Lecture 7a

Wednesday, October 19, 2005
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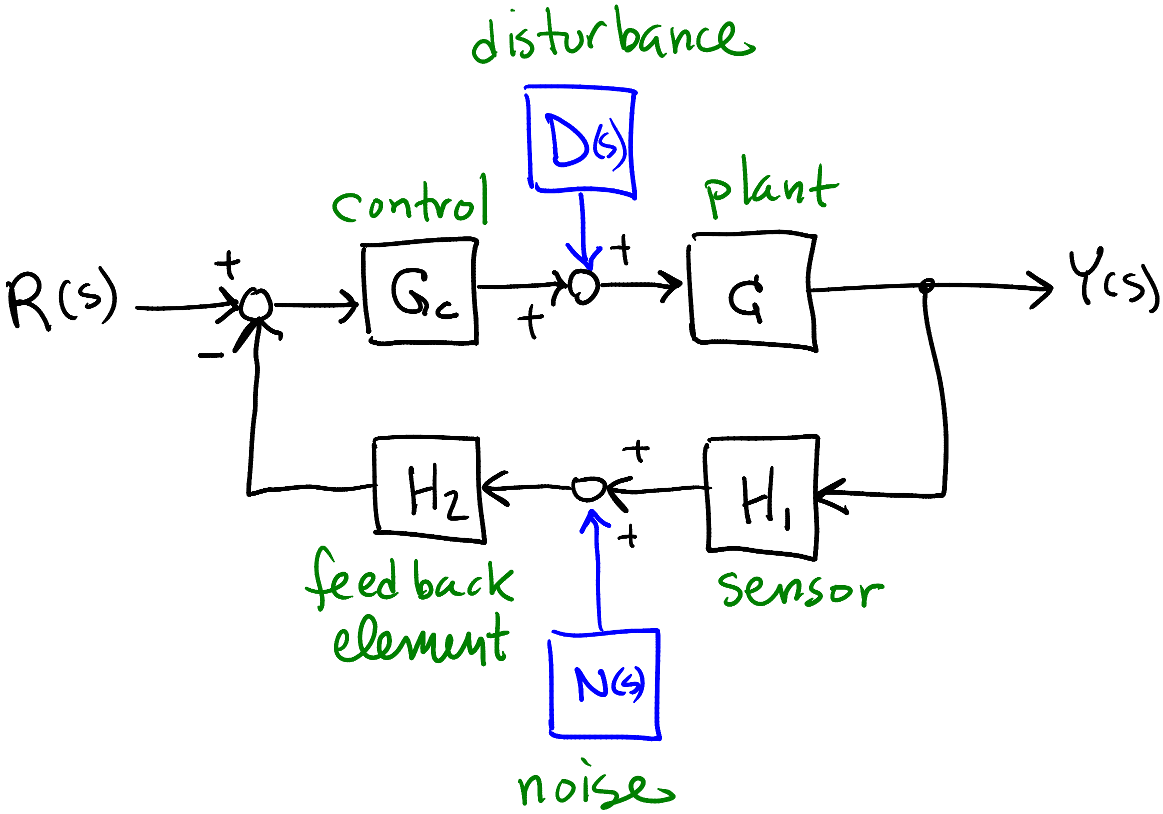
Lecture 7a: Sensitivity, Disturbances Noise

... in which we develop techniques for evaluating the effects of model uncertainty, disturbances and noise.

I. General Control Systems



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- Example Disturbances:
  - → wind
  - → power surge
  - → load change
- Example Noise Sources:

  [these typically affect the output of the sensor]

  - → quantization / resolution
  - → stray electrical signals
  - → device physics
- Facts:
  - → You don't really know G
  - → Your measurement device is lying
  - → You often don't know what the disturbances will be

II. Sensitivity

Question: what is the overall effect on the system of a change in one block?

$$\text{Sensitivity} = \frac{\%\text{ change in system}}{\%\text{ change in part}}$$

$$= \frac{\Delta T / T}{\Delta G / G} = \frac{\Delta T}{\Delta G} \cdot \frac{G}{T}.$$

Take the limit:

$$\boxed{S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T}}$$

Note: Usually look at $0 \le |S_G^T| \le 1$

Example: Say $T = G$.

$$S_G^T = \frac{\partial T}{\partial G} \frac{G}{T} = \frac{\partial T}{\partial T} \cdot \frac{T}{T} = \boxed{1}.$$

Example:

$$\underbrace{\to \boxed{G_c} \to \boxed{G} \to}_{T = G_c G}$$

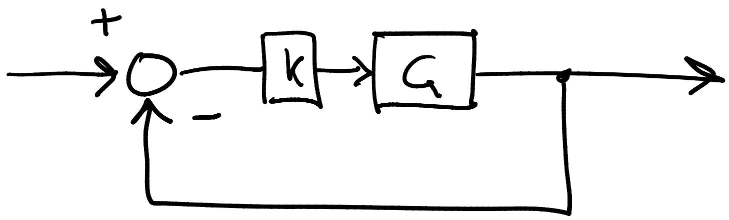
← this is called open-loop control

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{\partial G_c G}{\partial G} \cdot \frac{G}{G_c G}$$

$$= \cancel{G}_c \cdot \frac{G}{\cancel{G}_c G} = \boxed{1}.$$

Said differently, open loop control is terrible!

