CSMMA16

ASSIGNMENT

Due in by 12:00pm (mid-day) Wednesday 12th October 2016

Your solutions should include your R code. You are also required to submit annotated code as a .R file.
Annotated R code and output will count for a maximum of 70% of the total marks available.

Question 1
Construct plots of $\sin(x - \pi/4)$, and $\tan(x)$ for x between $-\pi$ and π on two separate plots in the same figure. Use abline to show the asymptotes.
Add appropriate titles and labels to your plots.

[8 marks]

Question 2
Calculate by hand the eigenvalues and eigenvectors of $\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$ and check your answer in R.
Comment on any differences.
Use R to obtain singular values and matrices in the singular value decomposition of A and compare these with the eigenvalues and eigenvectors in the eigen decomposition. Comment on the results.
Determine if A is positive definite.

[16 marks]

Question 3
Write your own R function to return the angle (in degrees) between two arbitrary vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

[3 marks]

Question 4

Find the optimal value of the function $y = (1 + 2.5t)e^{-0.3t}$ and the point $t.max$ at which this optima occurs by writing your own code in R implementing the Newton-Raphson algorithm. Use R to iterate until $t.max$ is found accurate to 4dp.

[9 marks]

Question 5

Consider a system with differential equation given by

$$\frac{d^2O}{dt^2} + 4\frac{dO}{dt} + 3O = 1 \ .$$

Determine the transient and steady state solutions for O, and hence find the particular solution if $O = 1$ and $dO/dt = 0$ at $t = 0$.

Using R, obtain a numerical solution and plot graphs showing the theoretical and numerical solutions for t in the interval $(0, 8)$. Comment on the plots.

[18 marks]

Question 6

Write a function to generate m datasets, each of sample size nn, from a binomial distribution with $p = 0.01$ and trial size 10 and to calculate the mean of each dataset. Do not write your own loop in R; instead use the `apply` function. Use this function in the `apply` function to generate 10000 means each from studies with sample sizes $nn =$

(a) 20 and (b) 100, respectively.

Plot histograms of the sample means for each sample size so that the two histograms are in the same figure. Use the Central Limit Theorem to explain your findings.

[13 marks]

Question 7

Write a function that uses the R command `t.test` to calculate and return a 95% confidence interval for the mean μ based on a randomly generated sample of size 25 from a normal distribution with mean of 10 and a standard deviation of 1. Use this function to generate 100 confidence intervals and count how often these intervals contain the mean μ. Comment on your results.

[8 marks]

Question 8

The following figure shows 3 faces from a face database.

The originals can be found in Blackboard under Assignments.

Calculate the mean face, and produce an image of it.

Also, obtain and produce **on one figure** images of the difference of each face from the mean face.

Based on the covariance matrix of the differences, calculate eigenfaces (equal to eigenvectors) and produce the first 3 eigenfaces on the same figure.

[25 marks]