Machine Learning (CSE 446): Decision Trees

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- If ϕ maps to $\{0, 1\}$, we call it a "binary feature (function)."
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- Feature functions can map to categorical values, ordinal values, integers, and more.

Features

Data derived from `https://archive.ics.uci.edu/ml/datasets/Auto+MPG`

mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
18.0	8	307.0	130.0	3504.	12.0	70	1
15.0	8	350.0	165.0	3693.	11.5	70	1
18.0	8	318.0	150.0	3436.	11.0	70	1
16.0	8	304.0	150.0	3433.	12.0	70	1
17.0	8	302.0	140.0	3449.	10.5	70	1
15.0	8	429.0	198.0	4341.	10.0	70	1
14.0	8	454.0	220.0	4354.	9.0	70	1
14.0	8	440.0	215.0	4312.	8.5	70	1
14.0	8	455.0	225.0	4425.	10.0	70	1
15.0	8	390.0	190.0	3850.	8.5	70	1
15.0	8	383.0	170.0	3563.	10.0	70	1
14.0	8	340.0	160.0	3609.	8.0	70	1
15.0	8	400.0	150.0	3761.	9.5	70	1
14.0	8	455.0	225.0	3086.	10.0	70	1
24.0	4	113.0	95.00	2372.	15.0	70	3
22.0	6	198.0	95.00	2833.	15.5	70	1
18.0	6	199.0	97.00	2774.	15.5	70	1
21.0	6	200.0	85.00	2587.	16.0	70	1
27.0	4	97.00	88.00	2130.	14.5	70	3
26.0	4	97.00	46.00	1835.	20.5	70	2
25.0	4	110.0	87.00	2672.	17.5	70	2
24.0	4	107.0	90.00	2430.	14.5	70	2

Goal: predict whether mpg is < 23 ("bad" $= 0$) or above ("good" $=$ 1) given other attributes (other columns).

201 "good" and 197 "bad"; guessing the most frequent class (good) will get 50.5% accuracy.

Contingency Table

values of y		values of feature ϕ			
		v_1	v_2	$\cdots$	v_K
	0				
	1				

Decision Stump Example

y	maker		
	america	europe	asia
0	174	14	9
1	75	56	70

$\downarrow$	$\downarrow$	$\downarrow$
0	1	1

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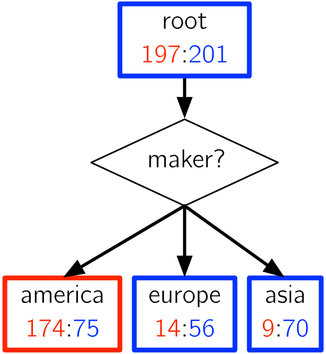

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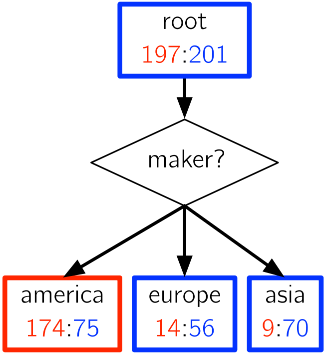

Errors: $75 + 14 + 9 = 98$ (about 25%)

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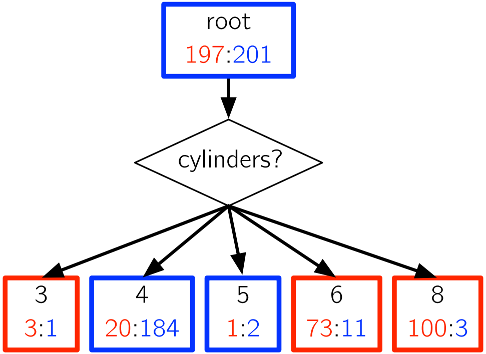

