CS 412 Intro. to Data Mining

Chapter 6. Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

What Is Pattern Discovery?

- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets
- Pattern discovery: Uncovering patterns from massive data sets
- Motivation examples:
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?
  - What code segments likely contain copy-and-paste bugs?
  - What word sequences likely form phrases in this corpus?

Pattern Discovery: Why Is It Important?

- Finding inherent regularities in a data set
- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: Discriminative pattern-based analysis
  - Cluster analysis: Pattern-based subspace clustering
- Broad applications
  - Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis

Basic Concepts: Frequent Itemsets (Patterns)

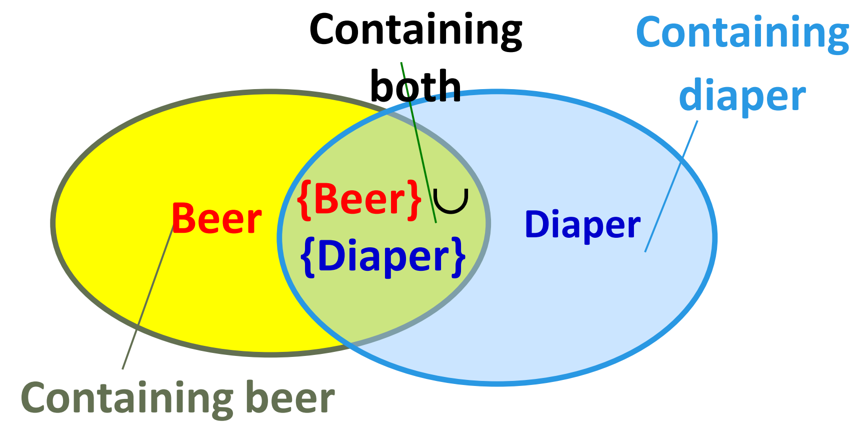
- Itemset: A set of one or more items
- k-itemset: $X = \{x_1, \ldots, x_k\}$
- *(absolute) support (count)* of X: Frequency or the number of occurrences of an itemset X
- *(relative) support*, *s:* The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is *frequent* if the support of X is no less than a *minsup* threshold (denoted as σ)

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- Let *minsup = 50%*
- Freq. 1-itemsets:
  - Beer: 3 (60%); Nuts: 3 (60%)
  - Diaper: 4 (80%); Eggs: 3 (60%)
- Freq. 2-itemsets:
  - {Beer, Diaper}: 3 (60%)

From Frequent Itemsets to Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



{Beer} ∪ {Diaper} = {Beer, Diaper}

Note: Itemset: X ∪ Y, a subtle notation!

- Association rules: $X \rightarrow Y$ (s, c)
  - Support, *s*: The probability that a transaction contains X ∪ Y
  - Confidence, *c: The* conditional probability that a transaction containing X also contains *Y*
  - c = sup(X ∪ Y) / sup(X)
- **Association rule mining**: Find all of the rules, $X \rightarrow Y$, with minimum support and confidence
- Frequent itemsets: Let *minsup = 50%*
  - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - Freq. 2-itemsets: {Beer, Diaper}: 3
- Association rules: Let *minconf = 50%*
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

(Q: Are these all rules?)

Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB_1 contain?
  - TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Assuming (absolute) *minsup* = 1
  - Let's have a try

1-itemsets: $\{a_1\}$: 2, $\{a_2\}$: 2, …, $\{a_{50}\}$: 2, $\{a_{51}\}$: 1, …, $\{a_{100}\}$: 1,

2-itemsets: $\{a_1, a_2\}$: 2, …, $\{a_1, a_{50}\}$: 2, $\{a_1, a_{51}\}$: 1 …, …, $\{a_{99}, a_{100}\}$: 1,

…, …, …, …

99-itemsets: $\{a_1, a_2, \ldots, a_{99}\}$: 1, …, $\{a_2, a_3, \ldots, a_{100}\}$: 1

100-itemset: $\{a_1, a_2, \ldots, a_{100}\}$: 1

  - In total: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1$ sub-patterns!

A too huge set for any computer to compute or store!

