Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
Dancing Slinky		Zhipeng Zhou

Dancing Slinky

Summary

This is a summary.

Keywords: keyword1, keyword2, keyword3

Contents

1 Introduction

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I love math.

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I love math.

Research Problem: When a slinky is twisted several times while keeping its bottom fixed, and then the top is released, a fascinating "dancing" phenomenon occurs. This wave-like motion, observable from the side-view, presents an intriguing physical problem that our research aims to explain by investigating the parameters affecting the slinky's motion.

1.1 Other Assumptions

There are other assumptions.

- This is a assumption.
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- This is a assumption.

2 Analysis of the Problem

Problem Description: Our research focuses on a specific physical phenomenon involving a slinky. When a slinky is twisted several times while its bottom end is kept fixed, and then the top end is suddenly released, the slinky begins to exhibit what can be described as a "dancing" motion. This wave-like phenomenon, particularly visible from a side view, demonstrates complex dynamics that arise from the conversion of torsional potential energy to various forms of mechanical energy.

The primary objectives of our research are:

- To explain the underlying physics of this dancing slinky phenomenon
- To identify and investigate the key parameters affecting the slinky's motion
- To develop a mathematical model that accurately describes the dynamics of the system

- To predict how variations in the initial conditions and physical properties of the slinky affect its subsequent motion

This problem combines elements of elastic mechanics, wave propagation, energy conversion, and non-linear dynamics, making it both theoretically interesting and experimentally accessible.

Figure 1: First example figure

This is Figure (1).

This is a cite[1].

$$E = mc^2 \tag{1}$$

$$E = mc^2$$

Figure 2: Second example figure

Figure 3: Third example figure

3 Preliminary Experiment

3.1 Experimental Materials and Setup

The primary material for the preliminary experiment is a standard Slinky. The approximate dimensions of the Slinky used are as follows:

- Outer Diameter: ≈ 76.36 mm
- Inner Diameter: ≈ 70.5 mm
- Height (uncompressed): ≈ 29.29 mm
- Cross-sectional wire diameter (thickness): ≈ 2.93 mm

Table 1: Caption

Title a	Title b	Title c	Title d
Aaa	Bbb	Ccc	Ddd
Aaa	Bbb	Ccc	Ddd
Aaa	Bbb	Ccc	Ddd

[Placeholder for Figure 2: Images showing Slinky measurements - Outer Diameter, Inner Diameter, Height, Cross-section length]

Figure 4: Slinky Dimensions

3.2 Experimental Procedure

The preliminary experiment was conducted using the following steps:

1. The bottom of the Slinky was fixed. This was achieved by using double-sided adhesive tape to attach it to a sheet of A4 paper, which was then secured to a flat surface.
2. The top of the Slinky was twisted by a varying number of turns. After stabilizing the Slinky, the top was released abruptly.
3. The subsequent motion and phenomena were observed.

[Placeholder for Figure 3: Image/video illustrating the experimental setup and procedure]

Figure 5: Experimental Setup and Procedure

3.3 Observed Phenomena and Initial Analysis

Several key phenomena were observed during the preliminary experiments:

- When the Slinky was twisted by a different number of turns and released, the wave-like motion observed from the side exhibited variations in wavelength, amplitude, frequency, and the time taken for the Slinky to return to its original state.
- A greater number of initial twists generally resulted in a more pronounced and observable phenomenon.
- The Slinky's final resting position often showed an angular displacement relative to its initial, untwisted orientation.
- If the number of twists exceeded a certain threshold, there was a risk of permanently deforming or damaging the Slinky.
- The twisting and subsequent unwinding motion of the Slinky appeared to be largely axially symmetric.

[Placeholder for Figure 4: Top-view images of the Slinky's motion - full process and detail]

Figure 6: Top-view of Slinky's Dancing Motion

3.4 Post-Experiment Reflections and Further Questions

The preliminary experiments led to several reflections and questions for further investigation:

- What is the minimum number of twists required to observe a distinct "dancing" effect?
- What are the precise physical mechanisms at play when the Slinky is twisted and released?
- What forms of energy are involved at the start of the motion, and how do these energies transform over time?
- Why does the Slinky exhibit a wave-like "dance"? What are the implications of this wave-like behavior?
- Can the frequency and amplitude of the observed waves be predicted, and how can they be influenced by changing initial parameters?
- Is there a quantifiable relationship between the initial number of twists and the characteristics of the resulting wave motion?

