DLD

Syllabus:

1. Number System
2. Logic Functions
3. Minimization
4. Design & Synthesis of Combinational Circuits
5. Sequential circuits

(1) Introduction:-

Base-1 → 0

Base 2 → 0, 1

Base 8 → 0, 1, 2, 3, 4, 5, 6, 7

Base N → ~~0, 1, 2, - - -, N-1.~~

Base 12 → 0, 1, 2, - - 9, A, B

Base 16 → 0, 1, 2, - - - 9, A, B, - - F.

A - 10
B - 11
C - 12
D - 13
E - 14
F - 15

Ex:- $(0126)_6$ → Wrong $(012\overset{\times}{6})$

(2) Conversion to Base 10:-

$b_1 \to b_2$

(i) $b_1 \to b_{10}$

(ii) $b_{10} \to b_2$

(1) $(abc)_x \to (?)_{10}$

$= ax^2 + bx + cx^0$

$= (ax^2 + bx + c)_{10}$

(2) $(123)_7 = (?)_{10}$

$1\times7^2 + 2\times7^1 + 3\times7^0$

$= 49 + 14 + 3$

$= (66)$

(3) $(12AB)_{16} = (?)_{10}$

$1\times16^3 + 2\times16^2 + A\times16^1 + B\times16^0$

$(16^3 + 2\times16^2 + 10\times16 + 11)_{10}$

(4) $(1010)_2 = (?)_{10}$

$(1010)_2$ with positions 3 2 1 0

$= 1\times2^3 + 1\times2^1 = 8 + 2 = (10)_{10}$

(5) 101010 (32 16 8 4 2 1)

$32 + 8 + 2 = (42)_{10}$

(3) Conversion from base 10 :-

(1) $(11)_{10} \to (?)_2$

$\Rightarrow (1011)_2$

11 = 8 4 2 1
1 0 1 1

(2) $(32)_{10} = (?)_2$ $(100000)_2$

32 = 32 16 8 4 2 1
1 0 0 0 0 0

(3) $(1056)_{16} = (?)_2$

~~1056 = 0001 0000 0101 0110~~
~~$= (1000001010110)_2$~~

$1056 = (0001000001010110)_2$

$000(1000001010110)_2 = (?)_{16}$

$= (1056)_{16}$

(4) $(273)_8 = (?)_2$

$273 = 010111011 = (10111011)_2$

(5) $(32)_{10} \to (\ )_2$

$(32)_{10} \to (20)_{16} \to (?)_2$

$(20)_{16} \to (00100000)_2$
$(100000)_2$

(6) $(65)_{10} \rightarrow (?)_2$

$(65)_{10} \rightarrow (?)_{16} \rightarrow (?)_2$

$(41)_{16} \rightarrow 01000001$

$\Rightarrow (1000001)_2$

⑥ If $(11)_2 + (22)_3 + (33)_4 + (44)_5 = (abc)_6$

Find a, b, c

a, b, c values are 0, 1, ---5

converts to base 10

$(11)_2 + (22)_3 + (33)_4 + (44)_5$

$3 + 8 + 15 + 24 = (50)_{10}$

$(50)_{10} = (abc)_6$

$\overset{a\,b\,c}{(1\,2\,2)}_6$

$$\begin{array}{r|l} 6 & 50 \\ \hline 6 & 8 - 2 \\ \hline & 1 - 2 \end{array}$$

⑦ Find the following solution, determine the possible values of 'x'

$(123)_5 = (x8)_y$

① condition

② converts to 10 base

$(123)_5 \rightarrow 25 + 10 + 3 = 38$

$x < y$; $y > 8$

$38 = xy + 8$

$\Rightarrow xy = 30$

$(x8)_y = 38$

$\cancel{(1,30)}\ \cancel{(2,15)}$

$x < y$ & $y > 8$

$\therefore (1,30), (2,15), (3,10)$

$\therefore x = 1, 2, 3$

1×30
2×15
3×10
5×6 ✗
6×5 ✗
10×3 ✗
15×2 ✗
30×1 ✗

(8) How many values of x and y are possible for

$(42)_9 = (x3)_y$

$x < y$
$y > 3$

$\Rightarrow (42)_9 \Rightarrow (38)_{10}$

$\therefore 38 = xy + 3$

$\Rightarrow xy = 35$

~~$x = 1, 5, 7$~~
~~$y = 5, 7, 35$~~

$x = 1, 5$
$y = 7, 35$

$(1,35)\ (5,7)$

1×35
5×7
7×5 ✗
35×1 ✗

(9) $(123)_x = (12x)_3$; $x =$ ——

a) 3 b) −3, 4 c) 3, −4 d) None ✓

$x > 3$ | $x < 3$

(10) Determine the base of the systems from following relations

a) $38 + 43 = 80$

b) Roots of $x^2 - 11x + 22 = 0$ are 3 & 6

c) $\frac{66}{6} = 11$

(a) $38_b + 43_b = 80_b$

$3b + 4b + 8 + 3 = 8b + 0$

$7b + 11 = 8b$

$\boxed{b = 11}$

(b) $ax^2 + bx + c = 0$

Product of roots $= c/a$

$3_b * 6_b = \frac{22_b}{1_b} \rightarrow (1)$

$\Rightarrow 3 \cdot 6 = \frac{2b + 2}{1}$

$2b = 18 - 2 = 16$

$\Rightarrow \boxed{b = 8}$

from (1) true

(c) $\frac{66_b}{6_b} = 11_b$ all values of $b > 6$

$\Rightarrow \frac{6b + 6}{6} = b + 1 \Rightarrow 6b + 6 = 6b + 6$

$\Rightarrow \boxed{b + 1 = b + 1}$ for all values of b

(12) Complementary number System :-

→ The main aim of complements is substraction can be done using addition only (adder not subtractor).

Base-b :-
- (b-1)'s complement
- b's complement

→ b's complement is called as radix complement.

→ (b-1)'s complement is called as Diminished radix complement.

For number x

(b-1)'s complement $= (b^n - 1) - x$

b's complement $= b^n - x$

Base-2 :-
- → 1's complement, $\bar{x} = 2^n - 1 - x$
- → 2's complement, $\bar{x} = 2^n - x$

Base-3
- → 2's complement, $\bar{x} = 3^n - 1 - x$
- → 3's complement, $\bar{x} = 3^n - x$

1's complement of Base 2 is different to 2's complement of base 3.

Base-10 :
- → 9's comp., $\bar{x} = 10^n - 1 - x$
- → 10's comp, $\bar{x} = 10^n - x$

Base-16 :
- → 15's comp. $\bar{x} = 16^n - 1 - x$
- → 16's comp. $\bar{x} = 16^n - x$

Range

no of bits	SM	1's	2's
3	-3 to +3	-3 to +3	-4 to 3
4	-7 to +7	-7 to +7	-8 to +7
n	$-(2^{n-1}-1)$ to $+(2^{n-1}-1)$	$-(2^{n-1}-1)$ to $+(2^{n-1}-1)$	$-(2^{n-1})$ to $+(2^{n-1}-1)$

(3) Find the max +ve number that can be represented by 10 bits in 2's complement.

-2^{n-1} to $+2^{n-1}-1$

-2^{10-1} to $+2^{10-1}-1$

-2^{9} to $+2^{9}-1$

$\Rightarrow 512-1 = 511$ 511 is the answer

$\therefore$ -512 to 511

30) ## Gray code :-

→ It is Reflexive, unit distance code, Cyclic code, belongs to non-weighted code.

→ 1 bit 2 combination

0 — 0
1 — 1

→ 2 bits 4 combinations

0	0	0
1	0	1
2	1	1
3	1	0

Mirror reflection

→ 3 bits 8 combin

0	0	0	0
1	0	0	1
2	0	1	1
3	0	1	0
4	1	1	0
5	1	1	1
6	1	0	1
7	1	0	0

→ 4 bit 16 combinations

0 -	0	0	0	0
1 -	0	0	0	1
2 -	0	0	1	1
3 -	0	0	1	0
4	0	1	1	0
5	0	1	1	1
6	0	1	0	1
7	0	1	0	0
8	1	1	0	0
9	1	1	0	1
10	1	1	1	1
11	1	1	1	0
12	1	0	1	0
13	1	0	1	1
14	1	0	0	1
15	1	0	0	0

→ (12) in BCD → 0001 0010

3321 0001 0010

gray 1010

Binary 1100

→ unit distance means for getting the next code only one bit in change. (counters)

0010
1110 → distance = 2

but in Gray code distance b/w successive no's = 1

→ Distance b/w last & first code = 1, so it is cyclic.

(31) Binary to Gray & vice-versa :-

① $(12)_{10} \to (1100)_2$ ← Binary → Gray

$b_4 \oplus b_3 \oplus b_2 \oplus b_1$

1 1 0 0

1 0 1 0

$G_4\ G_3\ G_2\ G_1$

Gray code of 12 = 1010

~~$b_4\ b_3\ b_2\ b_1$~~

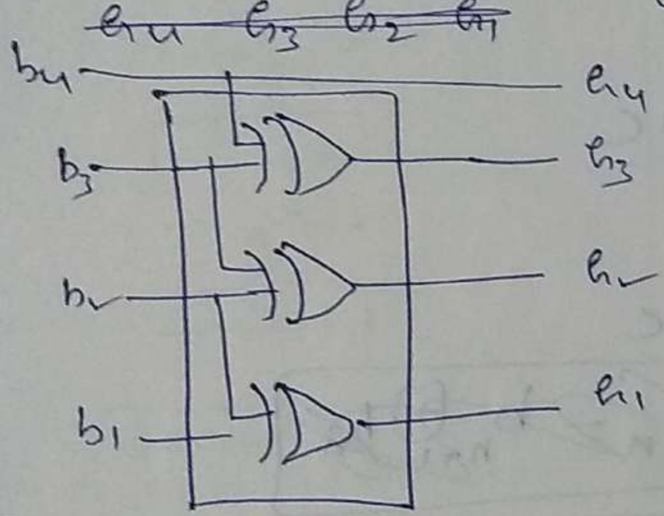
$\Rightarrow G_4 = b_4$

$G_3 = b_4 \oplus b_3$

$G_2 = b_3 \oplus b_2$

$G_1 = b_2 \oplus b_1$

~~$G_4\ G_3\ G_2\ G_1$~~



$$\boxed{G_n = b_{n+1} \oplus b_n}$$

② B → G

1 1 1 1

1 0 0 0

③ 200 = 1 1 0 0 1 0 0 0 BC

1 0 1 0 1 1 0 0 GC

Gray code → Binary

$b_4\ b_3\ b_2\ b_1$

1 0 1 0 GC

1 1 0 0 BC

$G_4\ G_3\ G_2\ G_1$

$G_4 = b_4$

$G_3 = G_4 \oplus b_3$

$G_2 = G_3 \oplus b_2$

$G_1 = G_2 \oplus b_1$

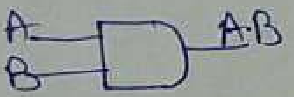
Logic Gates

Logic Gate:- Fundamental building block of a digital system.

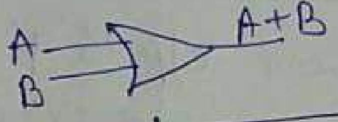
Basic Gates:- AND($\wedge$), OR($\vee$), NOT($\sim$).

Universal Gates:- NAND($\uparrow$), NOR($\downarrow$).

AND ($\wedge$)



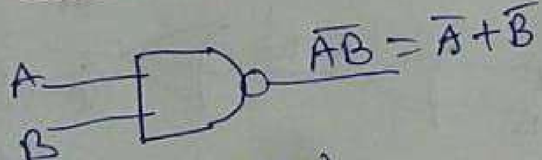
OR ($\vee$)



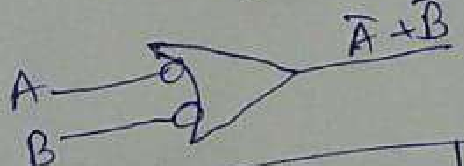
NOT ($\sim$ / $-$ / $'$)

A	B	$A \wedge B$	$A+B$	$\overline{A}$
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

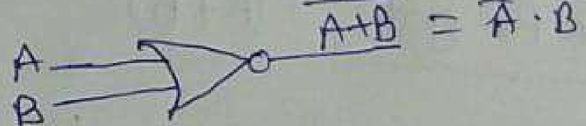
NAND($\overline{AB}$)



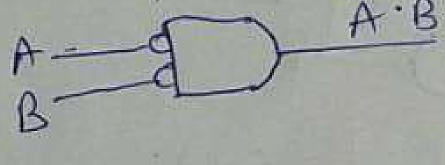
(or)



NOR($\overline{A+B}$)

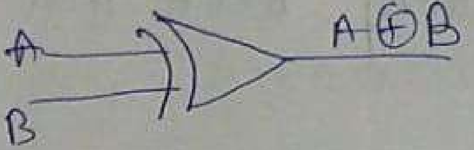


(or)

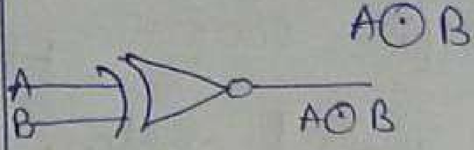


A	B	$\overline{AB}$	$\overline{A+B}$
0	0	1	1
0	1	1	0
1	0	1	0
1	1	0	0

EX-OR Gate ($\oplus$) :- $A \oplus B$



EX-NOR Gate ($\odot$) :-

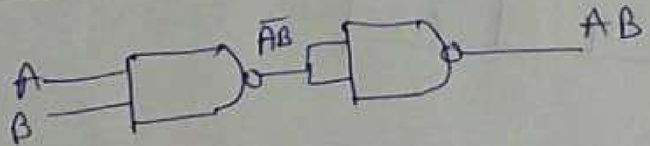


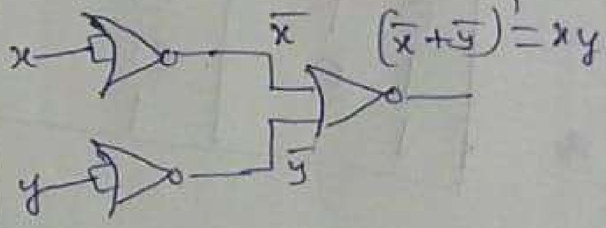
A	B	$A \oplus B$	$A \odot B$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

$\oplus$:- diff i/p then o/p is '1'.
$\odot$:- same i/p then o/p is '1'.

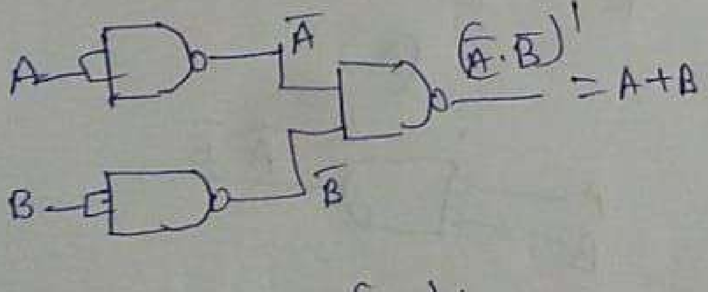
---

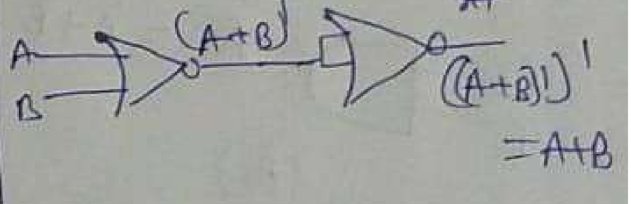
① AND gate :- (A·B)



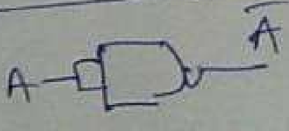


② OR gate :- (A+B)

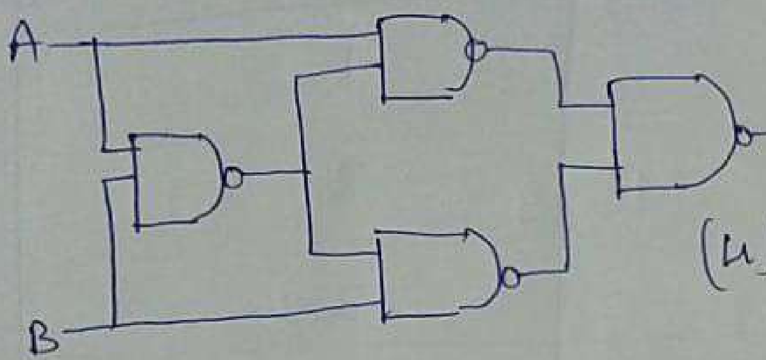




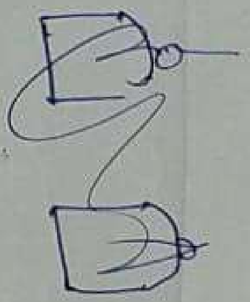
③ NOT gate ($\bar{A}$) :-

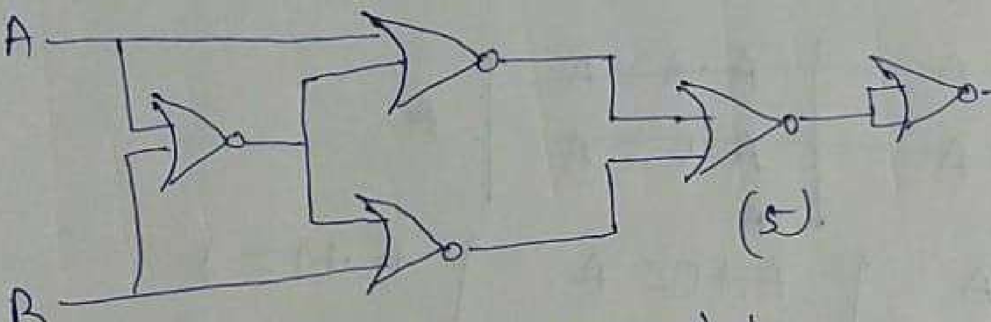


④ XOR $(A \oplus B)$:-



$A \oplus B = A'B + AB'$





⑤ XNOR $(A \odot B) = AB + A' \cdot B'$

Replace NAND (5) with NOR (4) gates.

Gate to be realized	Min. NAND required	Min. NOR required
NOT	1	1
AND	2	3
OR	3	2
NAND	1	4
NOR	4	1
EX-OR	4	5
EX-NOR	5	4

~~$A+A=A$~~ | ~~$A \cdot A=A$~~

~~$A+0=A$~~ | ~~$A \cdot 0=A$~~

① $A+A=A$ | $A+0=A$ | $A+1=1$

$A \cdot A=A$ | $A \cdot 0=0$ | $A \cdot 1=A$

② $A+\overline{A}=1$

$A \cdot \overline{A}=0$

③ $(A+B)'=\overline{A} \cdot \overline{B}=A' \cdot B'$

$(AB)'=A'+B'$

Syllabus

Logic functions, Minimization, Design and Synthesis of combinational and sequential circuits, Number representation and Computer Arithmetic (fixed and floating point).

1. Logic functions (32)
2. Minimization (40)
3. Design and Synthesis of combinational ckts (64)
4. Sequential ckt (51)
5. Number System (59)
6. practice questions (8)

① Logic functions

① Basic Properties of Switching Algebra:-

→ A basic element of switching algebra is boolean variable. Boolean variable value is either '0' or '1'.

{0, 1}.

Switching algebra operators. + OR

· AND

~ (NOT)

		+	·	~
0	0	0	0	1
0	1	1	0	0
1	0	1	0	1
1	1	1	1	0

Basic Properties:-

(i) **Idempotency:-** $x \cdot x = x$

$x + x = x$ goto ~~truth~~

(go to truth table)

$x + 1 = 1$
$x \cdot 0 = 0$

$x + 0 = x$
$x \cdot 1 = x$ → $+ \leftrightarrow \cdot$
$0 \leftrightarrow 1$

(ii) **Commutativity:-**

$x + y = y + x$

$xy = yx$

(iii) **Associativity:-**

$x + (y + z) = (x + y) + z$

$x(yz) = (xy)z$

(iv) **Complementation:-**

$x + \bar{x} = 1$

$x \cdot \bar{x} = 0$

(v) **Distributivity:-**

$x(y + z) = xy + xz$

* $x + yz = (x + y)(x + z)$

→ Associativity
- left Asso.
- Right Asso.

$a + \underline{b} + c$

$((a + b) + c)$ — LA

$(a + (b + c))$ — RA

$x\,(y+z) = x(y+z) = xy + xz$

* fast checking $x = 0 \Rightarrow 0(y+z) = 0y + 0z$

$0 = 0$

$x = 1 \Rightarrow 1(y+z) = 1 \cdot y + 1 \cdot z$

$y + z = y + z$

$\therefore x(y+z) = xy + xz$

→ In all the above it is happened

$+ \leftrightarrow \cdot$ & $0 \leftrightarrow 1$. It is called as principle of "Duality".

(2) Switching (SE) expressions and simplification:-

Switching expression is a finite no. of combinations of switching variables and constants {0,1} by means of switching operations (+, ·, NOT)

Ex:- $x + \bar{x}yz + x\bar{z}$, $a + bc + \bar{b}d$

properties for simplifying SE:-

(1) Absorption:-

$$\boxed{\begin{aligned} x + xy &= x \\ x + x'y &= x + y \end{aligned}}$$

$x + xy$
$x(1+y) = x$ ✓

$x + x'y$
$= (x + x')(x + y)$
$= 1(x+y)$
$= x + y$

(i) ~~Dual:- $x.(\ldots)$~~

$x(1+x)(x+y) = x$

$x(x+y) = x$

Dual $x\cdot(x'+y) = x\cdot y$

$xx' + xy = xy$

$0 + xy = xy \Rightarrow xy = xy$

Consensus Theorem:-

$$\boxed{xy + \bar{x}z + yz = xy + \bar{x}z}$$

$xy + \bar{x}z + yz(1) = xy + \bar{x}z + yz(x+\bar{x})$

$= xy + \bar{x}z + xyz + \bar{x}yz$

$= xy(1+z) + \bar{x}z(1+y)$

$= xy + \bar{x}z$

$\therefore \boxed{xy + \bar{x}z + yz = xy + \bar{x}z}$

S.C.

$xy + \bar{x}z + \underbrace{yz}_{\text{(redundant)}}$

$\Rightarrow xy + \bar{x}z$

$\Rightarrow$ inplace of x there will be a big expression.

(1) $x'y'z + yz + xz = ?\ z$

$\Rightarrow z(x'y' + y + x)$ (in TOC, not commutative)

$z\,((y+y')(y+x') + x)$

$z\,(x' + y + x) = z(\underbrace{x + x'}_{1} + y) = z(1+y) = z$

③ Demorgans law and Simplification :-

$\overline{(x\,y)} = \bar{x} + \bar{y}$ $\quad + \leftrightarrow \cdot$

$x \leftrightarrow \bar{x}$

$(x+y)' = \bar{x} \cdot \bar{y}$ $\quad 0 \leftrightarrow 1$

$$f(a, b, c, \dots z, 0, 1, \cdot, +)$$

$$\bar{f} = f(a', b', c', \dots \bar{z}, 1, 0, +, \cdot)$$

$$f_d = f(a, b, c, \dots z, 1, 0, +, \cdot)$$

$$f \to \bar{f}, \quad f \to f_d$$

$\bar{f}$: $0 \leftrightarrow 1$, $+ \leftrightarrow \cdot$, $x \leftrightarrow \bar{x}$

f_d (leave the variables as it is): $0 \leftrightarrow 1$, $+ \leftrightarrow \cdot$

Ex:- $f = \bar{x} + \bar{y}$

$\bar{f} = x \cdot y$; $f_d = \bar{x} \cdot \bar{y}$

① $(x+y)\left[x'(y'+z')\right]' + x'y' + x'z' = ?\ 1$

$(x+y)(x + yz) + x'y' + x'z'$

$\underline{(x+y)}\ \underline{(x+y)}(x+z) + x'y' + x'z'$

$(x+y)(x+z) + x'y' + x'z'$

$x + xz + xy + yz + x'y' + x'z'$

$x(1+z+y)+yz+x'y'+x'z'$

$x+yz+x'y'+x'z'$

$x+x'y'+yz+x'z'$

$(x+x')(x+y')+yz+x'z'$

$x+y'+yz+x'z'$

$\underline{x+x'z'}+\underline{y'+yz}$

$x+z'+y'+z$

$\underline{z+z'}+x+y'$

$1+x+y'=1$

$\therefore$

④ Switching functions :-

$f(a,b,c) = a+bc$

a	b	c	f	f'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

→ Boolean expression defining a function.

(5) Canonical Forms :- (SOP, POS)

→ A product term which contains each of 'n' variables as factors either in complemented or un-complemented form is called a "Min term".

→ A minterm given the value '1' for exactly one combination of the variables

→ the sum of all minterms of 'f' for which 'f' assumes '1' is called canonical sum of products (or) Disjunctive Normal Form (DNF).

Minterms are :- $abc + \bar{a}bc + \bar{a}\bar{b}\bar{c} + ab\bar{c}, \ldots$

$111 \quad 011 \quad 000 \quad 110$

$x \to 1$

$\bar{x} \to 0$

	a	b	c	f
0	0	0	0	0
1	0	0	1	1 ✓
2	0	1	0	1 ✓
3	0	1	1	0
4	1	0	0	1 ✓
5	1	0	1	0
6	1	1	0	1
7	1	1	1	0 ✓

Min terms are $\bar{a}\bar{b}c, \bar{a}b\bar{c}, a\bar{b}\bar{c}, ab\bar{c}$

SOP :- $a + bc + ac$

→ Canonical SOP ⊔ $abc + \bar{a}bc$

→ CSOP means min term containing all the variables.

$$\therefore f(a,b,c) = \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + ab\bar{c}$$
$$\Sigma(1,2,4,6)$$

6 Canonical Product of Sums (CPOS) :-

→ A sum term which contains each of 'n' variables as factors either in complemented or un-complemented form is called a "max term".

$$\bar{a}+b+c,\ a+\bar{b}+c,\ \bar{a}+\bar{b}+\bar{c}$$

→ A max. term gives the value '0' for exactly one combination of the variables.

→ The product of all max. terms of 'f' for which 'f' assumes '0' is called CPOS (or) Conjunctive Normal Form (CNF).

	a	b	c	f
0	0	0	0	0 ✓
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0 ✓
4	1	0	0	0 ✓
5	1	0	1	1
6	1	1	0	0 ✓
7	1	1	1	1

$$f(a,b,c) = (a+b+c)(a+\bar{b}+\bar{c}) \cdot (\bar{a}+b+c) \cdot (\bar{a}+\bar{b}+c)$$

$$\cancel{\Pi(0,1,3,\ldots)}\ \Pi(0,3,4,6)$$

(7) Examples of canonical forms :- Σ - minterms
π - Maxterms

	a	b	c	f_1	f_2	f_3
0	0	0	0	0	1	1
1	0	0	1	0	0	1
2	0	1	0	1	1	0
3	0	1	1	1	0	0
4	1	0	0	0	1	0
5	1	0	1	1	0	0
6	1	1	0	1	1	1
7	1	1	1	1	0	1

$f_1 = \Sigma(2,3,5,6,7)$ - CSOP

$f_1 = \pi(0,1,4)$ - CPOS

$f_2 = \Sigma(0,2,4,6)$

$= \pi(1,3,5,7)$

$f_3 = \Sigma(0,1,6,7)$

$= \pi(2,3,4,5)$

$f(x,y,z) = x'y + z' + xyz$, CSOP?

$\overset{011\,(3)}{x'yz} + \overset{\overset{(6)}{110}}{xyz'} + \overset{111\,(7)}{xyz}$

$x'yz'$ xyz' 100 (4)

010    $x'yz'$ 010 (2)

(2) $x'y'z'$ 000 (0)

$\therefore \Sigma(3,2,6,4,0,7)$

$\Rightarrow \Sigma(0,2,3,4,6,7)$ Then π?

$\Rightarrow \pi(1,5)$

(8) Functional properties :-

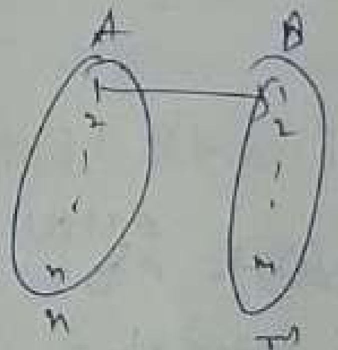
→ The canonical SOPs and POSs form a switching function is unique.

→ Two switching functions $f_1(x_1, x_2, \ldots x_n)$ & $f_2(x_1, x_2, \ldots, x_n)$ are said to be logically equivalent iff both functions have same value for each and every combination of $(x_1, x_2, \ldots, x_n)$.

→ Two switching functions are equivalent if their canonical POS (or) SOP are identical.

a	b	f_1	f_2
0	0	0	0
0	1	0	0
1	0	1	1
1	1	1	1

logically equivalent



(9) No. of functions :-

$\overset{1}{m}\ \overset{2}{m}\ \cdots\ \overset{n}{m}$

(how many fun m^n)

→ n - boolean variables are how many boolean functions are possible? 2^{2^n}

1 2 3 --- n | f

$2 \times 2 \times 2 \times \cdots \times 2$

2^n

2^n

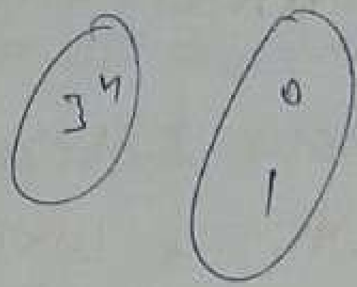
0

1

2^{2^n}

n - ternary var. then how many boolean functions are possible? 2^{3^n}

1	2	3	...	n	f
3	3	3	...	3^n	

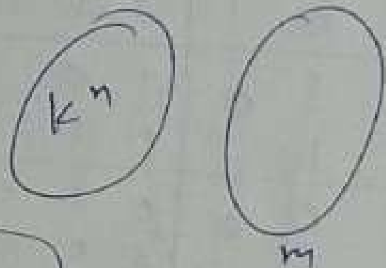


2^{3^n}

→ n-k-ary variable then how many m-ary functions are possible? $\underline{m^{k^n}}$

1 2 ... n

K K K



m^{K^n} : n → no. of variables, K → Type of variable, m → Type of function

m^{k^n}

→ 2 boolean var then how many boolean functions

$2^{2^2} = 2^4 = \underline{16}$

p — no. of vari

n — Type of variable

m — Type of function

(10) Counting the no. of functions and neutral functions :-

→ How many boolean functions are possible with 3 variables such that there are exactly 3 min terms.

3 var → 8 combination out of these 3 combi.

	a	b	c		
(0)	0	0	0	1	0
(1)	0	0	1	1	1
(2)	0	1	0	1	0
(3)	0	1	1	0	1
(4)	1	0	0	0	1
(5)	1	0	1	0	0
(6)	1	1	0	0	0
(7)	1	1	1	0	0

$^8C_3$

atmost '3' min terms

$$^8C_3 + {^8C_2} + {^8C_1} + {^8C_0}$$

at least '3' min terms

$$^8C_3 + {^8C_4} + {^8C_5} + {^8C_6} + {^8C_7} + {^8C_8}$$

K-var then 2^K combinations possible, 'm' min terms need then $\boxed{^{2^K}C_m}$

if ternary, $\boxed{3^{k} C_{m}}$

→ Neutral function means, The no. of min terms is equal to no. of max terms.

A	B	A	B	$\bar{A}$	$\odot$	$\oplus$	$\bar{B}$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	0
1	0	1	0	0	0	1	1
1	1	1	1	0	1	0	0

for 2 vars then 4 combinations

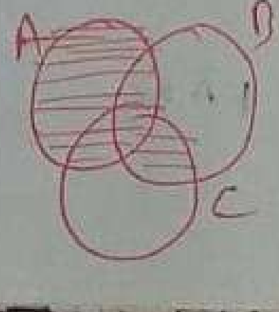
for two min terms then $4C_2$

n' variables then no. of i/ps $2^n C_{(2^n/2)}$

$$\boxed{\Rightarrow 2^n C_{2^{n-1}}}$$

(11) Venn Diagram Representation:

Find the boolean function that the following Venn diagram represents

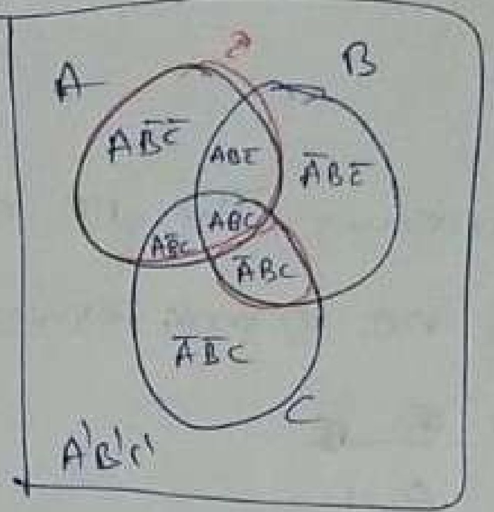


a) $A + A'B'C$

b) $A + BC$ ✓ — $A + A'BC$
$(A+A')(A+BC)$
$= 1(A+BC)$
$= A+BC$

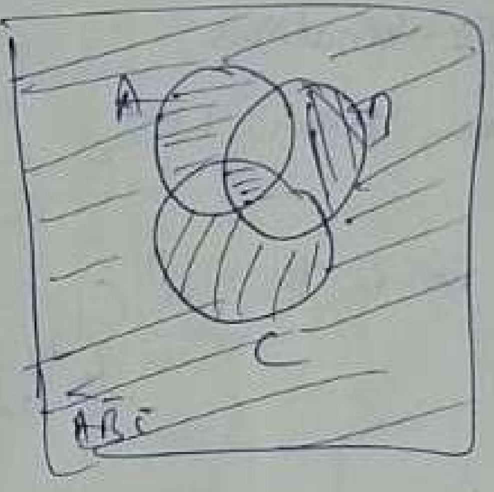
c) $A + A'BC$ ✓

d) $AB + C$

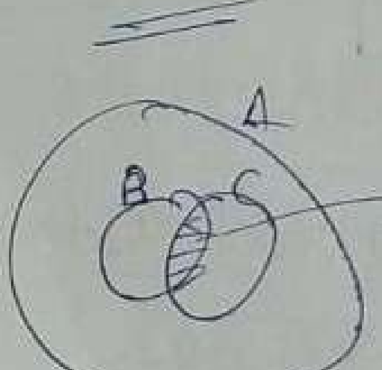


$A + \bar{A}BC$

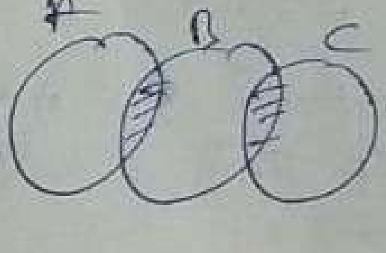
$(A+\bar{A})(A+BC) = A+BC$



$\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$

→ 

→ ABC

→ 

→ $AB\bar{C} + \bar{A}BC$

Forward path:- Any path starting from i/p and ending at o/p without performing a cycle.

Validity:- It means no path should contain a variable in both true and complemented form.

$AAB'C' + \underline{AABC} + AABA + \underline{ABC} + ABA + \boxed{AC'C} + AC'A$

× with validity rule

$AAB'C' + ABC(A+1) + AABA + ABA + AC'A$

$AAB'C' + ABC + AABA + ABA + AC'A$

⇒ Simplify this.

