{ij}^T u_i$
  - g is a glm link
  - Y_{ij} is jth observation in ith group
  - u_i is random effects for the ith group
  - $dim(X_{ij}^T) = (1, p)$ $dim(Z_{ij}^T) = (1, q)$
- $y_{ij}|u_i \sim f$ for $f \in$ exponential family
- $u_i \sim N(0, G_\theta)$

Estimation

$L(\beta, \theta) = f(\vec{y}|\beta\theta)$

$$= \int_{R^q} f(\vec{y}|u, \beta) f(u) du \text{ (*)}$$

Two main methods

- Laplace approximation of $*$
  - Use partial quasilikleihood
  - Laplace approximation to ingeral
  - Uses NR-like techniques, with a penalty on the coefficients
- Bayesian MCMC methods to sample from posterior distribution of β and θ

Degrees of Freedom

Extension of hat matrix to
For GLM

trace(H_b)
For GLMM

$$trace(H_{\beta,\theta}) \leq p + n * q$$

this is sometimes called "effective" df.

Example

Spinal implant procedure. Patients receive procedure and have follow up visits 1 and 2 days after. Binary outcome measuring success ("no pain") or failure ("pain").

$$y_{ij} = \begin{cases} 1 & \text{if patient i at visit j has no pain} \\ 0 & \text{if patient i at visit j has pain} \end{cases}$$

$$x_j = \begin{cases} 1 & \text{j=2 ,if patient's second visit} \\ 0 & \text{j=1,if pateint's first visit} \end{cases}$$

	No Pain	Pain
No pain	63	16
pain	12	35

Logistic Normal model

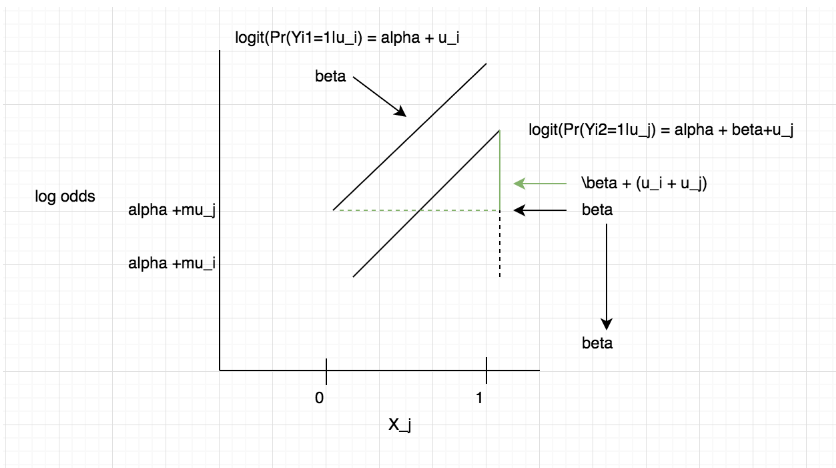
$$logit(Pr(Y_{ij} = 1|u_i)) = \alpha + \beta x_i + u_i$$
$$u_i \sim N(0, \sigma^2_{u_i})$$

or

$$Y_{ij}|u_i \sim Bernoulli(\pi_{ij})$$
$$\pi_{ij} = logit^{-1}(\alpha + \beta x_j + u_i)$$

$\beta \rightarrow$ change in log odds of success from visit 1 to 2 for a specific subject or, change in log odds for a subjects w/ same random intercept u_i.

$$logit(Pr(Y_i2 = 1|u_j)) = \alpha + \beta + u_j$$
$$-logit(Pr(Y_i2 = 1|u_i)) = \alpha + u_j$$
$$= \beta + (u_i + u_j)$$



GLMMs can be used to model overdisperison

- random terms (intercepts) add extra variability to outome
- $logit(Pr(Y_{ij} = 1|u_i)) = x\beta + u_i$
- $u_i \sim N(0, \sigma_u^2$
- $Y_{ij}|u_i \sim Bernoulli(\pi_{ij})$
- $Y_{ij}|u_i \sim Poisson(\lambda_{ij})$
- $log(\lambda_{ij}) = x\beta + u_i$