Sperry's supply-demand-loss model

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1 Introduction

Sperry and Love (2015 (What plant hydraulics can tell us about responses to climate-change droughts)) developed a model where a supply function ($E_{p-canopy}$) is derived which calculates the potential rate/amount of water able to be supplied from the soil to the atmosphere, i.e. potential transpiration. Transpiration is influenced by xylem pressure (ψ_{xylem}), hydraulic conductivity of the plant (K_{plant}), the hydraulic conductivity of the soil K_{soil} and the rate at which hydraulic conductivity is reduced as xylem pressure increases or soil conductivity decreases.

Below are parameters for the vulnerability-conductance curve,

```
p50 <- 2.5 # the matric potential where conductance is reduced by 50%
K_max <- 8 # maximum plant conductance - this is a trait in aDGVM2
res <- 1/K_max # resistance is simply the inverse of conductance
psi_canopy <- seq(0.0, 5, length=1000)
predawn_soil_mat_pot <- seq(0,2, by=0.5) # initial plant matric potential = soil matric potential
cum_can_transport <- matrix(0,0,nrow=1000, ncol=5) # this is the supply function

## make transpiration demand ---------------------------------------------------------------------------
# Maximum stomatal conductance
Gmax <- 12563.1 # (Sperry 2016, 2130 kg h^-1 m^-2) NOTE should be (kg h^-1 MPa^-1 m^-2) (12563.1 in Excel
G <- rep(Gmax, length=5)
VPD <- 0.5*0.001   # (Sperry 2016, leaf-to-air vapor pressure deficit 1 kPa) (0.001 converts to MPa)
# NOTE VPD converstion from kPa to MPa isn't documented in Sperry, I'm doing it as it makes sense and prod
# amounts of transpirtational demand.
#evap_demand <- G*VPD

# need to define an Pcrit, i.e. a matric potential we choose where we decide conductance is effectively ze
# We use this to get Ecrit, i.e. maximum transpiration beyond which leads to runaway cavitation
P_crit <- 4 # MPa - this is arbitrary and could be a plant trait.
# In Sperry (2016) a P_crit cutoff is chosen (either very low conductance or
# shallow slope of a tangent to the transpiration curve)

# get maximum transpiration possible based on Pcrit and soil matric potential
E_crit <- rep(0, lenght=5) # maximum transpiration beyond which leads to runaway cavitation
evap_demand <- rep(0, lenght=5) # evaporative demand
```

with the the conductance vulnerability curve we use in aDGVM2, which is analagous to Sperry's curve, defined as:

```
sperry_cond <- function(psi_canopy) { ((1 - (1 / (1 + exp(3.0*(p50 - psi_canopy)))))) / res }
```

The transpiration rate is the integral of the vulnerability-conductance curve between any the soil (pre-dawn) matric potential and $(p - canopy)$(canopy sap pressure) and is calculated as follows:

```
for(j in 1:length(predawn_soil_mat_pot))
  {
    for(i in 1:1000)
      {
        ffx <- integrate(sperry_cond, predawn_soil_mat_pot[j], psi_canopy[i] )
        cum_can_transport[i,j] <- pmax(0, ffx$value)
```

```
    }
}
```

Here we get the slope of the line which is tangent to the tanspiration curve at any particulat water potential. This slope is the conductance at this water potential.

```
cond_max_slope_sperry <- rep(0, lenght=5) # get the maximum slope of conductance given pre-dawn water pote
for(i in 1:length(predawn_soil_mat_pot))
  {
    cond_max_slope_sperry[i] <-  sperry_cond(predawn_soil_mat_pot[i])
    # for Sperry the maximum conductance is always the pre-dawn matric potential/soil matric potential
  }
```

We calculate the maximum transpiration beyond which leads to runaway cavitation E_{crit} based on a matric potential we choose where we decide conductance is effectively zero P_{crit}.

```
for(j in 1:5)
  {
    ffx <- integrate(sperry_cond, predawn_soil_mat_pot[j], P_crit )
    E_crit[j] <- pmax(0, ffx$value)
    evap_demand[j] <- G[j]*VPD
    if(evap_demand[j] > E_crit[j]) evap_demand[j] <- E_crit[j] # demand can't be greater than maximum supp
  }
```