(13) Nested function:

In the following simultaneous boolean expression, what are the values of w, x, y, z?

i) $x+y+z=1$		a) 0001 (w x y z)
ii) $xy + wz' = 0$		b) 1101
iii) $xw' + yz' = 1$	✓	c) 0101
		d) 1000

substitute options, c satisfy i, ii & iii

$f(A,B) = A' + B$ then find $f(f(x+y, y), z)$

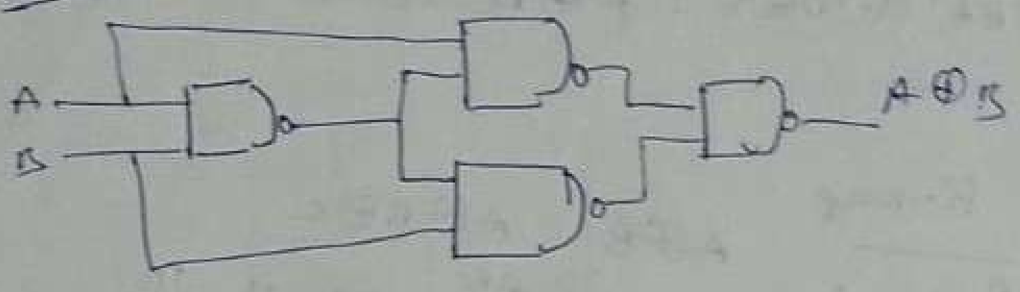
$f(x+y, y) = (x+y)' + y = x'y' + y = x' + y$

$f(f(x+y,y), z) = f(x'+y, z) = (x'+y)' + z = \underline{x \cdot y' + z}$

(10) Minimum no. of gates required for EX-OR & EX-NOR:-

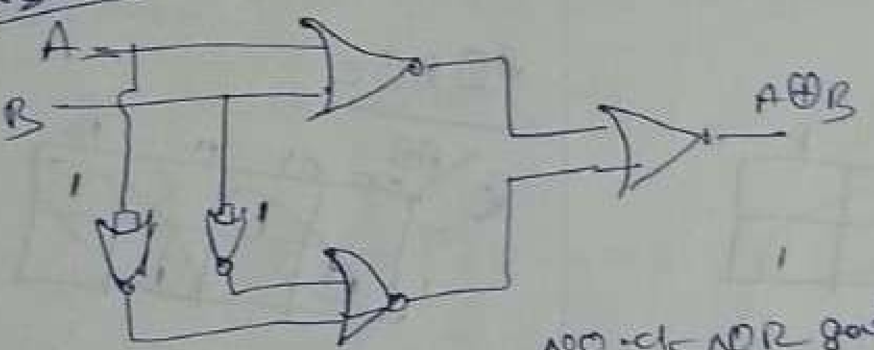
EX-OR

Using NAND



Min. No. of NAND gates are = 4

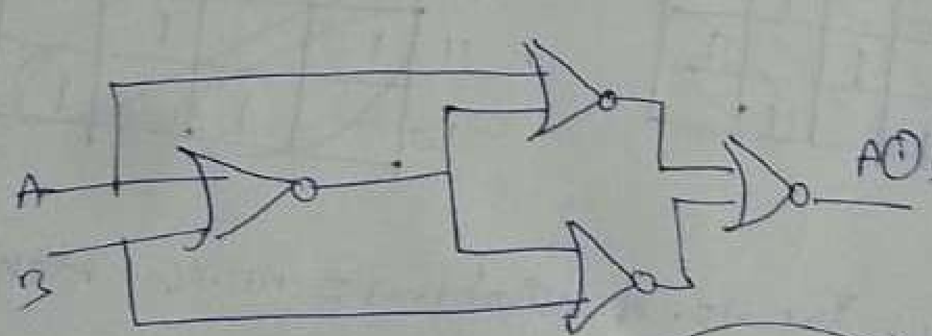
Using NOR



No. of NOR gates = 5

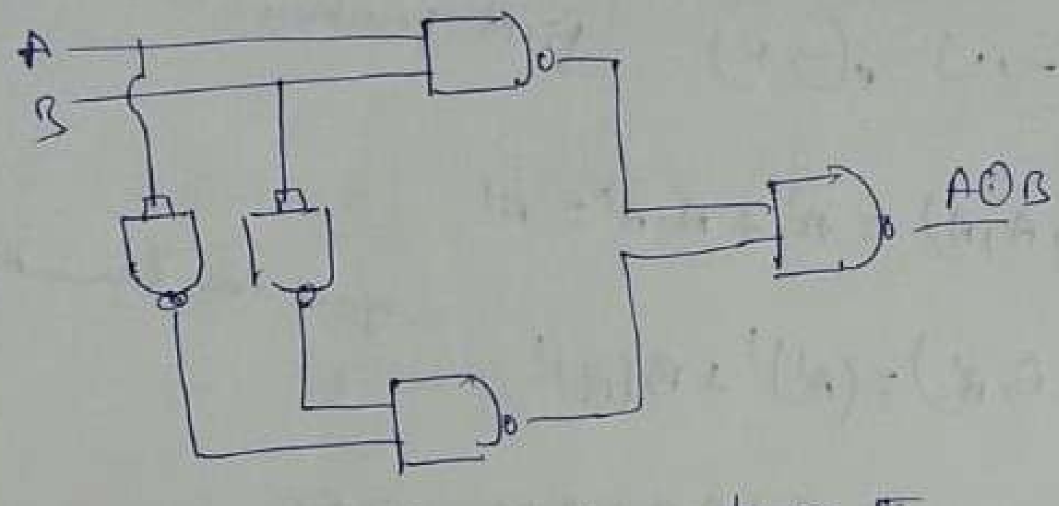
EX-NOR

Using NOR



4 gates

Using NAND



Min. no. of NAND gates = 5

	NAND	NOR
EX-OR	4	5
EX-NOR	5	4

(20) Functionally Completeness operations :-

A set of operations is said to be functionally complete (or) universal iff every switching function can be expressed by means of operations in it.

→ $\{+, \cdot, -\}$

$\{+, -\}$

$\{\cdot, -\}$ are functionally complete.

→ Functionally complete means, it is able to derive at least two operations.

$(-, \cdot)$ (or) $(-, +)$

(31) $f(A, B, C) = A' + BC'$

$(-, \cdot)$ $(-, +)$ first you have to derive the comp

Ex:- $f(A, A, A) = A' + A \cdot A' = A'$

$f(A', B, B') = (A')' + B(B')'$ you can derive And

$= A + B \cdot B = A + B \rightarrow FC$

(32) $f(A, B) = \overline{A} + B$

$f(A, A) = \overline{A} + A = 1$

$f(B, B) = \overline{B} + B = 1$ $(-, +)$

$f(A, 0) = \overline{A} + 0 = \overline{A}$

So, it takes the help from '0'

$f(\underbrace{f(A, 0)}_{A'}, B)$

$= f(\overline{A}, B)$

$= A + B \rightarrow +$

So, Partially FC.

FC
- FC: $(+, -)$, $(\cdot, -)$
- PFC: $(0, 1)$, $(+, -)$, $(\cdot, -)$
- NFC

(33) $f(A, B) = \overline{A} B$

$f(A, A) = \overline{A} \cdot A = 0$ $(-, \cdot)$

$f(B, B) = \overline{B} \cdot B = 0$

$\Rightarrow f(A, 0) = \overline{A} \cdot 0 = 0$

$f(A,1) = \bar{A}\cdot 1 = \bar{A}$

$\therefore$ with the help of '1' it is complement

$$f(\underbrace{f(A,1)}_{A'}, B) = (A')'B = AB$$

$\therefore$ Partially FC.

(24) $f(A,B,C) = AB + BC + CA$

$f(A,A,A) = A$

Since complement is absent in expression, can't derived

$\therefore$ NOT FC.

(25) $f(x,y) = \bar{x}y + x\bar{y}$

$f(x,x) = \bar{x}\cdot x + x\cdot\bar{x} = 0$; $f(y,y) = 0$

$f(x,1) = \bar{x}\cdot 1 + x\bar{1} = \bar{x} + 0 = \bar{x}$ — complement is derived.

$$f(\underbrace{f(x,1)}_{\bar{x}}, y) = (\bar{x})'y + \bar{x}\bar{y}$$

$$= xy + \bar{x}\bar{y}$$

$f(\bar{x}, \bar{y}) = x\bar{y} + \bar{x}y$

$f(1,\bar{y}) = \bar{x}\bar{y} + xy$

$f(x,\bar{x}) =$

$f(y,\bar{y}) =$

Here we can't derive $x+y$ or xy

SO NFC

(25) Any Boolean function can be defined with which of the following operations?

a) $\oplus$, NOT

b) $\oplus$, 1, OR

c) $\oplus$, 1, NOT

d) $\odot$, 1, NOT

$(\oplus, 1)$ gives $\to$ NOT

(a) $f(x,y) = x'y + xy'$

$f(x,1) = x' 1 + x \cdot 0$

$= x'$

(d) $f(x,y) = xy + \bar{x}\bar{y}$

$f(x,x) = xx + \bar{x}\bar{x} = x + \bar{x} = 1$

$f(y,y) = 1$ } No complement

$f(x,1) = x \cdot 1 + \bar{x}\bar{1} = x + 0 = x$

$f(y,1) = y$ } no

$f(x,0) = x0 + \bar{x}1 = \bar{x}$

By only $(\odot, 0)$ we can find complement.

$f(x',y) = x'y + (x')'y' = x'y + xy'$

$f(x,y') = xy' + x'y$

$f(x',y') = x'y' + xy$

NFC.

By observing the last problems

$$\boxed{\begin{aligned} x \oplus y &= x' \odot y = x \odot y' = x' \oplus y' \\ x \odot y &= x' \oplus y = x \oplus y' = x' \odot y' \end{aligned}}$$

27) Self - Dual functions :-

$0 \leftrightarrow 1$

$+ \leftrightarrow \cdot$

$\rightarrow f(A,B,C) = AB + BC + CA$

$f_d(A,B,C) = (A+B)(B+C)(C+A)$

$\Rightarrow f(A,B,C) = AB(C+C') + (A+A')BC + C(B+B')A$

$= ABC + ABC' + A'BC + AB'C$

	ABC	ABC'	$A'BC$	$AB'C$
Mutual Exclusive term	$A'B'C'$	$A'B'C$	$AB'C'$	$A'BC'$

→ A boolean function is self dual if

(i) It is neutral (no. of minterms = no. of max terms) $(2^{n-1} = 2^{n-1})$

(ii) The function doesnot contain two mutually exclusive terms

$ABC \rightarrow A'B'C'$

$A'BC' - AB'C$

28) No. of Self Dual functions :-

	A	B	C
0 -	0	0	0
1 -	0	0	1
2 -	0	1	0
3 -	0	1	1
4 -	1	0	0
5 -	1	0	1
6 -	1	1	0
7 -	1	1	1

(0,7) (1,6), (2,5) (3,4) all mutually exclusive term

0 - 7

1 - 6

2 - 5

3 - 4

(minterm, Mutual Exclusive term)

$(0,7)\ (1,6)\ (2,5)\ (3,4)$

$f = 2 \times 2 \times 2 \times 2$

$\Sigma(0,1,2,3)$

No. of self dual functions $= 2^4 = 16$

n variables, 2^n combinations

no. of pairs $= 2^{n-1}$

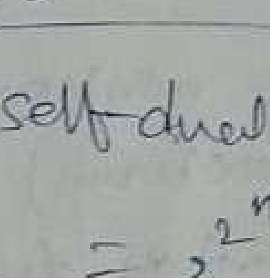
$2 \times 2 \times 2 \times \cdots \times 2^{n-1}$

$\therefore = 2^{2^{n-1}}$

$$\boxed{\text{No. of self dual functions with } n \text{ var} = 2^{2^{n-1}}}$$

→ No. of Neutral functions (N) $= {}^{2^n}C_{2^{n-1}}$

→ No. of self-dual function (SD) $= 2^{2^{n-1}}$



$\Sigma(0,7,1,2)$ → N ✓
— SD ✗

(29) Self-Dual functions are closed under

Complementation:-

which of the following functions are self-dual.

$f(A, B, C) = \Sigma(0, 2, 3)$ & 1 Mastered

$f(A, B, C) = \Sigma(0, 1, 6, 7)$ & $\underline{(1, 6)}$

$f(A, B, C) = \Sigma(0, 1, 2, 4)$ ✓

$f(A, B, C) = \Sigma(3, 5, 6, 7)$ ✓

0	7
1	6
2	5
3	4

(0, 7) (1, 6) (2, 5) (3, 4)

(0, 1, 2, 3)

	A B C	f	f'
0		1	0
1		1	0
2		1	0
3		1	0
4		0	1
5		0	1
6		0	1
7		0	1

complement of SD is also SD.

-ve LS

A	B	
0	0	0
0	1	0
1	0	0
1	1	1

H <-> L
1 <-> 0

0 — H — 1
(-ve) (+ve)

OR AND

∴ Every thing is dual.

→ PLS & NLS are dual to each other

gate take H as i/p
and o/p L, L as i/p
o/p H.

+ve (NOT)

A	
1	0
0	1

A	
H	L
L	H

-ve LS

A	
0	1
1	0

→ By default we follow the positive Logic System (PLS) everywhere.

(Q) 1.6) Let $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$, x_1, x_2, x_3, x_4 are boolean variables, $\oplus$ is XOR operator, which one of the following must always be true?

Sum of even no's = 0

A) $x_1 x_2 x_3 x_4 = 0$ 0 - 1's

B) $x_1 x_3 + x_2 = 0$ 2 - 1's

C) $\bar{x_1} \oplus \bar{x_3} = \bar{x_2} \oplus \bar{x_4}$ 4 - 1's

d) $x_1 + x_2 + x_3 + x_4 = 0$

$x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$

$\oplus$ is modx sum

$x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0 \rightarrow (1)$

all are 1's then (1) is zero

✗ A) $1 \cdot 1 \cdot 1 \cdot 1 \cdot = 0$ ✗

B) $1 \cdot 1 + 1 = 0$ ✗

d) $1 + 1 + 1 + 1 = 0$ ✗

C) ✓

$\bar{x_1} \oplus \bar{x_3} = x_1 \oplus x_3 = 1 \oplus 1 = 0$

$\bar{x_2} \oplus \bar{x_4} = x_2 \oplus x_4 = 1 \oplus 1 = 0$

$x_1 \oplus x_3 = x_2 \oplus x_4$

0	0	✓
0	1	✓
1	0	✓
1	1	✓

all are satisfied by (C)

$0\oplus 0 = 0 \oplus 0$

$0 \oplus 0 = 1 \oplus 1$

$1 \oplus 1 = 0 \oplus 0$

1 0 0 1

0 1 1 0

(33) 06) Consider no. is represented in 4-bit gray code ~~representation~~ let $h_3 h_2 h_1 h_0$ be the ~~gray~~ gray code representation of a number 'n' and let $g_3 g_2 g_1 g_0$ be the gray code of $(n+1) \bmod 16$ value of the no. which one of the following functions is correct?

a) $g_0(h_3 h_2 h_1 h_0) = \Sigma(1,2,3,6,10,13,14,15)$

b) $g_1(h_3 h_2 h_1 h_0) = \Sigma(4,9,10,11,12,13,14,15)$

c) $g_2(h_3 h_2 h_1 h_0) = \Sigma(2,4,5,6,7,12,13,15)$

d) $g_3(h_3 h_2 h_1 h_0) = \Sigma(0,1,6,7,10,11,12,13)$

$h_3 h_2 h_1 h_0$	$g_3 g_2 g_1 g_0$
0	1
1	2
2	3
|	
15	0

$(n+1) \bmod 16$

	h_3	h_2	h_1	h_0	g_3	g_2	g_1	g_0
0 –	0	0	0	0	0	0	0	1
1 –	0	0	0	1	0	0	1	1
2 –	0	0	1	1	0	0	1	0
3 –	0	0	1	0	0	1	1	0
4 –	0	1	1	0	0	1	1	1
5 –	0	1	1	1	0	1	0	1
6 –	0	1	0	1	0	1	0	0
7 –	0	1	0	0	1	1	0	0
8 –	1	1	0	0	1	1	0	1
9 –	1	1	0	1	1	1	1	1
10 –	1	1	1	1	1	1	1	0
11 –	1	1	1	0	1	0	1	0
12 –	1	0	1	0	1	0	1	1
13 –	1	0	1	1	1	0	0	1
14 –	1	0	0	1	1	0	0	0
15 –	1	0	0	0	0	0	0	0

$g_2(h_3 h_2 h_1, h_0) = \Sigma(0100, 1100, 1101, 1111, 1110, 1010, 1011, 1001)$

$= \Sigma(4, 12, 13, 15, 14, 10, 11, 9)$

$g_2(h_3 h_2 h_1 h_0) = \Sigma(2, 6, 7, 5, 4, 12, 13, 15)$

~~Suppose~~ $\underbrace{0100}_{h_3 h_2 h_1 h_0}$

	Identity	Commutative	Associativity
NAND	✗	✓	✗
NOR	✗	✓	✗
EXOR	✗	✓	✓
EXNOR	✗	✓	✓

① 0 / 1 ⇒ ② 00, 01 / 11, 10 ③ 000, 001, 011, 010 / 100, 101, 111, 110

⇒

② Minimization

③ K-Map Introduction:

→ Minimization is very difficult using algebraic formulas, so K-map is introduced.

→ The algebraic procedure of combining various terms and applying to the variables becomes very tedious as the no. of variables increased.

→ The Map method provides a systematic method for combining terms and deriving minimal expression.

→ A K-map is a modified form of the truth table in which the arrangement of combinations is particularly convinced for minimization.

→ Every 'n' variable map consists of "n" cells (squares), representing all possible combinations of variables.

3-var Map

z \ xy	00	01	11	10
0	0	2	6	4
1	1	3	7	5

4-var Map

yz \ wx	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

(1) → The function value associated with a particular combination is entered in the corresponding cell.

(2) → cycle code is used in the combination as column and row heading.

(3) → Because of these codes, two cells which have a common ~~code~~ side correspond to combinations that differ by the value of just a single variable.

(4) → These two cells play a major role in simplification process. Because they can be combined by means of rule $Aa + Aa' = A$

(1) $\Sigma(3,7) = \bar{x}yz + xyz$

3 var map

z \ xy	00	01	11	10
0				
1		1	1	

(2) cycle code means $00 \xrightarrow[\text{change}]{1\text{bit}} 01 \xrightarrow[\text{chan}]{1\text{b}} 11 \longrightarrow 10$

00 ~~—01~~ b/w any two adjacent

10's 1 bit in change.

(3)

z \ xy	00	01	11	10
0				
1				

$\bar{x}yz$ $\quad$ xyz

change

(4) $yz(\bar{x}+x) = yz$

(A) K-Map Simplification:-

=> Simplification and Minimization of function using K-map:-

1. A collection of "2^m" cells, each adjacent to 'm' cells of collection is called a subcube, and the subcube is said to cover these cells.

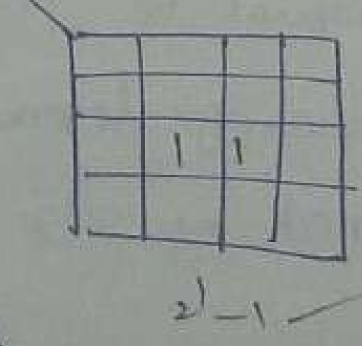
2. Each subcube can be expressed by a product containing "n-m" literals where 'n' is no. of variables on which function depends.

3. Any cell may be included in as many subcubes as desired.

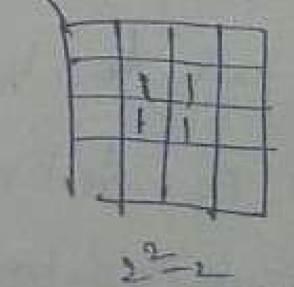
(1) $2^1 - 1$

$2^2 = 2$

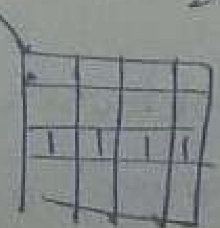
~~2^3 3~~



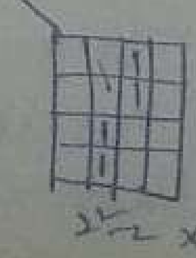
$2^1 - 1$



$2^2 - 2$

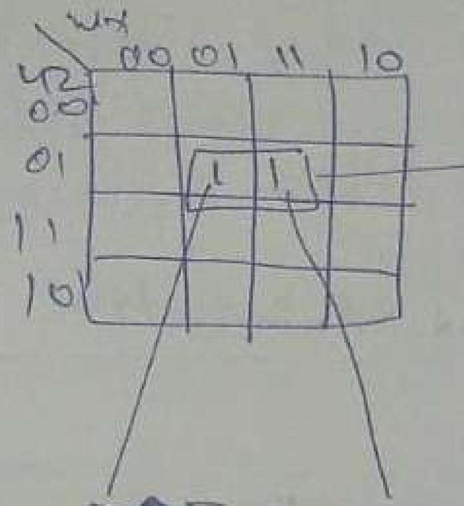


$2^2 - 2$



$2^2 - 2$ ×

(2)

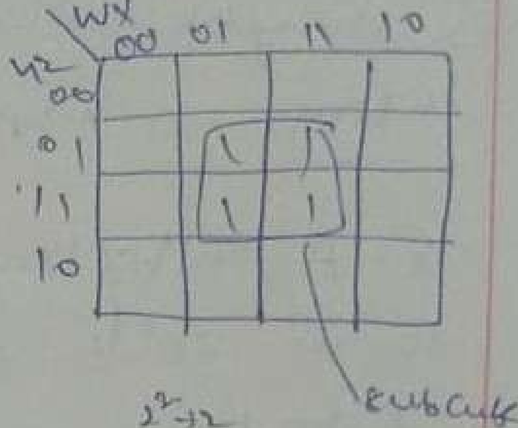


~~$\bar{w}x\bar{y}z + wx$~~

$\bar{w}x\underline{\bar{y}z} + wx\underline{\bar{y}z}$

$(\bar{w}+w)\,x\bar{y}z$

$x\bar{y}z$



$2^2 \to 2$

$2^m \to m$

$$\begin{matrix} \bar{w}x \cdot \bar{y}z \\ wx \cdot yz \end{matrix} = xz$$

$n - m = n - 2 = 2$

Minimizing:-

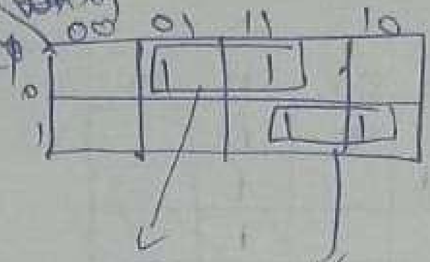
(1) A function 'f' can be expressed as sum of those product terms which corresponds to subcubes necessary to cover all it's '1' cells.

(2) The no. of product terms in the expression for 'f' is equal to the no. of sub cubes, while the no. of literals in each term is determined by size of corresponding subcubes.

(3) Therefore to obtain minimal expression, we must cover all '1' cells with smallest no. of subcubes such that each

Subcube is as long as possible.

$\Sigma(2,6,7,5)$



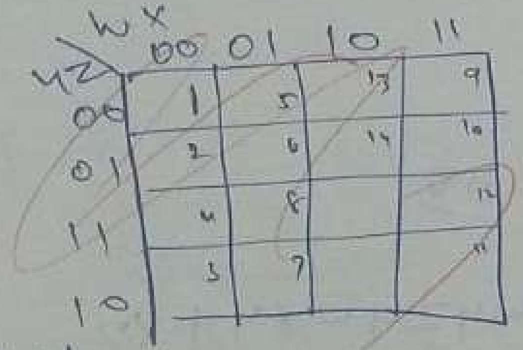
$y\bar{z}$ $\quad$ xz

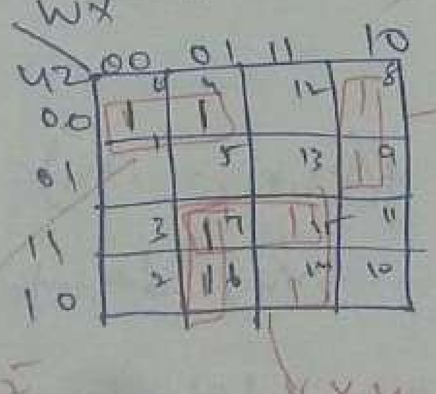
$$\boxed{\therefore \Sigma(2,6,7,5) = y\bar{z} + xz}$$

(5) Examples on K-map:-

(1) Minimize $f(w,x,y,z) = \Sigma(0,4,6,7,8,9,15,11)$

w	x	y	z
0	0	0	0
0	0	0	1





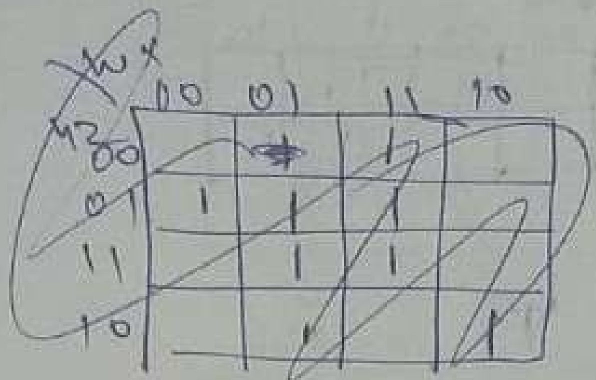
$w\bar{x}\bar{y}$

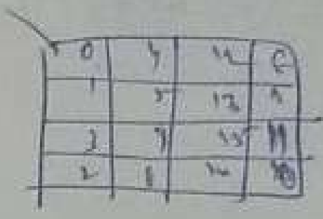
3 subcubes
3 terms

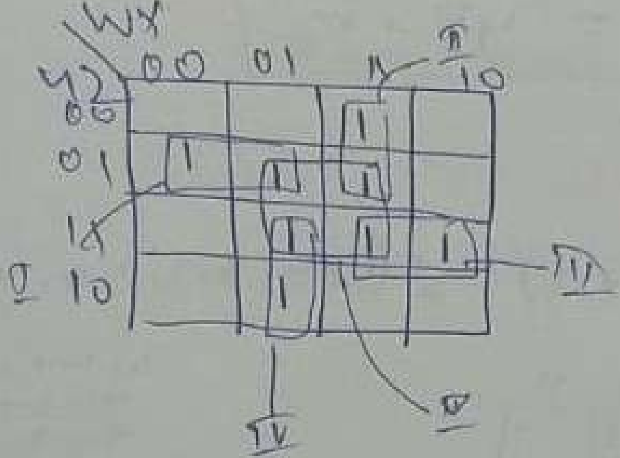
$w\bar{y}\bar{z}$ $\quad$ xy

$$\therefore f(w,x,y,z) = w\bar{x}\bar{y} + w\bar{y}\bar{z} + xy$$

(5) $f(w,x,y,z) = \Sigma(1,5,6,7,11,12,13,15)$







$\mathrm{I} + \mathrm{II} + \mathrm{III} + \mathrm{IV} + \mathrm{V}$

$w\bar{y}z + wx\bar{y} + wyz + \bar{w}xy + \cancel{xz}$

(6) Covering Functions:-

→ A switching function $f(x_1, x_2, \ldots, x_n)$ is said to cover $g(x_1, x_2, \ldots, x_n)$ denoted by "f" is superset of "g" if 'f' assumes true value whenever 'g' does.

Min terms	g	f
0	0	1
1	1	1
2	0	0
3	0	0

$\underbrace{\overset{f}{\{0,1\}} \supseteq \overset{g}{\{1\}}}_{\text{minterms}}$

Note:- If "f" covers "g" and at the same time "g" covers "f", then "f" and "g" are equivalent.

	g	f
0	0	0,1
1	1	1
2	0	0,1
3	0	0,1

$2 \times 2 \times 2 = 8$

→ If 'g' has 'x' min terms and 'g' is a function of 'n' variables, then no. of covering functions for 'g' is $\boxed{2^{(2^n - x)}}$.

Ex:-

$f(w,x,y,z) = wx + yz$

~~wx + wx~~ (w,x,y,z) is $2^4 = 16$

$wx \to \overset{w\,x\,y\,z}{1\,1\,-\,-} = 1100 + 1101 + 1110 + 1111$

$yz \to \overset{w\,x\,y\,z}{-\,-\,1\,1} = 0011 + 0111 + 1011 + \cancel{1111}$ repeated

$(wx+yz)$ has 7 Minterms

$\Rightarrow 2^n - m = 2^4 - 9 = 16 - \cancel{9} = \cancel{7}\,9$

$= 2^9$

(7) Implicants and Prime Implicants:-

→ If 'f' covers 'g', then 'g' is said to imply 'f'. This is denoted by $g \to f$

i.e. from 'f' we can derive 'g'

Ex:- $f(a,b,c) = ab+c$

a	b	c	ab (f_1)	c (f_2)	$ab+c$ (f)
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

f covers f_1, f_2

c \ ab	00	01	11	10
0			1	
1	1	1	1	1

every subcube is an implicant

Prime Implicant:- An implicant 'p' of a function 'f' is said to be prime implicant if

1. p is a Product term (i.e. subcube)
2. Deletion of any literal from 'p' results in a new product which is not covered by 'f'. (i.e., A subcube cannot be part of any other subcube).

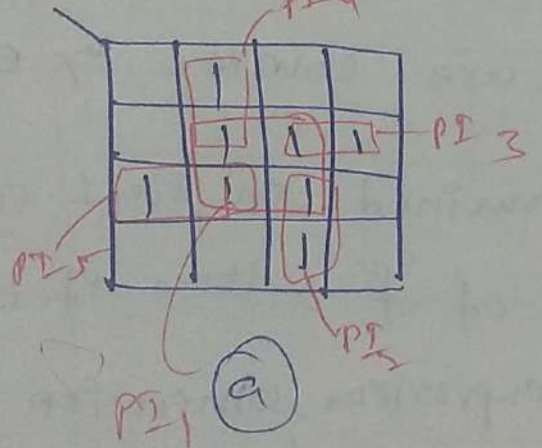
(8) Essential Prime Implicants:-

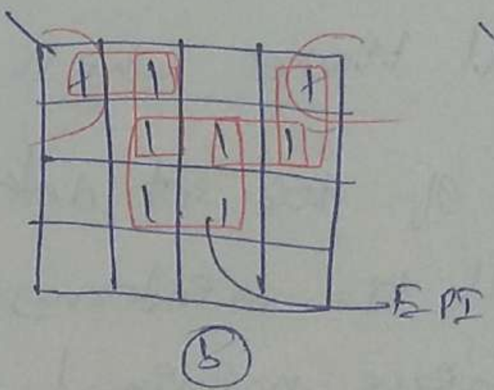
→ A prime implicant 'p' of a function 'f' is said to be an essential prime implicant, if it covers atleast one minterm of 'f' which is not covered by any other prime implicants.

Implicant → sub cube.

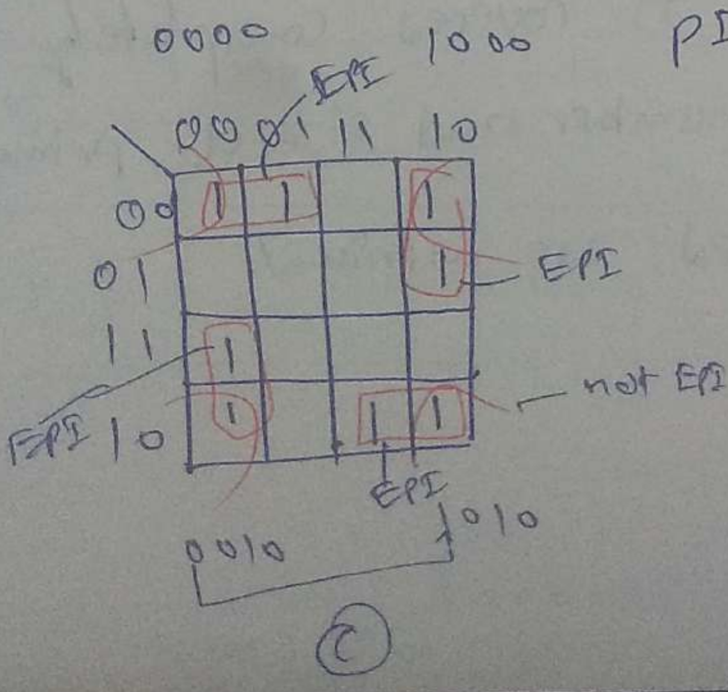
Prime implicants - biggest sub cube which is not a part of other subcubes.

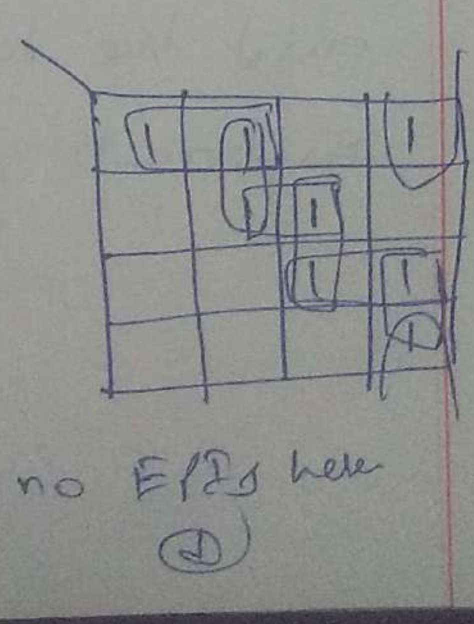
essential implicants -

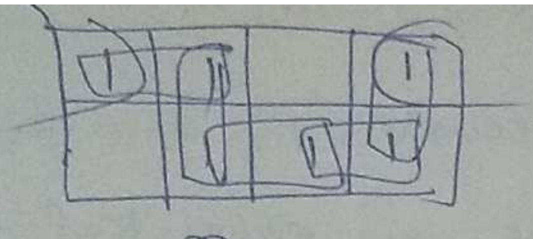




PI2, PI3, PI4, PI5 are essential PIs.







→ (d) & (e) are Cyclic K-maps.

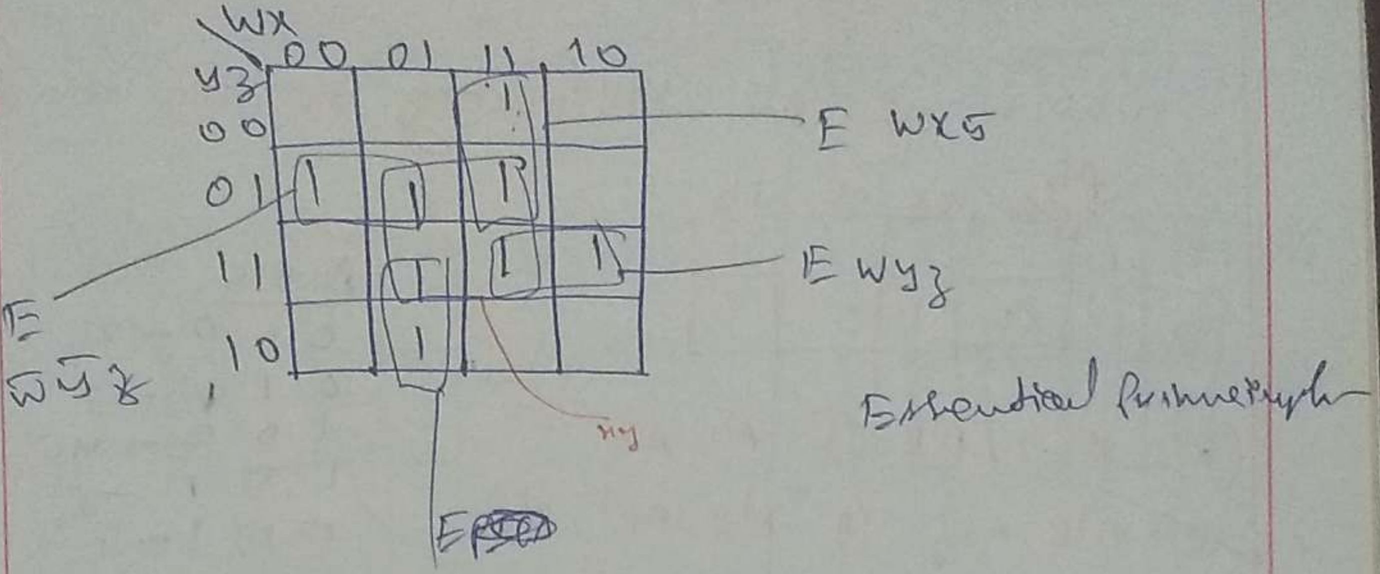
(9) Procedure for obtaining Minimal SOP:-

(1) Determine all essential prime implicants and include them in the minimal SOP.

