Financial Derivatives (FEM21011) – *Assignment 2*

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Instructions

This assignment should be made with the same group of four students as the first assignment, unless explicitly instructed by the lecturer. It is not allowed to cooperate with other groups, or to copy their results.

Your solutions for this assignment should be provided in a report of at most 10 pages. The report should provide a detailed and careful description of your methodology, the results and their interpretation. Make the report self-contained, so that it can be read without reading these instructions first. All relevant results should be included in the report. Additional results may be provided in an appendix (which does not count for the page limit), but don't 'hide' main results in the appendix purely because of space constraints. Do not copy-paste raw output into the report, but make proper tables and figures as you find these in academic papers and research reports. The order of your answers to the questions does not need to agree with to the order of the questions below, you are free to write your report in the structure that you deem most appropriate to report the findings. Preferably this is a 'standard' paper form (titlepage with names and abstract, introduction, some sections presenting the methods, data and results, and the conclusion, with possibly a list of references and an appendix). Feel free to use (academic) literature, but be sure to in that case add the citation(s) in the proper manner (I will run a plagiary scan). Give any additional assumptions you may need to make. Don't be shy in mentioning shortcomings / points for improvement.

Your grade will not only be awarded based on the answers you provide (so, e.g., whether the price is right or wrong), but also on how you present your results and the quality of your report.

You are free to choose the programming language that you use for the second part, but notify me in advance if you use any other program than Excel (or better, VBA), Matlab, C++, Java, R and Ox. Put your raw code in an appendix.

Part A: Transaction costs in 'small' Binomial Trees

We introduce transaction costs in the binomial tree model. The tree has the structure of that introduced in Section 2.4 of Baxter and Rennie. The size of the up and down moves are given on the bottom of page 41 and are a function of μ and σ. We set μ=0. The interest rate is set to zero, so r=0, and we normalize the bond price B=1 for all times. The initial stock price is denoted by S_0. We consider European call options with strike price K that mature 1 year from now. We introduce (symmetric) transaction costs that are proportional to the stock price. The cost for buying or selling a stock is a fraction c of the stock price. For example, if c=0.05 and the stock price is \$10, then buying or selling the stock costs 0.05x\$10=\$0.50.[1] Assume that the institution does not have to buy or sell the initial amount of the risky asset but already has it in their possession, such that there are no transaction costs here. We consider binomial trees of various number of steps n.

The input settings you use depend on the student numbers of the group members. Sort the last digits of all 4 student numbers. If one or more values are 0, set these to 1. Denote the sorted student numbers with S1, S2, S3, S4; these should be values in the set 1, 2, …, 9. The transaction cost are equal to c=S1/100 (a value in the set 0.01, 0.02, … 0.09). The initial stock price S_0 and strike price K are given by S_0=K=5x(S2+S3)/2 (round to the nearest integer to

[1] This is a way to model the bid-ask spread on financial markets. In this manner one can buy the stock for costs \$10+\$0.50=\$10.50, while selling the stock earns \$10-\$0.50=\$9.50. So c can be thought of the bid-ask spread divided by 2.

get a value in the set \$5, \$10, …, \$45). Finally, for the volatility take σ=max(0.15,S4/20) (a value in the set 0.15, 0.20, …, 0.45). Clearly state the settings you use in the report.

A1.) First, set n=1 to consider a simple stock price process where a stock starts at the initial stock price S_0 and can only move up or down at time 1. Derive the price of both a long and a short European call option. Discuss your findings and compare to the case without transaction costs. Also provide (information on) the replicating portfolios. Hint 1: Use that for European call options expiring in-the-money the replicating portfolio at expiry exists of one unit of the risky asset and a short position in riskless bonds equal to the exercise price. Hint 2: Make sure that you can cover both the costs of the replicating portfolio and the transaction costs.

A2.) Now set n=2. Derive the price of a long European call option. Discuss your findings and compare to the case without transaction costs and the n=1 case. Also here provide (information on) the replicating portfolios.

Part B: Transaction costs in 'large' Binomial Trees

We now extend the settings of Part A to larger binomial trees; that is, for larger values of n. It is only possible to analyze these trees by programming them.

Take the same set-up as in Part A, but only consider long European call options. Keep the input settings as parameters in your program (so define somewhere in your program the variables c, S_0, K, and σ, which you can easily change). Take the specific values (those that you got using your student numbers) as the baseline parameters in your report.

B1.) Answer question A2 for the long European call position again, but now using your program. (This is more a hint than a question. If the answers are not the same, there is something wrong with either your derivations or your code [or both..].)

B2.) For the baseline parameters, plot how the long European call option price depends on the number of steps n. Does the price settle down? Provide intuition for your findings.

B3.) Study how the price of the long European call option depends on the input variables.

Part C: Continuous time setting with and without transaction costs

We now consider Monte Carlo pricing of options based on the continuous time settings. The process of the stock under the real-world measure is given by

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where μ is the drift and σ the volatility of the stock price process.

C1.) In case of no transaction costs, price a European call option with the input settings from before using simulation, and compare to the analytical formula.

C2.) It turns out that in case of transaction costs the usual Black-Scholes formula can still be applied, but the volatility used in the equation has to be adjusted to $\tilde{\sigma} = \sqrt{\sigma\left(1 + \frac{2c\sqrt{n}}{\sigma\sqrt{T}}\right)}$, with n the number of re-adjustment times (take 250). For the case of transaction costs, compare the price of a European call option using this adjustment to your answer in Part B.

C3.) Finally, we'll price an exotic derivative for the case without transaction costs. The derivative we consider pays the stock price if the stock price at time 1 ends up below K, and 0 else. First, using general expressions for S_0, K, and σ, derive the analytical expression for the price of this derivative. Then, use simulation to price the derivative using the usual input settings, and compare to the price when plugging these values in your analytical expression.[2]

[2] In case you have difficulty placing part C3 in your report, feel free to put it in an appendix.