Further reflections on the nature of the "wave-like" dance: Initially, the focus was on the side-view "wave." It was noted that if viewed directly from one side, the Slinky appeared to exhibit a wave similar to a two-dimensional plane wave on that surface. However, observations from multiple angles revealed that a similar "wave" pattern was visible from all sides. This suggests the wave is not confined to a single plane but propagates along the Slinky's wire material. It was inferred that the "wave" likely propagates helically downwards along the Slinky wire.

Therefore, it is hypothesized that the side-view "wave" is a visual manifestation of the torsional (twisting) wave's projection as it travels along the Slinky.

[Placeholder for Figure 7: Image illustrating multi-angle observation or wave propagation concept]

Figure 7: Conceptualization of Wave Propagation

4 Theoretical Analysis

4.1 Introduction to the Phenomenon

The "dancing" phenomenon observed in a Slinky toy is understood as a result of the interplay between torsional waves and transverse vibrations. By altering the degree of twist, physical parameters of the Slinky, and the method of fixing, its vibrational behavior can be controlled and influenced. Understanding these factors is crucial for in-depth research into the wave characteristics and energy transfer mechanisms in elastic bodies.

4.2 Basic Assumptions

To develop a theoretical model for the Slinky's motion, the following basic assumptions are made:

1. The bottom end of the Slinky is fixed, and it is in its natural, unextended, and uncompressed state initially.
2. The Slinky has a uniform helical structure and is made of an elastic material (e.g., plastic).
3. Reflection effects of the waves at the ends are neglected for initial simplification, or handled by boundary conditions.
4. External forces (other than gravity, if considered) and boundary conditions are constant during the motion after release.
5. The motion includes damping effects, causing the wave amplitude to decrease over time. (This might be incorporated later or qualitatively discussed).

4.3 Parameter Definitions and Derivations

4.3.1 Initial Slinky Parameters (Untwisted State)

Let the initial parameters of the Slinky (before any twisting) be defined as follows:

- Radius: R_0
- Length: L_0
- Number of turns: N_0
- Pitch: $p_0 = L_0/N_0$
- Wire diameter: d
- Total length of the Slinky wire: $S = N_0\sqrt{(2\pi R_0)^2 + p_0^2}$
- Shear modulus of the material: G (describes the material's resistance to shear deformation)
- Density of the material: ρ

[Placeholder for Figure: Initial Slinky Parameters (Slide 15)]

Figure 8: Initial Parameters of the Slinky

4.3.2 Slinky Parameters After Twisting

When the Slinky is twisted by n additional turns, the total angle of twist is $\Theta = 2\pi n$. The parameters change as follows:

- New radius: R
- New length: L
- New total number of turns: $N = N_0 + n$ (assuming n is the number of *additional* twists causing the change in radius and length of the visible Slinky structure. The problem statement implies n is the total twist from the uncoiled state for the wire itself, which contributes to torsional stress.) More precisely, if n is the number of *full rotations* applied to the top end, this induces a torsional strain along the wire.
- New pitch: $p = L/N$
- Helix angle: α, such that $\tan\alpha = p/(2\pi R)$

[Placeholder for Figure: Slinky Parameters After Twisting (Slide 16)]

Figure 9: Slinky Parameters After Twisting and Helix Angle

4.3.3 Derivation of Radius and Length After Twisting

Assuming the total length S of the Slinky wire remains constant before and after twisting:

$$S = N_0\sqrt{(2\pi R_0)^2 + p_0^2} = N\sqrt{(2\pi R)^2 + p^2}$$

Using small angle approximations, where $\tan\alpha \approx \alpha$ and $\sec\alpha \approx 1$ (this is valid if the pitch is small compared to the circumference), we can simplify the relationship. The length of one turn of the wire is approximately $2\pi R\sec\alpha$. If the wire length remains constant:

$$N_0(2\pi R_0)\sec\alpha_0 \approx N(2\pi R)\sec\alpha$$

If $\sec\alpha_0 \approx 1$ and $\sec\alpha \approx 1$ (i.e., the helix angle is small, meaning the Slinky is relatively flat or tightly wound), then:

$$N_0R_0 \approx NR$$

Thus, the radius after twisting R can be expressed as:

$$R \approx \frac{N_0}{N}R_0 = \frac{N_0}{N_0+n}R_0$$

(Note: This derivation assumes $N_0 + n$ is the new effective number of coils for the radius calculation, which implies the twisting effectively adds or changes the coil structure. The provided slide indicates $N_0 R_0 = NR$ more directly from $N_0(2\pi R_0)\sec\alpha_0 \approx N(2\pi R)$ by cancelling 2π and assuming $\sec\alpha \approx 1$ for both states.)