Example:



$$T = \frac{KG}{1+KG}$$

$$\boxed{\frac{gf' - g'f}{g^2}}$$

$$\frac{\partial T}{\partial G} = \frac{(1+KG)K - K^2 G}{(1+KG)^2} = \frac{K}{(1+KG)^2}$$

So, $\frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{\cancel{K}}{(1+KG)^{\cancel{2}}} \cdot \cancel{G} \cdot \frac{\cancel{1+KG}}{\cancel{K}\cancel{G}}$

$$= \boxed{\frac{1}{1+KG}}$$

Example: Say $G = 1/s$ in the above.

Then $S_G^T = \frac{1}{1+K/s} = \frac{s}{s+K}$

This can be made very small by choosing K to be big!

You can also look at the sensitivity with respect to a parameter:

<u>Example</u>: Say $T = \frac{1}{s+\underbrace{a}_{\text{parameter}}}$

Then

$$S_a^T = \frac{\partial T}{\partial a}\frac{a}{T} = \frac{1}{(s+a)^2} \cdot a \cdot \frac{s+a}{1}$$

$$= \frac{a}{s+a} \leftarrow \text{so if } a \text{ is big this goes to } 1$$

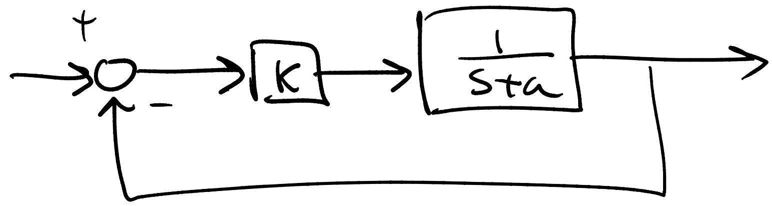
Another way to look at it: Say we have a unit step input. Then

$$Y(s) = \frac{1}{s} \cdot \frac{1}{s+a}$$

and $\quad y(\infty) = \lim_{s \to 0} sY(s) = \frac{1}{a}$

So the steady state is highly sensetive to changes in a.

Example:



$$T = \frac{K}{s+a+K}$$

$$S_a^T = \frac{-\cancel{K}}{(s+a+K)^{\cancel{2}}} \cdot a \cdot \frac{\cancel{s+a+K}}{\cancel{K}} = -\frac{a}{s+a+K}$$

This can be made very small by using a big K.

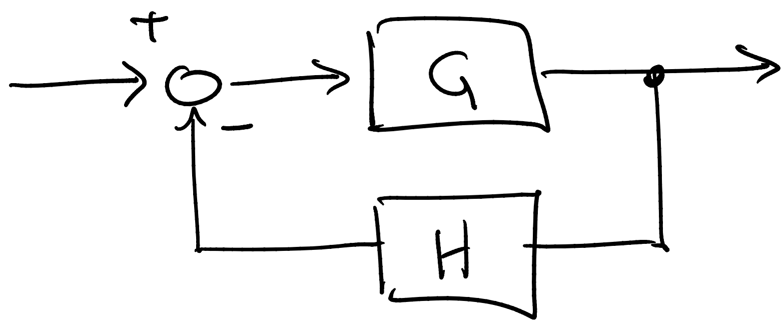
Another way to look at it: Say we have a unit step input. Then

$$Y(s) = \frac{1}{s} T(s) = \frac{1}{s} \cdot \frac{K}{s+a+K}$$

and $y(\infty) = \lim_{s \to 0} sY(s) = \frac{K}{a+K}$

which can be made very close to 1 and very insensitive to changes in a.

