CS6700 | Reinforcement Learning | Assignment 4

October 17, 2018

CS6700: Home Work 4
Avinash G. Kori | ED15B006

In [129]:
```
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt

def imshow(args):
    if len(args)==1:
        plt.imshow(args[0], interpolation='none')
    else:
        n=int(len(args)**0.5)
        plt.figure(figsize=(15, 15))
        for i in range(len(args)):
            plt.subplot(n + 1, n,i+1)
            plt.imshow(args[i])
    plt.show()
```

1 Question 1

1.0.1 Q1. 1> Value and policy iteration Implementation, in ipython notebook

In [130]:
```
class Question2(object):
    def __init__(self, variant = 1):
        """
            The objective of this question is to compare value iteration and policy it
            gridworld based on the actions, rewards and the state space given below.
            variant: variant of grid world
            variant = 1 (terminal state at (3, 0))
            variant = 2 (terminal state at (9, 9))
        """
        self.variant = variant
        # init Probabilities
        self.P = np.zeros((10, 10, 4, 10, 10))
        self.J = np.zeros((1, 1, 1, 10, 10))
        self.Jpi = np.zeros((1, 1, 1, 10, 10))
        # init all rewards with 0
```

```python
        self.g = np.zeros((10, 10, 4, 10, 10))
        # wormhole locations
        self.wormholeIN = np.array([
                    [(0,0)],
                    [(7,9)]
                    ])
        self.wormholeOUT = np.array([
                    [(2, 3), (2, 4), (2, 5), (2, 6)],
                    [(7, 1)]
                    ])
        # generate Probabilities
        self.P = self.generateP()
        # generate Rewards
        self.g = self.generateR()
        self.alpha = 0.95

    def generateP(self):
        """
            Generates and returns P matrix
            5th order Tensor
        """
        for ix in range(self.P.shape[0]):
            for iy in range(self.P.shape[1]):
                for action in range(self.P.shape[2]):
                    temp = np.zeros((10, 10))
                    if action == 0:
                        temp[ix, min(iy + 1, 9)] = 0.8
                        temp[ix, max(iy - 1, 0)] = 0.2/3.0
                        temp[max(0, ix - 1), iy] = 0.2/3.0
                        temp[min(ix + 1, 9), iy] = 0.2/3.0
                    elif action == 1:
                        temp[ix, min(iy + 1, 9)] = 0.2/3.
                        temp[ix, max(iy - 1, 0)] = 0.2/3.
                        temp[max(ix - 1, 0), iy] = 0.2/3.
                        temp[min(ix + 1, 9), iy] = 0.8
                    elif action == 2:
                        temp[ix, max(iy - 1, 0)] = 0.8
                        temp[ix, min(iy + 1, 9)] = 0.2/3.
                        temp[min(ix + 1, 9), iy] = 0.2/3.
                        temp[max(ix - 1, 0), iy] = 0.2/3.
                    else:
                        temp[max(ix - 1, 0), iy] = 0.8
                        temp[min(ix + 1, 9), iy] = 0.2/3.
                        temp[ix, min(iy + 1, 9)] = 0.2/3.
                        temp[ix, max(iy - 1, 0)] = 0.2/3.

                    if (ix, iy) == (0, 0):
                        temp = np.zeros((10, 10))
```

```
                    temp[2, 3] = temp[2, 4] = temp[2, 5] = temp[2, 6] = 1/4.0

                if (ix, iy) == (7, 9):
                    temp = np.zeros((10, 10))
                    temp[7, 1] = 1.0

#                     for wi in range(len(self.wormholeIN)):
#                         if (ix, iy) in self.wormholeIN[wi]:
#                             temp = np.zeros((10, 10))
#                             for o in self.wormholeOUT[wi]:
#                                 temp[o[0], o[1]] = 1.0/len(self.wormholeOUT[wi])

                if ((ix, iy) == (3, 0) and self.variant == 1) \
                   or (self.variant == 2 and (ix, iy) == (9, 9)):
                    temp = np.zeros((10, 10))

                self.P[ix, iy, action, :, :] = temp
        return self.P

    def generateR(self):
        """
            Generates and returns R matrix
            5th order Tensor
        """
        for ix in range(self.P.shape[0]):
            for iy in range(self.P.shape[1]):
                for action in range(self.P.shape[2]):
                    if self.variant == 2 and \
                    ((ix, iy, action) == (8, 9, 1) or \
                        (ix, iy, action) == (9, 8, 0)):
                        self.g[ix, iy, action, 9, 9] = 10

                    if self.variant == 1 and \
                     ((ix, iy, action) == (2, 0, 1) or \
                        (ix, iy, action) == (3, 1, 2)\
                        or(ix, iy, action) == (4, 0, 3)):
                        self.g[ix, iy, action, 3, 0] = 10
        return self.g

    def Toperator(self, J):
        """
            Applies T operator for current J
        """
        J = np.max(np.sum(self.P*(self.g + self.alpha*J),\
                              axis=(3, 4)), axis=2)
        return J

    def optPolicy(self, J):
```

```
        """
            Finds optimal policy for current states
        """
        optP = np.argmax(np.sum(self.P*(self.g + self.alpha*J),\
                                axis=(3, 4)), axis=2)
        return optP

    def Tpioperator(self, Policy, Jpi):
        """
            Applies Tpi operator for current Jpi
        """
        P = np.zeros((self.P.shape[0], self.P.shape[0],\
                      self.P.shape[0], self.P.shape[0]))
        G = np.zeros((self.g.shape[0], self.g.shape[0],\
                      self.g.shape[0], self.g.shape[0]))

        for i in range(len(Policy)):
            for j in range(len(Policy[0])):
                P[i,j, :, :] = self.P[i, j, Policy[i, j], :, :]
                G[i,j, :, :] = self.g[i, j, Policy[i, j], :, :]

        P = P.reshape(100, 100)
        G = G.reshape(100, 100)
        Jpi = Jpi.reshape(100, 1)

        Jpi = np.sum(P*G, axis=1)[:, None] + self.alpha*P.dot(Jpi)
        Jpi = Jpi.reshape(1, 1, 1, 10, 10)
        return Jpi

    def PolicyEvaluation(self, Policy, J, M = 1000):
        """
            Policy evaluation function
        """
        if M:
            for _ in range(M):
                J = self.Tpioperator(Policy, J)
        else:
#             I = np.zeros((self.P.shape[0], self.P.shape[0],\
#                           self.P.shape[0], self.P.shape[0]))
#             for i in range(self.P.shape[0]):
#                 for j in range(self.P.shape[0]):
#                     I[i,j, :, :] = np.eye(self.P.shape[0])

            I = np.eye(100)
            P = np.zeros((self.P.shape[0], self.P.shape[0],\
                          self.P.shape[0], self.P.shape[0]))
            G = np.zeros((self.P.shape[0], self.P.shape[0],\
                          self.g.shape[0], self.g.shape[0]))
```

```
            for i in range(len(Policy)):
                for j in range(len(Policy)):
                    P[i,j, :, :] = self.P[i,j, Policy[i, j], :, :]
                    G[i,j, :, :] = self.g[i,j, Policy[i,j], :, :]

            P = P.reshape(100, 100)
            G = G.reshape(100, 100)

            J = np.sum(P*G, axis=1)[None, :].dot(np.linalg.inv(I - self.alpha*P))
#             J = np.sum(P*G, axis = (2,3)).dot(np.linalg.inv(I - self.alpha*P).sum((2
#             print J.shape
            J = J.reshape(1,1,1,10,10)
        return J

    def PolicyUpdate(self, Jpi):
        """
            Tpi_new Jpi = TJpi finds new policy
        """
        Policy = self.optPolicy(Jpi)
        return Policy

    def ValueIteration(self, N = 1000):
        """
            Input Args:
                N: number of iterations
            returns:
                J: optimal J
                P: optimal policy
        """
        cost_hist, policy_hist = [], []
        for _ in range(N):
            self.J = self.Toperator(self.J)
            self.optP = self.optPolicy(self.J)
#             print "Value Iteration", _
#             print self.optP
            cost_hist.append(np.rot90(self.J.reshape(10, 10)))
            policy_hist.append(np.rot90(self.optP.reshape(10, 10)))
        return list(self.J.reshape(10, 10)), list(self.optP.reshape(10,10)), cost_hist

    def PolicyIteration(self, N = 1000, M = 100):
        """
            Input Args:
                N: number of iterations
            returns:
                J: optimal J
                P: optimal policy
        """
```

