1. In $\mathbb{R}^3$, what is the minimum number of linear equations needed to describe a) a point b) a line c) a plane?

2. The intersection of two solution sets for two systems is the solution set for the system comprising the equations of both. What are the possible solution set shapes for the intersection of three planes? (Think about pivots.)

3. Write $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ as a linear combination of the following vectors (if possible): i) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
   iii) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ iv) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.   Sketch the span of each pair of vectors.

4. a) What are all the ways $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ can be written as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$?
   b) What are all the ways $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ can be?
   c) What similarity do these answers have?

5. The following matrix A does not have a pivot in every row. Find a vector $\vec{b}$ so that $[A \mid \vec{b}]$ is inconsistent.
$$A = \begin{bmatrix} 1 & -2 & -1 \\ 1 & 0 & 3 \\ 2 & -1 & 4 \end{bmatrix}.$$

6. Suppose A is an $n \times n$ matrix such that $A\vec{x} = \vec{b}$ is consistent no matter the $\vec{b} \in \mathbb{R}^n$. Can A have any free columns? If so, give an example.

7. What vectors are in $\text{Span}\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$.

Logic
1. Fact: if it is raining, there are puddles on the ground.
   i) If there are puddles, is it raining?
   ii) If it is not raining, are there puddles?
   iii) If there are no puddles, is it raining?

For ones where you can't tell, give plausible scenarios either way.