Lecture 5a

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Lecture 5a: Combinations of Systems

... in which we send the output of one system to the input of another system, or back to itself.

I. Review of System Representation

A. General Nonlinear Systems

$$\dot{\vec{x}} = f(\vec{x})$$

B. Nonlinear Systems with I/O

$$\dot{\vec{x}} = f(\vec{x}, u)$$

$$y = g(\vec{x})$$

C. Linear Systems

a) $\dot{\vec{x}} = Ax$

b) $a\ddot{y} + b\dot{y} + cy = 0$

c) $as^2 Y(s) + bs Y(s) + c Y(s) = U(s)$

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D. Linear Systems With I/O

(a) $\begin{cases} \dot{\vec{x}} = A\vec{x} + Bu \\ y = C\vec{x} \end{cases}$

(b) $a\ddot{y} + b\dot{y} + cy = u$

(c) $T(s) = \dfrac{Y(s)}{U(s)}$

(a) $\leftrightarrow$ (b) via a choice of state (e.g. $x_1 = y$, $x_2 = \dot{y}$)

(a) $\rightarrow$ (c) via $T(s) = C(sI - A)^{-1}B$

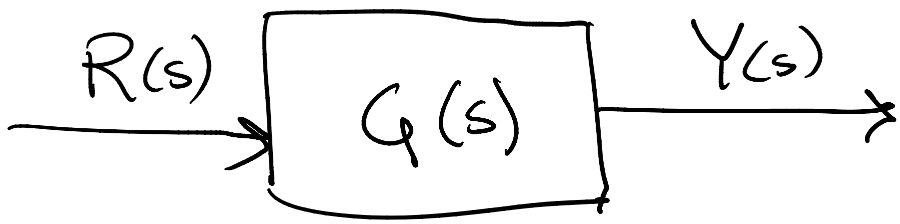
(c) $\rightarrow$ (a): we'll work this out soon

II. Block Diagrams

The transfer function

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\text{Output}}{\text{Input}}$$

is represented by a block



- Blocks can be combined in series, parallel, in loops and by summing inputs
- Each combination can be rewritten as a single block.

