Numbers

In this assignment we want you to look at the way computers store numbers, and the effect this may or may not have on the results of calculations. You will look at a number of computational tasks and in each case consider:

- whether the use of single or double precision variables significantly affects the accuracy of the outcome,
- whether the order or method of computation is similarly significant, and
- whether mathematically true identities are reflected in machine computation.

A series

The *harmonic numbers* are defined as the initial partial sums of the series

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$$

Specifically, let

$$H(n)=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n-1}+\frac{1}{n}.$$

From a mathematical point of view it makes no difference what order we add together the terms that make up say $H(10000)$. Does it matter practically?

Standard deviation

The standard deviation of a sequence of numbers $a_1, a_2, \ldots, a_n$ is an important measure of their spread. There are two similar formulas for the standard deviation depending on whether the numbers are thought of as representing a sample, or the entire population. We'll consider the population version. Let:

$$\bar{a}=\frac{a_1+a_2+\cdots+a_n}{n}=\frac{\sum a_i}{n}$$

be the mean of the a_i. Then the standard deviation σ can be expressed as:

$$\sigma=\sqrt{\frac{\sum\left(a_i-\bar{a}\right)^2}{n}}. \tag{1}$$

A mathematically equivalent form not requiring a pre-computation of $\bar{a}$ is:

$$\sigma=\sqrt{\frac{\sum a_i^2-\left(\sum a_i\right)^2/n}{n}}. \tag{2}$$

An identity

For any real numbers x and y, with $y \neq 0$ it is certainly true that:

$$x = \left(\frac{x}{y} - x \times y\right) \times y + x \times y \times y.$$

Is it always true that if we compute the quantity on the right hand side of this equation, we get the same result as when we compute that on the left?

Task

Write a report giving your answers to all the questions. Show all of your results, including a selection from your own experiments. As well as saying *what* you observed, you should also investigate and report on *why* the phenomena that you observed occur.

Harmonic numbers

- Write a program (or programs) that compute the harmonic numbers in both single and double precision. Do the answers agree?
- Modify your program so that it can calculate the sum either from largest term $(1/1)$ to smallest $(1/n)$, or in the reverse order. Do the answers agree? If not, which is likely to be more accurate? How could you test your hypothesis?

Standard deviation

- Implement the two methods for computing standard deviations and test them. Do they always agree? Does the level of precision matter?
- The standard deviation of a sequence is not changed if a fixed value is added to all the elements of the sequence. Do your implementations exhibit this behaviour?
- Which method should generally be preferred? Are there any situations where this might not be the case?

An identity

Not trusting your algebraic skills, write a program to test the given identity for various values of x and y. Can you find any where it (apparently) fails? What triggers this behaviour?

(2 points, Pair)