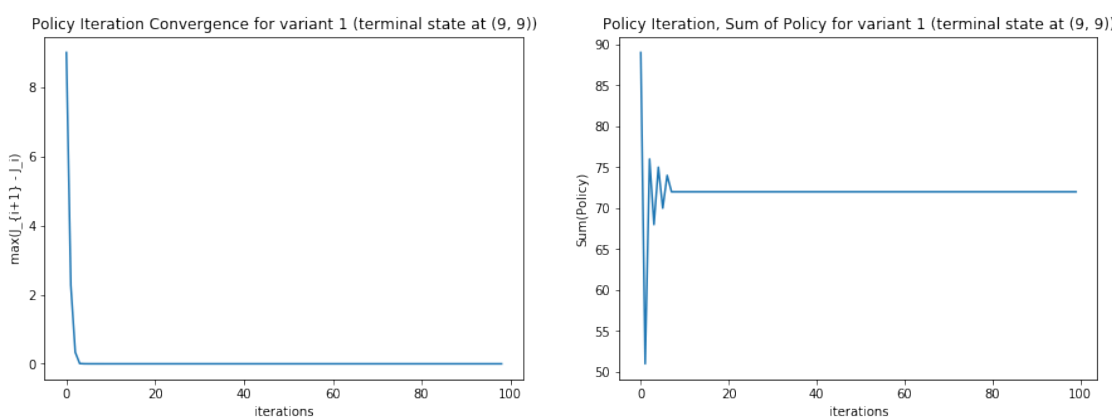
```
        cost_hist, policy_hist = [], []

        self.Policy = np.zeros((10, 10), dtype='int32')
        for _ in range(N):
            self.Jpi  = self.PolicyEvaluation(self.Policy, self.Jpi, M)
            self.Policy = self.PolicyUpdate(self.Jpi)
#             print "Policy Iteration", _
#             print self.Policy
            cost_hist.append(np.rot90(self.Jpi.reshape(10, 10)))
            policy_hist.append(np.rot90(self.Policy.reshape(10, 10)))

        return list(self.Jpi.reshape(10,10)), list(self.Policy.reshape(10,10)), cost_h

from unicodedata import *
def print_policy(pi, variant = 2):
    pi = np.array(pi)
    pi[0,0]=4
    pi[7,9]=4

    if variant == 1:
        pi[9, 9] = 5
    else:
        pi[3, 0] = 5

    plot = np.empty((10,10), dtype='str')

    print '--------------------------------'
    plot[np.where(pi == 0)] = '^'
    plot[np.where(pi == 1)] = '>'
    plot[np.where(pi == 2)] = 'v'
    plot[np.where(pi == 3)] = '<'
    plot[np.where(pi == 4)] = 'W'
    plot[np.where(pi == 5)] = 'T'

    print np.rot90(plot)

    print '--------------------------------'
```

2.a) Plot graph of maxs |Ji+1(s) Ji(s)| vs iterations and Ps i+1(s) 6= i(s) vs iterations for both value iteration and policy iteration.

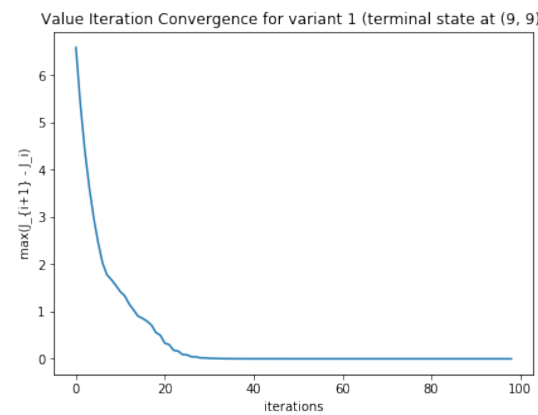
```
In [131]: print "================== Value Iteration Plots ==============="

          question2 = Question2(2)
          j, p, Cpv1hist, Ppv1hist = question2.PolicyIteration(N=100)

          diff_hist = np.diff(np.array(Cpv1hist), axis = 0)
          plt.figure(figsize = (15, 5))
```