(2) Remove from list of prime implicants all ~~those~~ which are covered by EPIs.

(3) If the set determined in step 1 covers all the minterms of 'f', then it is unique minimal expression. otherwise select the additional prime implicants so that the function 'f' is covered completely and the total number and size of prime implicants added are minimal.

Ex':- $\Sigma(1,5,6,7,11,12,13,15)$



$\bar{w}xy$

$= \bar{w}\bar{y}z + \bar{w}xy + wyz + wx\bar{y}$

Prime implicant chart:-

	1	5	6	7	11	12	13	15
$\bar{w}\bar{y}z$	1	1						
$wx\bar{y}$						1	1	
wyz					1			1
$\bar{w}xy$			1	1				
xz		1		1			1	1
	E		E		E	E		

(~~1~~,5, ~~1~~,7, ~~1~~,5, 13, 15)

$\bar{w}\bar{y}z$ E $\Rightarrow$ 1, 5

$\bar{w}xy$ E $\Rightarrow$ 6, 7

wyz E $\Rightarrow$ 11, 15

$wx\bar{y}$ E $\Rightarrow$ 12, 13

(10) Minimal SOP Ex:-

Which of the following PI are essential?

C \ AB	00	01	11	10
0		1		1
1	1	1		1

A	B	C	
0	1	0	– 2
0	1	φ	– 3
1	0	0	– 4
1	0	1	– 5
0	0	1	– 1

(a) $B'C$, $A'B$ (b) $A'C$, $A'B$

~~(c) $A'B$, AB'~~ (d) $A'B$, AB', $B'C$

Sol

C \ AB	00	01	11	10
0		1		1
1	1	1		1

$A\bar{B}$

$\bar{A}B$

$\bar{B}C$ $\bar{A}C$ (d)

$A\bar{B} + \bar{A}B + \bar{A}C$

$A\bar{B} + \bar{A}B + \bar{B}C$

2 Minimal Exp.

	1	2	3	4	5
$A'B$		1	1		
AB'				1	1
$A'C$	1		1		
$B'C$	1				1

$A'B$ – 2, 3

AB' – 4, 5

$(1, \cancel{2}, \cancel{3}, \cancel{4}, 5)$

(11) Minimal POS :-

$f(w,x,y,z) = \Sigma(5,6,9,10)$

yz \ wx	00	01	11	10
00	0	0	0	0
01	0	1	0	1
11	0	0	0	0
10	0	1	0	1

$(y+z)$

$(\bar{y}+\bar{z})$

$\bar{w}+\bar{x}$

SOP - 1's

POS's = 0's

$(w+x)$

$\Rightarrow$ POS $(y+z)(\bar{y}+\bar{z}) \cdot (\bar{w}+\bar{x}) \cdot (w+x)$

SOP

$\bar{w}x\bar{y}z + w\bar{x}\bar{y}z + \bar{w}xy\bar{z} + \bar{w}\bar{x}y\bar{z}$

$f'(w,x,y,z) = \pi(5,6,9,10)$

yz \ wx	00	01	11	10
00	1	1	1	1
01	1	0	1	0
11	1	1	1	1
10	1	0	1	0

$\bar{y}\bar{z}$

yz

$\bar{w}\bar{x}$

wx

SOP $= \bar{w}\bar{x} + wx + yz + \bar{y}\bar{z}$

POS $= (\bar{w}+\bar{x}+y+\bar{z}) \cdot (\bar{w}+x+y+z)$
$(w+\bar{x}+\bar{y}+z) \cdot (\bar{w}+x+\bar{y}+z)$

(12) Example on Min POS :-

Find the no. of literals in Min. POS and SOP for $f(w,x,y,z) = \pi(1,5,6,7,11,12,13,15)$.

yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	1
11	1	0	0	0
10	1	0	1	1

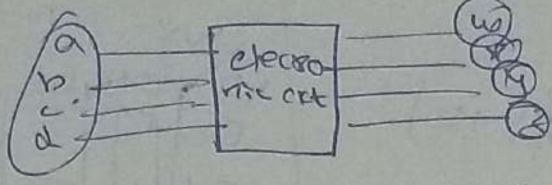
POS $= (\bar{w}+\bar{x}+y) \cdot (\bar{w}+\bar{y}+\bar{z}) \cdot$ (12)
$(w+\bar{x}+\bar{y}) \cdot (w+y+\bar{z})$

SOP $= w\bar{x}\bar{y} + wy\bar{z} + \bar{w}\bar{x}y + \bar{w}\bar{y}\bar{z}$
(12)

12 literal in both POS, SOP

no need of same count every time

(B) Introduction to Don't care:



BCD $\longrightarrow$ Excess 3

BCD — $0 \rightarrow 9$

$0 \rightarrow 3$
$1 \rightarrow 4$
$2 \rightarrow 5$
| |
$9 \rightarrow 12$

	a	b	c	d	w	x	y	z
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0
10 ↓ 15	ϕ	ϕ	ϕ	ϕ				

Dont care combinations :-

→ A function is said to be completely specified if it is given 0 (or) 1 for every combination of variables. There exists some functions which are not completely specified.

→ combinations for which the value of a function is not specified are called 'dont care combinations'

→ The value of a function for such combination is denoted by 'ϕ' (or) 'd'.

→ Since each dont care combination represents two values {0,1}, an in-completely specified function containing k-dont care combinations corresponds to a class of 2^k distinct functions.

→ Our task is to choose a function having minimal representation out of those 2^k functions.

→ we could assign a '0' (or) '1' to a dont care combination in such a way to increase (or) decrease size of a sub cube.

4 functi $= 2^2$

Ex:-

a	b	f	f_1	f_2	f_3	f_4
0	0	1	1	1	1	1
0	1	1	1	1	1	1
1	0	ϕ	0	0	1	1
1	1	ϕ	0	1	0	1

d

min. exp

$$\begin{bmatrix} 1 & 1 \\ \phi & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

ϕ

→ subcube containing any ϕ not possible.

(14) Don't care Ex1:-

$$w = \Sigma(5,6,7,8,9) + \phi(10,11,12,13,14,15)$$

yz \ wx	00	01	11	10
00			ϕ	1
01		1	ϕ	1
11		1	ϕ	ϕ
10		1	ϕ	ϕ

w

xz

xy

no-of function $= 2^6 = 64$

$$w = w + xz + xy$$

(15) Dont care Ex2:-

AB \ CD	00	01	11	10
00	0	0	1	0
01	X	X	1	X
11	0	1	1	0
10	0	1	1	0

CD

not essential

AD

$AD + CD$

(16) Dont care Ex 2.2 :-

yz \ wx	00	01	11	10
00	0	1	1	0
01	X	0	0	1
11	X	0	0	1
10	0	1	1	X

$\bar{x}z$

$x\bar{z} + \bar{x}z$

$x\bar{z}$

(17) Dont care Ex 2.3 :-

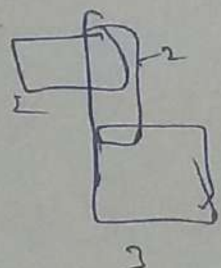
$z\bar{w}\bar{x}$

xy \ zw	00	01	11	10
00	X	1	0	1
01	0	1	X	0
11	1	X	X	0
10	X	0	0	X

$\bar{w}\bar{y}$

any one

$x\bar{z}\bar{w}$



(1+3) - 2

(18) Dont care Ex 2.4 :-

cd \ ab	00	01	11	10
00	1	1		1
01	X			
11	X			
10	1	1		X

$\bar{b}\bar{d}$

$\bar{a}\bar{d} + \bar{b}\bar{d}$

$\bar{a}\bar{d}$

(19) Dont care Exp.s :-

cd \ ab	00	01	11	10
00	1	x	x	1
01	x			1
11				
10	1			x

$\bar{b}\bar{d}$

$\bar{b}\bar{c}$

$\boxed{F = \bar{b}\bar{d} + \bar{b}\bar{c}}$

(20) Examples on dont care sets :-

xy \ wz	00	01	11	10
00	0	X	0	0
01	0	X	1	1
11	1	1	1	1
10	0	X	0	0

(Q) Which function doesnot implement the K-map given?

a) $(w+x)y$ (b) $xy + yw$ (c) $(w+x)(\bar{w}+y)(\bar{x}+y)$

~~(d)~~ None.

(a) POS

(c) POS

(b) SOP

xy \ wz	00	01	11	10
00	0	X	0	0
01	0	X	1	1
11	1	1	1	1
10	0	X	0	0

y

$(w+x)$

w x y z

$(w+x)$

(a) $(w+x)y = wy + xy$

POS check 0's

SOP check 1's

2 mixterms - 2^2 comb.

y → 3 mixterms - 2^3 comb.

(b)

xy \ wz	00	01	11	10
00	0	X	0	0
01	0	X	1	1
11	1	1	1	1
10	0	X	0	0

wy

xy

xy + yw
Beams 4c

xy + yw

xy → 2 rows → 2^2 can
4 can

yw → 4 can

(c) $\underbrace{(w+x)}_{4\ \text{cans}}\ \underbrace{(\bar{w}+y)}_{4}\ \underbrace{(\bar{x}+y)}_{4}$

xy \ wz	00	01	11	10
00	0	X	0	0
01	0	X	1	1
11	1	1	1	1
10	0	X	0	0

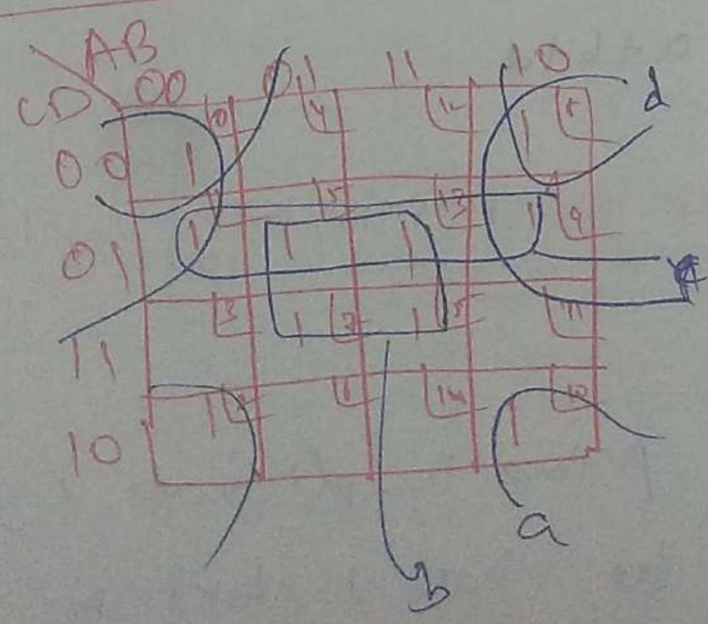
$(\bar{x}+y)$

$(w+x)$

$(\bar{x}+y)$

$(\bar{w}+y)$

(2) Finding Minimal Expressions:—



2-var K-map :-

A \ B	0	1
0	0	1
1	2	3

3-var K-map :-

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

4-var K-map :-

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

① The Boolean expression
$(x+y)(x+\bar{y}) + \overline{x\bar{y} + \bar{x}}$ simplifies to

ⓐ x ✓ ⓑ y ⓒ xy ⓓ $x+y$

Ans :-

$$x + x\bar{y} + xy + \overline{x\bar{y}} \cdot x$$

$$x\left(1 + \bar{y} + y + \overline{x\bar{y}}\right) = x$$

② In the Karnaugh map shown below, X denotes a don't care term. what is the minimal form of the function represented by the Karnaugh map?

cd \ ab	00	01	11	10
00	1	1		1
01	X			
11	X			
10	1	1		X

ⓐ $\bar{b}\bar{d} + \bar{a}\bar{d}$

ⓑ $\bar{a}\cdot\bar{b} + \bar{b}\bar{d} + \bar{a}\bar{b}\bar{d}$

ⓒ $\bar{b}\bar{d} + \bar{a}\bar{b}\bar{d}$

ⓓ $\bar{a}\bar{b} + \bar{b}\bar{d} + \bar{a}\cdot\bar{d}$

cd \ ab	00	01	11	10
00	1	1		1
01	X			
11	X			
10	1	1		X

$\rightarrow \bar{b}\bar{d}$

$\bar{a}\bar{d}$

$\therefore \bar{a}\bar{d} + \bar{b}\bar{d}$

option (a)

(3) What is the equivalent boolean expression in product-of-sums form for the Karnaugh map given in fig.?

CD \ AB	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

(a) $B\overline{D} + \overline{B}D$

(b) $(B+\overline{C}+D)(\overline{B}+C+\overline{D})$

(c) $(B+D)(\overline{B}+\overline{D})$

(d) $(B+\overline{D})(\overline{B}+D)$

CD \ AB	00	01	11	10
00	0			0
01		0	0	
11		0	0	
10	0			0

$\rightarrow (D+B)\cdot(\overline{D}+\overline{B})$

option (c)

(4) A combinational circuit has i/ps A, B and C and its Karnaugh map is given in below fig. The output of the circuit is?

C \ AB	00	01	11	10
0		1		1
1	1		1	

(a) $(\overline{A}B + A\overline{B})\cdot C$

(b) $(\overline{A}B + A\overline{B})\cdot \overline{C}$

(c) $A \oplus B \oplus C$

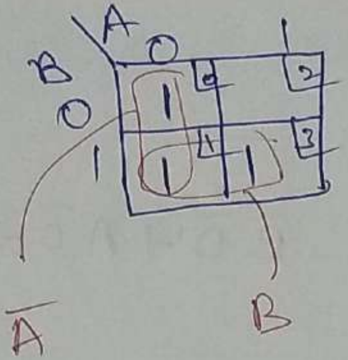
(d) $\overline{A}\cdot\overline{B}\cdot\overline{C}$.

$$\overline{A}B\overline{C} + \overline{A}\cdot\overline{B}C + ABC + A\overline{B}\,\overline{C}$$

$$(\overline{A}B + A\overline{B})\overline{C} + (\overline{A}\overline{B} + AB)C$$

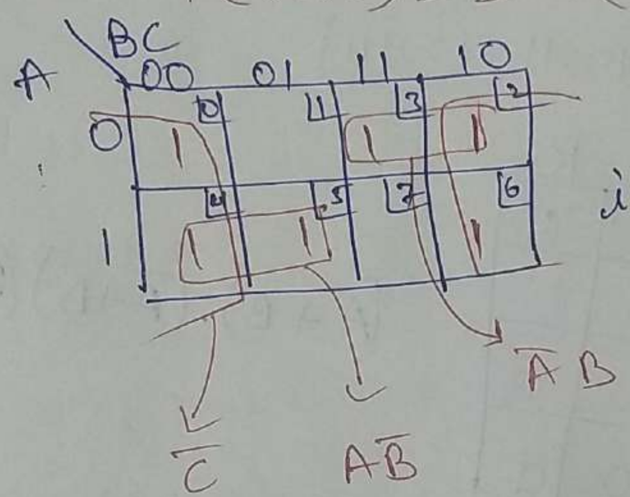
$$(A \oplus B)\overline{C} + A \odot B\, C = A \oplus B \oplus C$$

(5) $f(A,B) = \Sigma m(0,1,3)$



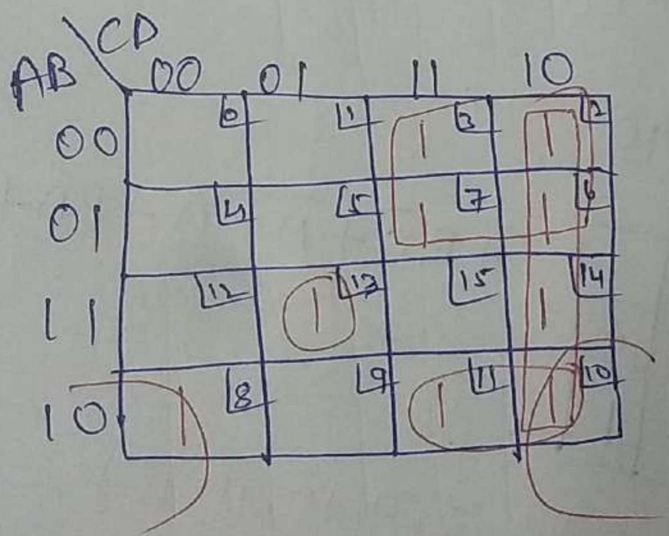
i.e. $\bar{A} + B$.

(6) Reduce $f(A,B,C) = \Sigma m(0,2,3,4,5,6)$



i.e. $A\bar{B} + \bar{A}B + \bar{C}$

(7) $f(A,B,C,D) = \Sigma m(2,3,6,7,8,10,11,13,14)$



$C\bar{D} + \bar{A}C + \bar{B}C + A\bar{B}\bar{D} + AB\bar{C}D$.

⑧ $f = \Sigma m(0,1,2,3,5,7,8,9,10,12,13)$.

AB \ CD	00	01	11	10
00	1	1	1	1
01		1	1	
11	1	1		
10	1	1		1

$\bar{B}\bar{D} + A\bar{C} + \bar{A}D$

⑨ $f = \Pi M(2,8,9,10,11,12,14)$

AB \ CD	00	01	11	10
00				0
01				
11	0			0
10	0	0	0	0

$(\bar{A}+B).(\bar{A}+D)(B+\bar{C}+D)$

⑩ $f(A,B,C,D) = \Sigma m(1,4,7,10,13) + \Sigma d(5,14,15)$

AB \ CD	00	01	11	10
00		1		
01	1	ϕ	1	
11		1	ϕ	ϕ
10				1

$\bar{A}B\bar{C} + \bar{A}\bar{C}D + BD + AC\bar{D}$

⑪ $f(A,B,C,D) = \Pi M(0,1,2,3,4,10,11,15)$.

AB \ CD	00	01	11	10
00	0	0	0	0
01	0			
11			0	
10			0	0

$(B+\bar{C})(A+B)(A+C+D)(\bar{A}+\bar{C}+\bar{D})$.

(12) $f(A,B,C,D) = \Sigma M(6,7,8,9) + d(10,11,12,13,14,15)$.

AB \ CD	00	01	11	10
00				
01			0	0
11	ϕ	ϕ	ϕ	ϕ
10	0	0	ϕ	ϕ

$\bar{A}(\bar{B}+\bar{C})$.

(13) Make a K-map of the following expression and obtain minimal expression in SOP & POS form.

$f = AB + A\bar{C} + C + AD + A\bar{B}C + ABC$

Sol $f = \Sigma m(2,3,6,7,8,9,10,11,12,13,14,15)$

$= \Sigma M(0,1,4,5)$

SOP

AB \ CD	00	01	11	10
00			1	1
01			1	1
11	1	1	1	1
10	1	1	1	1

$A+C$

POS

AB \ CD	00	01	11	10
00	0	0		
01	0	0		
11				
10				

$(A+C)$

(14) $f(A,B,C,D) = \Sigma m(0,2,7,8,12) + \Sigma d(10,13)$

AB \ CD	00	01	11	10
00	1			1
01			1	
11	1	ϕ		
10	1			ϕ

$\bar{B}\bar{D} + A\bar{C}\bar{D} + \bar{A}BCD$

(15) $f(w,x,y,z) = \pi M(0,5,7,10,12,13) \cdot \pi d(14,15)$

$wx \backslash yz$	00	01	11	10
00	0			
01		0	0	
11	0	0	ϕ	ϕ
10				0

$(w+x+y+z)$

$(\bar{x}+\bar{z})$

$(\bar{w}+\bar{x})$

$(\bar{w}+\bar{y}+z)$

$$(\bar{x}+\bar{z})(\bar{w}+\bar{x})(\bar{w}+\bar{y}+z)(w+x+y+z)$$

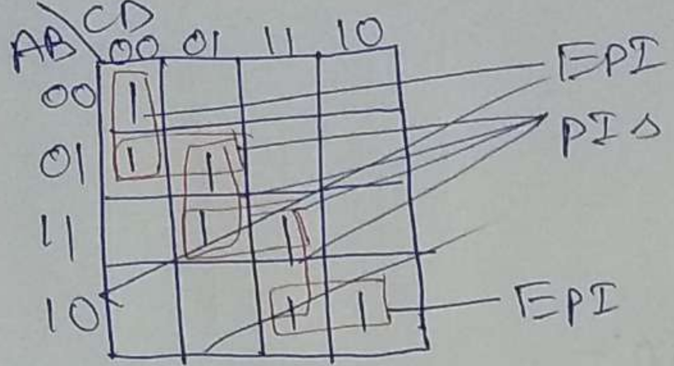
Prime Implicant :- (PI)

Each square (or) rectangle made up of the bunch of adjacent min terms is called a sub cube. Each of these sub cubes is called a prime implicant (PI).

Essential Prime Implicant (EPI):-

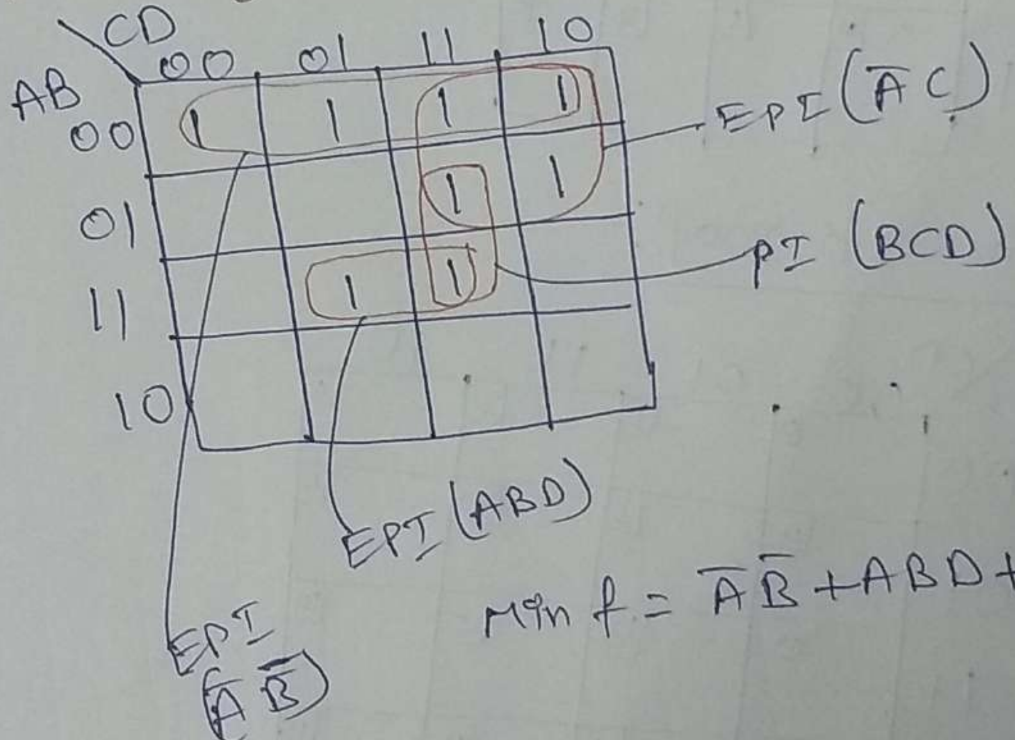
The prime implicant which contains at least one '1' which cannot be covered by any other prime implicant is called EPI.

(1) $f(A,B,C,D) = \Sigma m(0,4,5,10,11,13,15)$.



(2) Identify PI's and EPIs

$f = \Sigma m(0,1,2,3,6,7,13,15)$.



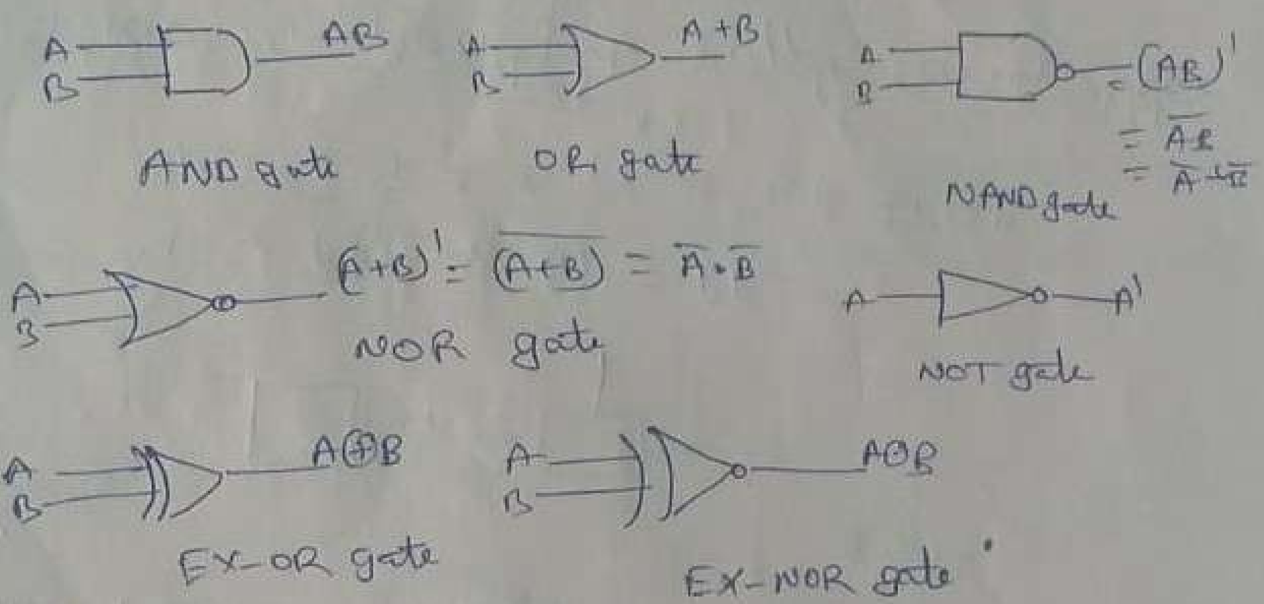
$\text{Min } f = \bar{A}\bar{B} + ABD + \bar{A}C$

3. Design and Synthesis of Combinational circuits (6H) ①

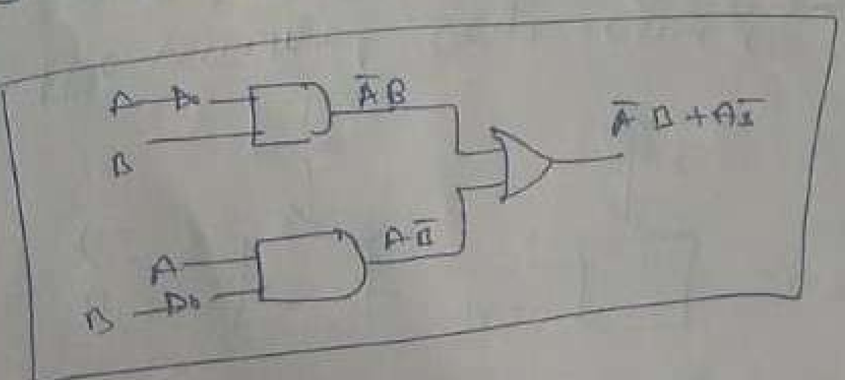
① Introduction to Logic design:-

→ The main application of switching theory is in the design of digital circuits. It is called Logic design

→ These circuits are designed using basic elements called gates.



①② $\bar{A}B + A\bar{B}$



→ The Logic ckt related to propagation delay & cost

→ Decrease no. of levels in the ckt decreases the propagation delay. (2)

→ Decrease the no. of gates decrease the cost of the circuit.

No. of gates in the ckt,

~~<100~~

<10 – SSI

10 – 100 – MSI

100 – 1000 – LSI

>1000 – VLSI

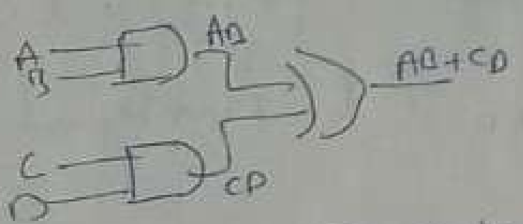
→ Basic gates are, OR, AND, NOT gates.

→ Universal gates are NAND, NOR gates.

(4) AND-OR, OR-AND Realization:-

SOP AND-OR realization

$f(A,B,C,D) = AB + CD$



AND – OR realization

POS OR-AND realization

$f(A,B,C,D) = (A+B)(C+D)$

~~OR-AND~~

OR – AND realization

cheap costly

→ Unless specified Fan In = 2

Fanout we can't consider

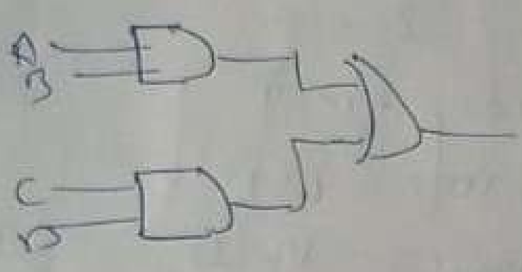
① $\bar{A}B+CD$

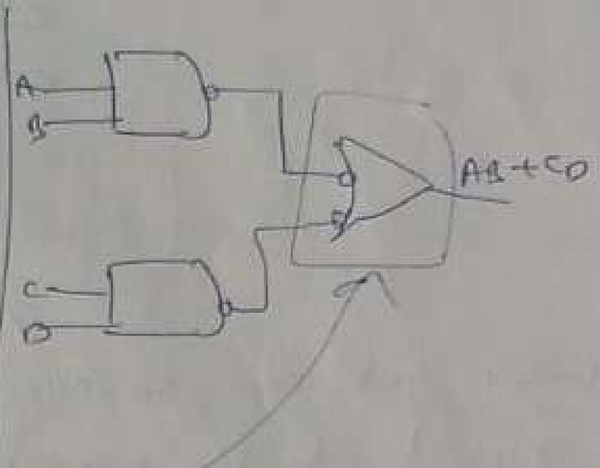
Realization means implement a function using gates.

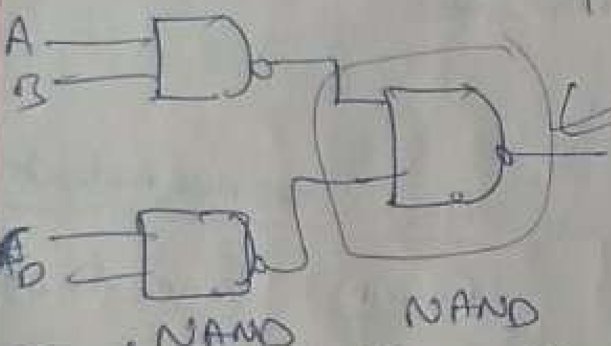
3

① AND-OR (SOP)

AB+CD







NOT also implements by

Implemented using NAND only

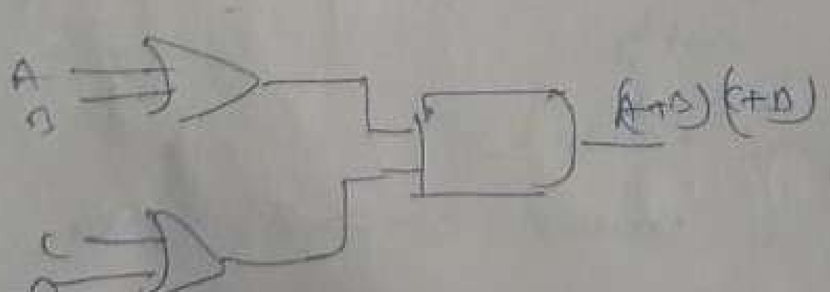
$(*,-)$ - FC

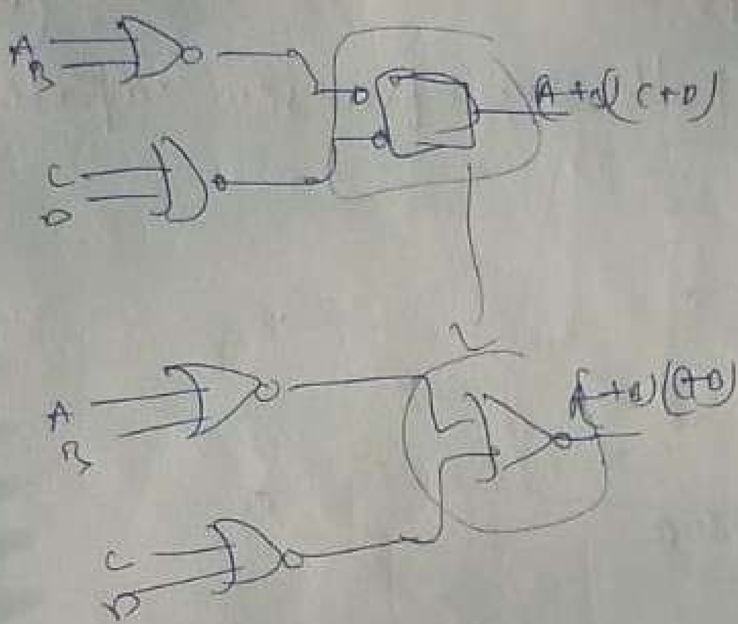
AND-OR = NAND-NAND

Functionally complete

② OR-AND is also same

(A+B) (C+D)





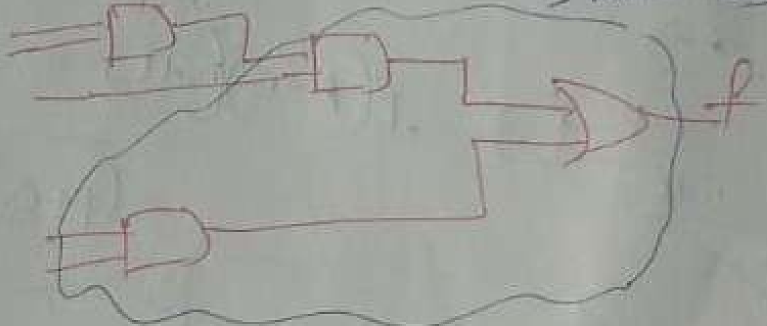
NOR-NOR

(↑, −) - FC

∴ OR-AND = NOR-NOR realization.

