- **Inner product of two waveforms.**

$$\langle x(t), y(t)\rangle = \int_0^T x(t)y^*(t)dt$$

두 신호의 닮은 정도.

- **Orthogonal and Energy**

$$\langle \psi_j(t), \psi_k(t)\rangle \;= \int_0^T \psi_j(t)\psi_k^*(t)dt = \begin{cases} K_j \text{ if } j = k \\ 0 \text{ if } j \neq k \end{cases}$$

K_j 는 $\psi_j(t)$ 의 *energy*다. 0이면 $\psi_j(t)$ 와 $\psi_k(t)$ 는 *orthogonal*이다.

- **Orthonormal**

Orthogonal set의 모든 j 에 대해서, $K_j = 1$ 이면, *orthonormal*이다.

- **Linear combination** 선형결합

$$f(t) \cong \sum_{n=1}^{N} f_n\psi_n(t)$$
$$= f_1\psi_1(t) + f_2\psi_2(t) + f_3\psi_3(t) + \cdots$$

$f(t)$는 *orthonormal* set의 linear *combination*으로 근사하게 표현될 수 있다.

근사 오차는 다음과 같다.

$$e(t) = f(t) - \sum_{n=1}^{N} f_n \;\psi_n(t)$$

오차를 최소화 하는 가중치 f_n은 다음과 같다.

$$f_n = \langle f(t), \psi_n(t)\rangle = \int_{t_1}^{t_2} f(t)\psi_n^*(t)dt$$

만약 *orthonormal* set이 아니면,

$$f_n = \langle f(t), \frac{\psi_n(t)}{\int_{t_1}^{t_2}|\psi_n(t)|^2dt}\rangle = \frac{\int_{t_1}^{t_2} f(t)\psi_n^*(t)dt}{\int_{t_1}^{t_2}|\psi_n(t)|^2dt}$$

- **Fundamental Frequency (Frequency Spacing)**

$$\langle \psi_n(t), \psi_m(t)\rangle \;= \int_{t_1}^{t_2} \psi_n(t)\psi_m^*(t)dt$$
$$= \int_{t_1}^{t_2} e^{jn\omega_0 t}e^{-jm\omega_0 t}dt\,, n \neq m$$

$$\rightarrow \omega_0 = \frac{2\pi}{t_2 - t_1}$$

- **Exponential Fourier Series**

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$F_n = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f(t)e^{-jn\omega_0 t}dt\,, \omega_0 = \frac{2\pi}{t_2 - t_1}$$

- **Output for periodic signal input to linear systems**

$$e^{jn\omega_0 t} \xrightarrow{H(\omega)} H(n\omega_0)e^{jn\omega_9 t}$$

$$\text{input } \; f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$
$$\text{output } \; g(t) = \sum_{n=-\infty}^{\infty} H(n\omega_0)F_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} G_n e^{jn\omega_0 t}$$
$$\rightarrow G_n = H(n\omega_0)F_n$$

G_n is Fourier series coefficient of $g(t)$

F_n is Fourier series coefficient of $f(t)$

- **Fourier Transform**

$$\mathcal{F}\{g(t)\} = G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt = \langle g(t), e^{j\omega t}\rangle$$

- **Inverse Fourier Transfrom**

$$\mathcal{F}^{-1}\{G(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega t}dt$$

- **rect function**

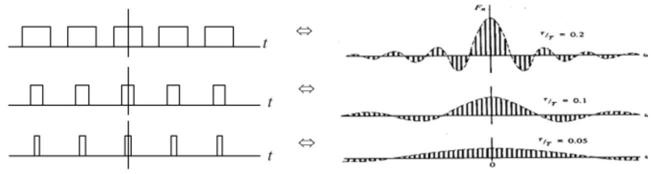
$$f(t) = \text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| < \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$$

