

Sperry's supply-demand-loss model

Liam Langan

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1 Introduction

Sperry and Love (2015 (What plant hydraulics can tell us about responses to climate-change droughts)) developed a model where a supply function ($E_{p-canopy}$) is derived which calculates the potential rate/amount of water able to be supplied from the soil to the atmosphere, i.e. potential transpiration. Transpiration is influenced by xylem pressure (ψ_{xylem}), hydraulic conductivity of the plant (K_{plant}), the hydraulic conductivity of the soil K_{soil} and the rate at which hydraulic conductivity is reduced as xylem pressure increases or soil conductivity decreases.

Below are parameters for the vulnerability-conductance curve,

```
p50 <- 2.5 # the matric potential where conductance is reduced by 50%
K_max <- 8 # maximum plant conductance - this is a trait in aDGVM2
res <- 1/K_max # resistance is simply the inverse of conductance
psi_canopy <- seq(0.0, 5, length=1000)
predawn_soil_mat_pot <- seq(0,2, by=0.5) # initial plant matric potential = soil matric potential
cum_can_transport <- matrix(0,0,nrow=1000, ncol=5) # this is the supply function

## make transpiration demand -----
# Maximum stomatal conductance
Gmax <- 12563.1 # (Sperry 2016, 2130 kg h-1 m-2) NOTE should be (kg h-1 MPa-1 m-2) (12563.1 in Excel)
G <- rep(Gmax, length=5)
VPD <- 0.5*0.001 # (Sperry 2016, leaf-to-air vapor pressure deficit 1 kPa) (0.001 converts to MPa)
# NOTE VPD conversion from kPa to MPa isn't documented in Sperry, I'm doing it as it makes sense and produces
# amounts of transpirational demand.
#evap_demand <- G*VPD

# need to define an Pcrit, i.e. a matric potential we choose where we decide conductance is effectively zero
# We use this to get Ecrit, i.e. maximum transpiration beyond which leads to runaway cavitation
P_crit <- 4 # MPa - this is arbitrary and could be a plant trait.
# In Sperry (2016) a P_crit cutoff is chosen (either very low conductance or
# shallow slope of a tangent to the transpiration curve)

# get maximum transpiration possible based on Pcrit and soil matric potential
E_crit <- rep(0, length=5) # maximum transpiration beyond which leads to runaway cavitation
evap_demand <- rep(0, length=5) # evaporative demand
```

with the the conductance vulnerability curve we use in aDGVM2, which is analagous to Sperry's curve, defined as:

```
sperry_cond <- function(psi_canopy) { ((1 - (1 / (1 + exp(3.0*(p50 - psi_canopy)))))) / res }
```

The transpiration rate is the integral of the vulnerability-conductance curve between any the soil (pre-dawn) matric potential and ($p - canopy$)(canopy sap pressure) and is calculated as follows:

```
for(j in 1:length(predawn_soil_mat_pot))
{
  for(i in 1:1000)
  {
    ffx <- integrate(sperry_cond, predawn_soil_mat_pot[j], psi_canopy[i] )
    cum_can_transport[i,j] <- pmax(0, ffx$value)
  }
}
```

```

    }
}

```

Here we get the slope of the line which is tangent to the transpiration curve at any particular water potential. This slope is the conductance at this water potential.

```

cond_max_slope_sperry <- rep(0, length=5) # get the maximum slope of conductance given pre-dawn water potential
for(i in 1:length(predawn_soil_mat_pot))
{
  cond_max_slope_sperry[i] <- sperry_cond(predawn_soil_mat_pot[i])
  # for Sperry the maximum conductance is always the pre-dawn matric potential/soil matric potential
}

```

We calculate the maximum transpiration beyond which leads to runaway cavitation E_{crit} based on a matric potential we choose where we decide conductance is effectively zero P_{crit} .

```

for(j in 1:5)
{
  ffx <- integrate(sperry_cond, predawn_soil_mat_pot[j], P_crit )
  E_crit[j] <- pmax(0, ffx$value)
  evap_demand[j] <- G[j]*VPD
  if(evap_demand[j] > E_crit[j]) evap_demand[j] <- E_crit[j] # demand can't be greater than maximum supply
}

