Lecture 15b

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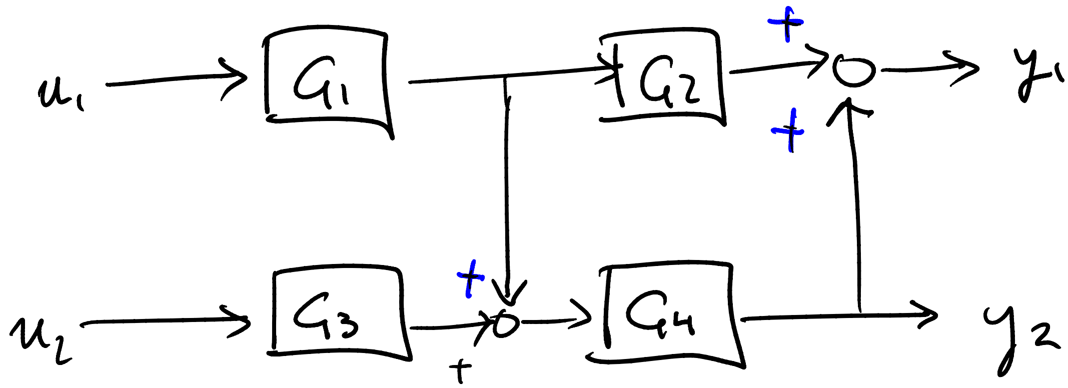
Lecture 15b: MIMO

... in which we briefly touch on how to represent and manipulate multi-input/multi-output systems.

I. Multi input/Multi-output

In MIMO systems, transfer functions are represented in matrix form. You see? Even if you don't use state space, you can't get away from linear algebra!

Example:



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$$y_1 = G_2 G_1 u_1 + G_4 (G_1 u_1 + G_3 u_2)$$

$$= [G_1 G_2 + G_1 G_4] u_1 + G_3 G_4 u_2$$

$$y_2 = G_1 G_4 u_1 + G_3 G_4 u_2$$

or

$$\vec{Y}(s) = \begin{pmatrix} G_1 G_2 + G_1 G_4 & G_3 G_4 \\ G_1 G_4 & G_3 G_4 \end{pmatrix} U(s)$$

$$= T(s) U(s).$$

You can't do $Y(s)/U(s)$ anymore, but otherwise, this looks familar.

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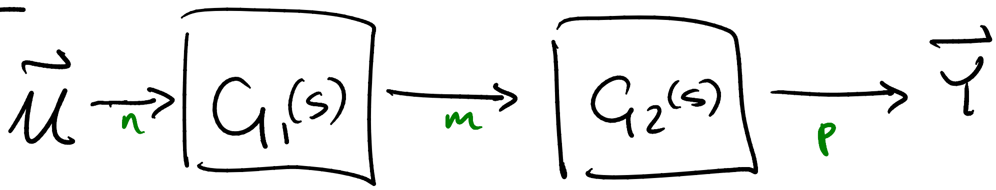
All compositions go through

Ex:

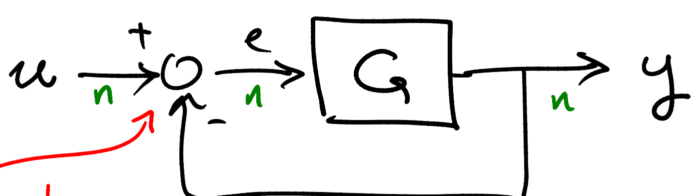


G_1 is $m \times n$

G_2 is $p \times m$

$\underbrace{G_1 G_2}_{\text{composition (matrix mult)}}$ is $p \times n$

Ex:



note:
u and y
must have
same dim.

$$Y = GE = G(U - Y) = GU - GY$$

$$\Rightarrow \quad Y + GY = GU$$

$$(I + G)Y = GU$$

$$Y = (I + G)^{-1} GU.$$

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II. MIMO in State Space

In state space, MIMO is very natural. You just change the dimensions of the I/O vectors.

$$\underbrace{\dot{\vec{x}}}_{n\times 1} = \underbrace{A}_{n\times n}\vec{x} + \underbrace{B}_{n\times m}\underbrace{\vec{u}}_{m\times 1}$$

m inputs

$$\underbrace{\vec{y}}_{p\times 1} = \underbrace{C}_{p\times n}\underbrace{\vec{x}}_{n\times 1}$$

p outputs

The system transfer function matrix is then

$$\underbrace{G(s)}_{p\times m} = \underbrace{C}_{p\times n}\underbrace{(sI-A)^{-1}}_{n\times n}\underbrace{B}_{n\times m}$$

as before.

By the way, an observer is a two input, n-output system!!

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III. Observers + full state feedback

To do full state feedback using the observer

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\hat{y} = C\hat{x}$$

we set $u = -K\hat{x} + r$ to obtain

$$\begin{pmatrix} \dot{\vec{x}} \\ \dot{\hat{x}} \end{pmatrix} = \left(\begin{array}{c|c} A & 0 \\ \hline 0 & A \end{array}\right)\begin{pmatrix} \vec{x} \\ \hat{x} \end{pmatrix} + \begin{pmatrix} B(-K\hat{x} + r) \\ B(-K\hat{x} + r) \end{pmatrix} + \begin{pmatrix} 0 \\ L(y - \hat{y}) \end{pmatrix}$$

$$L(C\vec{x} - C\hat{x})$$

$$= \underbrace{\begin{pmatrix} A & -BK \\ LC & A - BK - LC \end{pmatrix}}_{2n \times 2n}\begin{pmatrix} \vec{x} \\ \hat{x} \end{pmatrix} + \underbrace{\begin{pmatrix} B \\ B \end{pmatrix}}_{2n \times m} r.$$

Thus, a system in conjuction with an observer and a controller is ultimately just another linear system.

To understand its properties, let's look at

$$\tilde{A} = \begin{pmatrix} A & -BK \\ LC & A-BK-LC \end{pmatrix}$$

Suppose $(A-BK)\vec{v} = \lambda\vec{v}$. Then

$$\begin{pmatrix} A & -BK \\ LC & A-BK-LC \end{pmatrix}\begin{pmatrix} v \\ v \end{pmatrix} = \begin{pmatrix} Av - BKv \\ LCv + (A-BK-LC)v \end{pmatrix}$$

$$= \begin{pmatrix} (A-BK)v \\ (A-BK)v \end{pmatrix} = \lambda\begin{pmatrix} v \\ v \end{pmatrix}$$

So each eigenvalue λ of $A-BK$ is also an eigenvalue of $\tilde{A}$.

Also, suppose $w^T(A-LC) = \rho w^T$.
Then,

$$\begin{pmatrix} w^T & -w^T \end{pmatrix} \begin{pmatrix} A & -BK \\ LC & A-BK-LC \end{pmatrix}$$

$$= \begin{pmatrix} w^T(A-LC) \\ -w^T(A-LC) \end{pmatrix} = \rho \begin{pmatrix} w^T \\ -w^T \end{pmatrix}.$$

Since a matrix and its own transpose have the same eigenvalues, it turns out that every eigenvalue of $A-LC$ is also an eigenvalue of $\tilde{A}$.

Say $w^T M = \rho w^T$ and $Mx = \lambda x$

Then $w^T M x = \rho w^T x$

$\Rightarrow \lambda w^T x = \rho w^T x$

$\Rightarrow \lambda = \rho$ (or $w^T x = 0$).