def 4.13 **Characteristic Function**

$$\phi_X(u) = E[e^{juX}] = \int_{-\infty}^{\infty} e^{juX} p_X(x)dx = \sum_{n=0}^{\infty} \frac{(ju)^n}{n!} E[X^n]$$

$$p_X(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-juX}\phi_X(u)du$$

def 4.14 **Moment Generating Function**

$$\begin{aligned}\Psi_X(u) = E[e^{uX}] &= \sum_{n=0}^{\infty}\frac{u^n}{n!}E[X^n] \\ &= \phi_X(0) + \sum_{n=1}^{\infty}\frac{u^n}{n!}\phi_X^{(n)}(0) \\ &= 1 + \sum_{n=1}^{\infty}\frac{ju^n}{n!}E[X^n]\end{aligned}$$

$$\rightarrow \phi_X^{(k)}(0) = j^k E[X^k] \rightarrow j^{-k}\phi_X^{(k)}(0) = E[X^k] \rightarrow -j^k\phi_X^{(k)}(0) = E[X^k]$$

**Gaunssian Nomal R.V.**

$$p_X(x) = \frac{1}{\sqrt{2\pi}\sigma_X}\exp\left(\frac{-(x-\mu_X)^2}{2\sigma_X^2}\right)$$

def 4.16 **Expected value of function of two R.V.**

$$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)p_{XY}(x,y)dx\,dy$$

def 4.17 **Moments of function of two R.V.**

$$E[X^mY^n] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^m y^n p_{XY}(x,y)dx\,dy$$

def 4.18 **Mean Cross Product** (평균 외적)

$$E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy p_{XY}(x,y)dx\,dy$$

def 4.19 **Covariance** (공분산)

$$\mathrm{cov}[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = E[XY] - \mu_X\mu_Y$$

def 4.20 **Correllation Codefficient** (상관계수)

$$\rho_{XY} = \frac{\mathrm{cov}[X,Y]}{\sigma_X\sigma_Y}, |\sigma_{XY}| \le 1$$

def 4.21 **Characteristic function of two R.V. with a joint pdf**

$$\phi_{XY}(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j(ux+vy)}p_{XY}(x,y)dx\,dy$$

If two R.V. are independent,

$$\phi_{XY}(u,v) = \phi_X(u)\phi_Y(v)$$

def 4.22 **Conditional Exprectation of R.V. X given that Y=y**

$$E[X|Y=y] = \int_{-\infty}^{\infty} x p_{X|Y}(x|y)dx = \int_{-\infty}^{\infty} x\frac{p_{XY}(x,y)}{P_Y(y)}dx$$

def 4.23 **Expectation of R.V. Vector**

If

$$\vec{X} = (X_1, X_2, \cdots, X_n)^T$$

then mean vector

$$\vec{\mu}_{\vec{X}} = (\mu_1, \mu_2, \cdots, \mu_n)^{\mathrm{T}}$$

where

$$\begin{aligned}\mu_i &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} x_i p_{\vec{X}}(x_1, x_2, \cdots, x_n)dx_1\,dx_2\cdots dx_n \\ &= \int_{-\infty}^{\infty} x_i p_{x_i}(x)dx_i \text{ for } i = 1,2,\cdots,n\end{aligned}$$

def 4.24 **Covariance Matrix**

$$C_{\vec{X}\vec{X}^T} = E\left[(\vec{X}-\vec{\mu}_{\vec{X}})(\vec{X}-\vec{\mu}_{\vec{X}})^T\right]$$

def 4.25 **Correlation Matrix**

$$R_{\vec{X}\vec{X}^T} = E\left[(\vec{X})(\vec{X})^T\right]$$

**Given independent two R.V. X & Y, Z=X+Y?**

$$p_Z(z) = \int_{-\infty}^{\infty} p_X(x)p_Y(z-x)dx = p_X(z) * \mathrm{p_Y}(z)$$

- chracteristic

$$E[Z] = \mathrm{E[X]} + \mathrm{E[Y]}$$

$$\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 \text{ (증명문제 시험나옴)}$$

**Central Lmit Theorem**

$$p_Z(z) = p_{X_1}(z) * p_2(z) * \cdots * p_{X_n}(z)$$

$$\lim_{n\to\infty} p_Z(z) = \frac{1}{\sqrt{2\pi}\sigma_Z}\exp\left(\frac{-(x-\mu_Z)^2}{2\sigma_Z^2}\right)$$