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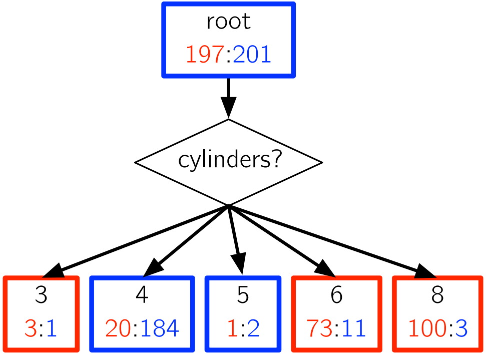


Errors: $1 + 20 + 1 + 11 + 3 = 36$ (about 9%)

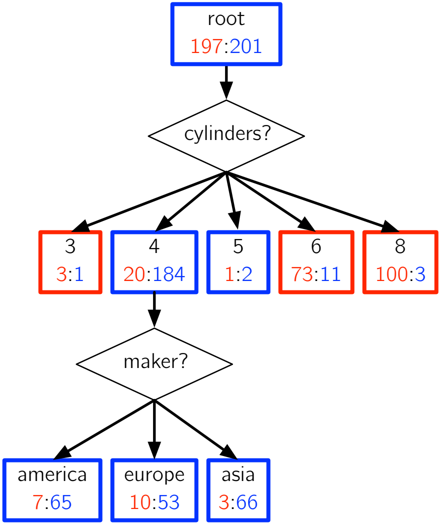
Key Idea: Recursion

A single feature **partitions** the data.

For each partition, we could choose another feature and partition further.

Applying this recursively, we can construct a **decision tree**.

Decision Tree Example



Error reduction compared to the cylinders stump?

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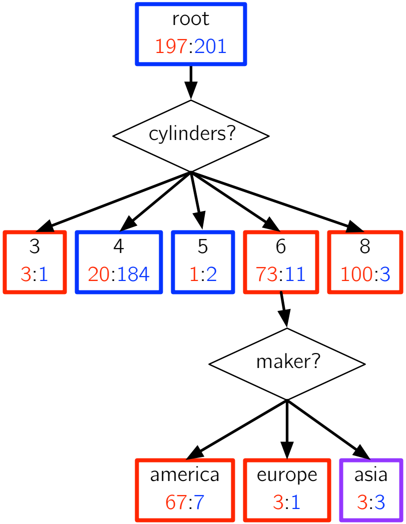

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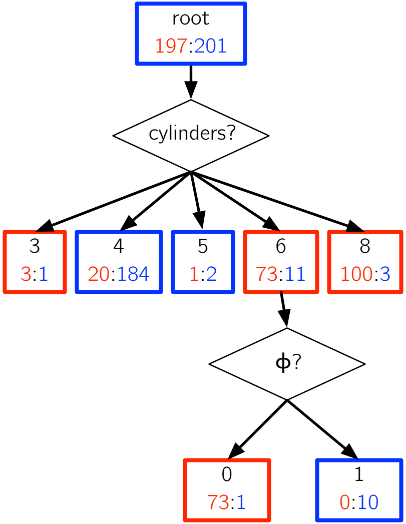

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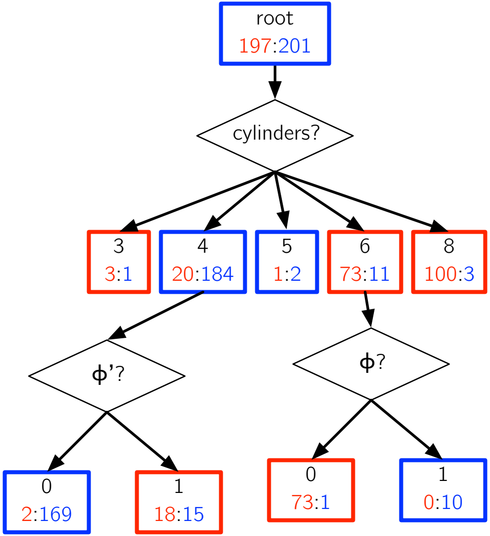

