Translating decision problems into planning tasks

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Directed Hamiltonian Path

Predefined relations (signature)

- $E(x,y)$: Directed edge linking nodes x and y.

Existentially quantified relations

- $F(x,y)$: F is an injective function that maps the number x (from `zero` to `max`) to a node y such that y is the x-th node in a sequence. It's a permutation on the vertices such that $E(F(x),F(x+1))$ for each $0 \leq x < n-1$.

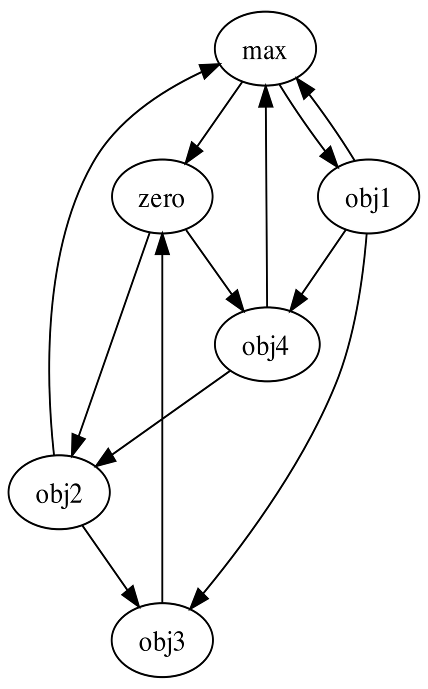
Formula

A DHP can be seen as a sequence such that:

- Every node in the sequence is succeeded by a node that is connected to it by an edge
- Every node appears in the sequence

$$(\exists F \in \mathsf{Inj})(\forall x)[x < \max \rightarrow (\exists x'yz)(E(y,z) \wedge F(x,y) \wedge \mathsf{SUC}(x,x') \wedge F(x',z))]$$

Finite structures (example)



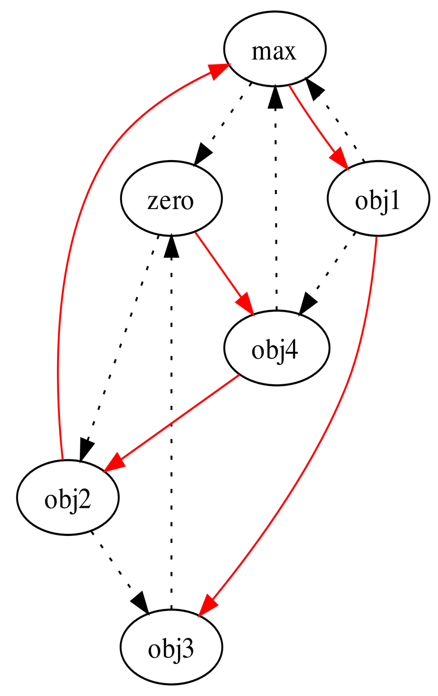
$\Rightarrow$

```
E(max, zero)
E(obj2, obj3)
E(obj3, zero)
E(zero, obj2)
E(obj4, obj2)
E(obj1, obj3)
E(zero, obj4)
E(obj1, max)
E(max, obj1)
E(obj1, obj4)
E(obj4, max)
E(obj2, max)
```

Solution (example)

```
F(zero, zero)
F(obj1, obj4)
F(obj2, obj2)
F(obj3, max)
F(obj4, obj1)
F(max, obj3)
```

$\Rightarrow$



k-Clique

Predefined relations (signature)

- $E(x,y)$: Undirected edge linking nodes x and y.
- $K(x)$: The size of the clique is at least $g(x)$. $g(\texttt{zero}) = 1$, $g(\texttt{obj1}) = 2$, ..., $g(\texttt{max}) = n$.

Existentially quantified relations

- $F(x,y)$: F is a total injective function that maps every node to a number from `zero` to `max`.