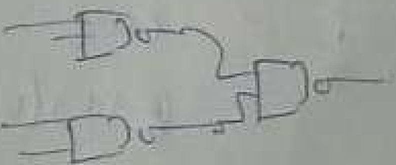
3 Minimum no. of NAND gates Example:-

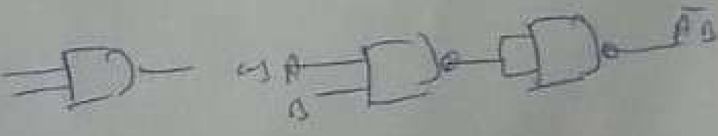
Identify min. no. of two i/p NAND gates required to represent the following



AND- OR

⇒ 3 NAND gates only

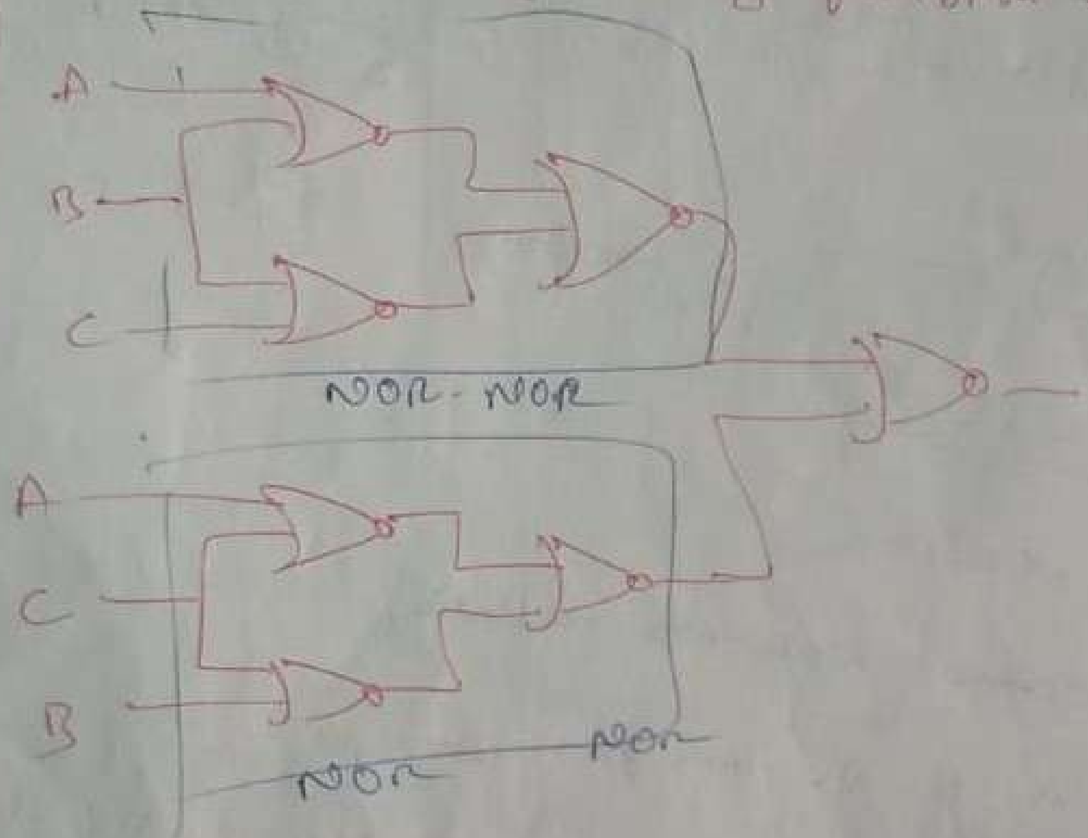




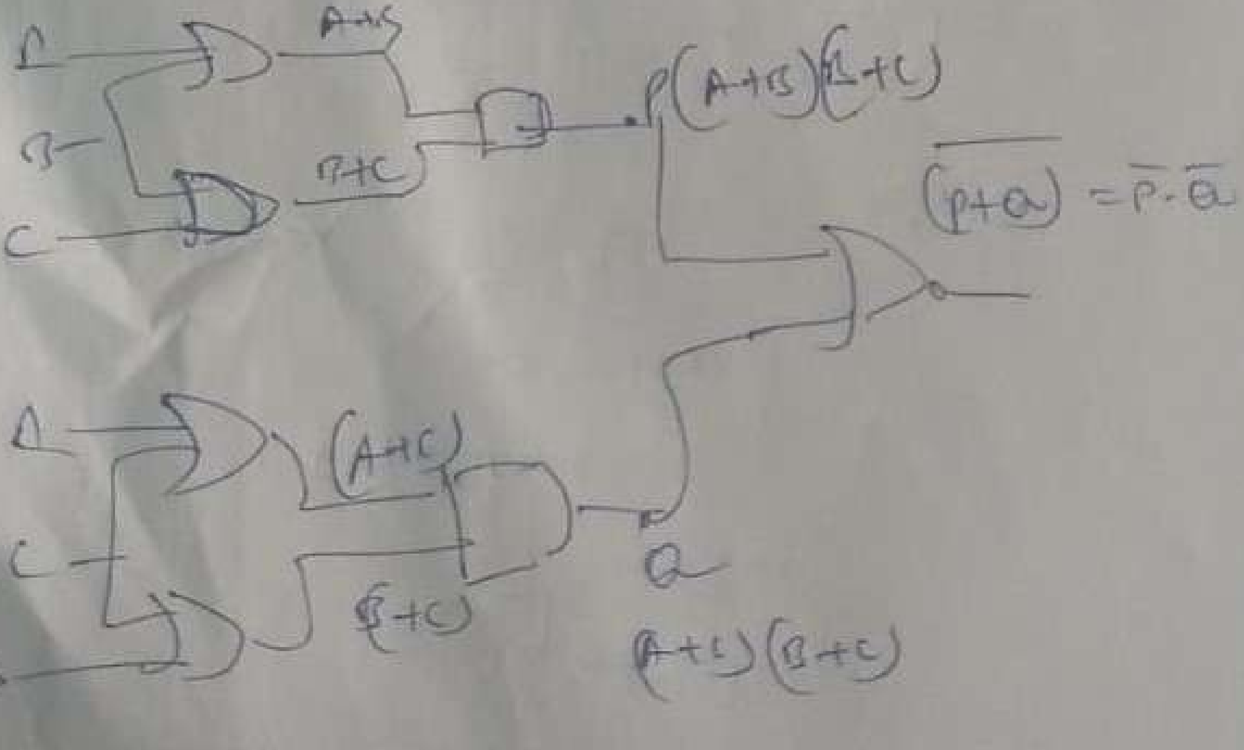
∴ 5 NAND gates required.

(4) NOR-NOR example:-

What does the following function represent? (5)



NOR - NOR = OR-AND



$P = (A+B)(B+C)$

$= AB + AC + B + BC$

$= AB + AC + B = AC + B$

$= \cancel{ABC}$

$Q = (A+C)(B+C)$

$= AB + AC + BC + CC$

$= AB + AC + C$

$= AB + C$

$\overline{PQ} = \overline{(AC+B)} \cdot \overline{(AB+C)}$

$= \bar{B} \cdot (\bar{A}+\bar{C}) \cdot \bar{C}(\bar{A}+\bar{B})$

$= (\bar{B}\bar{A}+\bar{B}\bar{C})(\bar{C}\bar{A}+\bar{C}\bar{B})$

$= \bar{B}\bar{A}\bar{C} + \bar{B}\bar{A}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{B}\bar{C}$

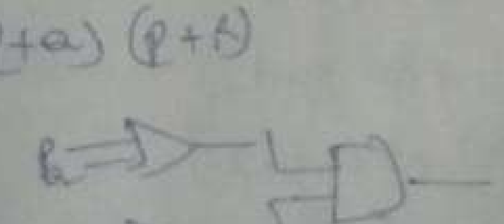
$= \bar{A}\bar{B}\bar{C} + \bar{B}\bar{C} = \bar{B}\bar{C}$

Minimum no: of NOR gates Example

Find the no. of 2 i/p 'NOR' gates required to represent $F(P,Q,R) = P + QR$

NOR-NOR = OR-AND (∴POS)

$(P+Q)(P+R)$



∴ 3 NOR gates req.

(6) Min no. of NOR gates required to implement
$x + x\bar{y} + x\bar{y}z$

(7)

First see is it in min form or not

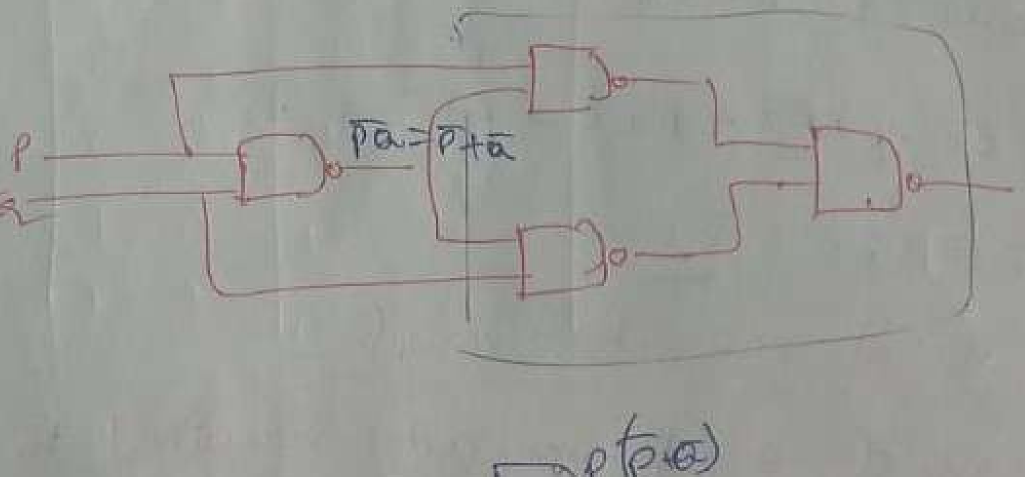
$$x\left(1+\bar{y}+\bar{y}z\right) = x$$

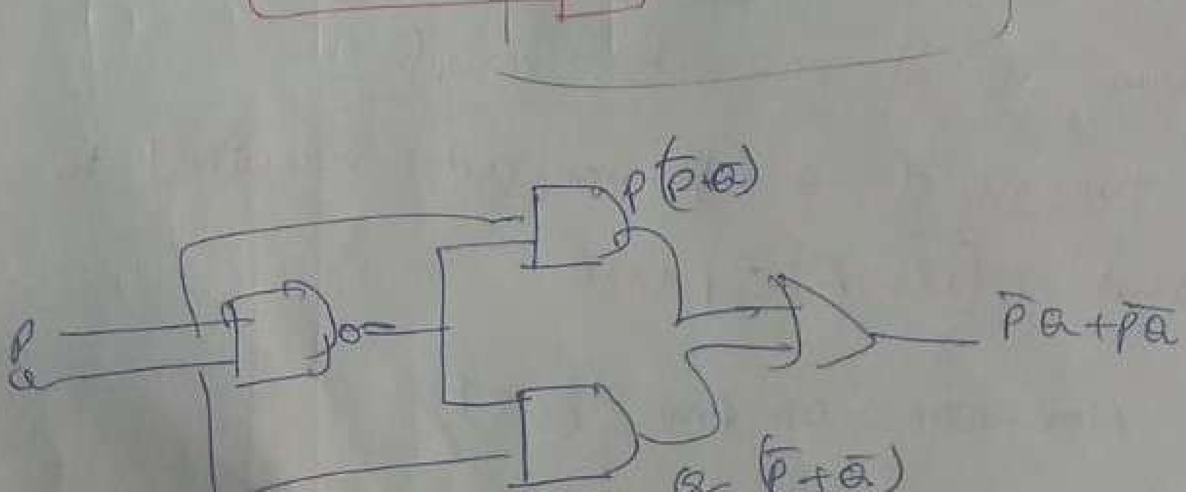
x ——————— x Min = 0.

Max = 2

(7) EX-OR and EX-NOR implementation with NOR and NAND gates

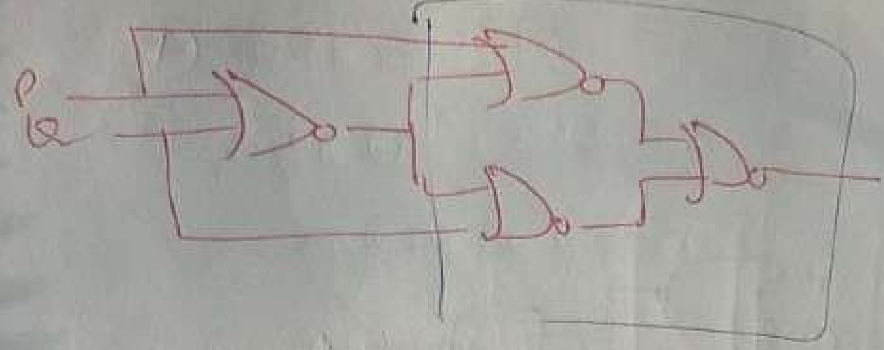
What does the following represent?

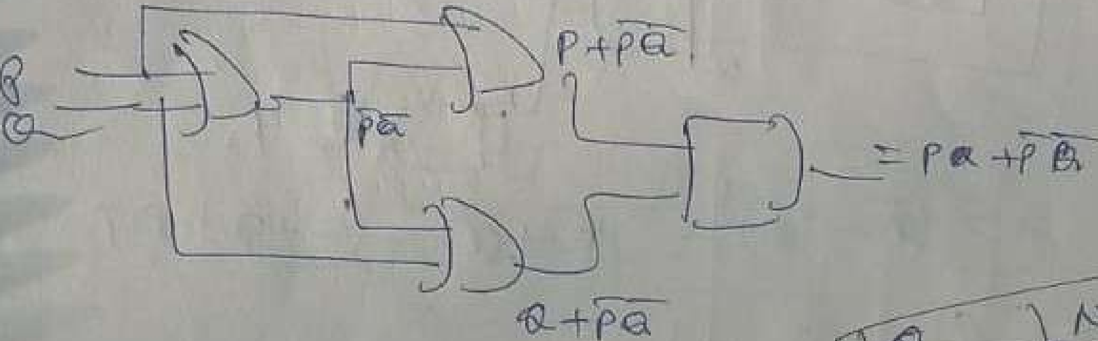




EXOR ⟷ 4 NAND gates

⟷ (4+1) NOR gates



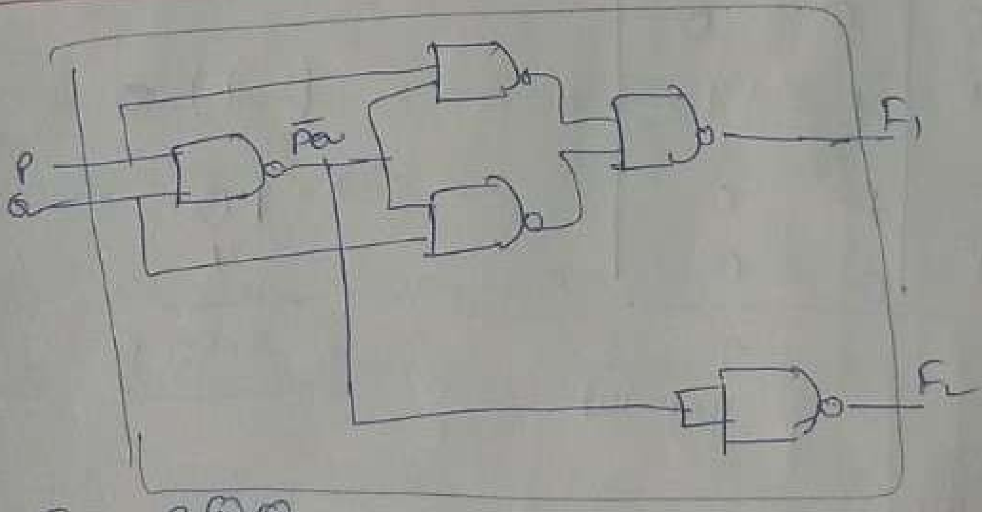


EX-NOR ⟷ 4 NOR gate

⟷ (4+1) NAND gate

	NOR	NAND
X-OR	5	4
X-NOR	4	5

(8) Half Adder:-



$F_1 = P \oplus Q$

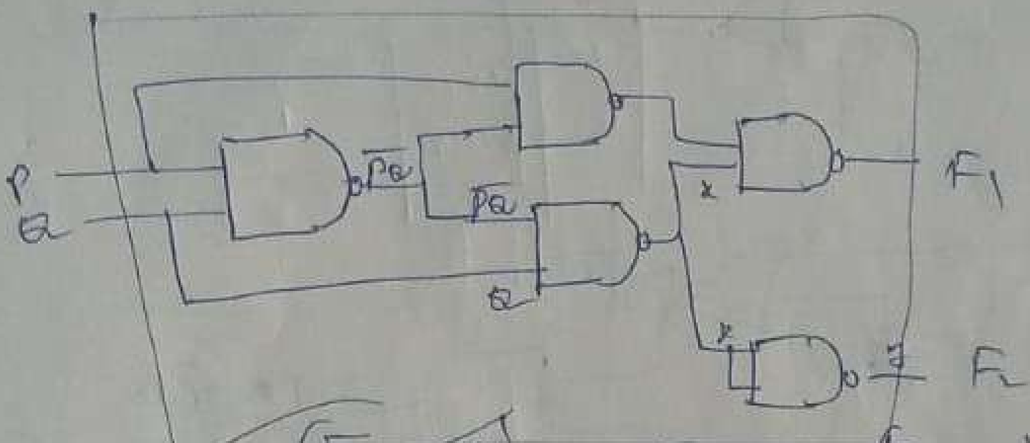
$F_2 = PQ$

P	Q	F_1	F_2
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1
		↑ Sum	↑ carry

Half Adder → Add two bits

Full Adder → Add three bits

(9) Half Subtractor :-



$\cancel{x = \overline{(\overline{PQ}\cdot Q)}} = PQ + \bar{Q} = \cancel{(\bar{Q}+Q)}\,\bar{Q}P$

$x = \overline{((\overline{P}+\overline{Q})\cdot Q)} = \overline{P}Q$

$y = \overline{\overline{PQ}} = PQ$

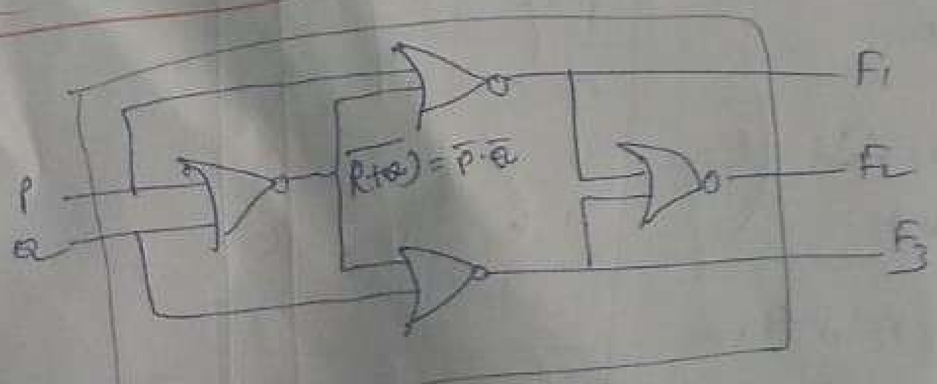
P	Q	F_1	F_2
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Borrow

(0 − 1)

⇒ 10
  1
  2 − 1 = 1

(10) comparator :-



W.K.T.

$$F_2 = \bar{P}\bar{Q} + PQ$$

$$F_1 = \overline{P + \bar{P}\bar{Q}} = \overline{P+\bar{Q}} = \bar{P}\cdot Q$$

$$F_3 = \overline{Q + \bar{P}\bar{Q}} = \overline{Q+\bar{P}} = \bar{Q}\cdot P = P\bar{Q}$$

P	Q	F_1	F_2	F_3
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

F_1: $P<Q$, F_2: $P=Q$, F_3: $P>Q$

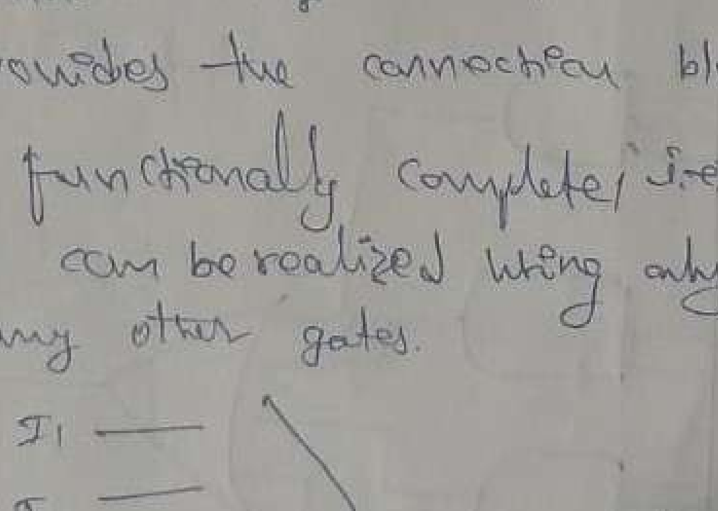
11) Introduction to MUX:-

→ A multiplexer is an electronic switch that can connect one out of 'n' i/ps to o/p.

→ It cannot change the logical level of the i/p, it only provides the connection b/w i/p and o/p.

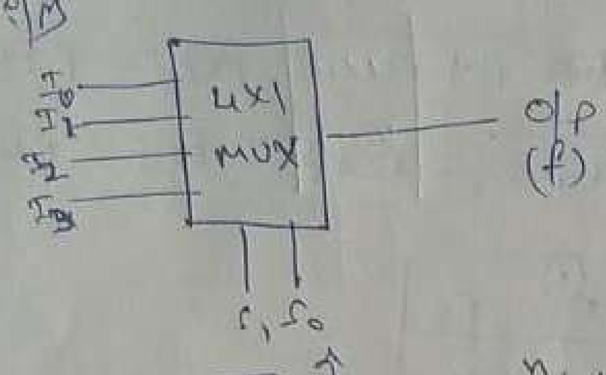
→ It is functionally complete, i.e., all boolean functions can be realized using only multiplexer without any other gates.

①→



(2) → take the i/p and produce same o/p

(3) ~~Do~~ by MUXs and implement any ckt. (11)



	S_1	S_0	o/p (f)
(0)	0	0	I_0
(1)	0	1	I_1
(2)	1	0	I_2
(3)	1	1	I_3

$2^n \times 1$ MUX ; when

n = no. of selection

2^n = no. of i/ps

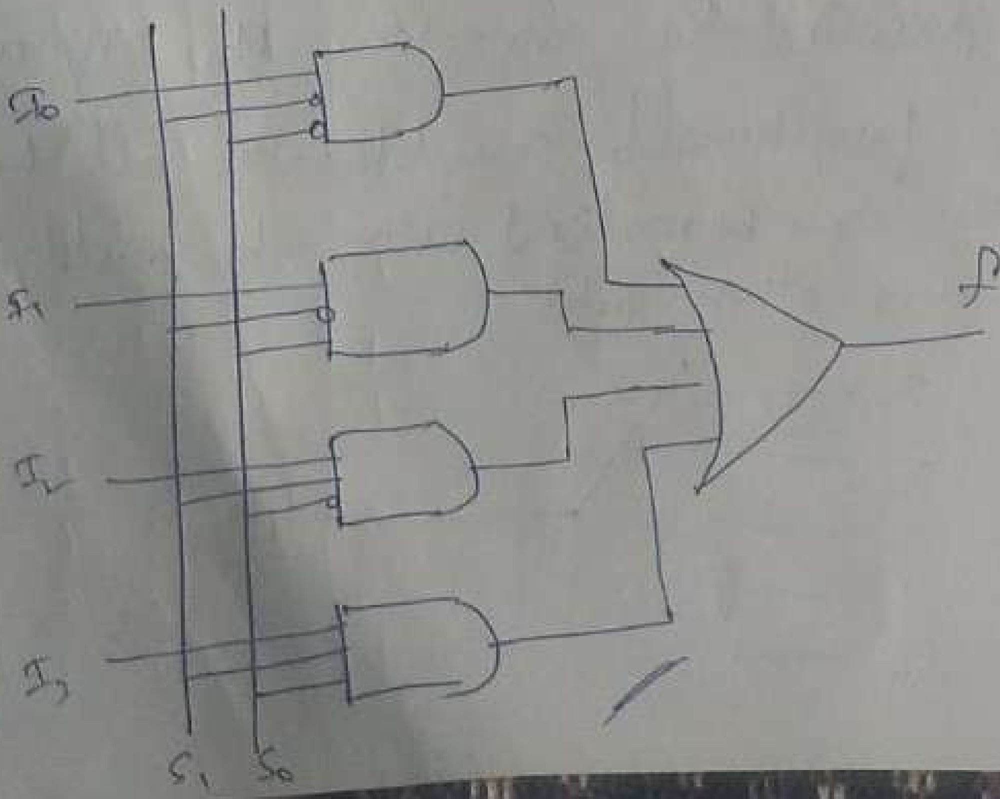
1 = o/p.

Characteristic table

$$\boxed{f = \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1 \bar{S}_0 I_2 + S_1 S_0 I_3}$$

Realization (Implementation)

f is SOP ⇒ AND-OR realization

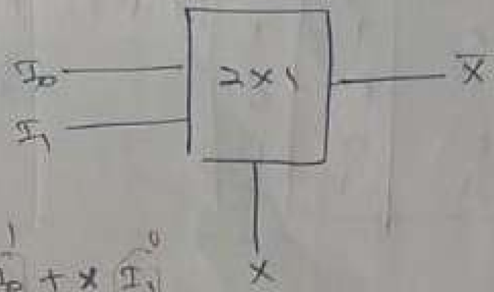


(12) Providing MUX is functionally complete :- (12)

(·, +, −) Deriving these three by using any set of operators, called as functionally complete.

(·, −) — FC
(+, −) — FC

NOT implementation using MUX

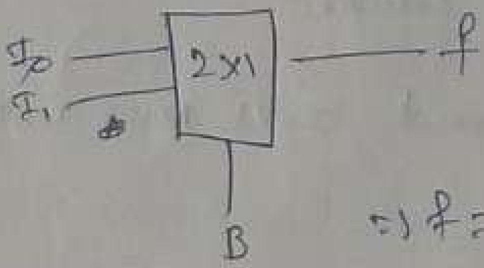


$$f_1 = \bar{x}\overset{1}{I_0} + x\overset{0}{I_1}$$

x	$\bar{x}$
0	1
1	0

$$\bar{x} = \bar{x}\cdot 1 + x\cdot 0$$

AND :-



$$\Rightarrow f = \bar{B} I_0 + B I_1$$

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

$$f = AB = B\overset{I_1}{(A)} + \bar{B}\overset{I_0}{(0)}$$

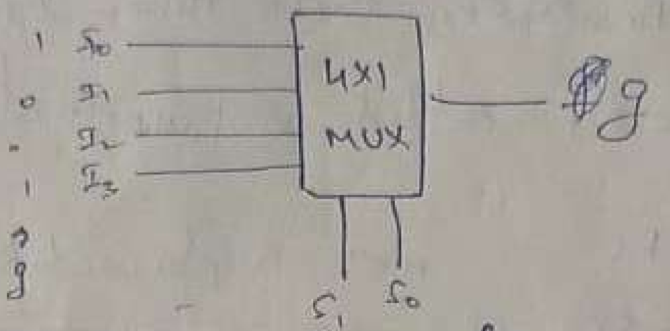
OR :-

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

$$f = A+B = \bar{A}B + A\bar{B} + AB$$
$$= \bar{B}A + B(A+\bar{A})$$
$$= \bar{B}\overset{I_0}{A} + B\overset{I_1}{(1)}$$

(13) Implementing Function with MUX ex:-

(12)



S_0 - LSB

S_1 - MSB

A B
B A → g h

A	B	g	h
0	0	1	1
0	1	0	0
1	0	0	1
1	1	1	0

$$g(A,B) = \bar{A}\bar{B} + AB$$

$$= \underbrace{\bar{A}\bar{B}(1)}_{I_0} + \underbrace{\bar{A}B(1)}_{I_1} + \underbrace{A\bar{B}(0)}_{I_2} + \underbrace{AB(1)}_{I_3}$$

⇒ $S_n \cdots S_2\, S_1\, S_0$

↑ MSB ↑ LSB

→ For $2^n \times 1$ MUX, it need 'n' selectors.

$2^2 \times 1$ MUX → 2 selectors

$2^4 \times 1$ MUX → 4 selectors

(1) → We can implement $2^2 \times 1$ MUX with 3 selectors.

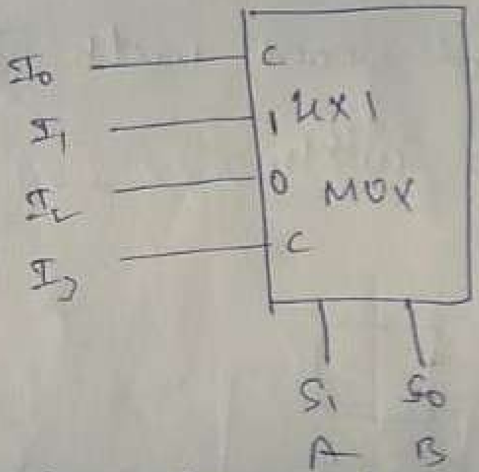
A	B	C	g
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(14)

$g = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$

$= \underset{I_0}{\bar{A}\bar{B}(C)} + \underset{I_1}{\bar{A}B(\bar{C}+C)} + \underset{I_2}{A\bar{B}(0)} + \underset{I_3}{AB(C)}$

$=$



$\therefore$ $2^n \times 1$ MUX can also implement >n selector variables.

(5) 4x1 MUX with 4 selector vars.

$\underset{I_0}{\bar{A}\bar{B}(CD)} + \underset{I_1}{\bar{A}B(\overline{CD})} + \underset{I_2}{A\bar{B}(CD)} + \underset{I_3}{AB(CD)}$

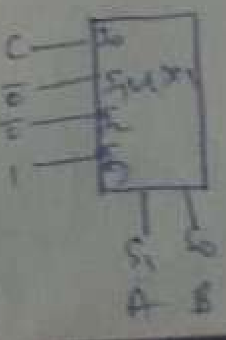
higher no. of selectors are possible but it needs more gates to implement this.

(14) Exa:- $f(A,B,C) = \Sigma(1,2,4,6,7)$

$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$

$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + (A\bar{B})(\bar{C}) + AB(\bar{C}+C)$

$= \underset{0\ 0}{\bar{A}\bar{B}}\underset{I_0}{(C)} + \underset{0\ 1}{\bar{A}B}\underset{I_1}{(\bar{C})} + \underset{1\ 0}{A\bar{B}}\underset{I_2}{(\bar{C})} + \underset{1\ 1}{AB}\underset{I_3}{(1)}$

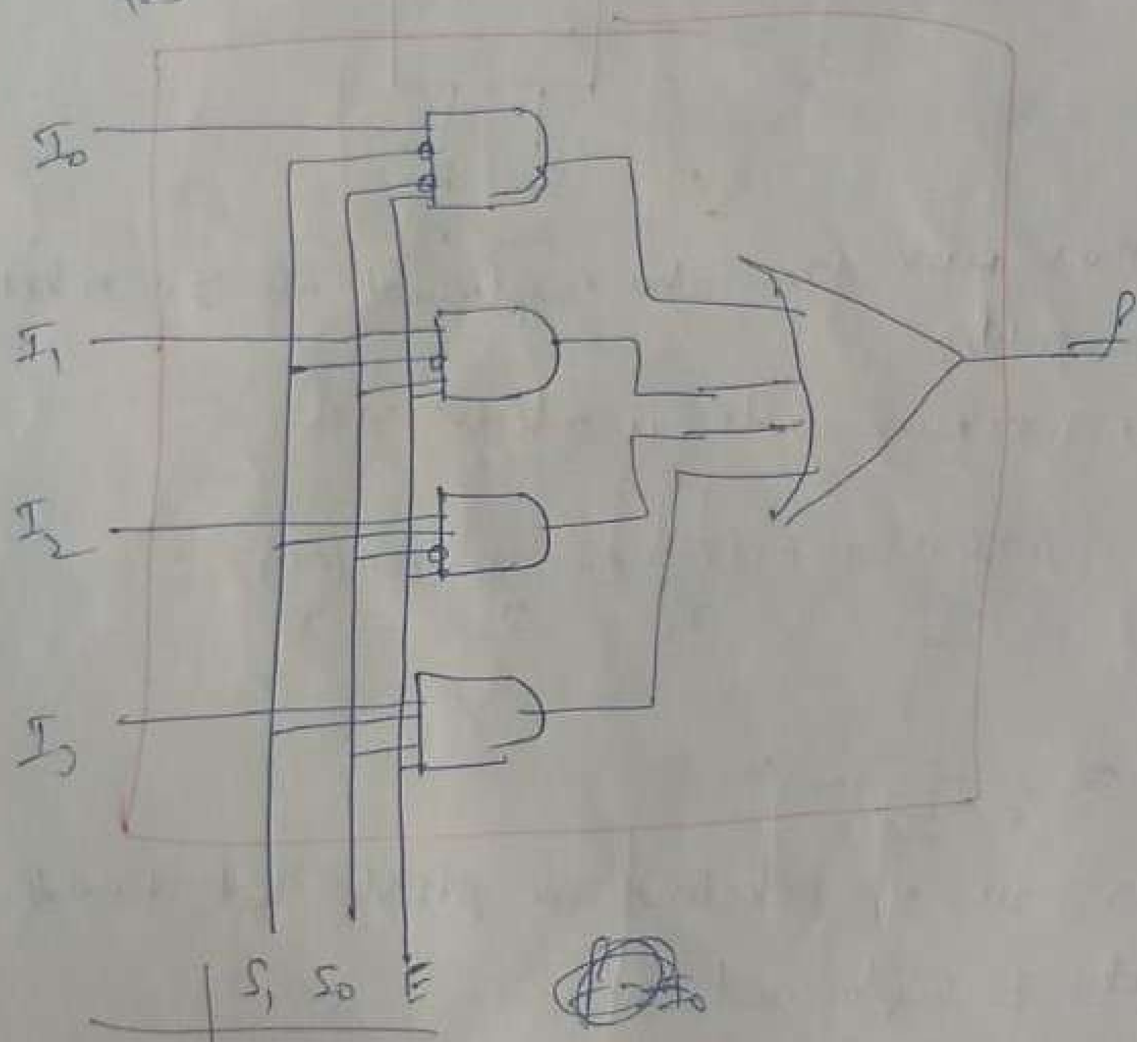


(15) Multiplexer with Enable input :- (E)

(15)

Whenever E=1 then the MUX reacts with the i/p and produce the corresponding result o/p. Otherwise not react with the i/p and o/p is 0.

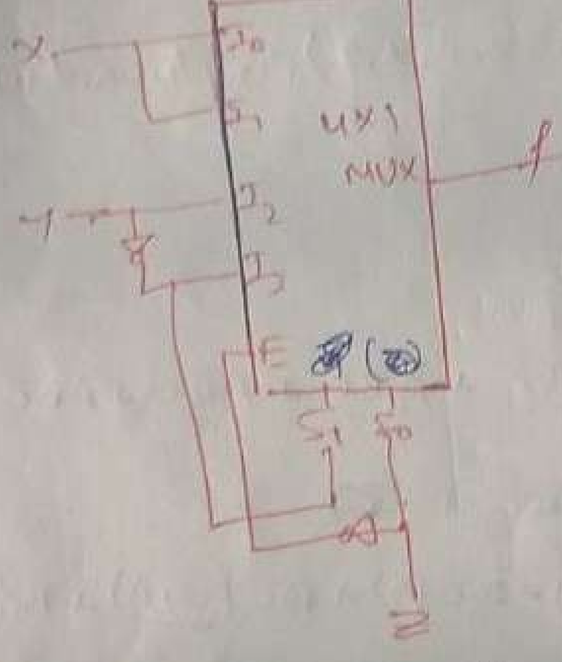
→ It Acts a switch. (E)



	S_1	S_0
I_0	0	0
I_1	0	1
I_2	1	0
I_3	1	1

$$f = E(I_0 \bar{S_1}\bar{S_0} + I_1 \bar{S_1} S_0 + I_2 S_1 \bar{S_0} + I_3 S_1 S_0)$$

(15) Minimize the function represented by following MUX (18)



→ always do write characteristic equation.

$$f = E(I_0\bar{S_1}\bar{S_0} + I_1\bar{S_1}S_0 + I_2 S_1\bar{S_0} + I_3 S_1 S_0)$$

here, $S_0 = z$; $S_1 = \bar{y}$; $E = \bar{z}$

$I_0 = x$; $I_1 = x$

$I_2 = y$; $I_3 = \bar{y}$

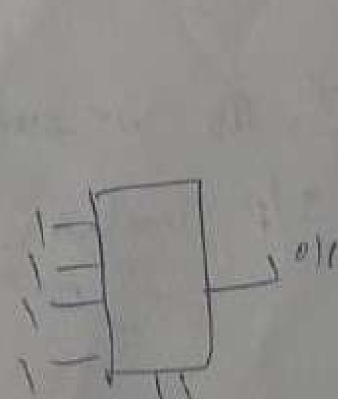
$$f = \bar{z}(xy\bar{z} + xyz + \underbrace{y\bar{y}z}_{0} + \bar{y}\bar{y}z)$$

$$= \bar{z}(xy\bar{z} + xyz + \bar{y}z)$$

$$= \bar{z}(xy(\bar{z}+z) + \bar{y}z)$$

$$= \bar{z}(xy + \bar{y}z)$$

$$= xy\bar{z}$$



(17) Relationship b/w select lines and inputs of a MUX:

$F(A, B, C) = \Sigma(2, 3, 5, 6, 7)$, Implement using 4x1 MUX

a) $S_1 = B$, $S_0 = C$

b) $S_1 = C$, $S_0 = B$

(a) $(2, 3, 5, 6, 7) = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$= \cancel{\bar{A}B}C + \cancel{BC}$

$\frac{S_1 \ S_0}{B \ \ C}$

$= \bar{B}\bar{C}(0) + \bar{B}C(A) + B\bar{C}\underbrace{(\bar{A} + A)}_{1} + BC\underbrace{(\bar{A} + A)}_{1}$

$= \underbrace{\bar{B}\bar{C}(0)}_{I_0} + \underbrace{\bar{B}C(A)}_{I_1} + \underbrace{B\bar{C}(1)}_{I_2} + \underbrace{BC(1)}_{I_3}$

$\Rightarrow I_0 = 0; I_1 = A; I_2 = 1; I_3 = 1$

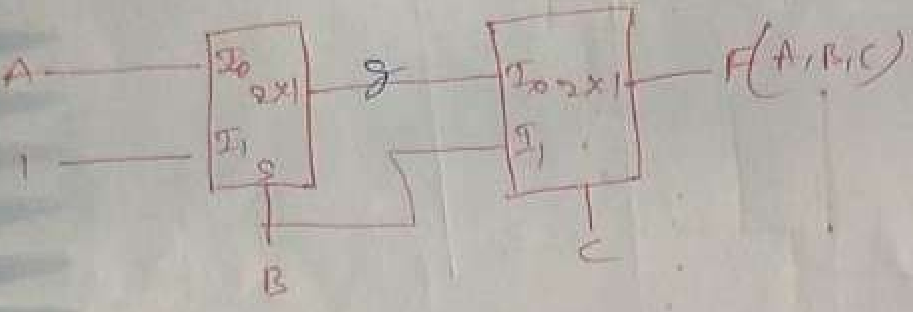
(b) $S_1 = C$, $S_0 = B$

	S_1 (C)	S_0 (B)		
I_0	0	0	$\bar{C}\bar{B}$	$I_0 = 0$
I_1	0	1	$\bar{C}B$	$I_1 = 1$
I_2	1	0	$C\bar{B}$	$I_2 = A$
I_3	1	1	CB	$I_3 = 1$

* $\rightarrow$ If we ~~select~~ change selectors then may change I/P values.

