CS6700 | Reinforcement Learning | Assignment 2

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0.0.1 Avinash G. Kori | ED15B006

```
In [1]: import numpy as np
        import seaborn as sns
        import matplotlib.pyplot as plt

        def imshow(args):
            if len(args)==1:
                plt.imshow(args[0], interpolation='none')
            else:
                n=int(len(args)**0.5)
                plt.figure(figsize=(15, 15))
                for i in range(len(args)):
                    plt.subplot(n + 1, n,i+1)
                    plt.imshow(args[i])
            plt.show()
```

1 Question 1

```
In [2]: class Question1(object):
            def __init__(self):
                """
                """
                # define probabilities
                self.P = np.array([
                        [[0.5   , 0.25 , 0.250 ],
                         [1./16., 3./4., 3./16.],
                         [1./4. , 1./8., 5./8. ]],
                        [[1./2. , 0.   , 1./2. ],
                         [1./16. , 7./8., 1./16.],
                         [0.    ,   1. , 0     ]],
                        [[1./4. , 1./4., 1./2. ],
                         [1./8. , 3./4., 1./8. ],
                         [3./4. ,1./16., 3./16.]]
                    ])
                # define rewards
                self.g = np.array([
```

```python
                [[10., 4., 8.],
                 [8.,  2., 4.],
                 [4.,  6., 4.]],
                [[14., 0., 18.],
                 [8., 16., 08.],
                 [0.,  0., 0. ]],
                [[10., 2., 8. ],
                 [6. , 4., 2. ],
                 [4. , 0., 8.]]
            ])
        # init J
        self.J = np.array([[0., 0., 0.]]).T # 3x1

    def Toperator(self):
        """
            Applies T operator for current J
        """
        self.J = np.max(np.sum(self.P*(self.g + self.J.reshape(1, 1, 3)), \
                                      axis=2), axis=1).reshape(3, 1)
        return self.J

    def optPolicy(self):
        """
            Finds optimal policy for current states
        """
        self.optP = (np.argmax(np.sum(self.P*(self.g + self.J.reshape(1, 1, 3)), \
                                     axis=2), axis=1)+1).reshape(3, 1)
        return self.optP

    def Iterate(self, N = 1000, display_ = True):
        """
            Input Args:
                N: number of iterations
                display: bool for displaying plots
            returns:
                J: optimal J
                P: optimal policy
        """
        # TODO: display fn
        cost_hist, policy_hist = [], []
        for _ in range(N):
            cost_hist.append(self.Toperator().reshape(3))
            policy_hist.append(self.optPolicy().reshape(3))
        return self.J.reshape(1,3)[0], self.optP.reshape(1,3)[0]


question1 = Question1()
cost10_, policy10_ = question1.Iterate(N = 10)
```

```
question1 = Question1()
cost20_, policy20_ = question1.Iterate(N = 20)
```

1.0.1 Q1.a Optimal Cost after 10 and 20 iteration

```
In [3]: print ("Optimal Cost after N = 10:  {} \n".format(cost10_) +\
               "Optimal Cost after N = 20:  {}".format(cost20_) + '\n')
```

```
Optimal Cost after N = 10:  [123.01726578 136.84919849 124.19373637]
Optimal Cost after N = 20:  [256.46264388 270.29457665 257.63911447]
```

1.0.2 Q1.b Optimal Policy after 10 and 20 iteration

```
In [4]: print ("Optimal Policy after N = 10:  {} \n".format(list(policy10_)) +\
               "Optimal Policy after N = 20:  {}".format(list(policy20_)) + '\n')
```

```
Optimal Policy after N = 10:  [2, 2, 2]
Optimal Policy after N = 20:  [2, 2, 2]
```

1.0.3 Q1.c

As the highest reward is associated with town B and action 2, and highest transition probability from any town to town B is also associated with action 2.

This is the reason why taking action 2 (Go to the nearest taxi stand and wait in line) irrespective of state is optimal policy

