Kelly-Optimal Betting in Advantage Play Blackjack Games

Luke Jackson LJACKSO.93@GMAIL.COM
Hanif Jetha HANIF.JETHA@MAIL.MCGILL.CA

Abstract

We perform repeated Monte-Carlo simulations of advantage play blackjack games to find win, push and blackjack probabilities for a player in a 2-player blackjack game at various counts. Having approximated these probabilties we calculate Kelly-optimal bets for a player playing basic strategy versus a dealer (and no other players). We find that a player playing basic strategy versus a single dealer should start betting when the count reaches 4, and find optimal Kelly bets for optimal play as the count increases (or decreases).

1. Introduction

Blackjack is one of the most popular casino games due to its simplicity and low house edge (relative to other games). One or more players plays strictly against the dealer, following a set of standard, but slightly modifiable rules. We omit the rules of blackjack here, but the interested reader can check (Wikipedia, 2014). Advantage play in blackjack is also extremely prevalent, due to the possibility of quickly trimming away the house edge using tricks and techniques varying in complexity, from basic card counting, to shuffle tracking and card identification. The most popular and widespread advantage play technique is card counting, where an advantage player increments the "count" for each low card (2-6), and decrements the count for each high card (10-A) exposed to the table. A high count indicates many high cards remain in the deck, and so the probability of hitting blackjack is higher. Due to the large number of cases that need be considered, calculating closed form probabilties is both tedious and error-prone. We approach the problem through Monte-Carlo simulation. By building a game engine, we can simulate blackjack games in our specific setting, where we are primarily interested in probabilities of events given counts. Given these probabilties, we then seek to calculate optimal bets using the n-outcome Kelly criterion. There has not been much work in this area as it is not on the frontier of any popular research area, but our experiments may prove useful for advantage play gamblers seeking to play (and bet) optimally.

2. Card Counting

Card counting is a relatively simple advantage play strategy where a player increases a "count" given values of exposed cards. As literature on this strategy is exhaustive and readily accesible, we simply state our counting rules:

Let C_i be the count at time step i.
Let $c_{i1}, c_{i2}, ..., c_{in}$ be the exposed cards at time step i.
Let $v(c_{ij})$ be the value function, which takes a card and returns a count, defined as follows:

$$v(c_{ij}) = \begin{cases} 1 & c_{ij} \in \{2, 3, .., 6\} \\ 0 & c_{ij} \in \{7, 8, 9\} \\ -1 & c_{ij} \in \{10, J, ..., A\} \end{cases}$$

Then $C_{i+1} = \sum_j v(c_{ij}) + C_i$

The player calulates the count at the end of each game (time step i) and uses this information to bet accordingly at time step $i + 1$.

3. Kelly Criterion

The Kelly Criterion tells a gambler the fraction of his capital (in this case "bankroll") to wager at a given time step. It is derived by maximizing the expected value of a utility function (in this case log_2[bankroll]). We omit the derivation here, but the interested reader can consult Kelly's original paper (J.L. Kelly, 1956) as well as Edward Thorp's extension to it (Thorp, 1997). The Kelly criterion for n outcomes is given by:

$$\max \sum p_i \log (1 + b_i x)$$

Where p_i is the probability of outcome i occuring, b_i is the profit multiple of your bet for that outcome (so if you won, but did not get blackjack, you would recieve

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your bet back, plus 1∗bet - so b_i in this case would be 1), and we sum over all outcomes.

4. Game Engine

We have built (in Java), a game engine capable of simulating 2-player single-deck blackjack games. The code is provided along with this report. In this engine we define a playing strategy for both players. We have implemented 2 player strategies and 2 dealer strategies. The code can easily be extended to simulate games with custom strategies. The player strategy can be either "Basic Strategy" or "Simple Strategy". "Basic Strategy" can be found at (Wikipedia, 2014). It is the strategy that loses the least money to the dealer in the long run (minimizes house edge). It was formulated by Edward Thorp in the 1960s and has now become prevalent in blackjack. "Simple strategy" is when the player holds if the value of his hand ≥ 17. The dealer plays "simple strategy" with 1 degree of freedom: "Hit on soft 17" is "simple strategy", but if the 17-value hand consists of an Ace and a 6, the dealer must hit. "Hold on soft 17" is defined likewise (but the dealer holds). When the dealer stands on "soft 17", the player's win probability increases as the player is more likely to draw a hand with higher value than the dealer's, and not bust. When the dealer hits on "soft 17", the player's win probability decreases as the dealer will more likely end up with higher value hands (but will also bust more often, although the increased "busting" is outweighed by the frequency of beating the other player). We have performed experiments with the player playing both "basic" and "simple" strategies vs. the dealer playing both "hit on soft 17" and "stand on soft 17" strategies, for a total of 4 experiments.