Expressing Patterns in Compressed Form: Closed Patterns

- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is *frequent*, and there exists *no super-pattern* Y ↄ X, *with the same support* as X
  - Let Transaction DB TDB_1: T_1: $\{a_1, …, a_{50}\}$; T_2: $\{a_1, …, a_{100}\}$
  - Suppose *minsup* = 1. How many closed patterns does TDB_1 contain?
    - Two: P_1: "$\{a_1, …, a_{50}\}$: 2"; P_2: "$\{a_1, …, a_{100}\}$: 1"
- Closed pattern is a lossless compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - You will still be able to say: "$\{a_2, …, a_{40}\}$: 2", "$\{a_5, a_{51}\}$: 1"

Expressing Patterns in Compressed Form: Max-Patterns

- Solution 2: **Max-patterns**: A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern Y ɔ X
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB_1: T_1: $\{a_1, …, a_{50}\}$; T_2: $\{a_1, …, a_{100}\}$
  - Suppose *minsup* = 1. How many max-patterns does TDB_1 contain?
    - One: P: "$\{a_1, …, a_{100}\}$: 1"
- Max-pattern is a lossy compression!
  - We only know $\{a_1, …, a_{40}\}$ is frequent
  - But we do not know the real support of $\{a_1, …, a_{40}\}$, …, any more!
- Thus in many applications, mining close-patterns is more desirable than mining max-patterns

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Efficient Pattern Mining Methods

- The Downward Closure Property of Frequent Patterns
- The Apriori Algorithm
- Extensions or Improvements of Apriori
- Mining Frequent Patterns by Exploring Vertical Data Format
- FPGrowth: A Frequent Pattern-Growth Approach
- Mining Closed Patterns

The Downward Closure Property of Frequent Patterns

- Observation: From $TDB_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - We get a frequent itemset: $\{a_1, \ldots, a_{50}\}$
  - Also, its subsets are all frequent: $\{a_1\}$, $\{a_2\}$, …, $\{a_{50}\}$, $\{a_1, a_2\}$, …, $\{a_1, \ldots, a_{49}\}$, …
  - There must be some hidden relationships among frequent patterns!
- The downward closure (also called "Apriori") property of frequent patterns
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
  - Apriori: Any subset of a frequent itemset must be frequent
- Efficient mining methodology
  - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?

A sharp knife for pruning!

Apriori Pruning and Scalable Mining Methods

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - Level-wise, join-based approach: Apriori (Agrawal & Srikant@VLDB'94)
  - Vertical data format approach: Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
  - Frequent pattern projection and growth: FPgrowth (Han, Pei, Yin @SIGMOD'00)

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find frequent (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k

F_k : Frequent itemset of size k

K := 1;

F_k := {frequent items}; // frequent 1-itemset

While (F_k != $\emptyset$) **do {** // when F_k is non-empty

C_{k+1} := candidates generated from F_k; // candidate generation

Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;

k := k + 1

}

return $\cup_k F_k$ // return F_k generated at each level

The Apriori Algorithm—An Example

minsup = 2

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

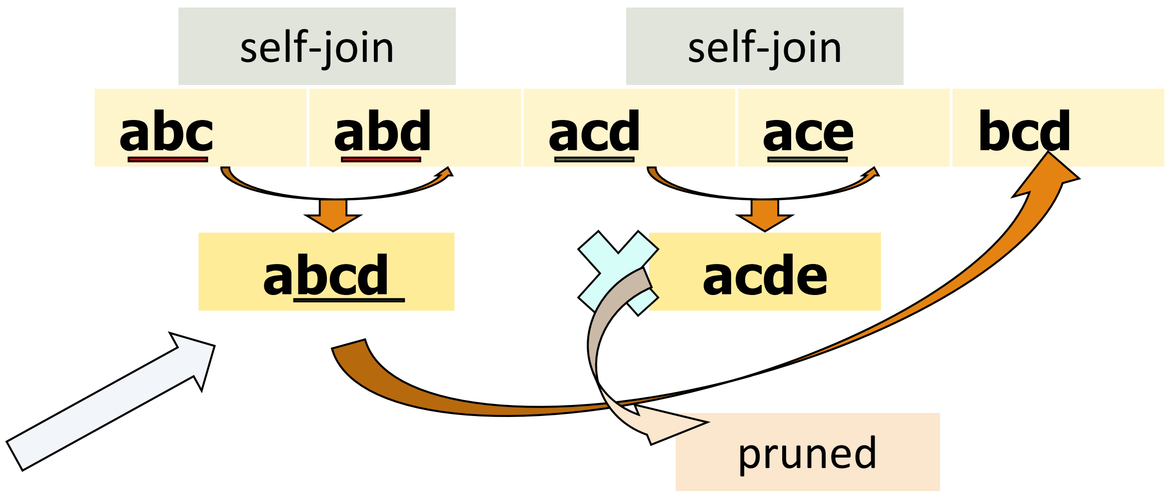
3rd scan

F_3

Itemset	sup
{B, C, E}	2

Apriori: Implementation Tricks

- How to generate candidates?
  - Step 1: self-joining F_k
  - Step 2: pruning
- Example of candidate-generation
  - $F_3 = \{abc, abd, acd, ace, bcd\}$
  - Self-joining: $F_3 * F_3$
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in F_3
  - $C_4 = \{abcd\}$



