When converting a contrained lagrange system to a DAE of index 1 to simulate it **baumgarte stabilization** is often neccesary to not accumulate numeric integration errors over time by updating $\ddot{c}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial \mathcal{L}}{\partial \mathbf{q}} = \mathbf{Q} \qquad\qquad \ddot{c} + 2\alpha\dot{c} + \alpha^2 c = 0$$

for some $\alpha > 0$, such that the second equation is stable with poles in $-\alpha$

Step by step list for **index reduction of** DAE:

1. Check if the DAE system is of index 1. If yes, stop.
2. Identify a subset of algebraic equations in the semi-explicit form that can be solved for a subset of algebraic variables z.
3. Apply $\frac{d}{dt}$ on the remaining algebraic equations containing some differential states x_j, this leads to terms $\dot{x}_j$ appearing in these differentiated equations.
4. Substitute the terms $\dot{x}_j$ by the corresponding expressions $f_j(x, z, u)$, this delivers new algebraic equations to replace those differentiated in step 2.
5. With this new DAE system, go to step 1.

Reducing the index of a DAE requires you to collect a set of consistency conditions for it to match the original DAE with +1 index. Generally speaking, when performing the index reduction, one ought to collect all the algebraic equations on which a time differentiation is performed, and add them to the list of consistency conditions.