- **Fourier transform of a rect function**

$$\mathcal{F}\left\{\text{rect}\left(\frac{t}{\tau}\right)\right\} = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau}\right)e^{-j\omega t}dt = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}dt$$
$$= \frac{e^{j\omega\tau\frac{1}{2}} - e^{-j\omega\tau\frac{1}{2}}}{j\omega}$$
$$= \frac{2j}{j\omega}\cdot\frac{e^{j\omega\tau\frac{1}{2}} - e^{-j\omega\tau\frac{1}{2}}}{j\omega} = \frac{2j}{j\omega}\sin\left(\omega\tau\frac{1}{2}\right)$$
$$= \frac{2j}{j\omega}\cdot\frac{\omega\tau}{2}\cdot\frac{sin\left(\omega\tau\frac{1}{2}\right)}{\omega\tau\frac{1}{2}} = \tau\,\text{sinc}\left(\omega\tau\frac{1}{2}\right)$$

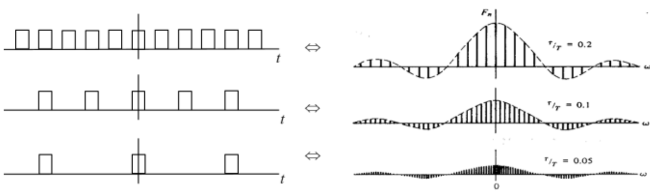
- **Fourier transform of periodic signals**

$$\boldsymbol{\mathcal{F}}\{f_T(t)\} = F(\omega)\cdot\sum_{n=-\infty}^{\infty}\frac{2\pi}{T}\delta(\omega-n\omega_0)$$



첫 번째 zero-cross = 2π/τ

∴Pulse Width가 작을수록 포락선은 늘어남.