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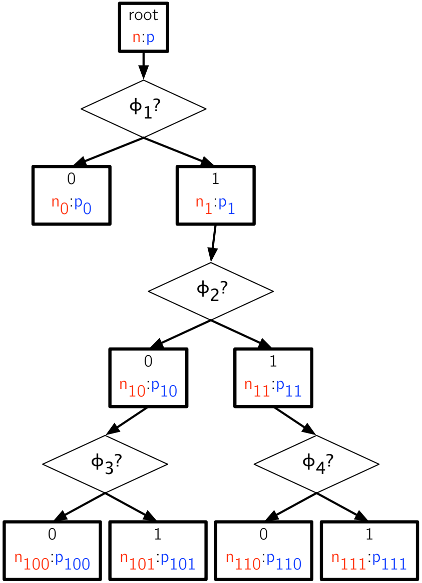
Decision Tree: Making a Prediction



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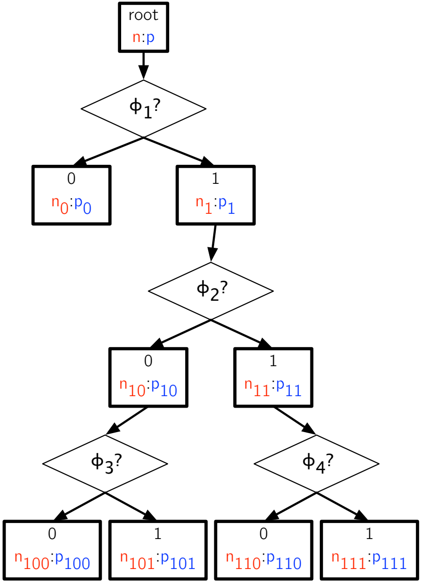

Data: decision tree t, input example x
Result: predicted class
if t *has the form* $\textsc{Leaf}(y)$ **then**
    return y;
else
    $\#$ $t.\phi$ is the feature associated with t;
    $\#$ t.child(v) is the subtree for value v;
    return $\textsc{DTreeTest}(t.\text{child}(t.\phi(x)), x)$;
end

Algorithm 1: $\textsc{DTreeTest}$

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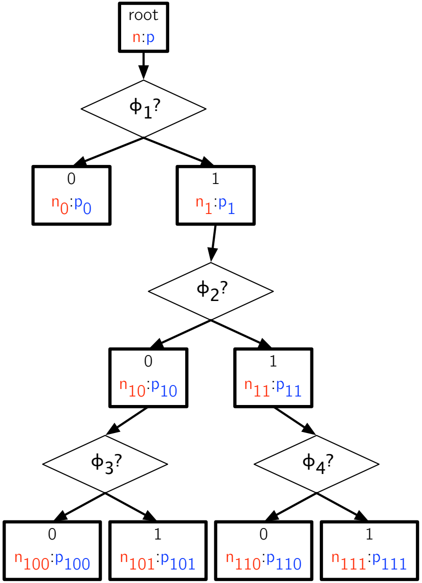

Equivalent boolean formulas:

$$(\phi_1 = 0) \Rightarrow [\![n_0 < p_0]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 0) \wedge (\phi_3 = 0) \Rightarrow [\![n_{100} < p_{100}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 0) \wedge (\phi_3 = 1) \Rightarrow [\![n_{101} < p_{101}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 1) \wedge (\phi_4 = 0) \Rightarrow [\![n_{110} < p_{110}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 1) \wedge (\phi_4 = 1) \Rightarrow [\![n_{111} < p_{111}]\!]$$

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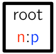
3^D formulas.

Growing a Decision Tree



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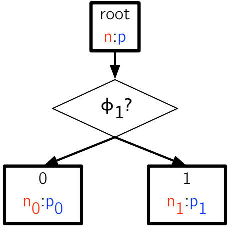

We chose feature ϕ_1. Note that $n = n_0 + n_1$ and $p = p_0 + p_1$.

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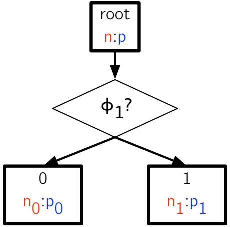
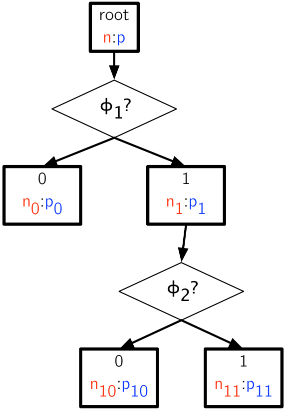


We chose not to split the left partition. Why not?

