
The colour of the day

Humanity colonised the planet Farbe almost 100 years ago. Since Farbe has a year of exactly 350 days, it was convenient to import the traditional seven day week. Mortally wounded shortly after landing, the captain's dying declaration was that each day should also have a colour: red, green, blue or gold. To keep the colonists' minds agile, the color of a day is never announced in advance, but must be worked out from first principles – it's a terrible social embarrassment for anyone over the age of 10 not to know the right colour, but even more embarrassing to have to ask. The rules to determine the colour of the day are as follows:

- The first day, day 1 (a Monday) had no colour.
- The colour of a prime day is red if it is a Monday or Thursday, green if a Tuesday or Friday, and blue if a Wednesday or Saturday. The first Sunday (day 7) was gold.
- If a day is not prime, then you need to count the non-gold colours of its factor days. If all three counts are different, then the colour is the most common. If two are equal, then the colour is the unequal one. If all three are equal, then the day is gold.

Gold days are particularly welcome, since by tradition they are Farberian holidays. In the extent of the colony's existence there has never been a three day holiday.

Task

- Will there ever be a three day holiday? If so, when will the first one occur?
- How many holidays will there be in the 1000th year of the colony's existence?
- What about the 10,000,000th year? (the colonists are nothing if not optimists).

Breaking news!

A closer examination of the chief's original declaration seems to suggest that "factor day" should have been read as "previous day of the same factor". That is, the appropriate days to consider when working out the colour of day 15 were not days 3 and 5 but days 12 and 10. Though this view is still controversial (the word "heresy" has been bandied about) it would probably be a good idea to repeat the task above under the new interpretation.

(2 points, Group)