We define a "loaded" deck to be a deck that has a non-zero count and cards removed from it. We create loaded decks for values of the count C from $C = -20$ to $C = 20$ by removing cards at random until the count reaches the desired value. We then simulate our two-player game using this loaded deck and record the outcome. We do this for $n = 10000$ iterations, for each loaded deck ($C = -20$ to $C = 20$) and each of the four possible combinations of strategies. The results of these experiments can be found in tables attached to the end of this report (note these are unchanged from the ones submitted on Wednesday).

Code for our game engine is available at https://github.com/ljackso/BlackJackStrategyOptimizer

5. Results

The following results tables can be found at the end of the document:

Basic Strategy - Hit on Soft 17 - (Table 1)
Basic Strategy - Stand on Soft 17 - (Table 2)
Simple Strategy - Hit on Soft 17 - (Table 3)
Simple Strategy - Stand on Soft 17 - (Table 4)

The figures demonstrate our results for the player playing "basic strategy" vs. the dealer playing both "hit on soft 17" and "stand on soft 17". We plot outcome probabilities vs. count and Kelly bets vs. count for each game.

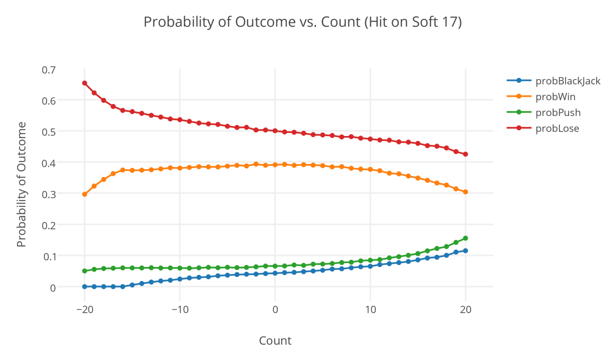

Figure 1. Probability of outcome at various count values, for a "basic strategy" game where the dealer hits on soft 17. We notice that as the count increases, the probability of losing begins to decrease, mostly due to increase in probability of obtaining Blackjack (the non-Blackjack win probability does not change as much)

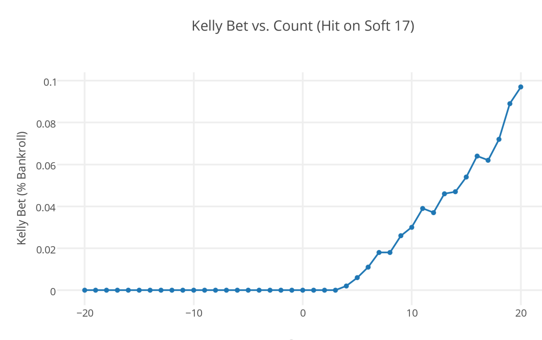

Figure 2. Kelly bet (as % of bankroll) at various count values, for a "basic strategy" game where the dealer hits on soft 17.

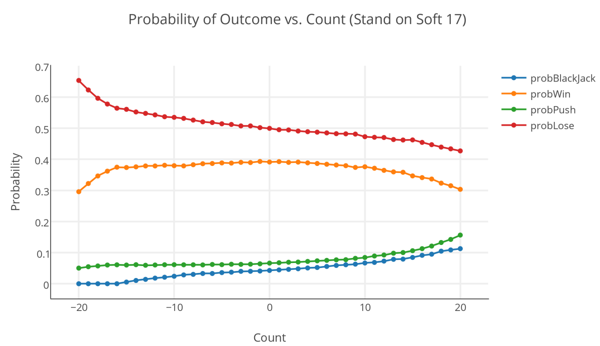

Figure 3. Probability of outcome at various count values, for a "basic strategy" game where the dealer stands on soft 17. We notice that the data is almost identical to the "hit on soft 17" game.