The axial length of the Slinky L is given by $L = Np = N(2\pi R\tan\alpha)$. If using the approximation $\alpha \approx p/(2\pi R)$, then $L \approx N \cdot 2\pi R \cdot \frac{p}{2\pi R} = Np$ which is definitional. From the slide: $L = Np = N \cdot 2\pi R \tan\alpha \approx N \cdot 2\pi R\alpha$. If $\alpha \approx \frac{p}{2\pi R}$, then $L \approx N \cdot 2\pi R \frac{p}{2\pi R} = Np$. Another derivation from the slides suggests that the change in length is:

$$L = \frac{N}{N_0} L_0 = \frac{N_0 + n}{N_0} L_0 = \left(1 + \frac{n}{N_0}\right) L_0$$

This implies that as the Slinky is twisted ($n > 0$), its axial length L increases. This seems to assume that the pitch p effectively changes proportionally to $1/R$ while $L = Np$.

4.4 Torsional Load and Stiffness

The relationship between torsional load (torque T) and the twist angle $\Theta = 2\pi n$ for a helical spring is given by $T = k_t\Theta$, where k_t is the torsional stiffness. The torsional stiffness k_t is given by:

$$k_t = \frac{Gd^4}{64RN_{wire}}$$

Where N_{wire} is the number of active coils in the wire. For our purpose, let's assume $N_{wire} = N$ (the number of structural turns). So, $T = \frac{Gd^4}{64RN}(2\pi n) = \frac{\pi G d^4 n}{32RN}$. Since $R = \frac{N_0}{N_0+n} R_0$ and $N = N_0 + n$:

$$T = \frac{\pi G d^4 n}{32\left(\frac{N_0}{N_0+n} R_0\right)(N_0 + n)} = \frac{\pi G d^4 n (N_0 + n)}{32 N_0 R_0}$$

This indicates that the torsional load increases with the number of twists n.

4.5 Torsional Wave Equation and Wave Speed

The propagation of a torsional wave in the Slinky can be described by the wave equation:

$$\frac{\partial^2\theta(x,t)}{\partial t^2} = c_\theta^2 \frac{\partial^2\theta(x,t)}{\partial x^2}$$

where $\theta(x,t)$ is the twist angle at position x along the Slinky's axis at time t. The torsional wave speed c_θ is given by:

$$c_\theta = \sqrt{\frac{GJ}{\rho I_p}}$$

where:

- G is the shear modulus.
- J is the polar moment of inertia of the wire's cross-section: $J = \frac{\pi d^4}{32}$.

- ρ is the density of the Slinky material.
- I_p is the mass moment of inertia per unit length of the Slinky. From the slide, this seems to be $I = \rho A R^2$, where $A = \frac{\pi d^2}{4}$ is the cross-sectional area of the wire. So, $I = \rho \frac{\pi d^2}{4} R^2$.

Substituting these into the wave speed equation:

$$c_\theta = \sqrt{\frac{G\frac{\pi d^4}{32}}{\rho\frac{\pi d^2 R^2}{4}}} = \sqrt{\frac{Gd^2}{8\rho R^2}}$$

Since twisting causes the radius R to decrease (as $R = \frac{N_0}{N_0+n} R_0$):

$$c_\theta = \sqrt{\frac{Gd^2}{8\rho\left(\frac{N_0}{N_0+n}R_0\right)^2}} = \sqrt{\frac{Gd^2(N_0+n)^2}{8\rho N_0^2 R_0^2}} = \frac{N_0+n}{N_0 R_0}\sqrt{\frac{Gd^2}{8\rho}}$$

This suggests that the torsional wave speed c_θ increases as the number of twists n increases.

