Lecture 8b

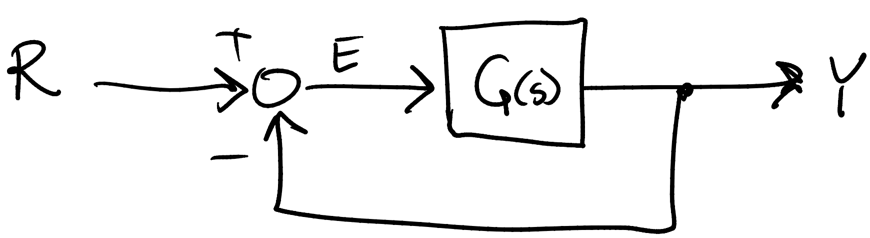
Monday, October 24, 2005
2:16 PM

Lecture 8b: Test Signals

	$r(t)$	$R(s)$
Impulse	$\delta(t)$	1
Step	1	$1/s$
Ramp	t	$1/s^2$
Parabola	t^2	$2/s^3$
later Sine Wave	$\sin \omega t$	$\omega / (s^2 + \omega^2)$

NOTE: $t^n \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{n!}{s^{n+1}}$

We will show that control system's ability to track a test signal has to do with the number of poles at the origin or type. Consider



The general form for $G(s)$ is

$$G(s) = \frac{\prod_{k=1}^{Q}(s_k + z_k)}{s^N \prod_{k=1}^{M}(s_k + p_k)}$$

where

- $z_1, \dots, z_Q$ are the <u>zeros</u>
- $p_1, \dots, p_M$ are the non-zero poles

and there are N poles at zero.

N = system type

$N+M$ = system order

Monday, October 24, 2005
3:05 PM

Now, consider the response to the various signals. For the above configuration

$$E(s) = R(s) - Y(s)$$
$$= R(s) - E(s)\,G(s)$$
$$\Rightarrow E(s) = \frac{1}{1+G(s)} R(s).$$

A. Impulse Response $R(s) = 1$

$$\lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{s}{1+G(s)} = \boxed{0}$$

$\Rightarrow$ Any stable system can track an impulse perfectly.

B. Step Response : $R(s) = 1/s$

$$\lim_{s \to 0} sE(s) = \frac{1}{1+G(0)}$$

$\Rightarrow$ If $N=0$, then $G(0)$ is finite,

so

$$\boxed{e(\infty) = \frac{1}{1+G(0)}} = \frac{1}{1+K_p}$$

$\Rightarrow$ If $N>0$, then $\lim_{s \to 0} G(s) = \infty$

so

$$\boxed{e(\infty) = \frac{1}{\infty} = 0}.$$

C. Ramp Response

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{1+G(s)} \cdot \frac{1}{s^2}$$

$$= \lim_{s\to 0} \frac{1}{s+sG(s)}$$

$$= \lim_{s\to 0} \frac{1}{sG(s)}$$

$\Rightarrow$ If $N=0$ then $e(\infty) = \infty$

$\Rightarrow$ If $N=1$ then $e(\infty) = \frac{1}{K_v}$ where

$$K_v \triangleq \lim_{s\to 0} sG(s)$$

$\Rightarrow$ If $N \geq 2$ then $e(\infty) = 0$

D. Parabola Response

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+G(s)}\frac{2}{s^3}$$

$$= \lim_{s\to 0} \frac{1}{s^2+s^2G(s)}$$

$$= \lim_{s\to 0} \frac{1}{s^2G(s)}$$

$\Rightarrow$ If $N=0$ or 1 then $e(\infty)=\infty$

$\Rightarrow$ If $N=2$ then $e(\infty)=\frac{1}{K_A}$

where $K_A = \lim_{s\to 0} s^2G(s)$

$\Rightarrow$ If $N\geq 3$ then $e(\infty)=0$.

Tuesday, October 25, 2005
10:24 AM

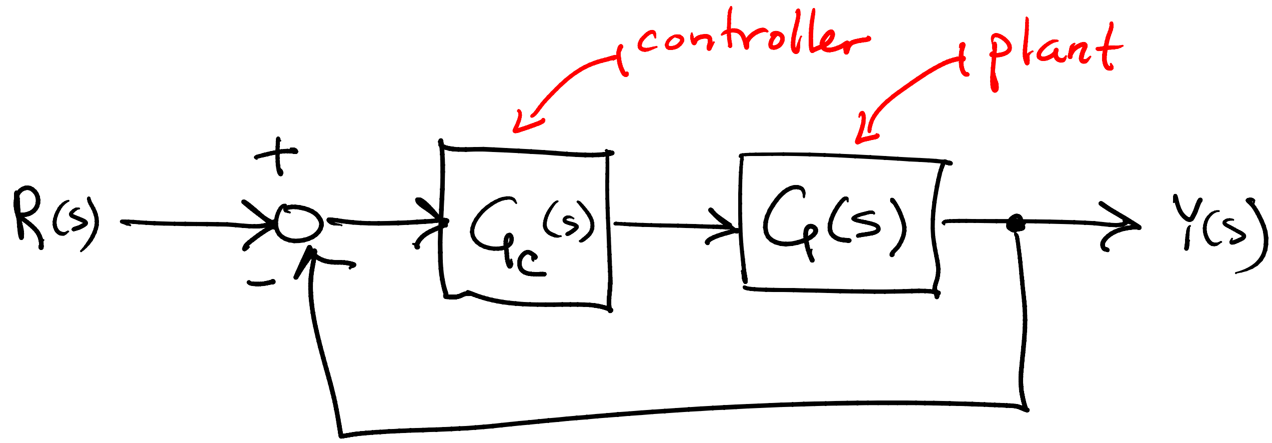
So the type of the system determines the kind of signals it can track.

N	1	$1/s$	$1/s^2$	$2/s^3$
0	0	$\frac{1}{1+K_p}$	∞	∞
1	0	0	$\frac{1}{K_V}$	∞
2	0	0	0	$\frac{1}{K_A}$
3	0	0	0	0
4	0	0	0	0
⋮	⋮	⋮	⋮	⋮

IV. Control Design for S.S.E.

If you know what kind of signal you need to track and the type of plant you have, you can design a controller to change the type.

Tuesday, October 25, 2005
10:30 AM



Example: Say $G(s) = \frac{1}{s+1}$ and you want to track a step input with 10% error. Then you want

$$\frac{1}{1+K_p} = 0.1 \Rightarrow K_p = 99.$$

But $K_p = G_c(s)\,G(s) = K \cdot \frac{1}{s+1}$

when $s \to 0$, this is $K_p = K = 99$. So just put $G_c = K$.

Example: Say $G(s) = \frac{1}{s+1}$ and you want to track a step exactly. Then you want to define $G_c(s)$ so that $G_c(s)\,G(s)$ is type 1 (or greater). For instance,

$$G_c(s) = \frac{K_I}{s}$$

would work. Although you may not get good control over the transient this way. A better controller would be

$$G_c(s) = K_p + \frac{K_I}{s}$$

$$= \frac{K_p s + K_I}{s}$$

which gives you control over damping as well.