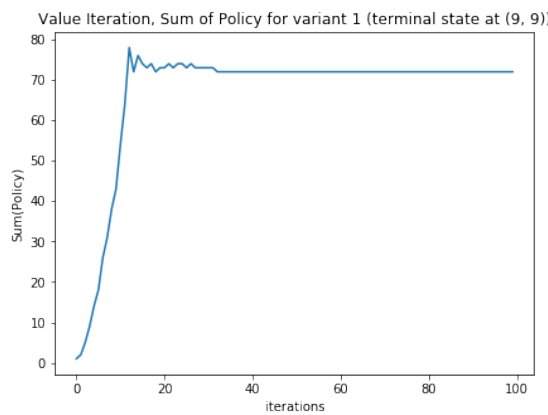
```python
plt.subplot(1, 2, 1)
plt.plot(np.max(diff_hist, axis = (1,2)))
plt.title('Policy Iteration Convergence for variant 1 (terminal state at (9, 9))')
plt.xlabel('iterations')
plt.ylabel('max(J_{i+1} - J_i)')

plt.subplot(1, 2, 2)
plt.plot(np.sum(np.array(Ppv1hist), axis = (1,2)))
plt.title('Policy Iteration, Sum of Policy for variant 1 (terminal state at (9, 9))')
plt.xlabel('iterations')
plt.ylabel('Sum(Policy)')
plt.show()

print "================== Policy Iteration Plots ==============="

question2 = Question2(2)
j, p, Cvv1hist, Pvv1hist = question2.ValueIteration(N=100)

diff_hist = np.diff(np.array(Cvv1hist), axis = 0)
plt.figure(figsize = (15, 5))
plt.subplot(1, 2, 1)
plt.plot(np.max(diff_hist, axis = (1,2)))
plt.title('Value Iteration Convergence for variant 1 (terminal state at (9, 9))')
plt.xlabel('iterations')
plt.ylabel('max(J_{i+1} - J_i)')

plt.subplot(1, 2, 2)
plt.plot(np.sum(np.array(Pvv1hist), axis = (1,2)))
plt.title('Value Iteration, Sum of Policy for variant 1 (terminal state at (9, 9))')
plt.xlabel('iterations')
plt.ylabel('Sum(Policy)')
plt.show()
```

```
================== Value Iteration Plots ===============
```



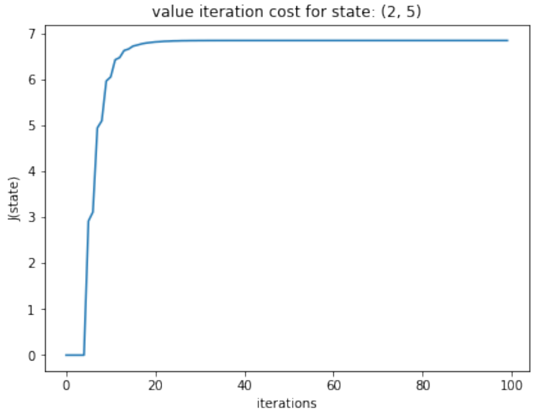
================== Policy Iteration Plots ===============

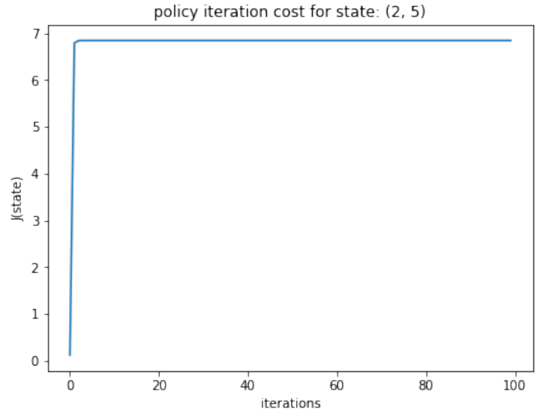


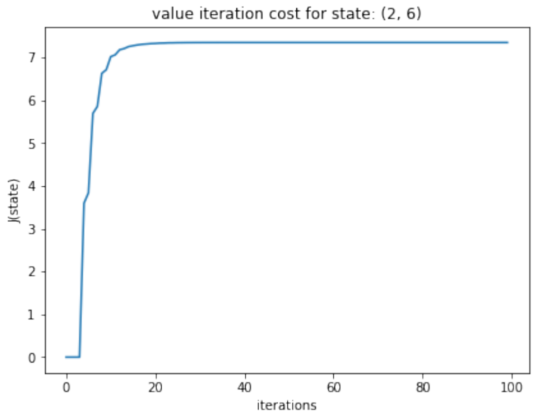


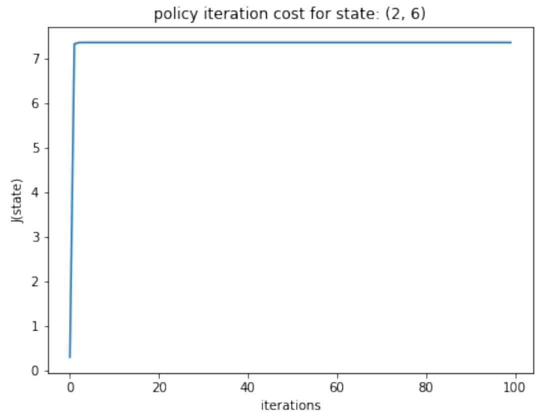
2.b) Compare value iteration and policy iteration by plotting J(s) vs iterations for three random states. Which converges faster? Why?

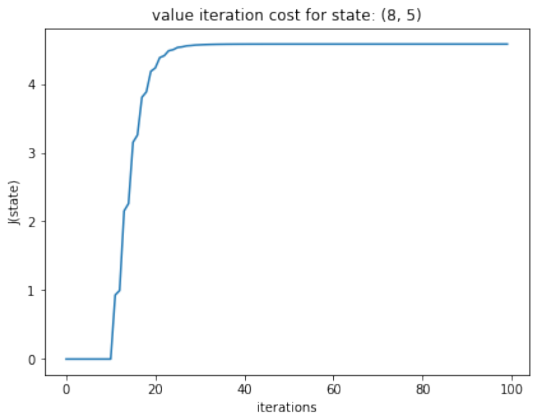
```
In [132]: states = zip(np.random.randint(9, size = 3), np.random.randint(9, size = 3))

          for (x,y) in states:
              plt.figure(figsize = (15, 5))
              plt.subplot(1, 2, 1)
              plt.plot(np.array(Cvv1hist)[:, x, y])
              plt.title('value iteration cost for state: ({}, {})'.format(x,y))
              plt.xlabel('iterations')
              plt.ylabel('J(state)')

              plt.subplot(1, 2, 2)
              plt.plot(np.array(Cpv1hist)[:, x, y])
              plt.title('policy iteration cost for state: ({}, {})'.format(x,y))
              plt.xlabel('iterations')
              plt.ylabel('J(state)')
              plt.show()
```

