PENSION REFORMS IN AN AGING SOCIETY: A FULLY DISPLAYED COHORT MODEL

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Abstract

We fully display a cohort model of an economy with an aging population, taking into account complications such as varying household size, habit formation, inheritance and credit constraints. Filling the model with numbers, we are able to compare different pension reforms: 1. the base run, where the contribution rate bears the entire burden of adjustment; 2. reduced accrual rates, 3. replacing wage indexation with price indexation and 4. raised retirement age. Whether the policy changes are anticipated or not, private reactions widely differ.

Keywords

Population Aging, Pension Models, Pension Reforms

I. Introduction

There is a growing practical and theoretical interest in the implications of an aging population, especially with regard to pension systems and their reform. Since the seminal work by Auerbach and Kotlikoff (1987), multi-cohort overlapping models have been used in such investigations. Most of these models are quite complex, need special software to produce numerical solutions and still leave out important details which may be of interest to researchers or experts. In the present paper I try to build my own cohort model, which considers certain important details, is relatively easy to program and all the details are given. Some readers may be disappointed that there is no single crucial assumption or unique observation distinguishing the present paper from others. Nevertheless, I am convinced that the present paper contributes to the better understanding of the impact of demographic transition on dynamic pension policy by choosing a relevant set of assumptions and observations.

Before building a model, one must set the questions to be answered. Different problems need different models. My basic question is as follows: how should the public PAYG pension system be reformed under the burden of an aging population? (i) Should only the contribution rate be raised to ensure the balance of the pension system? (ii) Or should the accrual rate be reduced to avoid a radical rise in the contribution rate? (iii) Or should the wage-indexation of pensions be replaced by the more modest wage-price or price

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indexation? (iv) Or should the retirement age be increased so that the original accrual rate and the indexation can be kept and the contribution rate remains moderate?

Such questions are of interest not only to modelers but to politicians as well. For example, in 2009, under the pressures of the Great Recession, the acting Hungarian government had to combine these options: reduce the accrual rate (by eliminating the 13th month pension), replace wage-price indexation by price indexation and propose raising the normal retirement age from 62 (in 2011) to 65 (in 2022). Another option, modelled by Catalán et al. (2010) is to extend the period of the past wages to the full working period used in calculating benefits. (Most models neglect the impact of a partial reference period.)

Of course, economists have often discussed such issues in their models, but they frequently neglected certain options, especially the change in indexation. To see the importance of indexation, let us calculate with an annual real wage increase of 2%. Then each half-step (from wage indexation to wage-price indexation and wage-price indexation to price indexation) diminishes pension outlays by 1% per year, altogether 10% in the long-run.

Here enters the choice of the model. Obviously, we must use a detailed cohort model rather than a crude two-generation model to go beyond the generalities of the traditional contrast young vs. old. Furthermore, we must model both factors of population aging: the fall in fertility and the rise in expected life expectancy. Within such a framework, we must sharply distinguish two expectancies: *period* expectancy and *cohort* expectancy. The former refers to the average age at death in a given period (this is used in practice, and needed in cross-sectional calculations), while the latter refers to the average age at death of those born in the same period (and is used in longitudinal calculations).

One basic reason for introducing a mandatory pension system is to insure risk-averse individuals against longevity risk. Modeling such risks is beyond my scope, and thus I only signal its lack. There are a few models which deal with longevity risk (e.g. De Nardi, Imrohoroglu and Sargent, 1999), but they also experienced great difficulties in keeping track of members of the same cohort with different ages at death. A common but unsatisfactory solution is to assume that the accumulated (public and private) savings of the early deceased are distributed among the survivors (e.g. Heer and Maussner, 2009), like in a PAYG system.

Turning to consumption smoothing, the diminishing size of the average household should be taken into account. (Blundell (1994) and Bütler (1997) empirically analyzed the connection between the life-cycle consumption and the household size.) To give a glaring example: nowadays a Chinese household with one child can and should save relatively much more than a similar household with two or three children thirty years ago. Notwithstanding this evidence, many models (but not mine) working on aging gloss over this relation.

Another dimension of consumption smoothing is that it refers more to relative rather than absolute numbers (*habit formation* in Carroll, Overland and Weil, 2000). Imagine a Chinese household whose adult members started their careers 30 years ago, when Chinese productivity was very low and stagnating. How could they smooth their absolute consumption path at all? Have they aimed way above the initial level and way below the terminal level? Possibly in order to save on additional parameters, a large part of the literature still neglects secular growth. In contrast, I insist on working with relative rather than absolute consumption levels.

We now turn to a basic ingredient of macroeconomics, namely, labor supply. There are modelers who attribute great significance to the elasticity of labor supply, and argue for lower contribution rates to increase labor supply. Other economists doubt its significance (see, Augusztinovics and Köllő, 2008 and Spieza, 2002). For the sake of simplicity, I take labor supply as a given, except in the case of a raised retirement age.

In a normal macromodel, a production function determines the output, the investment and the capital accumulation. Using the theory of marginal productivity, wages and interest rates are determined endogenously, even if the age-dependence of wages is given exogenously. In an economy with an aging population, such models describe the relative abundance of capital and a concomitant falling rate of interest and rising wages rate. This is in accord with Brooks (2000) but contradicts Poterba (2001).

One of the greatest innovations of Auerbach and Kotlikoff (1987) was to study intergenerational dynamics in the general equilibrium framework. They did not stop at maximizing the workers' lifetime utility functions for given wage and interest paths, but in the dynamic framework of capital accumulation, they harmonized the wage and interest paths with the individual decisions. This was a striking application of *perfect foresight*, but it is questionable as to whether it describes the actual economic dynamics well. Can one explain the recent world economic crisis on the basis of perfect foresight or more generally, rational expectations? At the same time, this insistence on the logical coherence makes the calculations very demanding: each of the three methods of determination of the steady state in Heer and Maussner (2009, Chapter 9.1.2, pp. 458–469) requires pages!

While we consider a closed economy, Börsch-Supan, Ludwig and Winter (2002) create a world-model: they claim that increased capital exports from developed countries to underdeveloped countries can solve the problems of population aging (see also Baker, Delong and Krugman, 2005).

Admitting the elegance of the dynamic general equilibrium approach, we rely on a short-cut: output changes exogenously, as a product of the labor force and productivity. One advantage of this short-cut is as follows: there is no need for the quite artificial backward recursion, and the problem of instability and indeterminacy disappears. In addition, I assume an *ad hoc* drop in the interest rate. (In a rather abstract analytical model, where every type of external change – in wage, fertility, life expectancy – was missing, Molnár and Simonovits (1998) compared perfect foresight with *naive expectations* and proved that the latter are more stable than the former.)

Most models work with a single interest rate and assume that there is no difference between the interest rate received while saving and that paid during dissaving. This is a very rough simplification and can be avoided by introducing credit constraints, i.e. excluding consumer credits. This makes the calculations much more difficult but worth the price. The existence of this constraint is especially important when one models the privatization and prefunding of the public pay-as-you-go pension system, and lower-paid workers cannot pay their new contributions by relying on credits. (Hubbard and Judd (1986) and Hubbard, Skinner and Zeldes (1995) were among the first to consider the issue of *credit constraint*, when consumers cannot finance their current consumption from future incomes. In a related approach, Kimball (1990) introduced *precautionary savings* into the model, when the uncertainty over future earnings forces the young to save more than is warranted in a deterministic world.)

Discussing the importance of longevity risk, we have already mentioned the issue of unwanted bequest. Here we return to the wanted bequest, having a strong interaction with the credit constraint. It is a basic, though hotly discussed question of life-cycle theory as to whether *intended bequest* is a small part of capital accumulation (Ando and Modigliani, 1963) or a significant one (Kotlikoff and Summers, 1981).

In addition to pension contributions, health care contributions and pure taxes also play important roles in defining net incomes. In our model, however, these features are neglected. In any development of the present model, the introduction of taxes is imperative and opens the door to model income redistribution within cohorts. Indeed, wage heterogeneity is important in analyzing the various pension reforms. For example, the privatization of the public pension system inevitably reduces the progressivity of the pension system (Fehr, 2000). Or, in reconsidering the Hungarian pension reform package, and insisting on an adequate minimal benefit, poorer pensioners' benefits cannot be reduced as steeply as those of others', and therefore the projected savings may be much less than suggested above.

Finally, we arrived at the far-sightedness of the individuals. Following the bulk of the literature, *in general* I also assume that workers foresee the dynamics of the economy. They are assumed to know the future paths of interest rates, contribution rates, accrual rates and plan their savings so as to maximize their lifetime utilities. This is a very heroic assumption which heavily influences our calculations. At least at the end, I give up this assumption, and assume that, at a given period, there is a sudden unexpected change in the government pension policy which the workers only recognize with a lag. Small wonder that in such a set-up, workers cannot smooth their consumption as nicely as in the idealized framework.