(18) cascading Multiplexers Ex1:- (L→R)

What is the function in canonical SOP? (18)



$g = \bar{B}A + B.1 = \bar{B}A + B$

$F = \bar{C}g + CI_1$

$= \bar{C}(\bar{B}A + B) + CB$

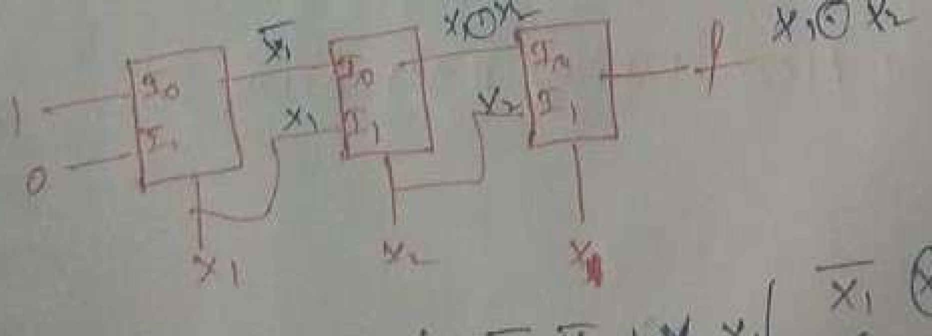
$= A\bar{B}\bar{C} + B\bar{C} + BC$

$= \underset{1\,0\,0}{A\bar{B}\bar{C}} + \overset{1\,1\,0}{AB\bar{C}} + \overset{1\,1\,1}{ABC}$
$\qquad \underset{0\,1\,0}{\bar{A}B\bar{C}} \qquad \underset{0\,1\,1}{\bar{A}BC}$

$= \Sigma(4, 6, 2, 7, 3)$

$= \Sigma(2, 3, 4, 6, 7) = \pi(0, 1, 5)$

(19) cascading Multiplexers Ex2:- (L→R)



$= \bar{x}_1 \cdot 1 + x_1 \cdot 0$

$= \bar{x}_1$

$\bar{x}_2\bar{x}_1 + x_2x_1$

$= x_1 \odot x_2$

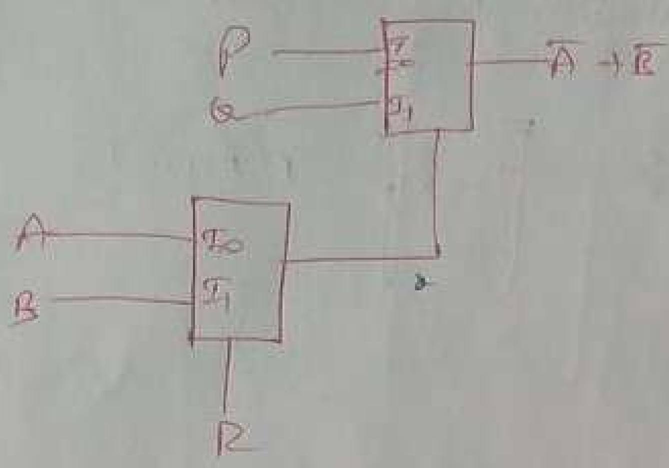
$\bar{x}_1(x_1 \odot x_2) + x_1x_2$

$\bar{x}_1(\bar{x}_1\bar{x}_2 + x_1x_2) + x_1x_2$

$\bar{x}_1\bar{x}_2 + x_1x_2 = x_1 \odot x_2$

(20) Cascading MUX ex3 :-

(19)



P, Q, R ?

$x = \bar{R}A + RB$

$\bar{x}P + xQ$ $= (\bar{R}A + RB)'P + (\bar{R}A + RB)Q$

~~$(\bar{R}A)P + \bar{R}A + RB)$~~

$= (R+\bar{A})(\bar{R}+\bar{B})P + (\bar{R}A + RB)Q$

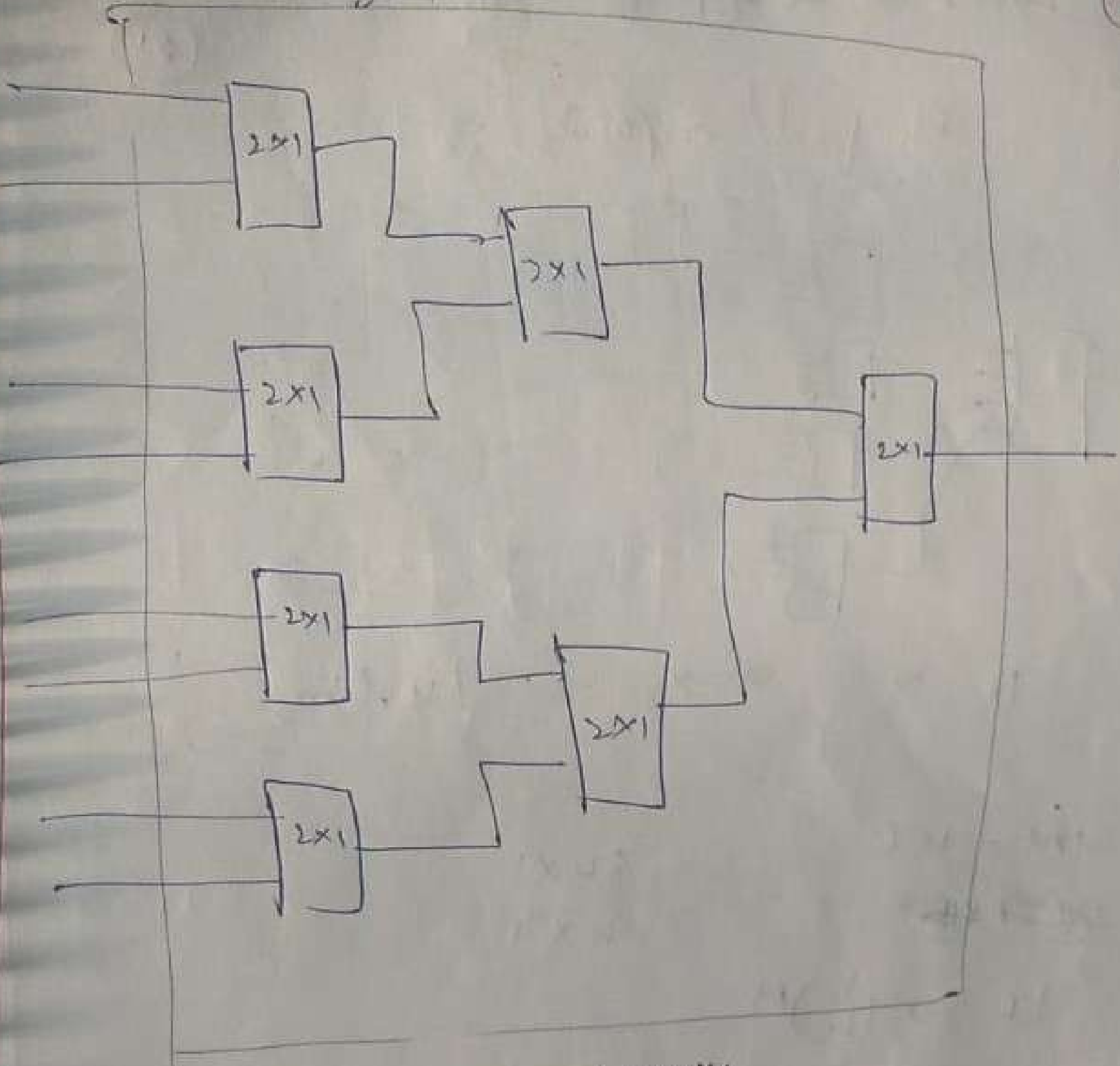
$= R\bar{B}P + \bar{A}\bar{R}P + \bar{A}\bar{B}P + A\bar{R}Q + BQR$

T.O. don't solve it like this go through option

(21) Expansion of Multiplexers :-

Design 8x1 MUX using 2x1 MUX.

8X1 MUX



3 levels & 7, 2X1 MUXs.

MUX 2X1 M covers 2 Lines 8X1 4X1

$\Rightarrow$ 1 M $\rightarrow$ 2L

1L $\rightarrow \frac{1}{2}$ M

8 L $-$ $\frac{8}{2}$ M = 4M

4L $-$ $\frac{4}{2}$ M $-$ 2M

2L $-$ $\frac{2}{2}$ M $-$ 1M

No of MUXs = 4+2+1 = 7

M = Multiplexer

MUX $\frac{32\times1}{4\times1}$

1M $\rightarrow$ 4L

1L $\rightarrow \frac{1}{4}$ M

32L $-$ $\frac{32}{4}$ M $-$ 8M

8L $-$ $\frac{8}{4}$ M $-$ 2M

2L $-$ $\left(\frac{2}{4}\right)$ M $-$ $\frac{1}{2}$ M $\simeq$ 1M

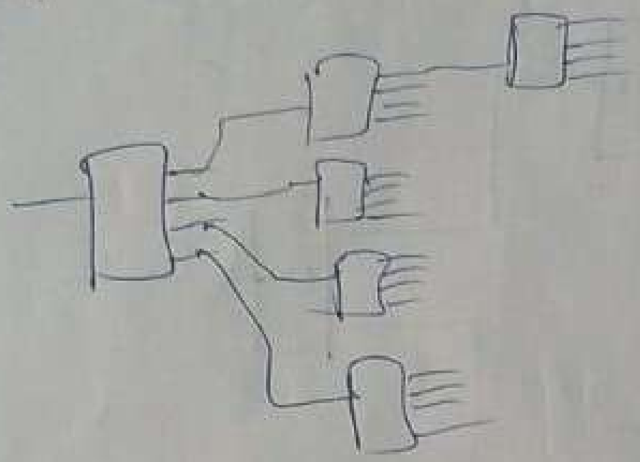
No. of MUXs = 8+2+1
= 11

→ The Last MUX uses it full capacity.

(21)

32×1

4×1 with full capacity



4 × 4 × 4 = 64 lines

1M — 4L

64×1

4×1

$1L \rightarrow \left(\frac{1}{4}\right)M$

$64L — \frac{64}{4}M — 16M$

$16L — \frac{16}{4}M — 4M$

$4L — \frac{4}{4}M — 1M$

∴ No. of MUX = 16+4+1 = 21

Generalise

How many levels?

How many MUX-s?

Implement M×1 MUX using N×1 MUX. (22)

N×1 MUX covers N Lines

$\Rightarrow$ N Lines — 1 MUX | NL — 1M

1L — $\left(\frac{1}{N}\right)$ MUX | 1L — $\left(\frac{1}{N}\right)$M

$\Rightarrow$ ML — $\left(\frac{M}{N}\right)$ MUX

Lev1 $\frac{M}{N}$ L — $\left(\frac{M}{N} \times \frac{1}{N}\right)$ MUX

Lev2 $\left(\frac{M}{N} \times \frac{1}{N}\right)$ L — $\left(\frac{M}{N} \times \frac{1}{N} \times \frac{1}{N}\right)$ MUX $= \left(\frac{M}{N^2}\right)$ MUX

Lev3 $\left(\frac{M}{N^2}\right)$ L — $\left(\frac{M}{N^3}\right)$ MUX

$\vdots$

$\frac{M}{N^k}$

when $\frac{M}{N^k} \le 1$ then stopped it.

$\Rightarrow N^k \ge M$

$k \log N \ge M \Rightarrow \boxed{k \ge \log_N^M} \Rightarrow k = \lceil \log_N^M \rceil$

(23)

No. of MUXs at level 'k' = $\frac{M}{N^k}$

Total Devices (MUXs) = $\boxed{\sum_{K=1}^{\lceil \log_N M \rceil} \left(\frac{M}{N^K}\right)}$

① 8X1 MUX using 2X1 MUX

M — 8X1, N — 2X1

No. of levels = $\lceil \log_N M \rceil = \lceil \log_2 8 \rceil = 3$

No. of Devices = $\sum_{K=1}^{3} \frac{M}{N^K}$

$= \frac{8}{2^1} + \frac{8}{2^2} + \frac{8}{2^3} = 4+2+1 = 7$

② 32X1 MUX using 4X1 MUX

No. of levels = $\lceil \log_4 32 \rceil = \lceil \frac{5}{2} \rceil = 3$

No. of Devices = $\frac{32}{4^1} + \frac{32}{4^2} + \frac{32}{4^3}$

$= 8 + 2 + \frac{1}{2} = 11$

③ 64X1 using 4X1

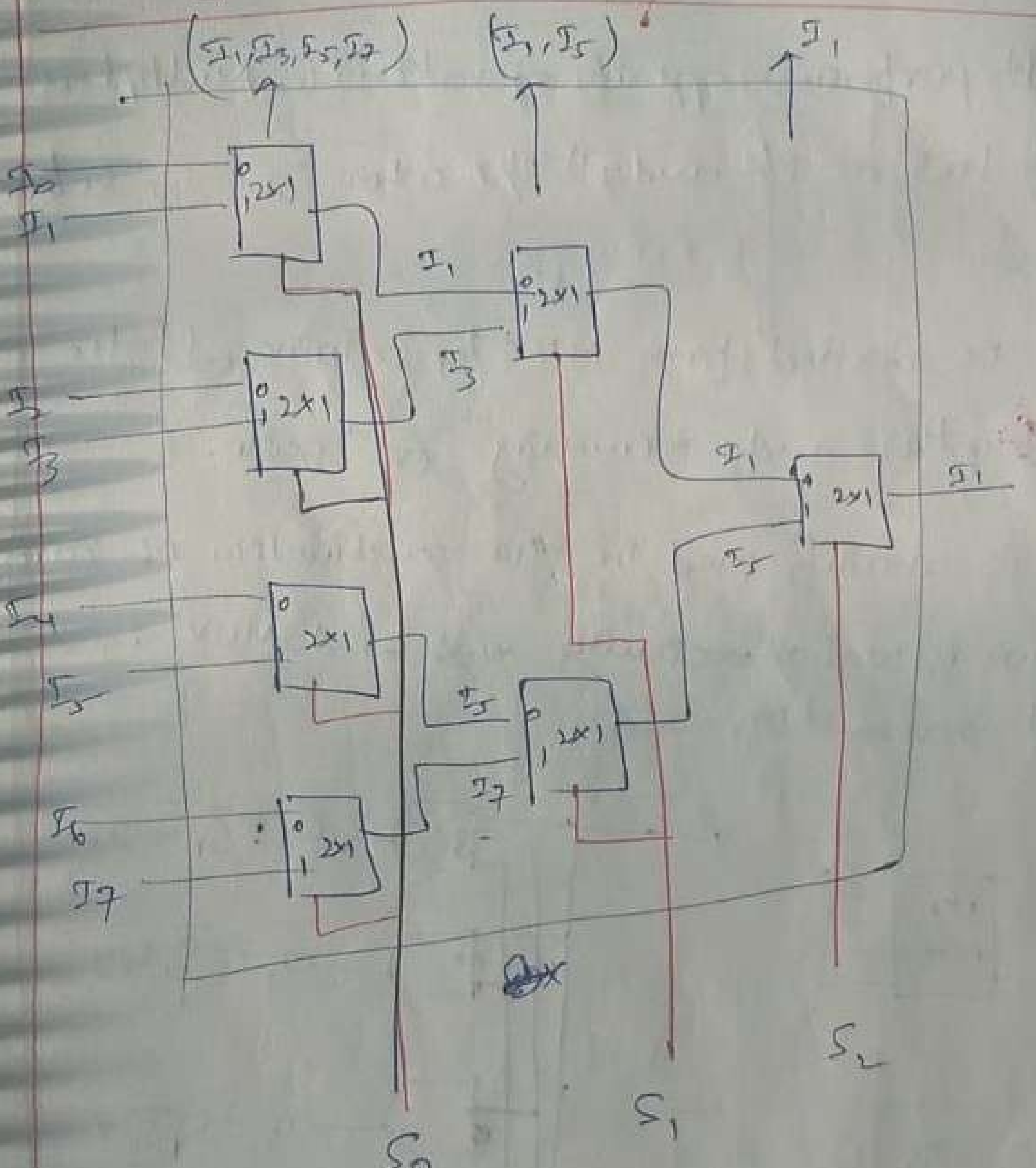
No. of levels = $\lceil \log_4 64 \rceil = \lceil \frac{6}{2} \rceil = 3$

No. of MUXs = $\frac{64}{4^1} + \frac{64}{4^2} + \frac{64}{4^3} = 16+4+1$

$= 21$

(24) Assigning select lines while expanding the MUX :-

(24)



For example :- S_2 S_1 S_0
0 0 1 → I_1 selected

S_2 S_1 S_0 → I_6 selected
1 1 0

S_0 I_2 I_4 I_6

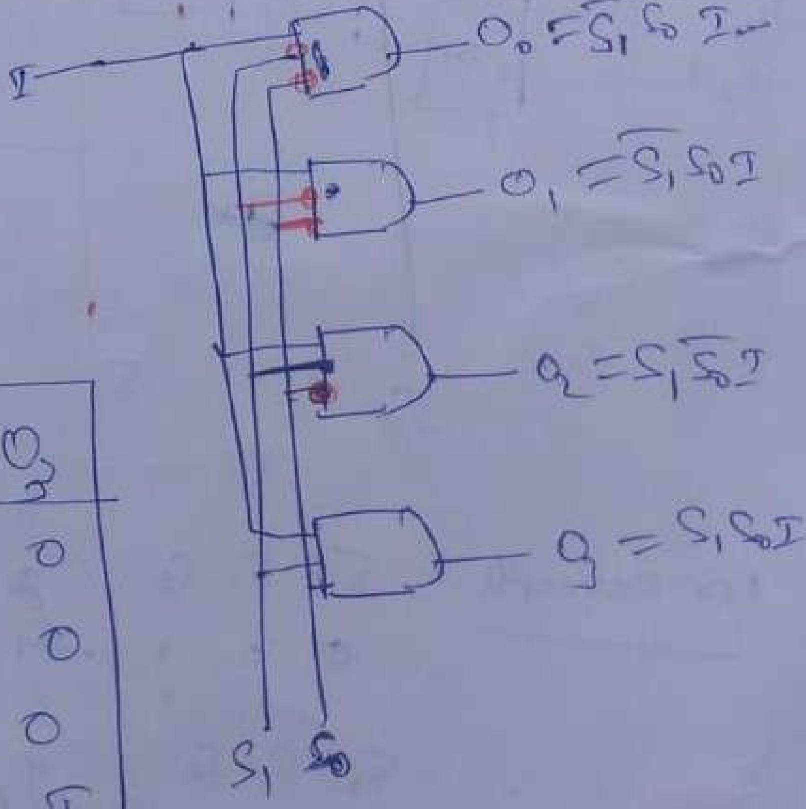
S_0 – 0

S_1 – 1 → I_2, I_6

S_2 – 1 I_6

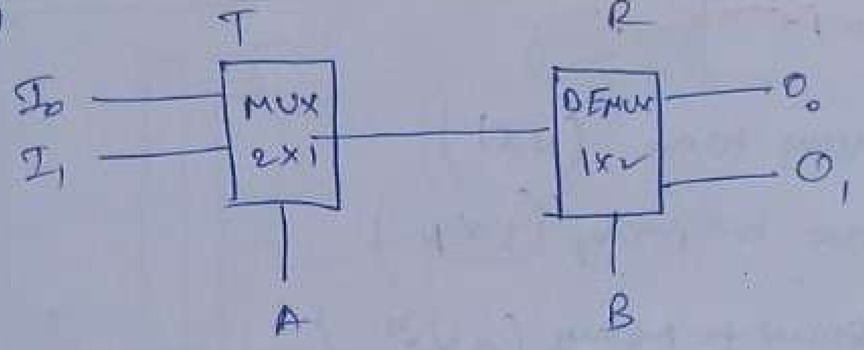
Introduction to DeMultiplexer :-

1. It performs opposite operation of Multiplexer.
2. It has one i/p and 2^n o/ps where 'n' is select lines.
3. It is derived from MUX by joining all the i/ps together and removing 'OR' gate.
4. It is mainly used in the construction of switches.
5. DeMUX is used at receiving end and MUX is used at transmitting end.



S_1	S_0	O_0	O_1	O_2	O_3
0	0	I	0	0	0
0	1	0	I	0	0
1	0	0	0	I	0
1	1	0	0	0	I

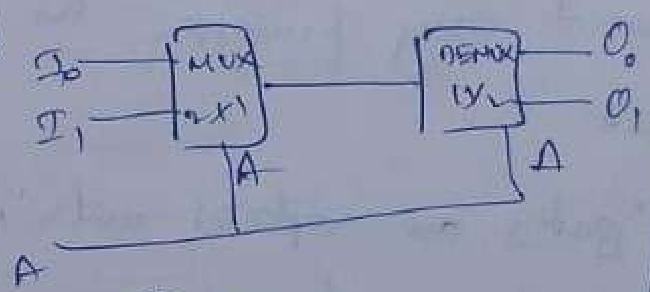
①



A	B	(S.C)
0	0	$(I_0 \to O_0)$
0	1	$(I_0 \to O_1)$
1	0	$(I_1 \to O_0)$
1	1	$(I_1 \to O_1)$

cross connection (C.C)

②

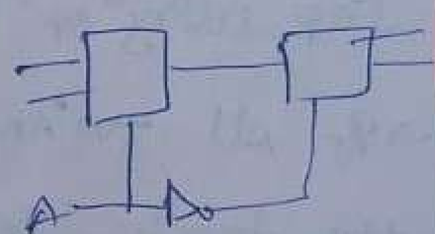


$I_0 - O_0$

$I_1 - O_1$

Straight connection

③

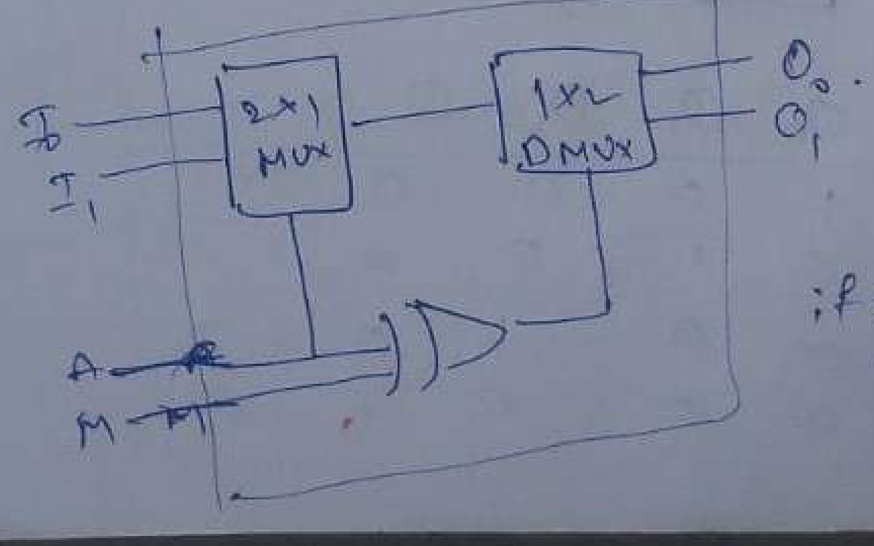


$I_0 - O_1$

$I_1 - O_0$

Cross connection

④ SC & CC



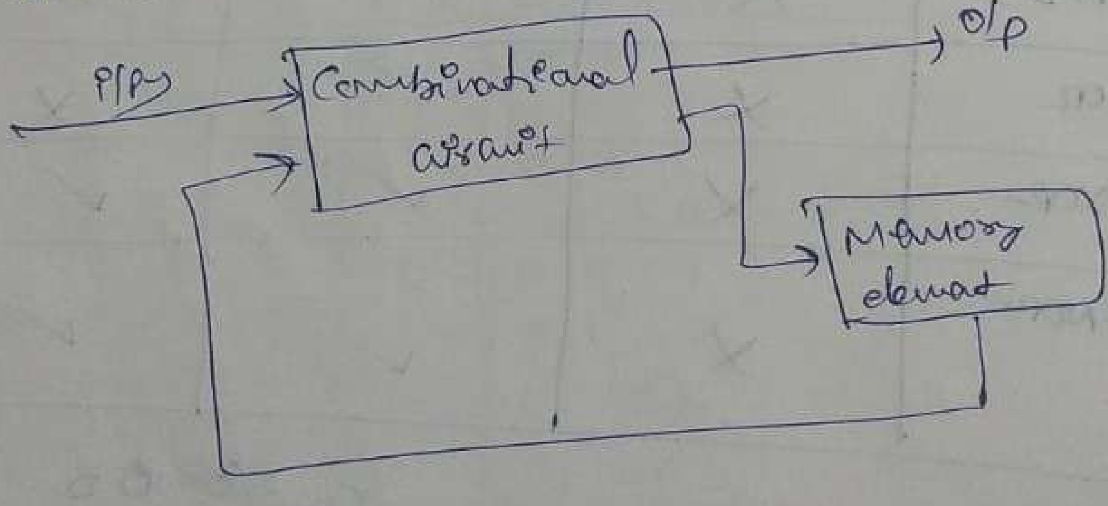
if M=0 – SC
M=1 – CC

(4) Sequential Circuits (51) (1)

(1) Introduction:-

(1) The external o/p of a sequential circuit depends on external i/ps and on present contents of "memory elements".

(2) The present contents of the memory elements are called present state and the new contents of memory elements are obtained by taking external i/ps and present state. This is called next state



(2) Latch and Flip-Flop:-

1 bit memory cell:

(i) A memory element is some medium in which one bit of information can be stored (or) retained until necessary and thereafter its contents can be replaced by new values.

(4) The basic binary (or) digital memory circuit (2) is known as Flip-Flop.

(5) It has 2 stable states which are known as '1' & '0'. so it is also called Bistable Multi vibrator.

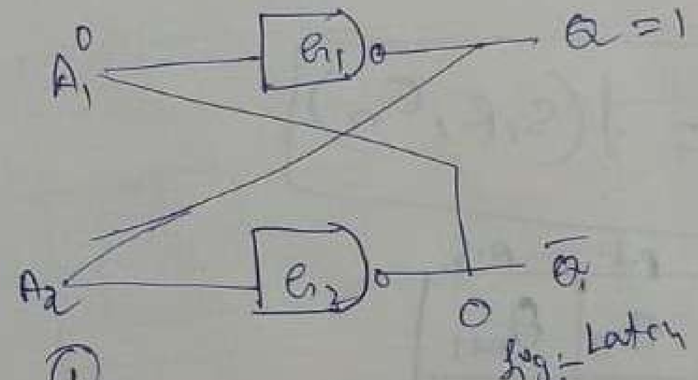


fig:- Latch

-> Latch means Lock.

if Q=1 ~~then~~ $A_2 = 1 \Rightarrow \bar{Q} = 0 \Rightarrow A_0 = 1 \Rightarrow Q = 1$

until the i/p changed the o/p doesnot change

-> using latch we build a FF.

(3) SR FF:-

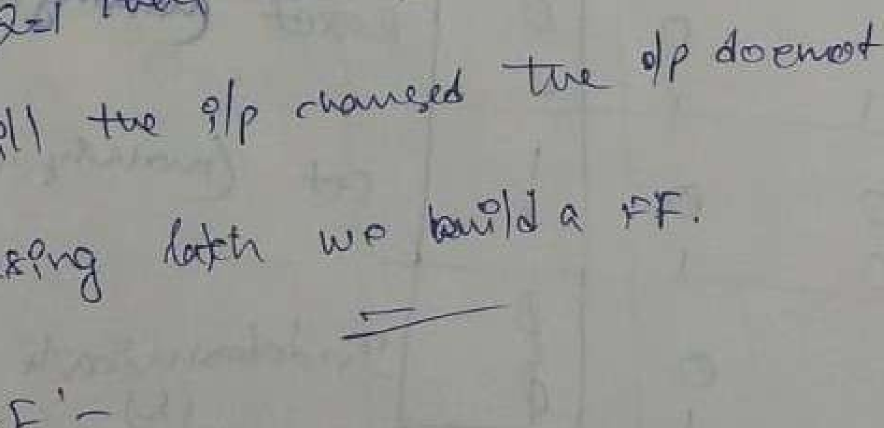


fig:- SR FF

$Q_n \longrightarrow Q_{n+1}$

Present State — Next State

Actually these are same state. whatever

Q

$\cancel{0}$, 1 — PS changed to — $0 \to 1$

$$\boxed{Q_{n+1} = f(S, R, Q_n)}$$

I/P's		P.S.	N.S.	
S	R	Q_n	Q_{n+1}	
0	0	0	0	No change (or) Latch $Q_{n+1} = Q_n$
0	0	1	1	
0	1	0	0	Reset (making it zero)
0	1	1	0	
1	0	0	1	Set (making it 1)
1	0	1	1	
1	1	0	ϕ	Indeterminate state (or) Invalid state.
1	1	1	ϕ	

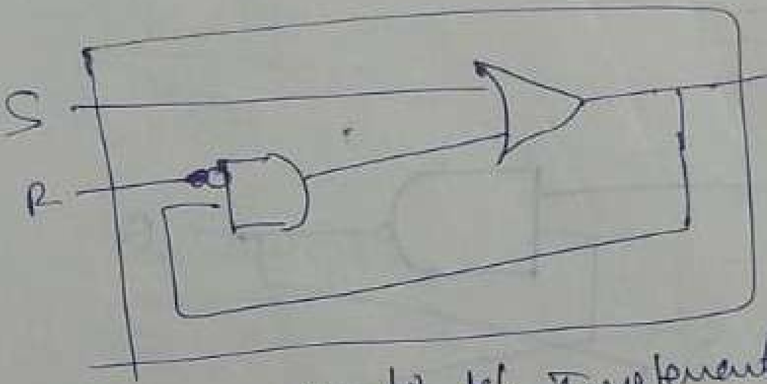
fig: characteristic table

Q_n \ SR	00	01	11	10
0				1
1	1			1

Q_n \ SR	00	01	11	10
0			ϕ	1
1	1		ϕ	1

$S\bar{R} + \cancel{Q_n \bar{R}}$

$\Rightarrow \boxed{Q_{n+1} = S + Q_n \bar{R}}$ → characteristic equation



Realisation (Implementation)

Excitation table:-

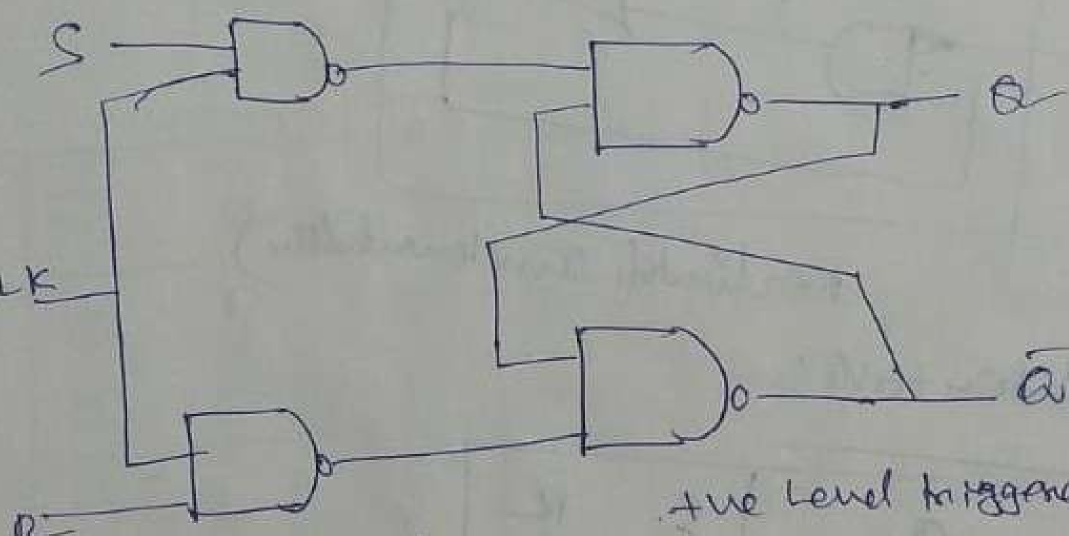
Q_n	Q_{n+1}	S	R
0	0	0	ϕ
0	1	1	0
1	0	0	1
1	1	ϕ	0

Function table :-

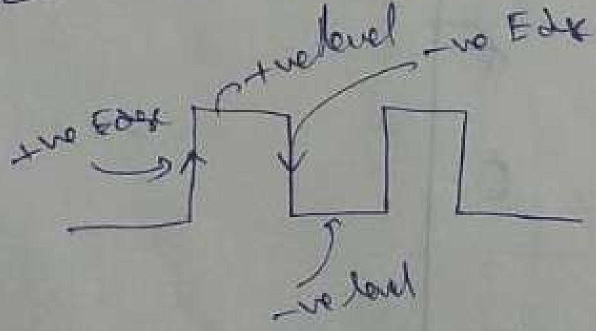
S	R	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	ϕ

i) FF diagram
ii) characteristic table
iii) characteristic equation
iv) Realization
v) Excitation table
vi) function table

(2) clocked FFs :-



+ve Level triggered FF.



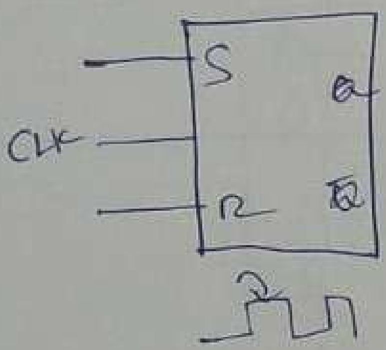
Transparent Mode
i/p effects the o/p

Latch Mode

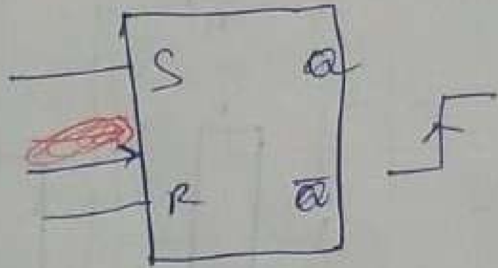
-> The FF which triggered at +ve level is called as +ve level FF.