2 Question 2

3 Action mapping

3.0.1 0 = (↑, violet) up, 1 = (→, blue) right, 2 = (↓, green) down, 3 = (←, yellow) left

```
In [5]: class Question2(object):
            def __init__(self, variant = 1):
                """
                    variant: variant of grid world
                    variant = 1 (terminal state at (3, 0))
                    variant = 2 (terminal state at (9, 9))
                """
                self.variant = variant
                # init Probabilities
                self.P = np.zeros((10, 10, 4, 10, 10))
                self.J = np.zeros((1, 10, 10, 1))
```

```python
        # init all rewards with -1
        self.g = np.zeros((10, 10, 4, 10, 10)) - 1
        # generate Probabilities
        self.P = self.generateP()
        # generate Rewards
        self.g = self.generateR()

    def generateP(self):
        """
            Generates and returns P matrix
            5th order Tensor
        """
        for ix in range(self.P.shape[0]):
            for iy in range(self.P.shape[1]):
                for action in range(self.P.shape[2]):
                    temp = np.zeros((10, 10))
                    if action == 0:
                        temp[ix, min(iy + 1, 9)] = 0.8
                        temp[max(0, ix - 1), iy] = 0.1
                        temp[min(ix + 1, 9), iy] = 0.1
                    elif action == 1:
                        temp[ix, min(iy + 1, 9)] = 0.1
                        temp[ix, max(iy - 1, 0)] = 0.1
                        temp[min(ix + 1, 9), iy] = 0.8
                    elif action == 2:
                        temp[ix, max(iy - 1, 0)] = 0.8
                        temp[min(ix + 1, 9), iy] = 0.1
                        temp[max(ix - 1, 0), iy] = 0.1
                    else:
                        temp[max(ix - 1, 0), iy] = 0.8
                        temp[ix, min(iy + 1, 9)] = 0.1
                        temp[ix, max(iy - 1, 0)] = 0.1

                    if (ix, iy) == (3, 2) or (ix, iy) == (4, 2)\
                      or (ix, iy) == (5, 2) or (ix, iy) == (6, 2):
                        temp = np.zeros((10, 10))
                        temp[0, 0] = 1

                    if (ix, iy) == (7, 1):
                        temp = np.zeros((10, 10))
                        temp[7, 9] = 1

                    if ((ix, iy) == (3, 0) and self.variant == 1) \
                       or (self.variant == 2 and (ix, iy) == (9, 9)):
                        temp = np.zeros((10, 10))

                    self.P[ix, iy, action] = temp
        return self.P
```

```python
def generateR(self):
    """
        Generates and returns R matrix
        5th order Tensor
    """
    for ix in range(self.P.shape[0]):
        for iy in range(self.P.shape[1]):
            for action in range(self.P.shape[2]):
                if self.variant == 2 and \
                ((ix, iy, action) == (8, 9, 1) or \
                    (ix, iy, action) == (9, 8, 0)):
                    self.g[ix, iy, action, 9, 9] = 100

                if self.variant == 1 and \
                 ((ix, iy, action) == (2, 0, 1) or \
                    (ix, iy, action) == (3, 1, 2)\
                    or(ix, iy, action) == (4, 0, 3)):
                    self.g[ix, iy, action, 3, 0] = 100
    return self.g

def Toperator(self):
    """
        Applies T operator for current J
    """
    self.J = np.max(np.sum(self.P*(self.g +\
                            self.J.reshape(1, 1, 1, 10, 10)),\
                           axis=(3, 4)), axis=2).reshape(10, 10)
    return self.J

def optPolicy(self):
    """
        Finds optimal policy for current states
    """
    self.optP = np.argmax(np.sum(self.P*(self.g +\
                                    self.J.reshape(1, 1, 1, 10, 10)),\
                                     axis=(3, 4)), axis=2).reshape(10, 10)
    return self.optP

def Iterate(self, N = 1000, display_ = True):
    """
        Input Args:
            N: number of iterations
            display: bool for displaying plots
        returns:
            J: optimal J
            P: optimal policy
            cost_history from 0 to N
```

```
            policy_history from 0 to N
        """
        cost_hist, policy_hist = [], []
        for _ in range(N):
            cost_hist.append(np.rot90(self.Toperator().reshape(10, 10)))
            policy_hist.append(self.optPolicy().reshape(10, 10))
        if display_: imshow(cost_hist)
        return (list(self.J.reshape(10, 10)), list(self.optP.reshape(10,10)),
                cost_hist, policy_hist)
```

3.0.2 Q2.a

- Stopping criteria is decided based on change in value of J from previous iteration
- $(J_{i+1} - J_i)$ < T (T is threshold for convergence)