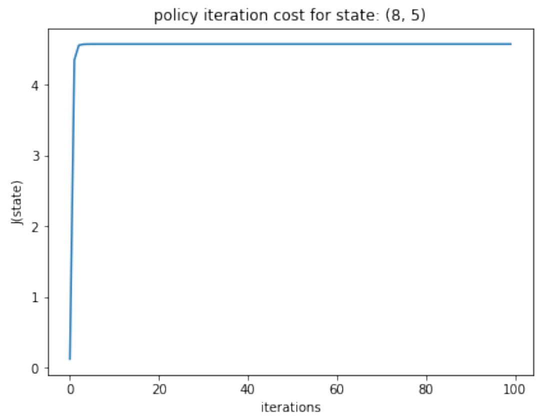

value iteration cost for state: (2, 5)
J(state)
iterations


policy iteration cost for state: (2, 5)
J(state)
iterations


value iteration cost for state: (2, 6)
J(state)
iterations


policy iteration cost for state: (2, 6)
J(state)
iterations


value iteration cost for state: (8, 5)
J(state)
iterations


policy iteration cost for state: (8, 5)
J(state)
iterations

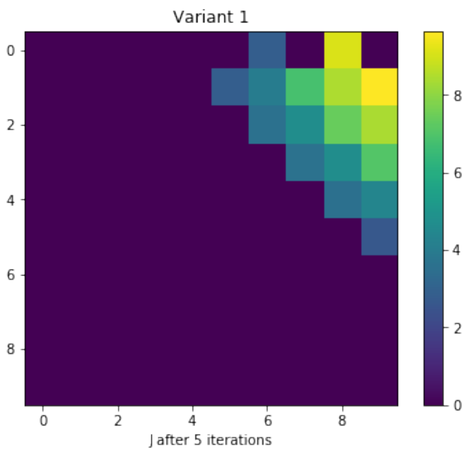
Convergence of policy iteration is faster: Value iteration takes about 20-30 iterations to converge, but policy iteration converges with in 5 iterations

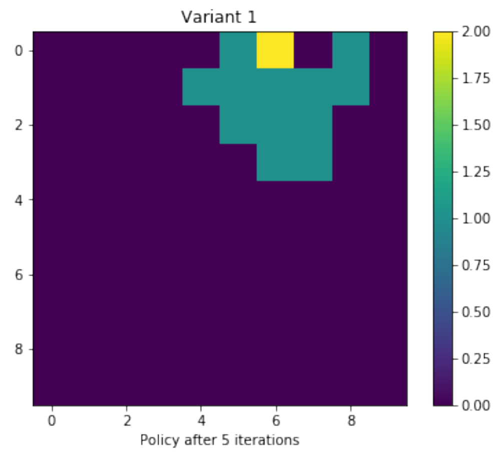
- In case of policy iteration each policy updated policy should be better than it's previous policy
- In case of policy iteration closed loop simultanious equations are solved to find optimal cost (policy evaluation step), but in case of value iteration simultanious equations are solved in iterative fashion

2.c) Show J(s) and greedy policy (s), s, obtained after 5 iterations, and after you stop value iteration. value iteration stops after 30 iterations...

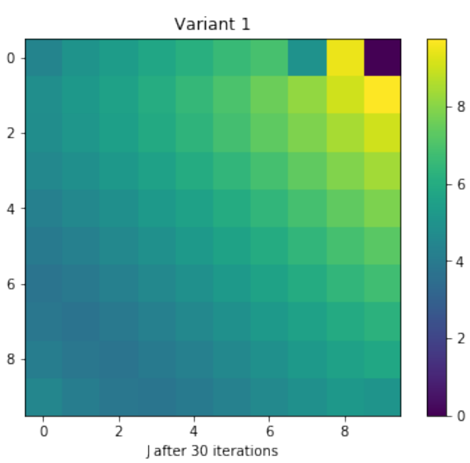
In [133]:
```
print ("=========== ValueIteration (N = 5) =============")
question2 = Question2(2)
j, p, Chist, Phist = question2.ValueIteration(N=5)

plt.figure(figsize = (15, 5))
plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.title("Variant 1")
plt.xlabel("J after 5 iterations")
plt.subplot(1, 2, 2)
plt.imshow(np.rot90(p))
plt.colorbar()
plt.title("Variant 1")
plt.xlabel("Policy after 5 iterations")
plt.show()
print_policy(p, 1)

print ("==== ValueIteration(N = 30) after convergence of value iteration =====")

question2 = Question2(2)
j, p, Chist, Phist = question2.ValueIteration(N=30)

plt.figure(figsize = (15, 5))
plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.title("Variant 1")
plt.xlabel("J after 30 iterations")
plt.subplot(1, 2, 2)
plt.imshow(np.rot90(p))
plt.colorbar()
plt.title("Variant 1")
plt.xlabel("Policy after 30 iterations")
plt.show()
print_policy(p, 1)
```

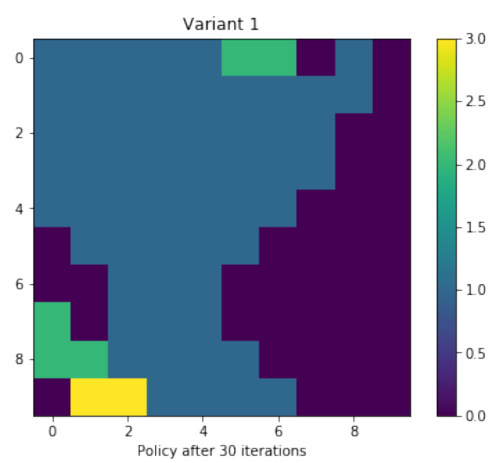
```
=========== ValueIteration (N = 5) =============
```






```
--------------------------------
[['^' '^' '^' '^' '^' '>' 'v' 'W' '>' 'T']
 ['^' '^' '^' '^' '>' '>' '>' '>' '>' '^']
 ['^' '^' '^' '^' '^' '>' '>' '>' '^' '^']
 ['^' '^' '^' '^' '^' '^' '>' '>' '^' '^']
 ['^' '^' '^' '^' '^' '^' '^' '^' '^' '^']
 ['^' '^' '^' '^' '^' '^' '^' '^' '^' '^']
 ['^' '^' '^' '^' '^' '^' '^' '^' '^' '^']
 ['^' '^' '^' '^' '^' '^' '^' '^' '^' '^']
 ['^' '^' '^' '^' '^' '^' '^' '^' '^' '^']
 ['W' '^' '^' '^' '^' '^' '^' '^' '^' '^']]
--------------------------------
==== ValueIteration(N = 30) after convergence of value iteration =====
```





