Soil Carbon Models

11/21/2016

Century Model

The flow diagram for the century model proposed by Parton et al. (1988) is shown in Figure 1. In this document, we re-write the model in terms of a set of differential equations. In this model, we have 5 pools, each with a different turnover times:

$$
\begin{aligned}
&\text{pool 1: Structural C,} && \kappa_1 \approx 1/3,\\
&\text{pool 2: Metabolic C,} && \kappa_2 \approx 1/0.5,\\
&\text{pool 3: Active Soil C,} && \kappa_3 \approx 1/1.5,\\
&\text{pool 4: Slow Soil C,} && \kappa_4 \approx 1/25,\\
&\text{pool 5: Passive Soil C,} && \kappa_5 \approx 1/1000,
\end{aligned}
$$

where κ denotes the decay rate which is defined as 1 over the turnover.

We denote the transfer rate from pool j to pool i by r_{ij}. The transfer rates are parameterized as a ratio of the decay rate: $r_{ij} = \alpha_{ij}\kappa_j$. From Figure 1, we have:

$$
\begin{aligned}
\alpha_{31} &= (1 - \mathrm{A})(1 - 0.45 \times \mathrm{SL} - 0.55 \times \mathrm{BL}),\\
\alpha_{41} &= 0.7 \times \mathrm{A},\\
\alpha_{32} &= 0.45,\\
\alpha_{43} &= 1 - F(\mathrm{T}) - 0.004,\\
\alpha_{53} &= 0.004,\\
\alpha_{34} &= 0.42,\\
\alpha_{54} &= 0.03,\\
\alpha_{35} &= 0.45,
\end{aligned}
$$

where 'SL' is the surface litter, 'BL' is the soil litter, 'A' is the Lignin fraction, 'T' is the soil silt + clay content, and $F(\mathrm{T}) = 0.85 - 0.68 \times \mathrm{T}$. The rest of the transfer coefficients are zero.

For each pool, we can write the following differential equation:

$$
\frac{dC_i(t)}{dt} = I_i(t) - \kappa_i C_i(t) + \sum_{j \neq i} \alpha_{ij}\kappa_j,
$$

where $I_i(t)$ is the external input flow to pool i. As far as I understand, in the century model, the input flows are due to the plant residues and only enter the first two pools. Denoting the total flow due to plant residue by I, we have:

$$\begin{aligned} I_1(t) &= (1 - \mathrm{L/N}) \times I, \\ I_2(t) &= \mathrm{L/N} \times I, \end{aligned}$$

where L/N denotes the Lignin to Nitrogen ratio. Combining all these differential equations into a single formula, we get:

$$\frac{dC(t)}{dt} = \begin{pmatrix} (1 - \mathrm{L/N}) \times I \\ \mathrm{L/N} \times I \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\kappa_1 & 0 & 0 & 0 & 0 \\ 0 & -\kappa_2 & 0 & 0 & 0 \\ \alpha_{31}\kappa_1 & \alpha_{32}\kappa_2 & -\kappa_3 & \alpha_{34}\kappa_4 & \alpha_{35}\kappa_5 \\ \alpha_{41}\kappa_1 & 0 & \alpha_{43}\kappa_3 & -\kappa_4 & 0 \\ 0 & 0 & \alpha_{53}\kappa_3 & \alpha_{54}\kappa_4 & -\kappa_5 \end{pmatrix} C(t).$$

