Lecture 13a

Monday, November 29, 2004
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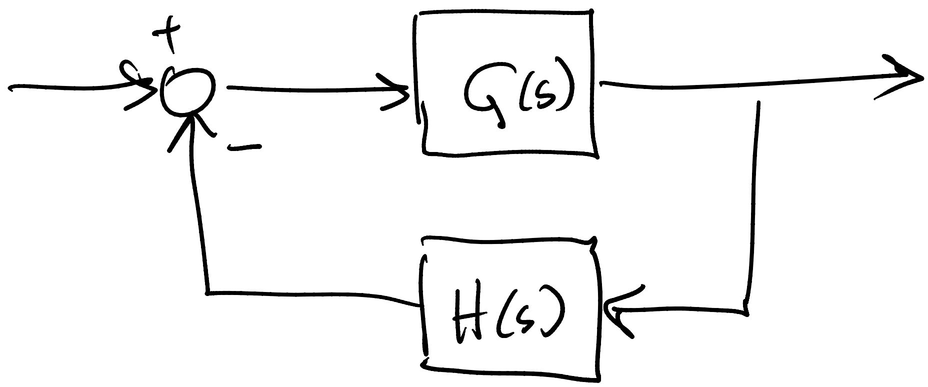
Lecture 13a : The Nyquist Criterion

...in which we relate the closed loop system to the open loop freq. response and open loop pole placement.



open loop

$GH(s)$

closed loop

$$T(s) = \frac{G(s)}{\underbrace{1+GH(s)}_{\text{char eqn.}}}$$

① Poles and zeros of $1+GH(s)$

Say $G(s) = \dfrac{p(s)}{q(s)}$

$$H(s) = \frac{n(s)}{d(s)}$$

Then

$$GH(s) = \frac{p(s)\,n(s)}{q(s)\,d(s)}$$

$$1+GH(s) = \frac{q(s)d(s) + p(s)n(s)}{q(s)d(s)}$$ and

$$T(s) = \frac{G(s)}{1+GH(s)} = \frac{p(s)\,n(s)}{q(s)d(s)+p(s)n(s)}$$

a) poles of $1+GH(s)$ = poles $GH(s)$ known

b) zeros of $1+GH(s)$ = poles of $T(s)$ unknown

② Note that a function $F(s)$ maps complex numbers to complex numbers. $F: \mathbb{C} \rightarrow \mathbb{C}$.

eg: $F(s) = \dfrac{1}{s+1}$

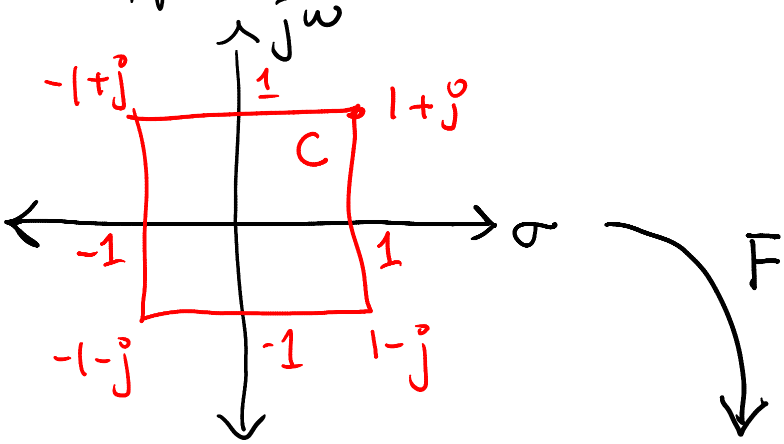
$$F(1+2j) = \frac{1}{1+2j+1} = \frac{2-2j}{8}$$