Candidate Generation: An SQL Implementation

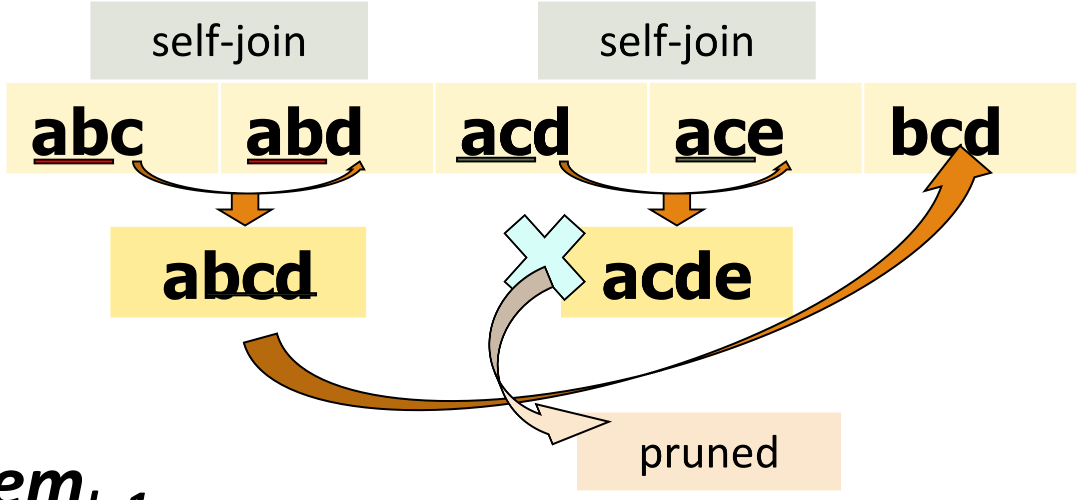
- Suppose the items in F_{k-1} are listed in an order
- Step 1: self-joining F_{k-1}

  insert into $\boldsymbol{C_k}$

  select ***p.item***$_1$, ***p.item***$_2$, ***…***, ***p.item***$_{k-1}$, ***q.item***$_{k-1}$

  from $\boldsymbol{F_{k-1}}$ ***as p***, $\boldsymbol{F_{k-1}}$ ***as q***

  where ***p.item***$_1$ **=** ***q.item***$_1$, ***…***, ***p.item***$_{k-2}$ **=** ***q.item***$_{k-2}$, ***p.item***$_{k-1}$ **<** ***q.item***$_{k-1}$



- Step 2: pruning

  for all ***itemsets c in*** $\boldsymbol{C_k}$ do

  for all ***(k-1)-subsets s of c*** do

  if *(s is not in* F_{k-1}*)* **then delete** *c* **from** C_k

Apriori: Improvements and Alternatives

- Reduce passes of transaction database scans
  - Partitioning (e.g., Savasere, et al., 1995)
  - Dynamic itemset counting (Brin, et al., 1997)

To be discussed in subsequent slides

- Shrink the number of candidates
  - Hashing (e.g., DHP: Park, et al., 1995)
  - Pruning by support lower bounding (e.g., Bayardo 1998)
  - Sampling (e.g., Toivonen, 1996)

To be discussed in subsequent slides

- Exploring special data structures
  - Tree projection (Agarwal, et al., 2001)
  - H-miner (Pei, et al., 2001)
  - Hypecube decomposition (e.g., LCM: Uno, et al., 2004)

Partitioning: Scan Database Only Twice

- Theorem: *Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB*

Here is the proof!

$$TDB_1 + TDB_2 + \cdots + TDB_k = TDB$$

$$sup_1(X) < \sigma|TDB_1| \quad sup_2(X) < \sigma|TDB_2| \quad \cdots \quad sup_k(X) < \sigma|TDB_k| \quad sup(X) < \sigma|TDB|$$

- Method: (A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*)
  - Scan 1: Partition database (how?) and find local frequent patterns
  - Scan 2: Consolidate global frequent patterns (how to?)
- Why does this method guarantee to scan TDB only twice?

Direct Hashing and Pruning (DHP)

- DHP (Direct Hashing and Pruning): Reduce the number of candidates (J. Park, M. Chen, and P. Yu, SIGMOD'95)
- Observation: A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
  - Candidates: a, b, c, d, e
  - Hash entries
    - {ab, ad, ae}
    - {bd, be, de}
    - …
  - Frequent 1-itemset: a, b, d, e
  - ab is not a candidate 2-itemset if the sum of count of {ab, ad, ae} is below support threshold