4.6 Relationship between Transverse Displacement and Twist Angle

Due to the helical structure of the Slinky, a change in the twist angle $\theta(x,t)$ can lead to a transverse displacement $y(x,t)$. This relationship is given by:

$$y(x,t) = R\theta(x,t)$$

4.7 Solution of the Wave Equation

We use the method of separation of variables. Let $\theta(x,t) = X(x)T(t)$. Substituting into the wave equation:

$$X(x)T''(t) = c_\theta^2 X''(x)T(t)$$

Separating variables:

$$\frac{T''(t)}{c_\theta^2 T(t)} = \frac{X''(x)}{X(x)} = -\lambda$$

where $-\lambda$ is the separation constant.

This gives two ordinary differential equations: 1. Spatial part: $X''(x) + \lambda X(x) = 0$ 2. Temporal part: $T''(t) + c_\theta^2 \lambda T(t) = 0$

4.7.1 Boundary Conditions and Spatial Solution

We define the boundary conditions:

- Fixed bottom end (at $x = 0$): $\theta(0,t) = 0 \Rightarrow X(0) = 0$

- Free top end (at $x = L$): Torque is zero. If torque is proportional to $\frac{\partial\theta}{\partial x}$, then $\frac{\partial\theta}{\partial x}\big|_{x=L} = 0 \Rightarrow X'(L) = 0$

For $X''(x) + \lambda X(x) = 0$ with $X(0) = 0$ and $X'(L) = 0$: The general solution is $X(x) = A\cos(\sqrt{\lambda}x) + B\sin(\sqrt{\lambda}x)$. From $X(0) = 0$, we get $A = 0$. So, $X(x) = B\sin(\sqrt{\lambda}x)$. From $X'(L) = 0$, we get $B\sqrt{\lambda}\cos(\sqrt{\lambda}L) = 0$. For non-trivial solutions ($B \neq 0$), $\cos(\sqrt{\lambda}L) = 0$. This implies $\sqrt{\lambda}L = (n - \frac{1}{2})\pi = \frac{(2n-1)\pi}{2}$ for $n = 1, 2, 3, \ldots$. So, the eigenvalues are $\lambda_n = \left(\frac{(2n-1)\pi}{2L}\right)^2$. The corresponding eigenfunctions are $X_n(x) = \sin\left(\frac{(2n-1)\pi x}{2L}\right)$.

4.7.2 Temporal Solution

The temporal equation is $T''(t) + c_\theta^2\lambda_n T(t) = 0$. Let $\omega_n^2 = c_\theta^2\lambda_n = c_\theta^2\left(\frac{(2n-1)\pi}{2L}\right)^2$. So, $\omega_n = c_\theta\frac{(2n-1)\pi}{2L}$ is the angular frequency. The solution is $T_n(t) = A_n\cos(\omega_n t) + B_n\sin(\omega_n t)$.

4.7.3 General Solution for Twist Angle

The general solution for $\theta(x, t)$ is a superposition:

$$\theta(x,t) = \sum_{n=1}^{\infty}[A_n\cos(\omega_n t) + B_n\sin(\omega_n t)]\sin\left(\frac{(2n-1)\pi x}{2L}\right)$$

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[Placeholder for Figure: Slinky Diagram for Wave Solution (Slide
24)]
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Figure 10: Slinky Parameters for Wave Solution

4.7.4 Initial Conditions and Determination of Coefficients

We need initial conditions:

- Initial twist angle: $\theta(x, 0) = \theta_0$ (assuming a uniform initial twist)
- Initial angular velocity: $\frac{\partial\theta}{\partial t}\big|_{t=0} = 0$

From $\frac{\partial\theta}{\partial t}\big|_{t=0} = 0$:

$$\sum_{n=1}^{\infty}[-\omega_n A_n\sin(0) + \omega_n B_n\cos(0)]\sin\left(\frac{(2n-1)\pi x}{2L}\right) = 0$$

$$\sum_{n=1}^{\infty}\omega_n B_n\sin\left(\frac{(2n-1)\pi x}{2L}\right) = 0$$

This implies $B_n = 0$ for all n.