-> Most popular FFs are Edge triggered FFs.

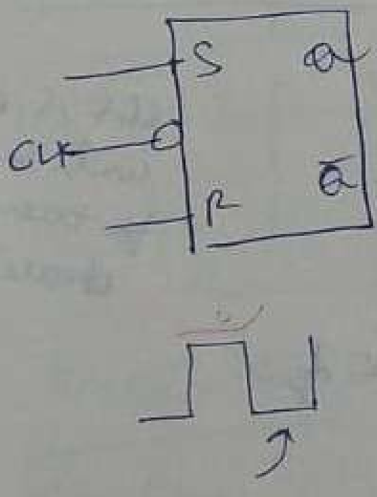
+ve Level



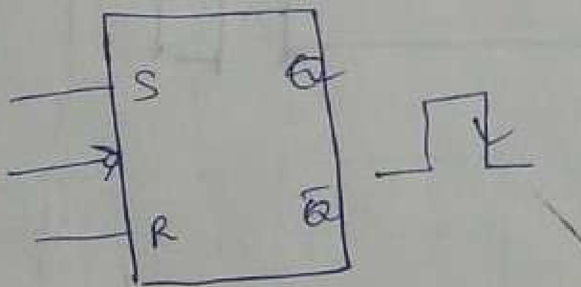
+ve Edge



-ve Level



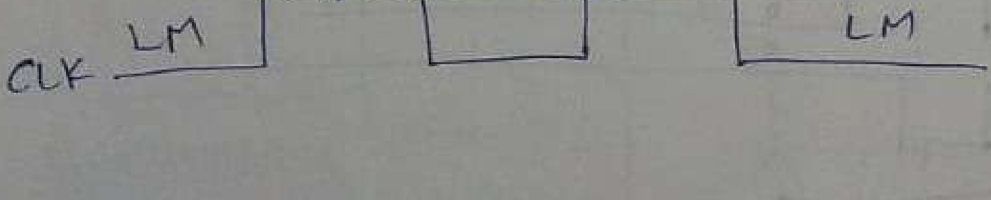
-ve Edge

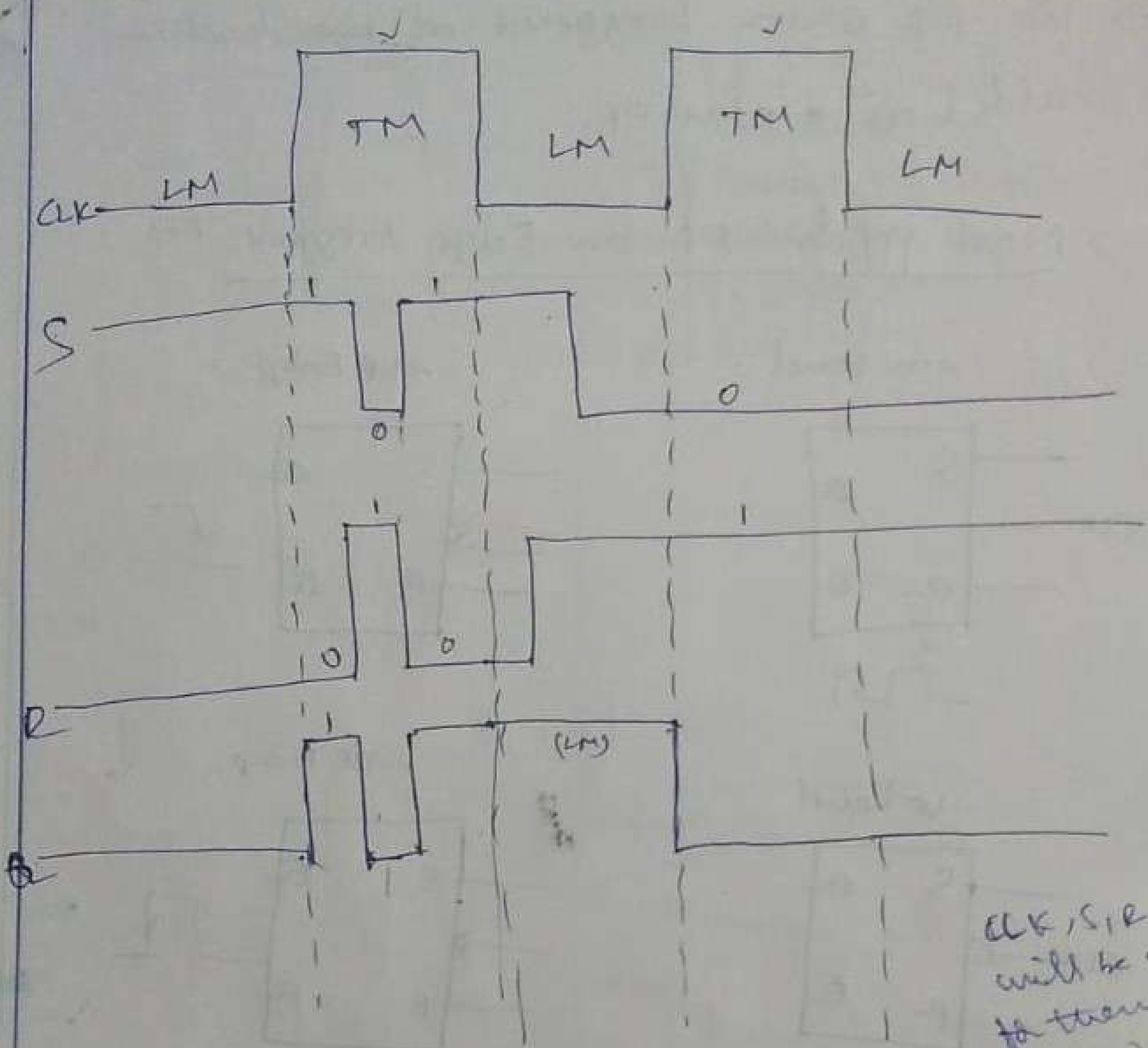


Transparent Mode
Latch Mode

⑤ Positive Level Triggered :-

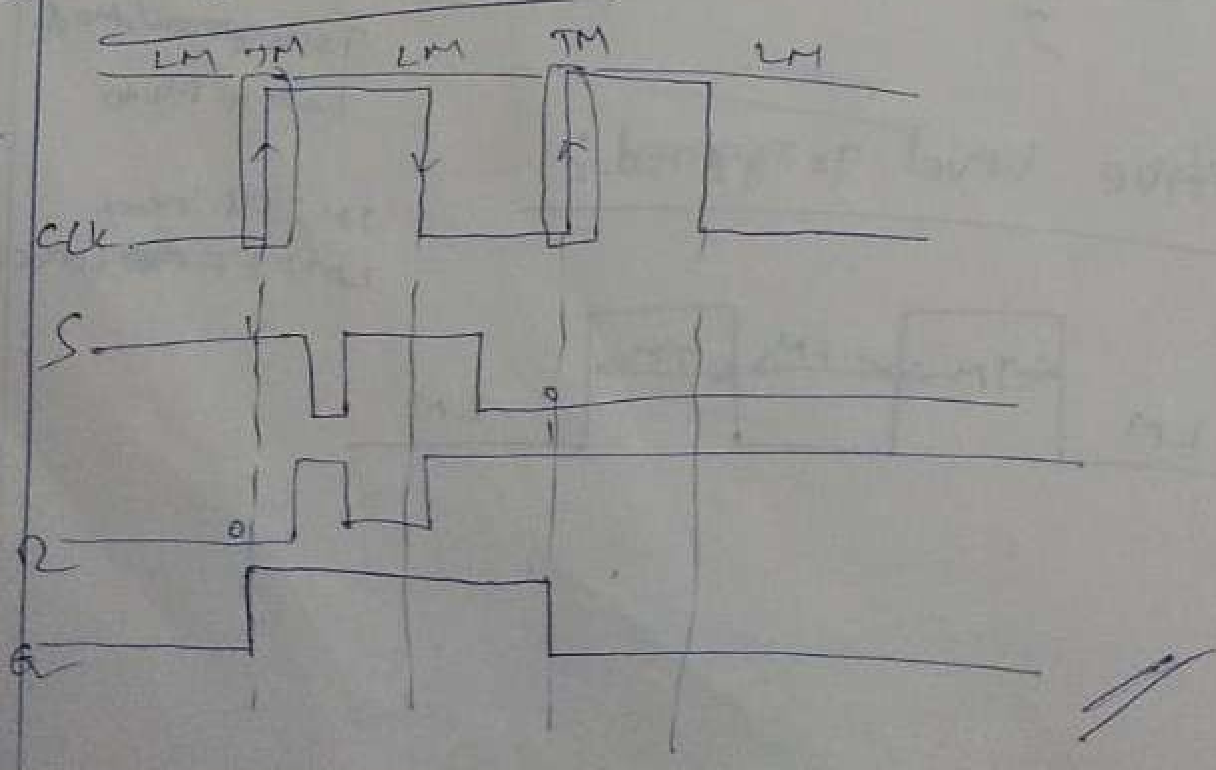
TM : FF react
LM : FF Locked.





CLK, S, R will be given for them we draw Q

(6) Edge Triggered FF:- +ve Edge



7 JK Flip Flop:-

- JK FF is same as SR FF (J=S, K=R). It is defined for J=K=1 also and the FF performs complementation.

Function table:-

J	K	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	$\bar{Q}_n$

characteristic table:

J	K	Q_n	Q_{n+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

characteristic Equation:-

Q_n \ JK	00	01	11	10
0			1	1
1	1			1

$$\boxed{Q_{n+1} = \bar{Q}_n J + Q_n \bar{K}}$$

$$\Rightarrow Q_{n+1} = J\bar{Q}_n + \bar{K} Q_n$$

Excitation table:-

Q_n	Q_{n+1}	J	K
0	0	0	ϕ
0	1	1	ϕ
1	0	ϕ	1
1	1	ϕ	0

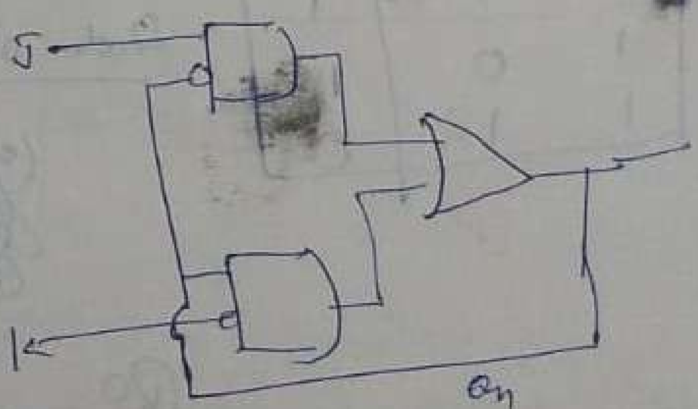
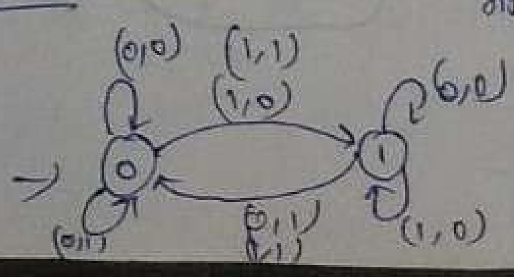


fig: Implementation

State Diagram:- (from CT)



T-FF :- Toggle

S

T=1 (J=K=1)
T=0 (J=K=0)

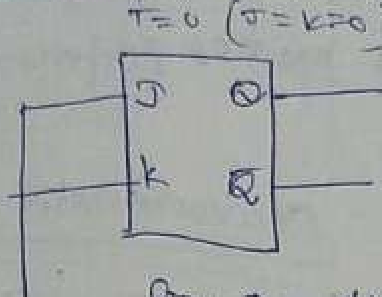


fig: Graphic Symbol

Function table

T	Q_{n+1}
0	Q_n
1	$\bar{Q}_n$

Latching
complementer

So called as Toggle FF

Characteristic table :-

T	Q_n	Q_{n+1}
0	0	0
0	1	1
1	0	1
1	1	0

Characteristic Equation

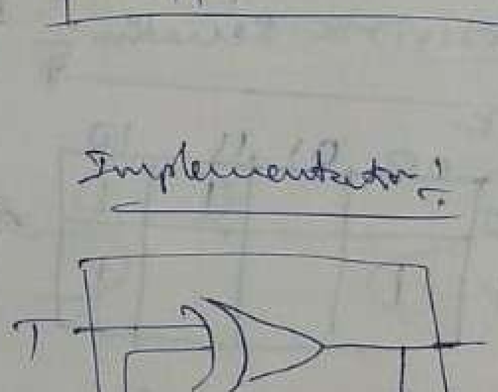
$$Q_{n+1} = \bar{T}\,Q_n + T\cdot\bar{Q}_n$$

$$\boxed{Q_{n+1} = T \oplus Q_n}$$

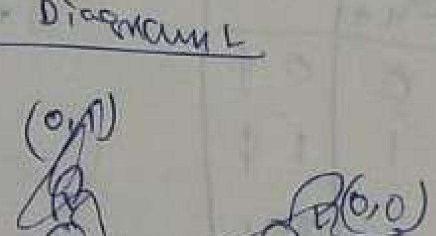
Excitation table :-

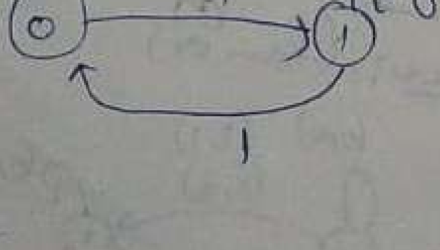
Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Implementation :-

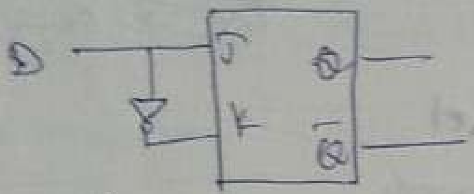


State Diagram





(9) D-FF :-



$D=0$ ($J=0$; $K=1$)

$D=1$ ($J=1$; $K=0$)

functional table :-

D	Q_{n+1}
0	0
1	1

characteristic table :-

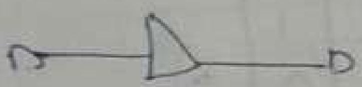
D	Q_n	Q_{n+1}
0	0	0
0	1	0
1	0	1
1	1	1

characteristic equation :

$$Q_{n+1} = D\bar{Q}_n + DQ_n$$

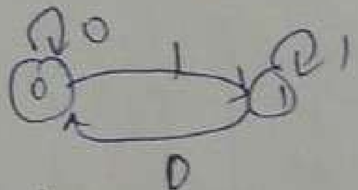
$$= D(\bar{Q}_n + Q_n) = D$$

$$\therefore \boxed{Q_{n+1} = D}$$



$\Rightarrow$ called as Buffer (delay).

State Diagram :- (or)



Excitation table :-

Q_n	Q_{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

(10) Example on Flip Flop :-

If the characteristic equation of a FF is

$Q_n = \bar{x}_1 \bar{Q} + \bar{x}_2 Q$. Define the behaviour of that.

11

Sol:- function table:-

x_1	x_2	Q_n	
0	0	1	set
0	1	$\bar{Q}$	Toggle
1	0	Q	Latch
1	1	0	Reset

$Q_n = \bar{x}_1 \bar{Q} + \bar{x}_2 Q$

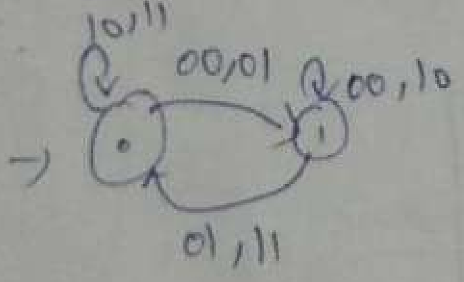
Characteristic table:-

x_1	x_2	Q	Q_n
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Excitation table:-

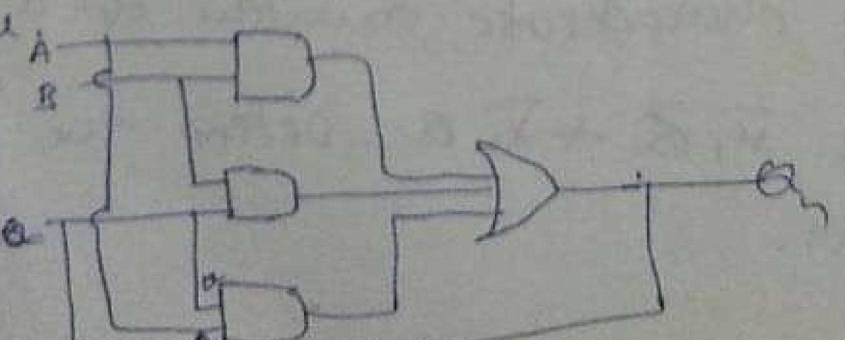
Q	Q_n	x_1	x_2
0	0	1	ϕ
0	1	0	ϕ
1	0	ϕ	1
1	1	ϕ	0

State Diagram:



(11) Example:

consider the following realization. construct excitation table



Sol $Q_n = AB + BQ + AQ$

$$\boxed{Q_n = AB + (B+A)Q}$$

~~Charact~~ Function table:-

A	B	Q_n	
0	0	0	Reset
0	1	Q	Latching
1	0	Q	Latching
1	1	1	Set

Characteristic table:-

A	B	Q	Q_n
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

~~State Diagram:-~~

Excitation table:-

Q	Q_n	A	B
0	0	0	ϕ
0	1	1	1
1	0	0	0
1	1	1	ϕ
0	0	1	0
1	1	0	1

Excitation table:-

Q	Q_n	A	B
0	0	[illegible]	[illegible]
0	1	1	1
1	0	0	0
1	1	1	ϕ

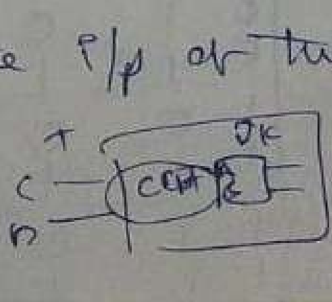
(0) → 0φ, 0, 10

00 → 00/01, 10 0φ ⇒ 00 ⇄ 10, 0φ

11 → 01, 10, 11 → 1φ ⇒ 11 ⇄ 01, 10

(12) Introduction to FF Inter Conversion:-

1. Get the characteristic table of the target FF.
2. Replace the next state using excitation of the given FF.
3. Obtain the expressions for the i/p of the given FF and realize them.



JK → T FF

T	Q_n	Q_{n+1}	J	K
0	0	0	0	ϕ
0	1	1	ϕ	0
1	0	1	1	ϕ
1	1	0	ϕ	1

for J

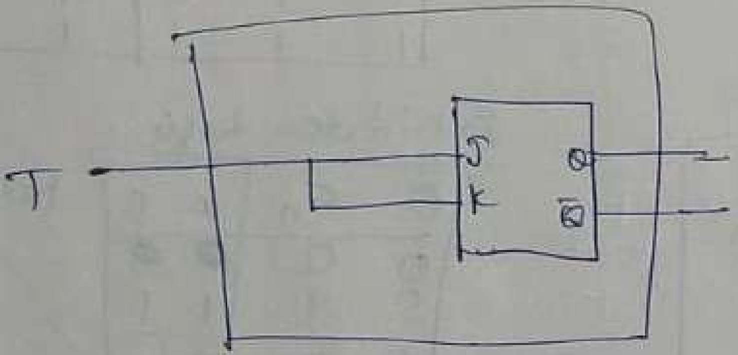
Q_n \ T	0	1
0		1
1	ϕ	ϕ

$J = T$

for K

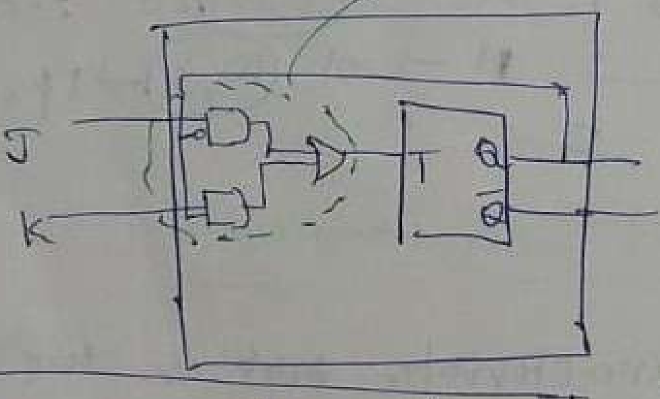
Q_n \ T	0	1
0	ϕ	ϕ
1		1

$K = T$



TFF → JK FF :-

conversion (combinational ckt used)



J	K	Q_n	Q_{n+1}	T
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	1

for T

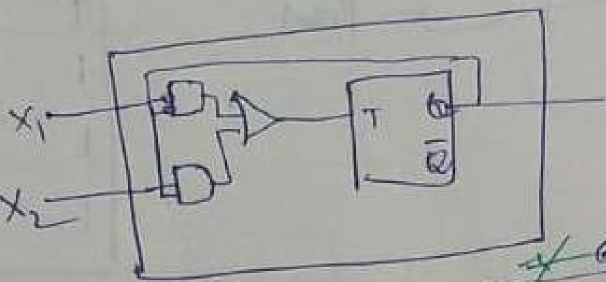
Q_n \ JK	00	01	11	10
0			1	1
1		1	1	

$$T = J\bar{Q}_n + KQ_n$$

(13) Interconversion of FF Ex:- (14)

A new FF $x_1 x_2$ has characteristic exp.

$Q_n = \overline{x_1}\,\overline{Q} + \overline{x_2}\,Q$. realise it using T-FF.



function table:-

x_1	x_2	Q_n	
0	0	1	set
0	1	$\overline{Q}$	toggle
1	0	Q	Latch
1	1	0	Reset

excitation table

x_1	x_2	Q	Q_n	T
0	0	0	1	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

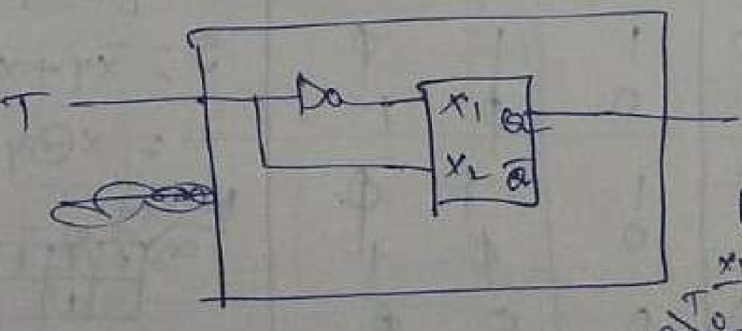
characteristic table.

Q_n \ x_1x_2	00	01	11	10
0	1	1		
1		1	1	

$$\boxed{T = \overline{x_1}\,\overline{Q} + \overline{x_2}\,Q}$$

(14) $x_1 x_2 \rightarrow T$ FF Ex

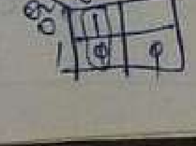
$Q_n = \overline{x_1}\,\overline{Q} + \overline{x_2}\,Q$.



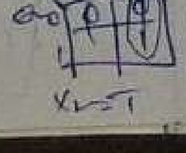
conv. table

T	Q	Q_n	x_1	x_2
0	0	0	φ	φ
0	1	1	φ	0
1	0	1	0	φ
1	1	0	φ	1

function table



$x_1 = \overline{T}$



$x_2 = T$

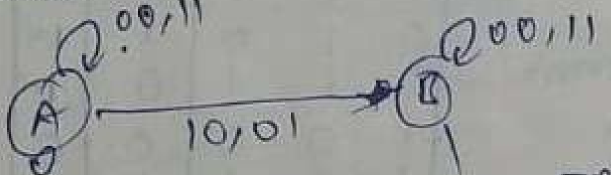
Function table of X_1X_2 :-

X_1	X_2	Q_n	
0	0	1	Set
0	1	$\bar{Q}$	toggle
1	0	Q	Latch
1	1	0	Reset

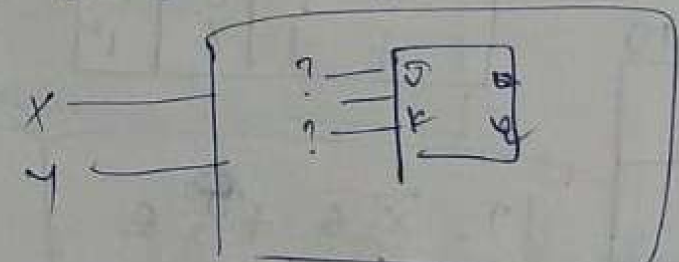
characteristic table of X_1X_2 :-

X_1	X_2	Q	Q_n
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

(15) An X,Y FF is represented by the following state diagram.



To realize it using JK FF. Find JK?



X	Y	Q_n	Q_{n+1}	J	K
0	0	0	0	0	φ
0	0	1	1	φ	0
0	1	0	1	1	φ
0	1	1	0	φ	1
1	0	0	1	1	φ
1	0	1	0	φ	1
1	1	0	0	0	φ
1	1	1	1	φ	0

for J

Q_n \ XY	00	01	11	10
0		1		1
1	φ	φ	φ	φ

$J = \bar{x}y + x\bar{y}$

$= x \oplus y$

for K

Q_n \ XY	00	01	11	10
0	φ	φ	φ	φ
1		1		1

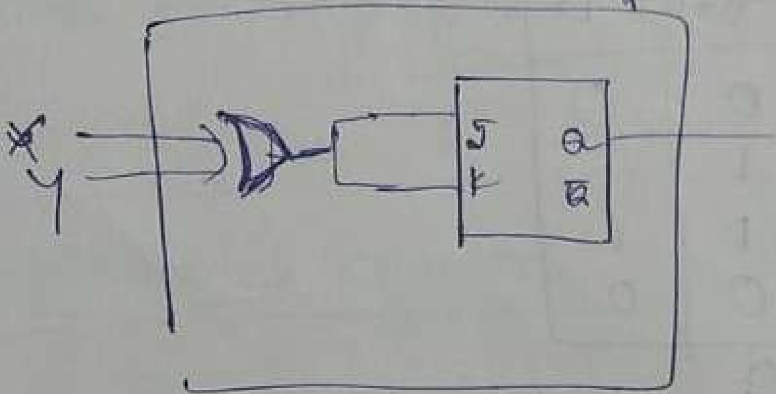
$K = x \oplus y$

XY FF function table :-

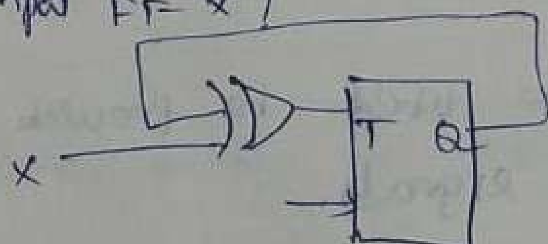
X	Y	Q_{n+1}
0	0	Q_n
0	1	$\overline{Q_n}$
1	0	$\overline{Q_n}$
1	1	Q_n

~~XY FF CT :-~~

~~X Y Qn Qn+1~~



(16) Ex4 What is the behaviour of the following one-input FF 'x'?



a) D FF b) T FF c) Inverted DFF d) Inv. TFF

characteristic table :-

X	Q_n	Q_{n+1}	T
0	0	0	0 — latch
0	1	0	1
1	0	1	1
1	1	1	0 — latch

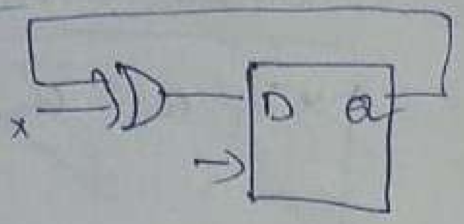
D FF

$T = x \oplus Q_n$

x Q_n
T
Q_{n+1}

calculate process.

(17) Ex5



a) D FF b) T FF c) Inv. D FF d) Inv. T FF

X CT:

$D = x \oplus Q_n$

x	Q_n	Q_{n+1}	D
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

(18) Introduction to counters :-

→ The counters are used to provide accurate timing and control signals.

→ These are of two types:

a) Synchronous b) Asynchronous

→ In Synchronous counters all the FFs respond to the same clock instances.

→ In asynchronous counters, the o/p of one FF drives the clock of another FF.

→ Synchronous counters are faster than Asynchronous counters.

→ Due to simplicity of design, Asynchronous counters are used in IC fabrication. (18)

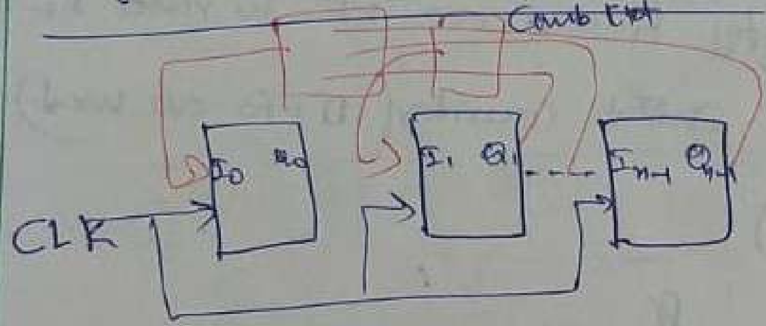
→ The simplified version of Synchronous counter is called Shift counters.

→ The basic element in shift counter is D-FF.

→ The Ring counter and Johnson counter are further simplified version of Shift counter.

(19) Asynchronous and Synchronous counters:-

Synchronous counter:-

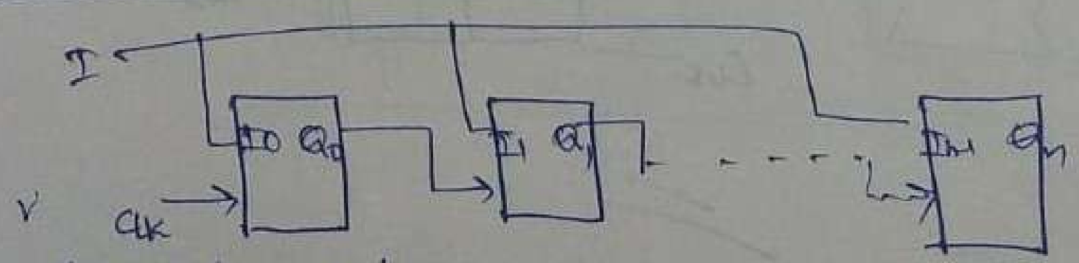


Propagation delay in synchronous counter is,

$$T_{pd\,syn} = T_{FF} + T_{combinational}$$

$$\therefore \boxed{T_{CLK} \geq T_{pd\,syn}}$$

Asynchronous counter:-



Propagation delay in Asynchronous counter is

$$T_{pd\,asyn} = N * T_{FF} + T_{combinational} \Rightarrow \boxed{T_{CLK} \geq T_{pd\,asyn}}$$

20) Shift Counters:-

→ $I_i = f(Q_0, Q_1, \ldots, Q_{N-1})$; $0 \le i \le N-1$

→ It's design is more complex. So simplifications are done as follows:

(i) $I_i = Q_{i-1}$: $1 \le i \le N-1$

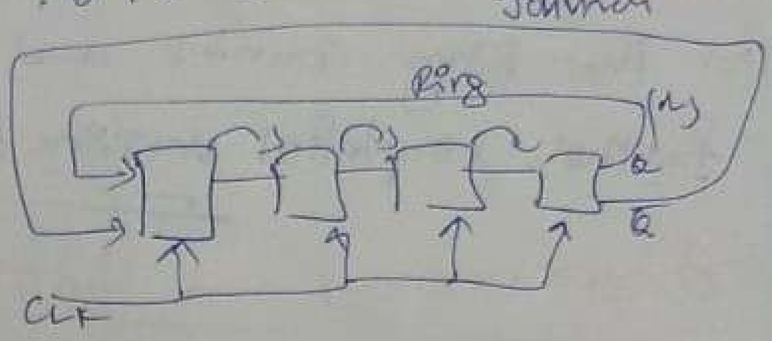
$I_1 = Q_0$

$I_2 = Q_1$

$I_3 = Q_2$

⋮

$I_n = Q_{n-1}$



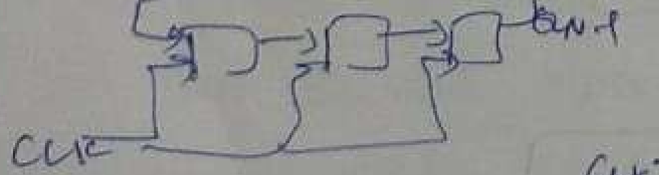
Here, data is shifted from one FF to next FF. So this is called shift counter (D FFs are used).

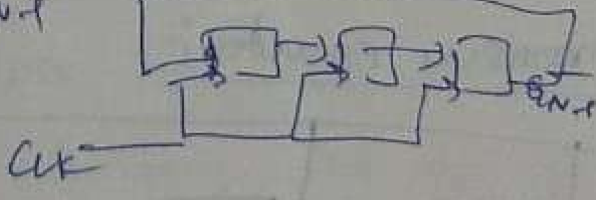
(ii) $I_0 = f(Q_{N-1})$

$I_0 = f(Q_{N-1})$

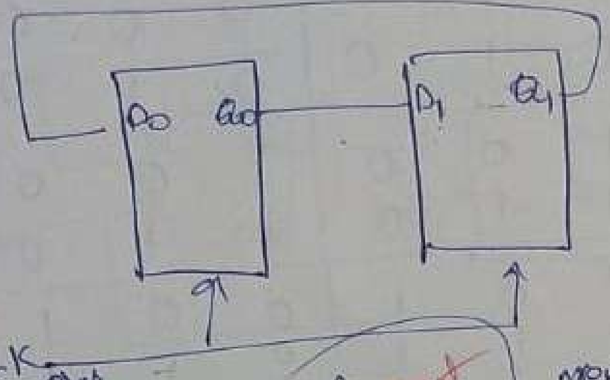
$I_0 = Q_{N-1}$ (Ring counter)

$I_0 = \overline{Q}_{N-1}$ (Johnson counter)





(21) Mod 2 Ring counter :-



P.S → i/p → N.S.

N.S.

