In order to evaluate this expression, we will transform the representation of the vector $\vec{r}_{G/O_b}$ such that it is expressed in terms of the NED-frame. In order to achieve this we will use the same sequence of Euler angle rotations as presented at the start of Section 3.1.1. More specifically we will describe the attitude of the ship by first considering a ship that is aligned with the NED-system (i.e the x-axis is parallel to the N axis, the y-axis is parallel to the E axis, and the z-axis is parallel to the D-axis), and then rotate the ship an angle ψ about the D-axis (such that ψ represents the ship´s heading). Next, we will rotate the ship an angle θ about the y-axis of the body frame (which no longer is parallel to the E-axis). The angle θ now represents the pitch angle of the ship. Finally we rotate the ship around an angle ϕ about the body-fixed x-axis (which no longer is parallel to the N-axis and also no longer contained in the N-E plane) The angle ϕ then represents the ship´s roll angle.

Using column vector notation and rotation transformation matrices for ease of notation we can then express Equation 22 as

$$\boldsymbol{r}^n_{O_b/O_n} = \boldsymbol{r}^n_{G/O_n} - \boldsymbol{R}_z(\psi)\boldsymbol{R}_y(\theta)\boldsymbol{R}_x(\phi)\boldsymbol{r}^b_{G/O_b} = \boldsymbol{r}^n_{G/O_n} - \boldsymbol{R}^n_b(\phi,\, \theta,\, \psi)\boldsymbol{r}^b_{G/O_b} \tag{23}$$

3.3 Rotations about an arbitrary axis

3.3.1 Angle-axis rotations

In the previous section we saw how any rotation transformation can be represented by a sequence of three principal rotation transformations (Euler angles). As an alternative, any rotation transformation can be represented by a single rotation α about some vector $\vec{v}$ (which is not necessarily aligned with any principal axis). This is referred to as angle axis rotations. Figure 6 illustrates the concept.

The vector $\vec{v}$ about which the rotation transformation occurs is characterized by the fact that

$$\boldsymbol{v}^a = \boldsymbol{R}^a_b \boldsymbol{v}^b = \boldsymbol{v}^b \tag{24}$$

In other words, the vector $\vec{v}$ is identical when represented in reference frame a and in the rotated reference frame b. This means that the rotation matrix $\boldsymbol{R}^a_b$ has an eigenvalue $\lambda = 1$ and that $\vec{v}$ is the corresponding eigenvector.

The parameters α and $\vec{v}$ are referred to as the angle axis parametrization of $\boldsymbol{R}^a_b$, where $\vec{v}$ is a unit vector (i.e. its length is 1).

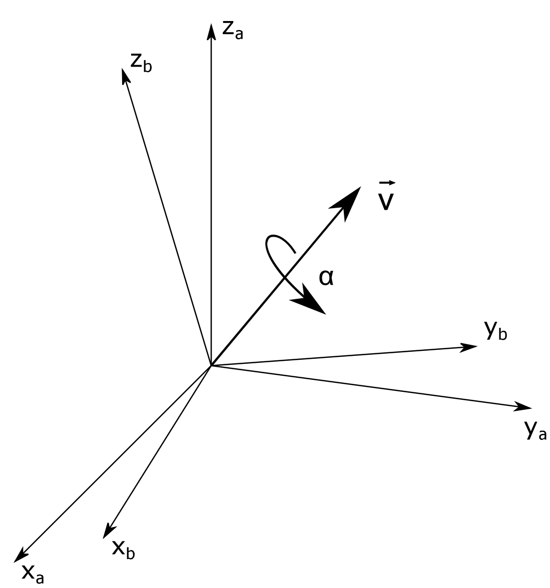

Figure 6: Illustration of the angle axis rotation transformation.