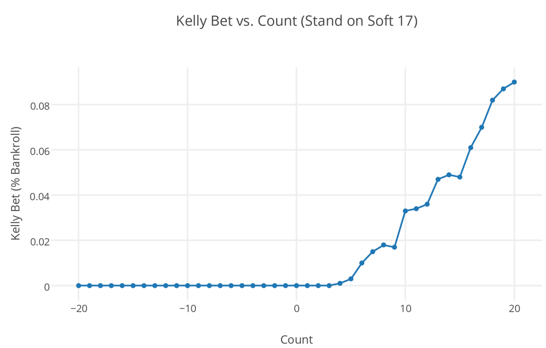

Figure 4. Kelly bet (as % of bankroll) at various count values, for a "basic strategy" game where the dealer stands on soft 17.

6. Discussion

Our results make intuitive sense. In the case of the player playing "basic strategy" vs. the dealer playing "simple strategy" and "hitting on soft 17", as the count increases, we observe that the probability of losing decreases, and more importantly, the probability of obtaining Blackjack sharply increases (which agrees with theoretical results). Furthermore, we find that the optimal Kelly bet is very close to 0 until the count reaches 4, at which point the Kelly criterion suggests the player bets 0.2% of his bankroll. As the count increases, the Kelly bet correspondingly increases as well, suggesting that the player bet 9.7% of his bankroll when the count reaches 20. Due to the results being so similar across our 4 experiments, we omit a detailed analysis of each experiment, but note that in the case of the dealer having to "stand on soft 17", the player loses a slight edge to the house, as is expected. The corresponding Kelly bet when the count reaches 4 is 0.1% of the player's bankroll, and 9.0% of the player's bankroll when the count reaches 20. This reduced Kelly bet reflects the lower expected value of this game.

7. Conclusion

We have derived optimal Kelly bets for advantage play Blackjack by approximating outcome probabilities using Monte Carlo simulations.

There are certain limitations to our research, such as we only experimented with a single shuffling technique (The Fisher-Yates shuffle) and we have only provided data for single deck games, although with the game engine we created, it would be possible to generate data for multiple deck games.

It is also clear to see from our results that Blackjack is a game where the house has a large advantage over the optimal player. Ultimately research like this can be instrumental as it demonstrates that there are many techniques available that can help players derive strategies not just to win each individual hand, but also to bet strategically with the aim of overcoming the house edge. It would be interesting to extend this work by experimenting with different shuffling techniques and frequencies, as well as adding multiple decks.

Overall this paper provides a brief insight into strategic betting in advantage play Blackjack games, and this betting scheme may be more profitable in the long run than traditional "ladder" betting schemes used by advantage players.

References

J.L. Kelly, Jr. A new interpretation of information rate. Technical report, AT&T Labs, 1956.

Thorp, Edward O. The kelly criterion in blackjack, sports betting, and the stock market. Technical report, Edward O. Thorp and Associates, 1997.

Wikipedia. Blackjack — wikipedia, the free encyclopedia, 2014. URL http://en.wikipedia.org/w/index.php?title=Blackjack&oldid=633811056. [Online; accessed 8-December-2014].