Itemsets	Count
{ab, ad, ae}	35
{bd, be, de}	298
……	…
{yz, qs, wt}	58

Hash Table

Exploring Vertical Data Format: ECLAT

- ECLAT (Equivalence Class Transformation): A depth-first search algorithm using set intersection [Zaki et al. @KDD'97]
- Tid-List: List of transaction-ids containing an itemset
- Vertical format: $t(e) = \{T_{10}, T_{20}, T_{30}\}$; $t(a) = \{T_{10}, T_{20}\}$; $t(ae) = \{T_{10}, T_{20}\}$
- Properties of Tid-Lists
  - $t(X) = t(Y)$: X and Y always happen together (e.g., $t(ac\} = t(d\})$
  - $t(X) \subset t(Y)$: transaction having X always has Y (e.g., $t(ac) \subset t(ce)$)
- Deriving frequent patterns based on vertical intersections
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - $t(e) = \{T_{10}, T_{20}, T_{30}\}$, $t(ce) = \{T_{10}, T_{30}\}$ → Diffset $(ce, e) = \{T_{20}\}$

A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

FPGrowth: Mining Frequent Patterns by Pattern Growth

- Idea: Frequent pattern growth (FPGrowth)
  - Find frequent single items and partition the database based on each such item
  - Recursively grow frequent patterns by doing the above for each partitioned database (also called *conditional database*)
  - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed
- Mining becomes
  - Recursively construct and mine (conditional) FP-trees
  - Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Example: Construct FP-tree from a Transational DB

TID	Items in the Transaction	Ordered, frequent items
100	$\{f, a, c, d, g, i, m, p\}$	$\{f, c, a, m, p\}$
200	$\{a, b, c, f, l, m, o\}$	$\{f, c, a, b, m\}$
300	$\{b, f, h, j, o, w\}$	$\{f, b\}$
400	$\{b, c, k, s, p\}$	$\{c, b, p\}$
500	$\{a, f, c, e, l, p, m, n\}$	$\{f, c, a, m, p\}$

1. Scan DB once, find single item frequent pattern: **Let min_support = 3**

   f:4, a:3, c:4, b:3, m:3, p:3

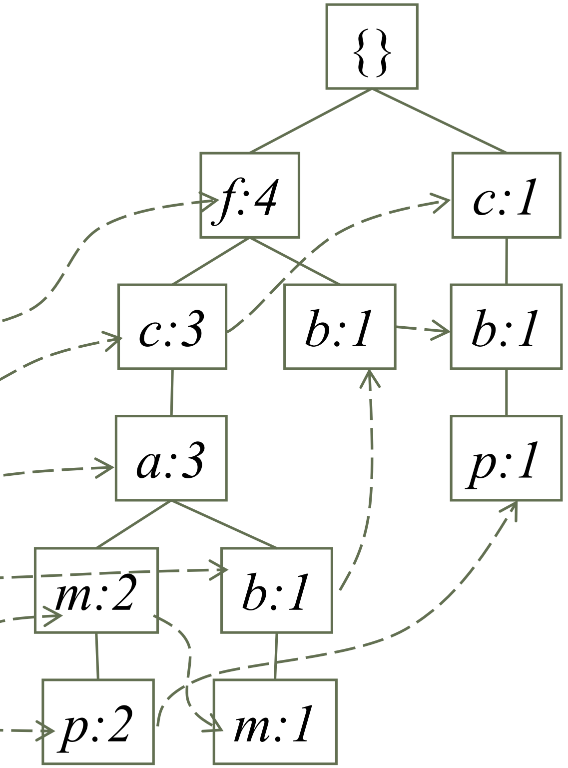
2. Sort frequent items in frequency descending order, f-list