Example: Sensor Problems



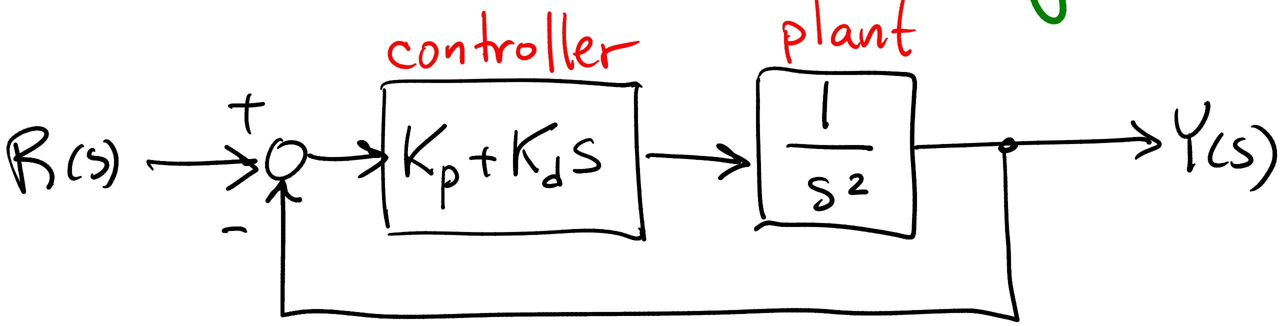
$$T = \frac{G}{1+GH}$$

$$S_H^T = \frac{-G^2}{(1+GH)^2} H \cdot \frac{\cancel{1+GH}}{\cancel{G}}$$

$$= -\frac{GH}{1+GH},$$

If GH is big then $|S_H^T| \approx 1$, and the system is highly sensitive to changes in H!

III. A More Detailed Analysis



$$T = \frac{K_p + K_d s}{s^2 + K_d s + K_p}$$

$$S_G^T = \frac{1}{1 + G_c G} = \frac{1}{1 + \frac{K_p + K_d s}{s^2}}$$

$$= \boxed{\frac{s^2}{s^2 + K_d s + K_p}}$$

We want $|S_G^T|$ to be small.

Let's take $s = j\omega$ and look at the sensitivity at different ω.

Then:

$$S_G^T(j\omega) = \frac{(j\omega)^2}{(j\omega)^2 + K_d j\omega + K_p}$$

$$= \frac{-\omega^2}{(K_p - \omega^2) + K_d \omega j}$$

As $\omega \to 0$, $S_G^T \to 0$. Good!

As $\omega \to \infty$,

$$\lim_{\omega \to \infty} S_G^T = \lim_{\omega \to \infty} \frac{-1}{\frac{K_p}{\omega^2} - 1 + \frac{K_d j}{\omega}} = \underline{1}.$$

If $K_d = 0$ (no damping term)

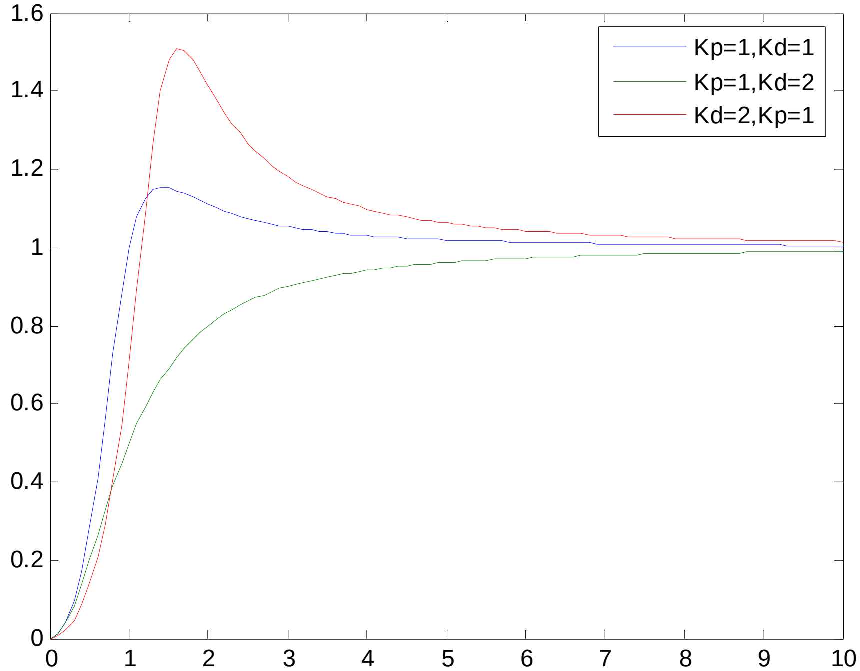
$$S_G^T = \frac{-\omega^2}{K_p - \omega^2}$$

which is ∞ when $\omega = \sqrt{K_p}$.

$\rightarrow$ So don't do this.

A plot of $|S_G^T|$ versus ω looks like

$|S_G^T|$ ↑



$\omega \rightarrow$

K_p bigger $\Rightarrow$ fast response, greater sensitivity

K_d bigger $\Rightarrow$ slower response, less sensitivity.