# EE 550 – SPRING 2011 - MIDTERM EXAM

## MARCH 09, 2011, 09:00 – 11:00

## CLOSED BOOK

***Students are allowed 1 sheet (8.5 x 11) of notes. No calculators, cell phones, computers etc. allowed. No notes. Answers should be written on the exam paper (attached extra pages as necessary).***

***DO NOT WRITE ON THE BACK OF THE PAPER!***

NAME: SOLUTIONS
LAST, first

Student ID Number: ______________________

| Problem | Max | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 12 | |
| 3 | 21 | |
| 4 | 25 | |
| 5 | 25 | |
| 6 | 15 | |
| TOTAL | 110 | |

**Question 1. (12 points)**

a) Explain the difference between Datagram and Virtual Circuit packet switching.

VC: PATH IS ESTABLISHED. ALL PKTS FOLLOW SAM PATH. (RESOURCES MAY OR MAY NOT BE ALLOCATED)

DG: EACH PACKET ROUTED INDEPENDENTLY.

b) A data frame uses a frame delimiter of 01111110 for the start and end of the frame. How is data transparency achieved? (i.e. you need to be able to transmit this bit pattern in the data part of the frame).

BIT-STUFFING: EG: INSERT A "0" AFTER ANY STRING OF 5 ONES IN DATA.

c) Compare two channels with the same mean bit error rate. In one, the bit errors are independent; in the other, the errors tend to occur in bursts. Which channel will have a higher frame error rate? Why?

BURSTY CHANNEL WILL HAVE LOWER FRAME ERROR SINCE THE BIT ERRORS WILL BE CONCENTRATED IN A SMALLER NUMBER OF FRAMES.

d) Describe the buffering requirements at the sender and the receiver for the following ink layer protocols: Stop and Wait, Go Back N, Selective Repeat.

| | SENDER | RCVR |
|---|---|---|
| S&W: | Buffer 1 packet. | No need to buffer, |
| GBN | Buffer Window of "N" packets | No need to buffer |
| SR | Buffer Window | Buffer Window. |

***Question 2. (12 points)***

An error detection scheme uses the following generator polynomial $x^5+x^3+1$. Data is sent in blocks of 8 data bits plus the CRC bits.

a) How many CRC bits are there?

$G(x) = x^5 + x^3 + 1.$ (101001)

There will be 5 CRC bits appended to data frame. (the remainder after "division" by G(x))

b) if the Data block is 1100 0101 what CRC bits are appended?

101001 ) 1100 0101 0000 0

...

$\boxed{10101}$ REMAINDER

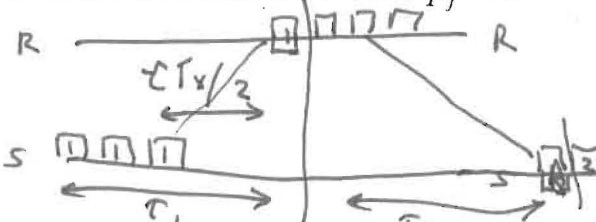
c) Give an example of an error pattern that cannot be detected by this code.

ANY MULTIPLE OF G(x)

e.g. G(x) itself! 101001.

***Question 3. (21 points)***

A modified stop and wait protocol operates over a full duplex channel as follows. The frame at the head of the queue is continuously transmitted until a positive acknowledgement is received. Thus if the first copy has an error, the receiver will receive another copy one frame time later. Acknowledgements are piggy-backed on data frames in the reverse direction (and every frame in the reverse channel will have an ACK for the last frame successfully received.) Assume that the round trip propagation delay ($\tau$ fames) is an integer multiple of the frame length (which is fixed). Assume that the probability of frame error in the forward direction is $p_f$ and that the probability of frame error in the reverse direction is $p_a$.




a) What is the efficiency for this protocol?