$$= \frac{1}{4} - \frac{1}{4}j.$$

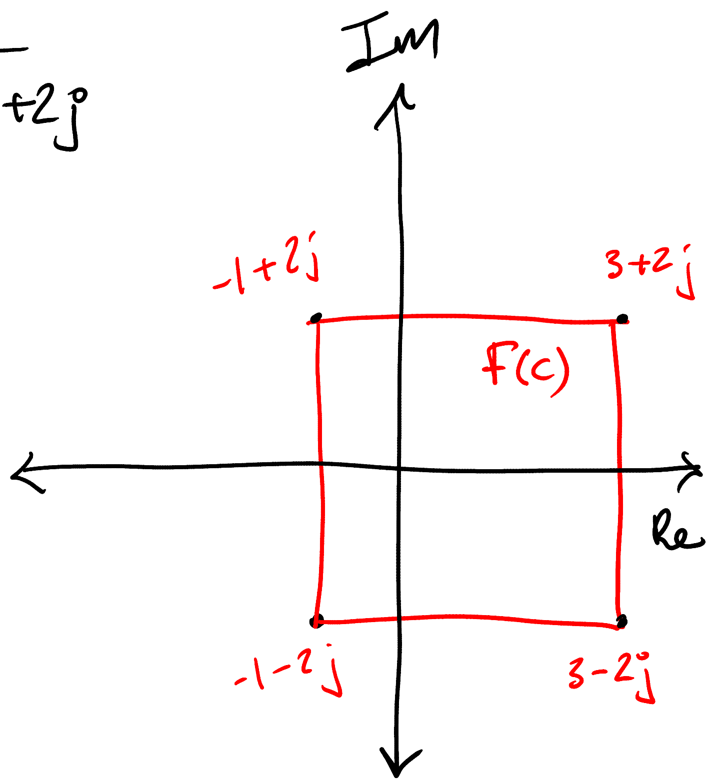
Therefore, If $c: \mathbb{R} \rightarrow \mathbb{C}$ is a contour (a curve in the complex plane), then F maps c to a new contour in the F-plane.

Ex: Say $F(s) = 2s+1$.

an suppose C is



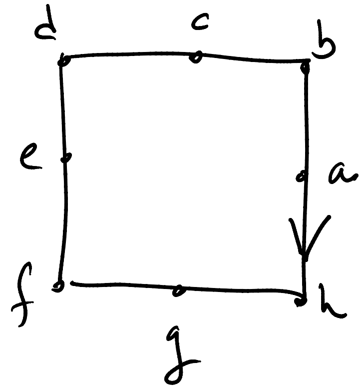
s	$F(s)$
$1+j$	$2+2j+1 = 3+2j$
$1-j$	$3-2j$
$-1+j$	$-1+2j$
$-1-j$	$-1-2j$