I have enumerated 13 dimensions of the pension models. Of course, they give rise to about eight thousand different combinations. I only pick six combinations, representing six different papers and compare them in Table 0. We shall see that none of them dominates the others or is dominated by the others. I hope that the last two combinations, representing my earlier and present works stand the comparison with the five other models.

Table 0 compares the characteristics of the present model with five other related ones: $+$ stands for the presence of the property, $-$ stands for its lack.

Table 0: Models and assumptions

Models Characteristics	AHKN (1989)	Bütler (1997)	Fehr (2000)	HM 2009	AS (2003)	AS (2012)
Decreasing birth rate	+	−	+	+	+	+
Increasing life expectancy	+	−	+	−	+	+
Longevity risk	−	−	−	+	−	−
Household size – consumption	+	±	−	−	−	+
Intracohort differences	−	−	+	−	−	−
Habit formation	−	−	−	−	+	+
Flexible labor supply	+	+	+	+	−	−
Endogenous wage	+	−	+	+	−	−
Endogenous interest rate	+	−	+	+	−	+
Credit constraint	−	±	−	−	−	+
Bequest	+	−	−	−	−	+
Taxes	+	+	+	+	−	−
Surprise in pension policy	−	+	−	+	−	±

Abbreviations: AHKN = Auerbach, Hagemann, Kotlikoff and Nicoletti (1989), HM = Heer and Maussner, AS (2003) = Simonovits (2003, Chapter 15) and AS (2012) = the present paper, respectively.

To give a single example of complications arising from this family of models, we mention the case of calculating the balanced path of contribution rates. Benefits depend on past net wages, containing past contribution rates, while the current contribution rate depends on current benefits, i.e. past contribution rates. Under such circumstances, it is not that easy to calculate the initial contribution rates to begin with.

Notwithstanding its complications, the present model is relatively simple, because its block-structure is *reducible*: the demography block is self-contained, the wage-and-pension block only depends on the demography but is independent of the consumption and asset block, etc. The model reflects the historically fast transition from a stationary population into a contracting stable population, occurring between 1950 and 2000. Although the model neglects a number of important features of such a transition, the results obtained nevertheless seem to be sensible, useful and novel.

As a summary of the literature, we repeat that Auerbach and Kotlikoff (1987) invented the approach also followed here: in an overlapping cohort model with age (year) groups, where fertility is decreasing, the pay-as-you-go public pension system interacts with optimizing individuals under a sophisticated system of perfect foresight. Auerbach, Hagemann, Kotlikoff and Nicoletti (1989) extended this analysis into several directions: varying household size, bequest, open economy and international comparison. The authors discussed alternative economic policies: a) fixing the per-capita government expenditure, b) raising the retirement age by two years, c) reducing benefits by 20%. The model is so rich that only the neglect of habit formation and credit constraints can be missed.

Finally, I would like to deal with the direct precursors of the present model. In my book I have modeled important issues of pension reforms: the impact of indexation, the pre-funding of the pay-as-you-go system (Simonovits, 2003, Chapters 14 and 15), in a shrinking population but neglected the rise in life expectancy. Oksanen (2004) and Beetsma and Oksanen (2007) studied the intergenerational equity of pension reforms in a three-generation model. Together with Oksanen (2009), my paper was motivated to overcome the limitations mentioned. Both papers describe the demographic structure more realistically, taking into account the bequests and the workers' optimal saving paths. The differences between them are quantitative rather than qualitative: Oksanen was able to incorporate fractional decades into his model, worked out surprises in full and surveyed the literature in detail. On the other hand, the present paper models intertemporal substitution and age-dependent wages and, moreover, contains the full mathematical model.

Further scenarios will be discussed in future developments: privatization of part of the public system, insurance against stochastic shocks, etc.

The structure of the remainder of the paper is as follows: Section II outlines a macromodel with given consumption paths. Section III introduces optimal consumption paths to derive asset dynamics. Section IV displays the numerical calculations. Section V concludes. The Appendix contains the more sophisticated proofs.

II. A macromodel

We start the presentation with a demographic block and continue with a wage and a pension block, yielding the macroblock.

Demographic block

Here we describe the demographic block, which is simple enough (every member of a given cohort dies at the same age) but works with cohorts rather than generations and takes care of dropping fertility and increasing life expectancy.

Let t be the index of calendar periods, $t = \ldots, -1, 0, 1, \ldots$ but also $\ldots, 1940, 1950, 1960, \ldots$. We shall always use the following principle of notation: when a quantity depends on age as well as on calendar time, then the first index refers to age, and the second to the calendar time. Let us denote the number of aged i in period t by $n_{i,t}$.

Concerning changing life expectancy, we must differentiate between so-called *period* t life expectancy (the traditional term): I_t and the so-called *cohort* t life expectancy: $\mathbf{I}_t$. The former stands for the average age at which people *die* in t, the latter stands for the average age at death of people *born* in t. We have the following relation between them: $\mathbf{I}_{t-I_t} = I_t$. For example, if the expected age of people at death is equal to 80 in 2050, then $\mathbf{I}_{1970} = I_{2050} = 80$. $\mathbf{I}_t$ appears in the longitudinal, individual level (in lifetime budget constraints), while I_t appears in the macrorelations (cross-sectional balances).

To get rid of the complexities of a two-sex world, we assume unisex single parents and children. If the parent was born in period t, then all his children are born at once, in period $t + H$, and their number is $2f_{t+H}$, f_{t+H} staying with him/her, the other f_{t+H} with her/him, where H is a positive integer. A child stays with his parent until age L, when he starts to work, L also being a positive integer. (Though both H and L have been increasing, we neglect this complication.) People born in t retire at age $\mathbf{J}_t$, generally

a time-varying integer. Again, $\mathbf{J}_{t-J_t} = J_t$. We assume that only workers have children in their household, i.e. $L < H < \mathbf{J}_t - L$.
To sum up, a person born in period t, starts working in period $t + L$, gives birth to f_{t+H} children in period $t + H$, separates from them in period $t + H + L$, retires in period $t + \mathbf{J}_t$ and dies in period $t + \mathbf{I}_t$.
We have the following demographic equations for $t \geq 0$:

$$n_{i,t} = \begin{cases} f_t n_{H,t} & \text{if } i = 0; \\ n_{i-1,t-1} & \text{if } i = 1, \ldots, I_t; \\ 0 & \text{if } i > I_t. \end{cases}$$

In words: the first, second and third rows give the number of newborns, those of the aged-i and the exiting cohort, respectively. In the transition of dynamic systems, we assume that the initial values of birth numbers, namely $n_{0,-I_0}, n_{0,-I_0+1}, \ldots, n_{0,-1}$ are given.
Denote $N_t = \sum_{i=0}^{I_t} n_{i,t}$ the population size in period t, then the growth factor of the total population is equal to

$$\nu_t = \frac{N_t}{N_{t-1}}.$$

For stable populations, where $f_t = f$ and $I_t = I$, $\nu_t = \nu = f^{1/H}$ holds.

Wage block

Let us denote by $w_{i,t}$ the household head's total wage at age i in period t. We assume that as time passes, the earning-age function is multiplied by the time-invariant productivity growth factor $g > 1$:

$$w_{i,t} = w_{i,L} g^{t-L} = w_i g^{t-L}, \qquad i = L{+}1, L{+}2, \ldots, J_t \quad \text{and} \quad t = -2, -1, 0, 1, 2, \ldots,$$

where w_i is the wage of the i-aged in period L and $w_{L,L} = w_L = 1$. (Note that the wage structure may depend on the demographic situation, as is persuasively argued by Akihiko (2006, Figure 35, p. 143), but we neglect this fact.) Later on, it will be useful to define $w_{i,t+i} = 0$ for $i > J_t$.
We define the aggregate total wage:

$$W_t = \sum_{i=L}^{J_t} n_{i,t} w_{i,t}.$$

The endogenous interest factor is determined as the product of a constant $\alpha > 1$ and the growth factor of the aggregate total wage:

$$R_t = \alpha \frac{W_t}{W_{t-1}}.$$

Note that this is a simplification that makes the present model a partial equilibrium model. In the dynamic general equilibrium theory, the interest factor is determined from the macroequilibrium conditions: either assuming perfect foresight concerning the interest rate (e.g. Auerbach and Kotlikoff, 1987) or naive expectations (cf. Molnár and Simonovits, 1998). In steady states, $R = \alpha\nu g$.