Growing a Decision Tree



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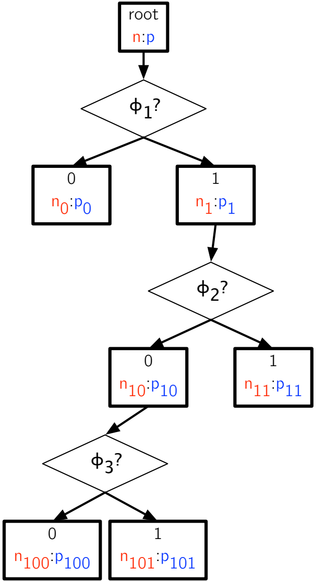

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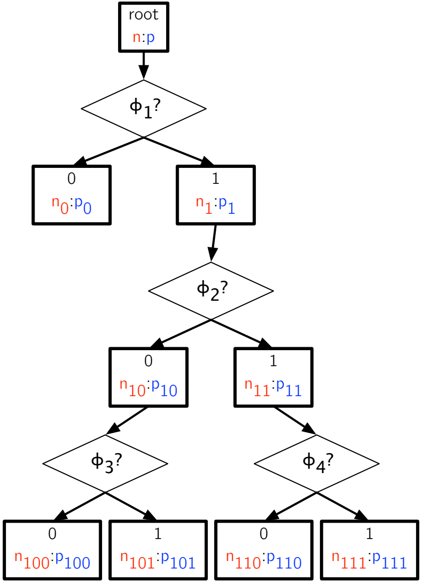
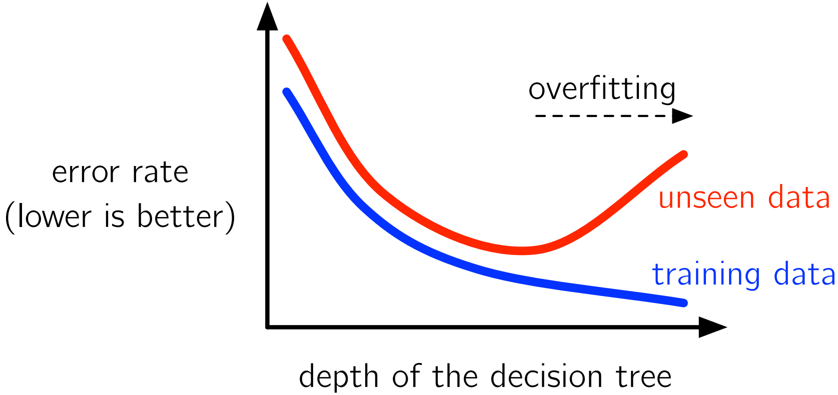


Greedily Building a Decision Tree (Binary Features)

Data: data D, feature set Φ
Result: decision tree
if *all examples in D have the same label y, or Φ is empty and y is the best guess* **then**
  return $\textsc{Leaf}(y)$;
else
  for *each feature ϕ in Φ* **do**
    partition D into D_0 and D_1 based on ϕ-values;
    let mistakes(ϕ) $=$ (non-majority answers in D_0) $+$ (non-majority answers in D_1);
  end
  let ϕ^* be the feature with the smallest number of mistakes;
  return $\textsc{Node}(\phi^*, \{0 \rightarrow \textsc{DTreeTrain}(D_0, \Phi \setminus \{\phi^*\}), 1 \rightarrow \textsc{DTreeTrain}(D_1, \Phi \setminus \{\phi^*\})\})$;
end

Algorithm 2: $\textsc{DTreeTrain}$

Danger: Overfitting



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Splitting your data into training/development/test requires careful thinking. Starting point: randomly shuffle examples with an 80%/10%/10% split.