P.S → i/p

D_1	D_0	Q_1	Q_0	Q_{1N}	Q_{0N}
0	0	0	0	0	0
1	0	0	1	1	0
0	1	1	0	0	1
1	1	1	1	1	1
(2)			(1)		(3)

Present, Next

$D_0 = Q_1$

$D_1 = Q_0$

00 ; 01 → 10 ; 11

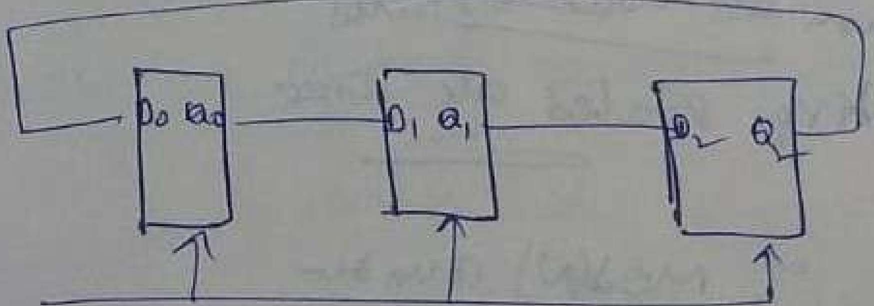
N – "MODN"

n = no. of states

mod2 means divisible by 2

Mod2 N=2

(22) Mod 3 Ring counter :-

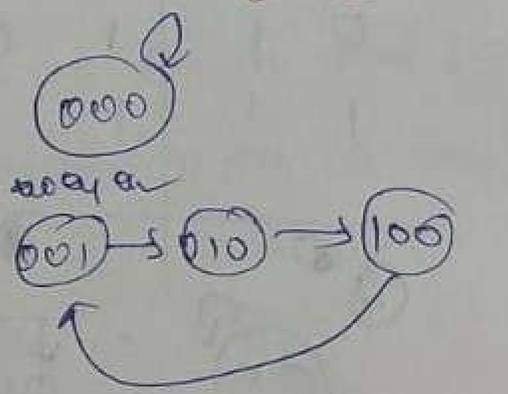


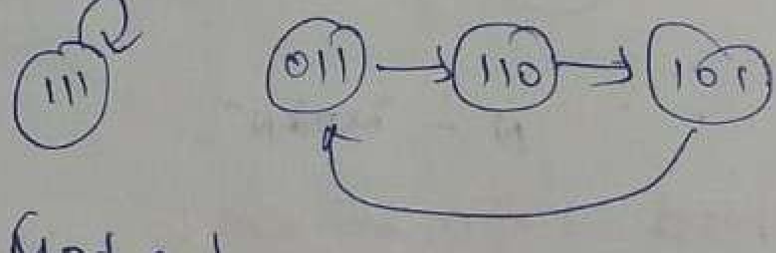
Present State			Next state					
Q_2	Q_1	Q_0	Q_{2N}	Q_{1N}	Q_{0N}	D_2	D_1	D_0
0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	1	0
0	1	0	1	0	0	1	0	0
0	1	1	1	1	0	1	1	0
1	0	0	0	0	1	0	0	1
1	0	1	0	1	1	0	1	1
1	1	0	1	0	1	1	0	1
1	1	1	1	1	1	1	1	1
(1)			(3)			(2)		

Q → D

D → Q

$$\boxed{\begin{aligned} D_0 &= Q_2 \\ D_1 &= Q_0 \\ D_2 &= Q_1 \end{aligned}}$$





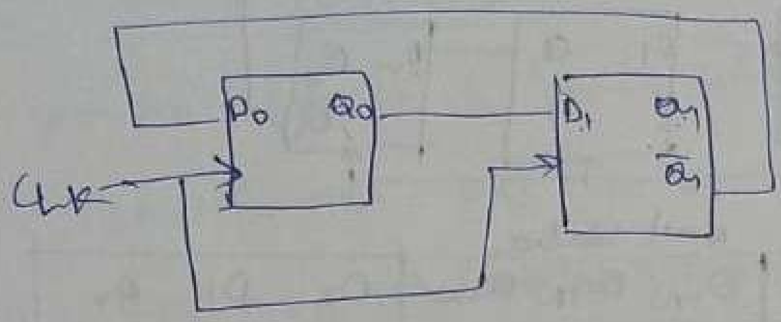
We are getting (Mod n).

Even though we have 8 states

we are counting states are three

∴ Mod(N) counter

(23) Mod 4 Johnson counter :-

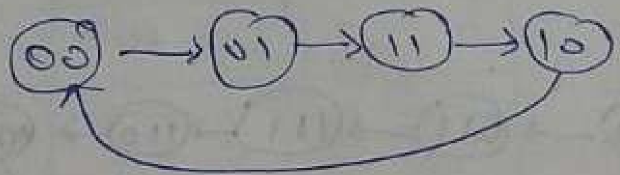


$D_0 = \overline{Q_1}$

$D_1 = Q_0$

P.S.		N.S.		i/p	
Q_1	Q_0	Q_{1N}	Q_{0N}	D_1	D_0
0	0	0	1	0	0
0	1	1	1	1	0
1	0	0	0	0	0
1	1	1	0	1	0

(1) (3) (2)

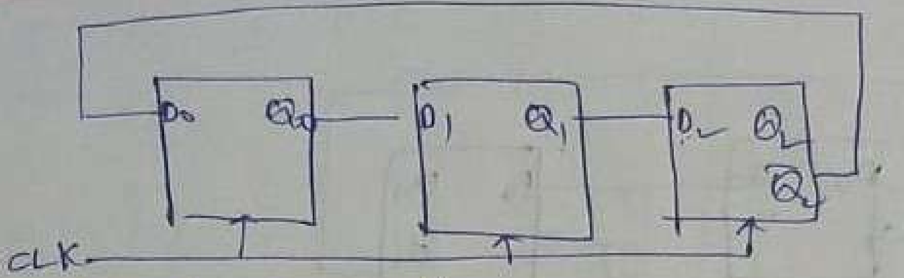


Acts as a Mod 4 counter that means

N FF ⇒ Mod(2N) counter ✓

Mod(2^N) ✗ n FF → 2n states

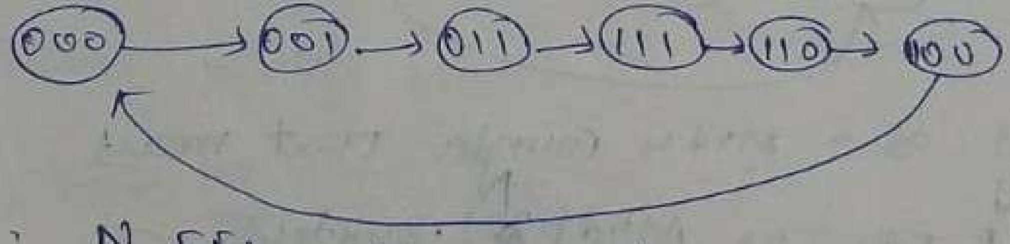
(24) MOD 6 Johnson counter:-



Present State			Next State					
Q_2	Q_1	Q_0	Q_{2N}	Q_{1N}	Q_{0N}	D_2	D_1	D_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	1	1
0	1	0	1	0	1	1	0	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	1	0	0	1	0
1	1	0	1	0	0	1	0	0
1	1	1	1	1	0	1	1	0

(1) (2)

$D_0 = \bar{Q}_2$; $D_1 = Q_0$; $D_2 = Q_1$



∴ N FFs acts as Mod 6 counter

that means Mod(2N) counter.

(25) MOD 4 gray counter using T-FF:-

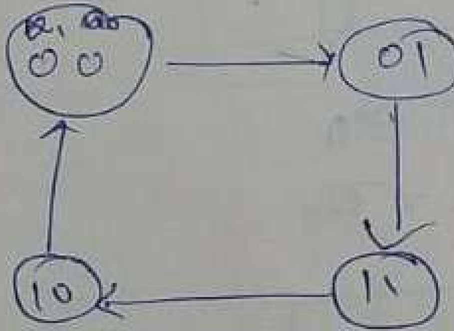
Design a counter:-

(1) get the state table from state diagram.

(2) Identify the FFs to be used and replace the concerned next state using excitation table of associated FF.

(3) Get the expressions for I/Ps and realize them.

Design a synchronous counter for the following using TFFs.



NOTE:- only one bit is changed when move from one state to next state.

P.S. Q_1	Q_0	N.S. Q_{1N}	Q_{0N}	excitation table T_1	T_0
0	0	0	1	0	1
0	1	1	1	1	0
1	1	1	0	0	1
1	0	0	0	1	0

(1) (2) (3)

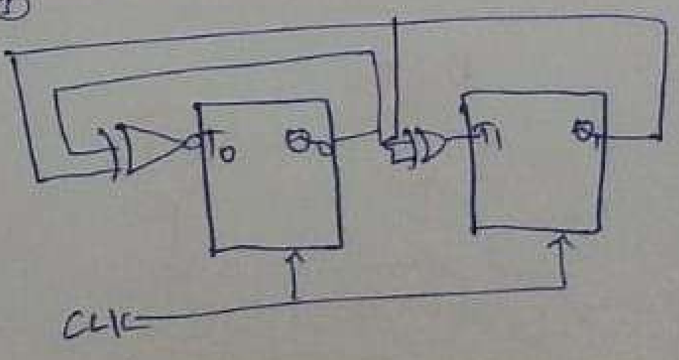
Step 3:

$\Rightarrow T_1 = \bar{Q}_1 Q_0 + Q_1 \bar{Q}_0 = Q_1 \oplus Q_0$

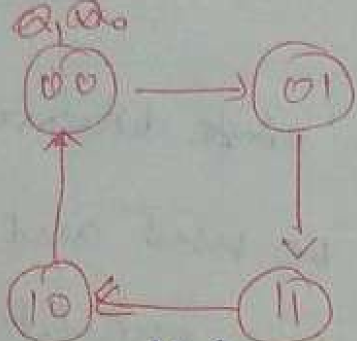
$\Rightarrow T_0 = \bar{Q}_1 \bar{Q}_0 + Q_1 Q_0 = Q_1 \odot Q_0$

Step 3:

$\therefore$ Min. no. of FFs $\lceil \log_2 N \rceil$



(26) MOD 4 Gray counter using D-FFs :-

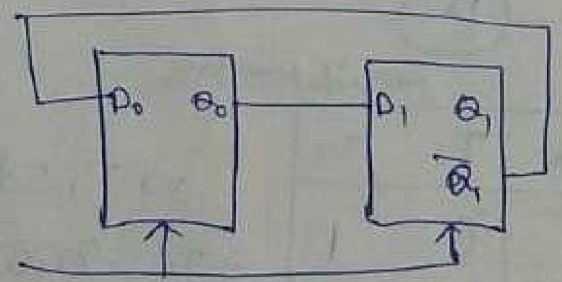


Excitation table of DFF

P.S.		N.S.			
Q_1	Q_0	Q_{1N}	Q_{0N}	D_1	D_0
0	0	0	1	0	1
0	1	1	1	1	1
1	1	1	0	1	0
1	0	0	0	0	0

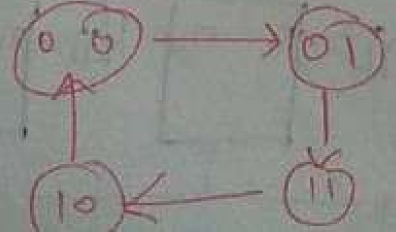
$D_1 = \overline{Q_0}\,\overline{Q_1} + Q_1 Q_0 = Q_0$

$D_0 = \overline{Q_1}\,\overline{Q_0} + \overline{Q_1} Q_0 = \overline{Q_1}$



compare to T-FFs, use D-FF for better performance no need a extra combinational ckt.

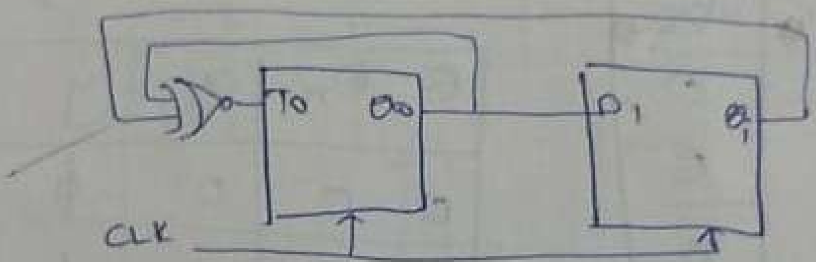
(27) MOD 4 Gray counter using 1D and 1T-FFs :-



Q_1	Q_0	Q_{1N}	Q_{0N}	D_1	T_0
0	0	0	1	0	1
0	1	1	1	1	0
1	1	1	0	1	1
1	0	0	0	0	0

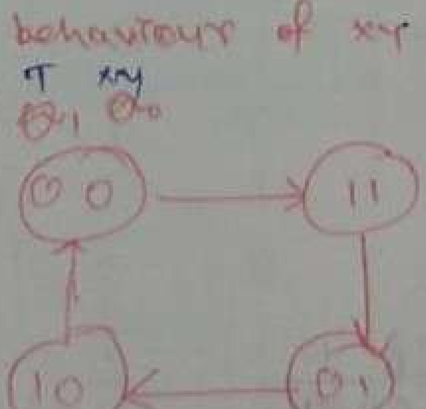
$D_1 = Q_0$

$T_0 = \bar{Q}_1\bar{Q}_0 + Q_1Q_0 = Q_1 \odot Q_0$



(28) Counter using two different FFs:-

consider the following state diagram which is to be designed using T FF for MSB and xy FF for LSB the behaviour of xy is given below:



F.T.T

x	y	Q_n
0	0	0
0	1	Q
1	0	$\bar{Q}$
1	1	1

MSB - - LSB

↑ ↑

T xy

P.S.		N.S.				
Q_1	Q_0	Q_{1N}	Q_{0N}	T	~~X~~ ϕ	Y
0	0	1	1	1	1	ϕ
1	1	0	1	1	ϕ	1
0	1	1	0	1	ϕ	0
1	0	0	0	1	0	ϕ

FT function table → CT characteristic table → ET excitation table

x	y	Q_n	Q_{n+1}
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

C.T.

Q	Q_n	x	y
0	0	0	ϕ
0	1	1	ϕ
1	0	ϕ	0
1	1	ϕ	1

for x

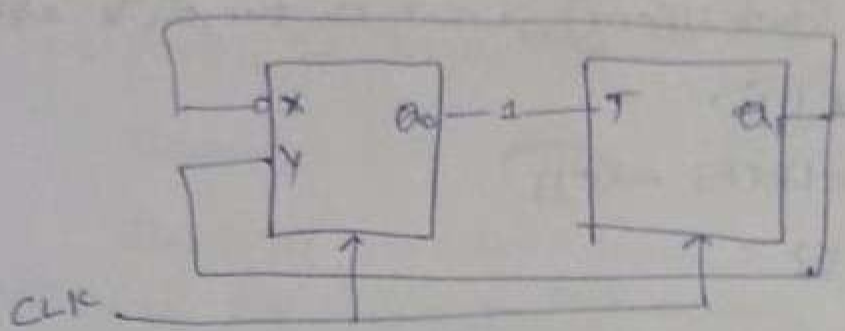
$Q_0 \backslash Q_1$	0	1
0	1	
1	ϕ	ϕ

$x = \bar{Q}_1$

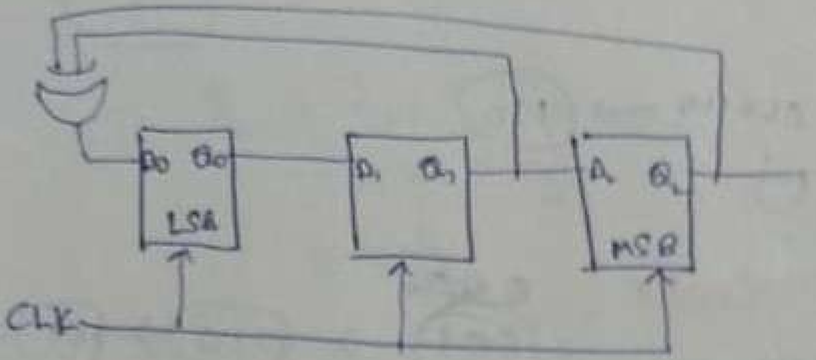
for y

$Q_0 \backslash Q_1$	0	1
0	0	ϕ
1		1

$y = Q_1$



(25) Model on Analysis Counting States and Sequence Generations:-



Sol:-

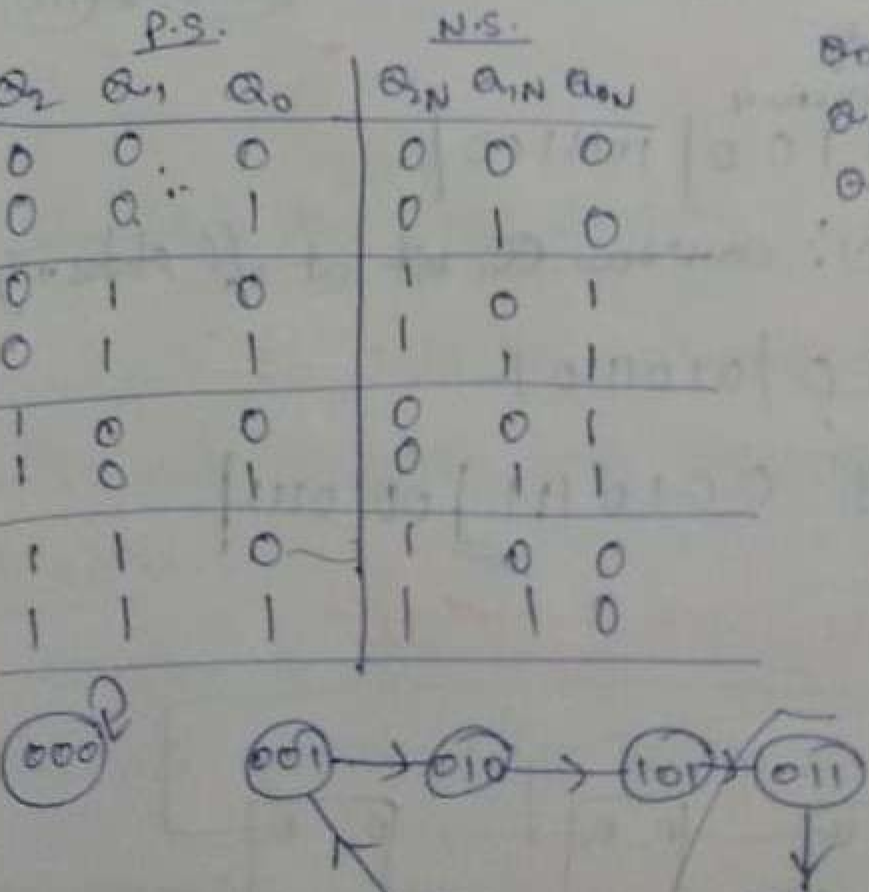
MSB		LSB
D_2	D_1	D_0

P.S.			N.S.		
Q_2	Q_1	Q_0	Q_{2N}	Q_{1N}	Q_{0N}
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	0	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	0

$Q_{0N} = D_0 = Q_1 \oplus Q_2$

$Q_{1N} = D_1 = Q_0$

$Q_{2N} = D_2 = Q_1$



acting as Mod 7 counter

∴ 7 states

(Q) If initial state is (001), what is the state after 4 clocks → (111) 29

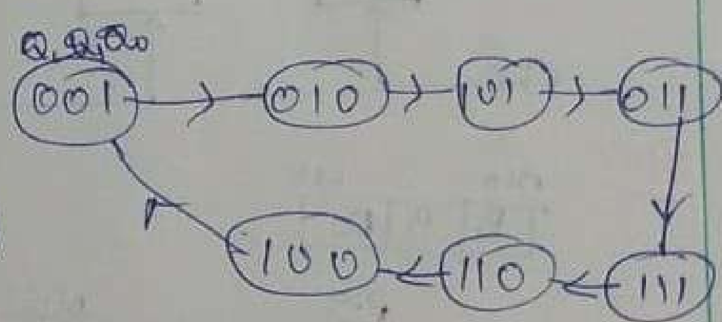
→ 001, 10 clocks → (011)
(7+3)

→ 001, 43 clocks → (010)
(42+1)

→ 001, 25 clocks → (111)
(21+4)

→ 001, 117 clocks → (110)
(7×16 + 5)

Sequence generator



if tap at Q_0:-

consider Q_0 bit at all states only

sequence
1 0 1 1 1 0 0 | 1011100 | . .

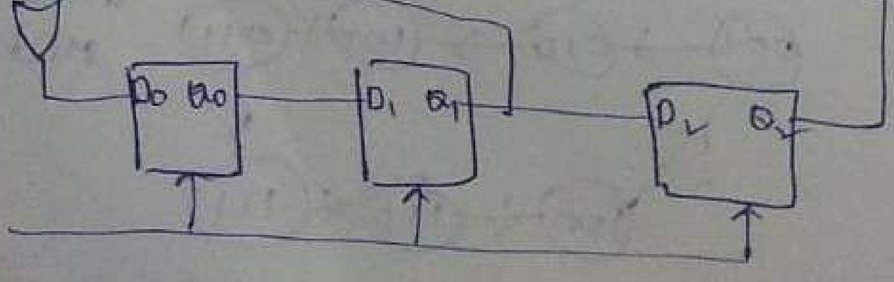
if tap at Q_1:- consider Q_1 bit at all states only

0 1 0 1 1 1 0 | 0101110 | - -

if tap at Q_2:- 0 0 1 0 1 1 1 | 0010111 | - - -

(30) ## Deriving the clock Frequency:-

Q)



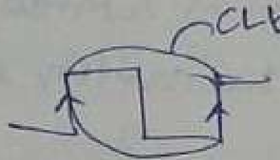
Time period for propogating the signal,

through FF, $T_{FF} = 15ns$

through comb. ckt, $T_{com} = 5ns$

which of the following clock frequency ensures proper counting.

~~a) 40 MHz~~ b) 60 MHz c) 90 MHz d) 300 MHz



Sol $T_{clk} \geq T_{FF} + T_{com}$

$\geq 15ns + 5ns$

$\geq 20ns$

$f_{clk} \leq \frac{1}{20ns}$ $\boxed{f_{clk} \leq \frac{1}{T_{clk}}}$

$\leq \frac{1}{20 \times 10^{-9}}$

$\leq 50 MHz$

$\frac{1000}{20 \times 10^{-6}}$

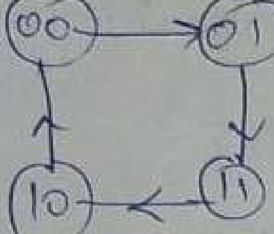
(31) Self Starting and free Running :-

→ A counter is said to be self-starting if it is possible to enter counting loops irrespective of the initial state.

→ A counter is said to be free running if it contains all possible states in the counting loop.

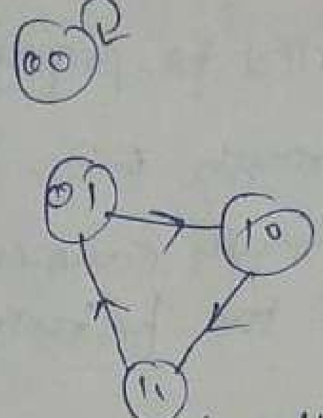
→ A free running counter is a self-starting counter but not vice-versa.

(a)



free running & self-starting

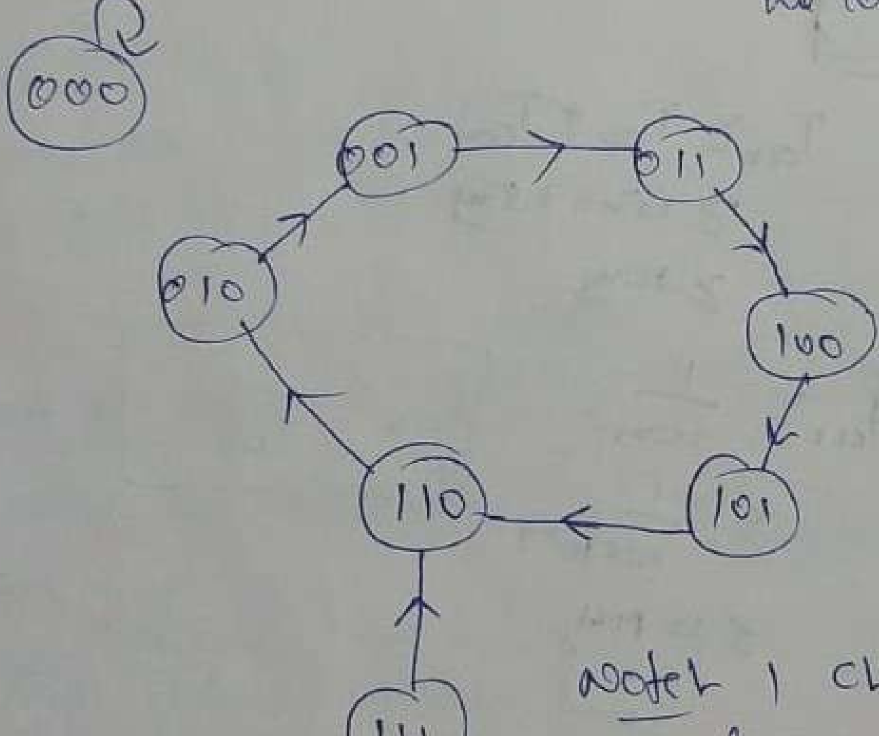
(b)



not self starting

(00) is outside of the counting states

(c)

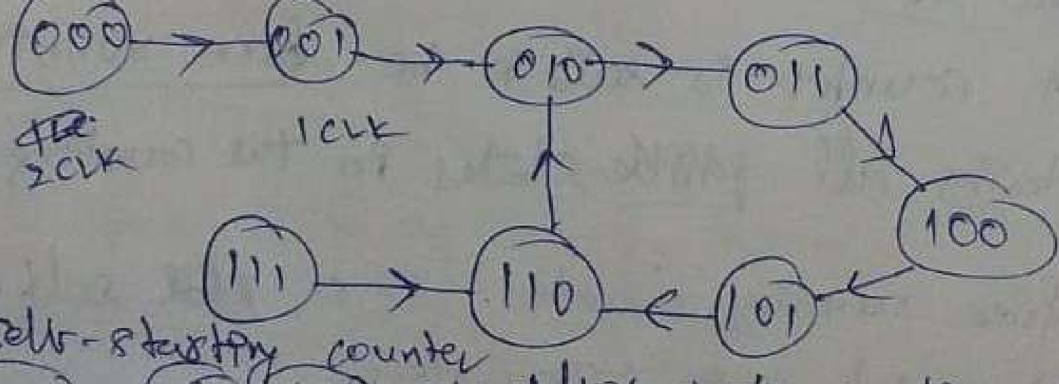


note: 1 CLK taken to enter into counting states

not self-starting counter

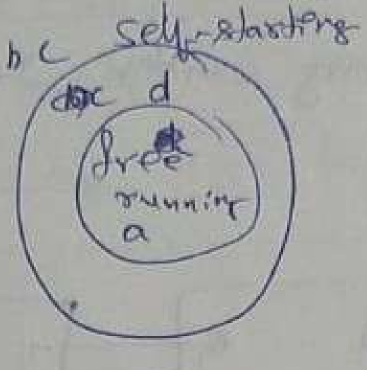
(000), (111) are outside of the counting states.

(d)



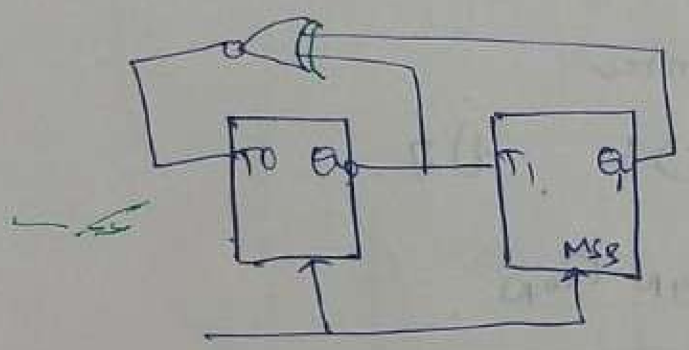
self-starting counter

(000), (001), (111) are outside of the counting states.



(32) Example on Selfstarting and free running counter:-

Q) Identify if the following counter is self-starting or free running.

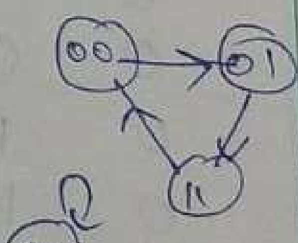


Sol

Q_1	Q_0	Q_{1N}	Q_{0N}	T_1	T_0
0	0	0	1	0	1
0	1	1	1	1	0
1	0	1	0	0	0
1	1	0	0	1	1

① ③ ②

⇒



neither self-starting nor free-running.

$T_1 = Q_0$

$T_0 = Q_0 \odot Q_1$

$Q_{0N} = Q_0 \; ; \; T_0 = 0$

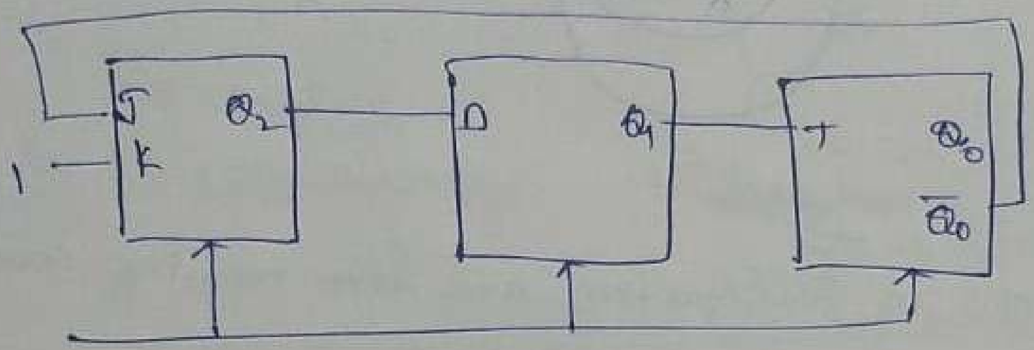
$= \overline{Q_0} \; ; \; T_0 = 1$

$Q_{1N} = Q_1 \; ; \; T_1 = 0$

$= \overline{Q_1} \; ; \; T_1 = 1$

33 Counter using 3 different Flip Flops (FFs):-

consider the following counter, Initially, $Q_2 Q_1 Q_0$ of 000



① What will be the states after 4 clocks

ⓐ 000 ⓑ 010 ⓒ 011 ⓓ 101

② Modulus of the counter

a) 4 b) 5 c) 6 d) 7.

Sol

Q_2	Q_1	Q_0	Q_{2N}	Q_{1N}	Q_{0N}
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	1	0	1
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	0	1	0

$Q_{1N} = D = Q_2$

$Q_{0N} = Q_0$; $T=0$, $Q_1=0$

$= \bar{Q}_0$; $T=1$, $Q_1=1$

TFF acts as a latch a toggle(NOT)

$Q_{2N} =$

J	K	Q_N
0	0	
0	1	
1	0	
1	1	

J	K	Q_N
0	1	0
1	1	$\bar{Q}_2$

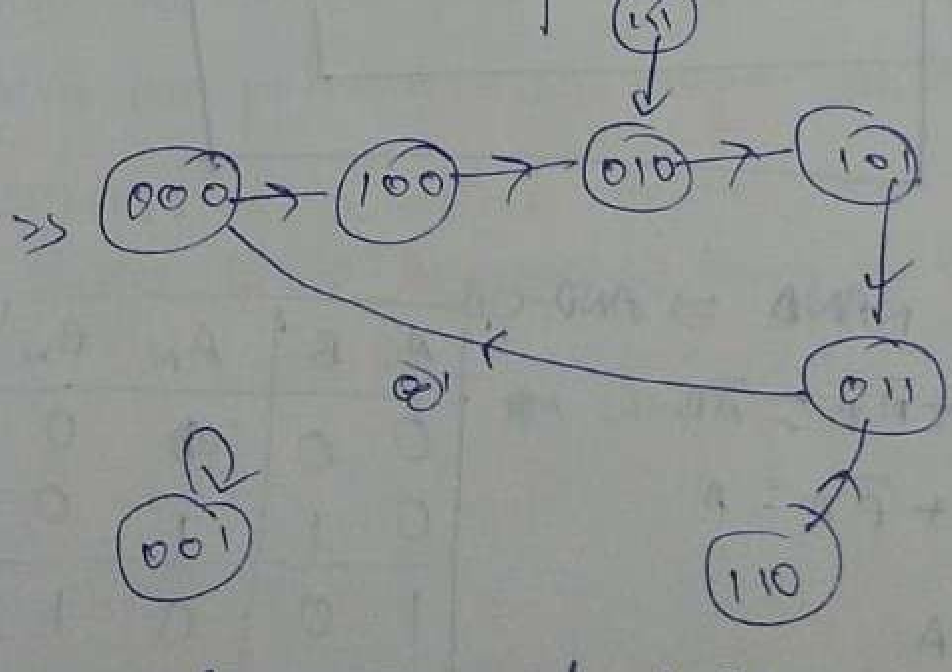
$\Rightarrow$

$\bar{Q}_0$	K	Q_{2N}
0	1	0
1	1	$\bar{Q}_2$

$\Rightarrow$

Q_0	K	Q_{2N}
1	1	0
0	1	$\bar{Q}_2$

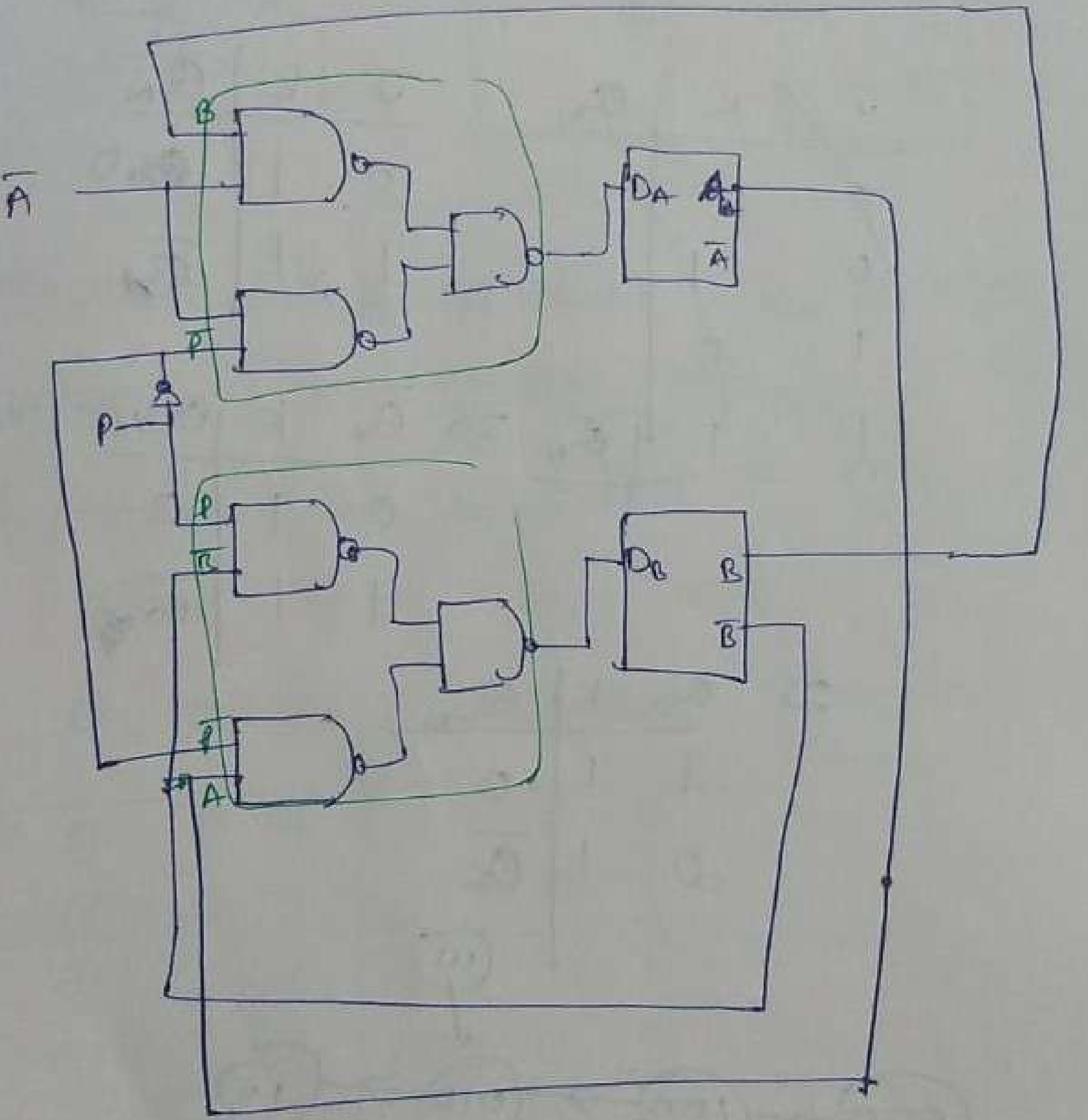
$\Rightarrow$



both not free running & self-starting

34 Example on Combinational ckts and FFs :-

consider the following counter, if $P = 0$, the counting sequence of AB is, ?