Now, using $\theta(x,0) = \theta_0$:

$$\theta_0 = \sum_{n=1}^{\infty} A_n \cos(0) \sin\left(\frac{(2n-1)\pi x}{2L}\right) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{(2n-1)\pi x}{2L}\right)$$

To find A_n, we use Fourier series properties:

$$A_n = \frac{2}{L} \int_0^L \theta_0 \sin\left(\frac{(2n-1)\pi x}{2L}\right) dx$$

$$A_n = \frac{2\theta_0}{L} \left[-\frac{2L}{(2n-1)\pi} \cos\left(\frac{(2n-1)\pi x}{2L}\right)\right]_0^L$$

$$A_n = \frac{2\theta_0}{L} \left(-\frac{2L}{(2n-1)\pi} \cos\left(\frac{(2n-1)\pi}{2}\right) - \left(-\frac{2L}{(2n-1)\pi} \cos(0)\right)\right)$$

Since $\cos\left(\frac{(2n-1)\pi}{2}\right) = 0$ for integer n:

$$A_n = \frac{2\theta_0}{L} \left(0 + \frac{2L}{(2n-1)\pi}\right) = \frac{4\theta_0}{(2n-1)\pi}$$

4.8 Final Expressions and Summary

4.8.1 Final Expression for Twist Angle

Substituting A_n and $B_n = 0$ into the general solution:

$$\theta(x,t) = \sum_{n=1}^{\infty} \frac{4\theta_0}{(2n-1)\pi} \cos(\omega_n t) \sin\left(\frac{(2n-1)\pi x}{2L}\right)$$

where $\omega_n = c_\theta \frac{(2n-1)\pi}{2L}$.

4.8.2 Final Expression for Transverse Displacement

Using $y(x,t) = R\theta(x,t)$:

$$y(x,t) = R \sum_{n=1}^{\infty} \frac{4\theta_0}{(2n-1)\pi} \cos(\omega_n t) \sin\left(\frac{(2n-1)\pi x}{2L}\right)$$

4.8.3 Summary of Derived Relationships

In the derived model, the key parameters after twisting by n turns are related as follows (where n here is the number of additional twists that alter N_0 to $N_0 + n$ for radius/length calculation, and θ_0 is related to the initial total twist from equilibrium):

- Radius: $R = \frac{N_0}{N_0+n} R_0$

- Axial Length: $L = \left(1 + \frac{n}{N_0}\right) L_0$
- Torsional Wave Speed: $c_\theta = \sqrt{\frac{Gd^2(N_0+n)^2}{8\rho N_0^2 R_0^2}} = \frac{N_0+n}{N_0 R_0}\sqrt{\frac{Gd^2}{8\rho}}$

When the Slinky is twisted multiple times and the top is released, the release of torsional load generates a torsional wave that propagates through the Slinky. Through the helical structure of the Slinky, this torsional wave is converted into transverse vibration, creating the observed "dancing" phenomenon.

5 Numerical Simulation and Model Validation

To further investigate the behavior of the Slinky and validate the derived theoretical model, a numerical simulation was performed using MATLAB.

5.1 Simulation Setup

The simulation incorporated the initial conditions as determined by our theoretical analysis. Crucially, a damping term was introduced into the model. This damping accounts for various energy dissipation mechanisms inherent in the physical system, such as:

- Energy loss along the propagation distance of the wave.
- Internal friction within the Slinky material.
- Air resistance acting on the moving Slinky.

The specific damping coefficient used in the simulation would be calibrated based on these factors or fitted from experimental data for more precise results.