We then calculate the matric potential where evaporative demand is met

```
## quick and dirty method to find the psi where demand is met.
demand_met_at_sperry <- rep(0, length=5)
psi_demand_met_at_sperry <- rep(0, length=5)
demand_met_at_slope_sperry <- rep(0, length=5)
loss_function_sperry <- rep(0, length=5)
regulated_transpiration <- rep(0, length=5)
regulated_leaf_psi <- rep(0, length=5)

for(i in 1:5)
  {
    demand_met_at_sperry[i] <- which(cum_can_transport[,i] >= evap_demand[i])
    psi_demand_met_at_sperry[i] <- psi_canopy[demand_met_at_sperry[i]] # the matric potential where demand
    demand_met_at_slope_sperry[i] <- sperry_cond(psi_demand_met_at_sperry[i]) # slope of tangent to suppy
    loss_function_sperry[i] <- demand_met_at_slope_sperry[i] / cond_max_slope_sperry
    regulated_leaf_psi[i] <- predawn_soil_mat_pot[i] + ((psi_demand_met_at_sperry[i] - predawn_soil_mat_po

    ffx <- integrate(sperry_cond, predawn_soil_mat_pot[i], regulated_leaf_psi[i] )
    regulated_transpiration[i] <- pmax(0, ffx$value)
    G[i] <- G[i]*loss_function_sperry[i]

  }
```

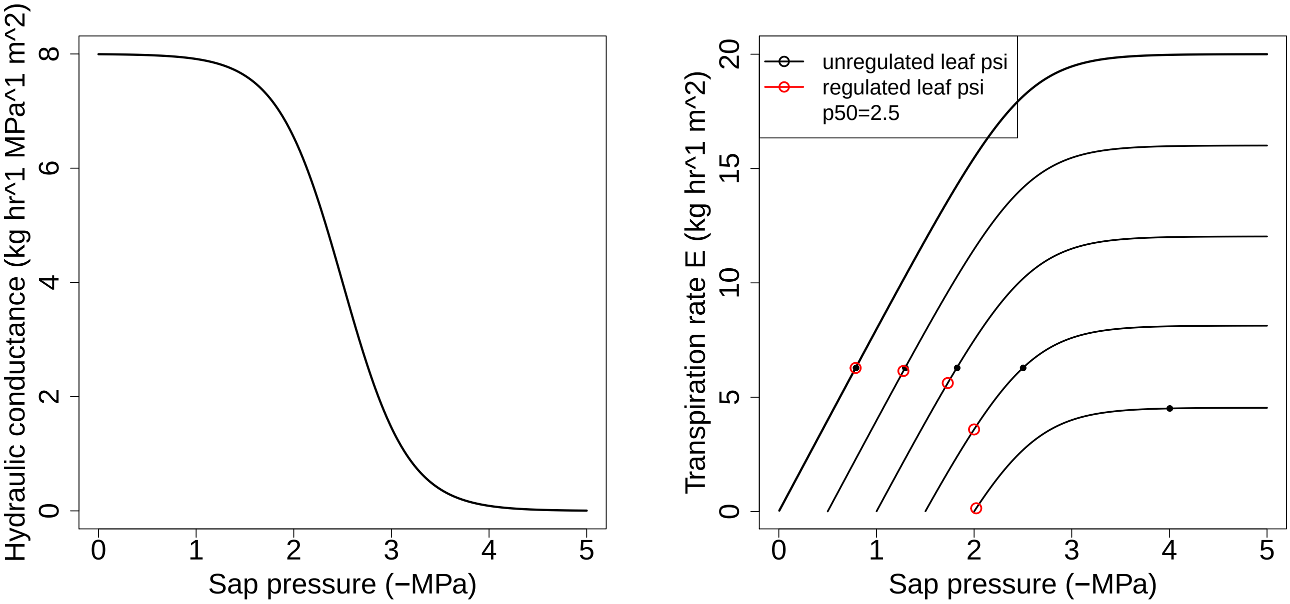

Figure 1: Hydraulic conductance and transpitation as a function of sap xylem pressure.