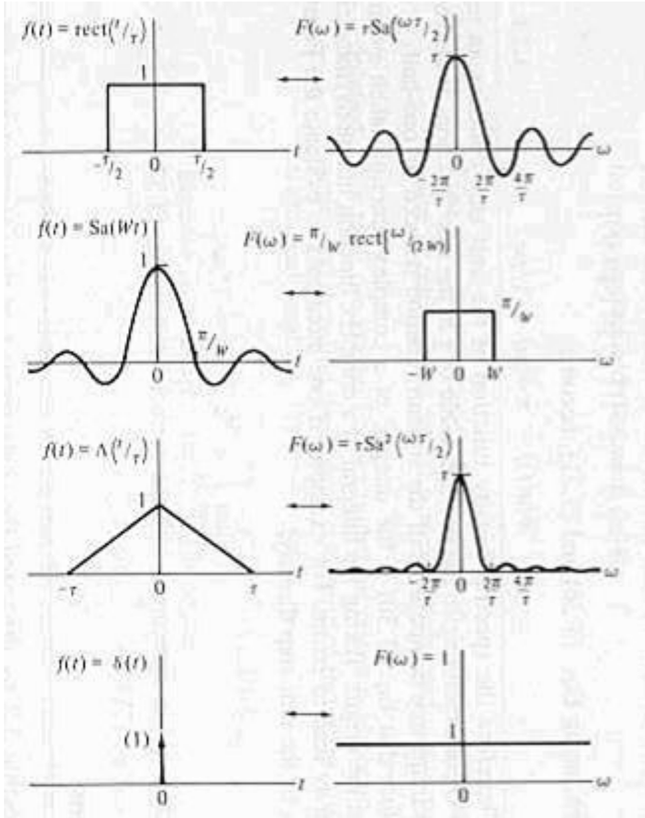


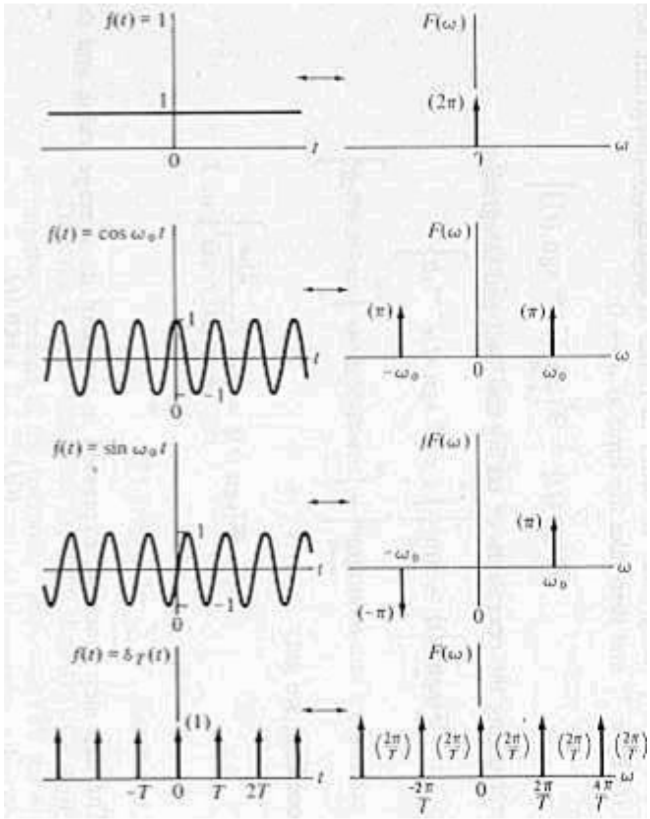
우항의 $2\pi/T$ 가 곱해지므로, 주기가 클수록 포락선이 작아짐

- **Properties of Fourier Transform**

$\mathcal{F}\{af(t)+bg(t)\}$	=	$aF(\omega)+bG(\omega)$
$\mathcal{F}\{f^*(t)\}$	=	$F^*(-\omega)=F(\omega)$
$\mathcal{F}\left\{\frac{d}{dt}f(t)\right\}$	=	$j\omega F(\omega)$
$\mathcal{F}\left\{\int_{-\infty}^{t}f(\tau)d\tau\right\}$	=	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
$\mathcal{F}\{f(\alpha t)\}$	=	$\frac{1}{\lvert\alpha\rvert}F\left(\frac{\omega}{\alpha}\right)$
$\mathcal{F}\{f(t-t_0)\}$	=	$F(\omega)e^{-j\omega t_0}$
$\mathcal{F}\{f(t)e^{j\omega_0 t}\}$	=	$F(\omega-\omega_0)$
$\mathcal{F}\{x(t)*h(t)\}$	=	$\mathcal{F}\{x(t)\}\cdot\mathcal{F}\{h(t)\}$

- **Fourier Transform table**





- **Low Pass Filter (LPF)**

$$H_{p,LPF}(\omega) = \mathrm{rect}\left(\frac{\omega}{4\pi B}\right)$$

$$\begin{aligned}
\mathcal{F}^{-1}\left(H_{p,LPF}\right) &= \frac{1}{2\pi}\int_{-2\pi B}^{2\pi B} e^{j\omega t} d\omega \\
&= \frac{1}{2\pi}\cdot\frac{1}{jt}\left[e^{j2\pi Bt} - e^{-j2\pi Bt}\right] \\
&= \frac{1}{2\pi}\cdot\frac{1}{jt}\cdot 2j\,\frac{\left[e^{j2\pi Bt} - e^{-j2\pi Bt}\right]}{2j} \\
&= \frac{1}{\pi t}\sin(2\pi Bt) = 2B\,\frac{\sin(2\pi Bt)}{2\pi Bt} \\
&= 2B\,\mathrm{sinc}(2Bt)
\end{aligned}$$

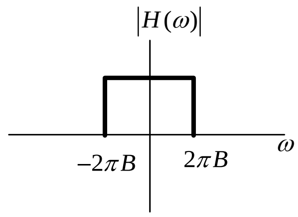
- **Delayed Low Pass Filter**

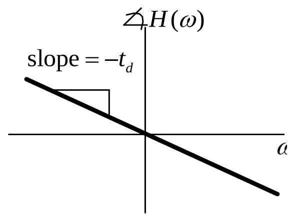
$$\begin{aligned}
H_p(\omega) &= \mathcal{F}\left\{2B\,\mathrm{sinc}\left(2B(t-t_d)\right)\right\} \\
&= \mathcal{F}\left\{2B\,\mathrm{sinc}\left(2B(t)\right)\right\}e^{-j\omega t_d} \\
&= \mathrm{rect}\left(\frac{\omega}{4\pi B}\right)e^{-j\omega t_d}
\end{aligned}$$