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A. Series

$$R(s) \to \boxed{G(s)} \to \boxed{H(s)} \to Y(s)$$

call this $X(s)$

To determine the resulting transfer function, use the definitions

$$G(s) = \frac{X(s)}{R(s)} \quad \& \quad H(s) = \frac{Y(s)}{X(s)}$$

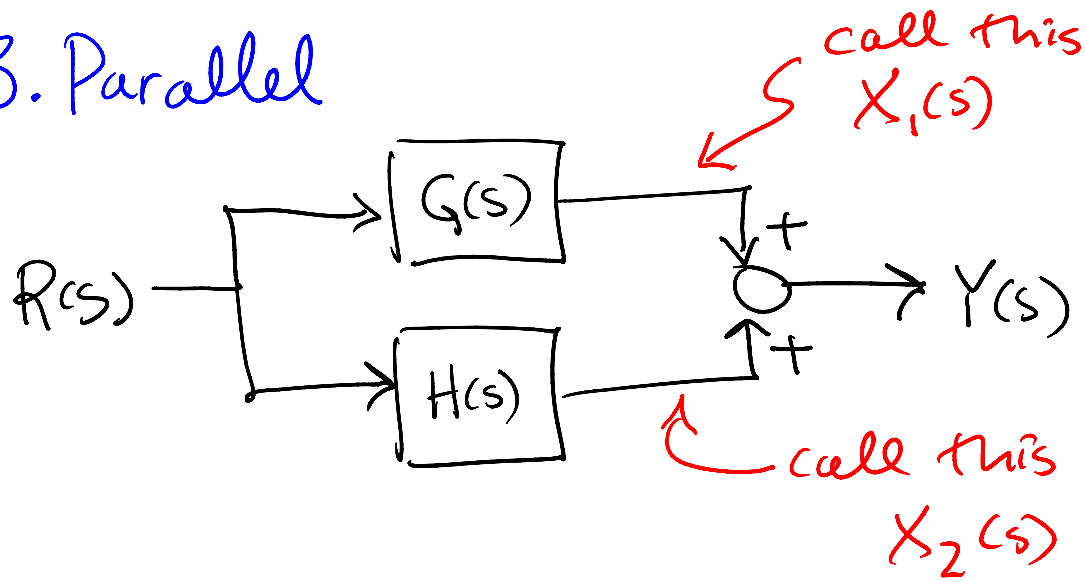
So

$$\frac{Y(s)}{R(s)} = \frac{H(s)\,X(s)}{X(s)/G(s)} = \boxed{G(s)H(s)}$$

So the resulting system is

$$R(s) \to \boxed{G(s)H(s)} \to Y(s).$$

B. Parallel



Once again, appeal to the definitions.

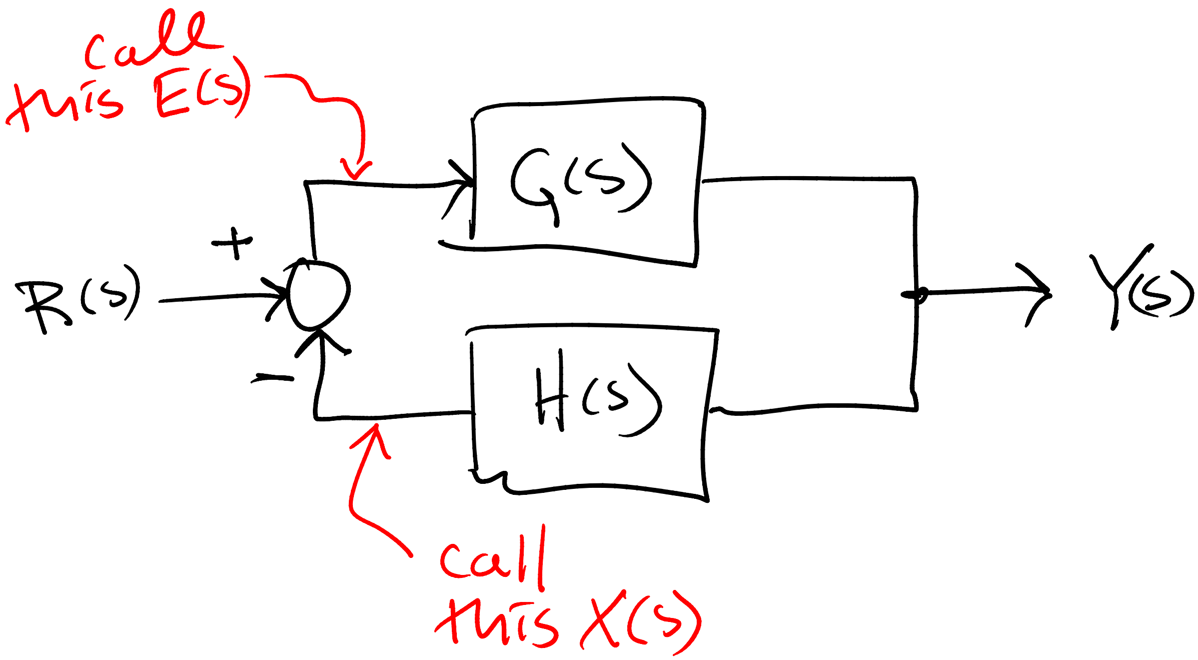
$$X_1 = RG \qquad Y = X_1 + X_2 = RG + RH$$
$$X_2 = RH$$

$$\frac{Y(s)}{R(s)} = G(s) + H(s)$$

So the resulting system is

$$R(s) \longrightarrow \boxed{G(s) + H(s)} \longrightarrow Y(s).$$

C. Feedback



$$
\begin{aligned}
Y &= EG \\
&= (R-X)G \\
&= (R-YH)G
\end{aligned}
\quad\Rightarrow\quad Y(1+GH) = RG
$$