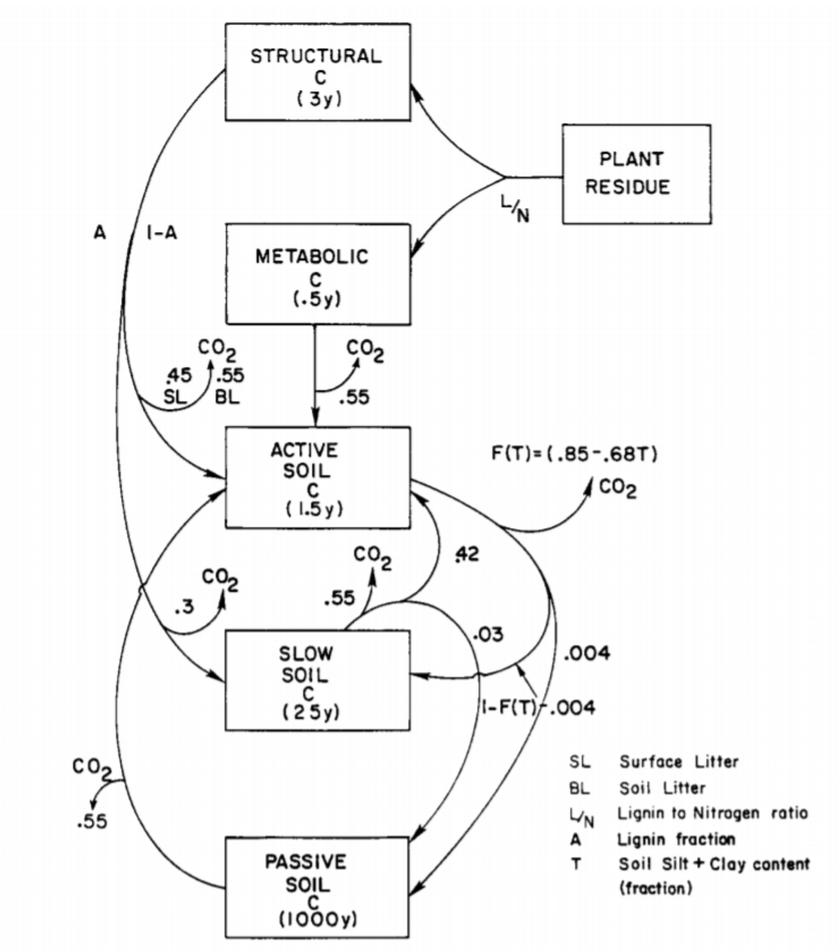

Figure 1: Flow diagram of century model

CN Model

The flow diagram of the CN model, developed by Thronton et al., is shown in Figure 2. There are six pools:

pool 1: Lit1, $\kappa_1 \approx 0.7$,

pool 2: Lit2, $\kappa_2 \approx 0.07$,

pool 3: Lit3, $\kappa_3 \approx 0.014$,

pool 4: SOM1, $\kappa_4 \approx 0.07$,

pool 5: SOM2, $\kappa_5 \approx 0.014$,

pool 6: SOM3, $\kappa_6 \approx 0.0005$.

The transfer rate coefficients are:

$$\alpha_{41} = 0.61, \quad \alpha_{52} = 0.45, \quad \alpha_{63} = 0.71, \quad \alpha_{54} = 0.72, \quad \alpha_{65} = 0.56.$$

So,

$$\frac{dC(t)}{dt} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\kappa_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\kappa_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\kappa_3 & 0 & 0 & 0 \\ \alpha_{41}\kappa_1 & 0 & 0 & -\kappa_4 & 0 & 0 \\ 0 & \alpha_{52}\kappa_2 & 0 & \alpha_{54}\kappa_4 & -\kappa_5 & 0 \\ 0 & 0 & \alpha_{63}\kappa_3 & 0 & \alpha_{65}\kappa_5 & -\kappa_6 \end{pmatrix} C(t).$$

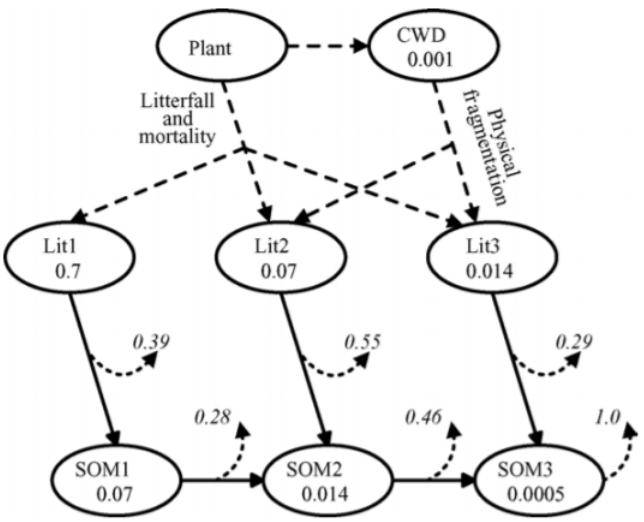

Figure 2: Flow diagram of CN model