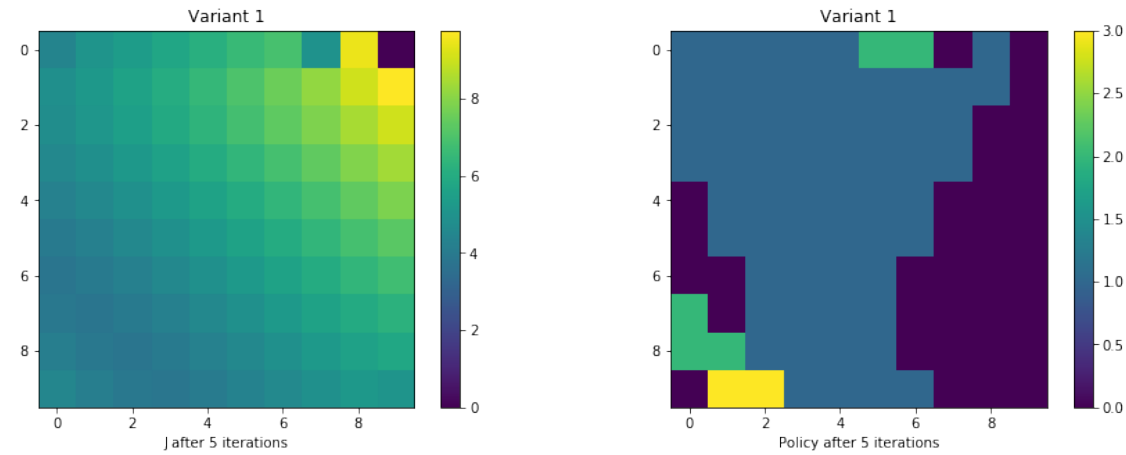
```
--------------------------------
[['>' '>' '>' '>' '>' 'v' 'v' 'W' '>' 'T']
 ['>' '>' '>' '>' '>' '>' '>' '>' '>' '^']
 ['>' '>' '>' '>' '>' '>' '>' '>' '^' '^']
 ['>' '>' '>' '>' '>' '>' '>' '>' '^' '^']
 ['>' '>' '>' '>' '>' '>' '>' '^' '^' '^']
 ['^' '>' '>' '>' '>' '>' '^' '^' '^' '^']
 ['^' '^' '>' '>' '>' '^' '^' '^' '^' '^']
 ['v' '^' '>' '>' '>' '^' '^' '^' '^' '^']
 ['v' 'v' '>' '>' '>' '>' '^' '^' '^' '^']
 ['W' '<' '<' '>' '>' '>' '>' '^' '^' '^']]
--------------------------------
```

2.d) Show J(s) and greedy policy (s), s, obtained after 5 iterations, and after you stop policy iteration. policy iteration stops after 5 iterations

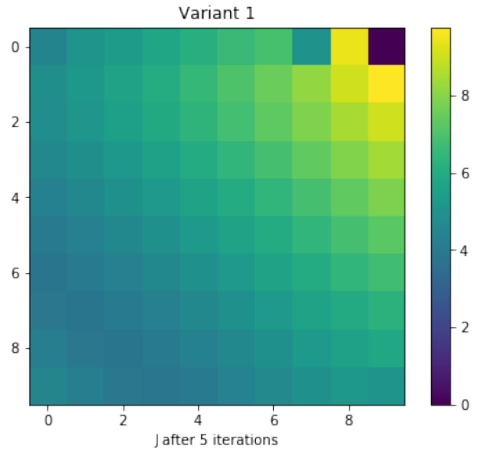
In [134]:
```python
print ("=========== PolicyIteration(N = 5) ===============")
question2 = Question2(2)
j, p, Chist, Phist = question2.PolicyIteration(N=5)

plt.figure(figsize = (15, 5))
plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.title("Variant 1")
plt.xlabel("J after 5 iterations")
plt.subplot(1, 2, 2)
plt.imshow(np.rot90(p))
plt.colorbar()
plt.title("Variant 1")
plt.xlabel("Policy after 5 iterations")
plt.show()
print_policy(p, 1)

print ("=== PolicyIteration(N = 5) after convergence of policy iteration =====")

question2 = Question2(2)
j, p, Chist, Phist = question2.PolicyIteration(N=5)

plt.figure(figsize = (15, 5))
plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.title("Variant 1")
```

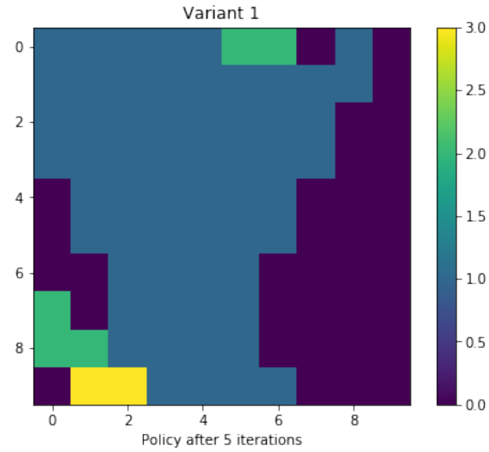
```
        plt.xlabel("J after 5 iterations")
        plt.subplot(1, 2, 2)
        plt.imshow(np.rot90(p))
        plt.colorbar()
        plt.title("Variant 1")
        plt.xlabel("Policy after 5 iterations")
        plt.show()
        print_policy(p, 1)
```

```
=========== PolicyIteration(N = 5) ===============
```




```
--------------------------------
[['>' '>' '>' '>' '>' 'v' 'v' 'W' '>' 'T']
 ['>' '>' '>' '>' '>' '>' '>' '>' '>' '^']
 ['>' '>' '>' '>' '>' '>' '>' '>' '^' '^']
 ['>' '>' '>' '>' '>' '>' '>' '>' '^' '^']
 ['^' '>' '>' '>' '>' '>' '>' '^' '^' '^']
 ['^' '>' '>' '>' '>' '>' '>' '^' '^' '^']
 ['^' '^' '>' '>' '>' '>' '^' '^' '^' '^']
 ['v' '^' '>' '>' '>' '>' '^' '^' '^' '^']
 ['v' 'v' '>' '>' '>' '>' '^' '^' '^' '^']
 ['W' '<' '<' '>' '>' '>' '>' '^' '^' '^']]
--------------------------------
=== PolicyIteration(N = 5) after convergence of policy iteration =====
```






```
--------------------------------
[['>' '>' '>' '>' '>' 'v' 'v' 'W' '>' 'T']
 ['>' '>' '>' '>' '>' '>' '>' '>' '>' '^']
 ['>' '>' '>' '>' '>' '>' '>' '>' '^' '^']
 ['>' '>' '>' '>' '>' '>' '>' '>' '^' '^']
 ['^' '>' '>' '>' '>' '>' '>' '^' '^' '^']
 ['^' '>' '>' '>' '>' '>' '>' '^' '^' '^']
 ['^' '^' '>' '>' '>' '>' '^' '^' '^' '^']
 ['v' '^' '>' '>' '>' '>' '^' '^' '^' '^']
 ['v' 'v' '>' '>' '>' '>' '^' '^' '^' '^']
 ['W' '<' '<' '>' '>' '>' '>' '^' '^' '^']]
--------------------------------
```

2.e) Explain the behaviour of J and greedy policy obtained by value iteration and policy iteration.