   F-list = f-c-a-b-m-p

3. Scan DB again, construct FP-tree

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

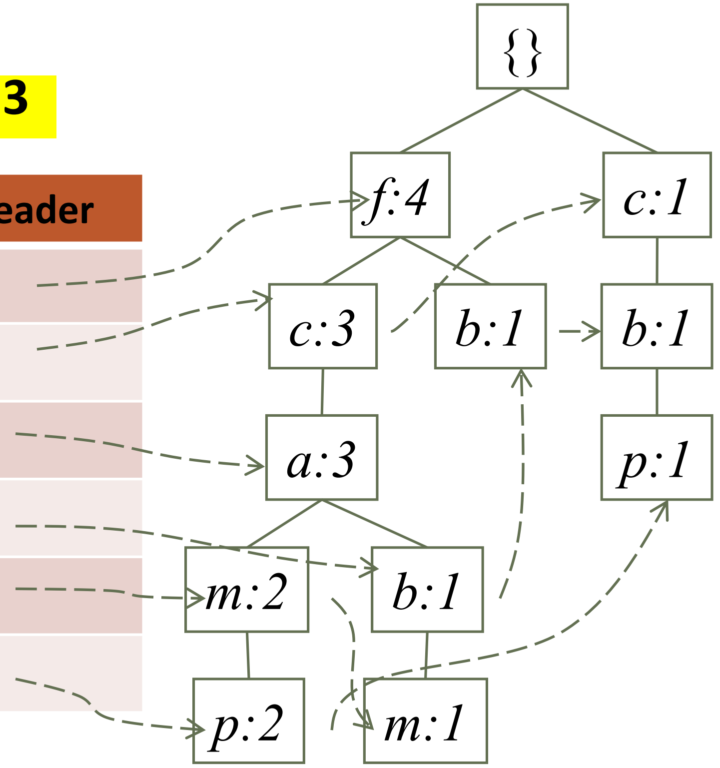


Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
  - Patterns containing p: p's conditional database: *fcam:2, cb:1*
  - Patterns having m but no p: m's conditional database: *fca:2, fcab:1*
  - …… ……
- *p*'s conditional pattern base: *transformed prefix paths* of item *p*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



***Conditional* pattern bases**

Item	*Conditional pattern base*
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

Mine Each Conditional Pattern-Base Recursively

***Conditional* pattern bases**

item	*cond. pattern base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

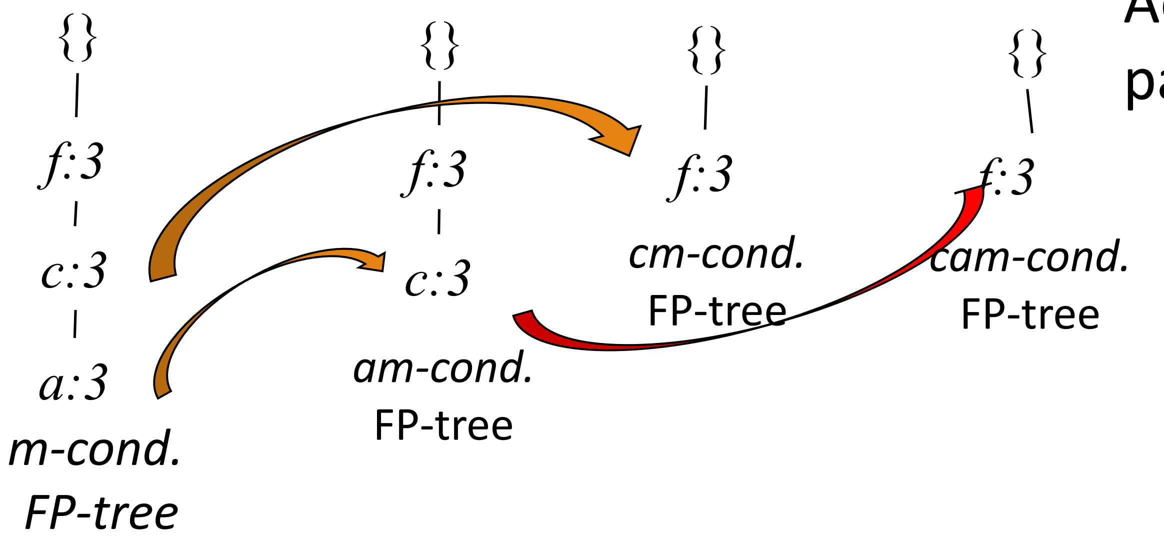
min_support = 3

- For each conditional pattern-base
  - Mine single-item patterns
  - Construct its FP-tree & mine it

