Sperry's supply-demand-loss model

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1 Introduction

Sperry and Love (2015 (What plant hydraulics can tell us about responses to climate-change droughts)) developed a model where a supply function (transpiration rate) (E) (kg hr^{-1} m^{-2}) is derived which calculates the potential rate/amount of water able to be supplied from the soil to the atmosphere. Transpiration (E) is influenced by the canopy sap pressure (p_canopy) (MPa) via changes to the hydraulic conductance of the plant (k) (kg hr^{-1} MPa^{-1} m^{-2}). Hydraulic conductance of the plant (k) is the conductance when there is no difference in matric potential between the soil and the leaf.

Below are parameters for the vulnerability-conductance curve:

```
##---------------------------------------------------------------------------------------------------
p50 <- 4.5 # the matric potential where conductance is reduced by 50% (trait aDGVM2)
k_max <- 8 # maximum plant conductance - (fn of p50)
res <- 1/k_max # resistance is simply the inverse of conductance
p_canopy <- matrix(seq(0.0, 8, length=1000), seq(0.0, 8, length=1000), nrow=1000, ncol=1000)
#p_canopy <- seq(0.0, 8, length=1000)
# this assumes initial plant matric potential is the same as the soil matric potential
predawn_soil_mat_pot <- seq(0,8, length=1000)
E_p_canopy <- matrix(0,0,nrow=1000, ncol=1000) # matrix to hold the supply function values
```

with the the conductance vulnerability curve (k_p_canopy) we use in aDGVM2, which is analagous to Sperry's curve, defined as:

```
k_p_canopy <- function(p_canopy) { ((1 - (1 / (1 + exp(3.0*(p50 - p_canopy)))))) / res }
```

We set the maximum stomatal conductance $Gmax$ to 12563.1 (kg h^{-1} MPa^{-1} m^{-2}) based on the value used by Sperry (2016, Excel table). The transpiration demand $E1$ is equal to

$$E1 = Gmax * D \tag{1}$$

where D (kPa) is the leaf to air vapor pressure defecit. The matric potential where, once passed, it is assumed that runaway cavitation is the result (P_crit) is arbitrary but is a point on the (k_p_canopy) curve where the slope of the tangent to the curve is very close to zero. The maximum transpiration rate beyond which runaway cavitation is assumed (E_crit). This is the transpiraiton rate at P_crit.

```
##---------------------------------------------------------------------------------------------------
# Maximum stomatal conductance
# (Sperry 2016, 2130 kg h^-1 m^-2) NOTE should be (kg h^-1 MPa^-1 m^-2) (12563.1 in Excel doc)
Gmax <- 12563.1
D <- 0.5*0.001 #(Sperry 2016, leaf-to-air vapor pressure deficit 1 kPa)(0.001 converts to MPa)
# NOTE VPD converstion from kPa to MPa isn't documented in Sperry, I'm doing it as it makes
# sense and produces realistic amounts of transpirtational demand.

# Pcrit, i.e. a matric potential we choose where we decide conductance is effectively zero.
# Used this to get Ecrit, i.e. maximum transpiration beyond which leads to runaway cavitation
P_crit <- 6 # MPa - this is arbitrary and could be a plant trait.
# In Sperry (2016) a P_crit cutoff is chosen (either very low conductance or
# shallow slope of a tangent to the transpiration curve)

# get maximum transpiration possible based on Pcrit and soil matric potential
```

```
E_crit <- rep(0, lenght=1000) # maximum transpiration beyond which leads to runaway cavitation
E1 <- rep(0, lenght=1000) # evaporative demand
```

E_crit is calculated as the integral of (*k_p_canopy*) between the predawn soil matric potential and *P_crit*. If the transpirational demand $E1$ is greater than the maximum transpiration rate *E_crit*, then demand is set to the maximum supply.

```
for(j in 1:1000)
{
  ffy <- integrate(k_p_canopy, predawn_soil_mat_pot[j], P_crit ) # supply up to P_crit
  E_crit[j] <- pmax(0, ffy$value)
  E1[j] <- Gmax*D
  if(E1[j] > E_crit[j]) E1[j] <- E_crit[j] # demand = maximum supply
}
```