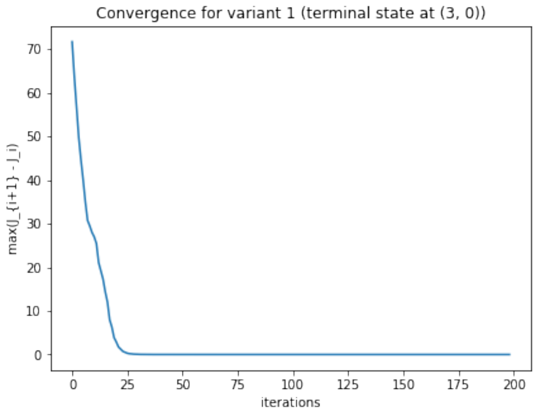
3.0.3 Q2.b

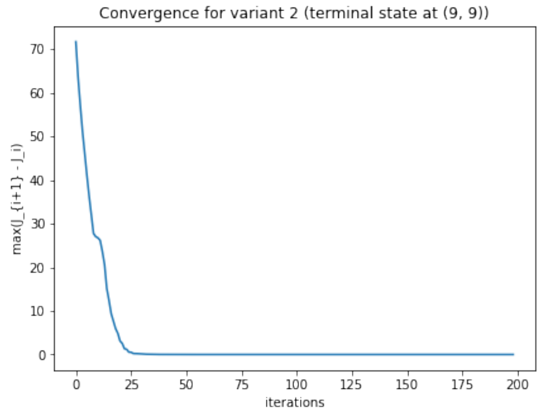
```
In [6]: question2 = Question2()
        j, p, Chist, Phist = question2.Iterate(N=200, display_ = False)

        diff_hist = np.diff(np.array(Chist), axis = 0)
        plt.figure(figsize = (15, 5))
        plt.subplot(1, 2, 1)
        plt.plot(np.max(diff_hist, axis = (1,2)))
        plt.title('Convergence for variant 1 (terminal state at (3, 0))')
        plt.xlabel('iterations')
        plt.ylabel('max(J_{i+1} - J_i)')

        question2 = Question2(2)
        j, p, Chist, Phist = question2.Iterate(N=200, display_ = False)

        diff_hist = np.diff(np.array(Chist), axis = 0)
        plt.subplot(1, 2, 2)
        plt.plot(np.max(diff_hist, axis = (1,2)))
        plt.title('Convergence for variant 2 (terminal state at (9, 9))')
        plt.xlabel('iterations')
        plt.ylabel('max(J_{i+1} - J_i)')
        plt.show()
```