Pension block

Here we outline our pension block, first a pay-as-you-go *public pension system* and income $y_{i,t}$ is either net wage $(1-\tau)w_{i,t}$ or pension benefit $b_{i,t}$. We have already met the retirement age $\mathbf{J}_t$ of the person born in period t (an integer) and the retirement age J_t in period $t+\mathbf{J}_t$, as determined by the government. We can now describe the pension system as follows. Our individual, born in t, contributes $\tau_{i,t+i}w_{i,t+i}$ to the public pension system at age $i = L, L+1, \ldots, \mathbf{J}_t$ and receives a *pension benefit* $b_{i,t+i}$ at age $i = \mathbf{J}_t + 1, \ldots, \mathbf{I}_t$. Thus his income path changes as follows:

$$y_{i,t+i} = \begin{cases} (1-\tau_{t+i})w_i g^{t-L+i} & \text{if } L \le i \le \mathbf{J}_t; \\ b_{i,t+i} & \text{if } \mathbf{J}_t < i \le \mathbf{I}_t. \end{cases}$$

We assume a pension system, where the *entry benefit* is given as a linear function of all past net wages, the coefficients being called *accrual rates*:

$$b_{\mathbf{J}_t+1,t+\mathbf{J}_t+1} = \sum_{j=L}^{\mathbf{J}_t} \theta_{t+j}(1-\tau_{j,t+j})w_{j,t+j}g^{\mathbf{J}_t-j+1} = \sum_{j=L}^{\mathbf{J}_t} \theta_{t+j}(1-\tau_{j,t+j})w_{j,L}g^{t+\mathbf{J}_t+1}.$$

Note that in practice, θ_{t+j} and $\tau_{j,t+j}$ may change in time, but in theory, if we assume time-invariant accrual rate θ and contribution rate τ, then the entry benefit is proportional to *valorized lifetime net earnings* $(1-\tau)\hat{w}_{t+\mathbf{J}_t+1}$:

$$b_{\mathbf{J}_t+1,t+\mathbf{J}_t+1} = \theta(1-\tau)\hat{w}_{t+\mathbf{J}_t+1},$$

where

$$\hat{w}_{t+\mathbf{J}_t+1} = \sum_{j=L}^{\mathbf{J}_t} w_{j,t+j}g^{\mathbf{J}_t-j+1} = g^{t-L+\mathbf{J}_t+1}\sum_{j=L}^{\mathbf{J}_t} w_{j,L}.$$

Note that Catalán et al. (2010) extended the analysis to the empirically relevant case wherein calculating the entry benefit only the earnings of the last few years are taken into account, with proportionally raised accrual rates.

The *continued benefits* are wage-price-indexed with shares ι_t and $1-\iota_t$, respectively:

$$b_{i+1,t+i+1} = b_{i,t+i}g^{\iota_t}, \qquad i = \mathbf{J}_t + 1, \ldots, \mathbf{I}_t - 1.$$

We consider the individual *pension wealth*, which plays a prominent role in the evaluation of the cohort-specific burden of unfunded pension systems. We shall define the *implicit pension wealth* $d_{i,t+i}$ of a person born in t at age i as the present value of outstanding pension benefits at the end of this period. To formulate the implicit pension debt, we need to introduce the *compounded interest factor* in the time interval $[v, z]$:

$$\rho_{v,z} = \prod_{t=v+1}^{z} R_t \quad \text{for} \quad z > v \quad \text{and} \quad \rho_{v,v} = 1.$$

For a worker who was born in period t, his earning $w_{h,t+h}$ in period $t+h$ will yield pension "part" $\theta_{t+h}(1-\tau_{h,t+h})w_{h,t+h}g^{i-h}$ in period $t+i$, for $h = L, \ldots, \mathbf{J}_t$ and $i = \mathbf{J}_t + 1, \ldots, \mathbf{I}_t$. Taking account of $w_{h,t+h} = w_{h,L}g^{t+h-L}$, summing up, and discounting to the period $t+j$ yields *pension wealth*

$$d_{j,t+j} = g^{t-L} \sum_{h=L}^{j} \theta_{t+h}(1-\tau_{h,t+h})w_{h,L} \sum_{i=\mathbf{J}_t+1}^{\mathbf{I}_t} g^i \rho^{-1}_{t+j,t+i}, \qquad j = L, \ldots, \mathbf{J}_t.$$

A pensioner's pension wealth is equal to the sum of the remaining claims between periods $t+i+1$ and $t+\mathbf{I}_t$:

$$d_{i,t+i} = \sum_{h=i+1}^{\mathbf{I}_t} b_{h,t+h}\rho^{-1}_{t+i,t+h}, \qquad i = \mathbf{J}_t + 1, \ldots, \mathbf{I}_t.$$

In aggregate relations, profiles rather than paths appear. (Pension wealth profiles are derived in the Appendix.) The pension system need not be balanced, i.e. the difference between the aggregate benefits and the aggregate contribution defines the *pension budget deficit*:

$$G_t = \sum_{i=L}^{I_t} n_{i,t}(b_{i,t} - \tau_t w_{i,t}).$$

The *explicit* debt of the public pension system follows the well-known dynamic equation:

$$D^E_t = R_t D^E_{t-1} + G_t.$$

Aggregating the individual implicit pension wealths, the aggregate *implicit* pension debt is as follows:

$$D^I_t = \sum_{i=L}^{I_t} n_{i,t} d_{i,t}.$$

Adding together the implicit and explicit pension wealths results in the *total pension wealth*

$$D_t = D^I_t + D^E_t.$$

If the retirement age J_t is constant, then the pension system is called *intergenerationally equitable* if the total pension debt grows parallel with the output: D_t/Y_t is constant. If the retirement age varies over time, then a more sophisticated definition is needed (see Beetsma and Oksanen, 2007).

In a lot of countries in a lot of periods, pension systems are purely pay-as-you-go, i.e. the annual pension deficit is equal to zero: $G_t = 0$. Then the contribution rate τ_t^{o} is equal to

$$\tau_t^{\mathrm{o}} = \frac{\sum_{i=J_t+1}^{I_t} n_{i,t} b_{i,t}}{\sum_{j=L}^{J_t} n_{j,t} w_{j,t}}.$$

In a general model, $(b_{i,t})$ give the initial conditions and then τ_t^{o} is determined. Note, however, that $(b_{i,t})$ depend in turn on $\tau_{-J_1+t}, \ldots, \tau_{-1+t}$. If we do not want to define initial contribution rates, we may assume that the system started in a steady state. Then with substitution, and assuming wage-indexed pensions with $\iota_t = 1$: $b_{i,t} = b_t$ and denoting the number of pensioners by P_t yields a steady-state equation and contribution rate:

$$\tau W_t = \theta P_t \hat{w}_t - \tau \theta P_t \hat{w}_t, \quad \text{i.e.} \quad \tau = \frac{\theta P_t \hat{w}_t}{W_t + \theta P_t \hat{w}_t}, \qquad t < 0.$$

Until now we have neglected consumption paths; from now on we can derive them from individual life-cycle optimization.

III. Optimal consumption paths

In this section, we first discuss a simple life-cycle model, where households maximize a standard lifetime utility function under a standard lifetime budget constraint. Second, we introduce complications such as habit formation, inheritance and credit constraint and shocks. This helps us derive asset dynamics, as well as opening the way to consider the partial prefunding of the unfunded pension system.

A simple household life-cycle model

As is usual, we build up the household consumption block from microeconomic variables. Let us denote $c_{i,t}$ the household head's consumption at age i in period t. At this stage, we take the adult consumption as given and delay its explanation for a while.
Consider an adult born in period t and his household in later periods. We assume that each child consumes μ times as his parent does ($0 < \mu \leq 1$). Let us denote the household consumption size by $m_{i,t}$ (cf. Meier and Wrede, 2005). Then we have

$$m_{i,t+i} = \begin{cases} 1 + \mu f_{t+H} & \text{if } H \leq i < H + L; \\ 1 & \text{if } L \leq i < H \text{ or } H + L \leq i \leq \mathbf{I}_t. \end{cases}$$

Here we introduce aggregate consumption in period t:

$$C_t = \sum_{i=L}^{I_t} n_{i,t} m_{i,t} c_{i,t}.$$

The end-of-period accumulated assets $a_{i,t+i}$ and the per-period saving $s_{i,t+i}$ of a household are defined as

$$a_{i,t+i} = R_{t+i}a_{i-1,t-1+i} + y_{i,t+i} - m_{i,t+i}\, c_{i,t+i}$$

and

$$s_{i,t+i} = a_{i,t+i} - a_{i-1,t-1+i} = a_{i-1,t-1}(R_{t+i} - 1) + y_{i,t+i} - m_{i,t+i}\, c_{i,t+i},$$

respectively. The initial and end values of assets are equal to zero: $a_{L-1,t} = 0 = a_{I_t,t}$. We define aggregate assets and saving respectively as

$$A_t = \sum_{i=L}^{I_t} n_{i,t}\, a_{i,t} \quad \text{and} \quad S_t = \sum_{i=L}^{I_t} n_{i,t}\, s_{i,t}.$$

By definition, $A_t = R_t A_{t-1} + S_t$.