$$\Downarrow$$

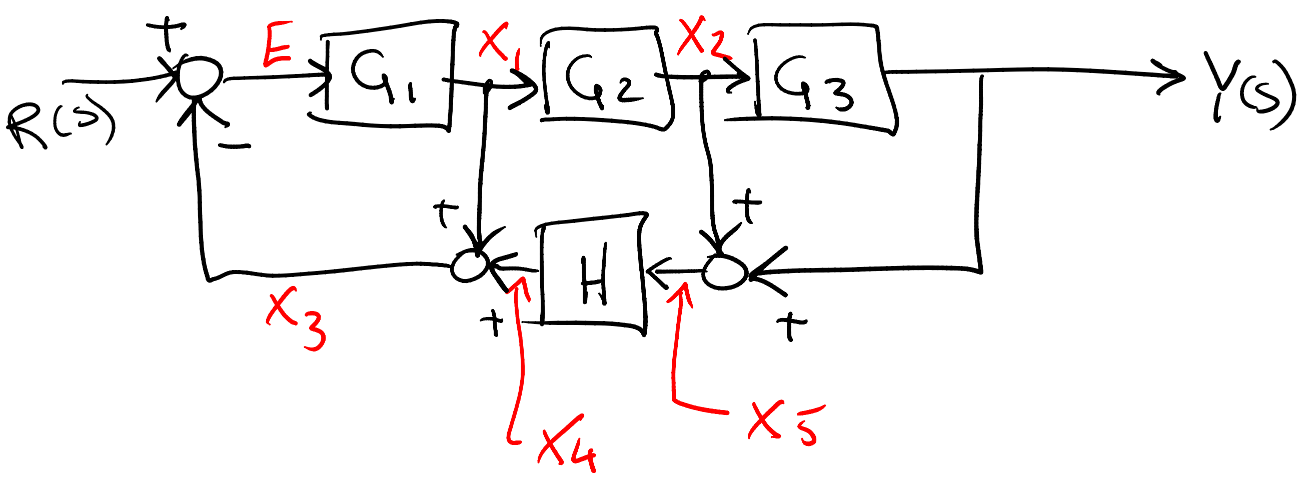
$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

So the system is

$$R(s) \longrightarrow \boxed{\frac{G(s)}{1+G(s)H(s)}} \longrightarrow Y(s).$$

- These three operations allow you to represent pretty much everything.
- Your textbook shows you an <u>algorithm</u> for reducing complex systems into simple ones
- I like to simply define auxillary signals and solve the resulting equations for $Y(s)/R(s)$.

<u>Example</u>: Find an expression for the transfer function of:



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$$Y = X_2 G_3$$
$$= X_1 G_2 G_3$$
$$= E G_1 G_2 G_3$$ (this is called the forward path)
$$= (R - X_3) G_1 G_2 G_3$$
$$= [R - (X_1 + X_4)] G_1 G_2 G_3$$
$$= [R - X_1 - X_5 H] G_1 G_2 G_3$$
$$= [R - X_1 - (X_2 + Y) H] G_1 G_2 G_3$$
$$= \left[R - \frac{Y}{G_2 G_3} - \frac{Y}{G_3} H - YH\right] G_1 G_2 G_3$$

$$\frac{Y}{X_1} = G_2 G_3$$

$$\Downarrow$$

$$\frac{Y}{X_2} = G_3$$

$$Y(1 + G_1 + G_1 G_2 H + G_1 G_2 G_3 H] = R G_1 G_2 G_3$$

$$\boxed{\frac{Y}{R} = \frac{G_1 G_2 G_3}{1 + G_1 + G_2 G_3 H + G_1 G_2 G_3 H}}$$