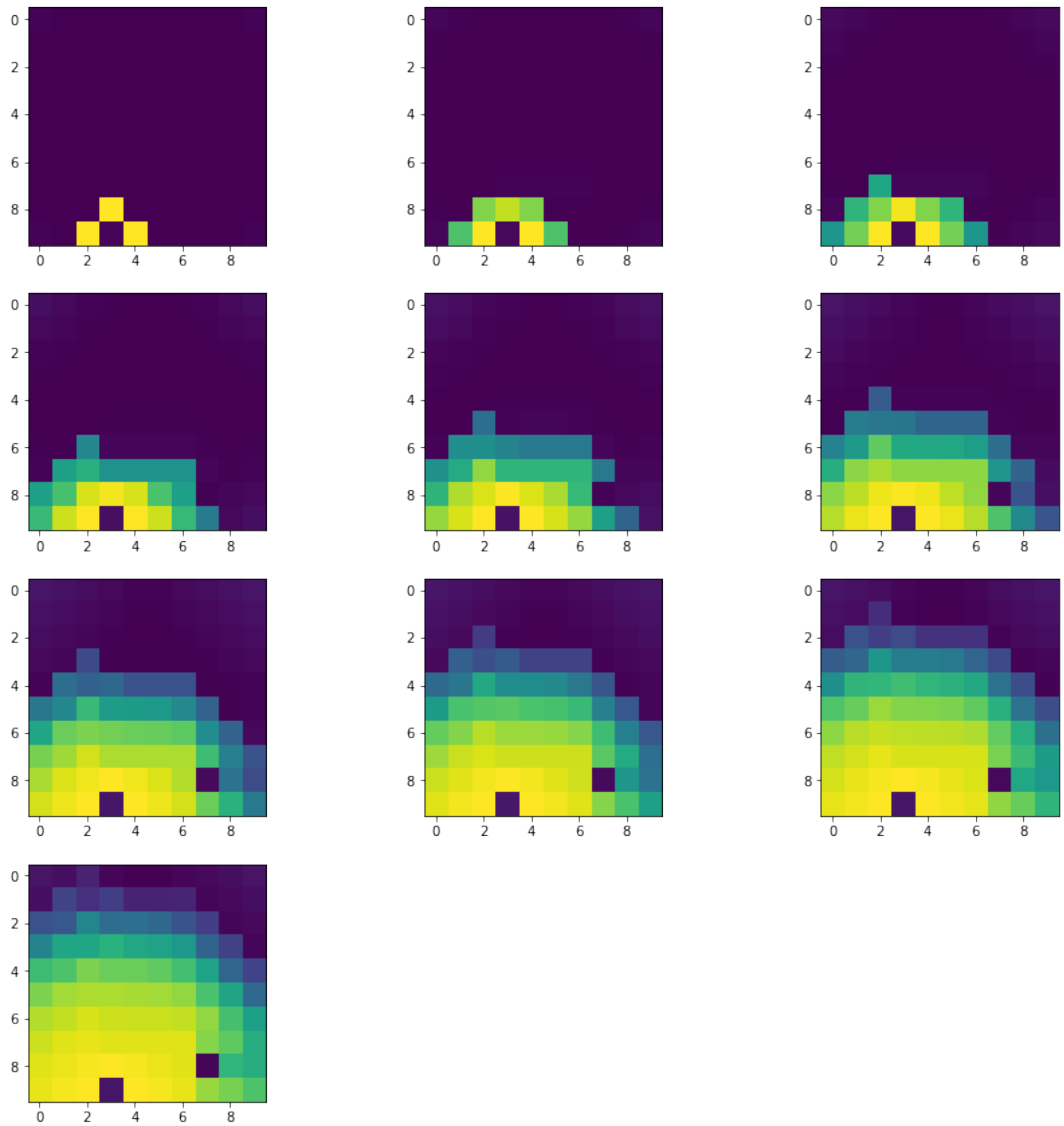
3.0.4 Q2.c

- Based on above plots convergence for Variant 1 can be considered around 25th iteration and for Variant 2 around 25th iteration

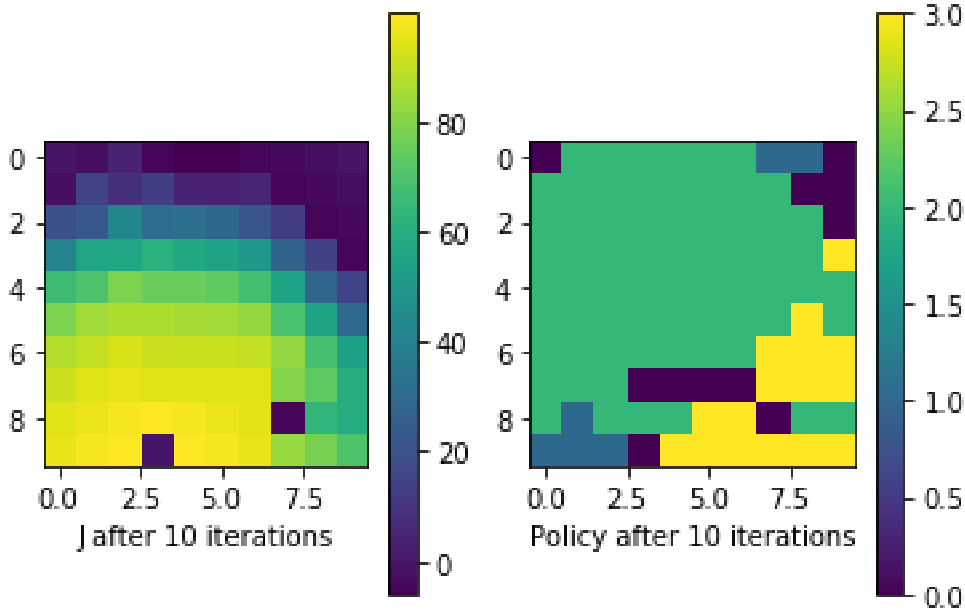
Q2.c Variant 1

In [11]:
```
question2 = Question2()
j, p, Chist, Phist = question2.Iterate(N=10, display_ = True)

plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.xlabel("J after 10 iterations")
plt.subplot(1, 2, 2)
plt.imshow(np.rot90(p))
plt.colorbar()
plt.xlabel("Policy after 10 iterations")
print ("Policy => 0: up, 1: right, 2: down, 3: left")
print (np.rot90(p))
```