- Policy in all the states lead to terminal state, it can even be observed in the policies around wormhole, around wormhole (7, 9) any policy is not directed towards (7, 9), while around wormhole (0, 0) all the policies are leading towards (0,0)

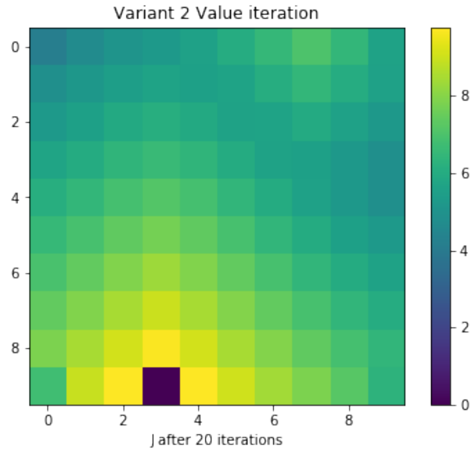
- Cost around terminal state high as compared to any other states
- Cost for all the states remain constant after convergence, Convergence in case of value iteration needs about 25 iteration while policy iteration just needs 5 iterations

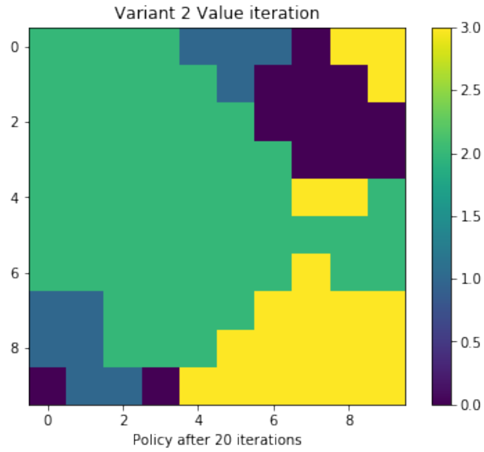
3.a) Show J(s) and policy (s), s, obtained after you stop value iteration and policy iteration and explain it's behaviour.

```
In [135]: print ("======== value iteration stops at 20 iterations ==============")

          question2 = Question2(1)
```

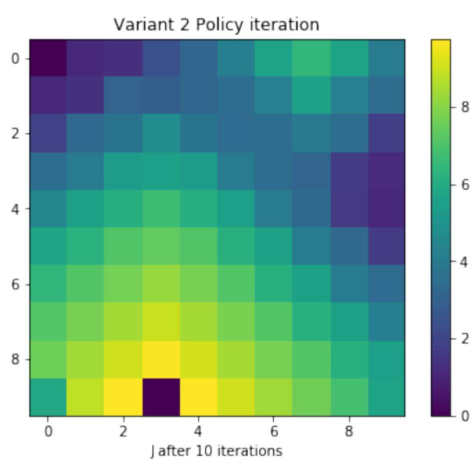
```
j, p, Chist, Phist = question2.ValueIteration(N=20)

plt.figure(figsize = (15, 5))
plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.title("Variant 2 Value iteration")
plt.xlabel("J after 20 iterations")
plt.subplot(1, 2, 2)
plt.imshow(np.rot90(p))
plt.colorbar()
plt.title("Variant 2 Value iteration")
plt.xlabel("Policy after 20 iterations")
plt.show()
print_policy(p, 2)

print ("======= policy iteration stops at 10 iterations ==============")

question2 = Question2(1)
j, p, Chist, Phist = question2.ValueIteration(N=11)

plt.figure(figsize = (15, 5))
plt.subplot(1, 2, 1)
plt.imshow(np.rot90(j))
plt.colorbar()
plt.title("Variant 2 Policy iteration")
plt.xlabel("J after 10 iterations")
plt.subplot(1, 2, 2)
plt.imshow(np.rot90(p))
plt.colorbar()
plt.title("Variant 2 Policy iteration")
plt.xlabel("Policy after 10 iterations")
plt.show()
print_policy(p, 2)
```

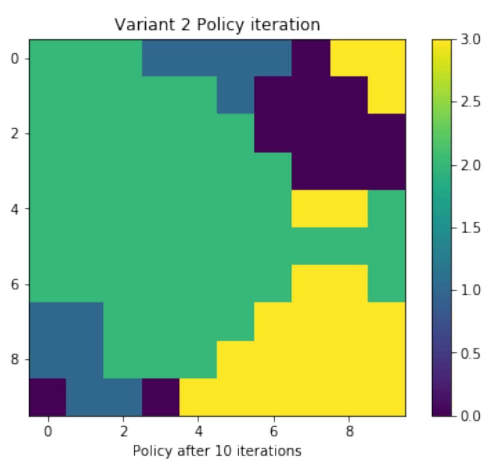
```
======== value iteration stops at 20 iterations ==============
```






```
--------------------------------
[['v' 'v' 'v' 'v' '>' '>' '>' 'W' '<' '<']
 ['v' 'v' 'v' 'v' 'v' '>' '^' '^' '^' '<']
 ['v' 'v' 'v' 'v' 'v' 'v' '^' '^' '^' '^']
 ['v' 'v' 'v' 'v' 'v' 'v' 'v' '^' '^' '^']
 ['v' 'v' 'v' 'v' 'v' 'v' 'v' '<' '<' 'v']
 ['v' 'v' 'v' 'v' 'v' 'v' 'v' 'v' 'v' 'v']
 ['v' 'v' 'v' 'v' 'v' 'v' 'v' '<' 'v' 'v']
 ['>' '>' 'v' 'v' 'v' 'v' '<' '<' '<' '<']
 ['>' '>' 'v' 'v' 'v' '<' '<' '<' '<' '<']
 ['W' '>' '>' 'T' '<' '<' '<' '<' '<' '<']]
--------------------------------
======= policy iteration stops at 10 iterations ==============
```





```
--------------------------------
[['v' 'v' 'v' '>' '>' '>' '>' 'W' '<' '<']
 ['v' 'v' 'v' 'v' 'v' '>' '^' '^' '^' '<']
 ['v' 'v' 'v' 'v' 'v' 'v' '^' '^' '^' '^']
 ['v' 'v' 'v' 'v' 'v' 'v' 'v' '^' '^' '^']
 ['v' 'v' 'v' 'v' 'v' 'v' 'v' '<' '<' 'v']
 ['v' 'v' 'v' 'v' 'v' 'v' 'v' 'v' 'v' 'v']
 ['v' 'v' 'v' 'v' 'v' 'v' 'v' '<' '<' 'v']
 ['>' '>' 'v' 'v' 'v' 'v' '<' '<' '<' '<']
 ['>' '>' 'v' 'v' 'v' '<' '<' '<' '<' '<']
 ['W' '>' '>' 'T' '<' '<' '<' '<' '<' '<']]
--------------------------------
```

