Sperry's supply-demand-loss model

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May 30, 2018

1 Introduction

Sperry and Love (2015 (What plant hydraulics can tell us about responses to climate-change droughts)) developed a model where a supply function (transpiration rate) (E) (kg hr^{-1} m^{-2}) is derived which calculates the potential rate/amount of water able to be supplied from the soil to the atmosphere. Transpiration (E) is influenced by the canopy sap pressure (p_canopy) (MPa) via changes to the hydraulic conductance of the plant (k) (kg hr^{-1} MPa^{-1} m^{-2}). Hydraulic conductance of the plant (k) is the conductance when there is no difference in matric potential between the soil and the leaf.

Below are parameters for the vulnerability-conductance curve:

```
##----------------------------------------------------------------------------------------------
p50 <- 2.5 # the matric potential where conductance is reduced by 50% (trait aDGVM2)
k_max <- 8 # maximum plant conductance - (fn of p50)
res <- 1/k_max # resistance is simply the inverse of conductance
p_canopy <- matrix(seq(0.0, 8, length=1000), seq(0.0, 8, length=1000), nrow=1000, ncol=1000)
#p_canopy <- seq(0.0, 8, length=1000)
# this assumes initial plant matric potential is the same as the soil matric potential
predawn_soil_mat_pot <- seq(0,8, length=1000)
E_p_canopy <- matrix(0,0,nrow=1000, ncol=1000) # matrix to hold the supply function values
```

with the the conductance vulnerability curve (k_p_canopy) we use in aDGVM2, which is analagous to Sperry's curve, defined as:

```
k_p_canopy <- function(p_canopy) { ((1 - (1 / (1 + exp(3.0*(p50 - p_canopy)))))) / res }
```

We set the maximum stomatal conductance $Gmax$ to 12563.1 (kg h^{-1} MPa^{-1} m^{-2}) based on the value used by Sperry (2016, Excel table). The transpiration demand $E1$ is equal to

$$E1 = Gmax * D \tag{1}$$

where D (kPa) is the leaf to air vapor pressure defecit. The matric potential where, once passed, it is assumed that runaway cavitation is the result (P_crit) is arbitrary but is a point on the (k_p_canopy) curve where the slope of the tangent to the curve is very close to zero. The maximum transpiration rate beyond which runaway cavitation is assumed (E_crit). This is the transpiraiton rate at P_crit.

```
##----------------------------------------------------------------------------------------------
# Maximum stomatal conductance
# (Sperry 2016, 2130 kg h^-1 m^-2) NOTE should be (kg h^-1 MPa^-1 m^-2) (12563.1 in Excel doc)
Gmax <- 12563.1
D <- 1.0*0.001 #(Sperry 2016, leaf-to-air vapor pressure deficit 1 kPa)(0.001 converts to MPa)
# NOTE VPD converstion from kPa to MPa isn't documented in Sperry, I'm doing it as it makes
# sense and produces realistic amounts of transpirtational demand.

# Pcrit, i.e. a matric potential we choose where we decide conductance is effectively zero.
# Used this to get Ecrit, i.e. maximum transpiration beyond which leads to runaway cavitation
P_crit <- 6 # MPa - this is arbitrary and could be a plant trait.
# In Sperry (2016) a P_crit cutoff is chosen (either very low conductance or
# shallow slope of a tangent to the transpiration curve)

# get maximum transpiration possible based on Pcrit and soil matric potential
```

```
E_crit <- rep(0, lenght=1000) # maximum transpiration beyond which leads to runaway cavitation
E1 <- rep(0, lenght=1000) # evaporative demand
Max_gs_test <- rep(0,length=1000)
```

E_crit is calculated as the integral of (*k_p_canopy*) between the predawn soil matric potential and *P_crit*. If the transpirational demand $E1$ is greater than the maximum transpiration rate *E_crit*, then demand is set to the maximum supply.

```
for(j in 1:1000)
{
  ffy <- integrate(k_p_canopy, predawn_soil_mat_pot[j], P_crit ) # supply up to P_crit
  E_crit[j] <- pmax(0, ffy$value)
  E1[j] <- Gmax*D
  if(E1[j] > E_crit[j]) E1[j] <- E_crit[j] # demand = maximum supply
  Max_gs_test[j] <- E1[j]/D
}
```