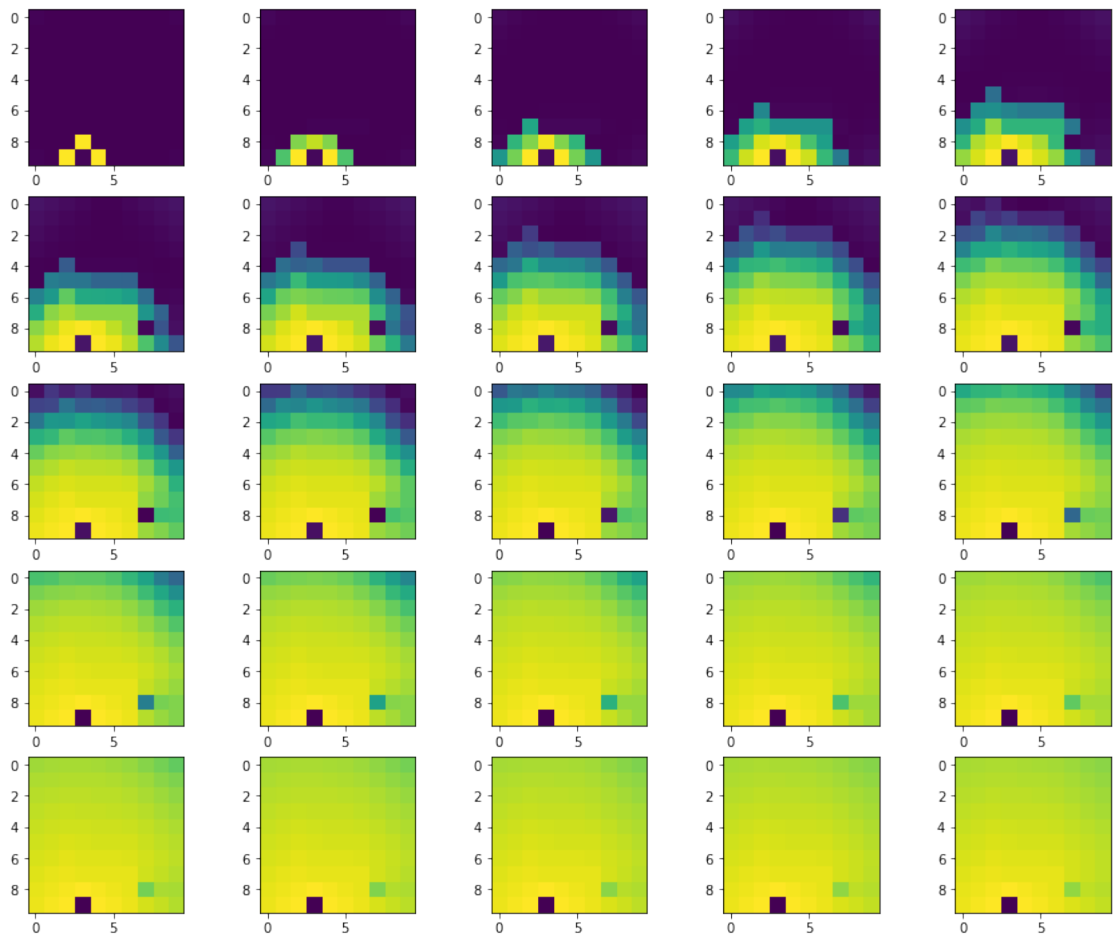


```
Policy => 0: up, 1: right, 2: down, 3: left
[[0 2 2 2 2 2 2 1 1 0]
 [2 2 2 2 2 2 2 2 0 0]
 [2 2 2 2 2 2 2 2 2 0]
 [2 2 2 2 2 2 2 2 2 3]
 [2 2 2 2 2 2 2 2 2 2]
 [2 2 2 2 2 2 2 2 3 2]
 [2 2 2 2 2 2 2 3 3 3]
 [2 2 2 0 0 0 0 3 3 3]
 [2 1 2 2 2 3 3 0 2 2]
 [1 1 1 0 3 3 3 3 3 3]]
```

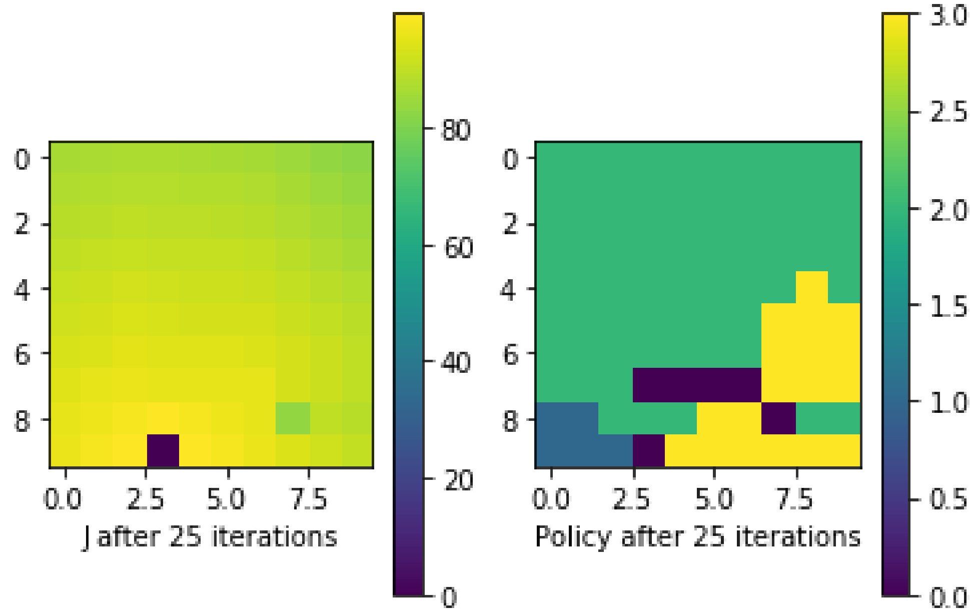


In [12]:
```
question2 = Question2()
j, p, Chist, Phist = question2.Iterate(N=25, display_ = True)

plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.xlabel("J after 25 iterations")
plt.subplot(1, 2, 2)
plt.imshow(np.rot90(p))
plt.colorbar()
plt.xlabel("Policy after 25 iterations")
print ("Policy => 0: up, 1: right, 2: down, 3: left")
print (np.rot90(p))
```