```

We then calculate the matric potential where evaporative demand is met

```

## quick and dirty method to find the psi where demand is met.
demand_met_at_sperry <- rep(0, length=5)
psi_demand_met_at_sperry <- rep(0, length=5)
demand_met_at_slope_sperry <- rep(0, length=5)
loss_function_sperry <- rep(0, length=5)
regulated_transpiration <- rep(0, length=5)
regulated_leaf_psi <- rep(0, length=5)

for(i in 1:5)
{
  # positions on transport curve where supply can meet demand
  # this throws a warning but returns the first number which is what I want
  demand_met_at_sperry[i] <- which(cum_can_transport[,i] >= evap_demand[i])
  # the matric potential where demand is met
  psi_demand_met_at_sperry[i] <- psi_canopy[demand_met_at_sperry[i]]
  # slope of tangent to supply/conductance curve where demand is met
  demand_met_at_slope_sperry[i] <- sperry_cond(psi_demand_met_at_sperry[i])
  # this is sperrys loss function (slope where demand is met / max slope)
  loss_function_sperry[i] <- demand_met_at_slope_sperry[i] / cond_max_slope_sperry
  # this is the adjusted leaf matric potential dP = dP' * (demand_met_at_slope_sperry / cond_max_slope_sperry)
  regulated_leaf_psi[i] <- predawn_soil_mat_pot[i] + ((psi_demand_met_at_sperry[i] - predawn_soil_mat_pot[i]) *
  # this is the adjusted leaf matric potential dP = dP' * (demand_met_at_slope_sperry / cond_max_slope_sperry)
  # dP' = predawn_matric_potential - unregulated_matric_potential, this doesn't work unless dP is added

  ffx <- integrate(sperry_cond, predawn_soil_mat_pot[i], regulated_leaf_psi[i] )
  regulated_transpiration[i] <- pmax(0, ffx$value) # regulated transpiration (E in Sperry)
  G[i] <- G[i]*loss_function_sperry[i] # regulated stomatal conductance
  # E = G * VPD so G = E/VPD
}

```

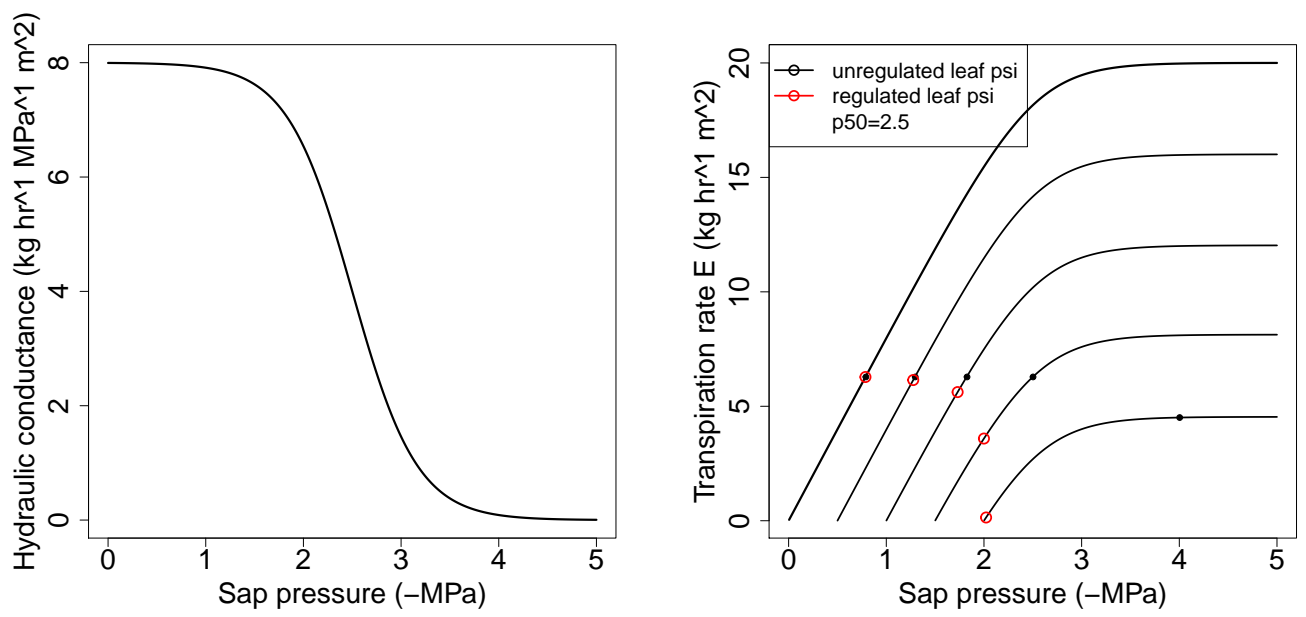


Figure 1: Hydraulic conductance and transpiration as a function of sap xylem pressure.