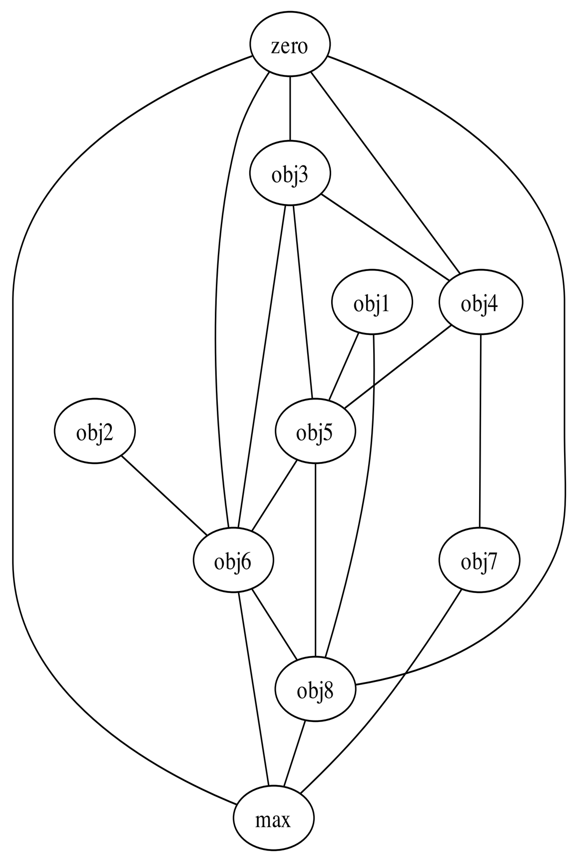
Formula

To find a clique of size k, we are interested in the first k values of $F(x)$.

- For every pair of distinct nodes x, y, if both $F(x)$ and $F(y)$ (which are distinct, because F is injective) are less than k, x and y must be connected by an edge.
- When the function F that satisfies this property is found, we have found a set of k nodes that are all connected by edges.

$$(\exists F \in \mathsf{Inj})(\forall x,y)[F(x) < k \wedge F(y) < k \rightarrow E(x,y)]$$

Finite structures (example)



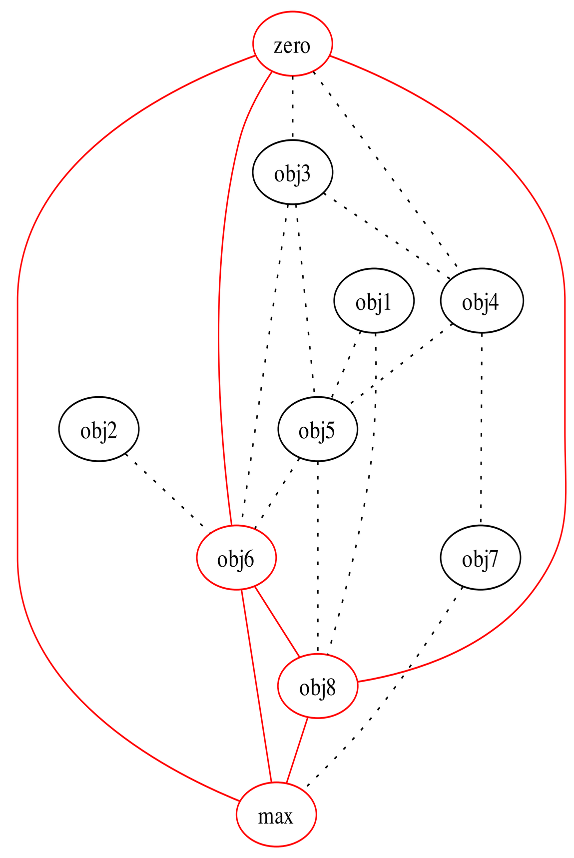
$\Rightarrow$

```
E(zero, obj3)
E(zero, obj4)
E(zero, obj6)
E(zero, obj8)
E(zero, max)
E(obj1, obj5)
E(obj1, obj8)
E(obj2, obj6)
E(obj3, obj4)
E(obj3, obj5)
E(obj3, obj6)
E(obj4, obj5)
...
```

Solution (example)

```
F(zero, zero)
F(obj1, obj4)
F(obj2, obj2)
F(obj3, max)
F(obj4, obj1)
F(max, obj3)
```

$\Rightarrow$



3-Colorability

Predefined relations (signature)

- $E(x,y)$: Undirected edge linking nodes x and y.