p-conditional PB: ***fcam:2, cb:1 → c: 3***

m-conditional PB: ***fca:2, fcab:1 → fca: 3***

b-conditional PB: ***fca:1, f:1, c:1 → ϕ***

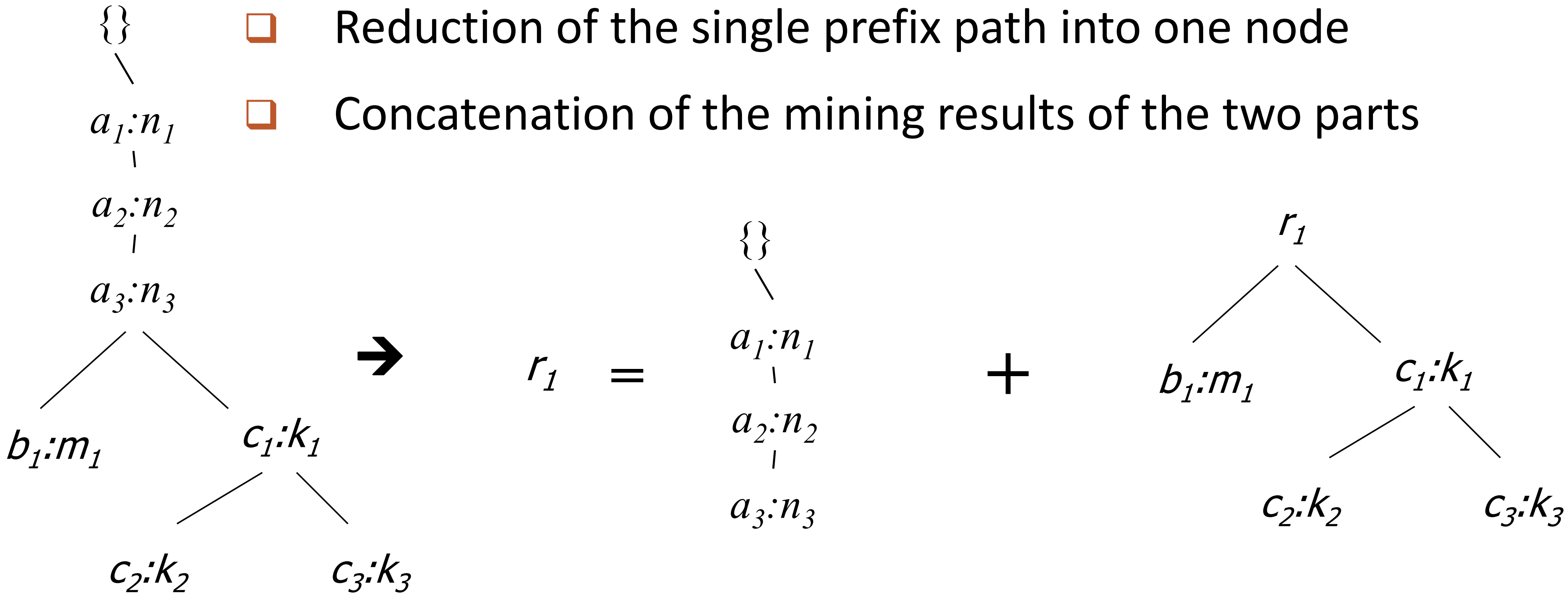


Actually, for single branch FP-tree, all frequent patterns can be generated in one shot

m: 3
fm: 3, cm: 3, am: 3
fcm: 3, fam:3, cam: 3
fcam: 3

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts



Scaling FP-growth by Database Projection

- What if FP-tree cannot fit in memory? — DB projection
  - Project the DB based on patterns
  - Construct & mine FP-tree for each projected DB
- Parallel projection vs. partition projection
  - Parallel projection: Project the DB on each frequent item
    - Space costly, all partitions can be processed in parallel
  - Partition projection: Partition the DB in order
    - Passing the unprocessed parts to subsequent partitions

Trans. DB
f_2 f_3 f_4 g h
f_3 f_4 i j
f_2 f_4 k
f_1 f_3 h
…

Assume only f's are freq. & the freq. item ordering is: f_1-f_2-f_3-f_4

Parallel projection

f_4-proj. DB	**f_3-proj. DB**
f_2 f_3	f_2
f_3	f_1
f_2	…
…	

Partition projection

f_4-proj. DB	**f_3-proj. DB**
f_2 f_3	f_1
f_3	…
f_2	
…	

f_2 will be projected to f_3-proj. DB only when processing f_4-proj. DB

CLOSET+: Mining Closed Itemsets by Pattern-Growth

- Efficient, *direct* mining of closed itemsets
- Ex. Itemset merging: If Y appears in every occurrence of X, then Y is merged with X
  - d-proj. db: {acef, acf} → acfd-proj. db: {e}, thus we get: acfd:2
- Many other tricks (but not detailed here), such as
  - Hybrid tree projection
    - Bottom-up physical tree-projection
    - Top-down pseudo tree-projection
  - Sub-itemset pruning
  - Item skipping
  - Efficient subset checking
- For details, see J. Wang, et al., "CLOSET+: ……", KDD'03

TID	Items
1	acdef
2	abe
3	cefg
4	acdf

Let minsupport = 2

a:3, c:3, d:2, e:3, f:3

F-List: a-c-e-f-d

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How to Judge if a Rule/Pattern Is Interesting?

- Pattern-mining will generate a large set of patterns/rules
  - Not all the generated patterns/rules are interesting
- Interestingness measures: Objective vs. subjective
  - Objective interestingness measures
    - Support, confidence, correlation, …
  - Subjective interestingness measures: One man’s trash could be another man’s treasure
    - Query-based: Relevant to a user’s particular request
    - Against one’s knowledge-base: unexpected, freshness, timeliness
    - Visualization tools: Multi-dimensional, interactive examination

Limitation of the Support-Confidence Framework

- Are *s* and *c* interesting in association rules: "A $\Rightarrow$ B" [*s*, *c*]? Be careful!
- Example: Suppose one school may have the following statistics on # of students who may play basketball and/or eat cereal:

	play-basketball	not play-basketball	sum (row)
eat-cereal	400	350	750
not eat-cereal	200	50	250
sum(col.)	600	400	1000