The following code calculates the maximum slope of the supply function from (*k_p_canopy*) across a range of predawn soil matric potentials. The slope of the supply function (*slope_supply*) is calculated between for a range of between the predawn matric potentials and *p_canopy*. The supply function (*E_p_canopy*) is the integral between the predawn soil matric potential and p_canopy, this is performed for 1000 levels of predawn matric potential and p_canopy, both (0-8 MPa). Non regulated stomatal/diffusive conductance ($non_regulated_Gs$) is simply the supply function divided by the vapor pressure defecit D. Sperry's loss function (loss_fun_sp) is simply the slope of the tangent to the supply curve line (which is the value of *k_p_canopy*) at any particular matric potential (slope_supply), divided by the maximum slope (max_slo_sp). In Sperry's model this is at the predawn matric potential (predawn_soil_mat_pot). The amount by which leaf pressure is adjusted (*delta_P*) is calcualted as the difference between *p_canopy* and the predawn soil matric potential times the loss function. (*delta_P*) is held at its maximum value once the maximum value is passed, this is the trick I was missing when Sperry writes about *delta_P* saturating. Regulated transpiration or a regulated supply function *regulated_trans* is calcualted as the integral of *k_p_canopy* between the predawn matric potential and the predawn matric potential plus *delta_P*. Regulated stomatal/diffusive conductance is then calcualted by dividing *regulated_trans* by the vapor pressure defecit D.

```
for(j in 1:1000)
{

  max_slo_sp[j] <- k_p_canopy(predawn_soil_mat_pot[j])

  for(i in 1:1000)
  {
   slope_supply[i,j] <- k_p_canopy(pmax(predawn_soil_mat_pot[j], p_canopy[i]))

   # half of these ingegrations are not necessary as predawn>=p_canopy
   ffx <- integrate(k_p_canopy, predawn_soil_mat_pot[j], p_canopy[i] )
   E_p_canopy[i,j] <- pmax(0, ffx$value)
   non_regulated_Gs[i,j] <- E_p_canopy[i,j]/D
   # E = G*VPD ---- G = E/VPD (VPD=1, 0.001 transforms to MPa)

   loss_fun_sp[i,j] <- slope_supply[i,j] / max_slo_sp[j]

   if(i==1) delta_P[i,j] <- pmax(0, ((p_canopy[i] - predawn_soil_mat_pot[j])
                                    *loss_fun_sp[i,j]))

   if(i>1)
   {
#max regulation is the point where delta P hits its maximum, held constant at max once max passed
     delta_P[i,j] <- pmax(0, pmax(delta_P[i-1,j],
                              ((p_canopy[i] - predawn_soil_mat_pot[j])*loss_fun_sp[i,j])))
   }

   # half of these ingegrations are not necessary as delta_P will be zero
```

```r
   ffx <- integrate(k_p_canopy, predawn_soil_mat_pot[j],
                    predawn_soil_mat_pot[j] + delta_P[i,j])
   regulated_trans[i,j] <- pmax(0, ffx$value)
   regulated_Gs[i,j] <- regulated_trans[i,j]/D # E=G*D -- G=E/D (D=1, 0.001 transforms to MPa)
#   G[i,j] <- G[i,j]*loss_fun_sp[i,j]
  }
}
```

The below code finds the the first position in the *E_p_canopy* dataset where supply is greater than or equal to demand.

```r
demand_place_holder <- rep(0, length=1000)
supply_limit_place_holder <- rep(0, length=1000)
min_diff_place_holder <- rep(0, length=1000)
supply_place_holder <- rep(0, length=1000)

for(i in 1:1000)
{
    a <-which(E_p_canopy[,i] >= E1[i])
    demand_place_holder[i] <-a[2]

    min_diff <- regulated_trans[,i] - E_p_canopy[demand_place_holder[i]]
    min_diff_max <- which(min_diff == max(min_diff))
    supply_limit_place_holder[i] <- min_diff_max[1]

    abs_min_diff <- abs(min_diff)
    abs_min_diff <- which(abs_min_diff == min(abs_min_diff))
    supply_place_holder[i] <- abs_min_diff[1]
}
```