Since working households pay pension contributions and pensioner households receive pension benefits, income $y_{i,t+i}$ differs from earning $w_{i,t+i}$. Therefore, we shall formulate the budget constraint with the former rather than the latter. Using the capital (present) values of incomes and consumption in period $t + L$, the lifetime budget constraint of the person born in period t is as follows:

$$\sum_{i=L}^{\mathbf{I}_t} \rho_{t+L,t+i}^{-1}(y_{i,t+i} - m_{i,t+i}\, c_{i,t+i}) = 0.$$

To determine the optimal consumption path $(c_{i,t+i})_{t=L}^{\mathbf{I}_t}$, we assume the following household lifetime utility function:

$$\sum_{i=L}^{\mathbf{I}_t} \delta^{i-L} u_i(c_{i,t+i}),$$

where $0 < \delta \leq 1$ is the discount factor and $u_i(c_{i,t+i})$ is the household head's per-period utility function at age i. To take care of the age-specific and time-variant household size, we assume that the per-period utility function at age i is equal to a time-invariant per-capita utility function multiplied by the size of the household and indicator variable $\beta_{i,t+i}$, equaling 1 if the consumer works and dropping to $0 < \beta < 1$ if he is retired (cf. Scholz, Sheshadri and Khitatrakun, 2006):

$$u_i(c_{i,t+i}) = \beta_{i,t+i}\, m_{i,t+i}\, u(c_{i,t+i}),$$

where

$$\beta_{i,t+i} = \begin{cases} 1 & \text{if } L \leq i \leq \mathbf{J}_t; \\ \beta & \text{if } \mathbf{J}_t < i \leq \mathbf{I}_t. \end{cases}$$

To obtain nice analytical results, we must assume a CRRA-utility function:

$$u(x) = \begin{cases} \dfrac{x^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 1; \\ \log x & \text{if } \gamma = 1. \end{cases}$$

(To take into account the inelastic intertemporal substitution, we exclude the case $0 \le \gamma < 1$.)
Then the optimal consumption path is given

$$c_{L,t+L} = \frac{\sum_{i=L}^{\mathbf{I}_t} \rho_{t+L,t+i}^{-1} y_{i,t+i}}{\sum_{i=L}^{\mathbf{I}_t} \delta^{(i-L)/\gamma} \rho_{t+L,t+i}^{1/\gamma-1} \beta_{i,t+i}^{1/\gamma} m_{i,t+i}}$$

and

$$c_{i,t+i} = \delta^{(i-L)/\gamma}(\rho_{t+L,t+i}\,\beta_{i,t+i})^{1/\gamma} c_{L,t+L}, \qquad i = L+1, L+2, \ldots, \mathbf{I}_t.$$

For future use (e.g. at reoptimization after shocks), it will be worthwhile to derive another form of optimal consumption path, which does not distinguish between initial and continued consumptions. On the other hand, it also relies on $a_{i-1,t-1}$ defining the capital value of the remaining life path. Because $\beta_{L,t} = 1$ but $\beta_{i,t}$ may be different from 1, $1/\beta_{i,t}$ appears in the denominator.
Consumption at age i in period t

$$c_{i,t} = \frac{R_t a_{i-1,t-1} + \sum_{j=i}^{\mathbf{I}_{t-i}} \rho_{t,t+j-i}^{-1} y_{j,t+j-i}}{\sum_{j=i}^{\mathbf{I}_{t-i}} \delta^{(j-i)/\gamma} \rho_{t,t+j-i}^{1/\gamma-1} (\beta_{j,t+j-i}/\beta_{i,t})^{1/\gamma} m_{j,t+j-i}}$$

and determine $a_{i,t}$ and $c_{i,t+1}$, respectively.

A complex life-cycle model

Having discussed the simple life-cycle block, it is time to introduce the complications: habit formation, inheritance and credit constraint.
Since the bulk of the income is wage, and other components like pensions and bequests more or less also follow wage dynamics, we assume the following *habit formation* mechanism. Optimizing their consumption paths, people care for relative rather than absolute consumption values, for example, this year's consumption relative to last year's one or to a trend. Therefore the utility function should also reflect the secular increase in productivity (see Carroll, Overland and Weil (2000) for habit formation with actual rather than secular consumption base). The simplest way of modeling this phenomenon is to work with a utility function, where the per capita consumption $c_{i,t+i}$ is discounted by the productivity level g^{t+i}

$$u_{i,t+i}(c_{i,t+i}) = \beta_{i,t+i}\, m_{i,t+i}\, u(c_{i,t+i}/g^{t+i}).$$

Hence the optimal consumption path is

$$c_{i,t+i} = \delta^{(i-L)/\gamma}(\rho_{t+L,t+i}\,\beta_{i,t+i})^{1/\gamma} c_{L,t} g^{i-L}, \qquad i = L+1, L+2, \ldots, \mathbf{I}_t,$$

where the initial adult consumption is given by

$$c_{L,t+L} = g^{t-L} \frac{\sum_{i=L}^{\mathbf{I}_t} \rho_{t+L,t+i}^{-1} y_{i,i}}{\sum_{i=0}^{\mathbf{I}_t} \delta^{(i-L)/\gamma} \rho_{t+L,t+i}^{1/\gamma-1} \beta_{i,t+i}^{1/\gamma} m_{i,t+i}\, g^{i-L}}.$$

Taking into account that $\beta_{i,t}$ may differ from 1, the recursive form of consumption at age i in period t is modified:

$$c_{i,t} = \frac{R_t a_{i-1,t-1} + \sum_{j=i}^{\mathbf{I}_{t-i}} \rho_{t,t+j-i}^{-1}\, y_{j,t+j-i}}{\sum_{j=i}^{\mathbf{I}_{t-i}} \delta^{(j-i)/\gamma} \rho_{t,t+j-i}^{1/\gamma-1}\, (\beta_{j,t+j-i}/\beta_{i,t})^{1/\gamma}\, m_{j,t+j-i}\, g^{j-i}}$$

and calculate $a_{i,t}$ and $c_{i,t}$ alternatingly.
The second complication is connected with the inheritance. As is well-known, parents leave a *bequest* to their children. We do not follow Barro's (1974) unrealistic infinite chain of bequest motives, rather we introduce a simple but sensible solution. Denoting the capital value of the lifetime earnings of a person deceased in period t as

$$\bar{w}_t = \sum_{j=L}^{I_t} \rho_{t-I_t+j,t}\, w_{j,t-I_t+j}$$

and assuming that each parent *leaves* a given share (say κ) of this variable at his death in t as a bequest, i.e. $q_t = \kappa\bar{w}_t$. The age of his heirs is $F_t = I_t - H$. Since the bequest is divided among f_{t-F_t} heirs, the per capita bequest *received* is $q_t^* = q_t/f_{t-F_t}$. Denote by $\hat{y}_{i,t}$ the *extended income* in period t which is the sum of standard income plus the signed bequest (bequest received has a positive, bequest left has a negative sign):

$$\hat{y}_{i,t} = y_{i,t} + \begin{cases} q_t/f_{t-F_t} & \text{if } i = F_t; \\ -q_t & \text{if } i = I_t; \\ 0 & \text{otherwise.} \end{cases}$$

Then the earlier identities are modified:

$$a_{i,t} = R_t a_{i-1,t-1} + \hat{y}_{i,t} - m_{i,t}\, c_{i,t}$$

and

$$s_{i,t} = a_{i-1,t-1}(R_t - 1) + \hat{y}_{i,t} - m_{i,t}\, c_{i,t},$$

respectively. With the extended income, the previous formula remains valid, only a hat should be put on $y_{i,t}$:

$$\hat{c}_{i,t} = \frac{R_t a_{i-1,t-1} + \sum_{j=i}^{\mathbf{I}_{t-i}} \rho_{t,t+j-i}^{-1}\, \hat{y}_{j,t+j-i}}{\sum_{j=i}^{\mathbf{I}_t} \delta^{(j-i)/\gamma}\, \rho_{t,t+j-i}^{1/\gamma-1}(\beta_{j,t+j-i}/\beta_{i,t})^{1/\gamma}\, m_{j,t+j-i}\, g^{j-i}}.$$

A third complication is the presence of *credit constraint*: the assets cannot be negative (Hubbard and Judd, 1986 and Hubbard, Skinner and Zeldes, 1995): $a_i \geq 0$, $i = L, L+1, \ldots, \mathbf{I}_t - 1, \mathbf{I}_t$. Credit constraints are especially stringent when children need to be fed from low starting earnings, further diminished by significant public pension contributions. Note, however, that Hubbard et al. neglected the "family composition changes" (p. 393), a cornerstone of our model.