Sol

NAND - NAND ⇒ AND-OR

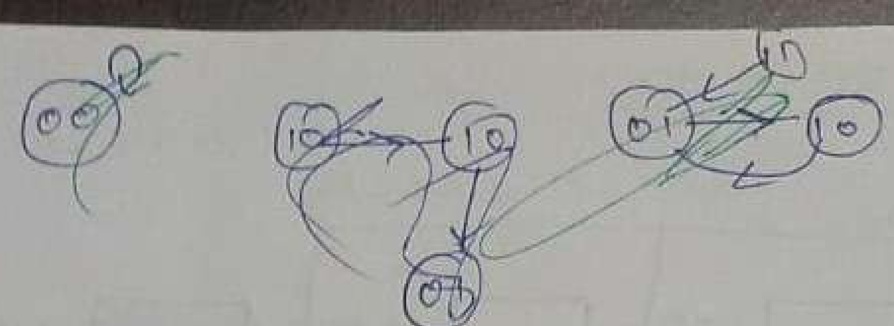
$$D_A = \bar{A}B + \bar{A}\bar{P} = \bar{A}B + \bar{A} = \bar{A}$$

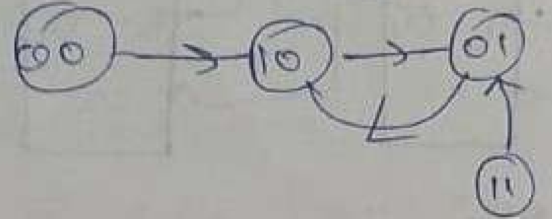
$$D_B = P\bar{B} + \bar{P}A = A$$

$$B_N = D_B = A$$

$$A_N = D_A = \bar{A}$$

A	B	A_N	B_N
0	0	1	0
0	1	1	0
1	0	0	1
1	1	0	1





self starting but not free running.

(35) Introduction to Asynchronous Counters:-

→ The asynchronous counters are also called as Ripple counter.

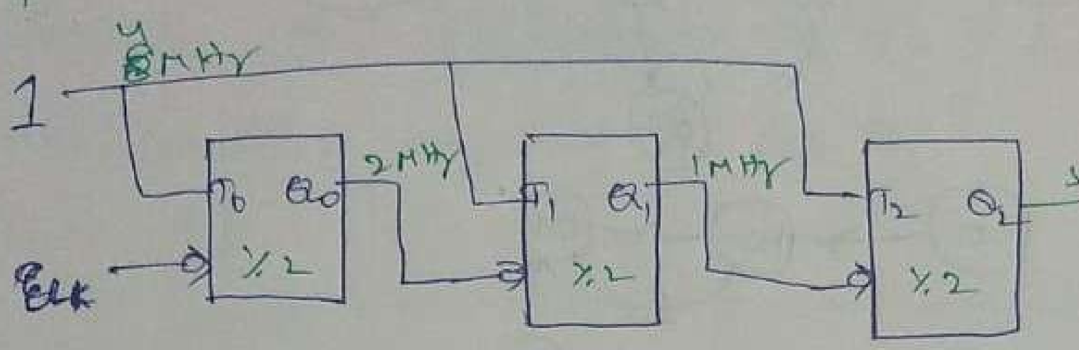
→ The basic FF is 'T' and basic counting is binary up counting. (1-2-3-4-5...0)

→ The up counters are used for implementing incrementation.

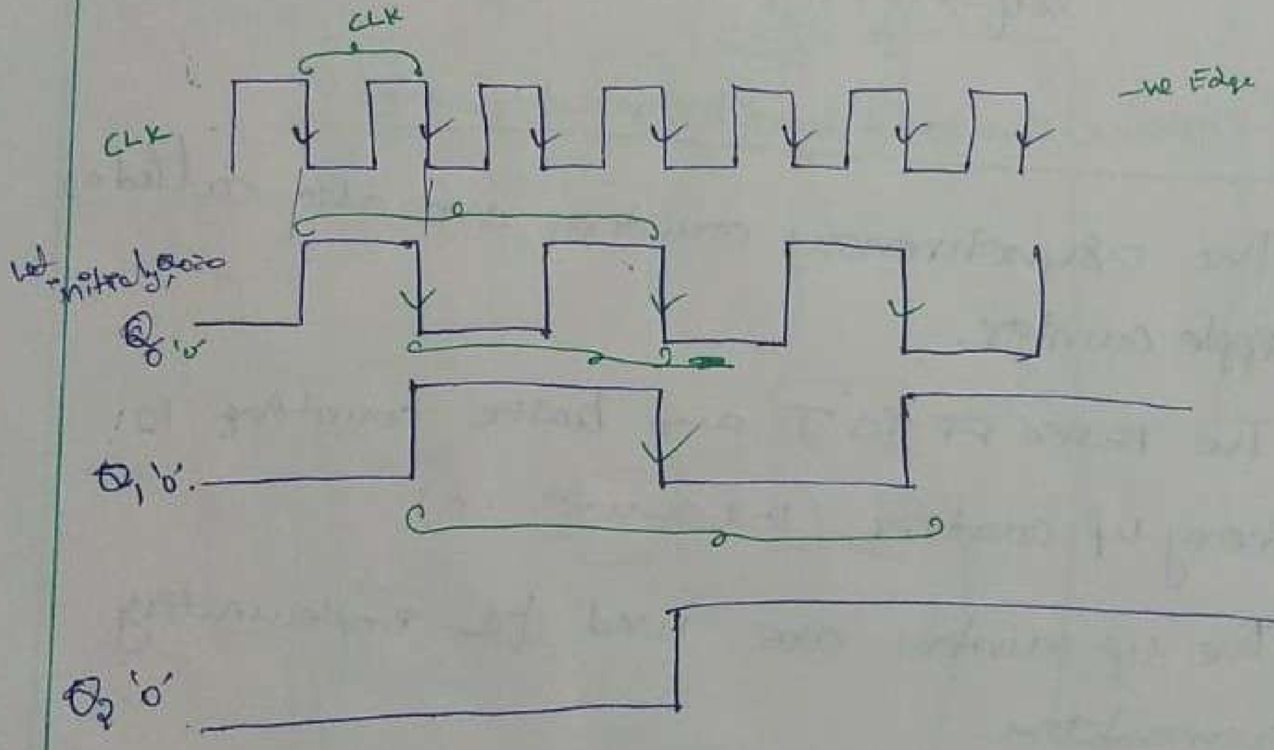
→ Due to the simpler design, Asynchronous counters are preferred in IC counter fabrication.

→ IC 7490 is a decade counter i.e. % 10 counter
IC 7492 is a Hexadecimal counter i.e., % 16 counter.

(36) MOD 8 up counter:-



"-ve Edge triggering FF."



$\therefore$ Time period, $Q_0 = 2 * T_{CLK}$

frequency $Q_0 = \frac{1}{2} * f_{CLK}$

Time period, $Q_1 = 2 * TQ_0$

freq, $Q_1 = \frac{1}{2} *$ freq Q_0

$= \frac{1}{4} *$ freq.$_{CLK}$

$TQ_2 = 2 * TQ_1$

freq $Q_2 = \frac{1}{2} *$ freq $Q_1 = \frac{1}{8}$ freq$_{CLK}$

if Clk = 4 MHz then

4M → 2M → 1M → 500 KHz

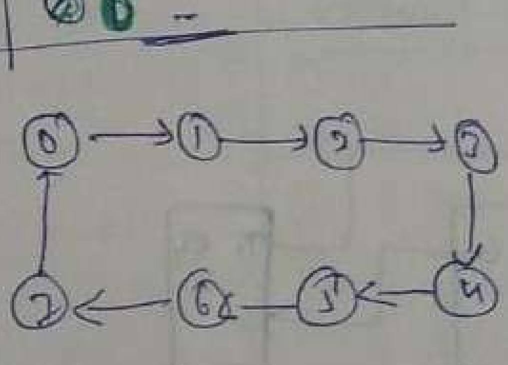
Exam point of view :-

$Q_{0N} = \overline{Q_0}$; every clock

$Q_{1N} = \overline{Q_1}$; $Q_0 : 1 \to 0$

$Q_{2N} = \overline{Q_2}$; $Q_1 : 1 \to 0$

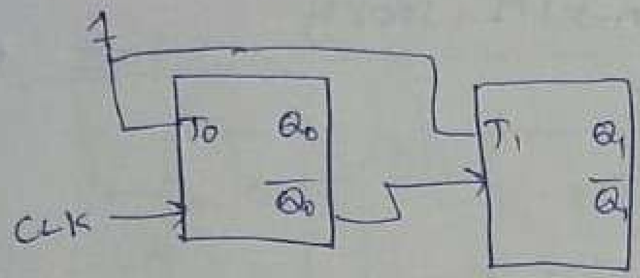
	P.S.			N.S.			
	Q_2	Q_1	Q_0	Q_{2N}	Q_{1N}	Q_{0N}	
(0)	0	0	0	0	0	1	(1)
(1)	0	0	1	0	1	0	(2)
(2)	0	1	0	0	1	1	(3)
(3)	0	1	1	1	0	0	(4)
(4)	1	0	0	1	0	1	(5)
(5)	1	0	1	1	1	0	(6)
(6)	1	1	0	1	1	1	(7)
(7)	1	1	1	0	0	0	(0)



free running & self-starting counter

Mod 8 counter (0 to 7)

(37) Mod 4 up counter :-



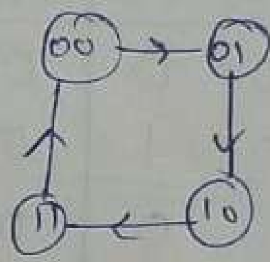
S.L

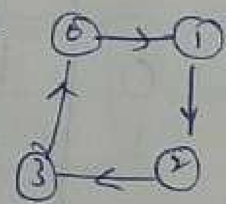
$Q_{0N} = \overline{Q_0}$; for every clock

$Q_{1N} = \overline{Q_1}$; $\overline{Q_0} : 0 \to 1$

$\Rightarrow Q_0 : 1 \to 0$

PS		NS	
Q_1	Q_0	Q_{1N}	Q_{0N}
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

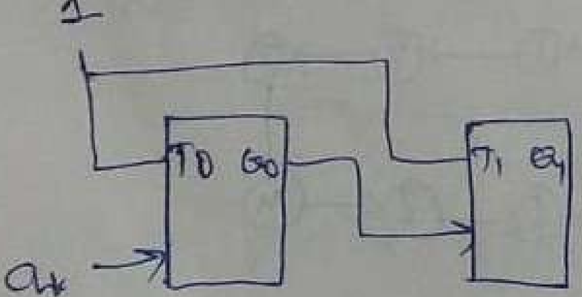




free running & self starting.

Mod 4 counter

(38) Mod 4 Down Counter :-

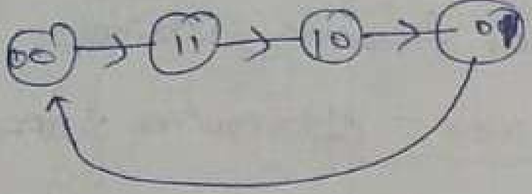


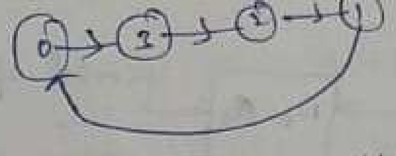
S.L

$Q_{0N} = \overline{Q_0}$; for every clk

$Q_{1N} = \overline{Q_1}$; $Q_0 : 0 \to 1$

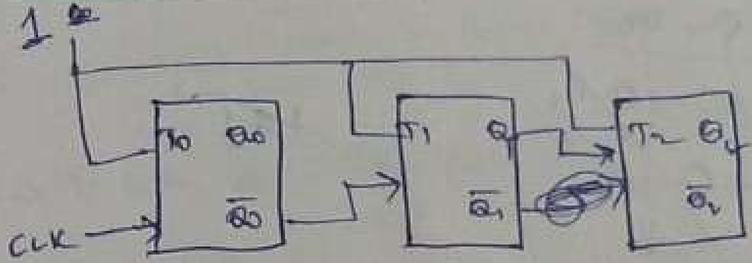
Q_1	Q_0	Q_{1N}	Q_{0N}
0	0	1	1
0	1	0	0
1	0	0	1
1	1	1	0





Free running self starting

(39) Mod 8 random Counter:-



$0 \rightarrow 1$ ✗

Sol:- If the initial state $Q_2 Q_1 Q_0 = 001$, what will be state after 4 clock cycles?

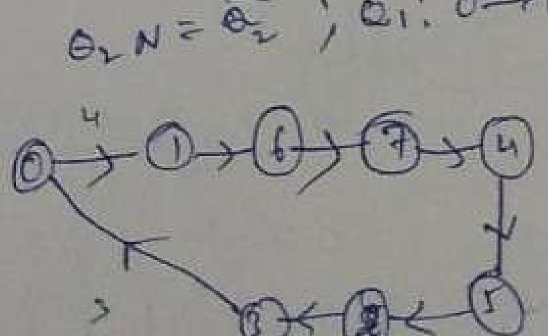
$Q_{0N} = \bar{Q}_0$; for every clock

$Q_{1N} = \bar{Q}_1$; $\bar{Q}_0 : 0 \rightarrow 1$

$Q_0 : 1 \rightarrow 0$

$Q_{2N} = \bar{Q}_2$; $Q_1 : 0 \rightarrow 1$

	Q_2	Q_1	Q_0	Q_{2N}	Q_{1N}	Q_{0N}
0	0	0	0	0	0	1
1	0	0	1	1	1	0
2	0	1	0	0	1	1
3	0	1	1	0	0	0
4	1	0	0	1	0	1
5	1	0	1	0	1	0
6	1	1	0	1	1	1
7	1	1	1	1	0	0



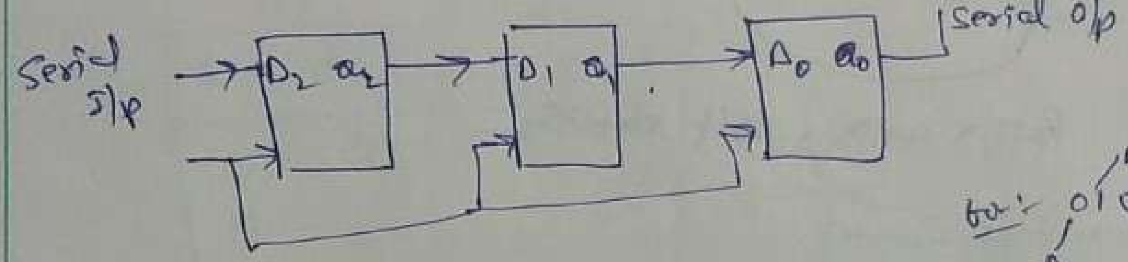
$\therefore$ 001

(40) Applications of FFs:-

① Shift registers → used as 'Sequential Memory'
Ex:- Accumulator in μp.

② counters → ① used to count no. of pulses
② used as frequency divider

(41) 3 bit Shift Right register:- (It requires 3 DFFs)



Ex:- 0 1 0 (D_2, D_1, D_0)

CLK	SIP	Q_2	Q_1	Q_0
0	-	0	0	0
1	0	0	0	0
2	1	1	0	0
3	0	0	1	0
4	1	1	0	1

~~101~~

for 101

Sol:-

CLK	SIP	Q_2	Q_1	Q_0
0	-	0	0	0
1	1	1	0	0
2	0	0	1	0
3	1	1	0	1

→ 'n' clocks are required for to put n bit data on register in SIP shift 'n' register

SO:- (Serial O/P)

CLK	Q_2	Q_1	Q_0	O/P
0	1	0	1	1 already Q_0 is the o/p
1	0	1	0	0
2	0	0	1	1

∴ 2 clocks are required for 3 bits

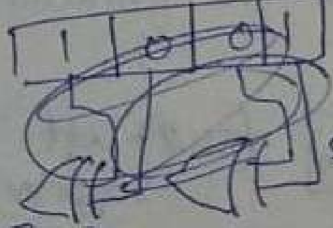
∴ (n-1) clock required for shift n reg.

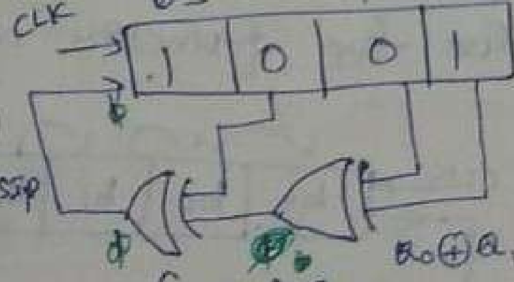
SIP – n clks
SIO – n-1, clks
total ⇒ (2n-1) clks
* (SIP+SIO)

→ If we go with PIPO then 1 clk is enough.

(42) Examples on Shift right Register:-

In the following right shift register, determine the no. of clocks required to bring it to the initial state of '1001'





Sol:

CLK	SIP	Q_3	Q_2	Q_1	Q_0
0	–	1	0	0	1
1	1	1	1	0	0
2	1	1	1	1	0
3	0	0	1	1	1
4	1	1	0	1	1
5	0	0	1	0	1
6	0	0	0	1	0

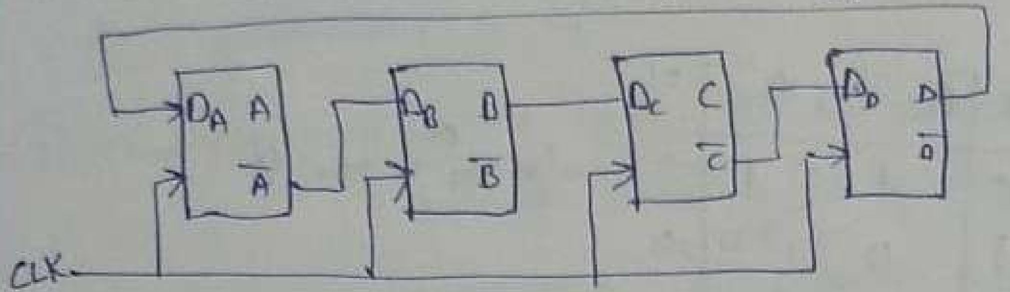
⇒

7	1	1	0	0	1

∴ After 7 clocks

(43) Ex2:-

Initial value of ABCD = 0000, then what are the sequence of numbers the following circuits counting



Sol:- i/ps

$D_A = D$

$D_B = \overline{A}$

$D_C = B$

$D_D = \overline{C}$

CLK	A	B	C	D	
0	0	0	0	0	- 0 initially
1	0	1	0	1	- 5
2	1	1	1	1	- 15
3	1	0	1	0	- 10
4	0	0	0	0	- 0
5	0	1	0	1	- 5

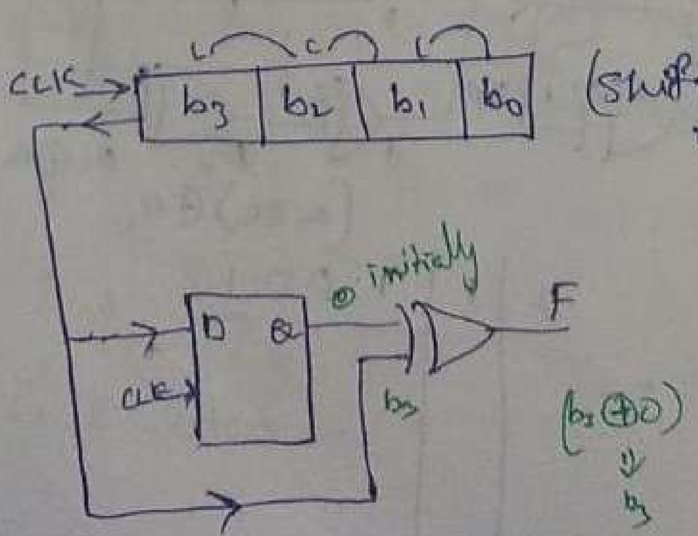
15

10

∴ 0 - 5 - 15 - 10

(44) Binary to Gray Converter:-

Determine the function of the following circuit.



B → G

$(10110)_2$

$(11011)_G$ =

$(11011)_G \rightarrow B$

10010

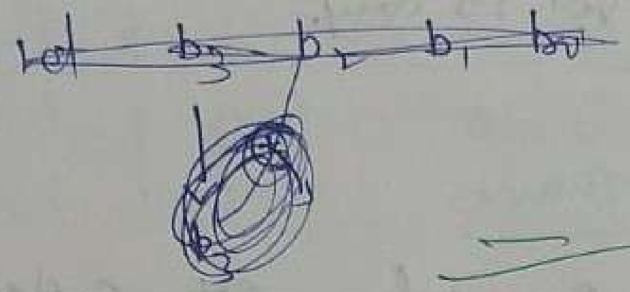
Sol:-

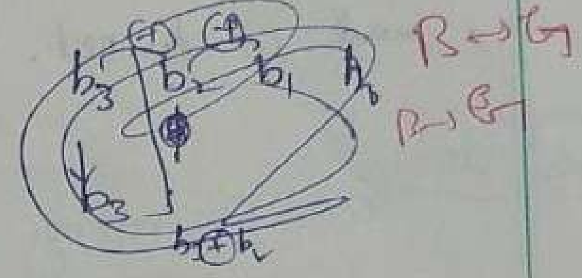
b_3	b_2	b_1	b_0
b_2	b_1	b_1	b_0
b_1	b_0	b_0	b_0
b_0	b_0	b_0	b_0

$0 \oplus b_3$ initially

$\Downarrow$

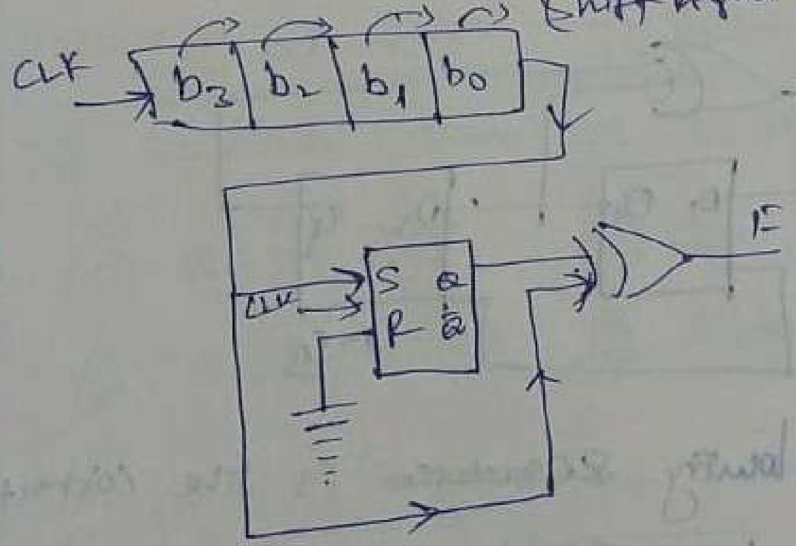
$(b_3)\ (b_3 \oplus b_2)\ (b_2 \oplus b_1)\ (b_1 \oplus b_0)$





(45) Finding 2's complement:-

What is the function of the following circuit?
(Shift Regrt register)



Sol: Initially at $Q = 0$,

$R = 0$ always

S	R	
0	0	- Latch
1	0	- Set

$\boxed{b \oplus 0 = b;\ b \oplus 1 = b'}$

$b_0 \oplus 0 = b_0$

0 1 0 0

1st - 0 0 1 0

2nd - 0 0 0 1

3 - 0 0 0 0

o/p F: 1 1 0 0

2's comp 0 1 0 0

0 1 | 0 0

1 1 | 0 0 1's

comp, 1's

exa: 0001

~~HEEP~~

2's comp ~~HEEO~~ (0001) -

0001
↓
0001

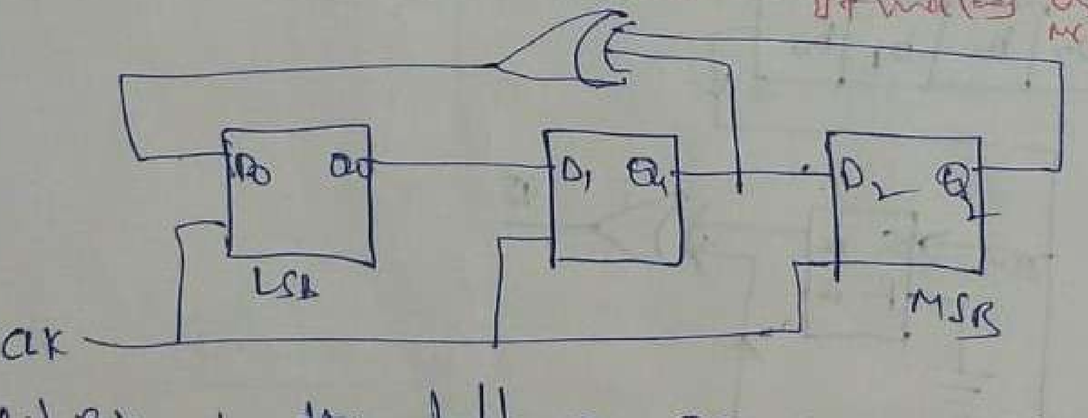
Hint: from LSB side, write as it is upto first 1, next complement. to get 2's comp.

(46) 2001

On counting sequence:-

Consider the ckt given below with initial state $Q_0 = 1, Q_1 = Q_2 = 0$. The state of the ckt is given by the value $4Q_2 + 2Q_1 + Q_0$.

It makes $Q_2\ Q_1\ Q_0$
MSB LSB



Which of the following ~~sequence~~ is the correct state sequence of the circuit (ckt)?

(a) 1, 3, 4, 6, 7, 5, 2 (b) 1, 2, 5, 3, 7, 6, 4

(c) 1, 2, 5, 6, 2, 5, 4 (d) 1, 6, 5, 7, 2, 3, 4

Sol

$Q_{2N} = D_2 = Q_1$

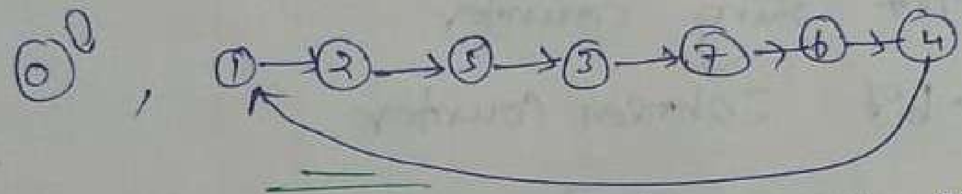
$Q_{1N} = D_1 = Q_0$

$Q_{0N} = D_0 = Q_1 \oplus Q_2$

	P.S.			N.S.		
	Q_2	Q_1	Q_0	Q_{2N}	Q_{1N}	Q_{0N}
0 –	0	0	0	0	0	0
1 –	0	0	1	0	1	0
2 –	0	1	0	1	0	1
3 –	0	1	1	1	1	1
4 –	1	0	0	0	0	1
5 –	1	0	1	0	1	1
6 –	1	1	0	1	0	0
7 –	1	1	1	1	1	0

(or) start with

Q_2	Q_1	Q_0
0	0	1

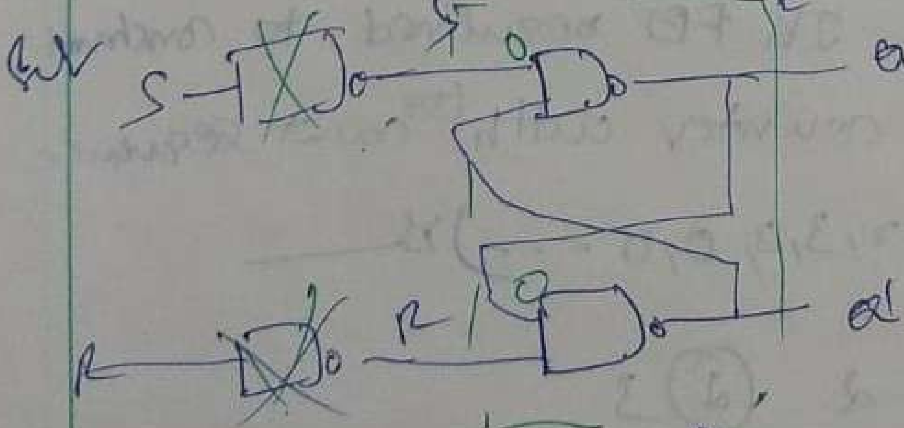


(47) 2004 on SR-Latch SR latch made by cross coupling two NAND gates

If S=R=0, then it will result in

(a) $Q=0, \bar{Q}=1$ (b) $Q=1, \bar{Q}=0$

(c) $Q=1, \bar{Q}=1$ (d) Indeterminate state.



consider this path and

(48)

(48) 2014 on counter

Q. Let $K = 2^n$, a circuit is built by giving the o/p of an n-bit binary counter as i/p to an n to 2^n bit decoder. This circuit is equivalent to a

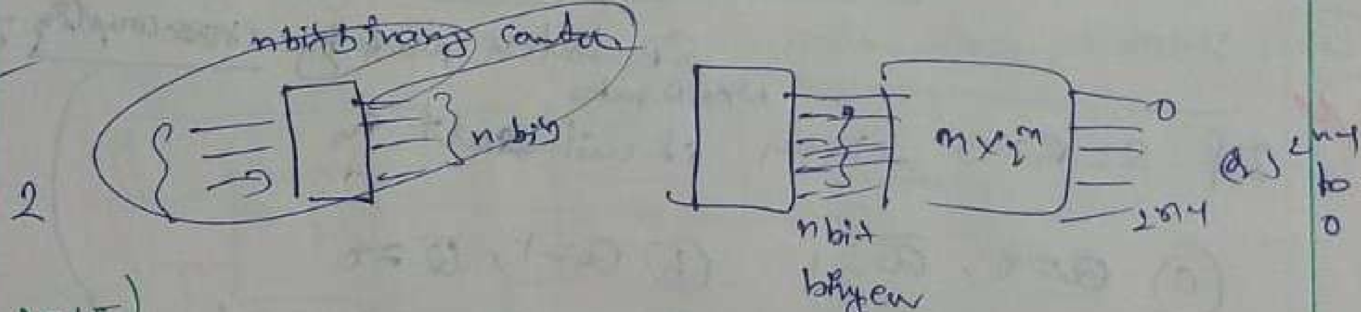
(a) K-bit binary up counter

(b) K-bit binary down counter

(c) K-bit Ring counter

(d) K-bit Johnson counter.

Sol



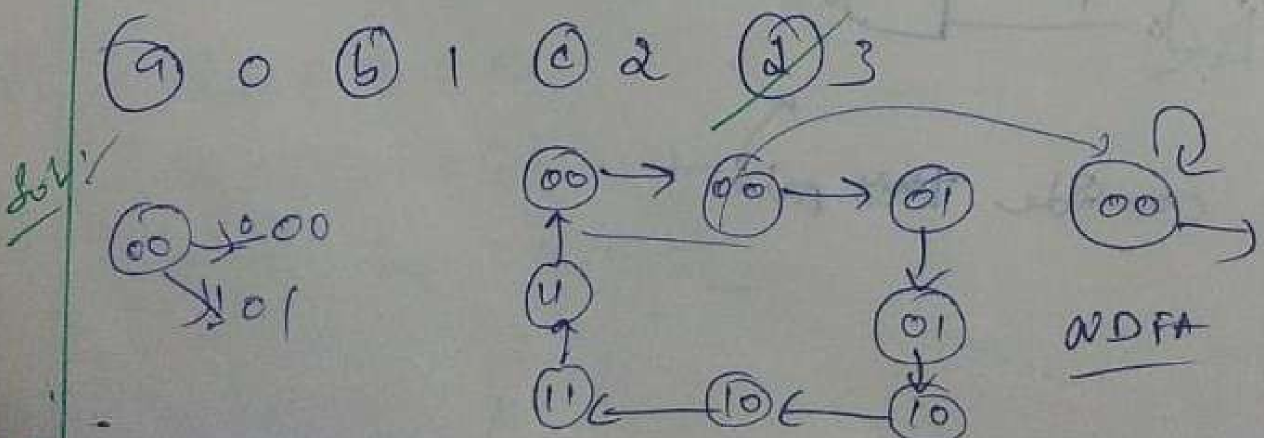
(49) 2015)

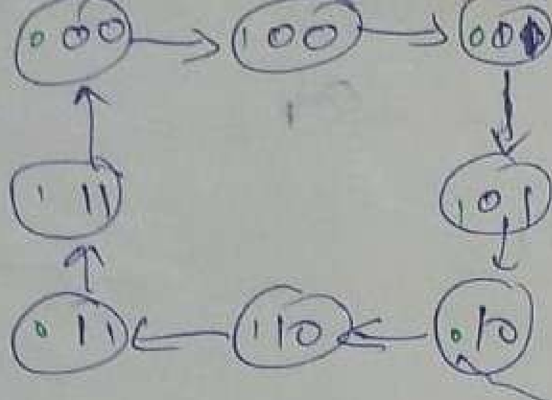
on Sequence Generation:-

The min. no. of JK FFs required to construct a synchronous counter with the count sequence $(0, 0, 1, 1, 2, 2, 3, 3, 0, 0 \ldots)$ is ____

(a) 0 (b) 1 (c) 2 (d) 3

Sol:





S.C.

two out going edge, add 1 b

4 out going edge add 2 bs

2^k out going edges add K b

K " " " add $\log_2 K$

DFA but not consider first FF

(50) 2015

On bit sequence

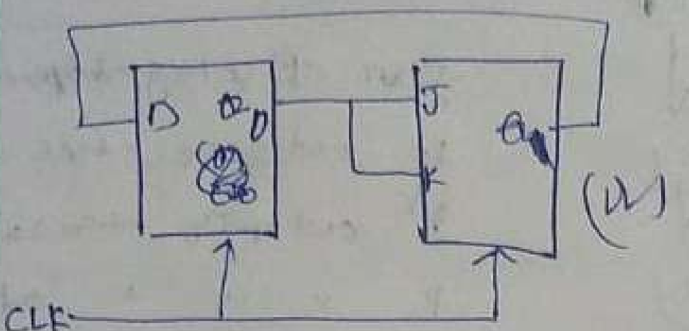
A +ve edge triggered D-ff is connected to a +ve Edge triggered JK FF as follows. The Q o/p of the DFF is connected to both J and K i/ps of the JK-FF, while the Q o/p the JK FF is connected to the i/p of the D-FF. Initially the o/p of the D-FF is set to logic one and the o/p of the JK-FF is cleared. Which one of the following is the bit sequence (including initial state) generated at the Q o/p of the JK FF, when FFs are connected to a free running common clock? Assume that J=K=1 is the toggle mode and J=K=0 is the state holding mode of the JK-FF. Both the FFs have non-zero propagation delays.

a) 011011011...

b) 0100100...

c) 01110110...

d) 011001100...



(ii)

$$Q_{1N} = D = Q_0$$

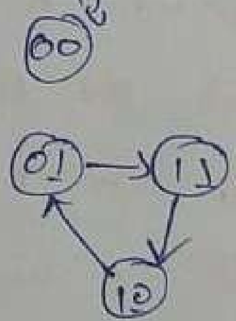
$$Q_{0N} = J\bar{Q}_0 + \bar{K}Q_0$$

$$= Q_1\bar{Q}_0 + \bar{Q}_1 Q_0$$

$$\boxed{Q_{0N} = Q_0 \oplus Q_1}$$

Q_1	Q_0	Q_{1N}	Q_{0N}
0	0	0	0
0	1	1	1
1	0	0	1
1	1	1	0

⇒



~~If we tap at JK Q_0 then 01 1 0110110 ...~~

Initial state is given, $\frac{Q_1 \quad Q_0}{1 \quad 0}$ then start at (10) and so on

(10) → (01) → (11) → (10) → (01) → (11) ...

If tap at JK Q_0,

0 1 1011011 ...

(57) 2015) m sequence generation

we want to design a synchronous counter that counts the the sequence 0-1-0-2-0-3 and then repeats the min. no. of JK FFs required to implement this counter is ______

a) 1 b) 2 c) 4 d) 5.

$00 \rightarrow 01 \rightarrow 00 \rightarrow 10 \rightarrow 00 \rightarrow 11$

$00 \rightarrow 01$, $00 \rightarrow 10$, $00 \rightarrow 11$

no of directions $= \lceil \log_2 3 \rceil = 2$

0,1,2,3

$\therefore 2 + 2 = 4$

(2) Minimization

(1) Introduction to Minimization of Boolean Expressions.

→ A switching function can usually be represented by using a no. of expressions.

Ex:- $f(x, y, z) = xy + yz + zx$

$= x'yz + xy'z + xyz' + xyz$

→ While simplifying a switching function $f(x_1, x_2, ---, x_n)$, our aim is to find an