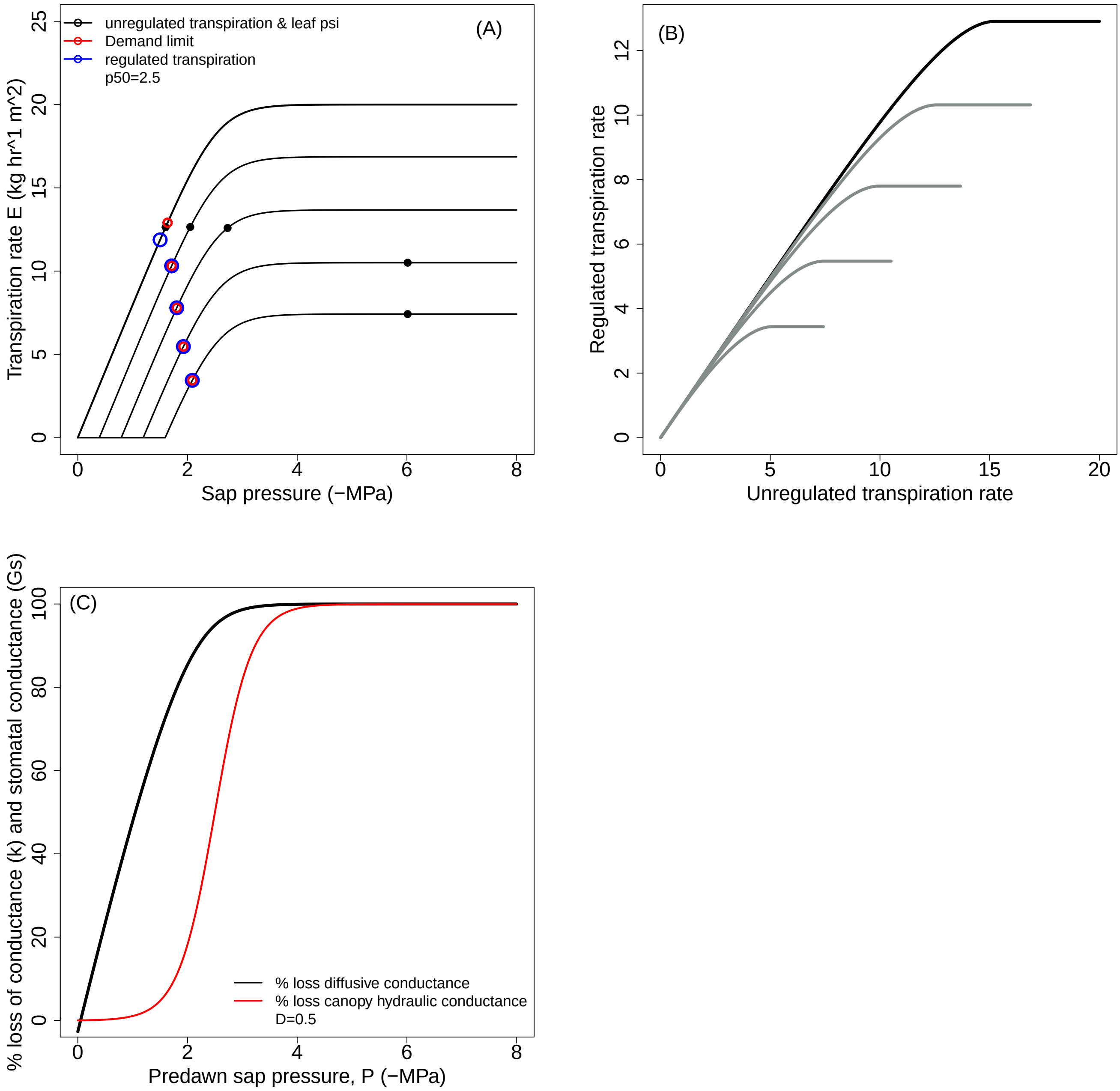

Figure 1: (A) Unregulated and regulated transpiration with supply demand limit. Where curves intersect the x-axis indicate the predawn/soil matric potential. (B) Regulated vs unregulated transpiration. the differing curves represent the responses for the differing predawn/soil matric potentials in (A). (C) Loss of stomatal conductance NOTE I haven't worked out how Sperry is producing his Fig.4 in Sperry and Love (2015). The differing curves correspond to the differing predawn/soil matric potentials in (A). (D) percentage loss of conductance.