The optimization problem under credit constraint is not that simple but in our numerical setting, Heikki Oksanen's algorithm is quite satisfactory.
Under low enough pensions and fertility and high enough wages, the credit constraint is not effective at all. In other cases we can choose an appropriate date between the arrival of the bequest and the age when the children leave the household:

$$t + F_t \le K_t \le t + L + H - 1 \quad \text{or} \quad t + L + H - 1 \le K_t \le t + F_t.$$

Similarly, let

$$F_t \le V_t \le L + H - 1 \quad \text{or} \quad L + H - 1 \le V_t \le F_t.$$

Cut the optimization process in two by K_t, or equivalently, V_t.
(1) Calculate the model for the time-periods $[t + L, K_t]$, or equivalently, ages $[L, V_t]$
(2) Calculate the model for the time-periods $[K_t, t + \mathbf{I}_t]$ or ages $(V_t, \mathbf{I}_t]$ with bequest received at the end K_t and bequest left at the end $t + \mathbf{I}_t$.
We can unify the two cases by introducing the notation

$$M_{i,t} = \begin{cases} V_t & \text{if } L \le i \le V_t; \\ \mathbf{I}_{t-i} & \text{if } V_{t-i} < i \le \mathbf{I}_{t-i}. \end{cases}$$

Here the extended incomes $\hat{y}_{i,t+i}$s are helpful, since the bequest to be received and left are incorporated into incomes. The generalized formula runs as follows:

$$\hat{c}_{i,t} = \frac{R_t a_{i-1,t-1} + \sum_{j=i}^{M_{i,t}} \rho_{t,t+j-i}^{-1} \, \hat{y}_{j,t+j-i}}{\sum_{j=i}^{M_{i,t}} \delta^{(j-i)/\gamma} \, \rho_{t,t+j-i}^{1/\gamma-1} \, (\beta_{j,t+j-i}/\beta_{i,t})^{1/\gamma} \, m_{j,t+j-i} \, g^{j-i}}$$

and determine $a_{i,t}$ and $M_{i,t}$, respectively.
Note that in our heuristic solution, the assets may become slightly negative around V_t. The simplest way to tackle this complication is to say that such small credits are possible. The perfect solution would demand a much more complex algorithm but in this paper we skip it.
Initial asset conditions are given as $(a_{i,-1})$. Of course, the simplest case is to assume that they were the result of previous optimization, most notably in a steady-state one. In more detail, we must go back to $-I_0$. Consider the optimization procedure of the consumer who was born in $t = -I_0$ and entered the labor market in $t = L - I_0$ with asset $a_{L-1,L-I_0-1} = 0$. Solving the optimization problem yields the asset path $(a_{i,-I_0+i})_{i=L}^{I_0}$, which can be converted into an asset profile

$$(a_{i,-I_0})_{i=L}^{I_0} = (a_{i,-I_0+i}/g^{i-L})_{i=L}^{I_0}.$$

Shocks

Until now we have neglected the shocks which may hit the system and necessitate subsequent reoptimization. Now we fill this gap. Suppose that the government abruptly changes its exogenous policy parameter values θ_t, ι_t and J_t and later on, even τ_t in period T. Denote the changed values by tilde. For notational simplification, in $\tilde{y}_{i,t+i}$ will drop the hat. Then the workers and pensioners must also change their remaining consumption paths. Due to the changes in the policy paths, the income $\tilde{y}_{i,t+i}$ also changes for $t \geq T$. The shocked optimum at T is as follows:

$$\tilde{c}_{i,T} = \frac{R_T a_{i-1,T-1} + \sum_{j=i}^{M_{i,T}} \rho_{T,T+j-i}^{-1} \, \tilde{y}_{j,T+j-i}}{\sum_{j=i}^{M_{i,T}} \delta^{(j-i)/\gamma} \, \rho_{T,T+j-i}^{1/\gamma-1} (\beta_{j,T+j-i}/\beta_{i,T})^{1/\gamma} \, m_{j,T+j-i} \, g^{j-i}}$$

and determine $a_{i,T}$s.

There are at least two ways of defining the *expected* incomes $\tilde{y}_{i,t}$: a) under *perfect foresight*, the workers correctly foresee the relevant future values of θ_t, ι_t and τ_t, respectively:

$$\theta_t^r = \theta_t, \qquad \iota_t^r = \iota_t \quad \text{and} \quad \tau_t^r = \tau_t; \qquad t = 0, 1, \ldots;$$

b) under *naive expectations*, the workers naively identify the relevant future values of θ_t, ι_t and τ_t by their trend values, respectively:

$$\theta_p^n = \theta_t, \qquad \iota_t^p = \iota_t \quad \text{and} \quad \tau_p^n = \tau_t; \qquad p = t+1, \ldots, \quad t = 0, 1, \ldots, .$$

For $e = r, n$, $\tilde{y}_{i,t}^e$ contains τ_v^e and θ_v^e, with $v < t$, respectively. First we shall use only perfect foresight, but then we shall also consider naive expectations.

IV. Numerical results

We have formulated our model but it is so complex that we can only analyze it with the aid of a computer. To simplify the presentation, as a prelude, we use decades rather than years. The drawback of this simplification that the new values for age and time are also integers, making the transition extremely abrupt.

Base run

First we describe our base run. We start displaying our numerical results with the *demographic block*. Assume that the end of childhood, the ages at birth and death are as follows: $L = 2$, $H = 3$ and $I = 6$.

Let us start the dynamics in $t = 0$ (calendar time 1950) and assume that the previous 7 decades have time indices $t = -7, -6, \ldots, -1$.

Fertility started to diminish uniformly in $t = 2$ (1970) from 1 to 0.79 in 3 decades, i.e. ended in $t = 5$ (2000). In formula:

$$f_t = \begin{cases} f1 & \text{if } t < T1^f; \\ f1 + \Delta f(t - T1^f) & \text{if } T1^f \leq t \leq T2^f; \\ f2 & \text{if } t > T2^f, \end{cases}$$

where $f2 - f1 = \Delta f(T2^f - T1^f)$. In words: the first, the second and the third rows give the fertility rate before, during and after the demographic transition, respectively.
Numerical values: $f1 = 1 > f2 = 0.79$, $\Delta f = 0.07$, $T1^f = 2$, $T2^f = 5$.
Life expectancy jumped in period T^I from $I1$ to $I2$.
In formula:

$$I_t = \begin{cases} I1 & \text{if } t < T^I; \\ I2 & \text{if } t \geq T^I. \end{cases}$$

The parameter values are as follows: $I1 = 6$ and $I2 = 7, T^I = 5$. For better understanding, we spell out this change: I_t jumps from $I_4 = 6$ in 1990 to $I_5 = 7$ in 2000.
For the time being, we set the retirement age low and time invariant: $J_{0,t} = 5$ but later on we shall modify it.
We shall assume that the initial population was stationary:

$$n_{0,-I_0} = n_{0,-I_0+1} = \ldots = n_{0,-1} = 1$$

and the fertility was unitary: $f_{-I_0} = \cdots = f_{-1} = 1$.
The left part of Table 1a displays the size of the subpopulations of children, workers and pensioners. In our setup, the jump in life expectancy only delays but does not counterbalance the impact of the drop in fertility, and the total population begins to decrease again. The real problem is the spectacular rise in the share of pensioners.
Making use of the fact that the pension block can be solved without solving the consumption block, first we concentrate on the former.
Let us assume that the wage-dynamics can be described by a quadratic function (Mincer, 1974):

$$w_{i,t} = (\omega_0 + \omega_1 i - \omega_2 i^2) g^{t-L}, \qquad i = L+1, \ldots, J_t.$$

We shall assume that the starting relative wage is 1: $\omega_0 + \omega_1 L - \omega_2 L^2 = 1$ and we shall work with a modest rise of about 20%, reached at the initial retirement age $i = J_0$. Numerically, $\omega_0 = 0.664$, $\omega_1 = 0.222$ and $\omega_2 = 0.022$.
Having chosen $\alpha = 1.015^{10}$, and $g = 1.0175^{10}$, we can now determine the aggregate total wage and the interest factor series.
To have a replacement rate of the usual order of magnitude, let us assume an accrual rate $\theta_t = 0.022 \times 10$. To get rid of awkward initial conditions on τ_t for $t = -7, -6, \ldots, 0, 1$, we determine them as steady state contribution rates. Because of aging, from $t = 2$ (1970), the equilibrium contribution rate τ_t° will rise from 0.18 to 0.39 by $t = 7$ (2020) and then oscillate a little bit around 0.355. Finally, let us underline the steep rise of the implicit pension debt in terms of total wages during 1930 and 2010: from 0.33 to 0.94. At the end it stabilizes at around 0.75. (Note that in our decade model, this stock/flow ratio is much lower than it would be in an annual model.)
Turning to the consumption path (Table 1b), we must define the parameters of the utility function. Striving for sensible outcomes, we experiment with $\gamma = 4 =$ elasticity of intertemporal substitution, $\beta = 0.7 =$ utility correction, $\delta^{1/10} = 1/R_A^{1/10} = 1/(\alpha g)^{1/10} = 0.9682768 =$ annualized discount factor. In addition, $\mu = 0.5 =$ equivalent consumption coefficient.