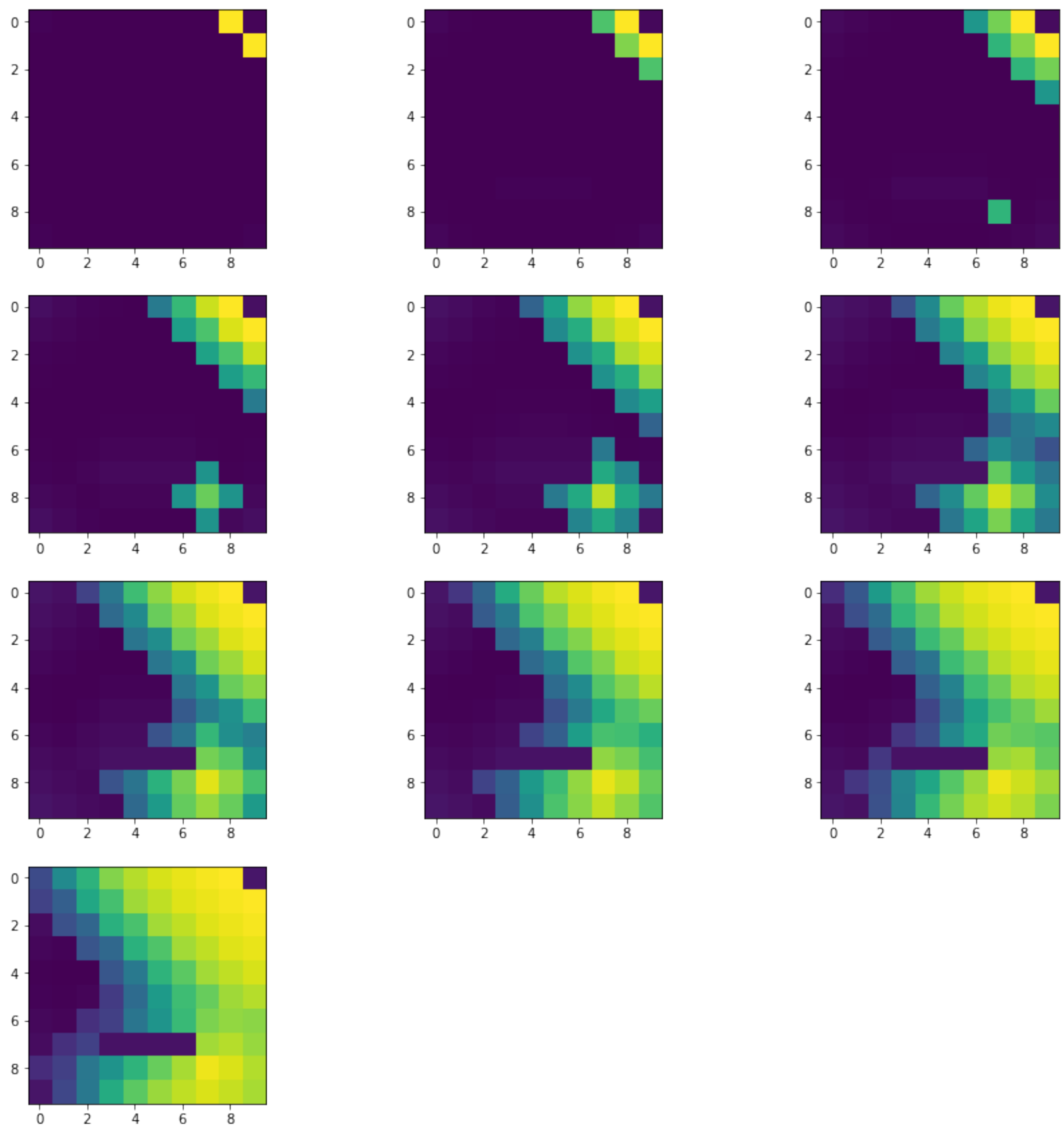
```
Policy => 0: up, 1: right, 2: down, 3: left
[[2 2 2 2 2 2 2 2 2 2]
 [2 2 2 2 2 2 2 2 2 2]
 [2 2 2 2 2 2 2 2 2 2]
 [2 2 2 2 2 2 2 2 2 2]
 [2 2 2 2 2 2 2 2 3 2]
 [2 2 2 2 2 2 2 3 3 3]
 [2 2 2 2 2 2 2 3 3 3]
 [2 2 2 0 0 0 0 3 3 3]
 [1 1 2 2 2 3 3 0 2 2]
 [1 1 1 0 3 3 3 3 3 3]]
```