- Policy in all the states lead to terminal state, it can even be observed in the policies around wormhole, around wormhole (7, 9) as all the policies are directing towards (7, 9), while around wormhole (0, 0) no policies are directing, as (3, 0) is terminal state
- Cost around terminal state and (7,9) wormhole is high as compared to any other states
- Cost for all the states remain constant after convergence, Convergence in case of value iteration needs about 25 iteration while policy iteration just needs 5 iterations

2 Question 2

```
In [136]: class Question1(object):
              """
              Consider a problem of a taxi driver, who serves three cities A, B and C. The taxi
              new ride by choosing one of the following actions.
              1. Cruise the streets looking for a passenger.
              2. Go to the nearest taxi stand and wait in line.
              3. Wait for a call from the dispatcher (this is not possible in town B because of
              For a given town and a given action, there is a probability that the next trip wil
              towns A, B and C and a corresponding reward in monetary units associated with each
              This reward represents the income from the trip after all necessary expenses have
              Please refer Table 1 below for the rewards and transition probabilities. In Table
              kij is the probability of getting a ride to town j, by choosing an action k while
              kij is the immediate reward of getting a ride to town j, by choosing an action k w
              town i.
              """
              def __init__(self, alpha = 0.95):
                  # define probabilities
                  self.P = np.array([
                          [[0.5   , 0.25 , 0.250 ],
                           [1./16., 3./4., 3./16.],
                           [1./4. , 1./8., 5./8. ]],
                          [[1./2. , 0.   , 1./2. ],
                           [1./16. , 7./8., 1./16.],
                           [0.    ,   1. , 0     ]],
```

```
                [[1./4.  , 1./4., 1./2. ],
                 [1./8.  , 3./4., 1./8. ],
                 [3./4.  ,1./16., 3./16.]]
            ])
        self.P = np.swapaxes(self.P, 1, 0) # S, a, S'

        # define rewards
        self.g = np.array([
                [[10., 4., 8.],
                 [8.,  2., 4.],
                 [4.,  6., 4.]],
                [[14., 0., 18.],
                 [8., 16., 08.],
                 [0.,  0., 0. ]],
                [[10., 2., 8. ],
                 [6.  , 4., 2. ],
                 [4.  , 0., 8.]]
            ])
        self.g = np.swapaxes(self.g, 1, 0) # S, a, S'
        # init J
        self.alpha = alpha
        self.J = np.array([[[0., 0., 0.]]]) # 1x1x3
        self.Jpi = np.array([[[0., 0., 0.]]]) # 1x1x3

    def Toperator(self, J):
        """
            Applies T operator for current J
        """
        J = np.max(np.sum(self.P*(self.g + self.alpha*J), axis=2), axis=1)
        return J

    def optPolicy(self, J):
        """
            Finds optimal policy for current states
        """
        optP = np.argmax(np.sum(self.P*(self.g + self.alpha*J), axis=2), axis=1)
        return optP

    def Tpioperator(self, Policy, Jpi):
        """
            Applies Tpi operator for current Jpi
        """
        P = np.zeros((self.P.shape[0], self.P.shape[0]))
        G = np.zeros((self.g.shape[0], self.g.shape[0]))
        for i in range(len(Policy)):
            P[i, :] = self.P[i, Policy[i], :]
            G[i, :] = self.g[i, Policy[i], :]
```

```
        Jpi = np.sum(P*G, axis=1)[:, None] + self.alpha*P.dot(Jpi.reshape(3, 1))
        Jpi = Jpi.reshape(1,1,3)
        return Jpi

    def PolicyEvaluation(self, Policy, J, M = 10):
        """
            Policy evaluation function
            M = None, performs policy evaluation
            M > 0, performs modified policy evaluation
        """
        if M:
            for _ in range(M):
                J = self.Tpioperator(Policy, J)
        else:
            I = np.eye(self.P.shape[0])
            P = np.zeros((self.P.shape[0], self.P.shape[0]))
            G = np.zeros((self.g.shape[0], self.g.shape[0]))
            for i in range(len(Policy)):
                P[i, :] = self.P[i, Policy[i], :]
                G[i, :] = self.g[i, Policy[i], :]
            J = np.sum(P*G, axis=1)[None, :].dot(np.linalg.inv(I - self.alpha*P))
            J = J.reshape(1,1,3)
        return J

    def PolicyUpdate(self, Jpi):
        """
            Tpi_new Jpi = TJpi finds new policy
        """
        Policy = self.optPolicy(Jpi)
        return Policy

    def ValueIteration(self, N = 1000):
        """
            Input Args:
                N: number of iterations
            returns:
                J: optimal J
                P: optimal policy
        """
        for _ in range(N):
#               print "Value Iteration,", _
            self.J    = self.Toperator(self.J)
            self.optP = self.optPolicy(self.J)
#               print self.J, self.optP
        return self.J.reshape(3), self.optP.reshape(3)

    def GaussSeidelValueIteration(self, N = 1000):
        """
```

```
            Input Args:
                N: number of iterations
            returns:
                J: optimal J
                P: optimal policy
        """
        for ii in range(N):
#               print "Value Iteration,", _
#               state    = np.random.randint(3)
            state = ii % 3
            self.J[:, :, state]  = np.max(np.sum(self.P[state, :, :]*\
                                        (self.g[state, :, :] + self.alpha*\
                                         self.J[0]), axis=1), axis=0)
            self.optP = self.optPolicy(self.J)
#               print self.J, self.optP
        return self.J.reshape(3), self.optP.reshape(3)

    def PolicyIteration(self, N = 1000):
        """
            Input Args:
                N: number of iterations
            returns:
                J: optimal J
                P: optimal policy
        """
        self.Policy = np.array([0, 0, 0])
        temp = np.array([0,0,0])
        for _ in range(N):
#               print "Policy Iteration,", _
            self.Jpi  = self.PolicyEvaluation(self.Policy, self.Jpi, M=None)
            self.Policy = self.PolicyUpdate(self.Jpi)
#               print self.Jpi, self.Policy
            if temp.all() == self.Policy.all(): break
            temp = self.Policy
        return self.Jpi.reshape(3), self.Policy.reshape(3)

    def ModifiedPolicyIteration(self, N = 1000, M = 5):
        """
            Input Args:
                N: number of iterations
            returns:
                J: optimal J
                P: optimal policy
        """
        self.Policy = np.array([2,2,2])
        temp = np.array([0,0,0])
        for _ in range(N):
#               print "Modified Policy Iteration,", _
```

```
            self.Jpi  = self.PolicyEvaluation(self.Policy, self.Jpi, M)
            self.Policy = self.PolicyUpdate(self.Jpi)
#             print self.Jpi, self.Policy
            if temp.all() == self.Policy.all(): break
            temp = self.Policy
        return self.Jpi.reshape(3), self.Policy.reshape(3)
```

```
question1 = Question1()
vcost10_, vpolicy10_ = question1.ValueIteration(N = 10)
pcost10_, ppolicy10_ = question1.PolicyIteration(N = 10)
mpcost10_, mppolicy10_ = question1.ModifiedPolicyIteration(N = 10)
```

```
question1 = Question1()
vcost20_, vpolicy20_ = question1.ValueIteration(N = 20)
pcost20_, ppolicy20_ = question1.PolicyIteration(N = 20)
mpcost20_, mppolicy20_ = question1.ModifiedPolicyIteration(N = 20)
```