$T_1 = E[\text{Time until success } S \to R]$

$$= \sum_{k=1}^{\infty} k(1-p_f)p_f^{k-1} T_x + \frac{\tau}{2} T_x$$

$$= \frac{T_x}{1-p_f} + \frac{\tau}{2} T_x$$

$T_2 = E[\text{time until success } R \to S \text{ for ack}]$

$$= \frac{T_x}{1-p_a} + \frac{\tau}{2} T_x$$

So $$\eta_1 = \frac{1}{1/(1-p_f) + 1/(1-p_a) + \tau}$$

b) What is the efficiency for regular Stop and Wait (in which the sender sends a single copy of the frame and waits for an acknowledgement or timeout)?

$$T_1 = \sum_{k=1}^{\infty} (k-1)(1-p_f)p_f^{k-1} T_o + T_x + \tau \frac{T_x}{2}$$

$$= \frac{p_f}{1-p_f} T_o + T_x + \frac{\tau T_x}{2}$$

$$T_o = 2T_x + \tau T_x$$

$$T_2 = \frac{p_a}{1-p_a} T_o + T_x + \frac{\tau T_x}{2}$$

So $$\eta_2 = \frac{1}{(2+\tau)\left\{\frac{p_f}{1-p_f} + \frac{p_a}{1-p_a}\right\} + (2+\tau)}$$

C) Compare for $p_f = p_a = 0.1$ and $\tau = 4$.

$$\eta_1 = \frac{1}{\frac{1}{0.9} + \frac{1}{0.9} + 4} = \frac{0.9}{5.6} \simeq 0.161$$

$$\eta_2 = \frac{1}{6\left\{1 + \frac{0.1}{0.9} + \frac{0.1}{0.9}\right\}} = \frac{0.9}{6.6} \simeq 0.136 \quad [< \eta_1]$$

**Question 4. (25 points)**

Consider a slotted time communication system (slot duration=packet transmission time). A multiplexer serves a collection of $M$ Bernoulli traffic sources. Each source generates a packet in a slot with probability $p$. You may assume infinite buffers.

a) Define $A(z) \triangleq \sum_{i=0}^{\infty} \alpha_i z^i$ where $\alpha_i = \Pr\{i \text{ packets arrive during a slot}\}$. Give an expression for $A(z)$ and find the mean.

$$\alpha_i = \binom{M}{i} p^i (1-p)^{M-i} \quad \text{(Binomial)}$$

$$\text{So } A(z) = \sum_{i=0}^{\infty} \binom{M}{i} p^i (1-p)^{M-i} z^i = (1-p+pz)^M.$$

$$E[A] = \left.\frac{dA(z)}{dz}\right|_{z=1} = \left. pM(1-p+pz)^{M-1}\right|_{z=1} = Mp.$$

b) Let $\pi_k = \Pr\{k \text{ packets in the buffer at the end of a slot}\}$ and considering the balance equations based on $\pi = \pi P$ write down the equation for state $i$.

$$\forall i \geq 0: \quad \pi_i = \pi_j P_{ji} = \pi_0 \alpha_i + \sum_{j=1}^{i+1} \pi_j \alpha_{i-j+1}$$

c) $P(z) \triangleq \sum_{i=0}^{\infty} \pi_i z^i$. Show that $P(z) = \dfrac{\pi_0 (1-z) A(z)}{A(z) - z}$

mult by $z^i$ & sum

$$\sum_{i=0}^{\infty} \pi_i z^i = \pi_0 \sum_{i=0}^{\infty} \alpha_i z^i + \sum_{i=0}^{\infty} \sum_{j=1}^{i+1} \pi_j \alpha_{i-j+1} z^i$$

$$P(z) = \pi_0 A(z) + \sum_{j=1}^{\infty} \sum_{i=j-1}^{\infty} \pi_j \alpha_{i-j+1} z^{i-j+1} z^{j-1}$$

$$P(z) = \pi_0 A(z) + \sum_{j=1}^{\infty} \pi_j z^{j-1} \sum_{k=0}^{\infty} \alpha_k z^k$$

$$P(z) = \pi_0 A(z) + \frac{P(z) - \pi_0}{z} A(z)$$

$$\Rightarrow \boxed{P(z) = \frac{\pi_0 A(z)(z-1)}{z - A(z)}}$$

d) Find $\pi_0$ and the mean buffer occupancy, $\overline{N}$.

look at $\lim_{z \to 1} P(z)$ this must be $= 1$.