**Transform of a R.V.**

$$Y = f(X) = g^{-1}(X)$$

$$P_Y(Y \le y) = P_X(X \le x) = P_X(g(Y) \le x)$$

$$p_Y(y) = p_X(g(y))\left[\frac{d}{dy}g(y)\right]$$

def 5.19 **Poisson Random Process**

$$P_X(i,t) = \frac{(\lambda t)^i}{i!} e^{-\lambda t} u(t)$$

$$p_X(i,t) = \sum_{r=0}^{\infty} \frac{(\lambda t)^r}{r!} e^{-\lambda t} \delta(i-r)$$

Ex 5.18 풀어보도록.

def 5.20 **Poisson Impulse**

$$d(t) = \sum_n \delta(t - t_n)$$

def 5.22 **Shot Noise**

$$S(t) = h(t) * D(t)$$

def 5.23 **Bernoulli Random Process**

+1과 -1(혹은 +1과 0)의 두 가지 값만을 가질 수 있는 이산 랜덤 시퀀스 $X(nT)$이다. +1의 확률은 $p$이며, -1의 확률은 $q = 1 - p$이다.

def 5.24 **Binary counting Process**

Bernoulli R.P. 에서 펄스의 발생을 계수하는 R.P. 펄스는 +1 혹은 0이다. $X(i) = X_i$는 $i$번 째 시점에서 펄스가 발생되면 1, 그렇지 않으면 0이다.

$$C(n) = C_n = \sum_{i=1}^{n} X_i$$

if $C(n) = k$

$$p(C_n = K) = \binom{n}{k} p^k q^{n-k}, \qquad \mathrm{k} = 0,1,\cdots,\mathrm{n}$$

def 5.25 **Independent intervals (or Independent increment)**

Bernoulli R.P.에서 연속되는 값들의 차이가 상호독립 즉, $\big(X(1) - X(0)\big), \big(X(2) - X(1)\big), \cdots, \big(X(k) - X(k-1)\big), \cdots, \big(X(n) - X(n-1)\big)$ 이 상호독립일 경우에, Independent intervals (or Independent increment)를 갖는다고 한다.

def 5.26 **Random Walk Process**

$$P_X(n,x) = \binom{n}{\frac{n+x}{2}} p^{\frac{n+x}{2}} q^{\frac{n-x}{2}}$$

$$p_X(n,x) = \sum_{i=-n}^{n} \binom{n}{\frac{n+i}{2}} p^{\frac{n+i}{2}} q^{\frac{n-i}{2}} \delta(x-i)$$
$$= \sum_{r=0}^{n} \binom{n}{r} p^r q^{n-r} \delta(x - 2r + n)$$

def 5.27 **Winner Random Process**

Random Walk P.R. 에서 p와 q의 확률이 같을 때.

def 5.28 **Markov Random Process**

$$P_S = \big[s_j(n) \big| s_a(n-1), s_b(n-1), s_c(n-1), \cdots\big]$$
$$= P_S\big[s_j(n) \big| s_a(n-1)\big]$$

$$P_{ij} = P\big[s_j(n) | s_i(n-1)\big] \; for \; 1 \le i,j \le k$$

def 5.29 **Markov Chain**

초기 상태와 유한하면서 셀수 있는 상태들 그리고 이와 관련된 전이 확률들을 갖는 Markov R.P.

def 5.30 **Transition Probability Matrix**

$$T = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

$$\sum_j P_{ij} = 1 \,, 0 \le P_{ij} \le 1$$

$i$번째 상태 $s$에서 시작하여, $m$번 천이 과정 후, $j$번째 상태 $s$에 있을 확률.