2-way contingency table

- Association rule mining may generate the following:
  - *play-basketball* $\Rightarrow$ *eat-cereal* [40%, 66.7%] (higher s & c)
- But this strong association rule is misleading: The overall % of students eating cereal is 75% > 66.7%, a more telling rule:
  - ¬ *play-basketball* $\Rightarrow$ *eat-cereal* [35%, 87.5%] (high s & c)

Interestingness Measure: Lift

- Measure of dependent/correlated events: **lift**

$$lift(B,C) = \frac{c(B \rightarrow C)}{s(C)} = \frac{s(B \cup C)}{s(B) \times s(C)}$$

- Lift(B, C) may tell how B and C are correlated
  - Lift(B, C) = 1: B and C are independent
  - > 1: positively correlated
  - < 1: negatively correlated
- For our example, $lift(B,C) = \frac{400/1000}{600/1000 \times 750/1000} = 0.89$

$$lift(B,\neg C) = \frac{200/1000}{600/1000 \times 250/1000} = 1.33$$

- Thus, B and C are negatively correlated since lift(B, C) < 1;
  - B and ¬C are positively correlated since lift(B, ¬C) > 1

Lift is more telling than s & c

	B	¬B	Σ_{row}
C	400	350	750
¬C	200	50	250
$\Sigma_{col.}$	600	400	1000

Interestingness Measure: χ^2

- Another measure to test correlated events: χ^2

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

- General rules
  - $\chi^2 = 0$: independent
  - $\chi^2 > 0$: correlated, either positive or negative, so it needs additional test
- Now, $c^2 = \frac{(400 - 450)^2}{450} + \frac{(350 - 300)^2}{300} + \frac{(200 - 150)^2}{150} + \frac{(50 - 100)^2}{100} = 55.56$
- χ^2 shows B and C are negatively correlated since the expected value is 450 but the observed is only 400
- χ^2 is also more telling than the support-confidence framework

	B	¬B	Σ_{row}
C	400 (450)	350 (300)	750
¬C	200 (150)	50 (100)	250
Σ_{col}	600	400	1000

Expected value

Observed value

Lift and χ^2 : Are They Always Good Measures?

- Null transactions: Transactions that contain neither B nor C
- Let's examine the dataset D
  - BC (100) is much rarer than B¬C (1000) and ¬BC (1000), but there are many ¬B¬C (100000)
  - Unlikely B & C will happen together!
- But, Lift(B, C) = 8.44 >> 1 (Lift shows B and C are strongly positively correlated!)
- χ^2 = 670: Observed(BC) >> expected value (11.85)
- *Too many null transactions may "spoil the soup"!*

	B	¬B	Σ_{row}
C	100	1000	1100
¬C	1000	100000	101000
$\Sigma_{col.}$	1100	101000	102100

null transactions

Contingency table with expected values added

	B	¬B	Σ_{row}
C	100 (11.85)	1000	1100
¬C	1000 (988.15)	100000	101000
$\Sigma_{col.}$	1100	101000	102100

Interestingness Measures & Null-Invariance

- *Null invariance:* Value does not change with the # of null-transactions
- A few interestingness measures: Some are null invariant

Measure	Definition	Range	Null-Invariant
$\chi^2(A, B)$	$\sum_{i,j=0,1} \frac{(e(a_i b_j)-o(a_i b_j))^2}{e(a_i b_j)}$	$[0, \infty]$	No
$Lift(A, B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$AllConf(A, B)$	$\frac{s(A \cup B)}{max\{s(A), s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A, B)$	$\frac{s(A \cup B)}{s(A)+s(B)-s(A \cup B)}$	$[0, 1]$	Yes
$Cosine(A, B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A, B)$	$\frac{1}{2}(\frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)})$	$[0, 1]$	Yes
$MaxConf(\text{A, B})$	$max\{\frac{s(A)}{s(A \cup B)}, \frac{s(B)}{s(A \cup B)}\}$	$[0, 1]$	Yes

X^2 *and lift are not null-invariant*

Jaccard, consine, AllConf, MaxConf, and *Kulczynski* are *null-invariant measures*

Null Invariance: An Important Property

- Why is null invariance crucial for the analysis of massive transaction data?
  - Many transactions may contain neither milk nor coffee!

milk vs. coffee contingency table

	$milk$	$\neg milk$	Σ_{row}
$coffee$	mc	$\neg mc$	c
$\neg coffee$	$m\neg c$	$\neg m\neg c$	$\neg c$
Σ_{col}	m	$\neg m$	Σ

- Lift and χ^2 are not null-invariant: not good to evaluate data that contain too many or too few null transactions!
- Many measures are not null-invariant!