Have to use L'hôpital rule $\quad \left. \dfrac{\pi_0 [\cancel{A'(z)(1-z)} - A(z)]}{A'(z) - 1} \right|_{z=1}$

write $A' = \dfrac{dA}{dz}$.

$$\pi_0 = 1 - A'(1) = 1 - Mp \quad (\text{or } 1 - \rho_{\text{utilization}})$$

To find $\bar{N}$ we recall

$$\bar{N} = \left. \frac{dP(z)}{dz} \right|_{z=1} = \frac{d}{dz} \left\{ \pi_0 \frac{A(z)(1-z)}{A(z) - z} \right\} \Big|_{z=1}$$

$$= \frac{[A'(z)(1-z) - A(z)][A(z) - z] - A(z)(1-z)[A'(z) - 1]}{(A(z) - z)^2} \Big|_{z=1}$$

Use L'hopitals rule twice to (eventually) find:

$$\boxed{\bar{N} = \bar{a} + \frac{\overline{a^2} - \bar{a}}{2(1-\bar{a})} = Mp + \frac{M(M-1)p^2}{2(1-Mp)}} \quad (\text{for } M \geq 2) \quad (\text{~~for M = 1~~})$$

e) Fill out the following table:

| $M$ | $p$ | $\rho = Mp$ | $N$=E[# in system] | |
|---|---|---|---|---|
| 1 | 0.8 | 0.8 | 0.8 | note for $M=1$ we have $P(z) = 1 - p + pz$ \| $\bar{N} = p$. |
| 2 | 0.4 | 0.8 | 1.6 | $\cancel{P(z) = \frac{(1-2p)(1-p+pz)^2}{(1-p)^2 - p^2 z}}$ |
| 8 | 0.1 | 0.8 | 2.2 | |

***Question 5. (25 points)***

A company has two branches (one in the US and one in China) which are connected by a line of capacity 120,000 bps. This line can be configured as 0 ,1, or 2 voice lines at 50,000 bps each and the remainder of the circuit is used for data. (The capacity is NOT dynamically allocated so the bandwidth allocated to voice is not available for data traffic.)

120 k bps

PART A – the VOICE SYSTEM

The voice traffic arrival rate (inbound and outbound combined) is one call every 3 minutes and the average call holding time is 3 minutes. The voice call arrival process is Poisson and the holding time is exponentially distributed. When a voice trunk is not available, the call is routed to the Public Switched Telephone Network and costs \$1 per call minute (these are considered overflow calls.

*a)* Give (or derive) an expression for the blocking probability for voice when one or two lines are available..

$\lambda = 1/3$ call / minute

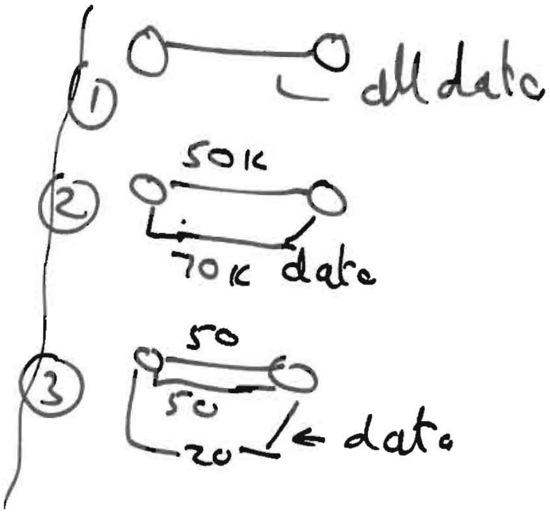
$\bar{x} = \frac{1}{\mu} = 3$ min $\quad \mu = 1/3$

$\lambda/\mu = 1$

(1) $B = 1$

(2) $\Rightarrow \pi_1 = \pi_0 = \frac{1}{2} \quad B = \frac{1}{2}$

(3) $\Rightarrow \pi_1 = \pi_0 = \frac{2}{5} \quad \pi_2 = \frac{1}{5}$




*b)* Fill out the following table. $\quad B = 1/5$

| *Number of trunks for voice* | *Voice Blocking Probability* | *Cost per minute for overflow calls* | Rate to PSTN |
|---|---|---|---|
| 0 | 1 | 1 | $\lambda$ |
| 1 | 1/2 | 1/2 | $\frac{1}{2}\lambda$ |
| 2 | 1/5 | 1/5 | $\frac{1}{5}\lambda$ |

~~Rat~~ Cost / min (0) $\lambda \cdot \bar{x}$

call duration

general (1) $\lambda B \bar{x}$ but $\lambda \bar{x} = \lambda \frac{1}{\mu} = 1$

cost $= B$

PART B – THE DATA SYSTEM

For the data traffic, the packet arrival process is Poisson and the packet arrival rate is $\lambda_d = 80$ packets per second with an average packet length of 1000 bits. The network average packet delay should be less than 0.1 seconds – in order to accomplish this some of the traffic may have to be sent to an external provider. The company has an arrangement with an external provider to carry traffic at a cost of \$0.10 per minute per ~~10,000~~ bps and the carrier guarantees to meet the average packet delay requirement of 0.1 seconds. (Capacity is available as 10,000; 20,000; 30,000 etc.)

c) Give an expression for the maximum traffic load, $\lambda$, (packets per second) that can be handled by the company's own circuit to maintain the required delay constraint when capacity $C$ bps is available.