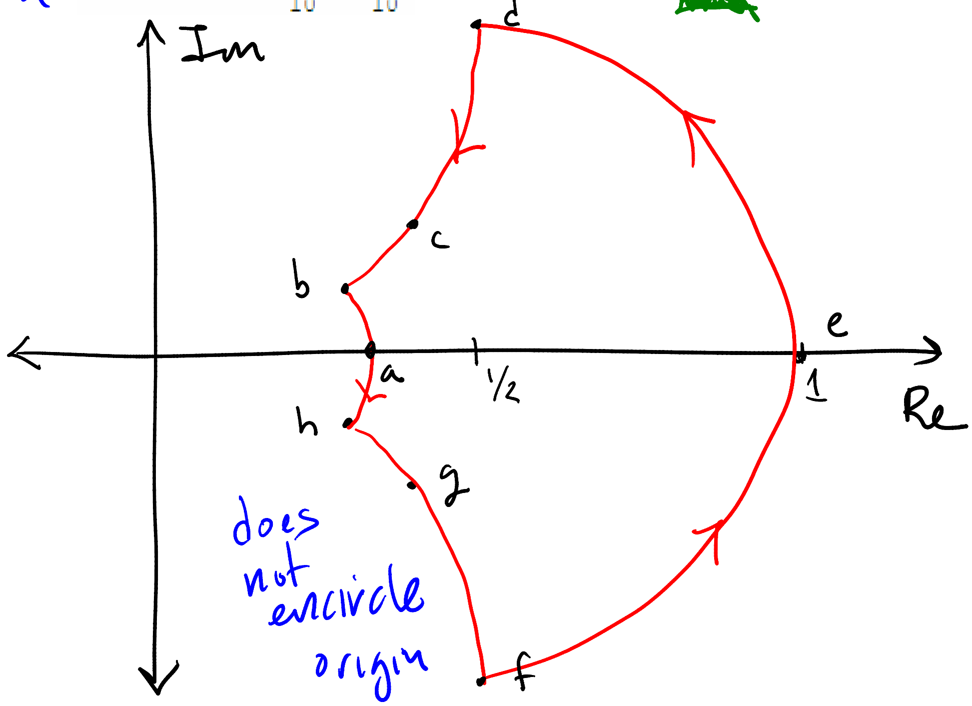


Ex: $F(s) = \dfrac{1}{s+2}$

	s	$F(s)$
a	1	$\frac{1}{3}$
b	$1 + i$	$\frac{3}{10} - \frac{i}{10}$
c	i	$\frac{2}{5} - \frac{i}{5}$
d	$-1 + i$	$\frac{1}{2} - \frac{i}{2}$
e	-1	1
f	$-1 - i$	$\frac{1}{2} + \frac{i}{2}$
g	$-i$	$\frac{2}{5} + \frac{i}{5}$
h	$1 - i$	$\frac{3}{10} + \frac{i}{10}$

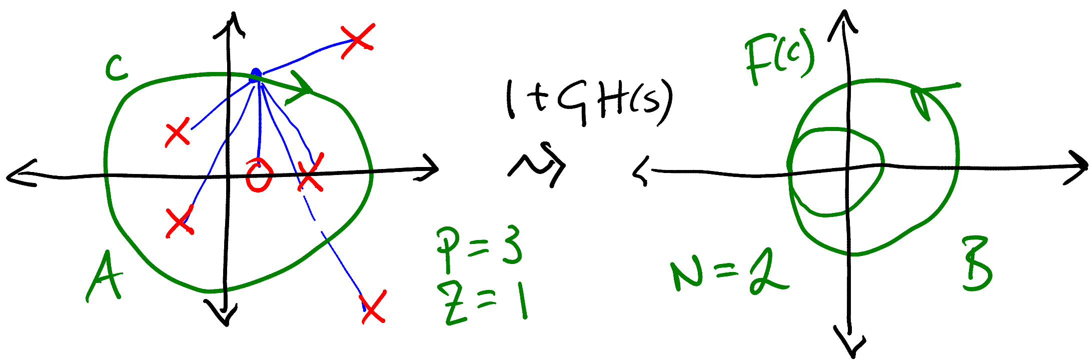
× pole at -2





does not encircle origin

We will consider $F(s) = 1 + GH(s)$



o → not known

x → known

$\angle F(s) = \sum \angle(s+z_i) - \sum \angle(s+p_i)$

If we map the contour c through $1+GH(s)$ then

- Each enclosed zero yields a clockwise rotation around the origin by contour B
- Each enclosed pole yields a counterclockwise encirclement around the origin by B
- poles and zeros outside A yield a net rotation of 0.

N = # of counter-clockwise rotations around 0 $= P - Z$

N = # of counterclockwise rotations of F(c)

P = poles inside c

Z = zeros inside c

So $\boxed{N = P - Z}$

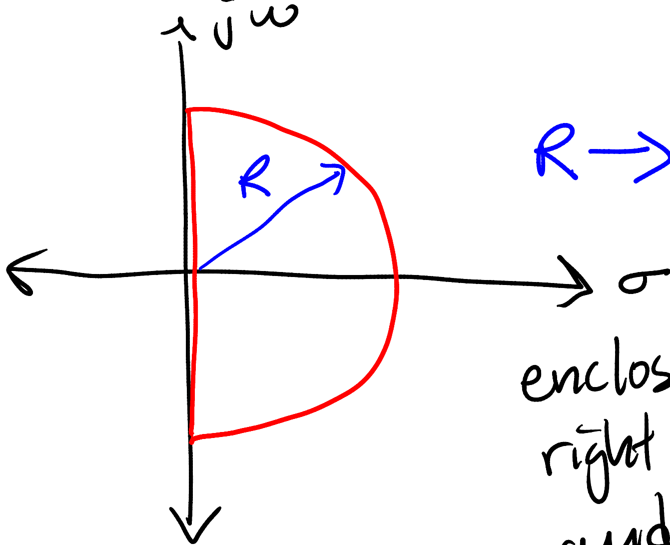
Recall,

poles $1+HG(s)$ = poles $HG(s)$ (known) ×

zeros $1+HG(s)$ = poles $T(s)$ (not known) o

so
P = # enclosed open loop poles
Z = # enclosed closed loop poles
= P − N ⟵ so we could compute Z !!