Null-transactions w.r.t. m and c

Data set	mc	$\neg mc$	$m\neg c$	$\neg m\neg c$	χ^2	$Lift$
D_1	10,000	1,000	1,000	100,000	90557	9.26
D_2	10,000	1,000	1,000	100	0	1
D_3	100	1,000	1,000	100,000	670	8.44
D_4	1,000	1,000	1,000	100,000	24740	25.75
D_5	1,000	100	10,000	100,000	8173	9.18
D_6	1,000	10	100,000	100,000	965	1.97

Comparison of Null-Invariant Measures

- Not all null-invariant measures are created equal
- Which one is better?
  - D_4—D_6 differentiate the null-invariant measures
  - Kulc (Kulczynski 1927) holds firm and is in balance of both directional implications

2-variable contingency table

	$milk$	$\neg milk$	Σ_{row}
$coffee$	mc	$\neg mc$	c
$\neg coffee$	$m\neg c$	$\neg m \neg c$	$\neg c$
Σ_{col}	m	$\neg m$	Σ

All 5 are null-invariant

Data set	mc	$\neg mc$	$m\neg c$	$\neg m \neg c$	$AllConf$	$Jaccard$	$Cosine$	$Kulc$	$MaxConf$
D_1	10,000	1,000	1,000	100,000	0.91	0.83	0.91	0.91	0.91
D_2	10,000	1,000	1,000	100	0.91	0.83	0.91	0.91	0.91
D_3	100	1,000	1,000	100,000	0.09	0.05	0.09	0.09	0.09
D_4	1,000	1,000	1,000	100,000	0.5	0.33	0.5	0.5	0.5
D_5	1,000	100	10,000	100,000	0.09	0.09	0.29	0.5	0.91
D_6	1,000	10	100,000	100,000	0.01	0.01	0.10	0.5	0.99

Subtle: They disagree on those cases

Analysis of DBLP Coauthor Relationships

Recent DB conferences, removing balanced associations, low sup, etc.

ID	Author A	Author B	$s(A \cup B)$	$s(A)$	$s(B)$	Jaccard	*Cosine*	*Kulc*
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Advisor-advisee relation: Kulc: high, Jaccard: low, cosine: middle

- Which pairs of authors are strongly related?
  - Use Kulc to find Advisor-advisee, close collaborators

Imbalance Ratio with Kulczynski Measure

- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications:

$$IR(A,B) = \frac{|s(A) - s(B)|}{s(A) + s(B) - s(A \cup B)}$$

- Kulczynski and Imbalance Ratio (IR) together present a clear picture for all the three datasets D_4 through D_6
  - D_4 is neutral & balanced; D_5 is neutral but imbalanced
  - D_6 is neutral but very imbalanced

Data set	mc	$\neg mc$	$m \neg c$	$\neg m \neg c$	Jaccard	*Cosine*	*Kulc*	IR
D_1	10,000	1,000	1,000	100,000	0.83	0.91	0.91	0
D_2	10,000	1,000	1,000	100	0.83	0.91	0.91	0
D_3	100	1,000	1,000	100,000	0.05	0.09	0.09	0
D_4	1,000	1,000	1,000	100,000	0.33	0.5	0.5	0
D_5	1,000	100	10,000	100,000	0.09	0.29	0.5	0.89
D_6	1,000	10	100,000	100,000	0.01	0.10	0.5	0.99

What Measures to Choose for Effective Pattern Evaluation?

- Null value cases are predominant in many large datasets
  - Neither milk nor coffee is in most of the baskets; neither Mike nor Jim is an author in most of the papers; ……
- *Null-invariance* is an important property
- Lift, χ^2 and cosine are good measures if null transactions are not predominant
  - Otherwise, *Kulczynski* + *Imbalance Ratio* should be used to judge the interestingness of a pattern
- Exercise: Mining research collaborations from research bibliographic data
  - Find a group of frequent collaborators from research bibliographic data (e.g., DBLP)
  - Can you find the likely advisor-advisee relationship and during which years such a relationship happened?
  - Ref.: C. Wang, J. Han, Y. Jia, J. Tang, D. Zhang, Y. Yu, and J. Guo, "Mining Advisor-Advisee Relationships from Research Publication Networks", KDD'10

Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

Summary: Mining Frequent Patterns, Association and Correlations

- Basic Concepts:
  - Frequent Patterns, Association Rules, Closed Patterns and Max-Patterns
- Frequent Itemset Mining Methods
  - The Downward Closure Property and The Apriori Algorithm
  - Extensions or Improvements of Apriori
  - Mining Frequent Patterns by Exploring Vertical Data Format
  - FPGrowth: A Frequent Pattern-Growth Approach
  - Mining Closed Patterns
- Which Patterns Are Interesting?—Pattern Evaluation Methods
  - Interestingness Measures: Lift and χ^2
  - Null-Invariant Measures
  - Comparison of Interestingness Measures

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