$\lambda_D = 80$ pkts/sec $\quad L = 1K. (10^3)$

(0) $C = 120K. (120\times10^3)$

$$T = \frac{\bar{x}}{1-\rho} = \frac{L/C}{1-\lambda L/C} = \frac{L}{C-\lambda L} \leq 0.1$$

*d)* Fill out the following table:

| *Capacity available on company circuit* | *Traffic Handling Capability of Company Circuit to meet delay* | *External capacity needed to maintain delay constraint* | *Cost per minute* A : B |
|---|---|---|---|
| 20,000 bps | 10 | 70 : 80 | 0.7 : 0.8 |
| 70,000 bps | 60 | 20 : 30 | 0.2 : 0.3 |
| 120,000 bps | 110 p/s (80) | 0 : 0 | 0 : 0 |

min cap

allows for delay constraint

PART C – TRADEOFF

*e)* How many channels should be dedicated to voice to minimize cost?

| *Capacity Allocated to Voice* | *Voice Cost* | *Data cost* | *Total Cost* |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 50,000 | 0.5 | 0.2 | 0.7 ← * |
| 100,000 | 0.2 | 0.7 | 0.9 |

(A)

(0) $C = 120$ K.

$$T = \frac{1}{120 - 80} = \frac{1}{40} < 0.1$$

when 120K available meet delay criterion.

(1) $C = 70$ K.

$$T = \frac{1}{70 - 80} \quad \text{problem!}$$

$$T = \frac{1}{70 - \lambda_{corp}} < 0.1$$

$$70 - \lambda_{copp} > 10$$

$$\lambda_{corp} < 60 \text{ pkts/sec}$$

$$\lambda_{overflow} = 20$$

$$T = \frac{1}{70 - 60} = \frac{1}{10} = 0.1 \checkmark$$

(2). $C = 20$K

$$T = \frac{1}{20 - \lambda_{copp}} < 0.1$$

$$\lambda_{corp} < 20 - 10 = 10.$$

$$\lambda_{overflow} = 70$$

***Question 6. (15 points)***

A device multiplexes a mix of data and control traffic over a network access link. We are interested in determining the performance characteristics of this link. The link has a capacity of $C$ bits per second. Data traffic consists of fixed length packets of length $L_d$ bits at a (Poisson) rate $\lambda_d$. Control traffic consists of fixed length packets of length $L_c$ bits at a Poisson rate $\lambda_c$.

a) Find expressions for the mean waiting time and time in system for the two types of traffic (when the buffer is managed as FCFS).

M|G|1 $\quad \lambda = \lambda_c + \lambda_D \quad \bar{X}_D = \frac{L_D}{C} \quad \bar{X}_c = \frac{L_c}{C}$

Assume ~~exponential~~ pkt length deterministic $\quad \overline{X^2} \quad \overline{X^2_D} \quad \overline{X^2_c} \quad \rho_D \quad \rho_c$

$$W = \frac{\lambda \overline{X^2}}{2(1-\rho)} \leftarrow \text{for control data} \qquad \rho = \rho_c + \rho_D$$

$$T_c = W + \bar{X}_c \qquad T_D = W + \bar{X}_D$$

b) Find an expression for mean waiting time and time in system for each class when control traffic is given priority over data traffic (HOL).

$$W_c = \frac{\lambda \overline{X^2}}{2} \frac{1}{1-\rho_c} \qquad W_D = \frac{\lambda \overline{X^2}}{2} \frac{1}{(1-\rho_c)(1-\rho_c-\rho_D)}$$

$$T_c = W_c + \bar{X}_c \qquad T_D = W_D + \bar{X}_D$$

c) What is the maximum value for $\lambda_c$ in both cases (for finite delay for control traffic).

FCFS $\rho < 1 \quad \rho = \frac{\lambda_c L_c}{C} + \frac{\lambda_D L_D}{C} < 1$

$$\boxed{\Rightarrow \lambda_c < \frac{C - \lambda_D L_D}{L_c}}$$

HOL $\rho_c < 1 \Rightarrow \frac{\lambda_c L_c}{C} < 1 \Rightarrow \lambda_c < \frac{C}{L_c}$