Rising contribution rates reduce individual net incomes. This reduction implies proportionally diminished consumption and saving decisions. For example, in $t = 0$ (1950), the adult consumption profile corresponds to the starting steady-state optimum. (Numbers are given in terms of the current total wages of the youngest worker, our numeraire). In the following decades, the profile adjusts itself to the new circumstances. The consumption profile stabilizes around $t = 13$ (2080) at a closing steady state.
Asset dynamics (Table 1.c) is a simple consequence of the income and consumption paths. Note that the critical age (when mid-life assets are zero) drops from 4 to 3. The issue of credit constraints becomes really important when privatization and prefunding will be studied in further studies. (If there were no credit constraints, then poorer workers could finance their new mandatory private contributions by takng up credits.)

Alternative policies with perfect foresight

We shall now consider other scenarios, where the government reacts to the demographic change that has been taking place since $t = 2$ (1970). Alternative pension reforms are introduced in $t = 6$ (2010). We shall study three different policies: (i) decreasing the accrual rates, (ii) replacing wage indexation of pensions with price indexation and (iii) raising the retirement age. For the time being, it will be assumed that all the policy changes are correctly anticipated by the public: *perfect foresight*.
(i) Decreasing the accrual rates
Here the pension accrual is immediately decreased from $\theta_0 = 0.022 \times 10$ to $\theta_T = 0.015 \times 10$ in $t = 6$, while keeping the retirement age constant. To avoid duplication, the data of early periods – with invariant data – are omitted. Thus from Table 2.a, the numbers of the unchanged demographic block are dropped.
Note that, with the relative drop in the (per capita) entry pension, with an overshooting to 0.39, the contribution rate stabilizes at a lower value, namely at 0.28. Also note that, because of the reduced contribution rates, the pension benefits are lower than in the base run but not proportionally to the reduced accrual rates: $b_{60,2100} = 0.493 < 0.637$ is a drop 23% rather than the drop 32% in accrual rates. Similarly, the implicit pension debt also diminishes since 2020, and faster than in the base run.
Table 2.b contains the data of the consumption block. (For brevity, the unchanged column of bequest is deleted.) It is worth starting the comparison in the 2010 row: due to the lower contribution rates, the two youngest cohorts' consumption significantly rises, the others' diminishes. For example, $c_{40,2010}$ drops from 0.697 to 0.657. Turning to the comparison of transitions: the older cohorts of transition lose especially.
(ii) Replacing wage indexation by price indexation
Here the wage indexation rule is replaced by a price indexation rule in period $t = 6$ (2010). Tables 3.a–3.b contain the relevant data. Similarly to the reduced accrual rate, the moderation of indexation also reduces the contribution rates: $\tau_{2100} = 0.338 < 0.355$ but raises the entry pensions: $b_{60,2100} = 0.656 > 0.637$. The consumption profiles in the closing steady states are similar, but there are dramatic changes during the transition: for example, $c_{60,2010}$ drops from 0.662 to 0.640, while $c_{20,2010}$ jumps from 0.547 to 0.575.

(iii) Raising the retirement ages
Here the retirement age J_t is immediately raised from 5 to 6 (in 2010). Tables 4.a–4.b contain the descriptions, reinstating the changed demographic block.
The radical rise in the retirement age restores and even improves the pensioner-to-worker ratio of 2/3.8=0.526 in 2000 to 1/4.58=0.218 in 2010. Small wonder that the contribution rate returns from the very high 0.38 toward the initial value, now 0.216. At the same time, longer employment increases entry pensions (from 0.637 to 0.978 in 2100), nevertheless reducing the high implicit pension debt ratio from 0.734 to 0.451.
The consumption also significantly increases with respect to the base run: $c_{20,2100} = 0.701 > 0.575$ etc. Now the transition cohorts' consumption rises rather than drops: $c_{40,2020} = 0.955$ vs. 0.683. It is another question that these cohorts must work much longer, and thus lose leisure.

Alternative policies with naive expectations

We turn now to the more realistic case, when the public is totally surprised by the changes in pension policy: naive expectations. To save space, we only report those decades when the change is unexpected and confine our attention to the consumption paths. Nevertheless, we add the consumption blocks of the other two scenarios: Tables 5.b–c.
When the decrease in the accrual rates is unexpected (Table 5a), then the consumption path of the cohort entering its 4th decade is changed between 2000 and 2030: $c_{40,2000}$ jumps from 0.694 to 0.717; $c_{50,2010}$ drops from 0.685 to 0.671 and the drop continues.

Removing complications

Having worked out our model, it is worth checking the impact of complications on the results. We will see that the impact is important. In Tables 6b and c, we simultaneously remove the four complications introduced in Section III: habit formation, inheritance, credit constraint and changing household size. (Table 6a would have been identical to Table 1a.) To save space, we confine the comparison to the base run. For example, the consumption values are uniformly higher (like $c_{20,1930}$ is 0.906 rather than 0.764) and total assets are negative ($A_{1930} = -0.049$) rather than positive (0.092). More detailed investigations would check the individual impact of the omitted factors.

V. Conclusion

We have just started to work on this model family, therefore we have not taken into account important details, for example, interest rates derived from capital accumulation and fine demographic details. More work is required before we will arrive at more definite results. Nevertheless, our early experiments have already testified the power of our approach: we have obtained meaningful results on the qualitative differences between alternative scenarios with or without foresight.

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Appendix: Some proofs

The Appendix contains two parts: the derivations of pension wealth profiles and of the optimal consumption path.

Pension wealth profiles

We shall need cross-sectional profiles rather than a longitudinal path. To do so we must deduct J_t, j and i respectively from the corresponding indices starting with t.
New benefit in time t

$$b_t = g^{t-L} \sum_{j=L}^{J_{t-1}} \theta_{t+j-J_{t-1}-1}(1-\tau_{t+j-J_{t-1}-1})w_{j,L}.$$

Pension wealth for workers

$$d_{j,t} = g^{t-L-j} \sum_{h=L}^{j} \theta_{t+h-j}(1-\tau_{t+h-j})w_{h,L} \sum_{i=J_{t-1}+1}^{\mathbf{I}_{t-j}} g^i \rho^{-1}_{t,t+i-j}, \qquad j = L, \dots, J_t.$$

Pension wealth for pensioners

$$d_{i,t} = \sum_{h=i+1}^{\mathbf{I}_{t-i}} b_{h,t+h-i}\rho^{-1}_{t,t+h-i}, \qquad i = J_{t-1}+1, \dots, I_t.$$

Optimal consumption paths

Here is the derivation of the optimal consumption path. Introduce the Lagrange function with a multiplier λ:

$$\mathcal{L} = \sum_{i=L}^{\mathbf{I}_t} [\delta^{i-L} u_i(c_{i,t+i}) + \lambda\rho^{-1}_{t+L,t+i}(y_{i,t+i} - m_{i,t+i}c_{i,t+i})].$$

The optimal household consumption path is determined from the implicit Euler equations:

$$\delta^{i-L}\beta_{i,t+i}u'(c_{i,t+i}) = \lambda\rho^{-1}_{t+L,t+i}, \qquad i = L, L+1, \dots, \mathbf{I}_t.$$

Comparing the multipliers for i and L, results in

$$\delta^{i-L}\beta_{i,t+i}\rho_{t+L,t+i}u'(c_{i,t+i}) = \beta_{L,t+L}u'(c_{L,t+L}).$$

Since $\beta_{L,t+L} = 1$, we drop it, but later on we shall need it again.
Substituting $c_{i,t+i}$s into the budget constraint, $c_{L,t+L}$ is determined, hence the entire consumption path is determined.

Tables

All the per capita variables are given in terms of the current wage of the youngest. All the aggregate variables are related to the current total wage bill.