If the delay t_d is too small, truncation error is significant and $H_p(\omega)$ has a large distortion.

$$\begin{aligned}
\left|H_p(\omega)\right| &= \left|\mathrm{rect}\left(\frac{\omega}{4\pi B}\right)e^{-j\omega t_d}\right| \\
&= \left|\mathrm{rect}\left(\frac{\omega}{4\pi B}\right)\right|\cdot\left|e^{-j\omega t_d}\right| \\
&= \mathrm{rect}\left(\frac{\omega}{4\pi B}\right)
\end{aligned}$$

$$\begin{aligned}
\angle H_p(\omega) &= \angle\left(\mathrm{rect}\left(\frac{\omega}{4\pi B}\right)e^{-j\omega t_d}\right) \\
&= \angle\left(\mathrm{rect}\left(\frac{\omega}{4\pi B}\right)\right) + \angle\left(e^{-j\omega t_d}\right)
\end{aligned}$$





- **Band Pass Filter (BPF)**

$$\begin{aligned}
&H_{p,BPF}(\omega) = H_{p,LPF}(\omega+\omega_0) + H_{p,LPF}(\omega-\omega_0) \\
&\rightarrow h_{p,BPF}(\mathrm{t}) = h_{p,BPF}(\mathrm{t})\cdot 2\cos(\omega_0 t)
\end{aligned}$$

$$\mathrm{B_{BPF}} = 2B_{LPF}$$

- **Sampling Operation**

$$\begin{aligned}
&s(t) = x(t)\cdot p(t) \\
&p(t) = \sum_{n=-\infty}^{\infty} P_n e^{jn\omega_0 t} \\
&P_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} p(t)e^{jn\omega_0 t}dt\,,\omega_0 = \frac{2\pi}{T} \\
&\rightarrow s(t) = x(t)\sum_{n=-\infty}^{\infty} P_n e^{jn\omega_0 t}
\end{aligned}$$

$$\begin{aligned}
&= \sum_{n=-\infty}^{\infty} P_n x(t)e^{jn\omega_0 t} \overset{\mathcal{F}}{\rightarrow} S(\omega) \\
&= \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} P_n x(t)e^{jn\omega_0 t}\right\} \\
&= \sum_{n=-\infty}^{\infty} P_n\mathcal{F}\left\{x(t)e^{jn\omega_0 t}\right\} \\
&= \sum_{n=-\infty}^{\infty} P_n X(\omega - n\omega_0)
\end{aligned}$$

- **Autocorrelation Function (ACF)**

$$\begin{aligned}
r_f(\tau) &= \int_{-\infty}^{\infty} f^*(t)\,f(t+\tau)dt = \int_{-\infty}^{\infty} f(t)\,f^*(t-\tau)dt \\
&= \int_{-\infty}^{\infty} f^*(t)\,f(t+\tau)dt = r_f^*(-\tau) \\
&\rightarrow \mathrm{r_{fg}}(\tau) = \mathrm{r_{gf}^*}(-\tau)
\end{aligned}$$

- **Energy Spectral Density (ESD)**

$$|F(\omega)|^2 = \mathcal{F}\left\{r_f(\tau)\right\}$$

- **Parseval's Theorem**

$$\begin{aligned}
&\text{Energy of } f(t) = \int_{-\infty}^{\infty}|f(t)|^2 dt = \int_{-\infty}^{\infty} f(t)f^*(t)dt \\
&= \int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F^{ast}(\omega)e^{-j\omega t}d\omega\right]dt \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} F^*(\omega)\left[\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt\right]d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} F^*(\omega)F(\omega)d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2 d\omega
\end{aligned}$$

$$\therefore \int_{-\infty}^{\infty}|f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2 d\omega$$

- **a**