1. $\det(c\vec{u},\vec{v}) = c\det(\vec{u},\vec{v})$
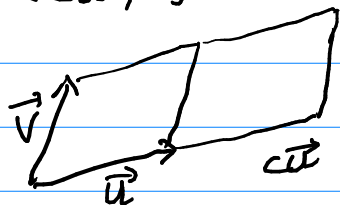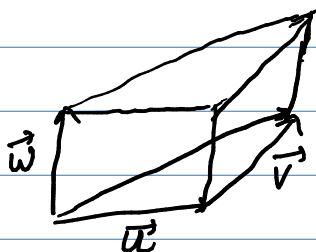
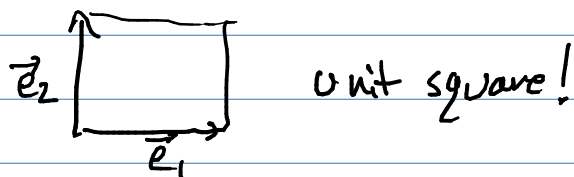$\vec{v}$ $\vec{u}$ $c\vec{u}$

2. $\det(\vec{u}+\vec{v},\vec{w}) = \det(\vec{u},\vec{w}) + \det(\vec{v},\vec{w})$

$\vec{w}$ $\vec{u}$ $\vec{v}$

3. same for second component

4. $\det(\vec{e_1},\vec{e_2}) = 1$

$\vec{e_2}$ unit square!

$\vec{e_1}$

5. $\det(\vec{u},\vec{v}) = -\det(\vec{v},\vec{u})$

$\vec{v}$ vs. $\vec{v}$

$\vec{u}$ $\vec{u}$
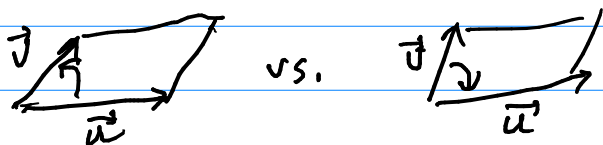
$\det\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \det\begin{pmatrix} a & c \\ 0 & d \end{pmatrix} + \det\begin{pmatrix} 0 & c \\ b & d \end{pmatrix}$

$= \det\begin{pmatrix} a & c \\ 0 & 0 \end{pmatrix} + \det\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} + \det\begin{pmatrix} 0 & c \\ b & 0 \end{pmatrix} + \det\begin{pmatrix} 0 & 0 \\ b & d \end{pmatrix}$

$= 0 + ad\det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + bc\det\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + 0$

$= ad - bc.$