5.2 Simulation of Torsional Wave Propagation

By inputting the derived parameters and the damping function, the simulation generated a visual representation of the torsional wave's propagation over time. An example of the simulated Slinky dynamics is shown below.

```
[Placeholder for Figure: MATLAB Simulation of Slinky Dynamics
(e.g., at Time = 2.42s)]
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Figure 11: MATLAB Simulation of Torsional Wave Propagation in the Slinky (Illustrative).

5.3 Validation Against Observations

The mathematical model developed based on theoretical principles was implemented in MATLAB. The resulting simulations demonstrate that the predicted phenomena are largely consistent with the actual behavior observed in preliminary experiments. This congruence between the simulated motion and physical observations suggests that the underlying theoretical framework and its derivations are fundamentally correct and accurately capture the essential dynamics of the dancing Slinky.

6 Experimental Investigation

6.1 Experimental Objectives

To investigate the influencing factors discovered in preliminary experiments and their relationship with the Slinky's motion, a series of experiments were designed for detailed exploration and theoretical validation. To better quantify the relationship, the number of Slinky twists was precisely controlled and investigated as the primary variable, referred to as the Slinky's twist angle.

Experimental Series 1: Investigating the relationship between wave characteristics (amplitude, wavelength, radius) and the Slinky's twist angle.

- Experiment 1: Amplitude - Time - Twist Angle - Length.
- Experiment 2: Wavelength - Time - Twist Angle - Length.
- Experiment 3: Radius - Twist Angle - Length.

Supplementary Experiment: Observing the Slinky's behavior under lubricated conditions.

Experimental Series 2: Building upon Series 1 to conduct further investigations related to the theoretical hypotheses of the twist angle, aiming for mutual verification and confirming experimental accuracy.

6.2 Experimental Apparatus

The following materials and equipment were used for the experiments (see Figure 12):

- Slinky toy
- 3D Printed Stand/Fixture
- Double-sided adhesive tape (for fixing the Slinky base)
- 360-degree high-precision servo motor (as the power source for twisting the Slinky, ensuring repeatable and precise twist angles for improved data credibility)
- Clothes pegs/clips (to hold the top of the Slinky)
- Square-gridded graph paper (for visual reference and scaling)
- Servo motor drive board
- Battery and associated circuit board
- Sony FX30 Camera (for recording the motion)

The servo motor is controlled by PC software, allowing for precise setting of the rotation angle (see Figure 13). The servo motor uses an absolute encoder and supports multi-turn positioning. It divides 360 degrees into 4095 steps, enabling fine control over the twist angle.

[Placeholder for Figure: Overall Experimental Apparatus (Slide 4.1)]

Figure 12: Experimental Apparatus Setup.

[Placeholder for Figure: Servo Motor Control Software Interface (Slide 4.1)]

Figure 13: Servo Motor Control Software Interface.

6.3 Experimental Procedure

The basic experimental steps were generally consistent across different tests (see Figure 14):

1. The Slinky was placed on the 3D printed fixture, ensuring its center was aligned with the rotation axis.
2. Markings were made on the Slinky's surface along its axis for tracking purposes.
3. The height of the clothes peg/clip holding the top of the Slinky was adjusted to match the Slinky's height, ensuring it gripped the top firmly.
4. Different twist angles were applied by rotating the Slinky using the servo motor.
5. The resulting phenomena and motion were observed and recorded.

Measurements in the experiment, such as pixel-based scale determination, were based on these setup images.

[Placeholder for Figure: Illustrations of Experimental Steps (Slide 4.2)]

Figure 14: Experimental Procedure Steps.

6.4 Material Parameters and Data Acquisition

6.4.1 Material Properties Lookup

Commonly used engineering material properties were referenced to inform the model. The table below shows some examples of material properties considered.

Specific parameters for materials similar to those used in Slinkys (e.g., PP Copolymer and 65Mn Spring Steel) were also compiled, as shown below.

`[Placeholder for Figure: Table of Material Properties (Slide 4.3, top part)]`

Figure 15: Reference Material Properties.

`[Placeholder for Figure: Table of Slinky Material Parameters (PP Copolymer, 65Mn Steel) (Slide 4.3, bottom part)]`

Figure 16: Example Slinky Material Parameters.

6.4.2 Data Acquisition Method

To accurately quantify the experimental results, the following data acquisition method was employed (see Figure 17):

- A high-speed camera (Sony FX30) was used to record videos of the Slinky's motion.
- Video editing software (e.g., DaVinci Resolve) was used to extract individual frames for analysis.
- High-resolution images (e.g., 3840x2160 pixels) with their inherent coordinate systems were used for pixel-based measurements of lengths and other parameters.
- These pixel measurements were converted to real-world lengths using a predetermined scale factor (e.g., a scale factor of 0.081 mm/pixel was determined for the setup).

`[Placeholder for Figure: Data Acquisition Method - Frame Extraction and Pixel Measurement (Slide 4.2)]`

Figure 17: Data Acquisition and Measurement Process.

6.5 Experimental Series 1: Investigating Influencing Factors

6.5.1 Experiment 1: Amplitude - Time - Twist Angle - Length

This experiment focused on observing how the amplitude of the Slinky's wave-like motion changes over time for different initial twist angles and Slinky lengths.

`[Placeholder for Figure: Exp 1 - Slinky motion at 360, 1080, 1800 degrees (Slide 4.2)]`

Figure 18: Experiment 1: Slinky Amplitude at Various Twist Angles.

`[Placeholder for Figure: Exp 1 - Data Indication and Sample Data Table (Slide 4.2)]`

Figure 19: Experiment 1: Amplitude Data Measurement and Sample Data.

6.5.2 Experiment 2: Wavelength - Time - Twist Angle - Length

This experiment aimed to understand the relationship between the observed wavelength, time, initial twist angle, and Slinky length.

Experimental Data Trends Analysis:

- As the initial twist angle increases, the initial wavelength of the Slinky tends to decrease.
- The wavelength observed near the top of the Slinky (closer to the point of release) is generally smaller, or alternatively, the wavelength increases as the wave propagates downwards along the Slinky.