Count	P(Blackjack)	P(Win)	P(Push)	P(Lose)	Kelly Bet (% Bankroll)
-20	0.0	0.29636	0.05032	0.65332	0.0
-19	0.0	0.32267	0.05543	0.6219	0.0
-18	0.0	0.34408	0.05818	0.5977399999999999	0.0
-17	0.0	0.36272	0.05909	0.57819	0.0
-16	0.0	0.37422	0.06026	0.56552	0.0
-15	0.00524	0.37305	0.05996	0.56175	0.0
-14	0.01016	0.37379	0.05999	0.55606	0.0
-13	0.01462	0.37506	0.06057	0.54975	0.0
-12	0.01824	0.37788	0.05974	0.5441400000000001	0.0
-11	0.02049	0.38143	0.05998	0.5381	0.0
-10	0.02432	0.38035	0.05949	0.53584	0.0
-9	0.02795	0.38246	0.05928	0.5303100000000001	0.0
-8	0.02981	0.38487	0.06032	0.525	0.0
-7	0.03154	0.38425	0.06186	0.52235	0.0
-6	0.03477	0.38406	0.06062	0.52055	0.0
-5	0.0367	0.38661	0.06209	0.5146	0.0
-4	0.0388	0.3892	0.06112	0.51088	0.0
-3	0.03985	0.38728	0.06163	0.5112399999999999	0.0
-2	0.04026	0.39339	0.06351	0.50284	0.0
-1	0.04191	0.38953	0.06589	0.50267	0.0
0	0.04313	0.39089	0.06578	0.5002	0.0
1	0.04492	0.39226	0.06629	0.49653	0.0
2	0.04584	0.38931	0.06943	0.49541999999999997	0.0
3	0.04803	0.3914	0.06831	0.4922599999999999	0.0
4	0.05016	0.38998	0.07203	0.48783	0.002
5	0.05254	0.38825	0.07241	0.4868	0.006
6	0.05634	0.38429	0.07421	0.48516000000000004	0.011000000000000003
7	0.05732	0.38497	0.07732	0.4803900000000001	0.01800000000000001
8	0.0604	0.37998	0.07832	0.48130000000000006	0.01800000000000001
9	0.06367	0.37726	0.08265	0.47642000000000007	0.026000000000000016
10	0.06515	0.37647	0.08469	0.47368999999999994	0.03000000000000002
11	0.07069	0.37202	0.08674	0.4705499999999999	0.03900000000000003
12	0.07371	0.36408	0.09222	0.46999	0.037000000000000026
13	0.07717	0.3621	0.09609	0.46464000000000005	0.046000000000000034
14	0.08079	0.35493	0.10058	0.4637	0.047000000000000035
15	0.08584	0.3484	0.10605	0.45971000000000006	0.05400000000000004
16	0.09161	0.34118	0.11471	0.4525	0.06400000000000004
17	0.0943	0.3325	0.12252	0.45067999999999997	0.06200000000000005
18	0.10042	0.32596	0.12857	0.44504999999999995	0.07200000000000005
19	0.11087	0.3137	0.14201	0.43342	0.08900000000000007
20	0.11527	0.30412	0.15549	0.42512000000000005	0.09700000000000007

Table 1. Basic Strategy - Hit on Soft 17

Count	P(Blackjack)	P(Win)	P(Push)	P(Lose)	Kelly Bet (% Bankroll)
-20	0.0	0.29596	0.05028	0.65376	0.0
-19	0.0	0.32223	0.05471	0.62306	0.0
-18	0.0	0.34653	0.05713	0.59634	0.0
-17	0.0	0.362	0.06017	0.5778300000000001	0.0
-16	0.0	0.37473	0.06087	0.5644	0.0
-15	0.0053	0.37374	0.06013	0.5608299999999999	0.0
-14	0.01051	0.37585	0.06126	0.55238	0.0
-13	0.01407	0.37872	0.0594	0.54781	0.0
-12	0.01808	0.37875	0.06027	0.5429	0.0
-11	0.02097	0.38095	0.06107	0.53701	0.0
-10	0.02401	0.37965	0.06126	0.53508	0.0
-9	0.02838	0.37895	0.0609	0.53177	0.0
-8	0.0301	0.38272	0.06093	0.52625	0.0
-7	0.03292	0.38588	0.06052	0.52068	0.0
-6	0.03291	0.38665	0.06201	0.51843	0.0
-5	0.03582	0.38849	0.06141	0.51428	0.0
-4	0.03713	0.38794	0.06271	0.51222	0.0
-3	0.03988	0.39033	0.06228	0.50751	0.0
-2	0.03998	0.38966	0.06286	0.5075	0.0
-1	0.04082	0.3932	0.06401	0.50197	0.0
0	0.04267	0.3914	0.06626	0.49966999999999995	0.0
1	0.04452	0.39265	0.06752	0.49531000000000003	0.0
2	0.04614	0.39024	0.06909	0.49453	0.0
3	0.04813	0.39113	0.0697	0.49104000000000003	0.0
4	0.05064	0.38869	0.07175	0.48892	0.001
5	0.05223	0.38663	0.0737	0.48744	0.003
6	0.05561	0.38421	0.07513	0.48505	0.010000000000000002
7	0.05883	0.38179	0.07686	0.48251999999999995	0.015000000000000006
8	0.06102	0.37964	0.07732	0.48202	0.01800000000000001
9	0.06303	0.37405	0.08167	0.48124999999999996	0.017000000000000008
10	0.0667	0.37609	0.08445	0.47276000000000007	0.03300000000000002
11	0.06852	0.37139	0.08915	0.47094	0.03400000000000002
12	0.07282	0.36457	0.09239	0.47021999999999997	0.036000000000000025
13	0.07845	0.35968	0.09814	0.46373	0.047000000000000035
14	0.07914	0.35843	0.10027	0.46216	0.04900000000000004
15	0.08458	0.34677	0.10619	0.46246	0.048000000000000036
16	0.0914	0.34128	0.11278	0.45454000000000006	0.06100000000000005
17	0.09497	0.33668	0.12116	0.44719	0.07000000000000005
18	0.1047	0.32342	0.13254	0.43934000000000006	0.08200000000000006
19	0.1086	0.31511	0.14249	0.43379999999999996	0.08700000000000006
20	0.11282	0.3036	0.15644	0.42713999999999996	0.09000000000000007