1) Find an optimal policy using policy iteration starting with a policy that will always cruise independent of the town. Solve it for discount factors ranging from 0 to 0.95 with intervals of 0.05. Tabulate the optimal policies and optimal values obtained for different values of . (5marks)

```
In [137]: V={'alpha':[], 'cost': [], 'policy': []}
          P={'alpha':[], 'cost': [], 'policy': []}
          PM={'alpha':[], 'cost': [], 'policy': []}

          # print "=================== Value Iteration ========================"
          for alpha in range(0, 95, 5):
              alpha = alpha*1.0/100.0

              question1 = Question1(alpha)
              vcost, vpolicy   = question1.ValueIteration(N = 10)
              V['alpha'].append(alpha)
              V['cost'].append(vcost)
              V['policy'].append(vpolicy)
          #     print alpha, vcost, vpolicy+1

          # print "==================== Modified Policy Iteration =================="
          for alpha in range(0, 95, 5):
              alpha = alpha*1.0/100.0

              question1 = Question1(alpha)
              pcost, ppolicy   = question1.PolicyIteration(N = 10)
              P['alpha'].append(alpha)
              P['cost'].append(vcost)
```

```
        P['policy'].append(vpolicy)
    #     print alpha, pcost, ppolicy+1

    print "==================== Policy Iteration ======================="
    print ('alpha        ', '       cost          ', '       policy')
    for alpha in range(0, 95, 5):
        alpha = alpha*1.0/100.0

        question1 = Question1(alpha)
        mpcost, mppolicy = question1.ModifiedPolicyIteration(N = 10)
        PM['alpha'].append(alpha)
        PM['cost'].append(vcost)
        PM['policy'].append(vpolicy)
        print str(alpha) + "     " + str(mpcost) + "     " + str(mppolicy+1)

    #     print (list(V['alpha']), list(V['cost']), list(V['policy']))
```

```
==================== Policy Iteration =======================
('alpha        ', '       cost          ', '       policy')
0.0     [16.   15.    4.5]      [2 2 3]
0.05     [16.5440639  15.75739147  5.2185664 ]     [2 2 3]
0.1     [17.15799375 16.59690146  6.00307659]     [2 2 3]
0.15     [17.85346742 17.53220299  6.86550504]     [2 2 3]
0.2     [18.64454899 18.57953341  7.82021028]     [2 2 3]
0.25     [19.54816605 19.75811775  8.88440947]     [2 2 3]
0.3     [20.58477005 21.0907643  10.07883526]     [2 2 3]
0.35     [21.7792532  22.60470363 11.42864865]     [2 2 3]
0.4     [23.16221888 24.33276447 12.96470426]     [2 2 3]
0.45     [24.77172862 26.31500611 14.72529119]     [2 2 3]
0.5     [26.65567963 28.60095664 16.75850298]     [2 2 3]
0.55     [28.87500064 31.25263978 19.12542434]     [2 2 3]
0.6     [31.50789165 34.34860957 21.90435971]     [2 2 3]
0.65     [34.65537473 37.98925346 25.19637055]     [2 2 3]
0.7     [38.44846791 42.30366766 29.13243307]     [2 2 3]
0.75     [43.05734347 47.45845719 33.88257685]     [2 2 3]
0.8     [48.70288385 53.66886382 39.66741754]     [2 2 2]
0.85     [55.67110533 61.21268058 46.77255266]     [2 2 2]
0.9     [64.33097876 70.44746992 55.56634954]     [2 2 2]
```

2.a) Find an optimal policy using modified policy iteration. Let mk = 5 k. Start with a policy that will always cruise independent of the town. Let = 0.9. What are the optimal values? (3 marks)

```
In [138]: question1 = Question1(alpha = 0.9)
          mpcost, mppolicy = question1.ModifiedPolicyIteration(N = 10, M = 5)

          print "=========== Modified Policy Iteration with M = 5 =================="
          print str(alpha) + "     " + str(mpcost) + "     " + str(mppolicy+1)
```

```
============ Modified Policy Iteration with M = 5 ==================
0.9     [64.33097876 70.44746992 55.56634954]     [2 2 2]
```

2.b) Do you find any improvement if you choose mk = 10 k? Explain. (2 marks)

```
In [139]: question1 = Question1(alpha = 0.9)
          mpcost, mppolicy = question1.ModifiedPolicyIteration(N = 10, M = 10)

          print "============ Modified Policy Iteration with M = 10 =================="
          print str(alpha) + "     " + str(mpcost) + "     " + str(mppolicy+1)
```

```
============ Modified Policy Iteration with M = 10 ==================
0.9     [ 91.91066062 101.63094332  83.3489995 ]     [2 2 2]
```

3.) Find an optimal policy using value iteration and Gauss-Seidel value iteration starting with a zero vector. Let = 0.9. What are the optimal values? (5 marks)

```
In [140]: question1 = Question1(alpha=0.9)
          vcost, vpolicy = question1.ValueIteration(N = 150)
          print "============ Value Iteration ============"
          print vcost, vpolicy+1


          question1 = Question1(alpha=0.9)
          gvcost, gvpolicy = question1.GaussSeidelValueIteration(N = 150)
          print "============ Gauss SeidelValue Iteration ============"
          print gvcost, gvpolicy+1
```

```
============ Value Iteration ============
[130.79243435 138.11318907 124.30186832] [2 2 2]
============ Gauss SeidelValue Iteration ============
[130.51816094 137.81893625 124.03704263] [2 2 2]
```

2.0.1 Reference

- Prashanth L. A. CS6700: Reinforcement learning Course notes, 2018
- Dimitri P. Bertsekas. Dynamic Programming and Optimal Control, vol. I. Athena Scientific, 2017.