Table 1.a. Population and pensions: Base run

Decades	Kids	Workers	Pension-ers	Annualized interest factor	Entry benefits	Contribu-tion rate	IPD/Wage
t	K_t	M_t	P_t	$R_t^{1/10}$	$b_t/w_{L,t}$	τ_t	D_t/W_t
1930	2.000	4.000	1.000	1.033	0.809	0.180	0.330
1940	2.000	4.000	1.000	1.033	0.809	0.180	0.330
1950	2.000	4.000	1.000	1.033	0.809	0.180	0.350
1960	2.000	4.000	1.000	1.033	0.809	0.180	0.402
1970	1.930	4.000	1.000	1.033	0.809	0.180	0.500
1980	1.790	4.000	1.000	1.033	0.809	0.180	0.663
1990	1.650	3.930	1.000	1.031	0.809	0.183	0.700
2000	1.525	3.790	2.000	1.029	0.809	0.379	0.911
2010	1.414	3.580	2.000	1.027	0.756	0.387	0.941
2020	1.303	3.315	2.000	1.025	0.703	0.390	0.932
2030	1.205	3.064	1.930	1.025	0.651	0.378	0.920
2040	1.117	2.828	1.790	1.025	0.609	0.354	0.907
2050	1.030	2.619	1.650	1.025	0.616	0.342	0.911
2060	0.952	2.421	1.525	1.025	0.627	0.347	0.841
2070	0.883	2.234	1.414	1.025	0.638	0.355	0.783
2080	0.814	2.069	1.303	1.025	0.643	0.357	0.745
2090	0.752	1.912	1.205	1.025	0.641	0.358	0.727
2100	0.697	1.765	1.117	1.025	0.637	0.359	0.734
2150	0.469	1.193	0.752	1.025	0.637	0.355	0.752

Table 1.b. Consumption: Base run

Decade	cons(2)	cons(3)	cons(4)	cons(5)	cons(6)	cons(7)	Bequest left	Saving rate
t	$c_2/w_{L,t}$	$c_{3,t}/w_{L,t}$	$c_{4,t}/w_{L,t}$	$c_{5,t}/w_{L,t}$	$c_{6,t}/w_{L,t}$	$c_{7,t}/w_{L,t}$	$q_t/w_{L,t}$	σ_t
1930	0.764	0.764	0.764	0.783	0.716	0	0.326	0.013
1940	0.764	0.764	0.764	0.783	0.716	0	0.326	0.013
1950	0.770	0.764	0.764	0.783	0.716	0	0.326	0.012
1960	0.783	0.770	0.764	0.783	0.716	0	0.326	0.007
1970	0.796	0.783	0.770	0.783	0.716	0	0.326	0.008
1980	0.749	0.796	0.783	0.816	0.716	0	0.326	0.013
1990	0.641	0.708	0.740	0.821	0.744	0	0	0.070
2000	0.551	0.636	0.717	0.734	0.744	0.737	0.361	−0.011
2010	0.547	0.543	0.697	0.707	0.662	0.734	0.342	0.016
2020	0.551	0.537	0.683	0.683	0.634	0.649	0.320	0.027
2030	0.568	0.540	0.700	0.669	0.613	0.622	0.304	0.025
2040	0.584	0.556	0.713	0.686	0.600	0.601	0.293	0.020
2050	0.587	0.573	0.719	0.700	0.616	0.589	0.288	0.015
2060	0.581	0.575	0.716	0.705	0.627	0.604	0.287	0.010
2070	0.576	0.570	0.712	0.702	0.632	0.615	0.286	0.007
2080	0.575	0.565	0.709	0.698	0.630	0.620	0.287	0.010
2090	0.574	0.563	0.708	0.696	0.626	0.618	0.287	0.012
2100	0.575	0.563	0.709	0.694	0.624	0.614	0.286	0.012
2150	0.576	0.566	0.710	0.697	0.625	0.613	0.287	0.012

Table 1.c. Assets: Base run

Decades t	asset(2) $a_{2,t}/w_{L,t}$	asset(3) $a_{3,t}/w_{L,t}$	asset(4) $a_{4,t}/w_{L,t}$	asset(5) $a_{5,t}/w_{L,t}$	asset(6) $a_{6,t}/w_{L,t}$	Total assets $A_t/w_{L,t}$
1930	0.056	0.156	0	0.201	0	0.092
1940	0.056	0.156	0	0.201	0	0.092
1950	0.050	0.156	0	0.201	0	0.090
1960	0.037	0.140	0	0.201	0	0.084
1970	0.024	0.133	0	0.201	0	0.080
1980	0.070	0.126	0	0.167	0	0.081
1990	0.176	0	0.074	0.160	0.257	0.148
2000	0.070	0	0.092	0.095	0.244	0.115
2010	0.066	0	0.118	0.130	0.199	0.121
2020	0.059	0	0.139	0.175	0.208	0.143
2030	0.054	0	0.140	0.225	0.226	0.159
2040	0.063	0	0.137	0.239	0.250	0.170
2050	0.072	0	0.138	0.238	0.256	0.173
2060	0.072	0	0.133	0.227	0.255	0.170
2070	0.069	0	0.129	0.215	0.250	0.164
2080	0.068	0	0.130	0.211	0.244	0.161
2090	0.068	0	0.131	0.214	0.242	0.161
2100	0.066	0	0.129	0.216	0.243	0.162
2150	0.069	0	0.132	0.217	0.244	0.163

Table 2.a. Pensions: Anticipated reduction of accrual rate

Decades t	Entry benefits $b_t/w_{L,t}$	Contribu-tion rate τ_t	IPD/Wage D_t/W_t
1990	0.809	0.183	0.700
2000	0.809	0.379	0.911
2010	0.756	0.387	0.941
2020	0.651	0.376	0.920
2030	0.552	0.337	0.897
2040	0.468	0.287	0.875
2050	0.441	0.254	0.870
2060	0.464	0.252	0.804
2070	0.485	0.266	0.677
2080	0.495	0.273	0.588
2090	0.497	0.277	0.542
2100	0.493	0.278	0.540
2150	0.469	0.273	0.580

Table 2.b. Consumption: Anticipated reduction of accrual rate

Decade t	cons(2) $c_2/w_{L,t}$	cons(3) $c_{3,t}/w_{L,t}$	cons(4) $c_{4,t}/w_{L,t}$	cons(5) $c_{5,t}/w_{L,t}$	cons(6) $c_{6,t}/w_{L,t}$	cons(7) $c_{7,t}/w_{L,t}$	Saving rate σ_t
1990	0.641	0.708	0.740	0.821	0.744	0	0.070
2000	0.551	0.636	0.694	0.734	0.744	0.737	−0.005
2010	0.553	0.543	0.657	0.685	0.662	0.734	0.031
2020	0.576	0.543	0.637	0.644	0.614	0.649	0.050
2030	0.616	0.565	0.656	0.624	0.578	0.602	0.052
2040	0.653	0.604	0.687	0.643	0.560	0.566	0.043
2050	0.668	0.640	0.706	0.674	0.577	0.549	0.031
2060	0.663	0.655	0.706	0.692	0.605	0.565	0.019
2070	0.653	0.650	0.700	0.693	0.620	0.593	0.013
2080	0.649	0.641	0.695	0.687	0.621	0.608	0.014
2090	0.647	0.636	0.693	0.682	0.616	0.609	0.017
2100	0.648	0.634	0.692	0.679	0.611	0.604	0.017
2150	0.650	0.638	0.695	0.682	0.611	0.599	0.018

Table 3.a. Population and pensions: From wage to price indexation

Decades	Entry benefits	Contribution rate	IPD/Wage
t	$b_t/w_{L,t}$	τ_t	D_t/W_t
1990	0.809	0.183	0.700
2000	0.809	0.379	0.884
2010	0.756	0.355	0.914
2020	0.711	0.360	0.908
2030	0.668	0.352	0.899
2040	0.631	0.333	0.889
2050	0.642	0.326	0.893
2060	0.650	0.331	0.823
2070	0.657	0.336	0.774
2080	0.660	0.337	0.744
2090	0.659	0.338	0.730
2100	0.656	0.338	0.738
2150	0.656	0.336	0.750

Table 3.b. Consumption: Anticipated transition from wage to price indexation

Decade	cons(2)	cons(3)	cons(4)	cons(5)	cons(6)	cons(7)	Saving rate
t	$c_2/w_{L,t}$	$c_{3,t}/w_{L,t}$	$c_{4,t}/w_{L,t}$	$c_{5,t}/w_{L,t}$	$c_{6,t}/w_{L,t}$	$c_{7,t}/w_{L,t}$	σ_t
1990	0.641	0.708	0.715	0.779	0.744	0	0.085
2000	0.565	0.636	0.706	0.709	0.707	0.737	0.005
2010	0.575	0.557	0.700	0.696	0.640	0.697	0.028
2020	0.576	0.564	0.688	0.687	0.625	0.627	0.029
2030	0.588	0.565	0.704	0.675	0.616	0.612	0.023
2040	0.600	0.577	0.712	0.690	0.605	0.604	0.019
2050	0.601	0.588	0.714	0.698	0.619	0.593	0.016
2060	0.596	0.589	0.712	0.700	0.626	0.607	0.012
2070	0.594	0.585	0.709	0.698	0.628	0.614	0.011
2080	0.593	0.582	0.708	0.696	0.626	0.616	0.013
2090	0.593	0.581	0.707	0.694	0.624	0.614	0.014
2100	0.593	0.581	0.707	0.693	0.622	0.612	0.014
2150	0.594	0.583	0.708	0.695	0.623	0.611	0.014