$$P_{ij}(m) = P\big[s_j(n+m) | s_i(n)\big] = \sum_{r=1}^{k} P_{ir}(m-l) P_{rj}(l)$$

def 5.31 **Transient State (일시적 상태)**

$$P_{ij}(l) \ne 0 \;\&\; P_{ji}(r) = 0 \text{ for } r = 0,1,2,\cdots$$

def 5.32 **Recurrent State (재귀상태)**

모든 상태는 일시적 상태가 아니면 재귀상태이다.

def 5.33 **Periodic State (주기상태)**

$c, 2c, 3c, \cdots$가 아닌 모든 r에 대해서

$$P_{ii}(r) = 0, \text{for } c > 1$$

def 5.34 **Steady State (정상상태)**

$$\lim_{m\to\infty} P[s_j(m)] = P_j \text{ for } j = 1,2,\cdots,k$$

def 5.35 **Regular Chain (정규상태)**

전이행렬의 거듭제곱이 양의 원소만을 가질 경우에 마르코프 사슬을 정규사슬이라 한다. 이는 정규사슬이 주기 상태를 가지지 않음을 의미한다.

def 6.1 **Expected Value of a continuous time R.P.**

$$\mu_X(t) = E[X(t)] \text{ for} -\infty < t < \infty$$
$$= \int_{-\infty}^{\infty} x(t) p_X(x(t)) d(x(t))$$

def 6.2 **Autocorrelation function**

$$R_{XX}(t_1,t_2) = E[X(t_1)X^*(t_2)]$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t_1)x(t_2)p_X(x(t_1),x(t_2))dx(t_1)\,dx(t_2)$$

def 6.3 **Autocovariance**

$$Cov_{XX}(t,t+\tau) = E[(X(t)-\mu_X(t))(X(t+\tau) - \mu_X(t+\tau))]$$
$$= R_{XX}(t,t+\tau) - \mu_X(t)\mu_X(t+\tau)$$
$$= \sigma_{XX}^2(t,t+\tau)$$

def 6.4 **Autocorrelation Coefficient**

$$\rho_{XX}(t,t+\tau) = \frac{Cov_{XX}(t,t+\tau)}{\sqrt{Cov_{XX}(t,t)Cov_{XX}(t+\tau,t+\tau)}}$$

def 6.5 **Strict Sense Staionary R.P.**

$$\rho_X[X(t_1),X(t_2),\cdots,X(t_n)]$$
$$= \rho_X[X(t_1+\tau),X(t_2+\tau),\cdots,X(t_n+\tau)]$$

def 6.6 **Wide Sense Stationary R.P.**

$$\mu_X(t) = \mu_X$$

$$R_{XX}(t,t+\tau) = R_{XX}(\tau)$$

def 6.7 **Autocovariance for W.S.S. R.P.**

$$Cov_{XX}(\tau) = R_{XX}(\tau) - \mu_X^2 = \sigma_{XX}^2(\tau)$$

def 6.8 **Autocorrelation Coefficient for W.S.S R.P.**

$$\rho_{XX}(\tau) = \frac{\sigma_{XX}^2(\tau)}{\sqrt{\sigma_{XX}^2(0)\sigma_{XX}^2(0)}} = \frac{\sigma_{XX}^2(\tau)}{\sigma_{XX}^2(0)}$$

def 6.9 **Mean function for Random sequence**

$$\mu_X(n) = E[X(n)] \text{ for } -\infty < n < \infty$$
$$= \int_{-\infty}^{\infty} x(n)p_X(x(n))d(x(n))$$

def 6.10 **Autocorrelation function for R.S.**

$$R_{XX}(n,n+k) = \mathrm{E}[X(n)X(n+k)] =$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(n)x(n+k)p_X(x(n),x(n+k))dx(n)\,dx(n+k)$$

