Discussion – Oct 10

1. For $\mathcal{B} = (1+x \quad 1-x)$, what is the coordinate rel. $\mathcal{B}$ of a) 1 b) x c) $2x+1$?
   d) is $\mathcal{B}$ a basis of $\mathbb{P}_1$?
2. What is the coordinate vector of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ from your point of view when you've rotated your head 45° counterclockwise?
3. What is the matrix of $T(\vec{x}) = \begin{bmatrix} 2x_1 \\ x_2 \end{bmatrix}$ from the same point of view as in 2?
4. The triangular lattice in $\mathbb{R}^2$ looks like ⋮⋱⋮⋱⋮ and models the packing of marbles in a tray.
   (a) What is a basis for it?
   (b) The lattice has 60° rotational symmetry. What is the matrix of 60° rotation relative to the basis in (a)?
   ((c) what other symmetries does the lattice have? Do linear transformations fail us?)
5. For $T: \mathbb{P}_2 \longrightarrow \mathbb{P}_3$ defined by multiplication by $(x-2)$,
   (a) what is the matrix of T relative to the standard polynomial bases?
   (b) Compute $T(x+2)$ by (i) definition (ii) the matrix in (a)
   (c) Compute $\operatorname{im} T$ and $\ker T$ using the matrix.
   (d) is T surjective and/or injective? (e) rank T = ?
6. $T: \mathbb{P}_1 \longrightarrow \mathbb{R}^2$ def. by $T(p(x)) = \begin{bmatrix} p(1) \\ p(2) \end{bmatrix}$
   $\mathcal{B} = (1 \quad x)$ $\mathcal{C} = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right)$. Matrix of T rel. these?
7. $T(\vec{x}) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \vec{x}$. Can you find a basis in which T's matrix is diagonal?