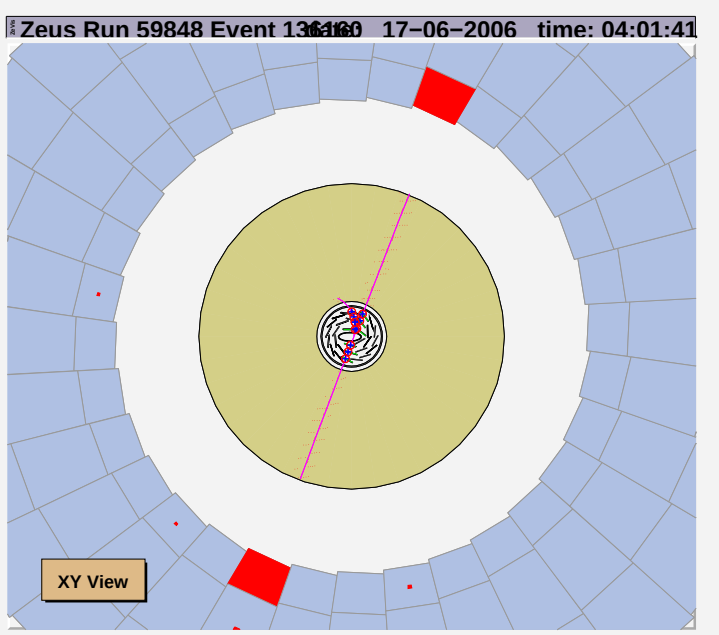
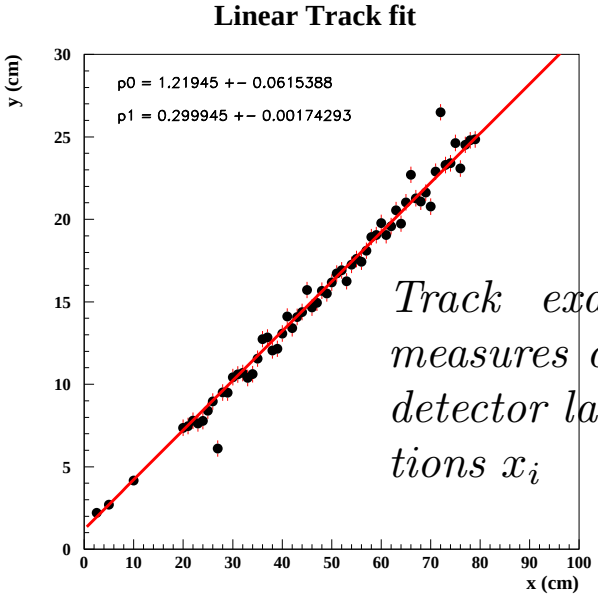
Linear Least square fit - introductory example

Example: Precise muon track fits for possible discovery $Z^* \longrightarrow \mu^+\mu^-$





Track example: Detector measures coordinate y_i in n detector layers at fixed positions x_i

- Necessary conditions for linear least square fit:
  - Measurements with gaussian uncertainties
  - Linear model, here: $y = a_0 + a_1 x + a_2 x^2$
- Fit construction:
  - $\chi^2 = \sum_i \frac{\left[y_i - (a_0 + a_1 x + a_2 x^2)\right]^2}{\sigma_i^2}$
  - Determine a_0, a_1, a_2 by finding χ^2 total minimum (normal equations)
- Check consistency
  - χ^2 and fit probability
  - Outlier rejection
- Detailed error analysis
  - Parameter errors and correlations (error ellipses), track trajectory error band
  - Momentum calculation (error propagation)
- Possible Extensions:
  - Apply constraint fits to both tracks, e.g. $p_t(\mu^+) = p_t(\mu^-)$ $\longrightarrow$ covered in session on extended fits
  - Analysis of obtained $\mu^+\mu^-$ mass spectrum containing background and possible signal events $\longrightarrow$ covered in session on non-linear least square fits

Overview of Linear least square fit section

Part I	Part II	Part III
• Reminder of χ^2-fit method • Linear χ^2-fit examples (Constant, straight line, parabola, etc.) • **Fit of a constant (averaging measurements)** • One single measurement: χ^2_{min} and $\chi^2_{min}+1$, Hesse matrix • Exercise: Two measurements: perform fit by adding χ^2-parabolas • Averaging many measurements, results • Exercise: Compare weighted vs unweighted average	• **χ^2-fit-quality test:** Example: χ^2 of two measurements and known true value • χ^2-function for n degrees of freedom and χ^2-fit probability Exercise: plot and study features of the χ^2-function vs n using the parameterised function New: Generate 1000 random experiments with n degrees of freedom and obtain χ^2 and χ^2-fit probability distributions • χ^2 for two measurements with unknown true value • New exercise: Track position measurement in test beam using 10 detector layers, in each detector 99% chance for signal hit and 1% for random noise hit → Generate 1000 tracks and corresponding hits and obtain χ^2, χ^2-fit probability and measured parameter distributions. Try to reject outliers: Method 1: reject track fits with small χ^2-fit probability, Method 2: iterative, repeat track-fit and downweight outliers • Exercise: **Outlier rejection**, case world average of m_W, study how the rejection of certain measurements change the average and the χ^2-fit probability • New exercise: Upscaling of errors a la PDG to obtain reasonable χ^2 • New exercise: Pulls of single measurements to the average	• General form of linear χ^2 • Solution by normal equations • Normal equation solution for **straight line fit** • Exercise: Learn qualitative features of straight line fits, e.g. importance of lever arm • Exercise: Straight line fit and detailed error analysis (error ellipse, trajectory error band) • New exercise: Coordinate transformation such that the coordinate center is in the middle of the points → study the effect on the parameter errors and correlation • New exercise: Add a very precise point at the origin of the track such that the p_0 parameter is basically fixed. Repeat the track-fit and study the effect on the slope and error • Exercise: **Parabola track fit**, complete analysis: *fit, outlier-rejection, parameter errors/correlation, trajectory uncertainty,* ***momentum calculation***