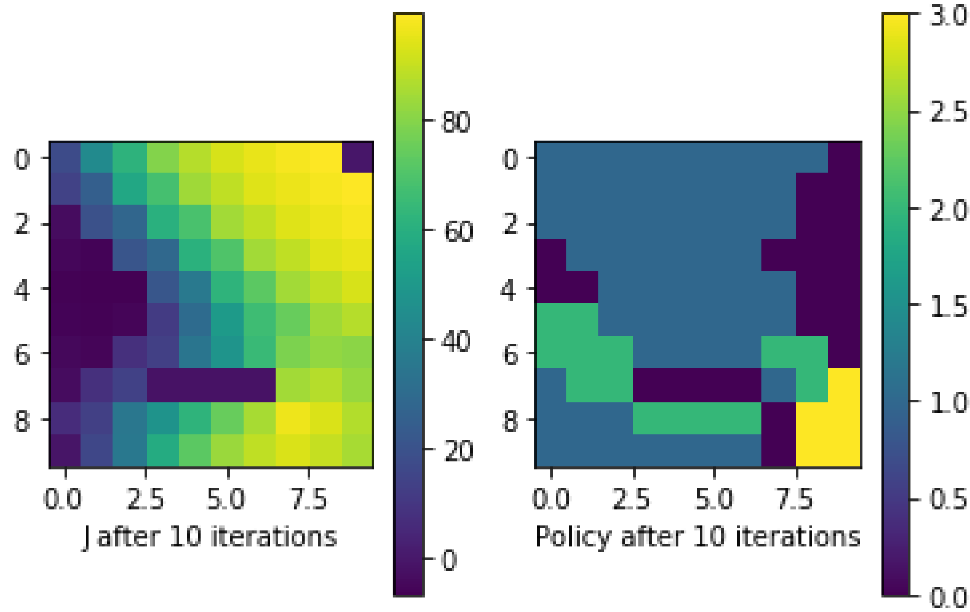
Q2.c Variant 2

```
In [13]: question2 = Question2(2)
         j, p, Chist, Phist = question2.Iterate(N=10, display_ = True)

         plt.subplot(1, 2, 1)
         plt.imshow(np.rot90(j))
         plt.colorbar()
         plt.xlabel("J after 10 iterations")
         plt.subplot(1, 2, 2)
         plt.imshow(np.rot90(p))
         plt.colorbar()
         plt.xlabel("Policy after 10 iterations")
         print ("Policy => 0: up, 1: right, 2: down, 3: left")
         print (np.rot90(p))
```

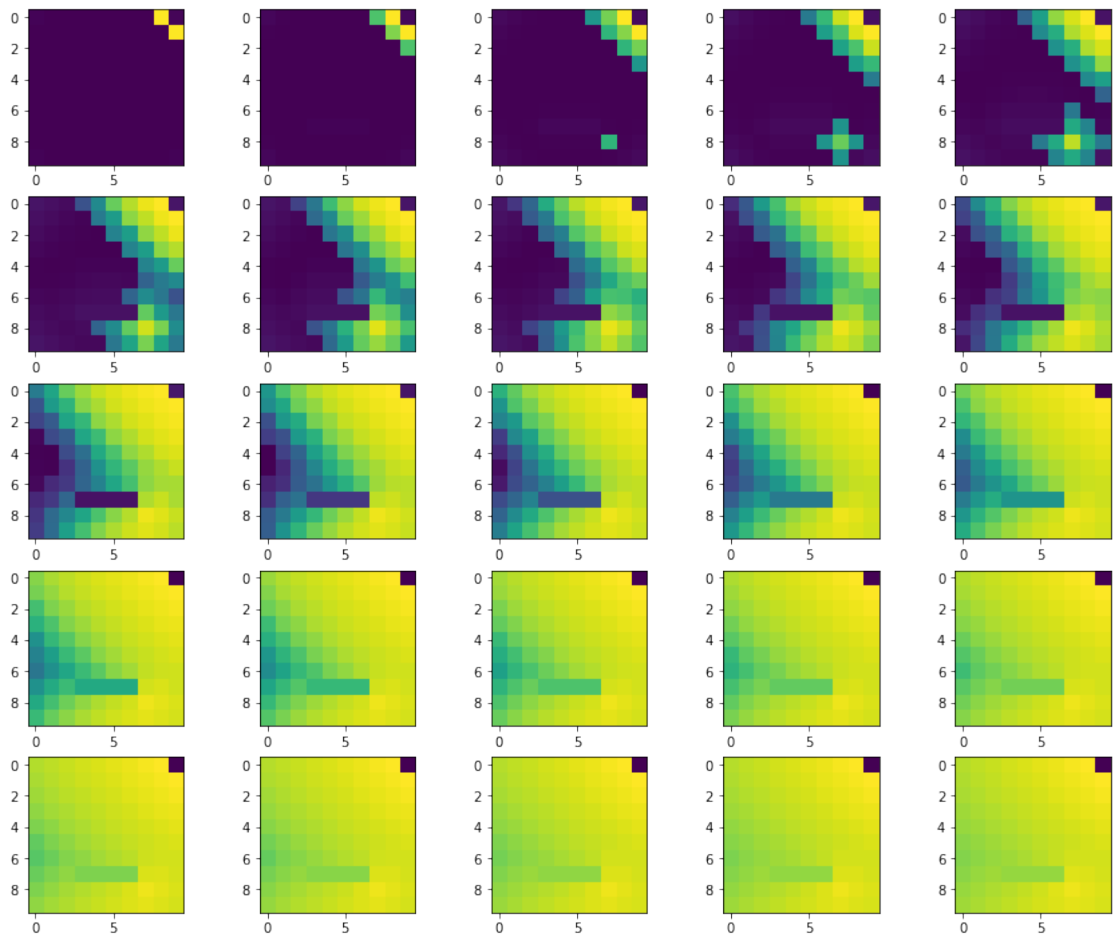


```
Policy => 0: up, 1: right, 2: down, 3: left
[[1 1 1 1 1 1 1 1 1 0]
 [1 1 1 1 1 1 1 1 0 0]
 [1 1 1 1 1 1 1 1 0 0]
 [0 1 1 1 1 1 1 0 0 0]
 [0 0 1 1 1 1 1 1 0 0]
 [2 2 1 1 1 1 1 1 0 0]
 [2 2 2 1 1 1 1 2 2 0]
 [1 2 2 0 0 0 0 1 2 3]
 [1 1 1 2 2 2 2 0 3 3]
 [1 1 1 1 1 1 1 0 3 3]]
```