Table 2. Basic Strategy - Stand on Soft 17

Count	P(Blackjack)	P(Win)	P(Push)	P(Lose)	Kelly Bet (% Bankroll)
-20,	0.0,	0.41346,	0.12103,	0.46551,	0.0
-19,	0.0,	0.40894,	0.11614,	0.47492,	0.0
-18,	0.0,	0.41126,	0.11175,	0.47699,	0.0
-17,	0.0,	0.40744,	0.10667,	0.48588999999999993,	0.0
-16,	0.0,	0.40652,	0.10292,	0.49056,	0.0
-15,	0.00525,	0.40027,	0.09935,	0.49513000000000007,	0.0
-14,	0.01047,	0.39433,	0.0976,	0.49760000000000004,	0.0
-13,	0.01493,	0.38675,	0.09641,	0.5019100000000001,	0.0
-12,	0.01812,	0.38565,	0.09328,	0.50295,	0.0
-11,	0.02212,	0.38071,	0.09235,	0.50482,	0.0
-10,	0.02533,	0.37586,	0.09185,	0.50696,	0.0
-9,	0.02648,	0.37653,	0.08891,	0.5080800000000001,	0.0
-8,	0.0304,	0.3708,	0.08969,	0.50911,	0.0
-7,	0.03273,	0.36894,	0.08923,	0.5091,	0.0
-6,	0.03445,	0.36652,	0.08829,	0.51074,	0.0
-5,	0.03644,	0.36835,	0.08672,	0.50849,	0.0
-4,	0.03878,	0.36155,	0.08748,	0.51219,	0.0
-3,	0.04042,	0.36282,	0.08555,	0.5112099999999999,	0.0
-2,	0.04139,	0.35964,	0.08682,	0.51215,	0.0
-1,	0.04171,	0.35823,	0.08774,	0.5123200000000001,	0.0
0,	0.04236,	0.36092,	0.08674,	0.50998,	0.0
1,	0.04517,	0.35616,	0.08757,	0.5111000000000001,	0.0
2,	0.04665,	0.35466,	0.08833,	0.51036,	0.0
3,	0.04781,	0.35572,	0.08648,	0.5099899999999999,	0.0
4,	0.05156,	0.35291,	0.08739,	0.50814,	0.0
5,	0.05386,	0.35243,	0.08831,	0.5054,	0.0
6,	0.05574,	0.34853,	0.09142,	0.50431,	0.0
7,	0.05854,	0.34901,	0.09012,	0.50233,	0.0
8,	0.05919,	0.34536,	0.09405,	0.5014,	0.0
9,	0.06395,	0.34565,	0.09385,	0.49654999999999994,	0.0
10,	0.06641,	0.3403,	0.09714,	0.49615,	0.0
11,	0.0691,	0.3385,	0.09919,	0.4932099999999999,	0.0
12,	0.07211,	0.33826,	0.10132,	0.48831,	0.0
13,	0.07801,	0.33732,	0.10449,	0.48018000000000005,	0.012000000000000004
14,	0.08047,	0.33474,	0.10955,	0.47524,	0.01800000000000001
15,	0.08565,	0.32889,	0.1138,	0.47165999999999997,	0.025000000000000015
16,	0.09074,	0.32746,	0.12016,	0.46164000000000005,	0.04200000000000003
17,	0.09503,	0.32287,	0.12717,	0.45493000000000006,	0.05100000000000004
18,	0.10173,	0.31712,	0.13329,	0.44786000000000004,	0.06400000000000004
19,	0.10957,	0.31093,	0.14485,	0.43465,	0.08400000000000006
20,	0.11514,	0.30049,	0.15935,	0.42501999999999995,	0.09300000000000007