Consider the contour



$R \to \infty$

encloses the whole right half plane and the im. axis

If we want no right half plane poles for $T(s)$ then we must have $Z = P - N = 0$.
or $P = N$.

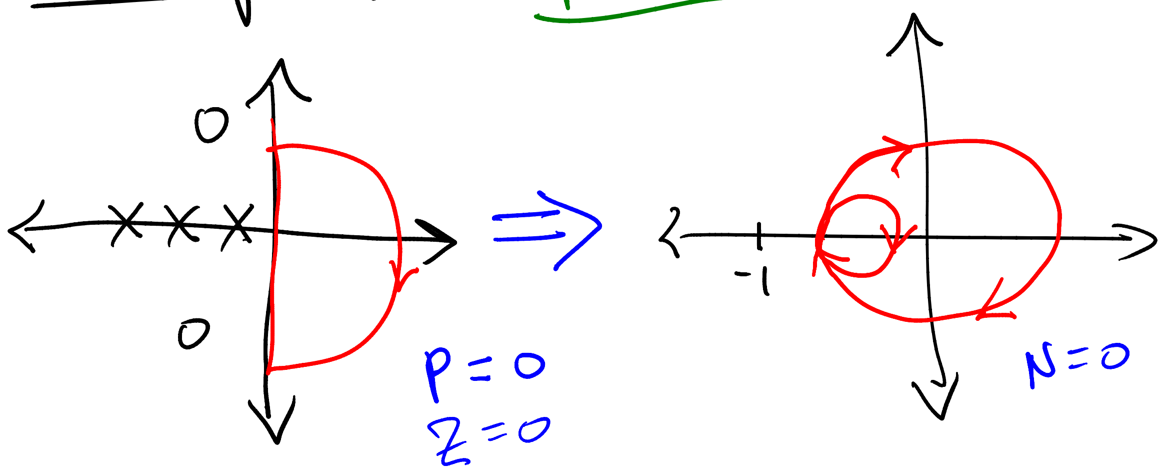
Easier: Put $F(s) = GH(s)$ instead of $1 + GH(s) \Rightarrow$ shift map to the left. Then we want N = # of encirclements of $-1 + 0j$.

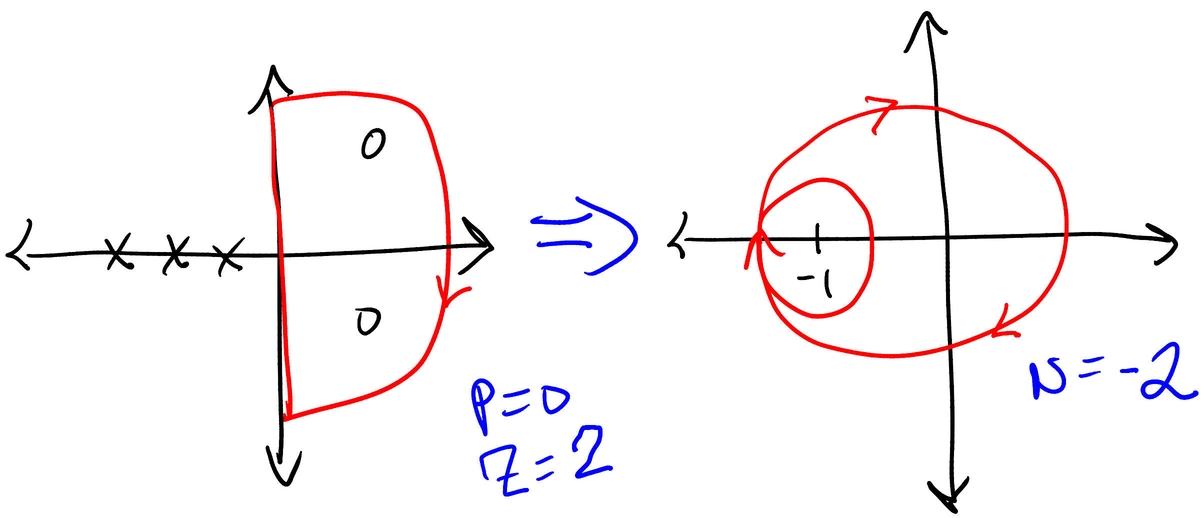
Nyquist Criterion:

- Let A be a contour that encircles the right half plane.
- Map A through $F(s) = GH(s)$ to get $F(A)$.
- The # of closed loop poles, Z, in the right half plane equals the # of open loop poles, P, that are in the right half plane minus the # of counterclockwise encirclements, N, of $-1+0j$ of the contour $F(A)$. $Z = P - N$

Examples:

$$\boxed{Z = P - N}$$





o = zeros of $1 + GH(s)$
= poles of $T(s)$ <u>unknown</u>

x = poles of $1 + GH(s)$
= poles of $GH(s)$ <u>known</u>

Example:

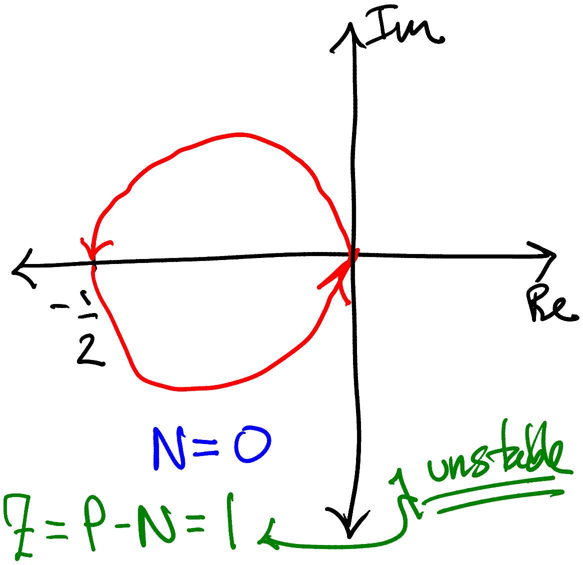
$$GH(s) = \frac{1}{s-2}$$

$P = 1$ = # of poles of $1+GH$
= # of poles of GH.

To plot a Nyquist plot, look at $GH(j\omega)$ for $-\infty < \omega < \infty$:

$$GH(j\omega) = \frac{1}{j\omega - 2} = \frac{-2 - j\omega}{\omega^2 + 4}$$

ω	$GH(j\omega)$
0	$-1/2$
1	$\frac{1}{5}(-2-j)$
2	$\frac{1}{8}(-2-2j)$
∞	0
$-\infty$	0
-1	$\frac{1}{5}(-2+j)$



