Note to self: discuss WL computer exam grading

items $X_1 \ldots X_n$

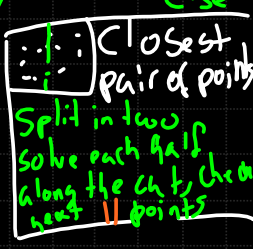
The conquered Compendium

Recursion – split the input: divide & conquer
– combine the results:

The master method: $T(n) = \begin{cases} \Theta(1) & \text{if } n=1 \\ a \cdot T(n/b) + f(n) & \text{else} \end{cases}$

1. $f(n)$ is $O(n^{\log_b(a)-\varepsilon})$ dominated by the leafs: $\Theta(n^{\log_b(a)})$
2. $f(n)$ equally distributed: $\Theta(n^{\log_b(a)} \cdot \log(n))$
3. $f(n)$ is $\Omega(n^{\log_b(a)+\varepsilon})$ in the root: $\Theta(f(n))$

Note: For case 3 also check the regularity condition: $a \cdot f(n/b) \leq c \cdot f(n)$ for some $c < 1$ – polynomial



algorithms: binary search: $\Theta(\log(n))$

merge sort: $\Theta(n \log(n))$

matrix multiplication: $7T(n/2) + O(n^2)$

Karatsuba's algorithm

integer multiplication: $3 \cdot T(n/2) + O(n)$ $\Theta(n^{\log_2(3)})$ $\Theta(n^{\log_2(3)})$

$T(n) = 5T(n/5) + \Theta(n)$

$a = 5$

$b = 5$

$f(n) = n$

$n^{\log_b(a)} = n^{\log_5(5)} = n^1$

$f(n)$ is $\Theta(n^1)$

So case 2 of the master method (work evenly spread)

So $T(n)$ is $\Theta(n^{\log_5(5)} \cdot \log(n)) = \Theta(n \log n)$

$T(n) = 2T(n/3) + n\log n$

$a = 2$

$b = 3$

$f(n) = n\log(n)$

$\log_b(a) = \log_3(2) = 0.\ldots$

$f(n)$ is $\Omega\left(n^{\log_b(a)}\right) = \Omega\left(n^{\log_3(2)}\right)$

So case 3. of the master method (work in the root)

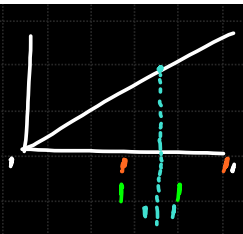
Regularity condition:

$a \cdot f\left(\frac{n}{b}\right) = 2 \cdot \frac{n}{3}\log\left(\frac{n}{3}\right) = \frac{2}{3}n \cdot (\log(n) - \log(3))$

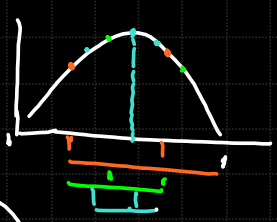
$\leq \frac{2}{3}n\log(n)$ So take $c = \frac{2}{3}$

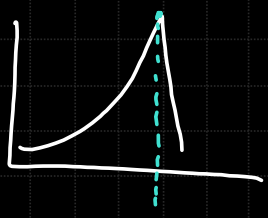
Thus $T(n)$ is $\Theta(f(n)) = \Theta(n\log n)$

binary search:



Ternary search:





$T(n) = 1 \cdot T(n/3/2) + O(1)$

$a = 1$

$b = 3/2$

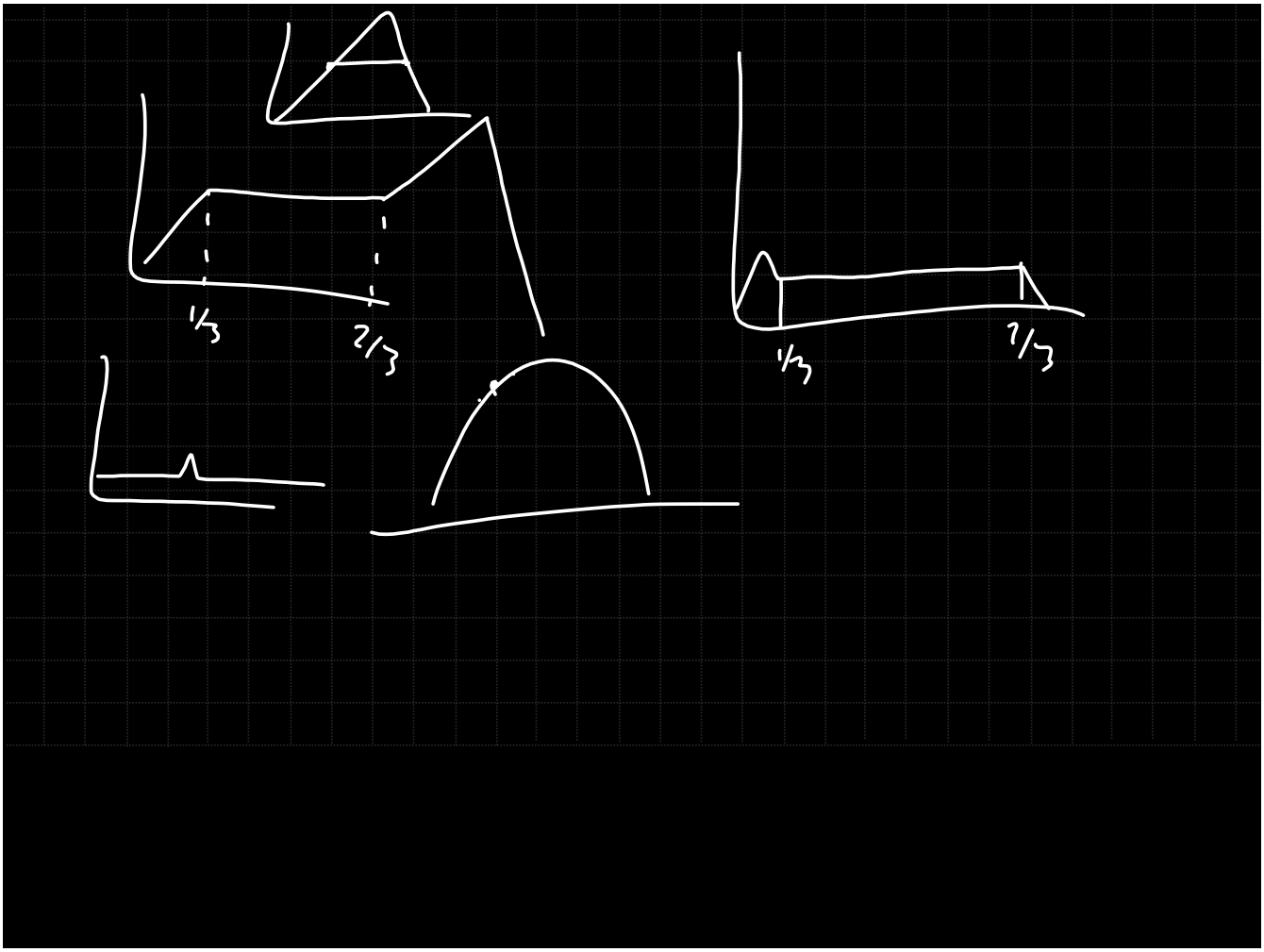
$f(n) = O(1)$

$\log_{3/2}(1) = 0$

$f(n)$ is $\Theta(n^{\log_b(a)})$

so case 2 (splits work evenly)

so $\Theta(n^0 \log(n))$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