Sperry's loss function (loss_fun_sp) is simply the slope of the tangent to the supply curve line (which is the value of *k_p_canopy*) at any particular matric potential (slope_supply), divided by the maximum slope (max_slo_sp). In Sperry's model this is at the predawn matric potential (predawn_soil_mat_pot).

```
for(j in 1:1000)
{

  max_slo_sp[j] <- k_p_canopy(predawn_soil_mat_pot[j])

  for(i in 1:1000)
  {
   slope_supply[i,j] <- k_p_canopy(pmax(predawn_soil_mat_pot[j], p_canopy[i]))

   ffx <- integrate(k_p_canopy, predawn_soil_mat_pot[j], p_canopy[i] )
   E_p_canopy[i,j] <- pmax(0, ffx$value)
   non_regulated_Gs[i,j] <- E_p_canopy[i,j]/D
   # E = G*VPD ---- G = E/VPD (VPD=1, 0.001 transforms to MPa)

   loss_fun_sp[i,j] <- slope_supply[i,j] / max_slo_sp[j]

   if(i==1) delta_P[i,j] <- pmax(0, ((p_canopy[i] - predawn_soil_mat_pot[j])
                                     *loss_fun_sp[i,j]))

   if(i>1)
   {
#max regulation is the point where delta P hits its maximum, held constant at max once max passed
     delta_P[i,j] <- pmax(0, pmax(delta_P[i-1,j],
                          ((p_canopy[i] - predawn_soil_mat_pot[j])*loss_fun_sp[i,j])))
   }

   ffx <- integrate(k_p_canopy, predawn_soil_mat_pot[j],
                    predawn_soil_mat_pot[j] + delta_P[i,j])
   regulated_trans[i,j] <- pmax(0, ffx$value)
   regulated_Gs[i,j] <- regulated_trans[i,j]/D # E=G*D -- G=E/D (D=1, 0.001 transforms to MPa)
   G[i,j] <- G[i,j]*loss_fun_sp[i,j]
  }
}
```

```
demand_place_holder <- rep(0, length=1000)
supply_limit_place_holder <- rep(0, length=1000)
min_diff_place_holder <- rep(0, length=1000)
supply_place_holder <- rep(0, length=1000)

for(i in 1:1000)
```

```
{
    a <-which(E_p_canopy[,i] >= E1[i])
    demand_place_holder[i] <-a[2]

    min_diff <- regulated_trans[,i] - E_p_canopy[demand_place_holder[i]]
    min_diff_max <- which(min_diff == max(min_diff))
    supply_limit_place_holder[i] <- min_diff_max[1]

    abs_min_diff <- abs(min_diff)
    abs_min_diff <- which(abs_min_diff == min(abs_min_diff))
    supply_place_holder[i] <- abs_min_diff[1]
}
```

From Sperry (2016) "Mathematically, P rises to a maximum before decreasing back to zero as E 0 increases to Ecrit. This decline in P is unrealis- tic (Saliendra et al., 1995), so it is assumed that P saturates at its maximum as E 0 increases. Eqn 5 expresses the outcome that xylem pressure is regulated in proportion to the damage caused by taking no action. (4) The regulated E corresponding to P is determined from the supply function. (5) The G is solved from E/D to determine how much it is reduced below Gmax. The model does not partition G into stomatal vs boundary layer com- ponents, but G is controlled by stomatal regulation. Cuticular water loss is assumed to be zero." I was unsure what Sperry ment with ΔP saturates however examining Fig.1B one can see the regulated response is too extreme. Plotting the regulated leaf matric potential against the demand defined matiric reveals that the regulated leaf matric potential reaches a maximum and then decreases. Fixing the regulated leaf matric potential to its maximum value once it has passed this value solves this issue. The next issue to solve is how sperry is calculating his percentage loss of stomatal conductance.