Table 4.a. Population and pensions: Anticipated jump in the retirement age

Decades	Kids	Workers	Pension-ers	Annualized interest factor	Entry benefits	Contribu-tion rate	IPD/Wage
t	K_t	M_t	P_t	$R_t^{1/10}$	$b_t/w_{L,t}$	τ_t	D_t/W_t
1990	1.650	3.930	1.000	1.031	0.809	0.183	0.575
2000	1.525	3.790	2.000	1.029	0.809	0.379	0.724
2010	1.414	4.580	1.000	1.054	0.970	0.186	0.460
2020	1.303	4.315	1.000	1.027	0.967	0.197	0.475
2030	1.205	3.994	1.000	1.025	0.964	0.212	0.476
2040	1.117	3.688	0.930	1.025	0.959	0.212	0.476
2050	1.030	3.409	0.860	1.025	0.956	0.212	0.475
2060	0.952	3.155	0.790	1.025	0.992	0.218	0.476
2070	0.883	2.914	0.735	1.025	0.984	0.218	0.439
2080	0.814	2.693	0.679	1.025	0.980	0.217	0.443
2090	0.752	2.493	0.624	1.025	0.978	0.215	0.448
2100	0.697	2.302	0.580	1.025	0.978	0.216	0.451
2150	0.469	1.556	0.390	1.025	0.978	0.215	0.456

Table 4.b. Consumption: Anticipated jump in the retirement age

Decade	cons(2)	cons(3)	cons(4)	cons(5)	cons(6)	cons(7)	Bequest left	Saving rate
t	$c_2/w_{L,t}$	$c_{3,t}/w_{L,t}$	$c_{4,t}/w_{L,t}$	$c_{5,t}/w_{L,t}$	$c_{6,t}/w_{L,t}$	$c_{7,t}/w_{L,t}$	$q_t/w_{L,t}$	σ_t
1990	0.641	0.708	0.763	0.822	0.744	0	0	0.063
2000	0.619	0.636	0.804	0.756	0.746	0.737	0.361	−0.051
2010	0.722	0.651	0.955	0.846	0.795	0.784	0.525	0.061
2020	0.712	0.712	0.955	0.941	0.833	0.717	0.486	0.027
2030	0.705	0.698	0.962	0.936	0.923	0.748	0.443	0.007
2040	0.705	0.691	0.947	0.943	0.918	0.828	0.406	−0.006
2050	0.703	0.692	0.934	0.928	0.924	0.823	0.376	−0.005
2060	0.700	0.689	0.924	0.916	0.910	0.829	0.351	−0.001
2070	0.700	0.686	0.924	0.906	0.898	0.816	0.350	0.003
2080	0.702	0.687	0.925	0.906	0.888	0.805	0.349	0.005
2090	0.702	0.688	0.925	0.907	0.889	0.797	0.350	0.008
2100	0.701	0.688	0.925	0.907	0.889	0.797	0.350	0.006
2150	0.702	0.688	0.925	0.907	0.890	0.798	0.350	0.007

Table 5.a. Consumption: Unanticipated reduction of accrual rate

Decade	cons(2)	cons(3)	cons(4)	cons(5)	cons(6)	cons(7)
t	$c_2/w_{L,t}$	$c_{3,t}/w_{L,t}$	$c_{4,t}/w_{L,t}$	$c_{5,t}/w_{L,t}$	$c_{6,t}/w_{L,t}$	$c_{7,t}/w_{L,t}$
1990	0.641	0.708	0.740	0.821	0.744	0
2000	0.551	0.636	0.717	0.734	0.744	0.737
2010	0.553	0.543	0.657	0.671	0.662	0.734
2020	0.576	0.543	0.637	0.644	0.602	0.649
2030	0.616	0.565	0.656	0.624	0.578	0.591
2040	0.653	0.604	0.687	0.643	0.560	0.566
2050	0.668	0.640	0.706	0.674	0.577	0.549

Table 5.b. Consumption: Unanticipated transition from wage to price indexation

Decade	cons(2)	cons(3)	cons(4)	cons(5)	cons(6)	cons(7)
t	$c_2/w_{L,t}$	$c_{3,t}/w_{L,t}$	$c_{4,t}/w_{L,t}$	$c_{5,t}/w_{L,t}$	$c_{6,t}/w_{L,t}$	$c_{7,t}/w_{L,t}$
1990	0.641	0.708	0.740	0.821	0.744	0
2000	0.551	0.636	0.717	0.734	0.744	0.737
2010	0.562	0.543	0.687	0.670	0.603	0.734
2020	0.579	0.551	0.688	0.674	0.601	0.592
2030	0.592	0.568	0.705	0.674	0.605	0.589
2040	0.603	0.580	0.715	0.691	0.604	0.593
2050	0.602	0.591	0.716	0.701	0.620	0.593

Table 5.c. Consumption: Unanticipated jump in the retirement age

Decade	cons(2)	cons(3)	cons(4)	cons(5)	cons(6)	cons(7)
t	$c_2/w_{L,t}$	$c_{3,t}/w_{L,t}$	$c_{4,t}/w_{L,t}$	$c_{5,t}/w_{L,t}$	$c_{6,t}/w_{L,t}$	$c_{7,t}/w_{L,t}$
1990	0.641	0.708	0.740	0.821	0.744	0
2000	0.551	0.636	0.717	0.734	0.744	0.737
2010	0.735	0.744	0.965	0.927	0.865	0.629
2020	0.710	0.725	0.955	0.951	0.913	0.779
2030	0.704	0.697	0.962	0.936	0.933	0.819
2040	0.704	0.690	0.945	0.943	0.918	0.836
2050	0.701	0.690	0.933	0.927	0.924	0.823

Table 6.b. Consumption: Base run without complications

Decade t	cons(2) $c_2/w_{L,t}$	cons(3) $c_{3,t}/w_{L,t}$	cons(4) $c_{4,t}/w_{L,t}$	cons(5) $c_{5,t}/w_{L,t}$	cons(6) $c_{6,t}/w_{L,t}$	cons(7) $c_{7,t}/w_{L,t}$	Saving rate σ_t
1930	0.906	0.906	0.906	0.906	0.829	0	−0.008
1940	0.906	0.906	0.906	0.906	0.829	0	−0.008
1950	0.907	0.906	0.906	0.906	0.829	0	−0.008
1960	0.908	0.907	0.906	0.906	0.829	0	−0.009
1970	0.862	0.908	0.905	0.906	0.829	0	0.002
1980	0.812	0.862	0.908	0.905	0.829	0	0.028
1990	0.758	0.809	0.859	0.904	0.824	0	0.071
2000	0.708	0.751	0.802	0.852	0.820	0.817	−0.052
2010	0.719	0.699	0.741	0.791	0.768	0.809	−0.047
2020	0.733	0.706	0.685	0.727	0.710	0.754	−0.038
2030	0.746	0.719	0.692	0.672	0.652	0.696	−0.023
2040	0.752	0.731	0.705	0.678	0.602	0.639	−0.005
2050	0.751	0.738	0.717	0.691	0.608	0.591	0.004
2060	0.746	0.736	0.723	0.703	0.620	0.597	−0.002
2070	0.743	0.731	0.722	0.709	0.630	0.608	−0.008
2080	0.743	0.728	0.717	0.708	0.636	0.618	−0.010
2090	0.744	0.728	0.714	0.703	0.635	0.624	−0.010
2100	0.744	0.729	0.714	0.700	0.631	0.622	−0.009
2150	0.745	0.730	0.716	0.703	0.630	0.617	−0.007

Table 6.c. Assets: Base run without complications

Decades t	asset(2) $a_{2,t}/w_{L,t}$	asset(3) $a_{3,t}/w_{L,t}$	asset(4) $a_{4,t}/w_{L,t}$	asset(5) $a_{5,t}/w_{L,t}$	asset(6) $a_{6,t}/w_{L,t}$	Total assets $A_t/w_{L,t}$
1930	−0.087	−0.096	−0.052	0.017	0	−0.049
1940	−0.087	−0.096	−0.052	0.017	0	−0.049
1950	−0.087	−0.096	−0.052	0.017	0	−0.049
1960	−0.088	−0.097	−0.052	0.017	0	−0.049
1970	−0.043	−0.099	−0.052	0.017	0	−0.039
1980	0.008	−0.001	−0.058	0.019	0	−0.007
1990	0.059	0.108	0.102	0.010	0.007	0.064
2000	−0.087	0.005	0.051	0.008	0	−0.003
2010	−0.107	−0.114	−0.014	0	−0.003	−0.049
2020	−0.123	−0.142	−0.089	−0.010	−0.007	−0.079
2030	−0.124	−0.160	−0.112	−0.021	−0.011	−0.092
2040	−0.106	−0.146	−0.115	−0.023	−0.016	−0.088
2050	−0.093	−0.120	−0.098	−0.024	−0.018	−0.077
2060	−0.093	−0.110	−0.082	−0.024	−0.019	−0.071
2070	−0.097	−0.114	−0.080	−0.023	−0.018	−0.072
2080	−0.100	−0.119	−0.083	−0.022	−0.018	−0.074
2090	−0.102	−0.123	−0.086	−0.022	−0.018	−0.076
2100	−0.103	−0.125	−0.090	−0.022	−0.017	−0.078
2150	−0.100	−0.121	−0.086	−0.023	−0.018	−0.075