Table 3. Simple Strategy - Hit on Soft 17

Count	P(Blackjack)	P(Win)	P(Push)	P(Lose)	Kelly Bet (% Bankroll)
-20,	0.0,	0.41288,	0.12184,	0.4652799999999999,	0.0
-19,	0.0,	0.40888,	0.1155,	0.47561999999999993,	0.0
-18,	0.0,	0.40971,	0.11327,	0.47702,	0.0
-17,	0.0,	0.40791,	0.10486,	0.48723000000000005,	0.0
-16,	0.0,	0.40885,	0.10232,	0.48883,	0.0
-15,	0.00523,	0.39856,	0.09976,	0.49644999999999995,	0.0
-14,	0.01023,	0.39546,	0.09702,	0.49729,	0.0
-13,	0.01456,	0.39104,	0.09695,	0.49744999999999995,	0.0
-12,	0.01901,	0.38699,	0.09393,	0.50007,	0.0
-11,	0.02129,	0.38117,	0.09151,	0.50603,	0.0
-10,	0.02513,	0.37868,	0.09241,	0.50378,	0.0
-9,	0.02677,	0.37404,	0.09147,	0.50772,	0.0
-8,	0.03017,	0.37308,	0.08883,	0.5079199999999999,	0.0
-7,	0.03151,	0.3715,	0.08892,	0.50807,	0.0
-6,	0.0337,	0.3675,	0.08767,	0.51113,	0.0
-5,	0.03519,	0.36634,	0.0874,	0.51107,	0.0
-4,	0.03739,	0.36428,	0.0862,	0.51213,	0.0
-3,	0.0405,	0.36367,	0.08579,	0.51004,	0.0
-2,	0.04009,	0.3603,	0.08778,	0.51183,	0.0
-1,	0.04201,	0.36102,	0.08658,	0.51039,	0.0
0,	0.04258,	0.35754,	0.08627,	0.5136099999999999,	0.0
1,	0.04403,	0.35892,	0.08822,	0.5088299999999999,	0.0
2,	0.04667,	0.35359,	0.08928,	0.51046,	0.0
3,	0.04907,	0.35529,	0.08792,	0.50772,	0.0
4,	0.0495,	0.35386,	0.09007,	0.50657,	0.0
5,	0.05264,	0.35118,	0.08784,	0.50834,	0.0
6,	0.05488,	0.34987,	0.09018,	0.50507,	0.0
7,	0.05721,	0.34948,	0.09243,	0.50088,	0.0
8,	0.05929,	0.34721,	0.09308,	0.50042,	0.0
9,	0.06365,	0.34178,	0.09531,	0.49926000000000004,	0.0
10,	0.06694,	0.3409,	0.09552,	0.49663999999999997,	0.0
11,	0.06959,	0.34064,	0.09915,	0.49062000000000006,	0.0
12,	0.07455,	0.33723,	0.10423,	0.48399000000000003,	0.002
13,	0.07689,	0.33538,	0.10612,	0.48161,	0.007
14,	0.08021,	0.33166,	0.10897,	0.47916000000000003,	0.011000000000000003
15,	0.08518,	0.3289,	0.11393,	0.47199,	0.024000000000000014
16,	0.0903,	0.32736,	0.11888,	0.46346,	0.03900000000000003
17,	0.09527,	0.32162,	0.12646,	0.45665,	0.04900000000000004
18,	0.10107,	0.3182,	0.13446,	0.44627000000000006,	0.06500000000000004
19,	0.10947,	0.31117,	0.14507,	0.43428999999999995,	0.08400000000000006
20,	0.11652,	0.30194,	0.15823,	0.42330999999999996,	0.09800000000000007

Table 4. Simple Strategy - Stand on Soft 17