def 6.11 **Autocovariance for R.S.**

$$Cov_{XX}(n,n+k)$$

$$= E[(X(n)-\mu_X(n))(X(n+k)-\mu_X(n+k))]$$

$$= R_{XX}(n,n+k) - \mu_X(n)\mu_X(n+k) = \sigma_{XX}^2(n,n+k)$$

def 6.12 **Autocorrelation Coefficient for R.S.**

$$\rho_{XX}(n,n+k) = \frac{Cov_{XX}(n,n+k)}{\sqrt{Cov_{XX}(n,n)Cov_{XX}(n+k,n+k)}}$$

def 6.13 **Strict Sense Stationary R.S.**

$$\rho_X[X(n_1),X(n_2),\cdots,X(n_n)]$$
$$= \rho_X[X(n_1+k),X(n_2+k),\cdots,X(n_n+k)]$$

def 6.14 **Wide Sense Stationary R.S.**

$$\mu_X(n) = \mu_X$$

$$R_{XX}(n,n+k) = R_{XX}(k)$$

def 6.15 **Autocovariance function for W.S.S. R.S.**

$$Cov_{XX}(k) = R_{XX}(k) - \mu_X^2 = \sigma_{XX}^2(k)$$

def 6.16 **Autocorrelation Coefficient for W.S.S. R.S.**

$$\rho_{XX}(k) = \frac{\sigma_{XX}^2(k)}{\sqrt{\sigma_{XX}^2(0)\sigma_{XX}^2(0)}} = \frac{\sigma_{XX}^2(k)}{\sigma_{XX}^2(0)}$$

def 6.17 **Crosscorrelation function**

$$R_{XY}(t_1,t_2) = E[X(t_1)Y(t_2)]$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t_1)y(t_2)p_X(x(t_1),y(t_2))dx(t_1)\,dy(t_2)$$

$$R_{XY}(\tau) = R_{YX}(-\tau)$$

$$R_{XX}(\tau) = R_{XX}(-\tau)$$

$$|R_{XY}(\tau)| \le \frac{1}{2}(R_{XX}(0) + R_{YY}(0))$$

$$|R_{XY}(\tau)|^2 \le R_{XX}(0)R_{YY}(0)$$

def 6.18 **Crosscovariance**

$$\begin{aligned} Cov_{XX}(t,t+\tau) &= E\big[\big(X(t)-\mu_X(t)\big)\big(Y(t+\tau)-\mu_Y(t+\tau)\big)\big] \\ &= R_{XY}(t,t+\tau)-\mu_X(t)\mu_Y(t+\tau) \\ &= \sigma_{XY}^2(t,t+\tau) \end{aligned}$$

def 6.19 **Crosscorrelation Coefficient**

$$\rho_{XY}(t,t+\tau)=\frac{Cov_{XY}(t,t+\tau)}{\sqrt{Cov_{XY}(t,t)Cov_{XY}(t+\tau,t+\tau)}}$$

def 6.25 **If two R.P. are independent**

$$P_{XY}\big(x(t),y(t)\big)=P_X\big(x(t)\big)P_Y\big(y(t)\big)$$

def 6.26 **If two R.P. are orthognal**

$$R_{XY}(t,t+\tau)=E[X(t)Y(t+\tau)]=0$$

def 6.27 **If two R.P. are uncorrelated**

$$Cov_{XY}(t,t+\tau)=R_{XY}(t,t+\tau)-\mu_X(t)\mu_Y(t+\tau)=0$$

$$\rightarrow R_{XY}(t,t+\tau)=\mu_X(t)\mu_Y(t+\tau)$$

def 6.28 **Power Spectral Density Function**

$$S_{XX}(\omega)=S_{XX}(2\pi f)=\int_{-\infty}^{\infty}R_{XX}(\tau)e^{-j\omega\tau}d\tau$$

$$R_{XX}(\tau)=\int_{-\infty}^{\infty}S_{XX}(f)e^{j\omega\tau}df=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{XX}(\tau)e^{j\omega\tau}d\omega$$

**Average power of the R.P.**

$$R_{XX}(0)=\int_{-\infty}^{\infty}S_{XX}(f)e^{j\omega\tau}df\bigg|_{\tau=0}=\int_{-\infty}^{\infty}S_{XX}(f)df\geq 0$$