$N = 0$

$Z = P - N = 1$ ← unstable

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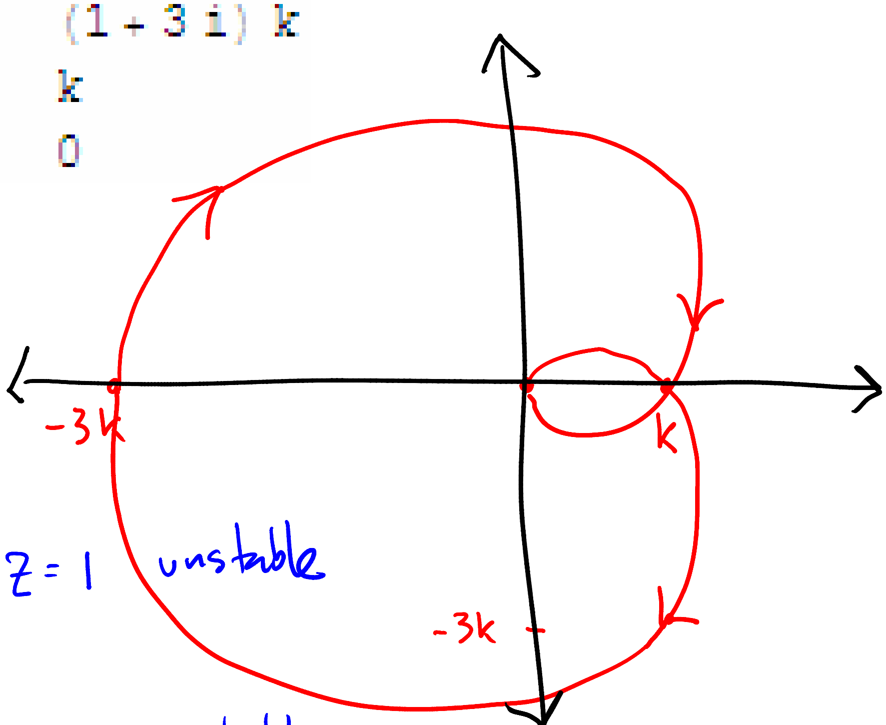
Example:

$$GH(s) = \frac{K(s-3)}{s^2+s+1} \leftarrow P=0$$

$$GH(j\omega) = \frac{K(j\omega-3)}{1-\omega^2+j\omega}$$

$-\infty$	0
-2	k
-1	(1 - 3 i) k
0	-3 k
1	(1 + 3 i) k
2	k
∞	0



$-3k < -1$:

$N = -1 \Rightarrow Z = 1$ unstable

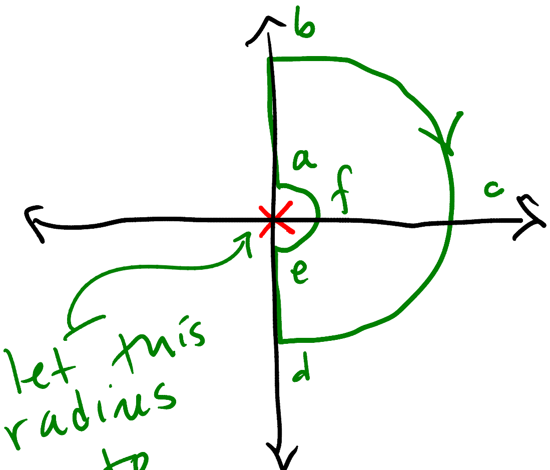
$-3k > -1$

$N = 0 \Rightarrow Z = 0$ stable

When $GH(s)$ has poles on the imaginary axis, the standard contour does not work. You need a modified contour:

Ex

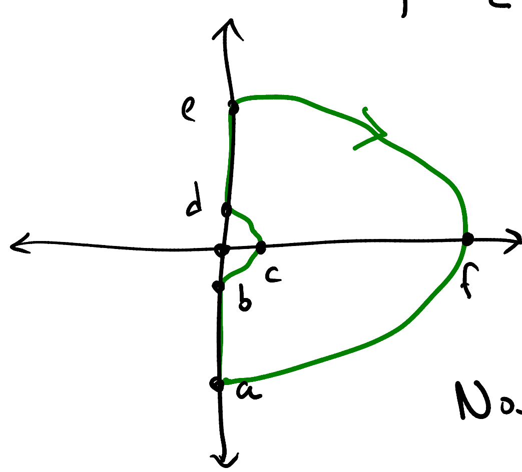
Say $G(s) = \frac{1}{s}$, $H(s) = 1$.



let this radius go to zero.

s	$G(s)$
$a = \varepsilon j$	$-\frac{1}{\varepsilon} j \to -\infty j$
$b = \infty j$	$-j/\infty \to 0$
$c = \infty$	$1/\infty \to 0$
$d = -\infty j$	$\frac{j}{\infty} \to 0$
$e = -\varepsilon j$	$\frac{1}{\varepsilon} j \to \infty j$
$f = \varepsilon$	$1/\varepsilon \to \infty$

$N = P - Z$

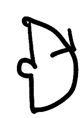


$P = 0$

$N = 0$

$\Downarrow$

$Z = P - N = 0$

Note:  ok too.