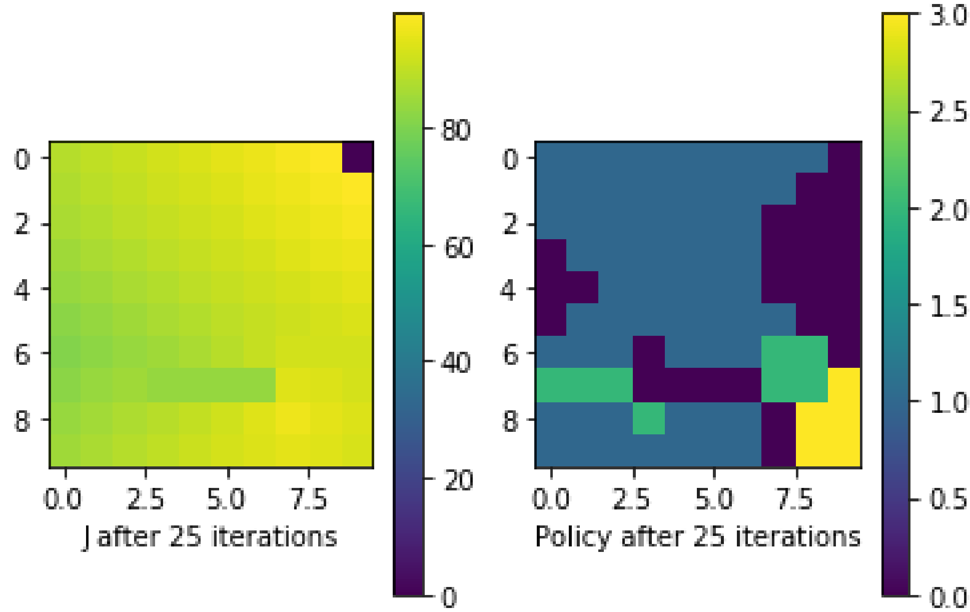


In [14]:
```
question2 = Question2(2)
j, p, Chist, Phist = question2.Iterate(N=25, display_ = True)

plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.xlabel("J after 25 iterations")
plt.subplot(1, 2, 2)
plt.imshow(np.rot90(p))
plt.colorbar()
plt.xlabel("Policy after 25 iterations")
print ("Policy => 0: up, 1: right, 2: down, 3: left")
print (np.rot90(p))
```



```
Policy => 0: up, 1: right, 2: down, 3: left
[[1 1 1 1 1 1 1 1 1 0]
 [1 1 1 1 1 1 1 1 0 0]
 [1 1 1 1 1 1 1 0 0 0]
 [0 1 1 1 1 1 1 0 0 0]
 [0 0 1 1 1 1 1 0 0 0]
 [0 1 1 1 1 1 1 1 0 0]
 [1 1 1 0 1 1 1 2 2 0]
 [2 2 2 0 0 0 0 2 2 3]
 [1 1 1 2 1 1 1 0 3 3]
 [1 1 1 1 1 1 1 0 3 3]]
```



3.0.5 Q2.d

- Once value iteration is converged the cost and policy per state remains constant
- After convergence action at each state leads to terminal state high reward (+100)

3.0.6 Reference

- Prashanth L. A. CS6700: Reinforcement learning Course notes, 2018
- Dimitri P. Bertsekas. Dynamic Programming and Optimal Control, vol. I. Athena Scientific, 2017.