```
[Placeholder for Figure: Exp 2 - Data Indication, Sample Data Table, and Trends (Slide 4.2)]
```

Figure 20: Experiment 2: Wavelength Data Measurement and Trends.

6.5.3 Experiment 3: Radius - Twist Angle - Length

This experiment measured the change in the Slinky's radius as a function of the applied twist angle. The experimental data was compared against the theoretically derived relationship $R = \frac{N_0}{N_0+n} R_0$.

Experimental Data Analysis: The observed results for the change in radius were found to be consistent with the theoretical calculations.

```
[Placeholder for Figure: Exp 3 - Radius vs. Twist Angle Data Table and Formula (Slide 4.2)]
```

Figure 21: Experiment 3: Slinky Radius vs. Twist Angle - Data and Theoretical Comparison.

6.6 Supplementary Experiment: Effect of Lubrication

Observations were made on the Slinky's motion when lubrication was applied between its coils.

Experimental Data Analysis:

- When inter-ring friction is reduced (e.g., by lubrication), the "dancing" phenomenon can become more pronounced, provided other factors are optimal.
- However, the effects of lubrication can be difficult to control. Excessive or viscous lubrication might introduce new damping effects or alter the coil interactions, potentially making the phenomenon less clear or changing its characteristics.

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[Placeholder for Figure:  Supplementary Experiment - Lubricated
        Slinky (failed, 1440, 1530 deg) (Slide 4.2)]
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Figure 22: Supplementary Experiment: Observations with Lubricated Slinky.

6.7 Experimental Series 2: Theoretical Validation (Further Work)

This series of experiments is designed to build upon the findings from Series 1. It involves more targeted investigations to further validate and refine the theoretical hypotheses concerning the twist angle and its impact on the Slinky's dynamics. The goal is to achieve a robust mutual verification between experimental results and theoretical predictions.

7 Calculating and Simplifying the Model

8 The Model Results

9 Validating the Model

10 Conclusions

11 Summary

12 Evaluate of the Mode

13 Strengths and weaknesses

13.1 Strengths

References

[1] A. Vaswani et al., "Attention is all you need," *Advances in Neural Information Processing Systems*, vol. 30, pp. 5998–6008, 2017.

Appendices

MEMORANDUM

To: MCM office

From: MCM Team 12345678

Subject: MCM

Date: May 20, 2025

This is a memorandum.

Appendix A First appendix

Here are simulation programmes we used in our model as follow.
MATLAB source code:

```
% MATLAB example code for MCM/ICM
function [output] = example_function(input)
    % This is a simple example function
    output = input.^2 + 2*input + 1;

    % Plot the results
    x = -10:0.1:10;
    y = x.^2 + 2*x + 1;
    plot(x, y);
    title('Example Function: f(x) = x^2 + 2x + 1');
    xlabel('x');
    ylabel('f(x)');
end

% Main script example
x = -5:5;
y = example_function(x);
disp('Results:');
disp([x; y]');
```

Appendix B Second appendix

Python source code:

```
#!/usr/bin/env python
# -*- coding: utf-8 -*-

import numpy as np
import matplotlib.pyplot as plt

def example_function(x):
    """
    A simple example function for demonstration
    f(x) = x^2 + 2x + 1
    """
    return x**2 + 2*x + 1

# Main script
if __name__ == "__main__":
    # Generate data
    x = np.linspace(-10, 10, 100)
    y = example_function(x)

    # Plot results
    plt.figure(figsize=(10, 6))
    plt.plot(x, y, 'b-', linewidth=2)
    plt.grid(True)
    plt.title("Example Function: f(x) = x^2 + 2x + 1")
    plt.xlabel("x")
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plt.ylabel("f(x)")

# Print some sample values
sample_x = np.array([-5, -2, 0, 2, 5])
sample_y = example_function(sample_x)

print("Sample Values:")
for i, j in zip(sample_x, sample_y):
    print(f"f({i}) = {j}")
```