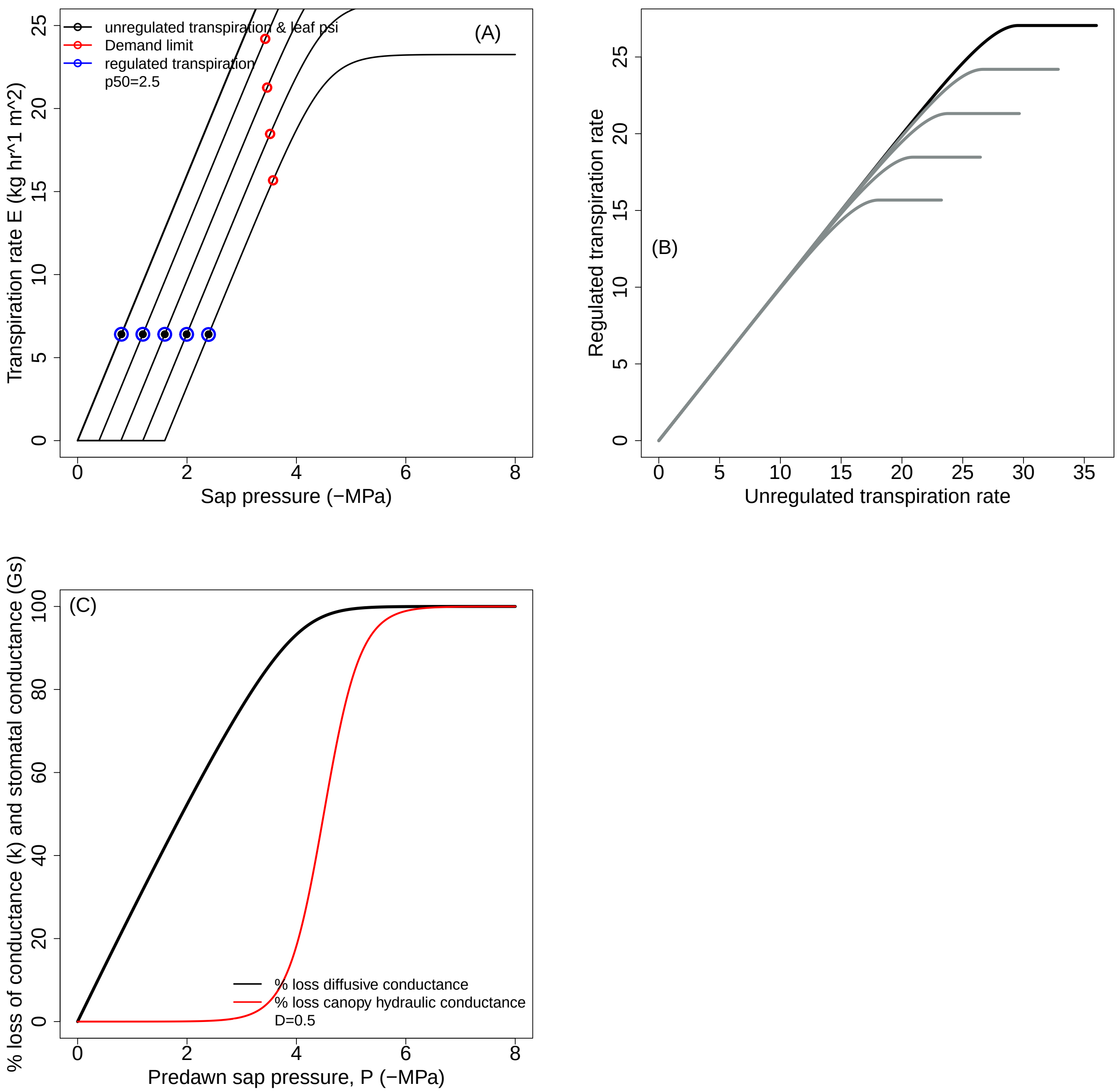

Figure 1: (A) Unregulated and regulated transpiration with supply demand limit. Where curves intersect the x-axis indicate the predawn/soil matric potential. (B) Regulated vs unregulated transpiration. the differing curves represent the responses for the differing predawn/soil matric potentials in (A). (C) Loss of stomatal conductance NOTE I haven't worked out how Sperry is producing his Fig.4 in Sperry and Love (2015). The differing curves correspond to the differing predawn/soil matric potentials in (A). (D) percentage loss of conductance.