Existentially quantified relations

- $R(x)$: node x has been colored **red**.
- $G(x)$: node x has been colored **green**.
- $B(x)$: node x has been colored **blue**.

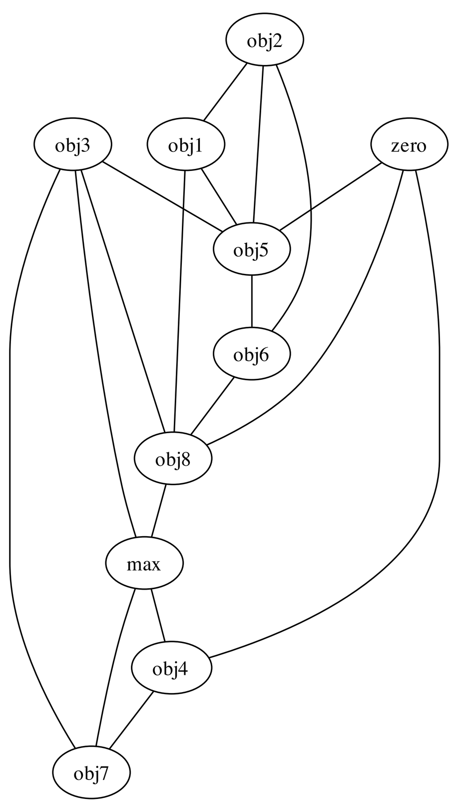
Formula

A valid coloring is one in which:

- Every node is colored
- Every node has only one color
- Adjacent nodes do not have the same color

$$\begin{aligned}(\exists R^1, G^1, B^1)(\forall x,y)[&(R(x) \vee G(x) \vee B(x)) \wedge \\ &R(x) \rightarrow \neg(G(x) \vee B(x)) \wedge \\ &G(x) \rightarrow \neg(R(x) \vee B(x)) \wedge \\ &B(x) \rightarrow \neg(R(x) \vee G(x)) \wedge \\ &E(x,y) \rightarrow \neg((R(x) \wedge R(y)) \vee \\ &(G(x) \wedge G(y)) \vee (B(x) \wedge B(y)))]\end{aligned}$$

Finite structures (example)



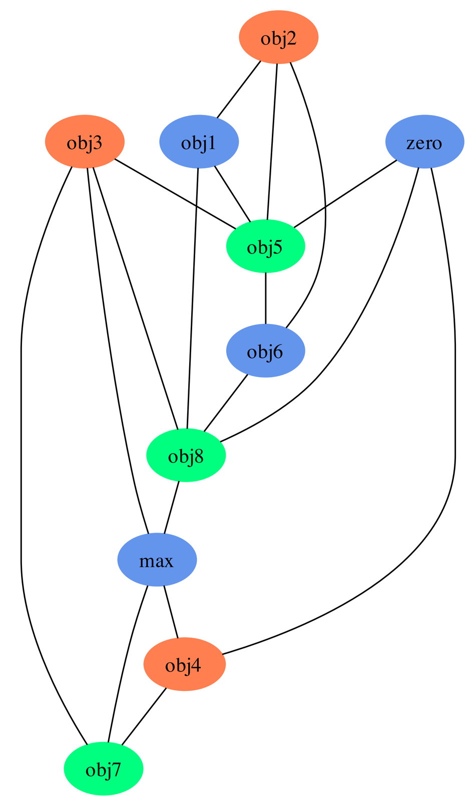
$\Rightarrow$

```
E(obj2, obj6)
E(max, obj3)
E(obj3, obj5)
E(obj3, obj8)
E(zero, obj4)
E(zero, obj8)
E(obj1, obj5)
E(obj1, obj8)
E(obj4, obj7)
...
```

Solution (example)

```
R(obj2)
R(obj3)
R(obj4)
G(obj5)
G(obj7)
G(obj8)
B(zero)
B(obj1)
B(obj6)
B(max)
```

$\Rightarrow$



Future work

- Implement other reductions and compare experimental results
- Extend the language: types
- Target the CO-NP, Σ_2^p, PH and PSPACE complexity classes

Doctoral Consortium

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